Regression Logic

EC 320: Introduction to Econometrics

Emmett Saulnier

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Prologue

Housekeeping

- Computational Problem Set 2 was due yesterday (Can still turn it in for partial credit!)
- Analytical Problem Set 2 is due Friday at midnight. We'll wrap up all of the material covered on that problem set in class today
- Nick Huntington-Cline has a great Introduction to R for Economists series on Youtube! Highly recommend watching the first 10 videos (or more) in that series

So far we've identified the fundamental problem econometricians face.

How do we proceed? **Regressions!**

- Running models
- Confounders
- Omitted Variable Bias

Regression Logic

Regression

Modeling is about reducing something really complicated into something simple that still represents some part of the complicated reality.

- It's about telling stories that are easy to understand, and thus, easy to learn from

Economists often rely on **(linear) regression** for statistical comparisons.

- *"Linear"* is more flexible than you think

- Describes the relationship between a dependent (endogenous) variable and one or more explanatory (exogenous) variable(s)

We will focus on the **simple univariate** case today.

Regression

Regression analysis helps us make *all else equal* comparisons.

- We can model the effect of X on Y while **controlling** for potential confounders
- Forces us to be explicit about the potential sources of selection bias
- Failure to control for confounding variables leads to **omitted-variable bias**, a close cousin of selection bias
- Why? The omitted variable, correlated with our covariate of interest, is sitting inside the error term causing chaos

Returns to Private College

Research Question: Does going to a private college instead of a public college increase future earnings?

- **Outcome variable:** earnings
- **Treatment variable:** going to a private college (binary)

Q: How might a private school education increase earnings?

Q: Does a comparison of the average earnings of private college graduates with those of public school graduates isolate the economic returns to private college education? Why or why not?

Returns to Private College

How might we estimate the causal effect of private college on earnings?

Approach 1: Compare average earnings of private college graduates with those of public college graduates.

- Prone to selection bias.

Approach 2: Use a matching estimator that compares the earnings of individuals the same admissions profiles.

- Cleaner comparison than a simple difference-in-means.
- Somewhat difficult to implement.
- Throws away data (inefficient).

Approach 3: Estimate a regression that compares the earnings of individuals with the same admissions profiles.

The Regression Model

We can estimate the effect of X on Y by estimating a **regression model**:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

- Y_i is the outcome variable.
- X_i is the treatment variable (continuous).
- β_0 is the **intercept** parameter. $\mathbb{E}[Y_i | X_i = 0] = \beta_0$
- β_1 is the **slope** parameter, which under the correct causal setting represents marginal change in X_i's effect on Y_i. $\frac{\partial Y_i}{\partial X_i} = \beta_1$
- u_i is an error (disturbance) term that includes all other (omitted) factors affecting Y_i.

The Error term

u_i is quite special. If we consider the data generating process of variable Y_i, u_i captures all the unobserved variables that explain variation in Y_i.

- Always some error to our models, we just aim for it to be small relative to the challenge we face
- Some aspects of the observed data that was collected may also have been inputted incorrectly (measurement error)

The error term is the price we are willing to accept for a more simplified model.

The Error Term

To be explicit, there are five items that contribute to the existence of this disturbance term.

- Omission of Explanatory Variables
- Aggregation of Variables
- Model Misspecificiation
- Functional Misspecification
- Measurement Error

Running Regressions

The intercept and slope are population parameters.

Using an estimator with data on X_i and Y_i, we can estimate a **fitted regression line**:

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = b_0 + b_1 X_i$$

- $\hat{Y}_i$ is the **fitted value** of Y_i.
- $\hat{\beta}_0$ is the **estimated intercept**.
- $\hat{\beta}_1$ is the **estimated slope**.

The estimation procedure produces misses called **residuals**, defined as $Y_i - \hat{Y}_i$.

