Water Balance Modelling Evaluation through Mapping the Hydrological Pattern to Information Space

Paper Script 1.0

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Outline

Abstract

- Motivation & Problem Statement
- Data & Approach
- Result
- Conclusion

Abstract: Motivation & Problem Statement

- General Picture
  Conceptual Water balance models have been developed and used for climatic change impact exploration and long-range stream flow forecast.
- Zoom in
  With a similar iteration pattern but different constitutive functions and insufficient inputs, most of the existing models are capable of generating satisfying outputs over a monthly time scale, where a fuzzy gap of hydrological simulation exists between the long range water-energy correlation pattern and the detailed precipitation run-off generation mechanism.

Abstract: Motivation & Problem Statement

- Find the Gap
  However, to smooth the interim of different time scale hydrological modelling, the legality of the iteration pattern and the constitutive functions of these models remain to be examined over the time crack.
- Theoretical Support of The Method
  The Aleatory-Epistemic Uncertainty Framework

Abstract: Data & Approach

- Data
  the experiment basins of MOPEX project
- Approach
  Use the Aleatory-Epistemic Uncertainty Evaluation Framework to examine the information flow and source sink terms of 6 conceptual monthly water balance models over time scales from one day to a year.

Abstract: Result

- the efficiency of the constitutive functions
- how the efficiency change with time scales
- how is the change rate connected with the catchment characteristics
- the state variable, its function and explanation

Abstract: Conclusion

Introduction

- General Picture & Zoom In
  - Water Balance Model
  - Evaluate Criterion
- Organization

General Picture

A major realm of hydrological community is to figure out the components of the catchment water balance over different time scales.

- Physically-based models
  - detailed information
- Conceptual models and patterns
  - Describe a coarser but valuable hydrological form
  - Available for mathematical analysis

Water Balance Model

- Budyko Curve: A Paradigm
- Brief Introduction of Water Balance Models

Evaluate Criterion

Organization

- Theoretical Analysis of 5 existed water balance models
  - Disintegration
  - Constitutive Function Analysis
- Brief Introduction of Aleatory Epistemic Uncertainty framework & Its Application Considerations
- Data & Method
- Result
- Discussion & Conclusion

Table: A Brief History of 5 Water Balance Models

Term	Developer	Time
TMWB	Thronthwaite & Mather	1948&1955
ABCD	Thomas	1981
VMWB	Vandewiele & Xu	1992
TPWB	Xiong & Guo	1996
DMWB	Zhang & Potter	2008

Basic Pattern –A Recursive Functional Equation Set

$$
\begin{cases}
Structure = \begin{cases} Output\_Generation \\ State\_Variable\_Renewal \end{cases} \\
Output\_Generation(Input, State, Paramter) = Output \\
State\_Variable\_Renewal(Input, State, Parameter) = New\_State \\
Simulation(Structure, Input, State, Parameter) \\
= \begin{cases} Output \\ Simulation(Structure, New\_Input, New\_State, Parameter) \end{cases}
\end{cases}
$$

Numerical Parts of 5 Water Balance Models

Term	Input	Output	State Variable	Parameter
TMWB	P, EP	E, Q	S_1, S_2	a, SC
ABCD	P, EP	E, Q_d, Q_g	S, G	a, B, c, d
VMWB	P, EP	E, Q	S	a, b, c
TPWB	P, EP	E, Q	S	c, SC
DMWB	P, EP	E, Q_d, Q_g	S, G	a, b, SC, d

Structures of 5 Water Balance Models

Term	Output Generation Section	Variable Renewal Section
TMWB	$E = \begin{cases} EP \,;\, P > EP \\ EP - P \,;\, else \end{cases}$ $Q = \lambda \underbrace{[S_2 + max(S_1 + P - E - SC, 0)]}_{WO}$	$S_{1new} = min(S_1 + P - E, SC)$ $S_{2new} = (1 - \lambda)WO$
ABCD	$E = \underbrace{[\frac{P+S}{2a} - \sqrt{(\frac{P+S+b}{2a})^2 - \frac{(P+S)b}{a}}]}_{EO}[1 - e^{-\frac{EP}{b}}]$ $Q_d = (1-c)(P+S-EO)$ $Q_b = dG$	$S_{new} = EOe^{-\frac{EP}{b}}$ $G_{new} = \frac{G+c(P+S-EO)}{1+d}$
VMWB	$E = min[EP(1 - a^{\frac{S+P}{EP}}), S+P]$ $Q = bS + c[P - EP(1 - e^{-\frac{P}{EP}})]$	$S_{new} = S + P - Q - E$
TPWB	$E = cEPtanh[(P+S)/EP]$ $Q = (S+P-E)tanh[(S+P-E)/SC]$	$S_{new} = S + P - Q - E$
DMWB	$Q_d = P - \underbrace{B\_Curve(P, SC - S + EP, a)}_{W}$ $Q_b = dG$ $R = W + S - B\_Curve(W + S, EP + SC, b)$ $E = B\_Curve(W + S, EP, b)$	$S_{new} = W + S - R - E$ $G_{new} = (1 - d)G + R$

