Chapter 8 (Edition 8): 8.1, 8.10, 8.12, 8.14, 8.51

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```
# loading libraries
library(gplots)
```

```
## Warning: package 'gplots' was built under R version 3.4.4
```

```
##
## Attaching package: 'gplots'
```

```
## The following object is masked from 'package:stats':
##
##     lowess
```

```
library(car)
```

8.1

Suppose that in the chemical process development experiment described in Problem 6.7, it was only possible to run a one-half fraction of the 2^4 design. Construct the design and perform the statistical analysis, using the data from replicate I.

```
#chemical data from 6.7
rep1 = c(90,74,81,83,77,81,88,73,98,72,87,85,99,79,87,80)
#rep2 = c(93,78,85,80,78,80,82,70,95,76,83,86,90,75,84,80)
A <- rep(x = c("-", "+"), times = 8)
B <- rep(x = c("-", "+"), each = 2, times = 4)
C <- rep(x = c("-", "+"), each = 4, times = 2)
D <- rep(x = c("-", "+"), each = 8)
#data
chemical = data.frame(A,B,C,D,rep1)

coded=function(x) #a function to code variable x
{
  ifelse(x=="+", 1, -1)
}
for (j in 1:4)
  chemical[, j]=as.numeric(coded(chemical[, j]))

fraction.chem=with(chemical, chemical[A * B * C * D == 1,])

#linear model
chem.lm = lm(rep1 ~ A*B*C*D, fraction.chem); summary(chem.lm)
```

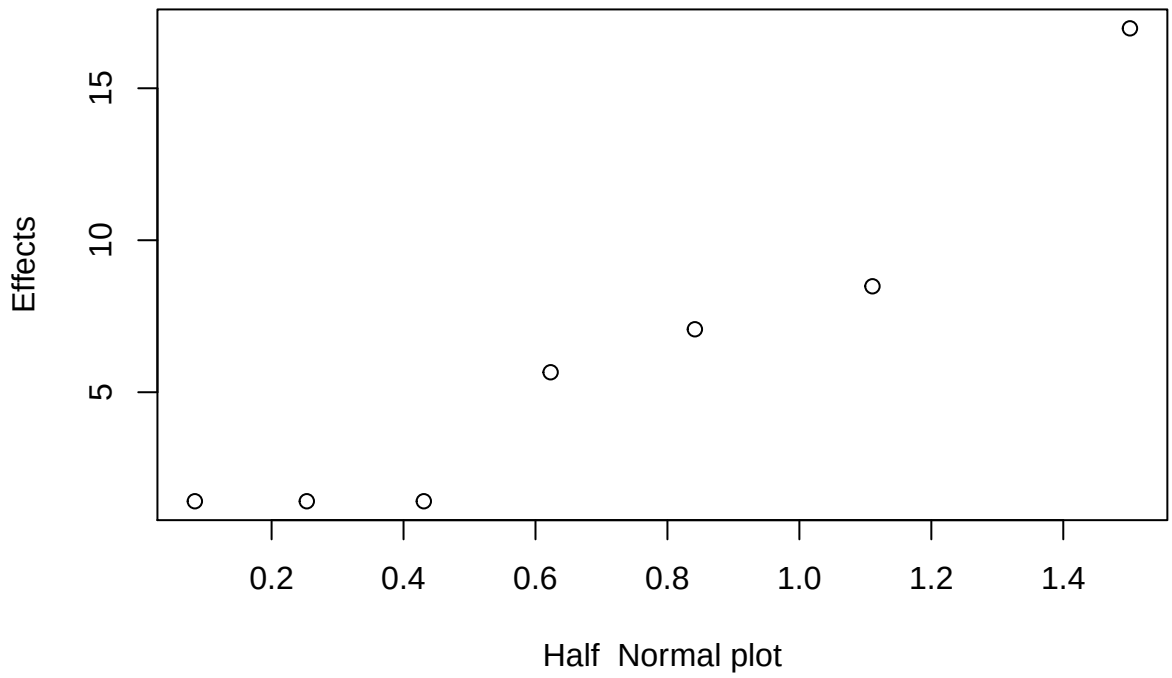
```
##
## Call:
## lm(formula = rep1 ~ A * B * C * D, data = fraction.chem)
##
## Residuals:
## ALL 8 residuals are 0: no residual degrees of freedom!
```

```
##
## Coefficients: (8 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)     85.0         NA      NA       NA
## A               -6.0         NA      NA       NA
## B               -0.5         NA      NA       NA
## C                2.0         NA      NA       NA
## D               -0.5         NA      NA       NA
## A:B              3.0         NA      NA       NA
## A:C             -0.5         NA      NA       NA
## B:C             -2.5         NA      NA       NA
## A:D               NA         NA      NA       NA
## B:D               NA         NA      NA       NA
## C:D               NA         NA      NA       NA
## A:B:C             NA         NA      NA       NA
## A:B:D             NA         NA      NA       NA
## A:C:D             NA         NA      NA       NA
## B:C:D             NA         NA      NA       NA
## A:B:C:D           NA         NA      NA       NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:    NaN
## F-statistic:   NaN on 7 and 0 DF,  p-value: NA
```

```
#alias
alias(chem.lm)
```

```
## Model :
## rep1 ~ A * B * C * D
##
## Complete :
##         (Intercept) A B C D A:B A:C B:C
## A:D     0           0 0 0 0 0   0   1
## B:D     0           0 0 0 0 0   1   0
## C:D     0           0 0 0 0 1   0   0
## A:B:C   0           0 0 0 1 0   0   0
## A:B:D   0           0 0 1 0 0   0   0
## A:C:D   0           0 1 0 0 0   0   0
## B:C:D   0           1 0 0 0 0   0   0
## A:B:C:D 1           0 0 0 0 0   0   0
```

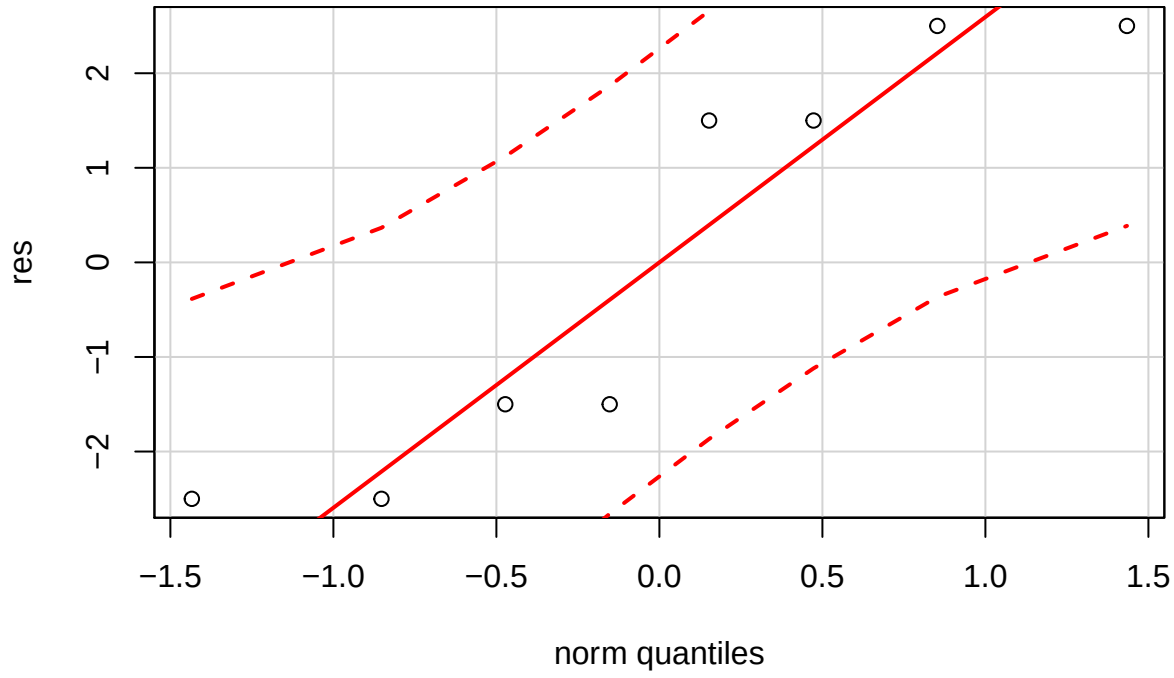
```
#normal probability plot
qqnorm(aov(rep1 ~ A * B * C * D, fraction.chem), label = TRUE)#A, C, AB, BC
```



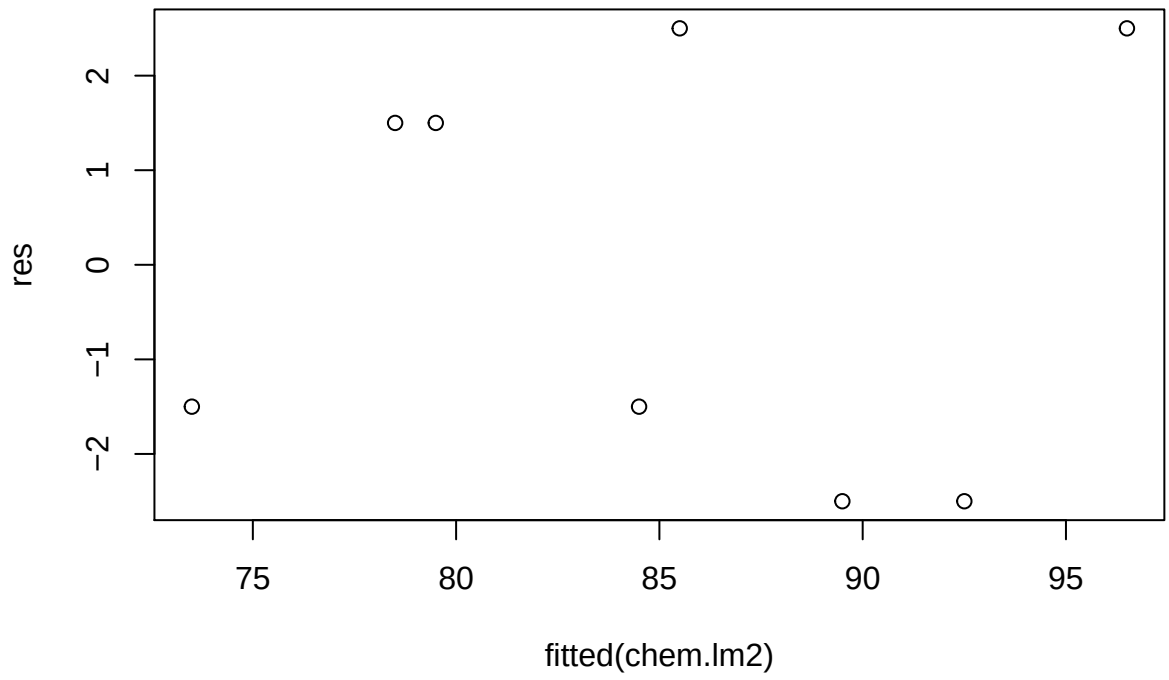

```
#refine model
chem.lm2 = lm(rep1 ~ A*B + A*D, fraction.chem); summary(chem.lm2)
```

```
##
## Call:
## lm(formula = rep1 ~ A * B + A * D, data = fraction.chem)
##
## Residuals:
##    1    4    6    7   10   11   13   16
## -2.5 -1.5  1.5  2.5 -1.5 -2.5  2.5  1.5
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   85.000      1.458  58.310 0.000294 ***
## A             -6.000      1.458  -4.116 0.054268 .
## B             -0.500      1.458  -0.343 0.764298
## D             -0.500      1.458  -0.343 0.764298
## A:B            3.000      1.458   2.058 0.175837
## A:D           -2.500      1.458  -1.715 0.228483
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.123 on 2 degrees of freedom
## Multiple R-squared:  0.9241, Adjusted R-squared:  0.7344
## F-statistic: 4.871 on 5 and 2 DF,  p-value: 0.1791
```

```r
#Residual Analysis
res = fraction.chem$rep1 - fitted(chem.lm2)
qqPlot(res)
```




```r
plot(fitted(chem.lm2), res)
```



We perform our analysis, check our aliases, and do half normal probability. We see that interaction effects AB + AD and their main effects have the largest effect on the response variable, we quickly check our residuals and everything seems good. Our model is good.

8.10

An article by J. J. Pignatiello Jr. and J. S. Ramberg in the Journal of Quality Technology (Vol. 17, 1985, pp. 198-206) describes the use of a replicated fractional factorial to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The data are shown in Table P8.1

(a) Write out the alias structure for this design. What is the resolution of this design?

We design a half factorial design with resolution V and generator ABCDE. The alias structure for this design is shown below:

```
# declaring data
A <- rep(x = c("-", "+"), times = 8)
B <- rep(x = c("-", "+"), each = 2, times = 4)
C <- rep(x = c("-", "+"), each = 4, times = 2)
D <- c("-", "+", "+", "-", "+", "-", "-", "+", "-", "+", "+", "-", "+", "-", "-", "+")
E <- rep(x = c("-", "+"), each = 8)
FH1 <- c(7.78, 8.15, 7.5, 7.59, 7.54, 7.69, 7.56, 7.56, 7.5, 7.88, 7.5, 7.63, 7.32, 7.56, 7.18, 7.81)
FH2 <- c(7.78, 8.18, 7.56, 7.56, 8, 8.09, 7.52, 7.81, 7.25, 7.88, 7.56, 7.75, 7.44, 7.69, 7.18, 7.5)
FH3 <- c(7.81, 7.88, 7.5, 7.75, 7.88, 8.06, 7.44, 7.69, 7.12, 7.44, 7.5, 7.56, 7.44, 7.62, 7.25, 7.59)
```

