Math 2300: Discrete Mathematics
Counting 1 - Definitions Textbook Section

Terms, Concepts, and Examples

- **Product Rule** - Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 * n_2$ ways to do the procedure.

  Example: Suppose there are 27 computers in a computer center and each computer has 15 ports. How many different ways are there to choose a specific port?

  Solution: Since the tasks are "independent" we can just multiply the number of ways to do each task.

  (computers) * (ports) = 27 * 15 = 405 was to choose a specific port

  Video Example of Product Rule

- **Sum Rule** - If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

  Example: A student in a Capstone course can choose a project from three different professors. The professors have 3, 7, and 4 possible projects, and no projects is on more than one professor's list. How many possible projects can the student choose?

  Solution: Since these projects are "disjoint" and don't overlap we can add up the total number of projects.

  3+7+4=14 possible projects

  Video Example of Sum Rule

- **Subtraction Rule** - If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

  Example: A tech company receives 200 applications for a position. 150 of the applicants were ITEC majors, 43 were business majors, and 25 were double majors in business and ITEC. How many application did not major in ITEC or business?

  Solution: The ITEC majors and business majors overlap, so to find the total number of folks who applied from ITEC OR Business we add them up and subtract the overlap.

  (ITEC majors) + (Business majors) - (double majors) = 150+43-25 = 159 people who applied from ITEC or Business. This leaves 200-158=42 people who applied from outside those majors.

  Video Example of Subtraction Rule

  Another Video Example of Subtraction Rule

- Often times we have to combine multiple rules together.

  Example: How many integers between 5 and 31 are divisible by 3?

Solution: To find this number divide 31 by 3 and round down. 31/3 rounded down is 10. So there are 10 integers below 31 that are divisible by 3. Since the list begins at 5, we need to subtract off one of those. This gives 9 total integers between 5 and 31 that are divisible by 3. Namely, 6, 9, 12, 15, 18, 21, 24, 27, and 30.

Example: How many strings of four digits are there that contain at least one 5?

Solution: This is easier to compute if we find the total number of strings and subtract out the strings that do not contain a 5 - this would make every string remaining contain at least one 5. The total number of strings is 10^4 and the number of string that do not contain a 5 is 9^4. Therefore, the number of strings that contain at least one 5 is $10^4 - 9^4 = 3439$.

Video Example of Counting Bit Strings

Practice Problems The 100 members of the Earth Club were asked what they felt the club's priorities should be in the coming year: clean water, clean air, or recycling. The repsonses were 45 for clean water, 30 for clean air, 42 for recycling, 13 for both clean air and clean water, 20 for clean air and recycling, 16 for clean water and recycling, and 9 for all three. For questions 1-4 refer to this poll.

1. How many members thought the priority should be clean air only?
2. How many members thought recycling should not be a priority?
3. How many members though the priority should be clean air and recycling, but not clean water?
4. How many members thought the priority should be anything but one of these three issues?
5. How many strings of length 8 can be formed from the letters, a, b, c, d, and e? How many of the strings have at least one e?
6. What is the value of k after the following code is executed?

```
k = 0
for i in range(1,5):
    for j in range(1,4):
        for l in range(1,6):
            for m in range(1,3):
                k+=1
print(k)
```

7. How many strings of eight uppercase English letters are there
   (a) if letters can be repeated?
   (b) if no letter can be repeated?
   (c) that start with X, if letters can be repeated?
   (d) that start with X, if no letter can be repeated?

(e) that start with the letters BO (in that order), if letters can be repeated?

(f) that start and end with the letters BO (in that order), if letters can be repeated?

(g) that start or end with the letters BO (in that order), if letters can be repeated?

8. How many positive integers less than 1000

(a) are divisible by 7?

(b) are divisible by 7 but not by 11?

(c) are divisible by both 7 and 11?

(d) are divisible by either 7 or 11?

(e) are divisible by exactly one of 7 and 11?

(f) are divisible be neither 7 nor 11?