Introduction to Power System Analysis

Izudin Džafić
International University of Sarajevo

Izudin Džafić
Ema Halilović
Emir Karamehmedović

Introduction to Power System Analysis

March 21, 2019

International University of Sarajevo

CIP

Dedications...

A book should have either intelligibility or correctness; to combine the two is impossible. - Bertrand Russel

Contents

Chapter 1
Basic Principles

This chapter represents a brief overview of the electric circuit laws and principles needed for the analysis of power networks. In order to achieve simplicity and easier understanding, it is common to formulate laws for Direct Current (DC) networks and then introduce Alternating Current (AC) networks and fundamental laws which are a generalization of the laws formulated for the DC networks. Complex representation of voltages, currents and power with units and phase behavior is presented through derivations and gradually more knotty examples.

1.1 Analysis of DC Networks

1.1.1 Fundamental laws

1.1.1.1 Ohm's law

The relationship between voltage (V), current (I) and resistance (R) in any DC electrical circuit was firstly discovered by the German physicist Georg Ohm. Provided that the environment temperature is constant, the ratio of potential difference across the ends of a conductor to the current flowing in that conductor will also be constant. Thus, the relation between voltage V, current I and resistance R (Figure 1.1) is

$$V = IR \tag{1.1}$$

Equation(1.1) represents impedance form of Ohm's law. Conductance form of Ohm's law is given as

$$I = VG \tag{1.2}$$

where $G = \frac{1}{R}$ is the conductance of the conductor. The unit for resistance is Ohm $[\Omega]$ whereas the unit for conductance is Siemens, $S = \frac{1}{\Omega}$.

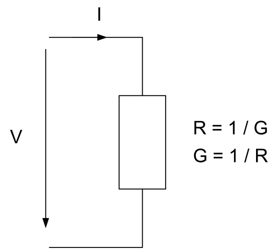

Fig. 1.1 DC network with a resistor and indicated current flow.

It is easier to remember Ohm's law relationship by using illustrations. The three quantities of V, I and R are organized in a triangle, as shown in Figure 1.2. Voltage is placed at the top, with current and resistance at the bottom. This arrangement represents the actual position of each quantity in the Ohm's law formulas. This triangle is called the Ohm's law triangle.

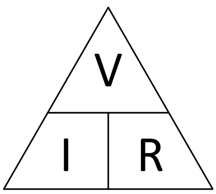

Fig. 1.2 Ohm's law triangle

1.1.2 Power in DC Circuits

Electrical power (P) in a circuit is the amount of energy that is consumed, absorbed, or produced within the circuit, in the observed time. The electric power associated with a complete electric circuit or a circuit component represents the rate at which energy is converted from the electrical energy to some other form, e.g., heat, mechanical energy, or energy stored in electric or magnetic fields. In a DC circuit, the power is given by the product of applied voltage and the electric current:

$$P = VI \tag{1.3}$$

Equation (1.3)can also be organized in a form of the power triangle, as shown in Figure 1.3.

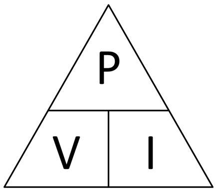

Fig. 1.3 Power triangle

1.1.2.1 Energy Dissipated in Resistor

With current flow, resistors dissipate electrical energy, i.e. electrical energy is converted to thermal energy (heat). Expression for the power dissipated in a resistor can be obtained by the use of Ohm's law.

$$P = VI = RI^2 = GV^2 \tag{1.4}$$

Energy that is dissipated in the resistor is proportional to the dissipation time

$$E_r = Pt = RI^2 t \tag{1.5}$$

1.1.2.2 Energy Stored in Electric Field

There is a fundamental differences between resistors and capacitors concerning the electrical energy. With resistors, as current flows, power is simply dissipated as heat. In a perfect capacitor, no energy is lost. The capacitor stores charge and is capable to give it back again. Energy is stored in the electric field between the capacitor plates. The unit of capacitance C is Farad (charge/voltage), and tells the amount of charge that can be stored between the capacitor plates at certain voltage drop. The amount of energy stored in a capacitor is given by

$$E_c = \frac{CV^2}{2} \tag{1.6}$$

Figure 1.4 shows a capacitor C in steady state (fully charged). No current flows through the circuit, whereas the voltage across the capacitor is constant.

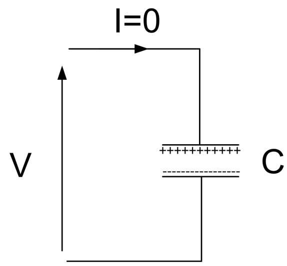

Fig. 1.4 Capacitor in steady state DC circuit

1.1.2.3 Energy Stored in Magnetic Field

There is also no energy lost in a perfect inductor. Here, energy is alternately stored in the magnetic field, and then given back to the circuit. Inductance is measured in Henry (magnetic flux/current), and tells the potential in creating magnetic field around a conductor carrying certain current. The energy stored in an inductor is:

$$E_l = \frac{LI^2}{2} \tag{1.7}$$

Figure 1.5 shows an inductor L in steady state - constant current flows through the inductor, while the voltage drop across the perfect inductor is zero.