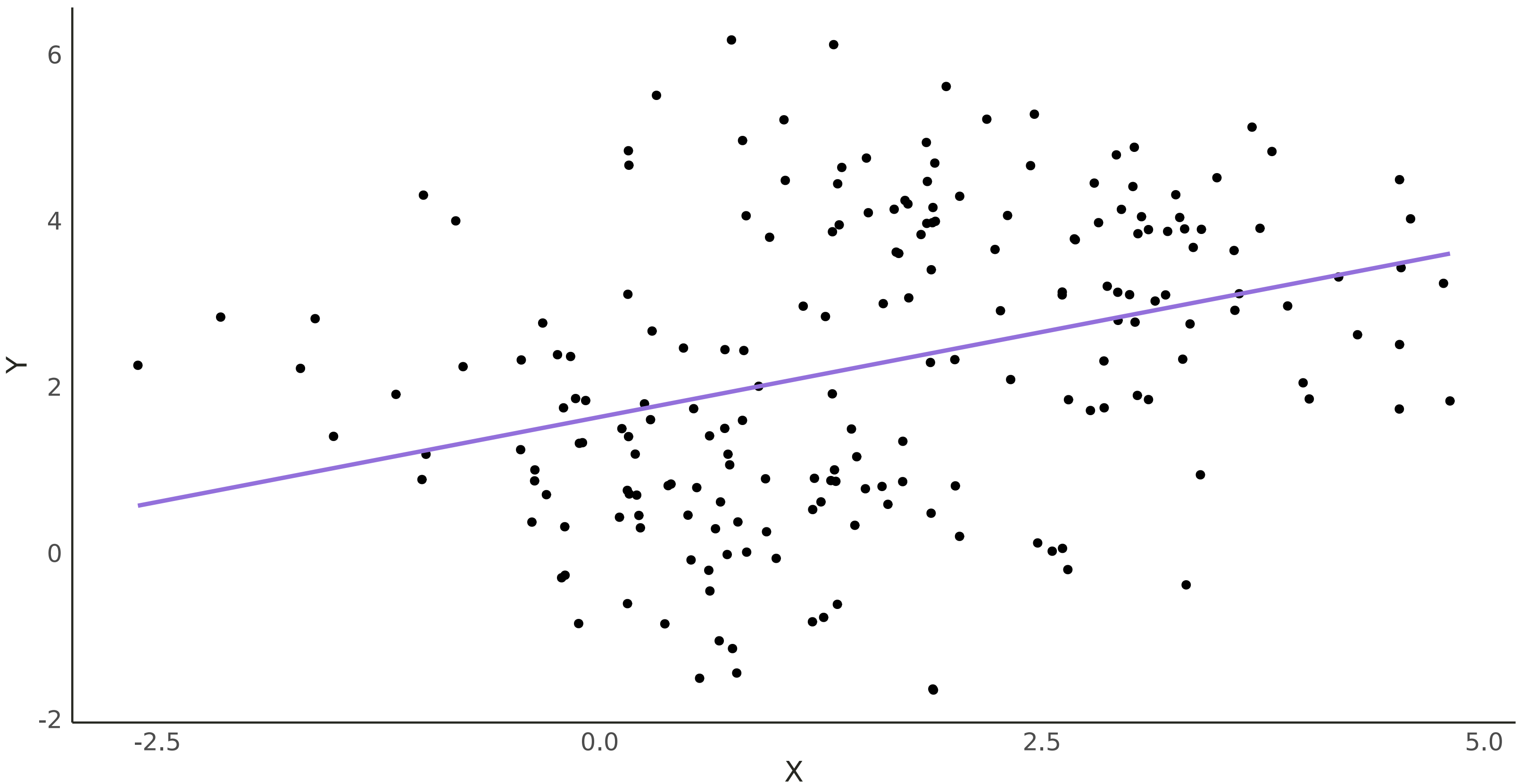
Running Regressions

In practice, we estimate the regression coefficients using an estimator called **Ordinary Least Squares** (OLS).

- Picks estimates that make $\hat{Y}_i$ as close as possible to Y_i given the information we have on X and Y.
- The residual sum of squares (RSS), $\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$, gives us an idea of how accurate our model is.
- **OLS** minimizes this sum.
- We will dive into the details next class.