```r
# creating table
A <- c(A, A, A)
B <- c(B, B, B)
C <- c(C, C, C)
D <- c(D, D, D)
E <- c(E, E, E)
FH <- as.numeric(c(FH1, FH2, FH3))

spring <- data.frame(cbind(A, B, C, D, E, FH))

# defining coded
coded=function(x)
{
  ifelse(x=="+", 1, -1)
}

# decoding data
for (j in 1:5)
  spring[, j]=as.numeric(coded(spring[, j]))

# defining fraction
#fraction <- with(spring, spring[A * B * C * D * E == 1,])

# linear regression
summary(lm(as.numeric(FH) ~ A * B * C * D * E, spring))
```

```
##
## Call:
## lm(formula = as.numeric(FH) ~ A * B * C * D * E, data = spring)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -8.000 -1.000  0.000  2.000  5.333
##
## Coefficients: (16 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.52083    0.47324  22.231  < 2e-16 ***
## A            2.89583    0.47324   6.119 7.69e-07 ***
## B           -1.85417    0.47324  -3.918 0.000441 ***
## C           -0.52083    0.47324  -1.101 0.279297
## D            0.64583    0.47324   1.365 0.181868
## E           -2.56250    0.47324  -5.415 5.94e-06 ***
## A:B         -0.06250    0.47324  -0.132 0.895758
## A:C          0.02083    0.47324   0.044 0.965160
## B:C          0.02083    0.47324   0.044 0.965160
## A:D               NA         NA      NA       NA
## B:D               NA         NA      NA       NA
## C:D               NA         NA      NA       NA
## A:E          0.72917    0.47324   1.541 0.133200
## B:E          1.81250    0.47324   3.830 0.000563 ***
## C:E         -0.27083    0.47324  -0.572 0.571123
## D:E          0.22917    0.47324   0.484 0.631508
## A:B:C             NA         NA      NA       NA
## A:B:D             NA         NA      NA       NA
```

```
## A:C:D             NA         NA      NA       NA
## B:C:D             NA         NA      NA       NA
## A:B:E       -0.31250    0.47324  -0.660 0.513762
## A:C:E        0.02083    0.47324   0.044 0.965160
## B:C:E       -0.64583    0.47324  -1.365 0.181868
## A:D:E             NA         NA      NA       NA
## B:D:E             NA         NA      NA       NA
## C:D:E             NA         NA      NA       NA
## A:B:C:D           NA         NA      NA       NA
## A:B:C:E           NA         NA      NA       NA
## A:B:D:E           NA         NA      NA       NA
## A:C:D:E           NA         NA      NA       NA
## B:C:D:E           NA         NA      NA       NA
## A:B:C:D:E         NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.279 on 32 degrees of freedom
## Multiple R-squared:  0.7666, Adjusted R-squared:  0.6572
## F-statistic: 7.008 on 15 and 32 DF,  p-value: 2.127e-06
```

```
# alias structure
alias(lm(as.numeric(FH) ~ A * B * C * D * E, spring))
```

```
## Model :
## as.numeric(FH) ~ A * B * C * D * E
##
## Complete :
##           (Intercept) A B C D E A:B A:C B:C A:E B:E C:E D:E A:B:E A:C:E
## A:D       0           0 0 0 0 0 0   0   1   0   0   0   0   0     0
## B:D       0           0 0 0 0 0 0   1   0   0   0   0   0   0     0
## C:D       0           0 0 0 0 0 1   0   0   0   0   0   0   0     0
## A:B:C     0           0 0 0 1 0 0   0   0   0   0   0   0   0     0
## A:B:D     0           0 0 1 0 0 0   0   0   0   0   0   0   0     0
## A:C:D     0           0 1 0 0 0 0   0   0   0   0   0   0   0     0
## B:C:D     0           1 0 0 0 0 0   0   0   0   0   0   0   0     0
## A:D:E     0           0 0 0 0 0 0   0   0   0   0   0   0   0     0
## B:D:E     0           0 0 0 0 0 0   0   0   0   0   0   0   0     1
## C:D:E     0           0 0 0 0 0 0   0   0   0   0   0   0   1     0
## A:B:C:D   1           0 0 0 0 0 0   0   0   0   0   0   0   0     0
## A:B:C:E   0           0 0 0 0 0 0   0   0   0   0   0   1   0     0
## A:B:D:E   0           0 0 0 0 0 0   0   0   0   0   1   0   0     0
## A:C:D:E   0           0 0 0 0 0 0   0   0   0   1   0   0   0     0
## B:C:D:E   0           0 0 0 0 0 0   0   0   1   0   0   0   0     0
## A:B:C:D:E 0           0 0 0 0 1 0   0   0   0   0   0   0   0     0
##           B:C:E
## A:D       0
## B:D       0
## C:D       0
## A:B:C     0
## A:B:D     0
## A:C:D     0
## B:C:D     0
## A:D:E     1
## B:D:E     0
```

```
## C:D:E     0
## A:B:C:D   0
## A:B:C:E   0
## A:B:D:E   0
## A:C:D:E   0
## B:C:D:E   0
## A:B:C:D:E 0
```

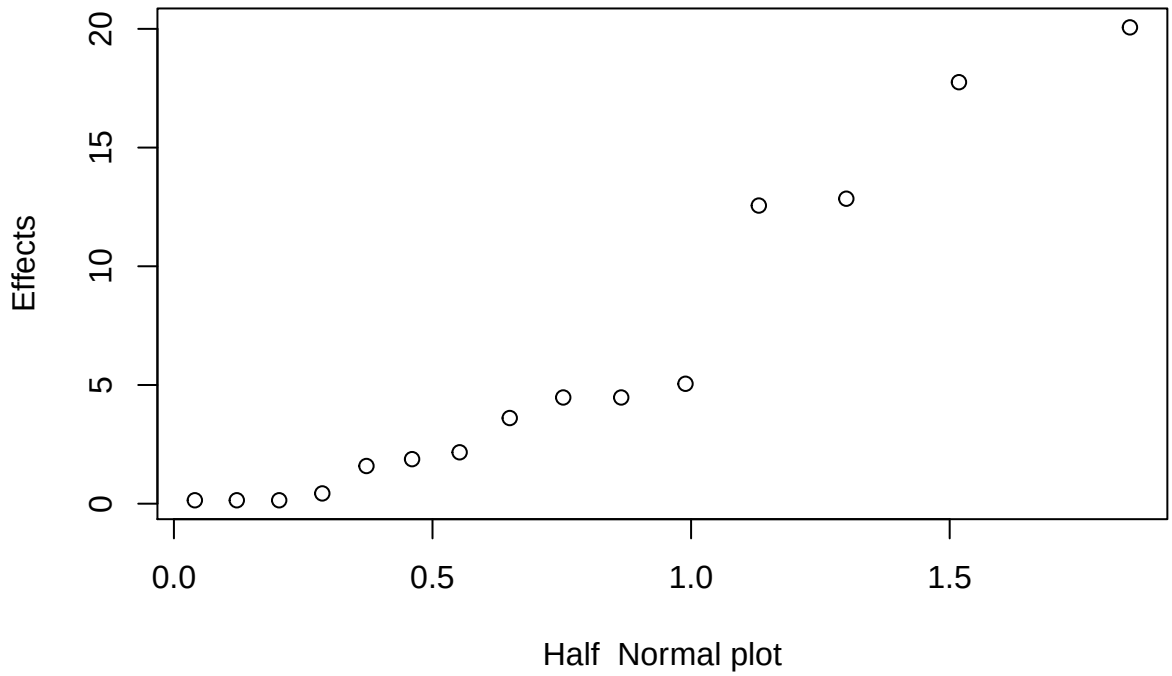
(b) Analyze the data. What factors influence the mean free height?

```
# linear regression
spring.lm <- lm(as.numeric(FH) ~ A * B * C * D * E, spring)
summary(spring.lm)
```

```
##
## Call:
## lm(formula = as.numeric(FH) ~ A * B * C * D * E, data = spring)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -8.000 -1.000  0.000  2.000  5.333
##
## Coefficients: (16 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 10.52083    0.47324  22.231  < 2e-16 ***
## A            2.89583    0.47324   6.119 7.69e-07 ***
## B           -1.85417    0.47324  -3.918 0.000441 ***
## C           -0.52083    0.47324  -1.101 0.279297
## D            0.64583    0.47324   1.365 0.181868
## E           -2.56250    0.47324  -5.415 5.94e-06 ***
## A:B         -0.06250    0.47324  -0.132 0.895758
## A:C          0.02083    0.47324   0.044 0.965160
## B:C          0.02083    0.47324   0.044 0.965160
## A:D               NA         NA      NA       NA
## B:D               NA         NA      NA       NA
## C:D               NA         NA      NA       NA
## A:E          0.72917    0.47324   1.541 0.133200
## B:E          1.81250    0.47324   3.830 0.000563 ***
## C:E         -0.27083    0.47324  -0.572 0.571123
## D:E          0.22917    0.47324   0.484 0.631508
## A:B:C             NA         NA      NA       NA
## A:B:D             NA         NA      NA       NA
## A:C:D             NA         NA      NA       NA
## B:C:D             NA         NA      NA       NA
## A:B:E       -0.31250    0.47324  -0.660 0.513762
## A:C:E        0.02083    0.47324   0.044 0.965160
## B:C:E       -0.64583    0.47324  -1.365 0.181868
## A:D:E             NA         NA      NA       NA
## B:D:E             NA         NA      NA       NA
## C:D:E             NA         NA      NA       NA
## A:B:C:D           NA         NA      NA       NA
## A:B:C:E           NA         NA      NA       NA
## A:B:D:E           NA         NA      NA       NA
## A:C:D:E           NA         NA      NA       NA
## B:C:D:E           NA         NA      NA       NA
```

```
## A:B:C:D:E         NA         NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.279 on 32 degrees of freedom
## Multiple R-squared:  0.7666, Adjusted R-squared:  0.6572
## F-statistic: 7.008 on 15 and 32 DF,  p-value: 2.127e-06
```

```
# half normal probability plot
qqnorm(aov(as.numeric(FH) ~ A * B * C * D * E, spring), label = TRUE)
```



The factors that influence mean free height are A, B, E, BE. We can create a reduced model using this information.

```
# new linear regression
spring.lm2 <- lm(as.numeric(FH) ~ A + B * E, spring)
summary(spring.lm2)
```

```
##
## Call:
## lm(formula = as.numeric(FH) ~ A + B * E, data = spring)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -6.6458 -2.0417  0.2292  2.1146  6.1042
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)  10.5208     0.4583  22.955  < 2e-16 ***
## A             2.8958     0.4583   6.318 1.26e-07 ***
## B            -1.8542     0.4583  -4.046 0.000213 ***
## E            -2.5625     0.4583  -5.591 1.43e-06 ***
## B:E           1.8125     0.4583   3.955 0.000282 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.175 on 43 degrees of freedom
## Multiple R-squared:  0.7059, Adjusted R-squared:  0.6785
## F-statistic:  25.8 on 4 and 43 DF,  p-value: 6.066e-11
```

(c) Calculate the range and standard deviation of the free height for each run. Is there any indication that any of these factors affects variability in the free height?

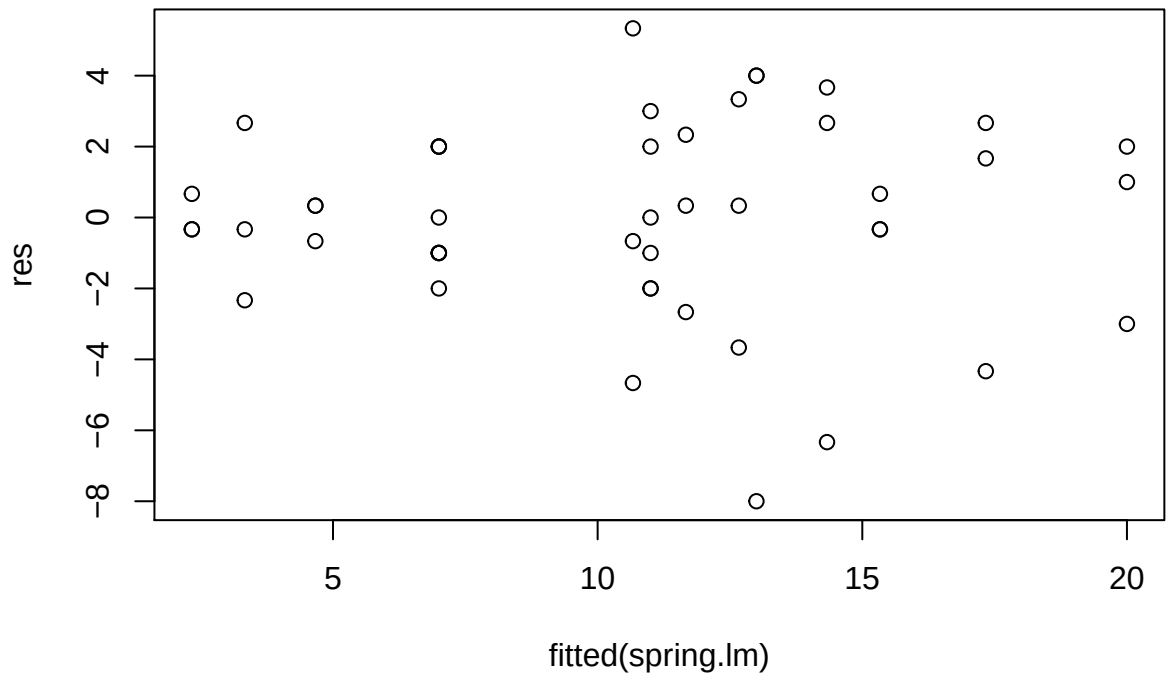
```
# min max and sd
data.frame("Free Height 1" = c(range(FH1), sd(FH1)),
           "Free Height 2" = c(range(FH2), sd(FH2)),
           "Free Height 3" = c(range(FH3), sd(FH3)),
           row.names = c("min", "max", "sd"))
```

