Math 2300: Discrete Mathematics
Logic 2 - Truth Tables and Logical Equivalences Textbook Section

Terms, Concepts, and Examples

- A **truth table** displays the relationships between truth values (T or F) of different propositions. Constructing a truth table for a compound statement depends upon the simple statements composing the compound statement.

  When making a truth table you need a row for each possible truth value. This means if there is one proposition in your statement, you will need 2 rows.

- Consider the **negation** operator. Recall, a statement and its negation have opposite truth values. Since it is just one statement and an operator, so its truth table will have 2 rows.

p	$\neg p$
T	F
F	T

- If there are two propositions, you will need 4 rows. As in **conjunction** and **disjunction**. Recall conjunction ($\wedge$) is true if both statements are true, and false otherwise. Disjunction ($\vee$) is false if both statements are false, and true otherwise.

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- **Exclusive or** ($\oplus$) is true when exactly one of p and q is true and it is false otherwise. **Implication** ($\implies$) is false when p is true and q is false and is true otherwise. The **biconditional** ($\iff$) is true when p and q have the same truth values and is false otherwise.

p	q	$p \oplus q$	$p \implies q$	$p \iff q$
T	T	F	T	T
T	F	T	F	F
F	T	T	T	F
F	F	F	T	T

- If there are three propositions you will need 8 rows. Note that the number of true-false cases has doubled with the addition of one statement. **The total number of cases (rows in your truth table) will be determined by using 2 to the power of the number of statements**. This means that if there are four statements, we would determine the number of cases by calculating 2^4.

Example: Complete the truth table for the compound proposition $(p \implies q) \wedge (\neg p \iff q)$

Solution: Notice there are 2 simple propositions so the truth table has 4 rows. There is also a column for each piece of the compound proposition.

p	q	$p \implies q$	$\neg p$	$\neg p \iff q$	$(p \implies q) \wedge (\neg p \iff q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Video Example of Truth Table

- The propositions p and q are called **logically equivalent** if they have the same truth table, this is noted by the symbol $\equiv$.

  Example: The propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

  Solution: Look at the truth table for both statements and compare. Notice that the fourth column and the last column are the same.

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

  Video Example of Logically Equivalent

- The above statements and their counterparts are called **DeMorgan's Laws**.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

  Example: Consider the statement "I went to the movie or I did not go bowling." Use DeMorgan's Law to find the negation of this statement.

  Solution: If p is the proposition "I went to the movie." and q is "I went bowling." then the statement above is written

$$p \vee \neg q$$

  Using DeMorgan's Law the negation would be

$$\begin{aligned} \neg(p \vee \neg q) &\equiv \neg p \wedge \neg(\neg q) \\ &\equiv \neg p \wedge q \end{aligned}$$

  In words, we would say, "I did not go to the movies and I went bowling."

  Video Example of DeMorgan's Law

Practice Problems

1. Construct a truth table for each of the compound propositions.

   (a) $(p \vee \neg q) \implies q$

   (b) $p \bigoplus (p \vee q)$

   (c) $(p \wedge q) \vee \neg r$

   (d) $(p \iff q) \oplus (p \iff \neg q)$

2. Determine which of the statements form logically equivalent pairs.

   (a) $\neg p \implies (q \implies r)$

   (b) $(p \vee q) \implies r$

   (c) $(p \wedge q) \implies r$

   (d) $(p \implies r) \wedge (q \implies r)$

   (e) $(p \implies q) \vee (p \implies r)$

   (f) $q \implies (p \vee r)$

   (g) $(p \vee q) \wedge (\neg p \vee r) \implies (q \vee r)$

   (h) $p \implies (q \vee r)$

   (i) $(p \implies r) \vee (q \implies r)$

   (j) $(p \implies q) \wedge (p \implies r)$

   (k) $p \implies (q \wedge r)$

   (l) $(p \implies q) \wedge (q \implies r) \implies (p \implies r)$

3. Show the following statements are logically equivalent $p \implies q$ and $\neg p \vee q$.

4. Use DeMorgan's Laws to find the negation of these statements.

   (a) 12 is even and 12 is prime.

   (b) Kwame will take a job in industry or go to graduate school.