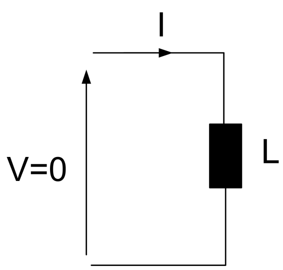

Fig. 1.5 Inductor in steady state DC circuit

1.1.3 Kirchhoff's Laws

1.1.3.1 Kirchhoff's current law

The first fundamental law of circuit analysis is Kirchhoff's current law. It states that at any node in a circuit, the sum of the currents arriving at the node is equal to the sum of the currents leaving the node. Alternatively one can interpret Kirchhoff's current law, also known as Kirchhoff's first law, as the sum of the currents injected to a node is equal to zero, if positive currents are considered those currents arriving at the node, as shown in Figure 1.6.

$$\sum I_i = 0 \tag{1.8}$$

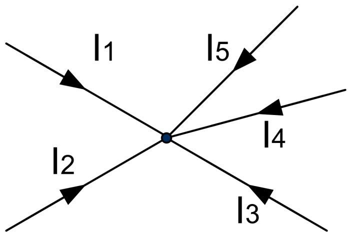

Fig. 1.6 Kirchhoff's current law

Example

Consider a part of network that consists of four lines whose diagram is shown in Figure 1.6. If $I_1 = 1A$, $I_2 = -2A$, $I_3 = 3A$, and $I_4 = 4A$, calculate current I_5 and its orientation.

Solution

According to Equation 1.8, it is found that:

$$I_1 + I_2 + I_3 + I_4 + I_5 = 0$$

$$I_5 = -(I_1 + I_2 + I_3 + I_4)$$

$$I_5 = -(1 - 2 + 3 + 4) = -6A$$

So, current $I_5 = -6A$. The negative sign means that the actual current flow direction is opposite to the one shown in Figure 1.6.

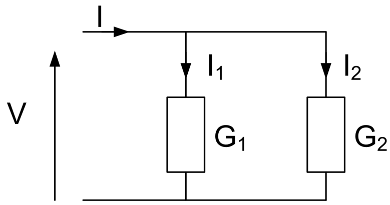

Fig. 1.7 Parallel connection of resistors.

1.1.3.2 Parallel connection of resistors

In case of parallel connection of two resistors, as shown on Figure 1.7, current I is equal to the sum of branch currents I_1 and I_2:

$$I - I_1 - I_2 = 0 \Rightarrow I = I_1 + I_2 \tag{1.9}$$

From equation 1.9the following is derived:

$$VG = VG_1 + VG_2 \Rightarrow G = G_1 + G_2 \tag{1.10}$$

Equivalent resistance of two resistors connected in parallel is obtained as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{1.11}$$

Similarly, in case when n resistors are connected in parallel, the equivalent resistance R is found to be:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \tag{1.12}$$

1.1.3.3 Kirchhoff's voltage law

The second fundamental law of circuit analysis is Kirchhoff's voltage law (Kirchhoff's second law). In any loop (path) around a circuit, the sum of the electromotive forces is equal to the sum of the voltage drops. In other words, this law states that the sum of voltages around a closed loop (Figure 1.8) is equal to zero:

$$\sum V_i = 0 \tag{1.13}$$

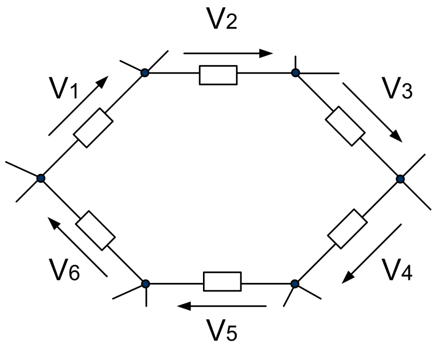

Fig. 1.8 Kirchhoff's voltage law

Example

Consider the electrical network shown in Figure 1.9. Using Kirchhoff's voltage law, calculate the voltage and polarization of DC generator E_3, where $E_1 = 12V$, $E_2 = 6V$, $I = 3A$, $R_1 = 10\Omega$, $R_2 = 20\Omega$.

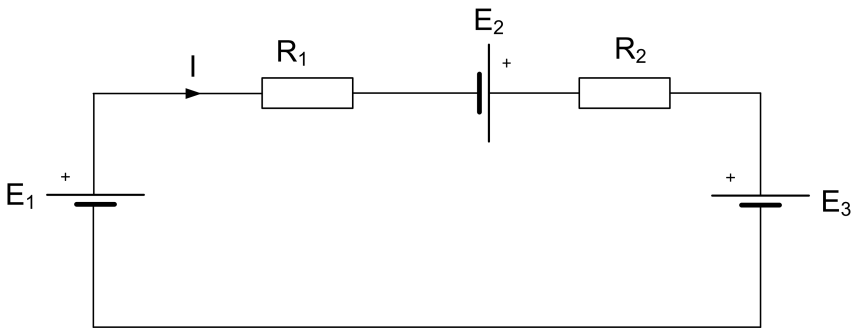

Fig. 1.9 Circuit analysis using Kirchhoff's voltage law

Solution

In order to apply Kirchhoff's second law, the following two steps are taken. The orientations for all DC generators and voltage drops are marked. Small arrows out of terminal with positive polarization are put. Polarizations of voltage drops depend on electrical current. The terminal in which electrical current arrives is marked as positive.

