Mathematical methods

We wish ALL THE BEST to the students those who are sitting for VCE 2013!!!

Mathematical problems for year 11 & 12 students, those who are studying mathematical methods subject. I do hope you are enjoying this page on mathematics questions. Again I appreciate your feedback regarding this page. As always you may contact me on kulana@gmail.com for any questions or comments. *This page is conducted by* ***Dr. Kulan Ranasinghe*** *(PhD in Mathematics and Statistics, BSc Mathematics Special (Frist Class Honours)), Director of SHAKYA Institute of Education; Research Fellow at The University of Melbourne. kulana@gmail.com / 0433 266 987.*

Question 1

For $f:(-5,2] \to R, f(x) = 1 - \frac{1}{3}x^2$, find the range of f.

Question 2

Evaluate the following definite integrals.

a) $\int_2^1 (x-1)^2\, dx$ ***b***) $\int_{\frac{\pi}{4}}^{-\frac{\pi}{4}} \cos 2x\ dx$

c) $\int_{k^2}^{4k^2} \left(2x^{\frac{1}{2}} - k\right) dx$ ***d***) $\int_{-1}^{1} (x+1)^3\, dx$

Question 3

If $\frac{1}{2}\log_3 x - 3\log_3(x+1) - 1 = \frac{1}{2}\log_3 y$, find y in terms of x.

Question 4

It is given that $\int_0^b f(x)dx = b^2$. Find

$$\int_0^{\frac{b}{2}} [f(2x) + x]\, dx$$

Question 5

Find the maximal domain for the functions with the given rules

a). $f(x) = \sqrt{1-|x|}$ ***b).*** $f(x) = \frac{3}{\sqrt{1+2x}}$

c). $f(x) = \frac{1}{6x-3} + 1$

Question 6

Let $f(x) = \frac{x^2+1}{\cos(x)}$, find $f'\left(\frac{\pi}{4}\right)$.

Question 7

Write down the exact values of x of the following equations (in terms of logarithm of base e)

a) $5^x = 2^{1-2x}$

b) $4^{-x} = 7$

Question 8

Find the inverse function of f, $f(x) = \frac{2x-3}{x-1}$.

Question 9

Let $f(x) = 1 - x + x^3$. A tangent to the curve at point $(a, f(a))$ is parallel to the line $y = 2x - 7$. Find all the possible points, $(a, f(a))$, on the curve which satisfy above.

Question 10

Differentiate each of the following with respect to t.

a) $(1-2t^2)^2$ ***b***) $(t+1)^{\frac{1}{3}}(t-1)$

c) $\frac{1-t^3}{t^3+1}$ ***d***) $\sqrt{1-2t^2} - t^4$

Question 11

It is given that $\Pr(A \cup B) = 0.82$, $\Pr(A) = 0.6$ and $\Pr(B) = 0.3$. Are the events A and B independent?

Question 12

Find the equation of the normal to the curve $y = 1 - 2x - 3x^2 - 4x^3$ at $x = 1$

Question 13

Solve each of the following equations for $x \in [-2\pi, 2\pi]$

a) $\sqrt{3}\cos(2x) = \sin(2x)$

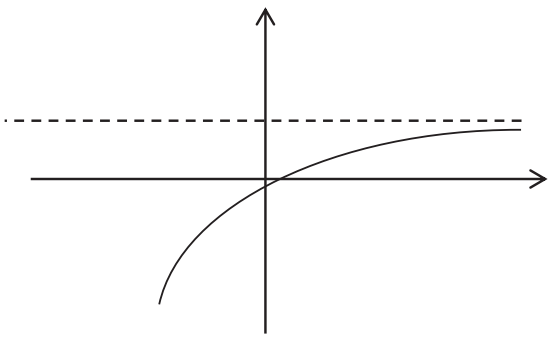
b) $\sqrt{2} - 2\cos(3x) = 0$

Question 14

Let $f(x) = \cos(x)$ and $g(x) = x^2$. Find $g\left(f\left(\frac{\pi}{6}\right)\right)$ and $f\left(g\left(\sqrt{\pi}\right)\right)$.

Question 15

Draw the inverse function of the given function in the same set of axes.



Question 16

Use the relationship $f(x+h) \approx f(x) + hf'(x)$, for small positive value of h, to approximate the value for $\sqrt[3]{27.06}$.

Question 17

Let $h: R\backslash\{3\} \to R, h(x) = \frac{x+2}{x-3}$. Find the inverse function , h^{-1}, of f. What is the domain of h^{-1} ?

Question 18

Find the anti-derivative of $\cos 3(2x) - 12x^3$.

Question 19

Let $g:[k,\infty) \to R, g(x) = 1 + (1-2x)^2$. Find,

a) the smallest value of k such that g is one-to-one.

b) *Range* g ***c***) $g^{-1}(x)$

Question 20

Let $\Pr(A) = 0.6$, $\Pr(B) = 0.5$ and $\Pr(B|A) = 0.2$. Find $\Pr(A \cap B)$ and $\Pr[(A \cup B)']$.

Question 21

Find the equation of the normal to the curve $y = 2\cos 2x + \sin 2x$ at $x = \frac{\pi}{4}$.

Question 22

Let

$$f(x) = \begin{cases} k + 2\sin x & , \quad x < 0 \\ x - 2 & , \quad x \geq 0 \end{cases}$$

a) Find k if f is continuous over R

b) Sketch the graph of f over $[-2\pi, 2\pi]$

c) Is f differentiable over $[-2\pi, 2\pi]$

Question 23

The random variable y has the following probability distribution.

y	-2	-1	0	1	2
$\Pr(Y = y)$	k	$2k$	$3k$	$2k$	k

Find,

a). The value of k ***b).*** $P_r(y \geq -1)$

c). $P_r(y > 1 \mid y \geq -1)$

---------- **Questions End Here** ----------

Dr. Kulan Ranasinghe *(PhD in Mathematics and Statistics, BSc Mathematics Special (Frist Class Honours)), Director of* ***SHAKYA Institute of Education****; Reseach Fellow at The University of Melbourne*