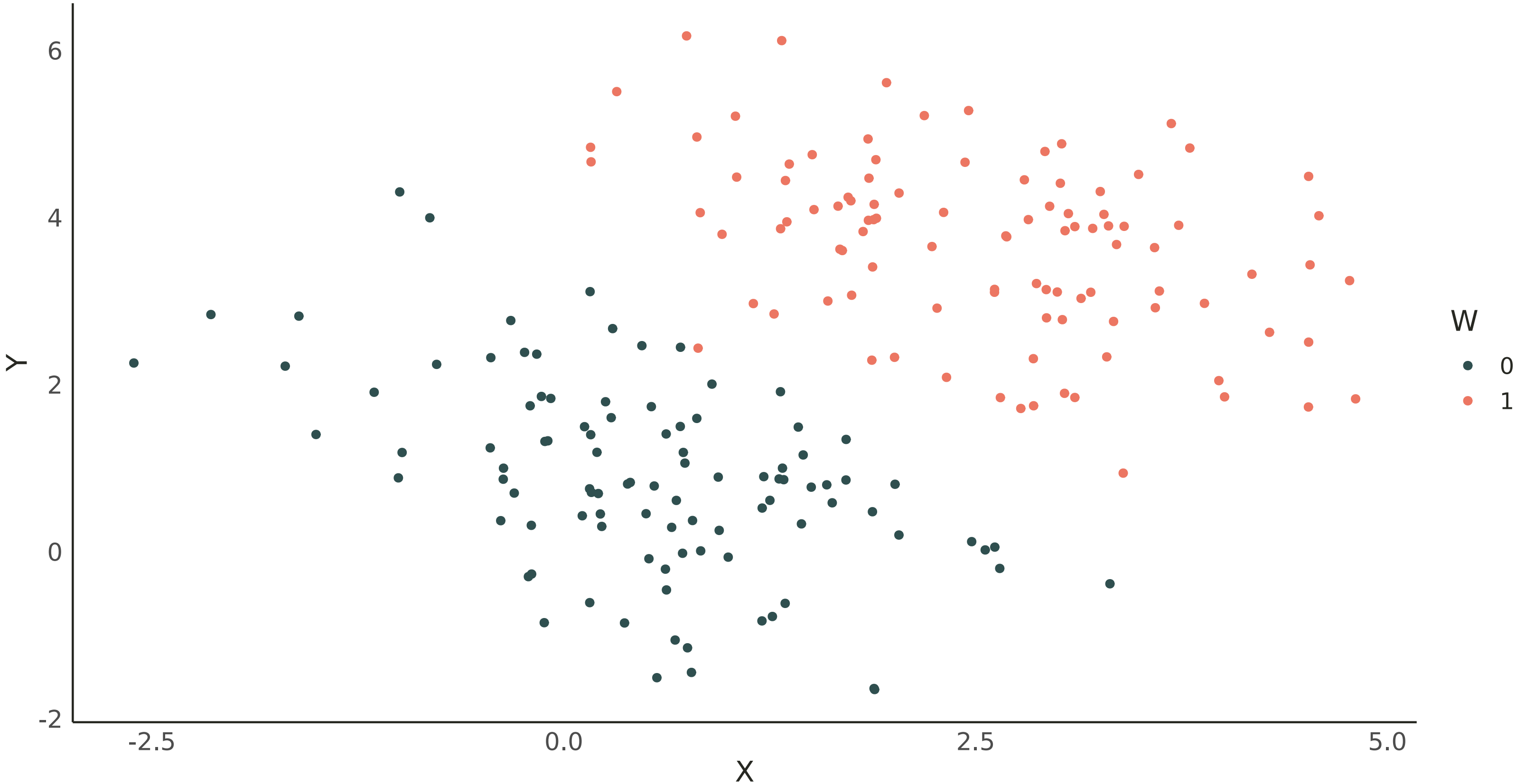
Running Regressions

OLS picks $\hat{\beta}_0$ and $\hat{\beta}_1$ that trace out the line of best fit. Ideally, we wound like to interpret the slope of the line as the causal effect of X on Y.



