Math 2300: Discrete Mathematics
Recursion 2 - Algorithms Textbook Section

Terms, Concepts, and Examples

- An algorithm is called **recursive** if it solves a problem by reducing it to an instance of the same problem with a smaller input.

  Python Examples of Recursive Algorithms

- A Recursive Algorithm for Computing $n!$

```
def rfactorial(n):
    z=1
    if n>1:
        z=n*rfactorial(n-1)
    return z
# Code to test above
print(rfactorial(5))
```

- A Recursive Algorithm for Computing b^n

```
def rpower(b,n):
    z=1
    if n>0:
        z=b*rpower(b,n-1)
    return z
# Code to test above
print(rpower(2,5))
```

  Video Example of Running a Recursive Algorithm in Python

- A Recursive Algorithm for Fibonacci Numbers

```
def rfib(n):
    if n < 2:
        return n
    else:
        return rfib(n-1) +rfib(n-2)
#Code to test above
print(rfib(6))
```

- The Merge Sort Algorithm

```
def mergeSort(A):
    if len(A) >1:
        mid = len(A) // 2 #Finding the mid of the array
        L = A[:mid] # Dividing the array elements
        R = A[mid:] # into 2 halves
```

```
        mergeSort(L) # Sorting the first half
        mergeSort(R) # Sorting the second half

        i = j = k = 0

        # Merge L and R into A
        while i < len(L) and j < len(R):
            if L[i] < R[j]:
                A[k] = L[i]
                i+=1
            else:
                A[k] = R[j]
                j+=1
            k+=1

        # Checking if any element was left
        while i < len(L):
            A[k] = L[i]
            i+=1
            k+=1

        while j < len(R):
            A[k] = R[j]
            j+=1
            k+=1

arr = [12, 11, 13, 5, 6, 7]
print(arr)
mergeSort(arr)
print(arr)
```

Video Tracing Merge Sort

Video Merge Sort Dance

Practice Problems

1. Write a recursive algorithm for finding the sum of the squares of the first n positive integers.

2. Write a recursive algorithm for finding the maximum of a finite list of integers, making use of the fact that the maximum of list of n integers is the larger of the last integer in the list and the maximum of the first $n-1$ integers in the list.

3. Write a recursive algorithm for finding the sum of the first n odd positive integers.

4. Use a merge sort to sort 4, 3, 2, 5, 1, 8, 7 into increasing order. Show all the steps used by the algorithm.

5. Write a recursive algorithm to find the nth term of the sequence defined by $a_0 = 1, a_1 = 3, a_2 = 5$ and
$$a_n = a_{n-1}a_{n-2}^2a_{n-3}^3$$