```
##     Free.Height.1 Free.Height.2 Free.Height.3
## min     7.1800000     7.1800000     7.1200000
## max     8.1500000     8.1800000     8.0600000
## sd      0.2248398     0.2810746     0.2442122
```

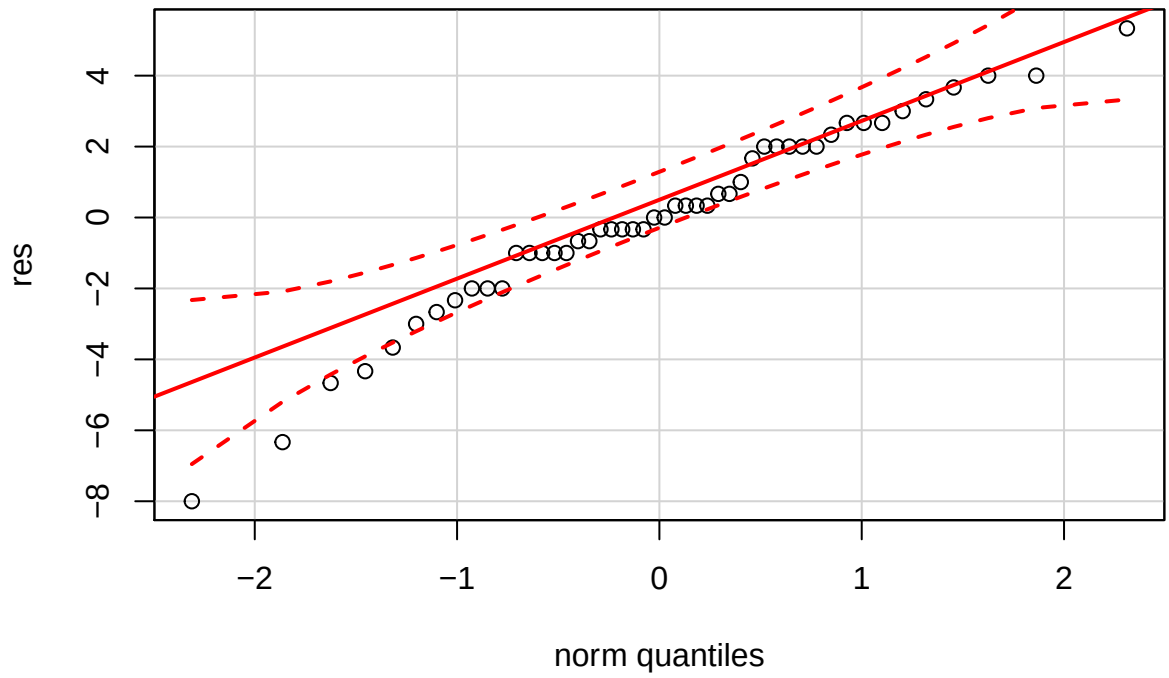
With similar values between all replicates, there's no obvious sign that range or standard devaition of the free height for each run affects variability in free height.

(d) Analyze the residuals from this experiment, and comment on your findings.

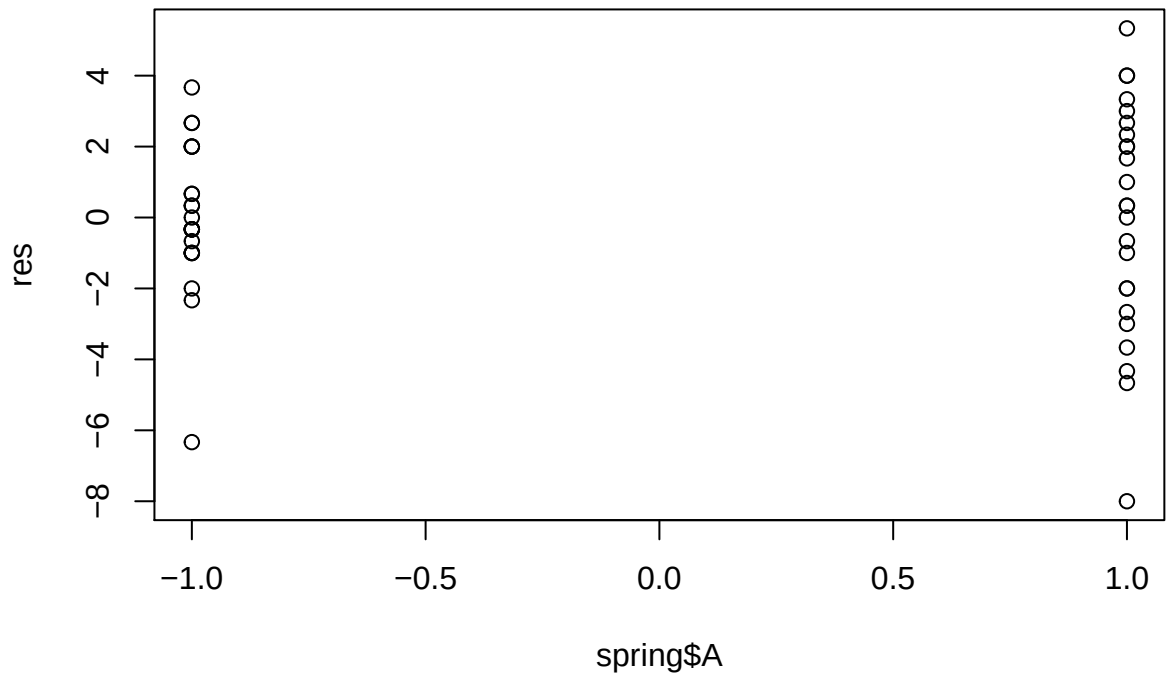
```
# plotting residuals
res <- as.numeric(spring$FH) - fitted(spring.lm)
plot(fitted(spring.lm), res)
```



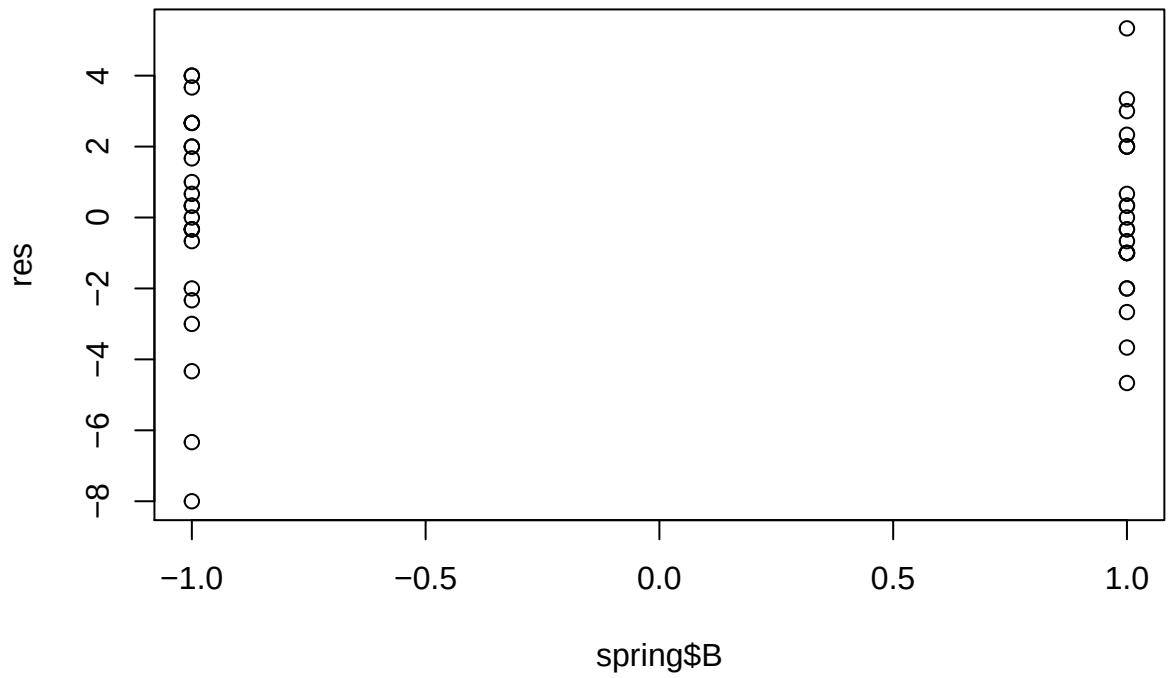