In the second step, the loop and its traversing orientation are marked as shown on Figure 1.10.

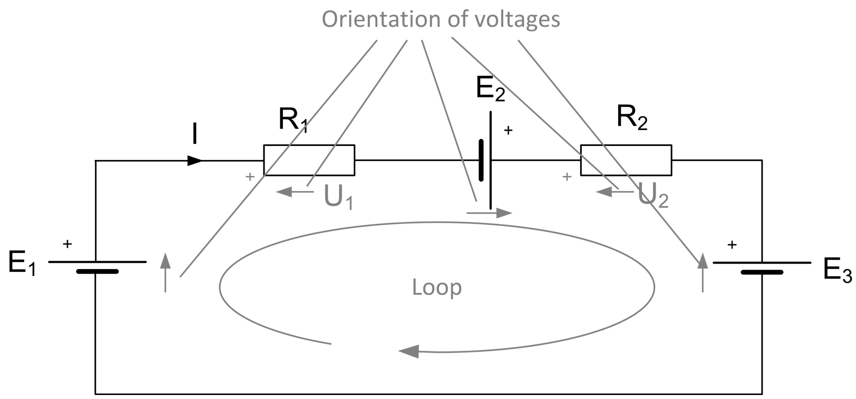

Fig. 1.10 Defining the orientation of DC generators and voltage drops across resistors

Kirchhoff's voltage law is then applied - for voltages having the same orientation like orientation of the loop, a positive sign is put, otherwise negative:

$$E_1 - U_1 + E_2 - U_2 - E_3 = 0$$

Applying Ohm's law, the following result is obtained

$$\begin{aligned} E_3 &= E_1 - IR_1 + E_2 - IR_2 \\ E_3 &= 12V - 2A \cdot 10\Omega + 6V - 2A \cdot 20\Omega \\ E_3 &= -42V \end{aligned}$$

The DC Generator $E_3 = 42V$, with polarity opposite to what is shown in Figure 1.9.

1.1.3.4 Serial connection of resistors

In order to find the equivalent resistance of two resistors in serial connection, Kirchhoff's second law is applied.

Kirchhoff's voltage law states that voltage V is equal to the sum of voltages V_1 and V_2:

$$V - V_1 - V_2 = 0 \Rightarrow V = V_1 + V_2 \tag{1.14}$$

According to Ohm's law:

$$IR = IR_1 + IR_2 \tag{1.15}$$

Reducing the current in Equation(1.15) gives the total resistance of serial connection of two resistors. It is equal to the sum of their resistances:

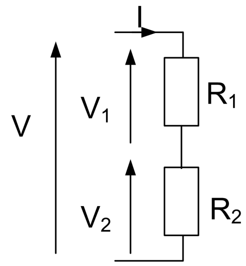

Fig. 1.11 Serial and parallel connection of two resistors

$$R = R_1 + R_2 \tag{1.16}$$

In case of n resistors, the equivalent resistance is

$$R = R_1 + R_2 + \cdots + R_n \tag{1.17}$$

1.2 Complex DC circuit analysis

1.2.1 Passive circuit reduction

Complex passive circuits [1] can be solved using network reduction. The reduction consists of successive application of Equations (1.12)and (1.17). These equations are actually special cases of Kirchhoff's current and voltage laws. Calculation procedure is explained by the following example.

Example

Calculate the resistance of the network given in Figure 1.12. Obtain the currents flowing through each resistor.

Solution

First step is to calculate the resistance of serial connection of resistors R_3 and R_4:

$$R_{34} = R_3 + R_4$$

[1] Passive circuit contains only resistors.

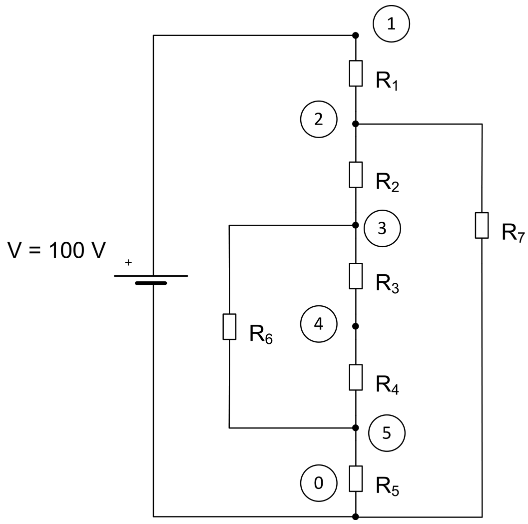

Fig. 1.12 Passive circuit

Serial connection of resistors R_3 and R_4 is connected in parallel with resistor R_6 . The resistance of them together is:

$$\frac{1}{R_{346}} = \frac{1}{R_{34}} + \frac{1}{R_6} \Rightarrow R_{346} = \frac{R_{34}R_6}{R_{34} + R_6}$$

