Math 2300: Discrete Mathematics
**Number Theory 1 - Divisibility** Textbook Section

---

**Terms, Concepts, and Examples**

- Consider any two integers $a$ and $b$, such that $a \neq 0$. We say that $a$ **divides** $b$ if there is an integer $c$ such that $b = ac$or equivalently if $\frac{b}{a}$ is an integer. If $a$ divides $b$ then we say $b$ is **divisible** by $a$ or $a$ is a factor of $b$ or $b$ is a multiple of $a$.

  The fact that $a$ divides $b$ is denoted $a|b$. If $a$ does not divide $b$ we write $a \nmid b$.

  Example: $4|24$ since $24 = 4 * 6$, but $3 \nmid 7$ since $\frac{7}{3}$ is not an integer.

  Video on Divisibility

- **Division Algorithm** - Let $a$ be an integer and $d$ a positive integer. Then there are unique integers $q$ and $r$, with $0 \leq r < d$, such that $a = dq + r$.

  In this definition, $d$ is called the **divisor**, $a$ is called the **dividend**, $q$ is called the **quotient** and $r$ is the **remainder**.

  *Example:* Find the quotient and remainder when 101 is divided by 11.

  Solution: $101 = 11 * 9 + 2$ so the quotient is 9 and the remainder is 2

  *Example:* Find the quotient and remainder when -123 is divided by 19.

  Solution: $-123 = 19 * (-7) + 10$ so the quotient is $-7$ and the remainder is 10.

  Video on Division Algorithm

- In the division algorithm, the notation used to express the quotient $q$ and remainder $r$ can also be written as

$$q = a \textbf{ div } d, \qquad r = a \textbf{ mod } d$$

  *Example*: Using the above examples we have $101 = 11 \cdot 9 + 2$, so

$$9 = 101 \textbf{ div } 11, \qquad 2 = 101 \textbf{ mod } 11$$

  Also, $-123 = 19 \cdot (-7) + 10$, so

$$-7 = -123 \textbf{ div } 19, \qquad 10 = -123 \textbf{ mod } 19$$

  Note that when $r = a$ **mod** $d$, $r$ is always a number between 0 and $d$.

  Video on Mod and Div (Example continued from Division Algorithm Video)

**Practice Problems**

1. Does 17 divide each of these numbers?

   (a) 68 (b) 84 (c) 357 (d) 1001

2. What are the quotient and remainder when

   (a) 19 is divided by 7?
   (b) -111 is divided by 11?
   (c) 789 is divided by 23?
   (d) 1001 is divided by 13?
   (e) 0 is divided by 19?
   (f) 3 is divided by 5?
   (g) -1 is divided by 3?
   (h) 4 is divided by 1?

3. Evaluate the quantities.

   (a) $-17$ **mod** 2
   (b) 144 **mod** 7
   (c) $-101$ **mod** 13
   (d) 199 **mod** 19

4. Find $a$ **div** $m$ and $a$ **mod** $m$ when

   (a) $a = 228, m = 119$
   (b) $a - 9009, m = 223$
   (c) $a = -10101, m = 333$
   (d) $a = -765432, m = 38271$