While the HBT effect was very confusing and even controversial for a long time, its classical electromagnetic analogue for photons (electromagnetic waves) or other waves is well known and routinely observed. Let us assume that we are dealing with two distant stars, as in the original HBT experiment, and can detect the light intensity in two separate coincidence detectors (see Fig. VII.5), which are only triggered if they simultaneously detect

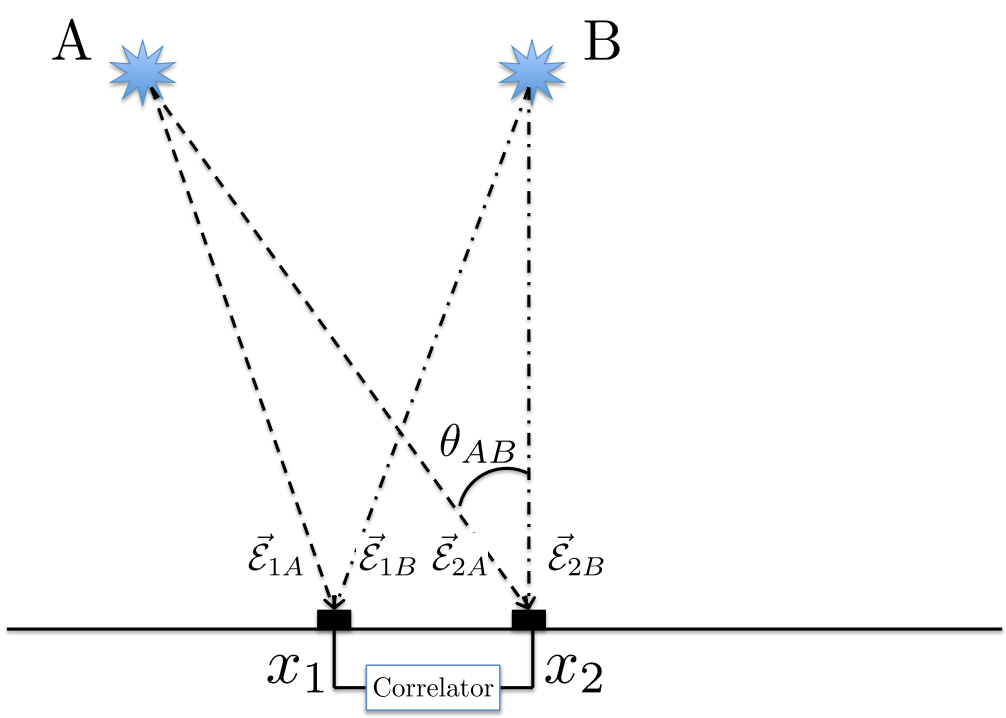

FIG. VII.5 Setup illustrating the HBT effect, which measures the interference of light coming from two distant stars A and B, which have angle θ_{AB} between them. The light hits the two synchronized coincidence detectors, which measure instantaneous intensity-intensity correlators.

light. Then the electric (and similarly the magnetic) field at the detectors 1 and 2 located at points x_1 and x_2 will be given by

$$\vec{\mathcal{E}}_1 = \vec{\mathcal{E}}_{1A} + \vec{\mathcal{E}}_{1B}, \qquad \vec{\mathcal{E}}_2 = \vec{\mathcal{E}}_{2A} + \vec{\mathcal{E}}_{2B}, \tag{VII.35}$$

where $\mathcal{E}_{1A}$ is the (complex) electric field coming from the star A and arriving at the detector 1. The light intensity on detector 1 will be (up to an overall constant)

$$I_1 = |\vec{\mathcal{E}}_1|^2 = |\vec{\mathcal{E}}_{1A}|^2 + |\vec{\mathcal{E}}_{1B}|^2 + \vec{\mathcal{E}}^*_{1A}\vec{\mathcal{E}}_{1B} + \vec{\mathcal{E}}^*_{1B}\vec{\mathcal{E}}_{1A}, \tag{VII.36}$$

and similarly for the detector B. If the photon sources are uncorrelated, the phases of the two fields $\vec{\mathcal{E}}_{1A}$ and $\vec{\mathcal{E}}_{1B}$ are random and time-averaging the intensity washes out any interference:

$$\overline{I_1} \propto |\vec{\mathcal{E}}_{1A}|^2 + |\vec{\mathcal{E}}_{1B}|^2 \qquad \Rightarrow \qquad \overline{I}_1 = I_{1A} + I_{1B}. \tag{VII.37}$$