The resistance of series connection of resistors R_2,$R_{346}$and R_5 is:

$$R_{2-6} = R_2 + R_{346} + R_5$$

Serial connection of resistors R_2, R_{346} and R_5 is connected in parallel with resistor R_7:

$$\frac{1}{R_{2-7}} = \frac{1}{R_7} + \frac{1}{R_{2-6}} \Rightarrow R_{2-7} = \frac{R_7 R_{2-6}}{R_7 + R_{2-6}}$$

Finally, the total resistance of the given network is:

$$R = R_1 + R_{2-7}$$

Total network current is flowing through resistor R_1. According to Ohm's law, it is equal to:

$$I_1 = \frac{V}{R}$$

Using Ohm's law and Kirchhoff's current and voltage laws, other network currents are calculated:

$$V_2 = V - R_1 I_1 \Rightarrow I_7 = \frac{V_2}{R_7}$$

$$I_2 = I_5 = I_1 - I_7$$

$$V_3 = V_2 - R_2 I_2 - R_5 I_5 \Rightarrow I_6 = \frac{V_3}{R_6}$$

$$I_3 = I_4 = I_2 - I_6$$

1.2.2 Direct application of Kirchhoff's laws

Kirchhoff's laws are, in general, used in any method for circuit analysis. In order to solve a network which has N_n nodes and N_l loops, $N_n - 1$ equations for Kirchhoff's current law, and $N_l - 1$ equations for Kirchhoff's voltage law need to be written. Node or junction is defined if there are three or more elements connected to it. Loops need to be defined after removing all current generators.

Example

Consider the network shown in Figure 1.13. The network contains two ampere meters (A_1 and A_2), two voltmeters (V_1 and V_2), three resistors ($R_1 = 1\Omega$, $R_2 = 2\Omega$, and $R_3 = 3\Omega$), three ideal DC batteries ($E_1 = 1V$, $E_2 = 2V$, and $E_3 = 3V$), and two DC current generators ($I_{G1} = 1A$ and $I_{G2} = 2A$). Calculate the values measured by voltmeters V_1 and V_2 and ampere meters A_1 and A_2.

Solution

In the first step, the number of loops and nodes in the network are found. Since ideal voltmeters have infinite internal resistance, they are for now removed. After this step, all nodes that are relevant for writing Kirchhoff's current law equations are defined. In this example, there are four nodes ($N_n = 4$) as shown in Figure 1.14.

Additionally, in order to define loops, all current generators are removed (open circuit). There are three loops in this network, as shown in Figure 1.15. Loop orientation can be arbitrarily selected.

Every resistor needs to be assigned corresponding voltage drop and its orientation. This part is explained in section 1.1.3.3. However, it is important to notice that voltage drop has positive sign on the terminal where electric

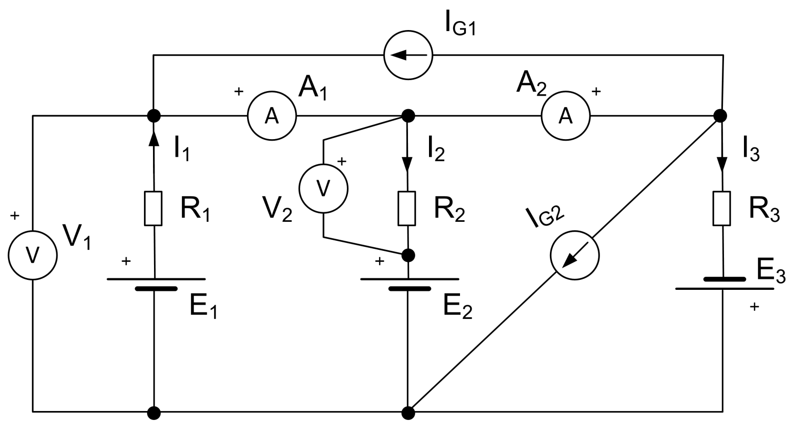

Fig. 1.13 DC network with two ampere meters and one voltmeter

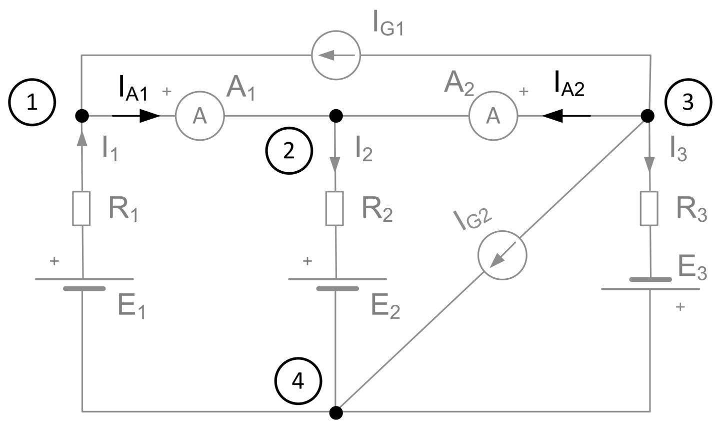

Fig. 1.14 Network with 4 nodes. Ampere meter currents are shown

current is incoming. Orientation of voltage generators is already given. Figure 1.16shows the orientation of all voltage sources and all voltage drops.