```
qqPlot(res)
```



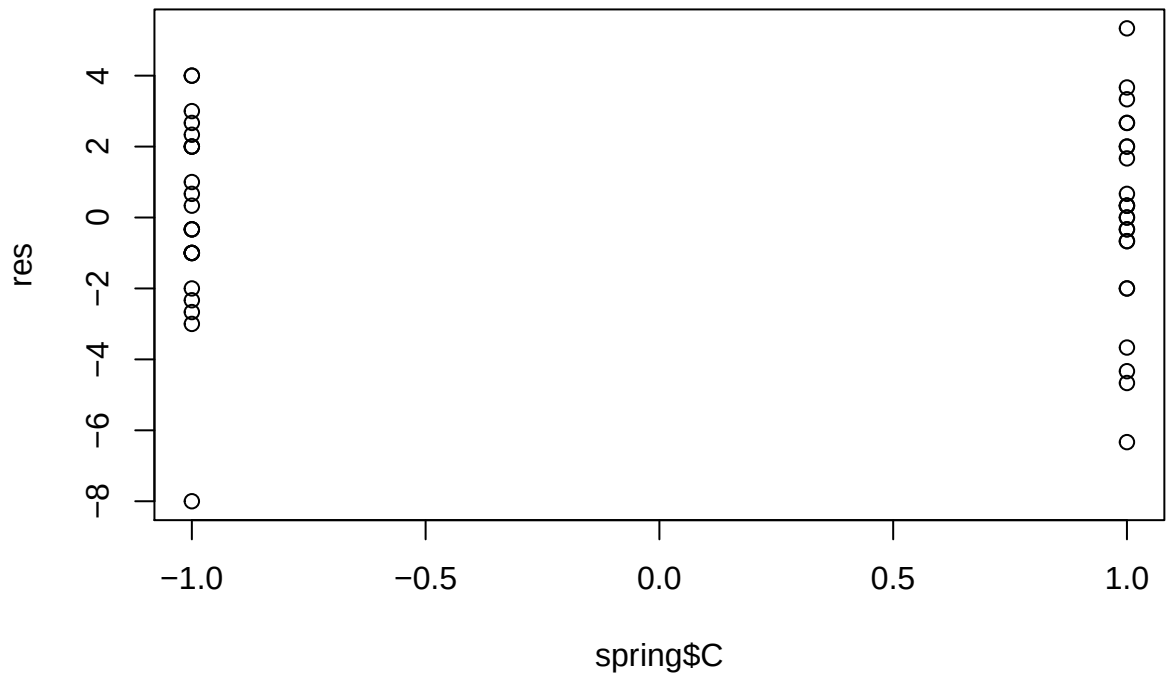

```
plot(spring$A, res)
```



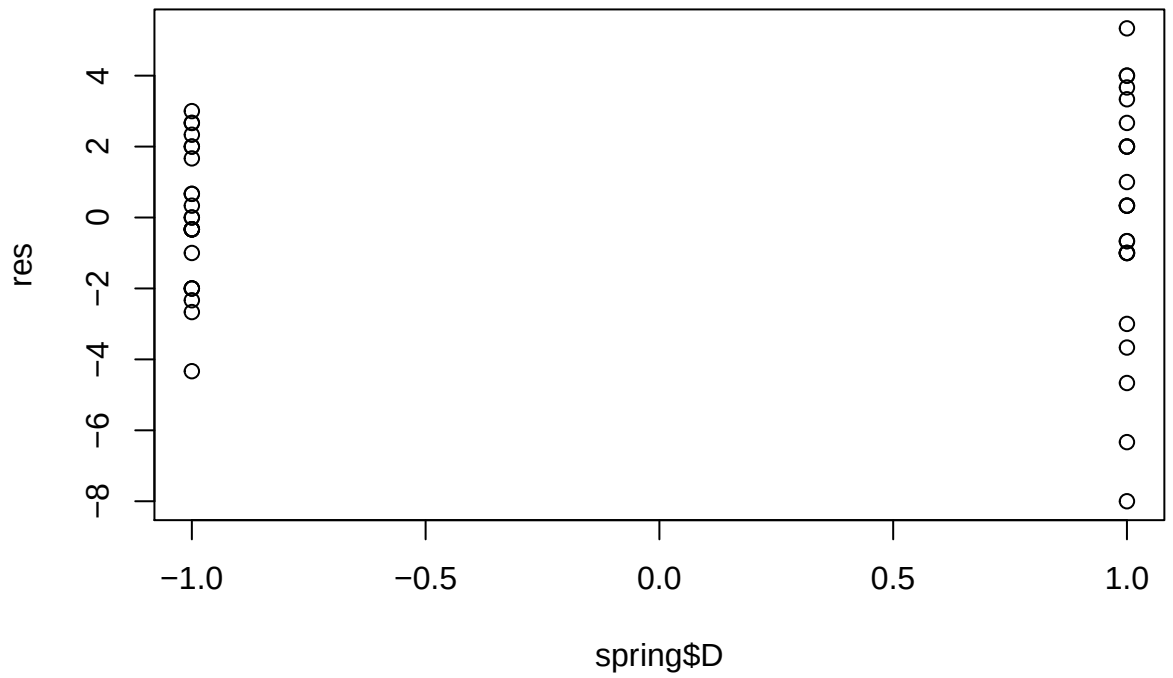

```
plot(spring$B, res)
```



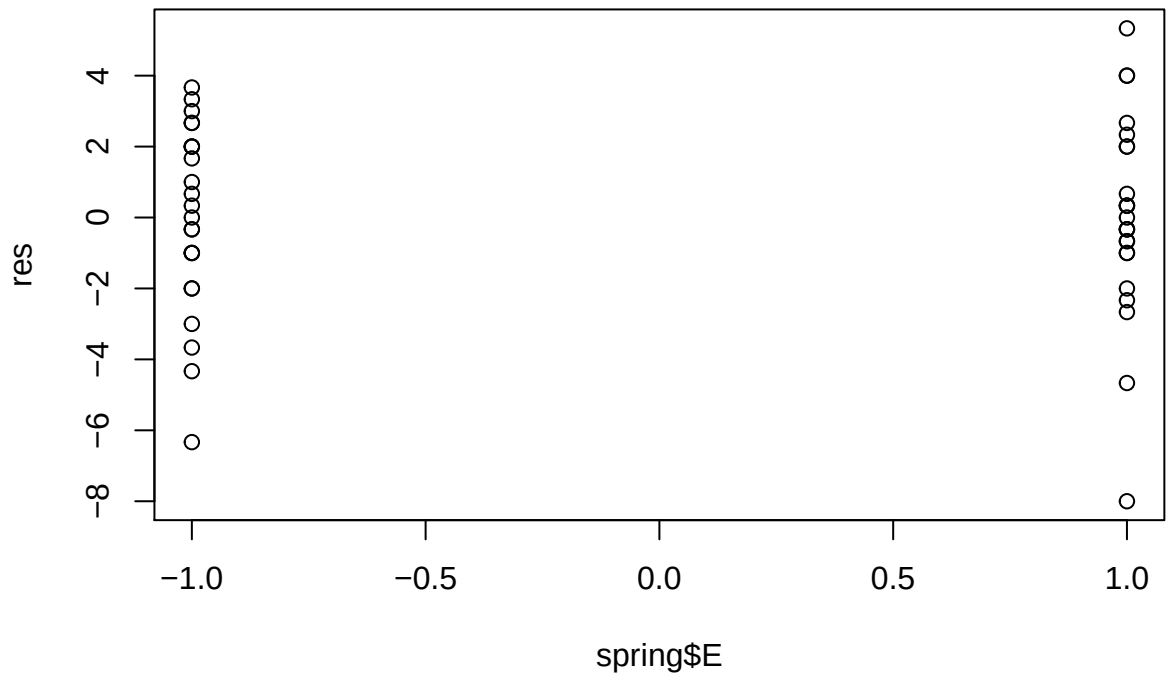

```
plot(spring$C, res)
```



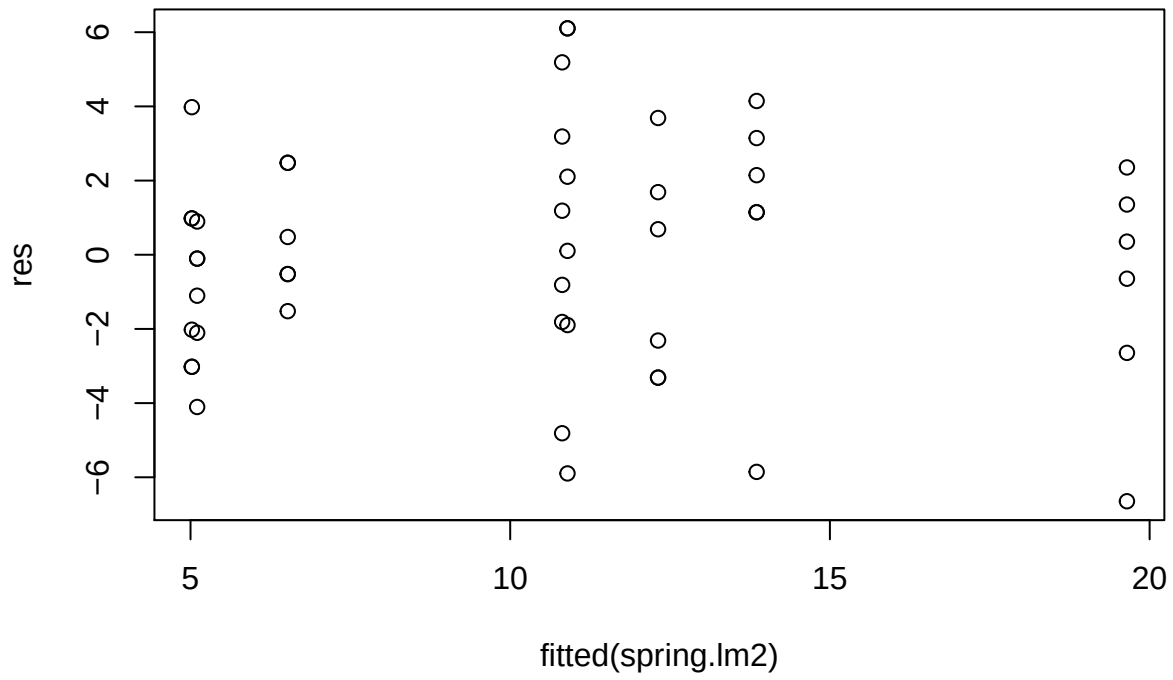

```
plot(spring$D, res)
```



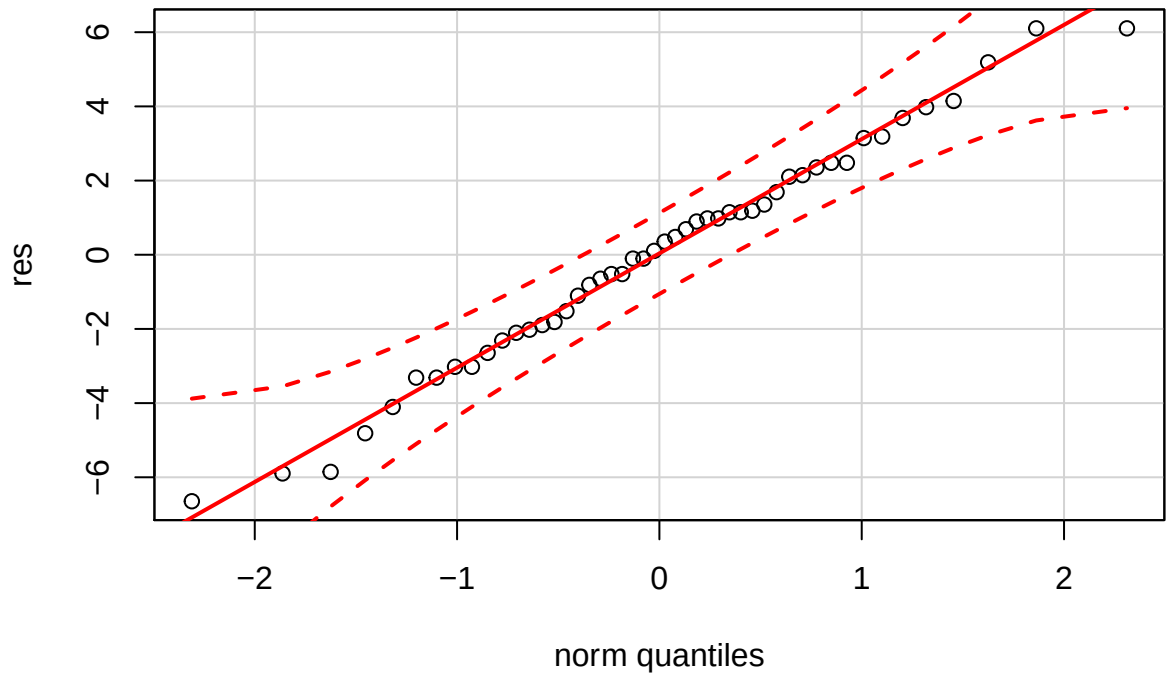

```
plot(spring$E, res)
```



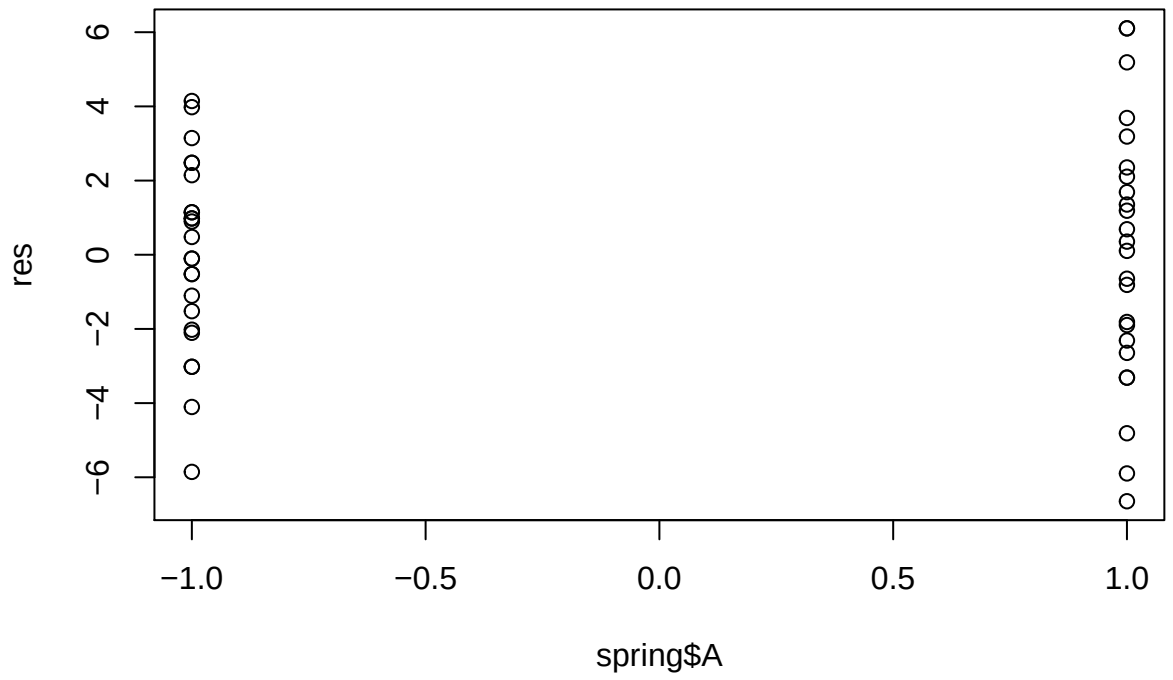

```
# plotting new model residuals
res <- as.numeric(spring$FH) - fitted(spring.lm2)
plot(fitted(spring.lm2), res)
```



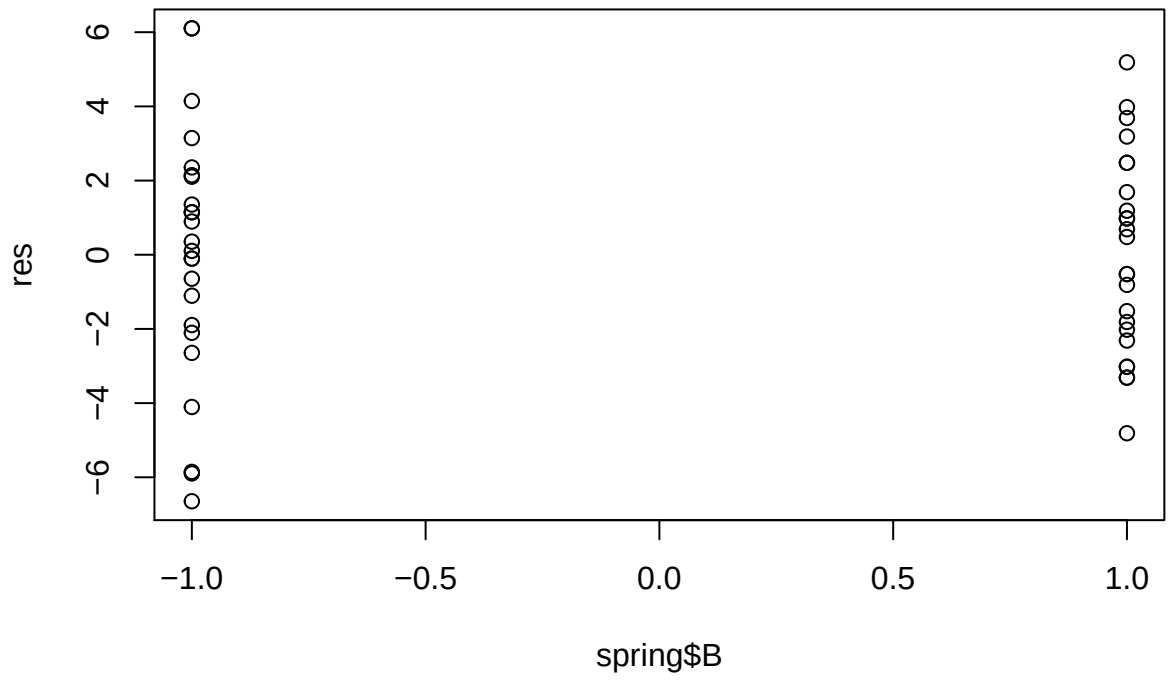