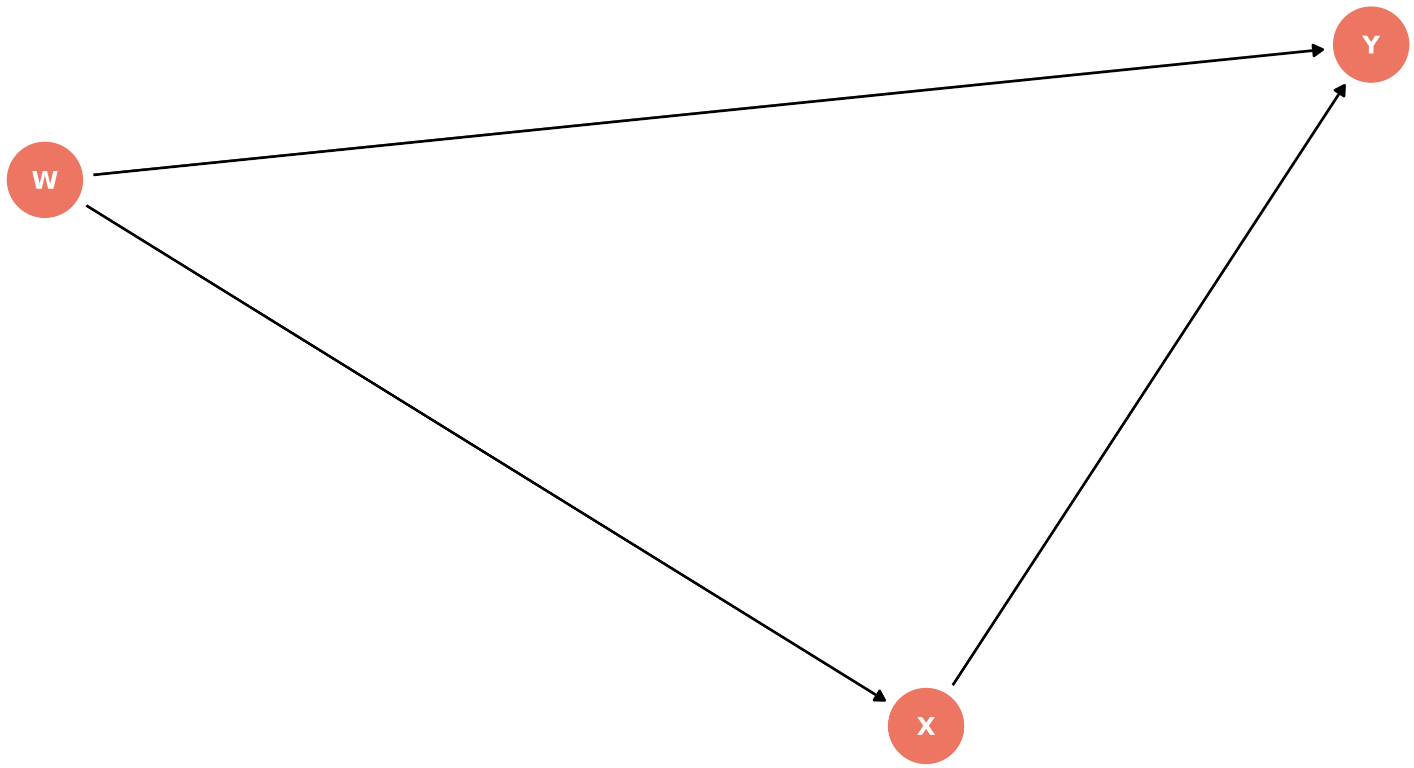
Confounders

However, the data are grouped by a third variable W. How would omitting W from the regression model affect the slope estimator?



Confounders

The problem with W is that it affects both Y and X. Without adjusting for W, we cannot isolate the causal effect of X on Y.



