Stochastic Learning and Its Application In Hydrology

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Outline

- Introduction to Stochastic Learning
- Two Cases
 - ICA
 - Support Vector Machine
- Discussion

Introduction

Definition: Machine learning is a scientific discipline that deals with the construction and study of algorithms that can learn from data. Such algorithms operate by building a model based on inputs and using that to make predictions or decisions, rather than following only explicitly programmed instructions.

Conditions to Use Stochastic Learning

- There is pattern
- There is not explicit mathematical expression
- Available Data

ICA

Independent Component Analysis (ICA) is a computational method for separating a multivariate signal into additive subcomponents. This is done by assuming that the subcomponents are non-Gaussian signals and that they are statistically independent from each other. ICA is a special case of blind source separation.

Problem Statement

Linear Regression of NDVI from P, R, T, EP, P_{log}

Result

ICA transform among all terms.

 $n_{independent_components} = n_{variables}$

$$M_o = 0.0015 P_{Mean} - 0.0074 Rh + 0.0005 P_{Lag} + 0.0349 Ta_{Mean} - 0.0006 Eps$$

$$\rho^2 = 0.6787$$

 $n_{independent_components} = 2$

$$\mathit{M}_{o} = 0.0008 P_{\mathit{Mean}} - 0.0049 Rh + 0.0008 P_{\mathit{Lag}} + 0.0144 \textit{Ta}_{\mathit{Mean}} - 0.0001 \textit{Eps}$$

$$\rho^2 = 0.5856$$

 $n_{independent_components} = 3$

$$M_o = 0.0009 P_{Mean} - 0.0046 Rh - 0.0146 Ta_{Mean} + 0.0008 Eps$$

$$\rho^2 = 0.4674$$



ICA transform among same dimension terms.

 $n_{independent_components} = n_{same_dimension_variables}$

$$\mathit{M}_{o} = 0.0015 P_{\mathit{Mean}} - 0.0074 Rh + 0.0005 P_{\mathit{Lag}} + 0.0349 \textit{Ta}_{\mathit{Mean}} - 0.0006 \textit{Eps}$$

$$\rho^2 = 0.6787$$

 $n_{independent_components} = 2$

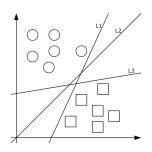
$$\mathit{M}_{o} = 0.0005 P_{\mathit{Mean}} + 0.0011 Rh + 0.0007 P_{\mathit{Lag}} + 0.0040 \textit{Ta}_{\mathit{Mean}} - 0.0001 \textit{Eps}$$

$$\rho^2 = 0.5800$$

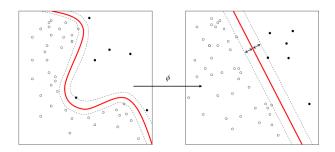
We recommend the last result for an intuitive deduction that the 2 independent components reflect the water and heat control.

Support Vector Machine

A support vector machine constructs a hyperplane or set of hyperplanes in a high or infinite dimensional space, which can be used for classification, regression, or other tasks. Intuitively, a good separation is achieved by the hyperplane that has the largest distance to the nearest training data point of any class (so-called functional margin), since in general the larger the margin the lower the generalization error of the classifier.



Non-linear Condition



Support Vector Regression

Minimize the sum of the empirical risk function and the structural risk function:

$$\frac{1}{I}\sum_{i=1}^{I}|y_i-f(x_i)|_{\epsilon}$$

structural risk function:

empirical risk function:

$$\frac{1}{2}w^Tw$$

Example