Now, Kirchhoff's equations can be written using Figure 1.14 and Figure 1.16.

From Figure 1.14 nodes 1, 2 and 3 are selected to write Kirchhoff's current law equations. For all currents that are coming in the node, positive sign is put, otherwise negative. For node 1:

$$I_1 - I_{A1} + I_{G1} = 0 \tag{1.18}$$

For node 2:

$$I_{A1} - I_2 + I_{A2} = 0 \tag{1.19}$$

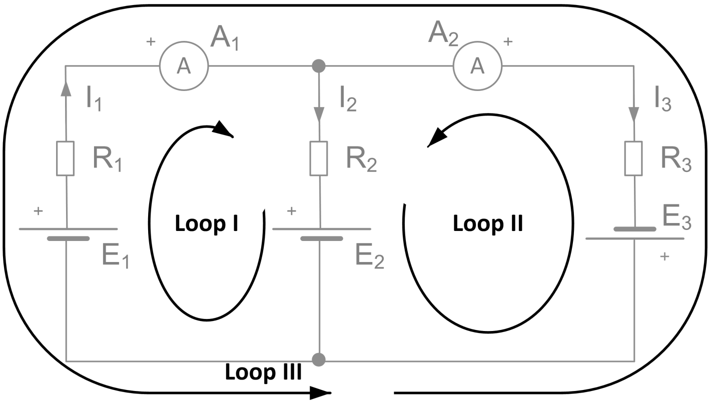

Fig. 1.15 Network with 4 nodes, ampere-meter currents are also shown

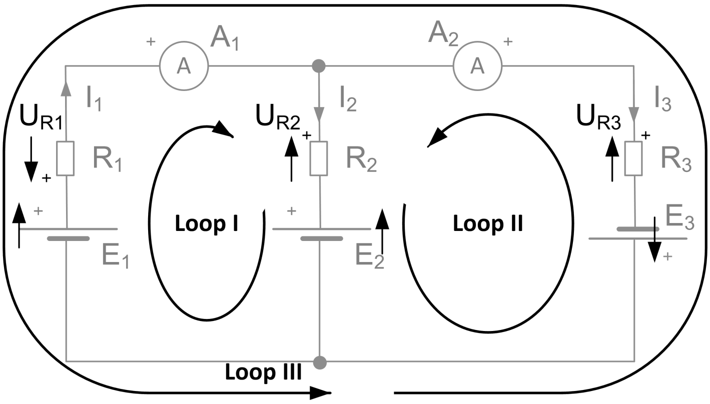

Fig. 1.16 Network with four nodes; ampere-meter currents are also shown

All currents are going out of node 3, so all currents have a negative sign:

$$-I_{A2} - I_{G2} - I_3 - I_{G1} = 0 \tag{1.20}$$

Figure 1.16 is used for writing Kirchhoff's voltage law equations. For Loop I:

$$E_1 - U_{R1} - U_{R2} - E_2 = 0 \tag{1.21}$$

and for loop II:

$$-E_2 - U_{R2} + U_{R3} - E_3 = 0 \tag{1.22}$$

Finally, using Ohm's law:

$$U_{R1} = I_1 R_1 \tag{1.23}$$

$$U_{R2} = I_2 R_2 \tag{1.24}$$

$$U_{R3} = I_3 R_3 \tag{1.25}$$

In the eight Equations (1.18)-(1.25) there are eight unknowns (I_1, I_2, I_3, I_{A1}, I_{A2}, U_{R1}, U_{R2}, and U_{R3},). After placing known values and solving this set of linear equations, the solution is obtained:

$$\begin{aligned}
I_1 &= 1.545A \\
I_2 &= -1.273A \\
I_3 &= 0.818A \\
I_{A1} &= 2.545A \\
I_{A2} &= -3.818A \\
U_{R1} &= 1.545V \\
U_{R2} &= -2.545V \\
U_{R3} &= 2.545V
\end{aligned}$$

Ampere meter A_1 shows $2.545A$. Ampere meter A_2 shows $3.818A$. Current orientation through ampere meter A_2 is opposite to the one shown in Figure 1.14. The orientation of current I_2 is also opposite to the one shown in Figure 1.14. In order to find voltmeter measurements, two additional equations for Kirchhoff's voltage law need to be written. Figure 1.17 shows a part of the original network with voltmeters and loops A and B.

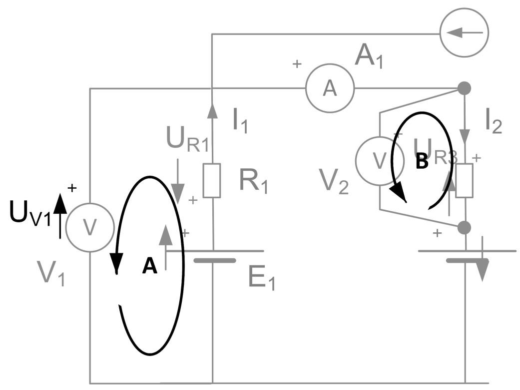

Fig. 1.17 A part of the original network with voltmeters

According to loop A:

$$E_1 - U_{R1} - U_{V1} = 0 \Rightarrow U_{V1} = E_1 - U_{R1} = 1V - 1.545V = -0.545V$$

For loop B:

$$U_{R2} - U_{V2} = 0 \Rightarrow U_{V2} = U_{R2} = -2.545V$$

So, both voltmeters should be placed with positive polarity down (opposite as shown in Figure 1.17.)