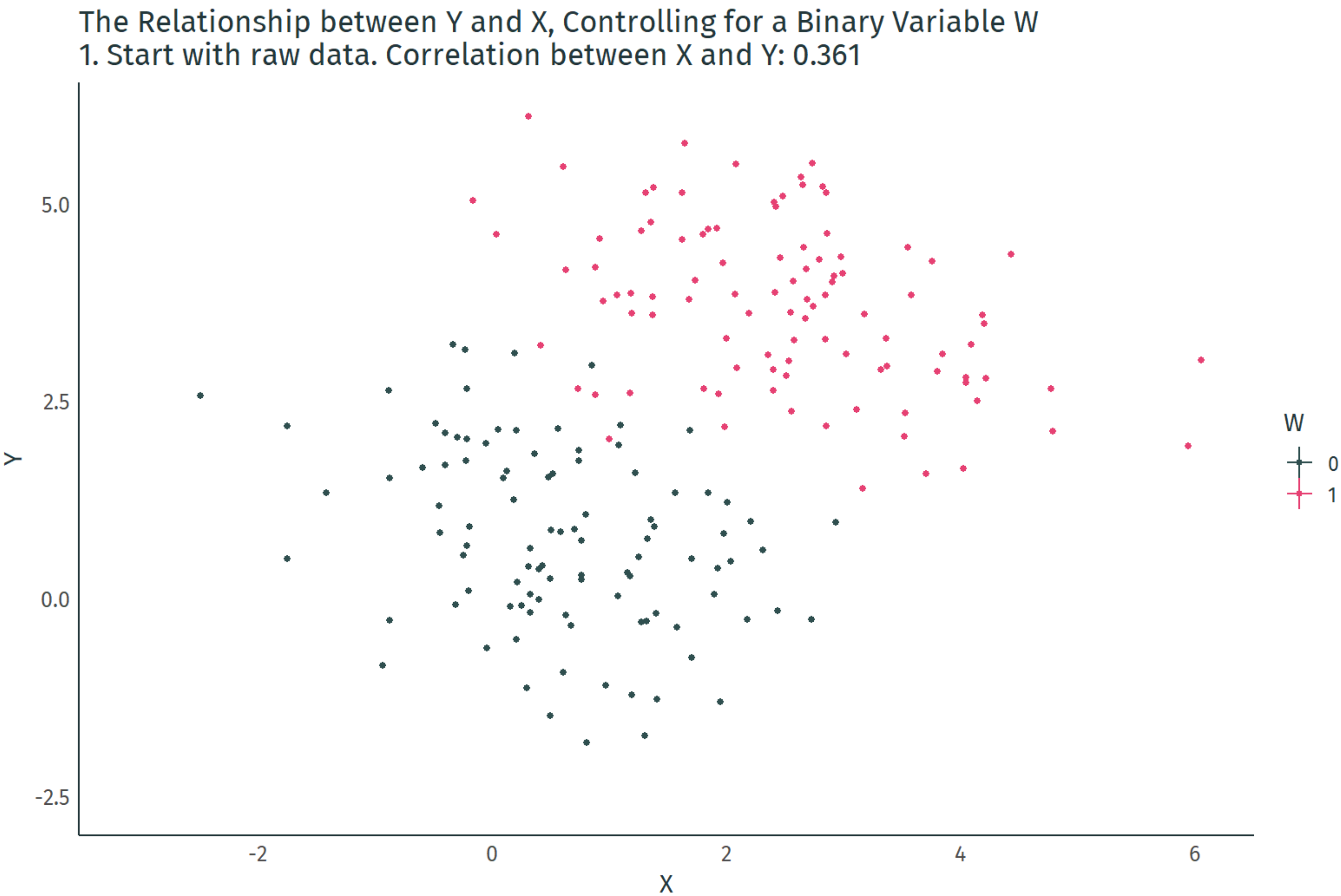
Controlling for Confounders

We can control for W by specifying it in the regression model:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + u_i$$

