Math 2300: Discrete Mathematics

Counting 3 - Permutations and Combinations Textbook Section

Terms, Concepts, and Examples

- **Permutation** - If n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are

$$P(n,r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

$r-$permutations of a set with n distinct elements.

In a permutation, the **order of the elements matters.**

Example: If there are 10 runners in a race, how many different ways can the gold, silver, and bronze medals be awarded?

Solution: In this problem the order you finish the race matters, so we use a permutation. There are $n = 10$ possible choices and we want to choose $r = 3$ of these.

$$P(10,3) = \frac{10!}{(10-3)!} = 10 * 9 * 8 = 720$$

So there are 720 ways to award the medals.

Video Permutation Introduction

Video Permutation Example

- **Combination** - The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \leq r \leq n$, equals

$$C(n,r) = \frac{n!}{r!(n-r)!}.$$

Example: How many ways can five cards be dealt from a standard 52-card deck?

Solution: In this problem the order you receive the cards in does not matter, so we use a combination. There are $n = 52$ possible choices and we want to choose $r = 5$ of these.

$$C(52,5) = \frac{52!}{5! * (52-5)!} = \frac{52!}{5! * 47!} = \frac{52 * 51 * 50 * 49 * 48}{5 * 4 * 3 * 2 * 1} = 2598960$$

So there are 2,598,960 ways to receive 5 cards .

Video Example of Combinations

Another Video Example of Combinations

Practice Problems

For problems 1-3, decide whether the situation being described is a permutation, a combination, or neither. Next, if it is a permutation or a combination, write it in the form $P(n, r)$ or $C(n, r)$, otherwise, explain why it is not a permutation or a combination. Finally, compute the number of possible ways each situation can occur.

1. You pick 3 other students from a class of 30 students to work together with on a group project.
2. A club with 15 members need to elect a President, a Vice President, a Secretary, and a Treasurer
3. A small business with 22 employees decides to delegate a project to a team of 4 people. They want two of the team members to be female, and two of them to be male. There are 12 female employees total in the company.
4. How many different 5-person basketball teams can be formed from a pool of 12 players?
5. Sixty people apply for 10 job openings. In how many ways can all the jobs be filled?
6. A poker hand consists of five cards. How many different poker hands contain all card of the same suit? (Such a hand is called a "flush".)
7. In how many ways can 100 senators be divided into groups of 20 each?
8. How many different three-letter words (i.e. sequences of letters) can be formed from the letters of the word JUPITER?
9. A bag of 10 apples contains 2 rotten apples and 8 good apples. A shopper selects a sample of 3 apples from the bag.
   (a) How many different samples are possible?
   (b) How many samples contain all good apples?
   (c) How many samples contain at least 1 rotten apple?
10. The student council at Gotham College is made up of four freshman, five sophomores, six juniors and seven seniors. A yearbook photographer would like to line up three council members from each class for a picture. How many different pictures are possible if each group of classmates stands together?
11. How many subsets with an odd number of elements does a set with 100 elements have?
12. There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?