A Systematic Perspective Toward the Constitutive Functions

Supply-Demand Framework

- Extreme Boundary Conditions
- Buckingham Π Theorem

A Stochastic Analytical Perspective Toward the Constitutive Functions

We should notice when applying the water balance models at a monthly or annual time scale, the information that inputs (here we mean precipitation and potential evapotranspiration) in a single iteration unit provides is the available water and energy in whole during that period, with the detailed hydrological processes lost in accumulation. We assume:

- Independence between local soil moisture and precipitation occurrence
- Consistent meteorological condition during the calculating period

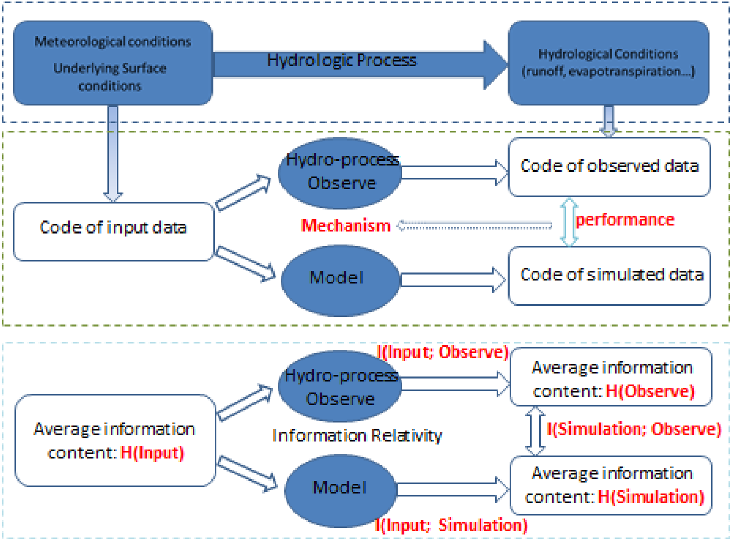
$$\frac{\partial f(s,t)}{\partial t} = \frac{\partial[\rho(s)f(s,t)]}{\partial s} - \lambda(t)f(s,t) + \lambda(t)\int_0^s g(z,t)p_{i|z}(s-z)dz + \lambda(t)p_0(0)e^{-\lambda(t)t}p_{i|0}(s) \quad (1)$$

$$p(r,t) = \lambda(t)\int_0^1 f(z,t)f_{r\_depth}(r+1-z)dz \quad (2)$$

We are constrained by:

- The accumulative EP equals the observation
- The accumulative P equals the observation

The Epistemic Aleatory Uncertainty Framework



Considerations

- Benchmark
- Seasonal Fluctuation Inconsistency

Method & Data

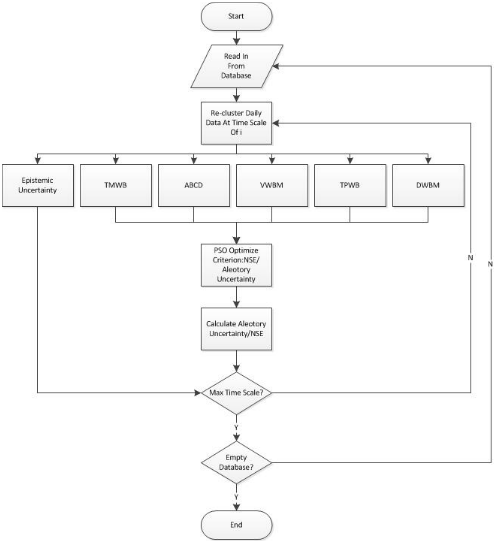


Figure: Programming Flow Chart

Auto Calibration

$$
\begin{cases}
Model(Parameter) = Simulation(Structure, Input, State, Parameter) \\
Evolution(Parameter\_Sets) = Evolved\_Parameter\_Sets \\
Optimization(Model, Parameter\_Sets, Observation, Evaluat\_Criterion) \\
= \begin{cases} Local\_best \\ Optimization(Model, Evolved\_Parameter\_Sets, Obsevation, Evaluat\_Criterion) \end{cases}
\end{cases}
$$