```
qqPlot(res)
```



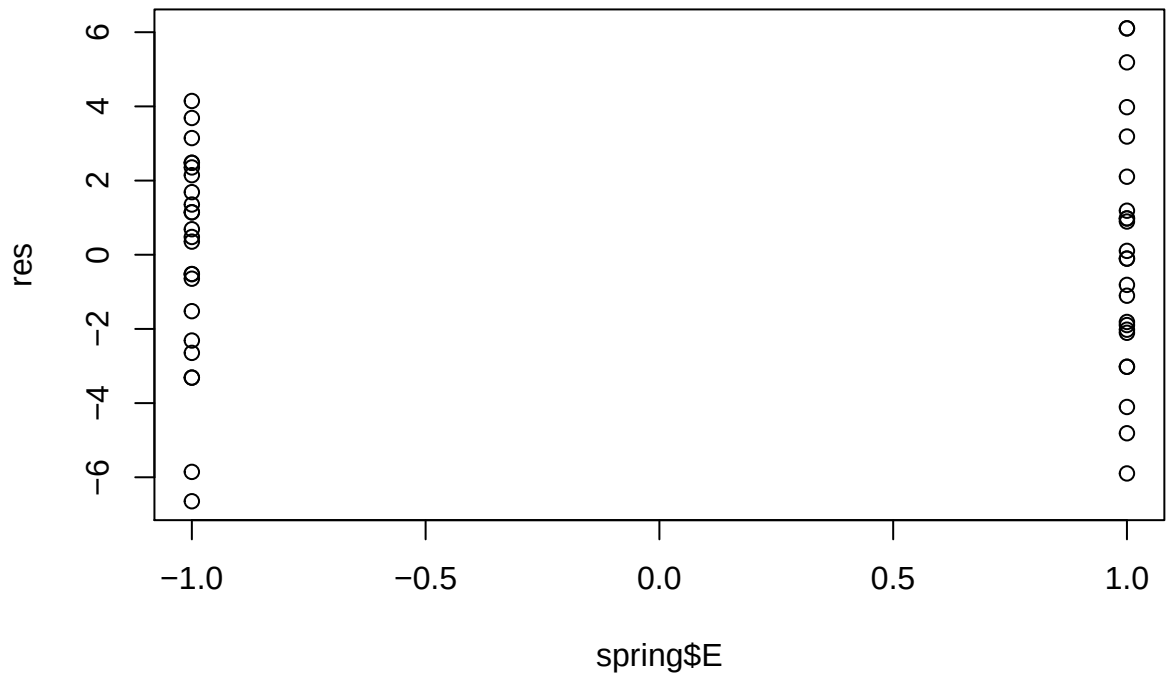

```
plot(spring$A, res)
```




```
plot(spring$B, res)
```




```
plot(spring$E, res)
```



Both models are normally distributed and have residuals who's variances are homogenous. The models are appropriate.

(e) Is this the best possible design for five factors in 16 runs? Specifically, can you find a fractional design for five factors in 16 runs with a higher resolution than this one?

It shouldn't be possible to find a fractional design with a higher resolution that our current one, since it's already a 5 resolution design.

8.12

Consider the leaf spring experiment in Problem 8.7. Suppose that factor E (quench oil temperature) is very difficult to control during manufacturing. Where would you set factors A, B, C, and D to reduce variability in the free height as much as possible regardless of the quench oil temperature used? Note refer to 8.10 instead of 8.7

```
# declaring data
A <- rep(x = c("-", "+"), times = 8)
B <- rep(x = c("-", "+"), each = 2, times = 4)
C <- rep(x = c("-", "+"), each = 4, times = 2)
D <- c("-", "+", "+", "-", "+", "-", "-", "+", "-", "+", "+", "-", "+", "-", "-", "+")
E <- rep(x = c("-", "+"), each = 8)
FH1 <- c(7.78, 8.15, 7.5, 7.59, 7.54, 7.69, 7.56, 7.56, 7.5, 7.88, 7.5, 7.63, 7.32, 7.56, 7.18, 7.81)
FH2 <- c(7.78, 8.18, 7.56, 7.56, 8, 8.09, 7.52, 7.81, 7.25, 7.88, 7.56, 7.75, 7.44, 7.69, 7.18, 7.5)
FH3 <- c(7.81, 7.88, 7.5, 7.75, 7.88, 8.06, 7.44, 7.69, 7.12, 7.44, 7.5, 7.56, 7.44, 7.62, 7.25, 7.59)

# defining coded
```

```r
coded=function(x)
{
  ifelse(x=="+", 1, -1)
}

# decoding data
for (j in 1:5)
  spring[, j]=as.numeric(coded(spring[, j]))
```

8.14

Consider the 2^5 design in Problem 6.24. Suppose that only a one-half fraction could be run. Furthermore, two days were required to take the 16 observations, and it was necessary to confound the 2^{5-1} design in two blocks. Construct the design and analyze the data.

```r
yield = c(7,9,34,55,16,20,40,60,8,10,32,50,18,21,44,61,8,12,35,52,15,22,45,65,6,10,30,53,15,20,41,63)
A <- rep(x = c("-", "+"), times = 16)
B <- rep(x = c("-", "+"), each = 2, times = 8)
C <- rep(x = c("-", "+"), each = 4, times = 4)
D <- rep(x = c("-", "+"), each = 8, times = 2)
E <- rep(x = c("-", "+"), each = 16)

experimento = data.frame(A,B,C,D,E,yield)

coded=function(x) #a function to code variable x
{
  ifelse(x=="+", 1, -1)
}
for (j in 1:5)
  experimento[, j]=as.numeric(coded(experimento[, j]))

fraction.experi=with(experimento, experimento[A * B * C * D * E== 1,])

#linear model
experi.lm = lm(yield ~ A*B*C*D*E, fraction.experi); summary(experi.lm)
```

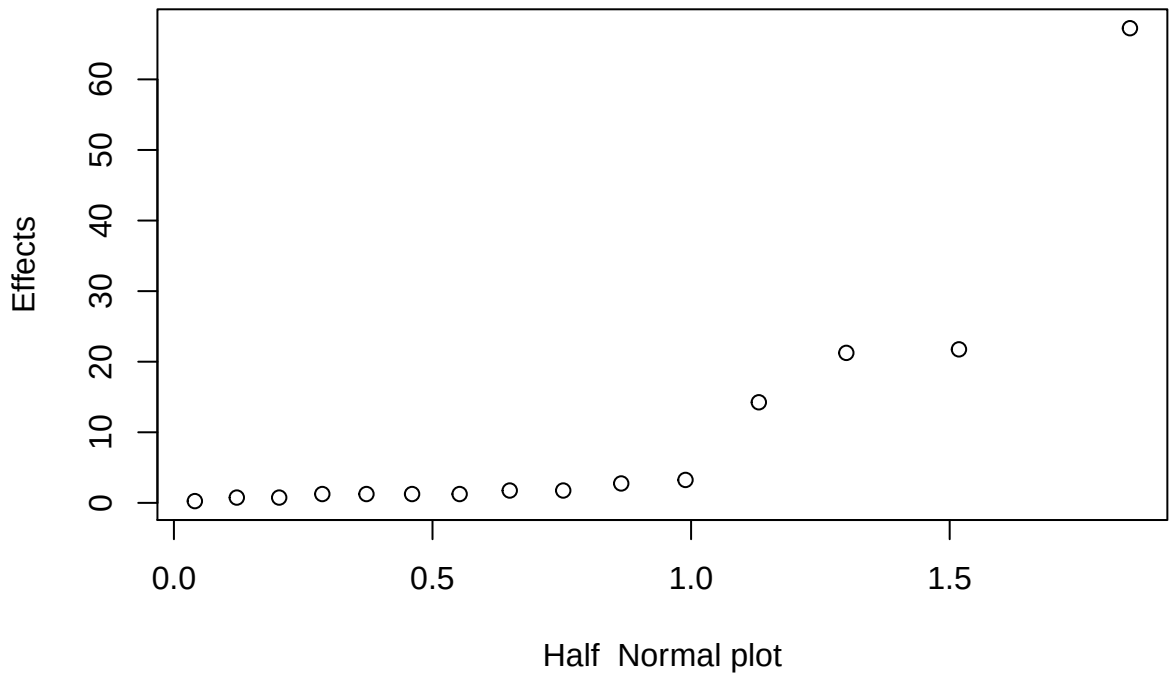
```
##
## Call:
## lm(formula = yield ~ A * B * C * D * E, data = fraction.experi)
##
## Residuals:
## ALL 16 residuals are 0: no residual degrees of freedom!
##
## Coefficients: (16 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  30.4375         NA      NA       NA
## A             5.4375         NA      NA       NA
## B            16.8125         NA      NA       NA
## C             5.3125         NA      NA       NA
## D            -0.3125         NA      NA       NA
## E             0.1875         NA      NA       NA
## A:B           3.5625         NA      NA       NA
## A:C           0.3125         NA      NA       NA
```

```
## B:C           0.4375         NA      NA       NA
## A:D           0.4375         NA      NA       NA
## B:D          -0.1875         NA      NA       NA
## C:D           0.3125         NA      NA       NA
## A:E           0.6875         NA      NA       NA
## B:E           0.0625         NA      NA       NA
## C:E           0.3125         NA      NA       NA
## D:E          -0.8125         NA      NA       NA
## A:B:C             NA         NA      NA       NA
## A:B:D             NA         NA      NA       NA
## A:C:D             NA         NA      NA       NA
## B:C:D             NA         NA      NA       NA
## A:B:E             NA         NA      NA       NA
## A:C:E             NA         NA      NA       NA
## B:C:E             NA         NA      NA       NA
## A:D:E             NA         NA      NA       NA
## B:D:E             NA         NA      NA       NA
## C:D:E             NA         NA      NA       NA
## A:B:C:D           NA         NA      NA       NA
## A:B:C:E           NA         NA      NA       NA
## A:B:D:E           NA         NA      NA       NA
## A:C:D:E           NA         NA      NA       NA
## B:C:D:E           NA         NA      NA       NA
## A:B:C:D:E         NA         NA      NA       NA
##
## Residual standard error: NaN on 0 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:    NaN
## F-statistic:   NaN on 15 and 0 DF,  p-value: NA
```

```r
#alias
alias(experi.lm)
```

```
## Model :
## yield ~ A * B * C * D * E
##
## Complete :
##           (Intercept) A B C D E A:B A:C B:C A:D B:D C:D A:E B:E C:E D:E
## A:B:C     0           0 0 0 0 0 0   0   0   0   0   0   0   0   0   1
## A:B:D     0           0 0 0 0 0 0   0   0   0   0   0   0   0   1   0
## A:C:D     0           0 0 0 0 0 0   0   0   0   0   0   0   1   0   0
## B:C:D     0           0 0 0 0 0 0   0   0   0   0   0   1   0   0   0
## A:B:E     0           0 0 0 0 0 0   0   0   0   0   1   0   0   0   0
## A:C:E     0           0 0 0 0 0 0   0   0   0   1   0   0   0   0   0
## B:C:E     0           0 0 0 0 0 0   0   0   1   0   0   0   0   0   0
## A:D:E     0           0 0 0 0 0 0   0   1   0   0   0   0   0   0   0
## B:D:E     0           0 0 0 0 0 0   1   0   0   0   0   0   0   0   0
## C:D:E     0           0 0 0 0 0 1   0   0   0   0   0   0   0   0   0
## A:B:C:D   0           0 0 0 0 1 0   0   0   0   0   0   0   0   0   0
## A:B:C:E   0           0 0 0 1 0 0   0   0   0   0   0   0   0   0   0
## A:B:D:E   0           0 0 1 0 0 0   0   0   0   0   0   0   0   0   0
## A:C:D:E   0           0 1 0 0 0 0   0   0   0   0   0   0   0   0   0
## B:C:D:E   0           1 0 0 0 0 0   0   0   0   0   0   0   0   0   0
## A:B:C:D:E 1           0 0 0 0 0 0   0   0   0   0   0   0   0   0   0
```