The situation changes if we first measure the instantaneous cross-correlations of the two intensities $I_1 I_2$ and then time average this product. Then it is easy to see that

$$\overline{I_1 I_2} = \overline{I_1}\,\overline{I_2} + \overline{\vec{\mathcal{E}}^*_{1A}\cdot\vec{\mathcal{E}}_{2A}}\,\overline{\vec{\mathcal{E}}_{1B}\cdot\vec{\mathcal{E}}^*_{2B}} + \overline{\vec{\mathcal{E}}^*_{2A}\cdot\vec{\mathcal{E}}_{1A}}\,\overline{\vec{\mathcal{E}}_{2B}\cdot\vec{\mathcal{E}}^*_{1B}}. \tag{VII.38}$$

The interference contribution now does not depend on the relative phase between the electric fields $\vec{\mathcal{E}}_A$ and $\vec{\mathcal{E}}_B$. For a spherical wave emitted by the star A, the corresponding electric fields on the two detectors are given by

$$\vec{\mathcal{E}}_{1A} = \vec{\mathcal{E}}_A \mathrm{e}^{ikR_{1A}}, \qquad \vec{\mathcal{E}}_{2A} = \vec{\mathcal{E}}_A \mathrm{e}^{ikR_{2A}}, \tag{VII.39}$$

where R_{1A} (R_{2A}) is the distance from the star A with the detector 1 (2) and k is the wave vector of the light. A similar expression holds for the star B. Plugging this equation into the expression for the density-density correlation function, we find

$$\overline{I_1 I_2} = \overline{I_1}\,\overline{I_2} + 2\overline{I}_{1A}\overline{I}_{1B}\cos\left[k(R_{1A} + R_{2B} - R_{1B} - R_{2A})\right]. \tag{VII.40}$$

Now let us use the fact that the stars are very far from each other. Then from the vector relation

$$\vec{R}_{2A} = \vec{R}_{1A} + \vec{r}, \tag{VII.41}$$

where $\vec{r}$ is the vector connecting the detectors 1 and 2, we find

$$R_{2A} \approx R_{1A} + r\cos(\theta_A), \quad R_{2B} \approx R_{1B} + r\cos(\theta_B), \tag{VII.42}$$

where θ_A and θ_B are the altitude angles characterizing the two stars. Therefore

$$\overline{I_1 I_2} = \overline{I_1}\,\overline{I_2} + 2\overline{I}_{1A}\overline{I}_{1B}\cos\left[kr(\cos(\theta_A)) - \cos(\theta_B))\right]. \tag{VII.43}$$

In order to increase the wavelength of the interference pattern one can choose nearby stars with $\theta_A \approx \theta_B$. In this case we can approximate $\cos(\theta_B) \approx \cos(\theta_A) - \sin(\theta_A)\theta_{AB}$ such that

$$\overline{I_1 I_2} = \overline{I_1}\,\overline{I_2} + 2\overline{I}_{1A}\overline{I}_{1B}\cos\left[kr\,\theta_{AB}\sin(\theta_A)\right]. \tag{VII.44}$$

The interference pattern should oscillate with a period given by $\lambda/(\theta_{AB}\sin(\theta_A))$, where λ is the wavelength of the light. Choosing θ_{AB} and θ_A to be sufficiently small, we can substantially increase the oscillation period compared to λ. The first experiment detecting these correlations was done by R. Hanbury Brown and R. Q. Twiss in 1956. Assuming for simplicity that all four intensities $\overline{I}_{1A}, \overline{I}_{1B}, \overline{I}_{2A}, \overline{I}_{2B}$ are identical and equal to Q, we find that the intensity-intensity correlation function oscillates around a mean value $4Q^2$ with an amplitude $2Q^2$. We note that real stars themselves are not point sources and have some angular resolution $\delta\theta_A \approx \delta\theta_B$. This resolution limits the minimal angular distance between the starts required to see the interference $\theta_{AB} \gg \delta\theta_A$ and bounds the distance between the two detectors required to see the interference: $r_{\max} = \lambda/(\delta\theta_A \sin(\theta_A))$ (cf. Problem 5). Physically, detectors separated by a distance $r \lesssim r_{\max}$ can not resolve different points within the same star, such that the interference between different photons coming from the same star is constructive.

Interestingly, the HBT effect has the exact same origin as the Fock term leading to magnetism. When computing the energy shift due to interactions in the first order of perturbation theory, we assumed that the orbitals did not change and ignored tunneling, such that unperturbed particles never saw each other. Nevertheless, there was an interference contribution to the energy shift, where we can interpret the interaction coupling as a detector that is sensitive to the simultaneous location of two independent particles. As in the HBT effect, the Fock term is generally oscillating, depending on the details of the functions ψ_1 and ψ_2, and it exactly doubles (cancels) the conventional density-density interaction in the case of bosons (fermions).