1.2.3 Node voltage method

A closer look at the network shown in Figure 1.18 will justify the node voltage method. The circuit shown in the Figure is modified network shown in Figure 1.12. Resistor R7 is here connected between nodes 2 and 4, and because of this, it is not possible to apply the circuit reduction method. Also, using direct application of Kirchhoff's laws would result in a very complicated and confusing procedure. It is necessary to write network equations and solve them using other method.

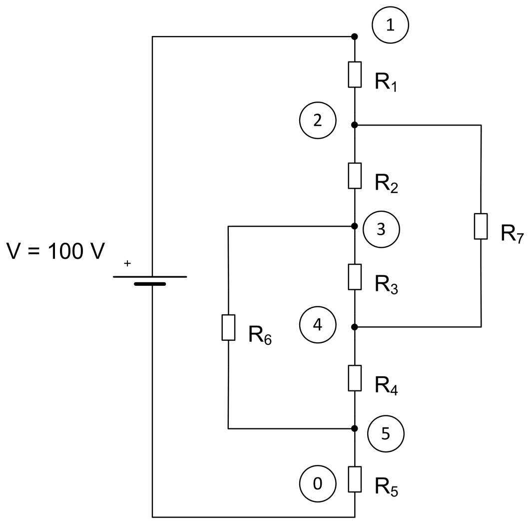

Fig. 1.18 Passive circuit that cannot be reduced

The nodal solution of the system is based on Kirchhoff's current law. Thus, all network resistances must be converted to conductances, and all voltage sources must be converted to current sources.

Figure 1.19 shows a portion of a circuit with a number of nodes; 0 designates the reference node (ground). If node voltages of nodes k and i are denoted by V_k, and V_i respectively, and the conductance of the branch between them is denoted by G_{ki}, then the current flowing in this branch from node k to node i can be calculated as:

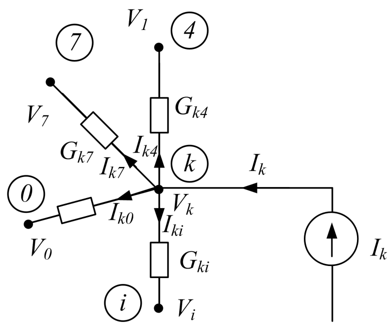

Fig. 1.19 Part of a network with node k

$$I_{ki} = G_{ki}(V_k - V_i) \tag{1.26}$$

Kirchhoff's current law states that the current injected to node k must be equal to the sum of the currents leaving node k:

$$I_k = \sum_{i=0}^{n} I_{ki} = \sum_{i=0}^{n} G_{ki}(V_k - V_i) \tag{1.27}$$

Assuming the system is linear and $V_0 = 0$ (ground), it is valid:

$$\begin{aligned} I_k &= \sum_{\substack{i=0 \\ i \neq k}}^{n} G_{ki} V_k - \sum_{\substack{i=1 \\ i \neq k}}^{n} G_{ki} V_i \\ &= G_{kk} V_k - \sum_{\substack{i=1 \\ i \neq k}}^{n} G_{ki} V_i \end{aligned} \tag{1.28}$$

where

$$G_{kk} = \sum_{\substack{i=0 \\ i \neq k}}^{n} G_{ki} V_k$$

is the sum of all conductances that are connected to node k.

If this equation is written for all nodes except the reference, then the node voltage equations are:

$$
\begin{aligned}
I_1 &= G_{11}V_1 - G_{12}V_2 \quad -G_{13}V_3 - \cdots - G_{1n}V_n \\
I_2 &= -G_{21}V_1 + G_{22}V_2 \quad -G_{23}V_3 - \cdots - G_{2n}V_n \\
I_3 &= -G_{31}V_1 - G_{32}V_2 \quad +G_{33}V_3 - \cdots - G_{3n}V_n \\
&\vdots \qquad\qquad\qquad \vdots \\
I_n &= -G_{n1}V_1 - G_{n2}V_2 \quad -G_{n3}V_3 - \cdots + G_{nn}V_n
\end{aligned}
\tag{1.29}
$$

Writing Equations(1.29) in matrix form gives:

$$
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} G_{11} & -G_{12} & -G_{13} & \cdots & -G_{1n} \\ -G_{21} & G_{22} & -G_{23} & \cdots & -G_{2n} \\ -G_{31} & -G_{32} & G_{33} & \cdots & -G_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -G_{n1} & -G_{n2} & -G_{n3} & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} \tag{1.30}
$$

where,

- I_k is the sum of currents that flow into or from the node k. Current entering a node is positive in sign, while current leaving the node is negative.
- G_{kk} is the sum of all conductances connected to node k,
- G_{ki} is equal to sum of all conductances between node k and node i.

