CSE 470N B

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2nd February 2023

Lecture notes from the 2023 undergraduate course Quantum Computing, given by Professor James D Kiper at Miami University at Benton Hall in the academic year 2022-2023. This course covers introductory quantum computing concepts. Credit for the material in these notes is due to Professor James D Kiper, while the structure is loosely taken from the in-class lectures. The credit for the typesetting is my own.

Disclaimer: This document will inevitably contain some mistakes–both simple typos and legitimate errors. Keep in mind that these are the notes of an undergraduate student in the process of learning the material, so take what you read with a grain of salt. If you find mistakes and feel like telling me, I will be grateful and happy to hear from you, even for the most trivial of errors. You can reach me by email, in English, at sayahie@miamioh.edu.

For more notes like this, visit my GitHub profile.

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Thu, 26 January 2023, 2:50pm – 4:10pm

Lecture 1: Week 1, Thursday

Definition 1.1

A *bit* is a binary digit that can take on one of two values, 0 or 1.

Definition 1.2

A *qubit* is analogous to a bit in a quantum computer, but can take on a superposition of the values 0 and 1–it can be in a state of 0 and 1 at the same time.

Definition 1.3

The *planetary model of the atom* is a model of the atom in which the electrons orbit the nucleus in a circular orbit. The planetary model of the atom was developed by Niels Bohr in 1913.

The Stern-Gerlach experiment, first successfully performed in 1922, demonstrated that the magnetic field of an electron can be used to separate the electron into two different states, one with a magnetic field pointing up and one with a magnetic field pointing down. Silver atoms with random spatial orientations were sent straight between two magnets, with the atoms hitting a detector on the other side. The detector was able to detect which direction the atoms were moving in, and the results showed that the atoms were split into two groups, one with a magnetic field pointing up and one with a magnetic field pointing down–'the magnetism was quantised'. This was not expected–the initial hypothesis was that the atoms would form a continuous pattern instead of falling onto two points on the detector, as the spatial orientations were random.

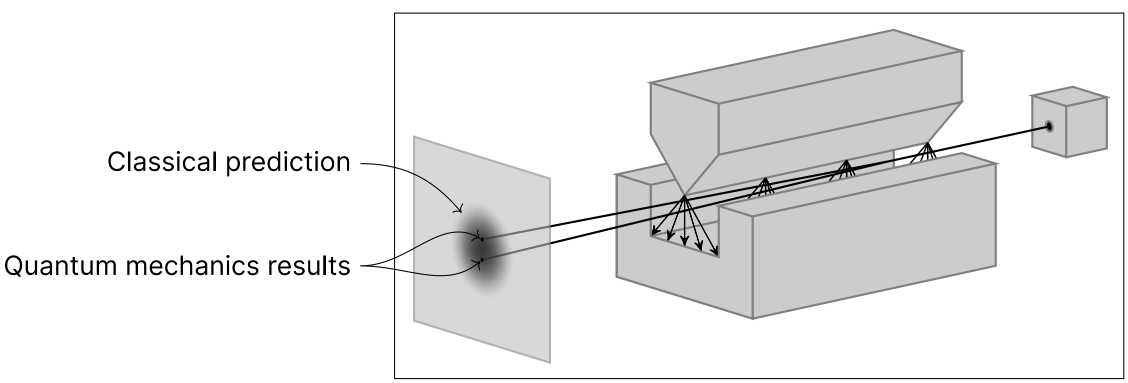

Figure 1: Stern-Gerlach Experiment. Figured designed by Clemens Koppensteiner

Note:-

An electron orbiting in a circular orbit generates a magnetic field.

Particles have some properties, such as ‘colour’ (with two possible values: black or white), and ‘hardness’ (with two possible values: soft, hard). We can build detectors that, when given many particles, show a long-run probability of detecting a particle with a certain property. These detectors can be repeated (eg, a colour detector followed by another colour detector) without the probability changing. These detectors demonstrate that the properties are also probabilistically independent (as in, the results are not correlated between a particle’s colour, hardness, etc).

Definition 1.4

The *uncertainty principle* states that the probability of measuring a certain property of a particle is inversely proportional to the probability of measuring a different property of the same particle. This is demonstrated in Figure 2. In other words, the more certain we are of measuring one property of a particle, the less certain (or more uncertain) we are of measuring a different property of the same particle.

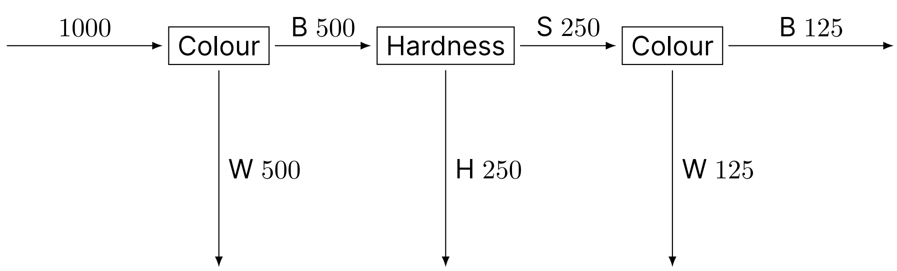

Figure 2: Repeated detectors that detect colour and hardness, demonstrating the uncertainty principle. By measuring the hardness, we became uncertain of the colour; the 250 ‘black’ & ‘soft’ particles were redetected as 125 black and 125 white.