```r
#normal probabiliy plot
qqnorm(aov(yield ~ A * B * C * D * E, fraction.experi), label = TRUE)# A,B,C, and AB
```

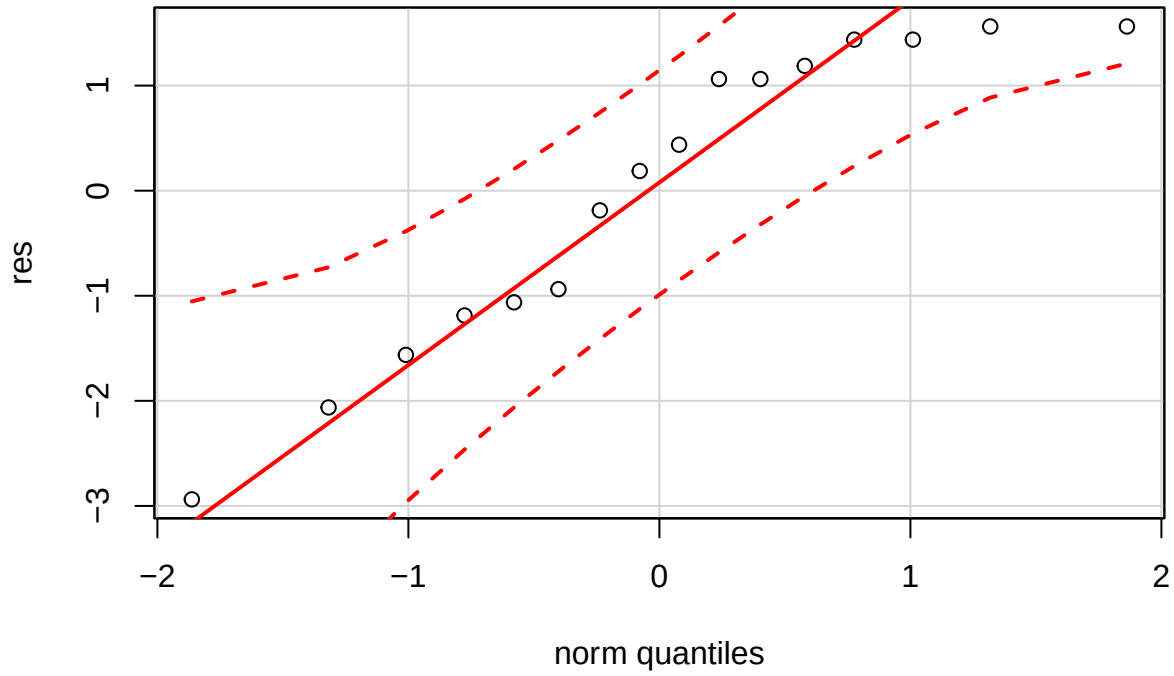



```r
#new linear model
experi.lm2 = lm(yield ~ A + B + C + A*B, fraction.experi); summary(experi.lm2)
```

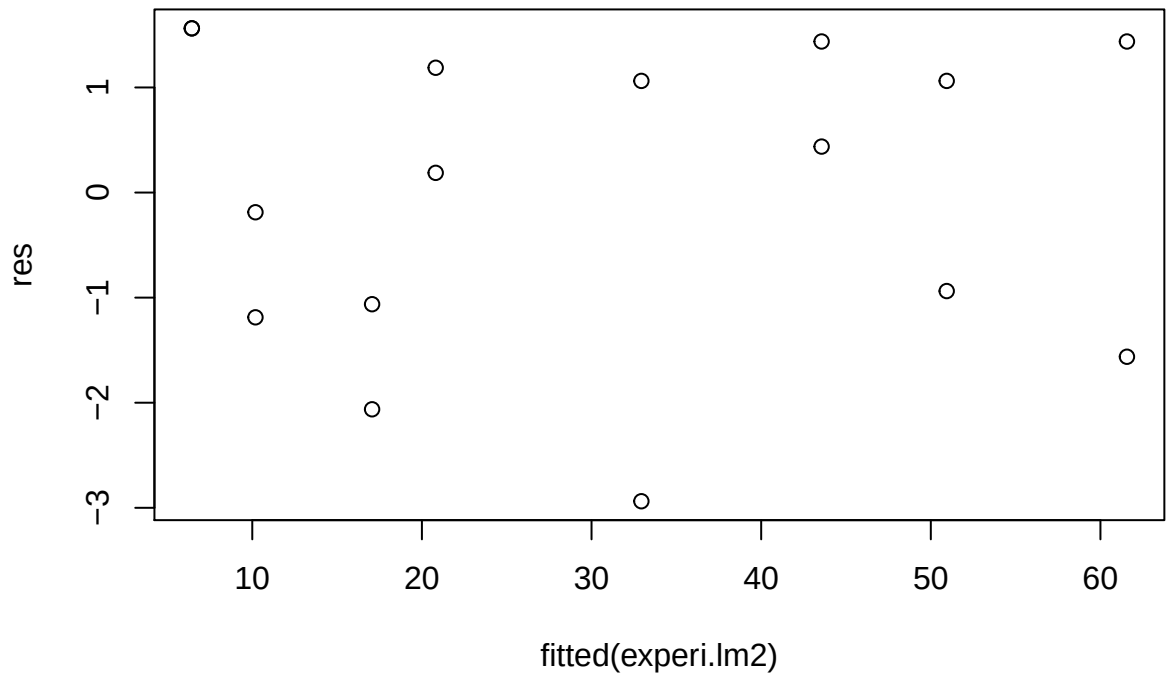
```
##
## Call:
## lm(formula = yield ~ A + B + C + A * B, data = fraction.experi)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9375 -1.0938  0.3125  1.2500  1.5625
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  30.4375     0.4243  71.733 4.80e-16 ***
## A             5.4375     0.4243  12.815 5.90e-08 ***
## B            16.8125     0.4243  39.623 3.21e-13 ***
## C             5.3125     0.4243  12.520 7.51e-08 ***
## A:B           3.5625     0.4243   8.396 4.11e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.697 on 11 degrees of freedom
## Multiple R-squared:  0.9944, Adjusted R-squared:  0.9924
```

```
## F-statistic: 490.4 on 4 and 11 DF,  p-value: 2.606e-12
```

```
res = fraction.experi$yield - fitted(experi.lm2)
qqPlot(res)
```




```
plot(fitted(experi.lm2), res)
```



We begin with our analysis by checking the null model and checking if the main effects are aliased with their interaction effects. We see the aliased chart has good results. We use half normal probability and see that the largest effects are A,B,C, and AB. We fit that into a new refined model, check our p-values, normality and p-values and our results look appropriate. We can state that our refined model is good.

8.51

A 16-run fractional factorial experiment in nine factors was conducted by Chrysler Motors Engineering and described in the article "Sheet Molded Compound Process Improvement," by P. I. Hsieh and D. E. Goodwin (Fourth Symposium on Taguchi Methods, American Supplier Institute, Dearborn, MI, 1986, pp. 13-21). The purpose was to reduce the number of defects in the finish of sheet-molded grill opening panels. The design, and the resulting number of defects, c, observed on each run, is shown in Table P8.14. This is a resolution III fraction with generators E = BD, F = BCD, G = AC, H = ACD, and J = AB.

(a) Find the defining relation and the alias relationships in this design.

(b) Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

```
A <- rep(x = c("-", "+"), times = 8)
B <- rep(x = c("-", "+"), each = 2, times = 4)
C <- rep(x = c("-", "+"), each = 4, times = 2)
D <- rep(x = c("-", "+"), each = 8)
E <- c("+","+","+","+","-","-","-","-","-","-","-","-","+","+","+","+")
F <- c("+","+","-","-","+","+","-","-","-","-","+","+","-","-","+","+")
G <- c("+","+",rep(x =c("-","+"), each = 4), rep(x =c("-"), each = 4),"+","+")
```

```r
H <- c("+","-","+","-","-","+","-","+","+","-","+","-","-","+","-","+")
J <- c("+","-","-","+","+","-","-","+","+","-","-","+","+","-","-","+")
K <- c("-","+","+","-","+","-","-","+","-","+","+","-","+","-","-","+")
FTMOD <- c(1.363,1.555,1.417,1.076,1.363,1.363,1.123,1.259,0.968,1.083,1.556,1.242,1.363,1.130,1.160,1.3
#data
hardata = data.frame(A,B,C,D,E,F,G,H,J,K,FTMOD)

coded=function(x) #a function to code variable x
{
  ifelse(x=="+", 1, -1)
}
for (j in 1:10)
  hardata[, j]=as.numeric(coded(hardata[, j]))

fraction.hardata=with(hardata, hardata[A * B * C * D * E * F * G * H * J * K== 1,])

#linear regression
hardata.lm <- lm(FTMOD ~ A*B*C*D*E*F*G*H*J*K, hardata)

#alias
alias(hardata.lm)
```

```
## Model :
## FTMOD ~ A * B * C * D * E * F * G * H * J * K
##
## Complete :
##                     (Intercept) A B C D E F G H J K A:D A:E B:E A:F F:H
## G:H                 0           0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:J                 0           0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:J                 0           1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:J                 0           0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## D:J                 0           0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## E:J                 0           0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## F:J                 0           0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## G:J                 0           0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## H:J                 0           0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:K                 0           0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:K                 0           0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:K                 0           0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## D:K                 0           0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## E:K                 0           0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## F:K                 0           0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## G:K                 0           1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## H:K                 0           0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## J:K                 0           0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C               0           0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:D               0           0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:D               0           0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:D               0           0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:E               0           0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:E               0           0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:E               0           0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:D:E               0           0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:D:E               0           0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
```

```
## C:D:E              1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F              0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:C:F              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:F              0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:D:F              0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:D:F              1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F              0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:E:F              0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:E:F              0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F              0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F              0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:G              0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:C:G              0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:G              1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:G              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:D:G              0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:D:G              0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:E:G              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:E:G              0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:G              0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## D:E:G              0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:G              0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:F:G              0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:F:G              0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## D:F:G              0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:G              1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:H              0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:H              1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:H              0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:D:H              0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:D:H              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:D:H              0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:E:H              0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:H              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:E:H              0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## D:E:H              0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:H              0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:F:H              0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:F:H              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## D:F:H              0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## E:F:H              0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:G:H              0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:G:H              0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:G:H              0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## D:G:H              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## E:G:H              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## F:G:H              0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:J              1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:J              0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:J              0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:D:J              0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:D:J              0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:D:J              0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:E:J              0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
```

```
## B:E:J             0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:E:J             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## D:E:J             0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:F:J             0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:F:J             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:F:J             0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## D:F:J             0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:J             0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:G:J             0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:G:J             0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## C:G:J             0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:G:J             0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## E:G:J             0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## F:G:J             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:H:J             0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:H:J             0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:H:J             0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:H:J             0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## E:H:J             0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## F:H:J             0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## G:H:J             1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:K             0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:K             0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:K             0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:K             0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:D:K             0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:D:K             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:E:K             0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:E:K             0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:E:K             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## D:E:K             0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:F:K             0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:F:K             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:F:K             0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## D:F:K             0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## E:F:K             0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:G:K             1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:G:K             0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:G:K             0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## D:G:K             0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## E:G:K             0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## F:G:K             0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:H:K             0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:H:K             1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:H:K             0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## D:H:K             0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## E:H:K             0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## F:H:K             0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## G:H:K             0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:J:K             0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:J:K             0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:J:K             1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:J:K             0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## E:J:K             0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
```

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## F:J:K              0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## G:J:K              0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## H:J:K              0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:E            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:D:E            0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:D:E            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E            0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:D:F            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F            0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:D:F            0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:E:F            0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:C:E:F            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:D:E:F            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:F            0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:G            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:G            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:D:G            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:D:G            0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:G            0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:C:E:G            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:E:G            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:G            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:D:E:G            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:G            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:F:G            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:F:G            0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:F:G            0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:D:F:G            0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:D:F:G            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:D:F:G            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:G            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:G            0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:G            0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:G            0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:H            0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:D:H            0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:E:H            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:E:H            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:D:E:H            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:H            0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:D:E:H            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:F:H            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:F:H            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:F:H            0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:D:F:H            0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:D:F:H            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:D:F:H            0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
```

```
## A:E:F:H            0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:H            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:H            0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## D:E:F:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:G:H            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:G:H            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:G:H            0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:D:G:H            0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:D:G:H            0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:D:G:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:E:G:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:E:G:H            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:E:G:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## D:E:G:H            0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:F:G:H            0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:F:G:H            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:F:G:H            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## D:F:G:H            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:G:H            0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:J            0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:J            0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:J            0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:D:J            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:E:J            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:J            0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:C:E:J            0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:D:E:J            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:D:E:J            0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:D:E:J            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:F:J            0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:F:J            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:F:J            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:D:F:J            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:J            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:D:F:J            0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:E:F:J            0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:J            0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:E:F:J            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:J            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:G:J            0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:G:J            1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:G:J            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:D:G:J            0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:D:G:J            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:D:G:J            0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:E:G:J            0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:G:J            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:E:G:J            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## D:E:G:J            0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:G:J            0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:F:G:J            0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:F:G:J            0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## D:F:G:J            0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## E:F:G:J            0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
```

```
## A:B:H:J           0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:H:J           0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:C:H:J           1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:H:J           0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:D:H:J           0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:D:H:J           0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:E:H:J           0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:E:H:J           0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:E:H:J           0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## D:E:H:J           0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:H:J           0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:F:H:J           0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:F:H:J           0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## D:F:H:J           0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:H:J           1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:G:H:J           0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:G:H:J           0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:G:H:J           0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## D:G:H:J           0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## E:G:H:J           0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## F:G:H:J           0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:K           1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:K           0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:D:K           0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:D:K           0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:E:K           0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:K           0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:E:K           0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:D:E:K           0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:K           0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:K           0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:F:K           0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:F:K           0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:F:K           0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:D:F:K           0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:K           0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## C:D:F:K           0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:K           1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:K           0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:E:F:K           0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## D:E:F:K           0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:G:K           0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:G:K           0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:G:K           0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:D:G:K           0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:D:G:K           0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:D:G:K           0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:E:G:K           0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:E:G:K           0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:E:G:K           0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## D:E:G:K           0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:F:G:K           0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:F:G:K           0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:F:G:K           0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
```

```
## D:F:G:K          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## E:F:G:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:H:K          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:H:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:H:K          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:H:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:D:H:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:D:H:K          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:E:H:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:E:H:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:E:H:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## D:E:H:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:F:H:K          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:F:H:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:F:H:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## D:F:H:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:H:K          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:G:H:K          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:G:H:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:G:H:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:G:H:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## E:G:H:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## F:G:H:K          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:J:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:J:K          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:J:K          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:J:K          0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:D:J:K          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:D:J:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:E:J:K          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:E:J:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:E:J:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## D:E:J:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:J:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:F:J:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:F:J:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## D:F:J:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## E:F:J:K          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:G:J:K          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:G:J:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:G:J:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## D:G:J:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## E:G:J:K          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## F:G:J:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:H:J:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:H:J:K          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:H:J:K          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## D:H:J:K          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## E:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## F:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## G:H:J:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:D:E        0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:C:D:F        0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:E:F        0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
```

```
## A:B:D:E:F          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:D:E:F          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:D:E:F          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:G          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:E:G          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:D:E:G          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:G          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:D:E:G          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:G          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:D:F:G          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:D:F:G          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:G          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:G          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:E:F:G          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:C:E:F:G          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:D:E:F:G          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:D:E:F:G          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:D:E:F:G          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:H          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:H          0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:D:E:H          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:H          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:H          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:F:H          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:F:H          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:H          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:H          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:H          1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:H          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:E:F:H          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:D:E:F:H          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:D:E:F:H          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:D:E:F:H          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:G:H          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:G:H          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:G:H          0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:D:G:H          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:E:G:H          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:G:H          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:C:E:G:H          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:D:E:G:H          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:D:E:G:H          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:D:E:G:H          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:F:G:H          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:F:G:H          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:F:G:H          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:D:F:G:H          1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:G:H          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:D:F:G:H          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:E:F:G:H          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:G:H          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## C:E:F:G:H          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:G:H          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:J          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
```

