Math 2300: Discrete Mathematics
Set Theory 2 - Subsets Textbook Section

Terms, Concepts, and Examples

- A set A is said to be a **subset** of B if and only if every element of A is also an element of B. We use $A \subseteq B$ to indicate A is a subset of B.

  We can express the definition in symbols as

  $$\forall x (x \in A \implies x \in B)$$

- There are some special subset relationships that work no matter the set A.

  1. $\emptyset \subseteq A$ for any set A

  2. $A \subseteq A$ for any set A

  Video Example of Subsets

- A set A is said to be a **proper subset** of B if and only if $A \subseteq B$ and $A \neq B$. We use $A \subset B$ to indicate A is a proper subset of B.

  Example If $U = \{1, 3, 5, 7, 9, 11, 13\}$, then which of the following are subsets and which are proper subsets of U.

  $B = \{2, 4\}$

  $A = \{0\}$

  $C = \{1, 9, 5, 13\}$

  $D = \{5, 11, 1\}$

  $E = \{13, 7, 9, 11, 5, 3, 1\}$

  $F = \{2, 3, 4, 5\}$

  Solution: B is not a subset since $2 \in B$ but not in U. A is not a subset since $0 \in A$ but not in U. F is not a subset since $4 \in F$ but not in U.

  The sets C and D are proper subsets ($\subset$) of U since each element of those sets is also in U but $|U|$ is larger than $|C|$ and $|D|$. Every set that is a proper subset is also a subset, so C, D and E are subsets ($\subseteq$) of U.

  Video Example of Proper Subsets

  Example List all the possible subsets of the set $\{2, 5, 9\}$. Solution: Subsets of this set could have 0 elements, 1 element, 2 elements or 3 elements. This would include $\emptyset$, $\{2\}$, $\{5\}$, $\{9\}$, $\{2, 5\}$, $\{2, 9\}$, $\{5, 9\}$ and $\{2, 5, 9\}$. If we combine all these subsets into a single set (with sets as its elements) we create a new set.

- Given a set A, the **power set** of A is the set of all possible subsets of A. We use $\mathcal{P}(A)$ to denote the power set of A. If the set A has cardinality $|A| = n$, then $|\mathcal{P}(A)| = 2^n$.

  Video Example of Power Set

Practice Problems

1. Use correct set notation (using $=$ or $\subseteq$) to indicate which of the following sets are equal and which are subsets of one of the other sets.

$A = \{3, 6, 9\}$ $B = \{6, 9, 3, 6\}$
$C = \{3, 6, 9, ...\}$ $D = \{3, 6, 7, 9\}$
$E = \{9, 12, 15, ...\}$ $F = \{9, 7, 6, 2\}$

2. Assume the universal set is the set of integers ($\mathbb{Z}$). Let

$A = \{-3, -2, 2, 3\}$ $B = \{x \in \mathbb{Z} \mid x^2 = 4 \text{ or } x^2 = 9\}$
$C = \{x \in \mathbb{Z} \mid x^2 + 2 = 0\}$ $D = \{x \in \mathbb{Z} \mid x > 0\}$

   (a) Is the set A a subset of the set B?

   (b) Is the set C a subset of the set D?

   (c) Is the set A a subset of the set D?

3. Let the universal set be $U = \{1, 2, 3, 4, 5, 6\}$, and let

$A \;=\; \{1, 2, 4\}$ $B \;=\; \{1, 2, 3, 5\}$ $C = \{x \in U \mid \; x^2 \leq 2\}$

Fill in each blank with one or more of the symbols $\subset$, $\subseteq$, $=$, $\neq$, $\in$ or $\notin$ so that the resulting statement is true. For each blank, include all symbols that will result in a true statement. If none of these symbols makes a true statement, write nothing in the blank.

   (a) A_____B (f) $\emptyset$_____A
   (b) 5_____B (g) $\{5\}$_____B
   (c) A_____C (h) $\{1, 2\}$_____C
   (d) $\{1, 2\}$_____A (i) $\{4, 2, 1\}$_____A
   (e) 6_____A (j) B_____$\emptyset$

4. Find the power set of the set $A = \{6, 11\}$.