Finally, Equation (1.30) can be written as

$$
\underbrace{I}_{n\times 1} = \underbrace{G}_{n\times n}\underbrace{V}_{n\times 1} \tag{1.31}
$$

1.2.3.1 DC voltage generators

Node voltage method requires current generators and circuit conductances. DC generator can be converted to current generator using Norton's theorem. Figure 1.20 shows DC generator with EMF E and internal resistance R.

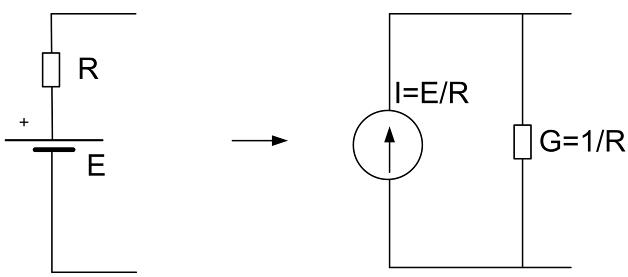

Fig. 1.20 Norton's equivalent of a voltage generator

Relations between Norton's equivalent current generator and voltage generator are given as,

$$\begin{aligned} I_{Nort} &= \frac{E}{R} \\ G_{Nort} &= \frac{1}{R} \end{aligned} \tag{1.32}$$

In case there is an ideal voltage source connected between a node and the reference, it is easy to conclude that the voltage of that node is equal to EMF of that voltage source. Voltage for that node is known, and Kirchhoff's current law equation needs to be replaced with

$$V_i = E_i$$

where i is the node where the ideal voltage source is connected. For example, if node 1 is connected to an ideal voltage source, the node voltage equations in matrix form are written as:

$$\begin{bmatrix} E_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -G_{21} & G_{22} & -G_{23} & \cdots & -G_{2n} \\ -G_{31} & -G_{32} & G_{33} & \cdots & -G_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -G_{n1} & -G_{n2} & -G_{n3} & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix} \tag{1.33}$$

or

$$\underbrace{I_m}_{n\times 1} = \underbrace{G_m}_{n\times n} \underbrace{V}_{n\times 1} \tag{1.34}$$

where m denotes modified matrices.
Network equations can be formulated in a variety of forms, but node voltage method is commonly used for power system problems. Some of many advantages of the node voltage method are:

- simple numbering of the nodes directly from the system diagram, easy data preparation,
- number of variables and equations is usually less than with other methods used for power system analysis,
- parallel branches do not increase the number of variables or equations,
- node voltages are available directly from the solution.

Example

Obtain network currents for the network shown in Figure 1.21. Given is the following data: $G_{12} = 0.5S$, $G_{13} = 0.3S$, $G_{23} = 0.5S$, $G_{24} = 0.2S$, $G_{34} = 0.4S$, $G_{20} = 0.05S$, $E_1 = 100V$, $I_4 = 5A$ and $I_3 = 2A$.

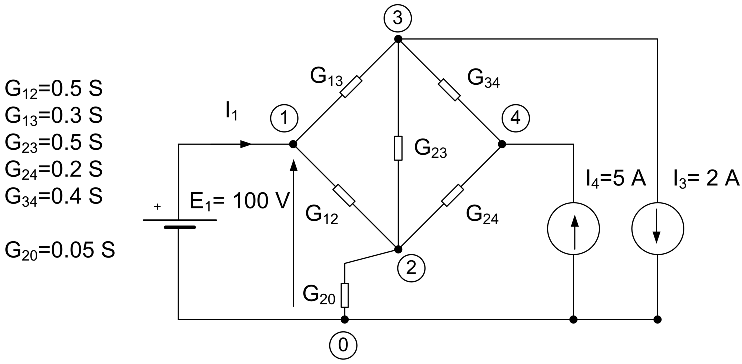

Fig. 1.21 DC Network

Solution

Node voltage equations for the given network are:

$$\begin{bmatrix} E_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -G_{12} & G_{20}+G_{12}+G_{23}+G_{24} & -G_{23} & -G_{24} \\ -G_{13} & -G_{23} & G_{13}+G_{23}+G_{34} & -G_{34} \\ 0 & -G_{24} & -G_{34} & G_{24}+G_{34} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$\begin{bmatrix} 100 \\ 0 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.5 & 1.25 & -0.5 & -0.2 \\ -0.3 & -0.5 & 1.2 & -0.4 \\ 0 & -0.2 & -0.4 & 0.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

which gives the following network voltages:

$$\begin{aligned} V_1 &= 100V \\ V_2 &= 96.78V \\ V_3 &= 99.24V \\ V_4 &= 106.75V \end{aligned}$$

