Chapter 8 (Edition 8): 8.1, 8.10, 8.12, 8.14, 8.51

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May 10, 2018

```r
# loading libraries
library(gplots)
```

```
## Warning: package 'gplots' was built under R version 3.4.4

##
## Attaching package: 'gplots'

## The following object is masked from 'package:stats':
##
##     lowess
```

8.1

Suppose that in the chemical process development experiment described in Problem 6.7, it was only possible to run a one-half fraction of the 2^4 design. Construct the design and perform the statistical analysis, using the data from replicate I.

```r
#chemical data from 6.7
rep1 = c(90,74,81,83,77,81,88,73)
rep2 = c(93,78,85,80,78,80,82,70)
rep3 = c(98,72,87,85,99,79,87,80)
rep4 = c(95,76,83,86,90,75,84,80)
chemical = data.frame(rep1,rep2,rep3,rep4)

#2^(4-1) with I=ABCD (Resolution IV)
fraction.chem=with(chemical, chemical[rep1 * rep2 * rep3 * rep4 == 1,])
```

8.10

An article by J. J. Pignatiello Jr. and J. S. Ramberg in the Journal of Quality Technology (Vol. 17, 1985, pp. 198-206) describes the use of a replicated fractional factorial to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The data are shown in Table P8.1

(a) Write out the alias structure for this design. What is the resolution of this design?

We design a half factorial design with resolution V and generator ABCDE. The alias structure for this design is shown below:

$[A] \rightarrow A+BCDE$ $[B] \rightarrow B+ACDE$ $[C] \rightarrow C+ABDE$ $[D] \rightarrow D+ABCE$ $[E] \rightarrow E+ABCD$ $[AB] \rightarrow AB+CDE$ $[AC] \rightarrow AC+BDE$ $[AD] \rightarrow AD+BCE$ $[AE] \rightarrow AE+BCD$ $[BC] \rightarrow BC+ADE$ $[BD] \rightarrow BD+ACE$ $[BE] \rightarrow BE+ACD$ $[CD] \rightarrow CD+ABE$ $[CE] \rightarrow CE+ABD$ $[DE] \rightarrow DE+ABC$

(b) Analyze the data. What factors influence the mean free height?

```r
# declaring data
A <- rep(x = c("-", "+"), times = 8)
```

```r
B <- rep(x = c("-", "+"), each = 2, times = 4)
C <- rep(x = c("-", "+"), each = 4, times = 2)
D <- c("-", "+", "+", "-", "+", "-", "-", "+", "-", "+", "+", "-", "+", "-", "-", "+")
E <- rep(x = c("-", "+"), each = 8)
FH1 <- c(7.78, 8.15, 7.5, 7.59, 7.54, 7.69, 7.56, 7.56, 7.5, 7.88, 7.5, 7.63, 7.32, 7.56, 7.18, 7.81)
FH2 <- c(7.78, 8.18, 7.56, 7.56, 8, 8.09, 7.52, 7.81, 7.25, 7.88, 7.56, 7.75, 7.44, 7.69, 7.18, 7.5)
FH3 <- c(7.81, 7.88, 7.5, 7.75, 7.88, 8.06, 7.44, 7.69, 7.12, 7.44, 7.5, 7.56, 7.44, 7.62, 7.25, 7.59)

# creating table
A <- c(A, A, A)
B <- c(B, B, B)
C <- c(C, C, C)
D <- c(D, D, D)
E <- c(E, E, E)
FH <- as.numeric(c(FH1, FH2, FH3))

spring <- data.frame(cbind(A, B, C, D, E, FH))

# defining coded
coded=function(x)
{
  ifelse(x=="+", 1, -1)
}

# decoding data
for (j in 1:5)
  spring[, j]=as.numeric(coded(spring[, j]))

# defining fraction
fraction <- with(spring, spring[A * B * C * D * E == 1,])

# linear regression
summary(lm(as.numeric(FH) ~ A * B * C * D * E, fraction))
```

```
##
## Call:
## lm(formula = as.numeric(FH) ~ A * B * C * D * E, data = fraction)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.0000 -1.0000 -0.1667  2.0000  5.3333
##
## Coefficients: (24 not defined because of singularities)
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.95833    0.69846  11.394 4.34e-09 ***
## A            3.62500    0.69846   5.190 8.94e-05 ***
## B           -0.04167    0.69846  -0.060    0.953
## C           -0.79167    0.69846  -1.133    0.274
## D            0.87500    0.69846   1.253    0.228
## E                 NA         NA      NA       NA
## A:B         -0.37500    0.69846  -0.537    0.599
## A:C          0.04167    0.69846   0.060    0.953
## B:C         -0.62500    0.69846  -0.895    0.384
## A:D               NA         NA      NA       NA
```

