Math 2300: Discrete Mathematics
Logic 1 - Propositions Textbook Section

Terms, Concepts, and Examples

- A **proposition** is a statement that is either true or false.

  Examples:

Propositions	Not a Proposition
GGC is located in Gwinnett County.	How are you?
$5 + 2 = 8$	$x + 5 = 3$
It is raining today.	She is very talented.
2 is a prime number.	

  Solution: The left-hand column represents statements that have truth values (some are true like the first and last), some are false like the second one and some are both true and false depending on the day. The right-hand column are either not statements (like the first one), or have ambiguous truth values (like the second and last) because we don't know which x or "She" the statement refers to.

- More complex statements can be built from elementary statements using **logical operators**. Some common logical operators are negation, conjunction, disjunction, exclusive or, implication, and biconditional.

- Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation** of p. The negation of p is denoted by $\neg p$ and read as "not p."

  Example:

Propositions	Negation
GGC is located in Gwinnett County.	GGC is **not** located in Gwinnett County.
$5 + 2 = 8$	$5 + 2 \neq 8$
2 is a prime number.	2 is **not** a prime number.

  Video Example of Negation

- Let p and q be propositions. The proposition "p and q" denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

- Let p and q be propositions. The proposition "p or q" denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

- Let p and q be propositions. The proposition "p **exclusive or** q" denoted by $p \oplus q$ and often denoted "xor", is true when exactly one of p and q is true and it is false otherwise.

- Let p and q be propositions. The proposition "p implies q" denoted by $p \implies q$ is called **implication**. It is false when p is true and q is false and is true otherwise.

  $p \implies q$ is read in a variety of equivalent ways:

  - if p then q
  - p only if q
  - p is sufficient for q
  - q whenever p

- Let p and q be propositions. The **biconditional** $p \iff q$ (read p if and only if q), is true when p and q have the same truth values and is false otherwise.

- **Translations** - Look for patterns corresponding to logical operators in the sentence and use them to define elementary propositions.

  Example: $\boxed{\text{If}}$ you are are older than 13 $\boxed{\text{or}}$ you are with your parents $\boxed{\text{then}}$ you can attend a PG-13 movie.

  Solution: The propositions are the simple sentences:

  $a =$ you are older than 13

  $b =$ you are with your parents

  $c =$ you can attend a PG-13 movie

  The sentence translates to symbols as $(a \vee b) \implies c$.

  Video Example of Translation

  Example: Use the propositions d : "David plays poker." and s : "Sue plays blackjack."

  Put the symbolic statement $d \implies \neg q$ into words.

  Solution: If David plays poker, then Sue does not play blackjack. Alternatively, we could write it as "Sue does not play blackjack whenever David plays poker." or "David plays poker only if Sue does not play blackjack."

  Another Video Example of Translation

Practice Problems

1. Let p and q be the propositions:

$$p : \text{I bought a lottery ticket this week.}$$
$$q : \text{I won the million dollar jackpot.}$$

   Express each of these propositions as an English sentence.

   (a) $\neg p$
   (b) $p \vee q$
   (c) $p \implies q$
   (d) $p \wedge q$
   (e) $p \iff q$
   (f) $\neg p \implies \neg q$
   (g) $\neg p \wedge \neg q$
   (h) $\neg p \vee (p \wedge q)$

2. Let p, q, and r be the propositions

$$p : \text{You get an A on the final exam.}$$
$$q : \text{You do every exercise in the course.}$$
$$r : \text{You get an A in this class.}$$

Write these propositions using p, q, and r and logical connectives (i.e. boolean operators), including negations.

(a) You get an A in this class, but you do not do every exercise in this course.

(b) You get an A on the final, you do every exercise in this course, and you get an A in this class.

(c) To get an A in this class, it is necessary for you to get an A on the final.

(d) You get an A on the final, but you don't do every exercise in this course; nevertheless, you get an A in this class.

(e) Getting an A on the final and doing every exercise in this course is sufficient for getting an A in this class.

(f) You will get an A in this class if and only if you either do every exercise in the course or you get an A on the final.