Assume a unit measure of firms living on $[0,1]$. An individual firm faces problem of when to change prices. Firms differ in the quality of their good which is given by $a_{i,t}$. Assume that $a_{i,t}$ follows a growth rate process in logs

$$\log(a_{i,i}) = \log(a_{i,t-1}) + \epsilon_{i,t} \tag{1}$$

where $\epsilon_{i,t}$ is normally distributed with mean 0, standard deviation σ_ϵ^2 and is i.i.d. across agents and time.

Given their current price, $P_{i,t}$, firms face a demand for their good which is given by[1]

$$D_{i,t} = a_{i,t}^{\gamma-1} \left(P_{i,t}\right)^{-\gamma}. \tag{2}$$

We assume that the amount of labor required to produce good of quality $a_{i,t}$ is $a_{i,t}$.[2] If $\bar{W}$ is the wage rate in this economy, then the per-period profit of a firm is given by

$$[P_{i,t} - a_{i,t}\bar{W}]D_{i,t}.$$

Firms can change their prices $P_{i,t}$ every period but must pay a cost $\bar{W}^{1-\gamma}\kappa$ in order to do so. Let β be the discount factor of these firms. The firm's objective is to maximize

$$\max_{P_{i,t}} \sum_{t=0}^{\infty} \beta^t \left([P_{i,t} - a_{i,t}\bar{W}] a_{i,t}^{\gamma-1} \left(P_{i,t}\right)^{-\gamma} - \bar{W}^{1-\gamma}\kappa 1_{P_{i,t} \neq P_{i,t-1}} \right) \tag{3}$$

given $P_{i,-1}$. $1_{P_{i,t} \neq P_{i,t-1}}$ is an indicator capturing if the firm changes its price.

1. Let $\mu_{i,t} = \frac{P_{i,t}}{a_{i,t}\bar{W}}$ be the markup (price over marginal cost) of firm i. Show that you can write the firm's maximization problem as problem as

$$\max_{\mu_{i,t}} \bar{W}^{1-\gamma} \sum_{t=0}^{\infty} \beta^t \left([\mu_{i,t} - 1]\mu_{i,t}^{-\gamma} - \kappa 1_{\mu_{i,t} \neq \frac{\mu_{i,t-1}}{\exp(\epsilon_{i,t})}} \right) \tag{4}$$

   given $\mu_{i,-1}$

2. Let $V(\mu)$ be the value of a firm entering with markup μ (this is after it's current quality shock ϵ is realized so $\mu = \frac{\mu\_}{\exp(\epsilon_i)}$ where $\mu\_$ is last period's markup) which chooses not to change prices this period. Let J be the value of a firm which elects to change prices this period (why doesn't this depend on μ?). Write down the Bellman equation(s) of the firm.

3. Let $\beta = 0.9981$ (targeting a monthly discount factor), $\gamma = 3$, $\kappa = 0.01$ and $\sigma_\epsilon = 0.1$. Write a program to solve the problem of the firm. (Use Gauss-Hermite quadrature to compute the expectation over ϵ_i)

4. When do firms change their prices? (You should find that firms change their prices once markups are outside of a certain range of the optimal markup)

5. Write a program to simulate $100,000$ firms to compute an ergodic distribution of firm markups. Adjust κ and σ_ϵ so that in the ergodic distribution 21.9% of firms change their price every month and the average size of price changes is 0.095. Compute the average markup

$$\left(\int_i \mu_{i,t}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$$

   in the steady state.

6. Suppose that wage costs unexpectly increase by 5% (this will reduce the markups of all firms proportionally by 5%). Starting from this new distribution simulate the path of of the average markup. How long does it take firms to adjust prices to pass along these new costs?

[1] For those who are curious this is the result of monopolistic competition and a Dixit-Stiglitz aggregator with elasticity of substitution given by γ. We have normalized the aggregate price level to 1.

[2] Higher quality goods take more effort to produce.