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## A:B:C:E:J          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:J          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:J          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:J          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:J          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:D:F:J          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:J          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:J          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:E:F:J          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:C:E:F:J          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:J          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:D:E:F:J          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:D:E:F:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:D:E:F:J          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:G:J          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:G:J          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:D:G:J          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:G:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:E:G:J          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:E:G:J          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:C:E:G:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:D:E:G:J          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:G:J          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:D:E:G:J          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:F:G:J          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:F:G:J          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:F:G:J          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:D:F:G:J          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:D:F:G:J          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:D:F:G:J          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:E:F:G:J          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:G:J          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:G:J          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## D:E:F:G:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:H:J          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:H:J          0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:D:H:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:D:H:J          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:H:J          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:C:E:H:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:E:H:J          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:D:E:H:J          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:D:E:H:J          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:H:J          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:F:H:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:F:H:J          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:F:H:J          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:D:F:H:J          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:D:F:H:J          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:D:F:H:J          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:H:J          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:H:J          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:H:J          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:H:J          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
```

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## A:B:G:H:J          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:C:G:H:J          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:G:H:J          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:D:G:H:J          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:D:G:H:J          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:D:G:H:J          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:E:G:H:J          0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:E:G:H:J          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:E:G:H:J          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## D:E:G:H:J          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:G:H:J          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:F:G:H:J          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:F:G:H:J          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## D:F:G:H:J          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:G:H:J          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:D:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:C:D:E:K          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:F:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:D:F:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:C:D:F:K          1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:K          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:E:F:K          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:K          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:D:E:F:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:F:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:D:E:F:K          0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:G:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:D:G:K          0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:D:G:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:C:D:G:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:E:G:K          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:E:G:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:G:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:D:E:G:K          0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:G:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## C:D:E:G:K          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:G:K          0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:F:G:K          0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:F:G:K          0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:D:F:G:K          0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:G:K          0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:G:K          0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:E:F:G:K          0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:E:F:G:K          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:E:F:G:K          0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## D:E:F:G:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:H:K          0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:H:K          0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:C:D:H:K          0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:D:H:K          0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
```

```
## A:B:E:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:E:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:E:H:K          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:H:K          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:D:E:H:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:H:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:F:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:F:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:D:F:H:K          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:H:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:H:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:H:K          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:E:F:H:K          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:E:F:H:K          1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:H:K          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:G:H:K          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:G:H:K          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:G:H:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:G:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:D:G:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:D:G:H:K          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:G:H:K          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:E:G:H:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:E:G:H:K          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## D:E:G:H:K          1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:F:G:H:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:F:G:H:K          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:F:G:H:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## D:F:G:H:K          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## E:F:G:H:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:J:K          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:D:J:K          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:D:J:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:E:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:E:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:E:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:D:E:J:K          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:J:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:J:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:F:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:F:J:K          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:F:J:K          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:D:F:J:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:J:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:J:K          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:E:F:J:K          1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:J:K          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## D:E:F:J:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:G:J:K          0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:G:J:K          0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:G:J:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
```

```
## A:D:G:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:D:G:J:K          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:G:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:E:G:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:E:G:J:K          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:E:G:J:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## D:E:G:J:K          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:F:G:J:K          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:F:G:J:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:F:G:J:K          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## D:F:G:J:K          1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F:G:J:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:H:J:K          0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:H:J:K          0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:H:J:K          0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:D:H:J:K          0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:D:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:E:H:J:K          0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:E:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:E:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## D:E:H:J:K          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:F:H:J:K          0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:F:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:F:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## D:F:H:J:K          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## E:F:H:J:K          0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:G:H:J:K          0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:G:H:J:K          0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:G:H:J:K          0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## D:G:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## E:G:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## F:G:H:J:K          0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:E:F        0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:D:E:G        0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:D:F:G        0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:E:F:G        0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:D:E:F:G        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:D:E:F:G        0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:D:E:F:G        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:D:E:H        0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:H        1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:H        0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:H        0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:F:H        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:D:E:F:H        0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:G:H        0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:G:H        0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:G:H        0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:G:H        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:G:H        0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:G:H        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:D:F:G:H        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:G:H        0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
```

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## B:C:D:F:G:H        0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:E:F:G:H        0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:C:E:F:G:H        1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:G:H        0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:D:E:F:G:H        0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:D:E:F:G:H        0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:D:E:F:G:H        0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:D:E:J        1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:J        0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:E:F:J        0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:J        0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:D:E:F:J        0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:F:J        0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:D:G:J        0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:E:G:J        0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:D:E:G:J        0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:G:J        0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:G:J        0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:F:G:J        0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:F:G:J        0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:G:J        0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:G:J        0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:G:J        1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:G:J        0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:C:E:F:G:J        0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:D:E:F:G:J        0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:D:E:F:G:J        0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:D:E:F:G:J        0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:D:H:J        0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:E:H:J        0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:D:E:H:J        0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:H:J        0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:D:E:H:J        0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:H:J        0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:D:F:H:J        0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:D:F:H:J        0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:H:J        0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:H:J        0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:C:E:F:H:J        0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:E:F:H:J        0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:D:E:F:H:J        0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:D:E:F:H:J        0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## C:D:E:F:H:J        0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:G:H:J        0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:D:G:H:J        0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:D:G:H:J        0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:D:G:H:J        0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:E:G:H:J        0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:E:G:H:J        0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:E:G:H:J        0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:D:E:G:H:J        0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:D:E:G:H:J        0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:D:E:G:H:J        1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:G:H:J        0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
```

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## A:C:F:G:H:J        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:F:G:H:J        0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:D:F:G:H:J        0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:D:F:G:H:J        1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:G:H:J        0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:E:F:G:H:J        0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:E:F:G:H:J        0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:G:H:J        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:G:H:J        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:D:E:K        0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:K        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:K        0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:K        0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:F:K        0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:F:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:D:G:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:E:G:K        0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:G:K        0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:D:E:G:K        1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:G:K        0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:F:G:K        0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:F:G:K        1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:G:K        0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:D:F:G:K        0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:E:F:G:K        0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:G:K        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:G:K        0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:G:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:D:E:F:G:K        0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:D:E:F:G:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:D:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:E:H:K        0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:D:E:H:K        0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:C:D:E:H:K        0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:D:E:H:K        1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:D:F:H:K        0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:D:F:H:K        0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:C:D:F:H:K        0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:E:F:H:K        0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:E:F:H:K        0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:H:K        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:D:E:F:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:D:E:F:H:K        0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:G:H:K        0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:G:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:D:G:H:K        0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:D:G:H:K        0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:E:G:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:E:G:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:E:G:H:K        0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:D:E:G:H:K        0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:G:H:K        0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
```

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## C:D:E:G:H:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:G:H:K         0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:F:G:H:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:F:G:H:K         0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:D:F:G:H:K         0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:D:F:G:H:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:G:H:K         0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:G:H:K         0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:E:F:G:H:K         1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:E:F:G:H:K         0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## D:E:F:G:H:K         0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:D:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:E:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:D:E:J:K         0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:C:D:E:J:K         0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:D:E:J:K         0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:F:J:K         0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:D:F:J:K         0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:D:F:J:K         0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:C:D:F:J:K         1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:J:K         0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:J:K         0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:E:F:J:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:D:E:F:J:K         0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:F:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:G:J:K         0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:G:J:K         0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:C:D:G:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:D:G:J:K         0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:E:G:J:K         0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:E:G:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:E:G:J:K         0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:G:J:K         0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:D:E:G:J:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:G:J:K         0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:G:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:F:G:J:K         0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:F:G:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:D:F:G:J:K         0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:G:J:K         0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:G:J:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:G:J:K         0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:E:F:G:J:K         0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## C:E:F:G:J:K         1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:E:F:G:J:K         0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:H:J:K         0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:D:H:J:K         0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:D:H:J:K         0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:C:D:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:E:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:E:H:J:K         0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:D:E:H:J:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
```

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## B:D:E:H:J:K         0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## C:D:E:H:J:K         0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:F:H:J:K         0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:F:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:F:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:D:F:H:J:K         0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:H:J:K         0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:F:H:J:K         0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:E:F:H:J:K         0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:E:F:H:J:K         0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## C:E:F:H:J:K         0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## D:E:F:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:G:H:J:K         0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:G:H:J:K         0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:G:H:J:K         0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:D:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:D:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:E:G:H:J:K         0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:E:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## C:E:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## D:E:G:H:J:K         0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:F:G:H:J:K         0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:F:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## C:F:G:H:J:K         0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## D:F:G:H:J:K         0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## E:F:G:H:J:K         0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:G       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:D:E:F:H       0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:G:H       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:G:H       0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:E:F:G:H       0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:G:H       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:D:E:F:G:H       0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:F:G:H       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:D:E:F:J       0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:D:E:G:J       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:D:F:G:J       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:G:J       0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:G:J       0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:F:G:J       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:D:E:F:G:J       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:E:H:J       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:D:F:H:J       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:C:E:F:H:J       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:D:E:F:H:J       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:C:D:E:F:H:J       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:C:D:E:F:H:J       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:D:G:H:J       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:E:G:H:J       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:D:E:G:H:J       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:D:E:G:H:J       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:G:H:J       0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:G:H:J       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
```