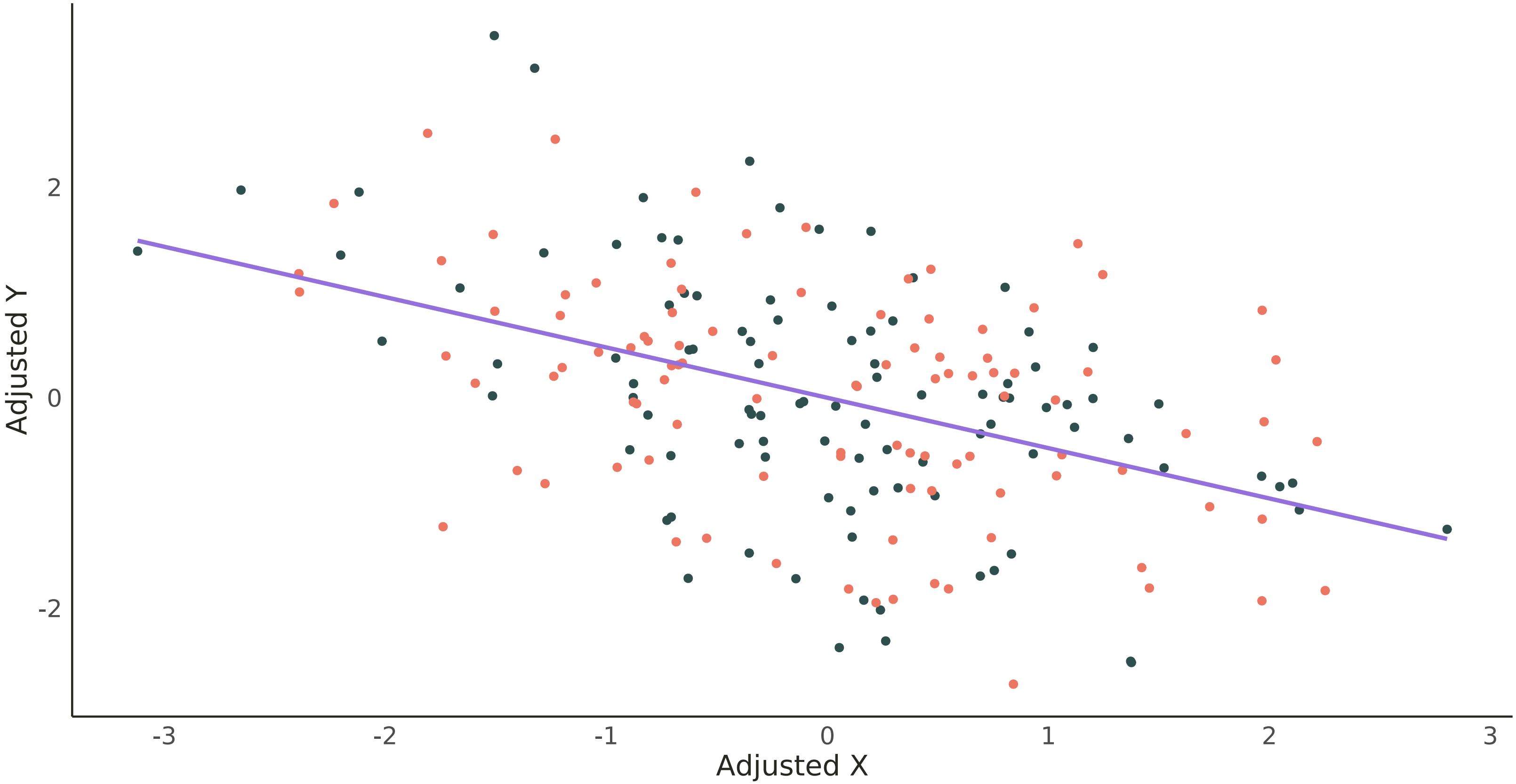
- W_i is a **control variable**.
- By including W_i in the regression, we can use OLS can difference out the confounding effect of W.
- **Note:** OLS doesn't care whether a right-hand side variable is a treatment or control variable, but we do.