"Evolution"

Any formal system could be expressed in the language of number theory.
The mathematical fact of the evolution in the A.I. optimization algorithms is to map the pseudo-random numbers generated by some number theory laws to the parameter space under the guide of the former generations' performance for producing some "unexpected" but useful new parameters.

An Evolution Paradigm: PSO

$$
\begin{aligned}
& Evolution(Parameter) \\
=& Inertial\_Coefficient \times Parameter + \\
& Self\_Experience\_Coefficient \times (Hitorical\_Best - Parameter) + \\
& Social\_Experience\_Coefficient \times (Global\_Best - Parameter)
\end{aligned}
$$

Programming Language

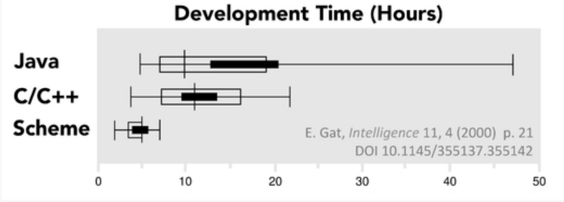
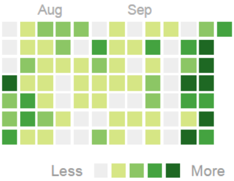


Figure: Development Time of Languages

All the codes are available at the github URL:



https://github.com/morepenn

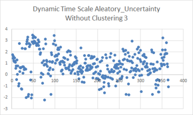
Results

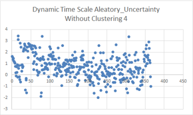
- Aleatory Uncertainty
  - Excluding State Variable
  - Including State Variable
- Epistemic Uncertainty

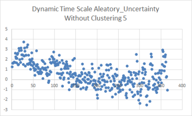
Results: Aleatory Uncertainty Excluding State Variable

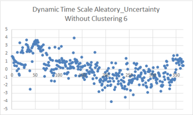
The stationary point of the Aleatory Uncertainty -Time Scale Function is the exact time scale point that we could ignore the impact of iterative variable.

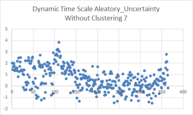
Results:
Unclustered Aleatory Uncertainty Excluding Former Inputs

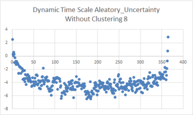


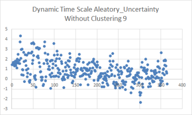




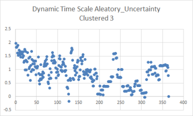


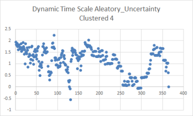


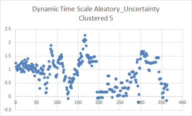


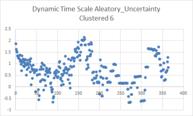


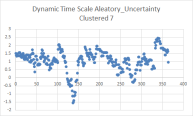
Results:
Clustered Aleatory Uncertainty Excluding Former Inputs

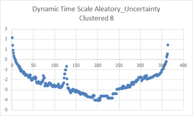


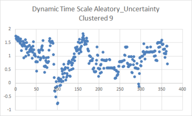












Results

when $T > 365$, the influence of the state variable become insignificant in experimental basin 6.

Discussion

The shape of different defined Aleatory Uncertainty (Epistemic Uncertainty)- Modelling Time Scale at different basins.

Conclusion

Difficulties

- Calculating Speed
- Data

Remaining Work
Aleatory Uncertainty

- The impact of former input in calculating Aleatory Uncertainty at different time scales
- Did the state-variables in the models take the same effect as the former input in calculating Aleatory Uncertainty at different time scales

- The Efficiency of the Constitutive Functions
- The Shape of the Epistemic Uncertainty-Modelling Time Scale Curve
- Its Relation with the Basin Characteristics