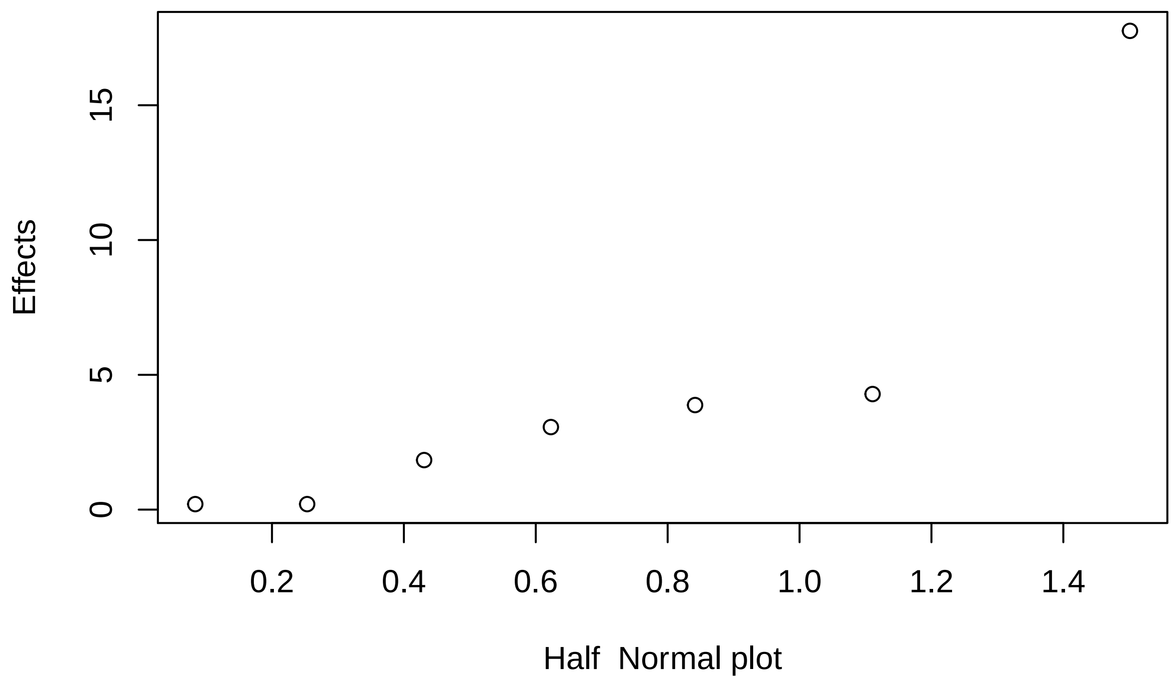
```
## B:D                NA          NA      NA       NA
## C:D                NA          NA      NA       NA
## A:E                NA          NA      NA       NA
## B:E                NA          NA      NA       NA
## C:E                NA          NA      NA       NA
## D:E                NA          NA      NA       NA
## A:B:C              NA          NA      NA       NA
## A:B:D              NA          NA      NA       NA
## A:C:D              NA          NA      NA       NA
## B:C:D              NA          NA      NA       NA
## A:B:E              NA          NA      NA       NA
## A:C:E              NA          NA      NA       NA
## B:C:E              NA          NA      NA       NA
## A:D:E              NA          NA      NA       NA
## B:D:E              NA          NA      NA       NA
## C:D:E              NA          NA      NA       NA
## A:B:C:D            NA          NA      NA       NA
## A:B:C:E            NA          NA      NA       NA
## A:B:D:E            NA          NA      NA       NA
## A:C:D:E            NA          NA      NA       NA
## B:C:D:E            NA          NA      NA       NA
## A:B:C:D:E          NA          NA      NA       NA
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.422 on 16 degrees of freedom
## Multiple R-squared:  0.6587, Adjusted R-squared:  0.5094
## F-statistic: 4.412 on 7 and 16 DF,  p-value: 0.006646
```

```r
# alias structure
alias(lm(as.numeric(FH) ~ A * B * C * D * E, fraction))
```

```
## Model :
## as.numeric(FH) ~ A * B * C * D * E
##
## Complete :
##           (Intercept) A B C D A:B A:C B:C
## E         1           0 0 0 0 0   0   0
## A:D       0           0 0 0 0 0   0   1
## B:D       0           0 0 0 0 0   1   0
## C:D       0           0 0 0 0 1   0   0
## A:E       0           1 0 0 0 0   0   0
## B:E       0           0 1 0 0 0   0   0
## C:E       0           0 0 1 0 0   0   0
## D:E       0           0 0 0 1 0   0   0
## A:B:C     0           0 0 0 1 0   0   0
## A:B:D     0           0 0 1 0 0   0   0
## A:C:D     0           0 1 0 0 0   0   0
## B:C:D     0           1 0 0 0 0   0   0
## A:B:E     0           0 0 0 0 1   0   0
## A:C:E     0           0 0 0 0 0   1   0
## B:C:E     0           0 0 0 0 0   0   1
## A:D:E     0           0 0 0 0 0   0   1
## B:D:E     0           0 0 0 0 0   1   0
## C:D:E     0           0 0 0 0 1   0   0
```

```
## A:B:C:D   1           0 0 0 0 0   0   0
## A:B:C:E   0           0 0 0 1 0   0   0
## A:B:D:E   0           0 0 1 0 0   0   0
## A:C:D:E   0           0 1 0 0 0   0   0
## B:C:D:E   0           1 0 0 0 0   0   0
## A:B:C:D:E 1           0 0 0 0 0   0   0
```

```r
# half normal probability plot
qqnorm(aov(as.numeric(FH) ~ A * B * C * D * E, fraction), label = TRUE)
```