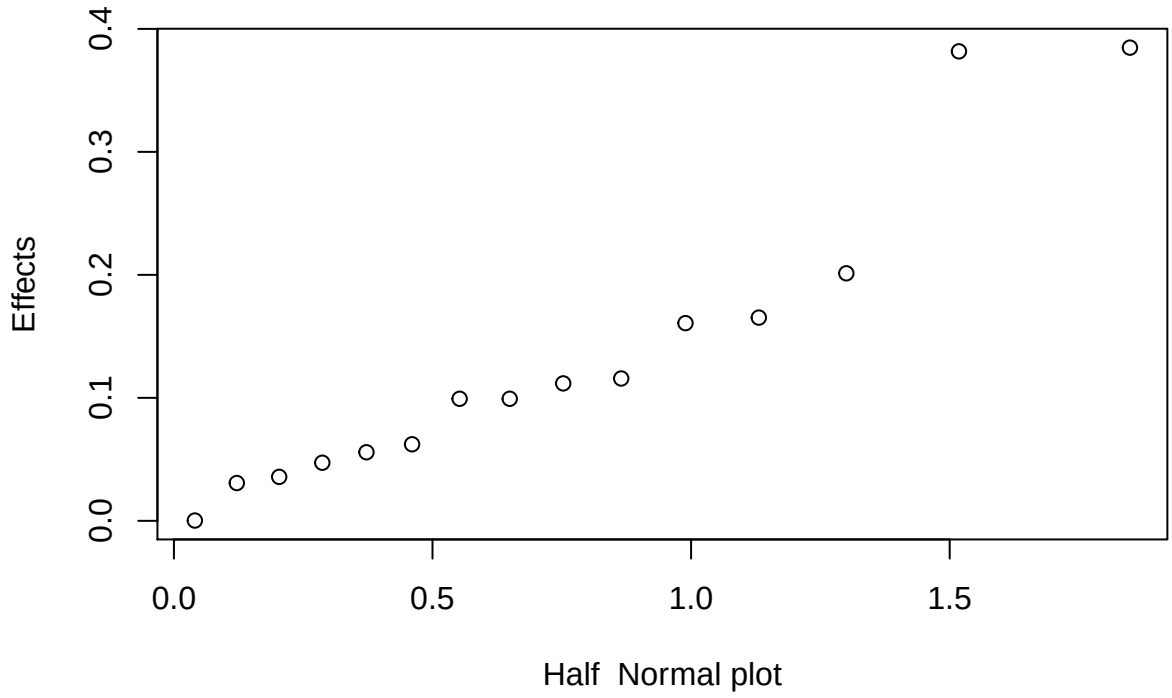
```
## A:B:D:F:G:H:J       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:G:H:J       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:D:F:G:H:J       0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:G:H:J       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:E:F:G:H:J       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:C:E:F:G:H:J       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:G:H:J       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:D:E:F:G:H:J       0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:D:E:F:G:H:J       0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:D:E:G:K       0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:G:K       0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:G:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:G:K       0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:F:G:K       0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:D:E:F:G:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:D:E:H:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:H:K       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:E:F:H:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:D:E:F:H:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:D:E:F:H:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:D:E:F:H:K       0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:D:G:H:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:E:G:H:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:D:E:G:H:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:C:D:E:G:H:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:D:E:G:H:K       0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:F:G:H:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:D:F:G:H:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:D:F:G:H:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:C:D:F:G:H:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:E:F:G:H:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:G:H:K       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:C:E:F:G:H:K       0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:G:H:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:D:E:F:G:H:K       0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:F:G:H:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:D:E:J:K       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:D:F:J:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:J:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:E:F:J:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:C:D:E:F:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:D:E:F:J:K       0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:G:J:K       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:E:G:J:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:D:E:G:J:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:D:E:G:J:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## B:C:D:E:G:J:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:F:G:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:D:F:G:J:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:C:D:F:G:J:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## B:C:D:F:G:J:K       0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:E:F:G:J:K       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:C:E:F:G:J:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
```

```
## B:C:E:F:G:J:K       0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:G:J:K       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## B:D:E:F:G:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## C:D:E:F:G:J:K       0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B:C:E:H:J:K       0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:D:E:H:J:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C:D:E:H:J:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:H:J:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:C:F:H:J:K       0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:D:F:H:J:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:F:H:J:K       0          0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:C:D:F:H:J:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:E:F:H:J:K       0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:H:J:K       0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:H:J:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:D:E:F:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:D:E:F:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## C:D:E:F:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:G:H:J:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:G:H:J:K       0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:D:G:H:J:K       0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:D:G:H:J:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:E:G:H:J:K       0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:G:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## B:C:E:G:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:D:E:G:H:J:K       0          0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:G:H:J:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## C:D:E:G:H:J:K       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:F:G:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:C:F:G:H:J:K       0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:F:G:H:J:K       0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:D:F:G:H:J:K       0          0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:F:G:H:J:K       0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## C:D:F:G:H:J:K       0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:E:F:G:H:J:K       1          0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:E:F:G:H:J:K       0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:E:F:G:H:J:K       0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## D:E:F:G:H:J:K       0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:D:E:F:G:H     0          0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:G:J     0          0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:H:J     0          0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:D:E:G:H:J     0          0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:C:D:F:G:H:J     0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:C:E:F:G:H:J     0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:G:H:J     0          0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:C:D:E:F:G:H:J     0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## B:C:D:E:F:G:H:J     0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:G:K     0          0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:H:K     0          0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:D:E:G:H:K     0          0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:D:F:G:H:K     0          1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:G:H:K     0          0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:E:F:G:H:K     0          0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
```

```
## A:C:D:E:F:G:H:K     0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## B:C:D:E:F:G:H:K     0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:F:J:K     0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:E:G:J:K     0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:G:J:K     0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:B:C:E:F:G:J:K     0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:D:E:F:G:J:K     0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:C:D:E:F:G:J:K     0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## B:C:D:E:F:G:J:K     0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:H:J:K     0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:H:J:K     0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:H:J:K     1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:H:J:K     0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:F:H:J:K     0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:F:H:J:K     0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## A:B:C:D:G:H:J:K     0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:G:H:J:K     0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:B:D:E:G:H:J:K     1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:G:H:J:K     0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:C:D:E:G:H:J:K     0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## A:B:C:F:G:H:J:K     0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:B:D:F:G:H:J:K     0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:C:D:F:G:H:J:K     1          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:F:G:H:J:K     0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:B:E:F:G:H:J:K     0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:C:E:F:G:H:J:K     0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:C:E:F:G:H:J:K     0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## A:D:E:F:G:H:J:K     0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## B:D:E:F:G:H:J:K     0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## C:D:E:F:G:H:J:K     0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:E:F:G:H:J   0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
## A:B:C:D:E:F:G:H:K   0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## A:B:C:D:E:F:G:J:K   0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   1   0
## A:B:C:D:E:F:H:J:K   0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:E:G:H:J:K   0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:D:F:G:H:J:K   0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## A:B:C:E:F:G:H:J:K   0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:B:D:E:F:G:H:J:K   0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## A:C:D:E:F:G:H:J:K   0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## B:C:D:E:F:G:H:J:K   0          0 0 0 0 0 0 0 0 0 0 0 0   0   0   0   1
## A:B:C:D:E:F:G:H:J:K 0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## A:B                 0          0 0 0 0 0 0 0 0 1 0 0 0   0   0   0   0
## A:C                 0          0 0 0 0 0 0 0 1 0 0 0 0   0   0   0   0
## B:C                 0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## B:D                 0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## C:D                 0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## C:E                 0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## D:E                 0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:F                 0          0 0 0 1 0 0 0 0 0 0 0 0   0   0   0   0
## C:F                 0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## D:F                 0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## E:F                 0          0 0 0 0 0 0 1 0 0 0 0 0   0   0   0   0
## A:G                 0          0 0 0 0 0 0 0 0 0 0 1 0   0   0   0   0
## B:G                 0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
```

```
## C:G                0          0 1 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:G                0          0 0 0 0 0 0 0 0 0 0 0 0   0   1   0   0
## E:G                0          0 0 0 0 0 1 0 0 0 0 0 0   0   0   0   0
## F:G                0          0 0 0 0 1 0 0 0 0 0 0 0   0   0   0   0
## A:H                0          0 0 1 0 0 0 0 0 0 0 0 0   0   0   0   0
## B:H                0          0 0 0 0 0 0 0 0 0 1 0 0   0   0   0   0
## C:H                0          1 0 0 0 0 0 0 0 0 0 0 0   0   0   0   0
## D:H                0          0 0 0 0 0 0 0 0 0 0 0 0   1   0   0   0
## E:H                0          0 0 0 0 0 0 0 0 0 0 0 1   0   0   0   0
```

```
qqnorm(aov(FTMOD ~ A*B*C*D*E*F*G*H*J*K, hardata), label = TRUE)#K and F
```



(c) Fit an appropriate model using the factors identified in part (b) above.

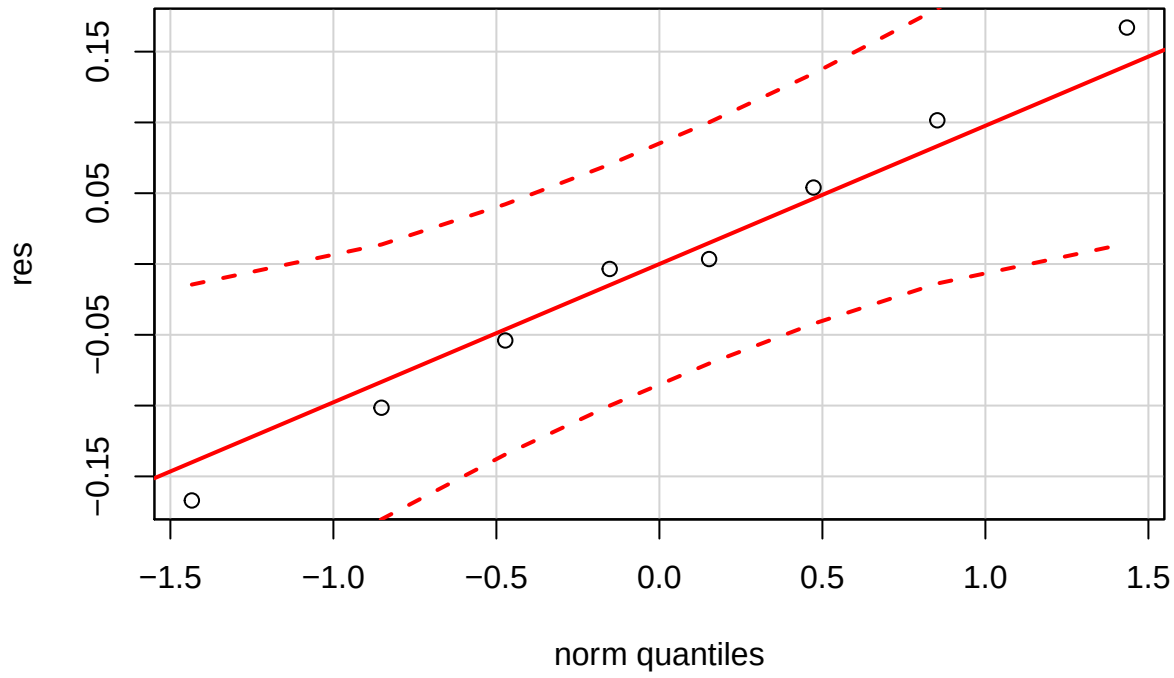
```
#refined model
hardata.lm2 <- lm(FTMOD ~ F * K, fraction.hardata); summary(hardata.lm2)
```

```
##
## Call:
## lm(formula = FTMOD ~ F * K, data = fraction.hardata)
##
## Residuals:
##       3       4       5       6       9      10      15      16
##  0.1670  0.0540  0.0035  0.1015 -0.0540 -0.1670 -0.1015 -0.0035
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.22325    0.05069  24.130 1.75e-05 ***
```

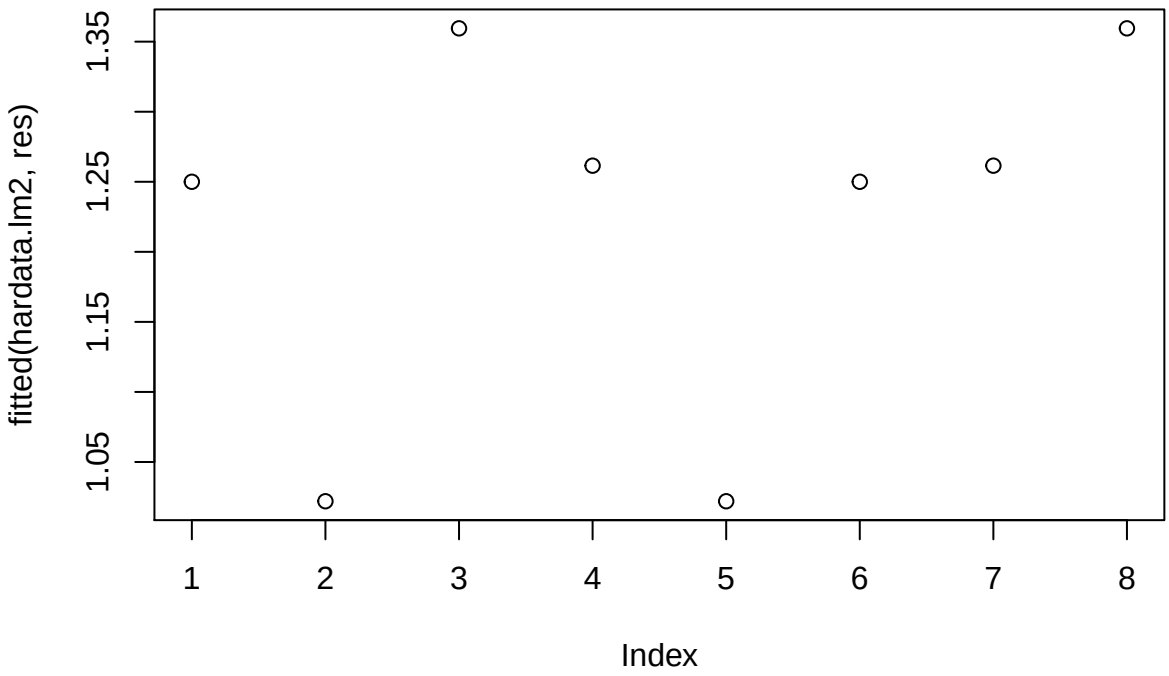
```
## F            0.08725    0.05069   1.721    0.160
## K            0.08150    0.05069   1.608    0.183
## F:K         -0.03250    0.05069  -0.641    0.556
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1434 on 4 degrees of freedom
## Multiple R-squared:  0.5983, Adjusted R-squared:  0.297
## F-statistic: 1.986 on 3 and 4 DF,  p-value: 0.2584
```

(d) Plot the residuals from this model versus the predicted number of defects. Also, prepare a normal probability plot of the residuals. Comment on the adequacy of these plots.

```
#residual analysis
res <- fraction.hardata$FTMOD - fitted(hardata.lm2)
qqPlot(res)
```




```
plot(fitted(hardata.lm2, res))
```



(e) In part (d) you should have noticed an indication that the variance of the response is not constant. (Considering that the response is a count, you should have expected this.) The previous table also shows a transformation on c, the square root, that is a widely used variance stabilizing transformation for count data. (Refer to the discussion of variance stabilizing transformations in Chapter 3.) Repeat parts (a) through (d) using the transformed response and comment on your results. Specifically, are the residual plots improved?

(f) There is a modification to the square root transformation, proposed by Freeman and Tukey ("Transformations Related to the Angular and the Square Root," Annals of Mathematical Statistics, Vol. 21, 1950, pp. 607-611) that improves its performance. FandT's modification to the square root transformation $\frac{\sqrt{c}+\sqrt{c+1}}{2}$ is Rework parts (a) through (d) using this transformation and comment on the results. (For an interesting discussion and analysis of this experiment, refer to "Analysis of Factorial Experiments with Defects or Defectives as the Response," by S. Bisgaard and H. T. Fuller, Quality Engineering, Vol. 7, 1994-95, pp. 429-443.)