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Lecture 2: Week 2, Tuesday

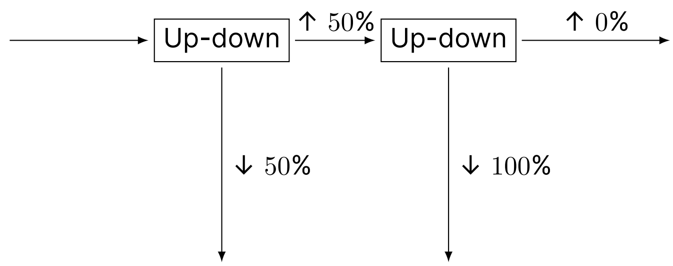

Figure 3: Two repeated detectors of whether a particle's spin is up or down. The same property is being measured; the percentages heading into the second detector are known.

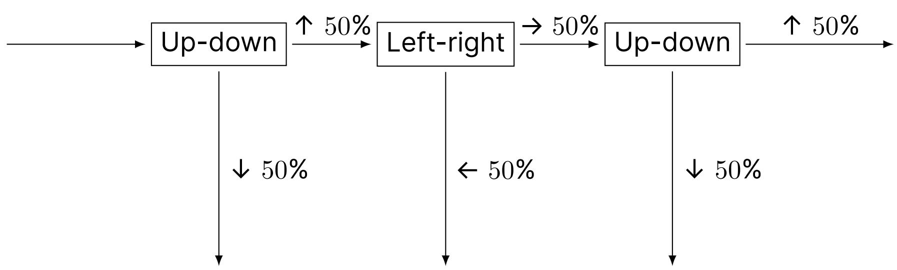

Figure 4: Three repeated detectors, now detecting three different properties of a particle. Now, the percentages for spin up or down are not known; the first and last detectors are probabilistically independent because of the middle detector.

'To talk about an electron that is both spin up and spin right is nonsensical.'

Definition 2.5

Tunneling is a phenomenon in quantum mechanics where an object on the quantum scale can penetrate barriers in a manner that's contradictory to what classical mechanics predicts. In other words, an object can sometimes move through something that should seemingly stop its movement.

Definition 2.6

Quantum decoherence is when the wave function that describes the quantum state of a particle 'collapses' (ie, the quantum state can no longer be predicted or described by the wave function). With decoherence, information about the system is lost into the environment; if a quantum system were perfectly isolated (ie, if nothing could interact with it), it would maintain coherence indefinitely.

With n qubits, a quantum algorithm can search up to 2^n states simultaneously. This is the advantage of quantum computers–modelling complex systems and searching through a large set of possibilities is where quantum computers can be useful.

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Lecture 3: Week 2, Thursday

'The state of a quantum system corresponds to a vector in a vector space of complex numbers.'

Definition 3.7

A *vector* is a list of numbers. The length (or magnitude; denoted by $||v\rangle|$) of a vector can be found by calculating the square root of the sum of squares of the horizontal and vertical components. Scalar multiplication can be performed by multiplying every value of a vector by the scalar. Vector addition can be performed by adding every element in a vector with the element in the corresponding position in another vector. Vector multiplication (also referred to as finding a dot product, or an inner product; denoted by the product $\langle v|w\rangle$) can be done between two vectors with the same dimensions. If the result of the multiplication is 0, the two vectors are *orthogonal*.

Example. Row: $\langle v| = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

Column: $|w\rangle = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

Magnitude: $|v\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$

$||w\rangle| = \sqrt{a^2 + b^2}$

Scalar multiplication: $|v\rangle = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$4 \cdot |v\rangle = 4 \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix}$

Vector addition: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 7 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ 7 \end{bmatrix}$

Multiplication: $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix} = -2 + 6 + 28 = 32$

Note:-

For the purposes of this course, we must be able to find the length of a vector, preform scalar multiplication, perform vector addition, and check for orthogonality.

Definition 3.8

A set of *basis vectors* is a set of vectors that can be combined in a linear combination to make any other vector in the vector space.

Example. Possible basis vectors with two dimensions:

$$\begin{bmatrix} 76.9513 \\ \pi \end{bmatrix} = 76.9513 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \pi \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Possible basis vectors with three dimensions:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Note:-

$$|{\rightarrow}\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|{\leftarrow}\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|{\nearrow}\rangle = \begin{bmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{bmatrix}$$

$$|{\swarrow}\rangle = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

Notes