(c) Calculate the range and standard deviation of the free height for each run. Is there any indication that any of these factors affects variability in the free height?

(d) Analyze the residuals from this experiment, and comment on your findings.

(e) Is this the best possible design for five factors in 16 runs? Specifically, can you find a fractional design for five factors in 16 runs with a higher resolution than this one?

8.12

Consider the leaf spring experiment in Problem 8.7. Suppose that factor E (quench oil temperature) is very difficult to control during manufacturing. Where would you set factors A, B, C, and D to reduce variability in the free height as much as possible regardless of the quench oil temperature used?

8.14

Consider the 2^5 design in Problem 6.24. Suppose that only a one-half fraction could be run. Furthermore, two days were required to take the 16 observations, and it was necessary to confound the 2^{5-1} design in two blocks. Construct the design and analyze the data.

8.51

A 16-run fractional factorial experiment in nine factors was conducted by Chrysler Motors Engineering and described in the article "Sheet Molded Compound Process Improvement," by P. I. Hsieh and D. E. Goodwin (Fourth Symposium on Taguchi Methods, American Supplier Institute, Dearborn, MI, 1986, pp. 13-21). The purpose was to reduce the number of defects in the finish of sheet-molded grill opening panels. The design, and the resulting number of defects, c, observed on each run, is shown in Table P8.14. This is a resolution III fraction with generators E = BD, F = BCD, G = AC, H = ACD, and J = AB.

(a) Find the defining relation and the alias relationships in this design.

(b) Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

(c) Fit an appropriate model using the factors identified in part (b) above.

(d) Plot the residuals from this model versus the predicted number of defects. Also, prepare a normal probability plot of the residuals. Comment on the adequacy of these plots.

(e) In part (d) you should have noticed an indication that the variance of the response is not constant. (Considering that the response is a count, you should have expected this.) The previous table also shows a transformation on c, the square root, that is a widely used variance stabilizing transformation for count data. (Refer to the discussion of variance stabilizing transformations in Chapter 3.) Repeat parts (a) through (d) using the transformed response and comment on your results. Specifically, are the residual plots improved?

(f) There is a modification to the square root transformation, proposed by Freeman and Tukey ("Transformations Related to the Angular and the Square Root," Annals of Mathematical Statistics, Vol. 21, 1950, pp. 607-611) that improves its performance. FandT's modification to the square root transformation $\frac{\sqrt{c}+\sqrt{c+1}}{2}$ is Rework parts (a) through (d) using this transformation and comment on the results. (For an interesting discussion and analysis of this experiment, refer to "Analysis of Factorial Experiments with Defects or Defectives as the Response," by S. Bisgaard and H. T. Fuller, Quality Engineering, Vol. 7, 1994-95, pp. 429-443.)