Math 2300: Discrete Mathematics
**Logic 5 - Negations** Textbook Section

## Terms, Concepts, and Examples

- A conditional statement is of the form $p \implies q$. When you think of the truth table, this is False when $p$ is True and $q$ is False. If you compare truth tables, you will see

$$p \implies q \equiv \neg p \vee q$$

We can use DeMorgan's Laws to see that the **negation of a conditional** will be a conjunction (an AND statement).

$$\neg(p \implies q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$$

*Example:* Negate the statement "If a dog wags its tail, then it doesn't bite."

Solution: This would symbolically represented as $w \implies \neg b$ where $w$ is "A dog wags its tail." and $b$ is "A dog bites."

The negation would be
$$\neg(w \implies \neg b) \equiv \neg(\neg w \vee \neg b) \equiv w \wedge b$$

In words, "A dog wags its tail and it bites." Think of negation as the "counterexample" to the statement. The thing you have to find to prove someone wrong.

Video Example of Negating Conditionals

- Consider the statement "Nothing is perfect." We translate this symbolically as $\neg(\exists x P(x))$. Another way to express the same meaning is "Everything is imperfect." This would translate to $\forall x \neg P(x)$.

Since these two statements have the same meaning we say that

$$\neg(\exists x P(x)) \equiv \forall x \neg P(x)$$

We can do something similar and say

$$\neg(\forall x P(x)) \equiv \exists x \neg P(x)$$

Think of these as DeMorgan's Laws for quantifiers.

- A helpful table for negating expressions. Each row shows a statement logically equivalent to the negation.

| | |
|---|---|
| $\neg(p \wedge q)$ | $\neg p \vee \neg q$ |
| $\neg(p \vee q)$ | $\neg p \wedge \neg q$ |
| $\neg(p \implies q)$ | $p \wedge \neg q$ |
| $\neg\forall x P(x)$ | $\exists x \neg P(x)$ |
| $\neg\exists x P(x)$ | $\forall x \neg P(x)$ |

- Some examples:

| Statement | Negation |
|---|---|
| Jan is rich and happy. | Jan is not rich or not happy. |
| Mel walks or takes the bus to class. | Mel doesn't walk and doesn't take the bus to class. |
| If you drive more than 400 miles,<br>you will need to buy gasoline. | You drove more than 400 miles<br>and you didn't buy gasoline. |

Video Example of Negations

- Recall this statement from the last set of notes

$$\forall x \exists y (x + y = 0)$$

Our goal is to be able to write the negation of this statement (a statement with the opposite truth value).

For all integers $x$, there exists an integer $y$ such that $x + y = 0$.

The negation of this statement would be

$$\neg(\forall x \exists y (x + y = 0)) = \exists x \forall y \neg(x + y = 0) = \exists x \forall y (x + y \neq 0)$$

In words, There exists an integer $x$ such that for all integers $y$, $x + y \neq 0$.

Video Example of Quantified Negations

**Practice Problems**

1. Find the negation of these statements.
   (a) If I am a cloud, then I have a silver lining.
   (b) The light is green, so we can go.
   (c) If it is purple, then it is not a carrot.
2. Find the negation of these statements.
   (a) All dogs have fleas.
   (b) There is a horse that can add.
   (c) No monkey can speak French.
3. Express each statement using quantifiers. Then form the negation of the statement. Next, express the negation in simple English.
   (a) Every student in this class has taken exactly two mathematics classes at this school.
   (b) No one has climbed every mountain in the Himalayas.
   (c) There is a student in this class who has never seen a computer.