Math 2300: Discrete Mathematics

Set Theory 1 - Set Definitions Textbook Section

Terms, Concepts, and Examples

Discrete Mathematics is the study of discrete structures used to represent discrete objects.

- Many objects are built using sets. A **set** is a (unordered) collection of objects. These objects are called called **elements** or **members** of the set.

  Examples:

  Vowels in the English alphabet - $V = \{a, e, i, o, u\}$

  First seven prime numbers - $P = \{2, 3, 5, 7, 11, 13, 17\}$

- We can represent a set multiple ways.

  1. **Roster method** - listing the members of the set
  2. **Set builder notation** - defining a set using a given property - $\{x \mid x \text{ has property } P\}$

  Example: Represent the even integers between 50 and 63 using the roster method and set builder notation.

  Solution: Roster method $E = \{50, 52, 54, 56, 58, 60, 62\}$

  Set builder notation $E = \{x \mid 50 \leq x \leq 63 \text{ and } x \text{ is even}\}$

  If enumeration of the members is too cumbersome or hard we often use ellipses.

  Example: a set of integers between 1 and 100 $A = \{1, 2, 3, \ldots, 100\}$

  Video Example of Set Representations - Roster Method

  Video Example of Set Representations - Set Builder Notation

- There are some important sets that get used regularly so they have special symobls to represent them.

  1. $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ the set of Natural numbers
  2. $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ the set of integers
  3. $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$ the set of positive integers
  4. $\mathbb{R}$ is the set of all real numbers

- If x is an element of a set A, then we write $x \in A$. If x is not an element of set A, then we write $x \notin A$

  Video Example Using Element Notation

- Two sets are **equal** if and only if the have the same elements.

  Example: $\{1, 2, 3\} = \{1, 3, 2\} = \{1, 2, 1, 3, 2\}$

  $\{1, 2, 3, 4\} \neq \{1, 2, 2, 4\}$

  Note: Duplicates don't contribute anything new to a set, so remove them. The order of the elements in a set doesn't contribute anything new.

- The **universal set** is denoted by U and is the set of all objects in consideration. The **empty set** is the set with no elements and is denoted by $\{\}$ or $\emptyset$.

  Video Example of Empty Set

- The number of elements in a set is the **cardinality** of the set and is denoted $|A|$.

  Examples: $|\emptyset| = 0$, $|\{1, 2, 3, 4\}| = 4$, $|\{x \in \mathbb{Z} \mid -2 < x \leq 4\} = 6$

  Video Example of Cardinality

Practice Problems

1. Use the roster method to specify elements of each of the following sets:
   (a) The set of integers that are solutions of the equation $x^2 - 5x = 0$.
   (b) The set of natural numbers that are less than or equal to 10.
   (c) The set of integers that are greater than -2.

2. Each of the following sets is defined using the roster method. For each set determine four elements of the set other than the ones listed using the roster method.

   $A = \{1, 4, 7, 10, ...\}$ $\quad$ $B = \{2, 4, 8, 16, ...\}$
   $C = \{..., -8, -6, -4, -2, 0\}$ $\quad$ $D = \{..., -9, -6, -3, 0, 3, 6, 9, ...\}$

3. Describe the following sets using the roster method.

   $\{3n + 1 \mid n \in \mathbb{N}\}$ $\quad$ $\{x \in \mathbb{Z} \mid x^2 \leq 4\}$

4. Describe the following sets using set builder notation.

   $A = \{1, 5, 9, 13, ...\}$ $\quad$ $B = \{..., -8, -6, -4, -2, 0\}$ $\quad$ $C = \{1, 3, 9, 27, ...\}$

5. Assume the universal set is the set of integers ($\mathbb{Z}$). Let

   $A = \{-3, -2, 2, 3\}$ $\quad$ $B = \{x \in \mathbb{Z} \mid x^2 = 4 \text{ or } x^2 = 9\}$
   $C = \{x \in \mathbb{Z} \mid x^2 + 2 = 0\}$ $\quad$ $D = \{x \in \mathbb{Z} \mid x > 0\}$

   (a) Is the set A equal to the set B?
   (b) Is the set C equal to the set D?