Controlling for Confounders



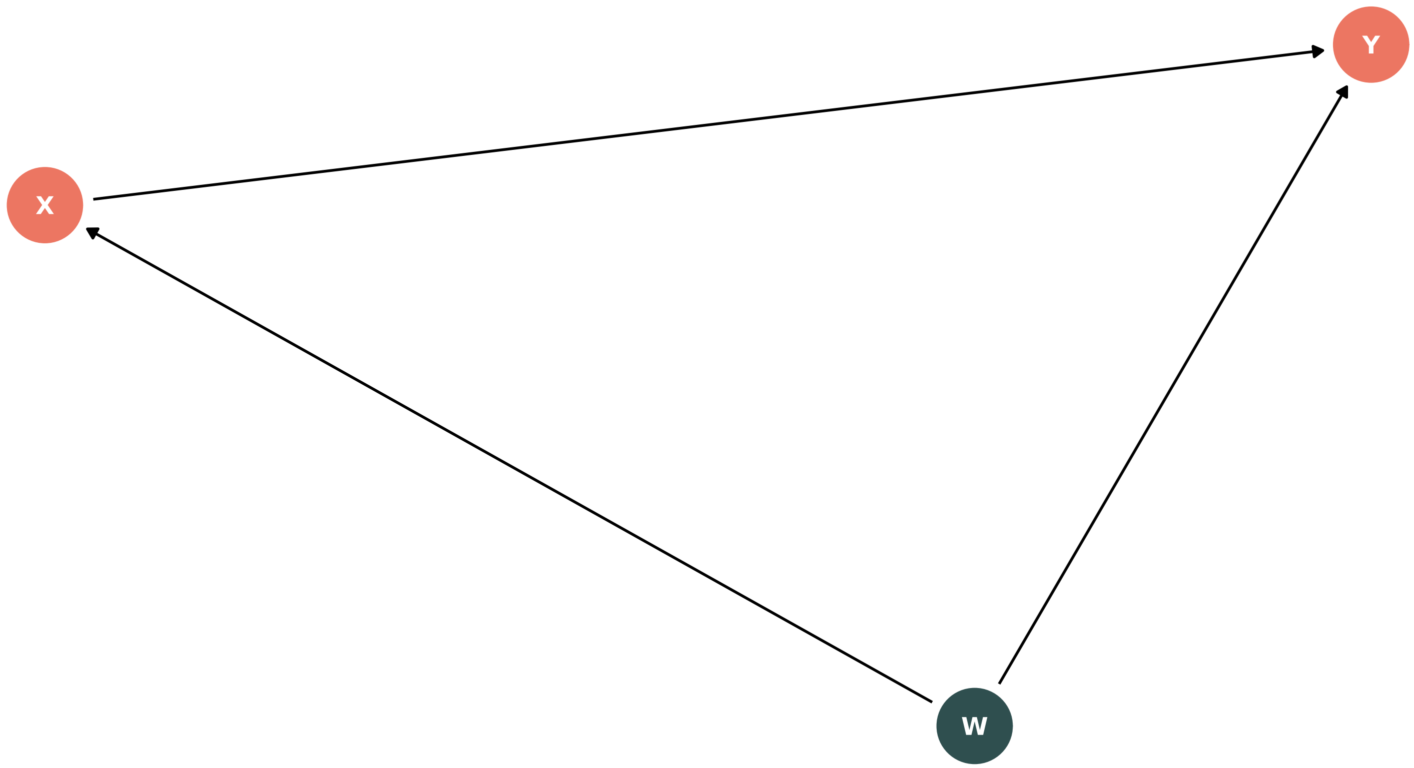
Controlling for Confounders

Controlling for W "adjusts" the data by **differencing out** the group-specific means of X and Y. **Slope of the estimated regression line changes!**



Controlling for Confounders

Can we interpret the estimated slope parameter as the causal effect of X on Y now that we've adjusted for W?



Controlling for Confounders

Example: Returns to schooling

Last class:

> **Q:** Could we simply compare the earnings those with more education to those with less?
> **A:** If we want to measure the causal effect, probably not.

What omitted variables should we worry about?

Controlling for Confounders

Example: Returns to schooling

Three regressions ***of*** wages ***on*** schooling.

Outcome variable: log(Wage)

Explanatory variable	**1**	**2**	**3**
Intercept	5.571	5.581	**5.695**
	(0.039)	(0.066)	**(0.068)**
Education	0.052	0.026	**0.027**
	(0.003)	(0.005)	**(0.005)**
IQ Score		0.004	**0.003**
		(0.001)	**(0.001)**
South			**-0.127**
			(0.019)

Omitted-Variable Bias

The presence of omitted-variable bias (OVB) precludes causal interpretation of our slope estimates.

We can back out the sign and magnitude of OVB by subtracting the slope estimate from a ***long*** regression from the slope estimate from a ***short*** regression:

$$\text{OVB} = \hat{\beta}_1^{\text{Short}} - \hat{\beta}_1^{\text{Long}}$$

Dealing with potential sources of OVB is one of the main objectives of econometric analysis!

OVB vs. Irrelevant Variables

So if we risk bias as a result of excluding a variable, why not throw every possible variable and transformation of variables (log-linearized, squared, inverted) at the model?

- Time consuming
- Data not always available
- Irrelevant variables actually make matters **worse**

OVB vs. Irrelevant Variables

How can more variables cause trouble? **Loss of efficiency** in estimator while still unbiased.

- This is the classic **multicollinearity** problem
- If an irrelevant variable is highly correlated with your explanatory variable of interest, the standard error will increase
- Inference of the coefficient's significance becomes muddled by higher standard error term
- More details on what this looks like statistically next week

Summary

What to remember

- Regressions are models of how we imagine the data generating process plays out
- They are usually simplifications of real life observations
- A linear regression fits a line through the data to reveal the relationship between treatment and outcome
- Confounders, omitted variables and irrelevant variables all pose risks to the identification challenge involved in estimating a population parameter of interest in our regression model
- OLS is the most common algoritm for estimating regressions, and that is what our next lecture will focus on