Network currents are:

$$I_{13} = (V_1 - V_3)\,G_{13} = (100 - 99.24)\cdot 0.3 = 0.228A$$

$$I_{12} = (V_1 - V_2)\,G_{12} = (100 - 96.78)\cdot 0.5 = 1.61A$$

$$I_{43} = (V_4 - V_3)\,G_{34} = (106.75 - 99.24)\cdot 0.4 = 3A$$

$$I_{42} = (V_4 - V_2)\, G_{24} = (106.75 - 96.78) \cdot 0.2 = 1.994A$$

$$I_{32} = (V_3 - V_2)\, G_{23} = (99.24 - 96.78) \cdot 0.5 = 1.23A$$

$$I_1 = I_{12} + I_{13} = 1.838A$$

1.3 Analysis of AC Networks

Power system that is commonly used around the world is AC system. Therefore, all DC current laws are in the following generalized to describe the current and voltage relations in AC networks. AC networks are composed of capacitors, inductors and resistors, and electrical quantities are sinusoidal temporal functions. All laws previously formulated for DC networks retain their formulation, except that now relevant quantities are time-dependent. Time dependent quantities are denoted by small letters.

1.3.1 Complex Impedance in AC Circuits

Behavior of components in AC circuit is not as trivial as in DC circuits. Impedance can be complex and frequency dependent. To illustrate the issue, a simple RL circuit shown in Figure 1.22 is considered.

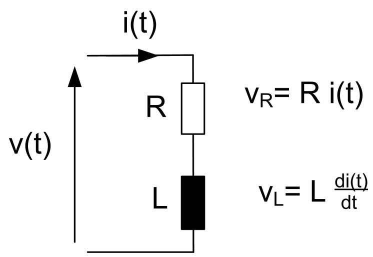

Fig. 1.22 RL circuit

Kirchhoff's voltage law states:

$$Ri(t) + L\frac{di(t)}{dt} = v(t) \tag{1.35}$$

where

$$v(t) = V_m cos(\omega t + \varphi_v) \tag{1.36}$$

To solve the differential Equation(1.35) for *i(t)*, it will be assumed that the solution has the form:

$$i(t) = Acos(\omega t + \varphi_v) + Bsin(\omega t + \varphi_v)$$

Using this, equation1.35 can be written as:

$$R\left(Acos(\omega t + \varphi_v) + Bsin(\omega t + \varphi_v)\right) + L\left(-A\omega sin(\omega t + \varphi_v) + B\omega cos(\omega t + \varphi_v)\right)$$

$$= V_m cos(\omega t + \varphi_v) \tag{1.37}$$

Equating the coefficients of $cos(\omega t + \varphi_v)$ and coefficients of $sin(\omega t + \varphi_v)$, two equations are obtained:

$$R \cdot A + \omega L \cdot B = V_m \tag{1.38}$$

$$-\omega L \cdot A + R \cdot B = 0 \tag{1.39}$$

From here:

$$A = \frac{R \cdot V_m}{R^2 + (\omega L)^2} \tag{1.40}$$

$$B = \frac{\omega L \cdot V_m}{R^2 + (\omega L)^2} \tag{1.41}$$

$i(t)$ is then:

$$i(t) = \frac{R \cdot V_m}{R^2 + (\omega L)^2} cos(\omega t + \varphi_v) + \frac{\omega L \cdot V_m}{R^2 + (\omega L)^2} sin(\omega t + \varphi_v)$$

$$i(t) = \frac{V_m}{R^2 + (\omega L)^2} \left[Rcos(\omega t + \varphi_v) + \omega L sin(\omega t + \varphi_v)\right]$$

$$i(t) = \frac{V_m}{R^2 + (\omega L)^2} \sqrt{R^2 + (\omega L)^2} cos(\omega t + \varphi_v - arctan\frac{\omega L}{R})$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} cos\left(\omega t + \varphi_v - arctan\frac{\omega L}{R}\right) \tag{1.42}$$

Using the equation 1.42, maximum value of current $i(t)$, or the RL circuit, is found:

$$I_m = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} = \frac{V_m}{Z}$$

Impedance Z for the circuit is found to be:

$$Z = \sqrt{R^2 + (\omega L)^2}$$

Obviously, solving large networks using this approach would be very complex and time consuming. Exponential functions are much more convenient for manipulation than sinusoidal functions, since both the voltage drop and current through any component in the network can be represented as a complex variable.

1.3.1.1 Complex Impedance of Resistor

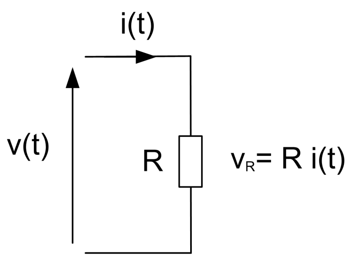

Fig. 1.23 Impedance and admittance

Ohm's law relates voltage and current of the resistor as:

$$v_r(t) = i_r(t)R$$

It is assumed that voltage generator has the form

$$v_r(t) = V_m sin(\omega t)$$

It follows that

$$\underline{Z}_r = \frac{V_m sin(\omega t)}{I_m sin(\omega t)} = R \tag{1.43}$$

In an AC circuit involving only resistors, the current and voltage are in phase with each other. This means that the peak voltage is reached at the same moment as peak current (Figure 1.24).

1.3.1.2 Complex Impedance of Inductor

Voltage-current relation for an inductor is given by

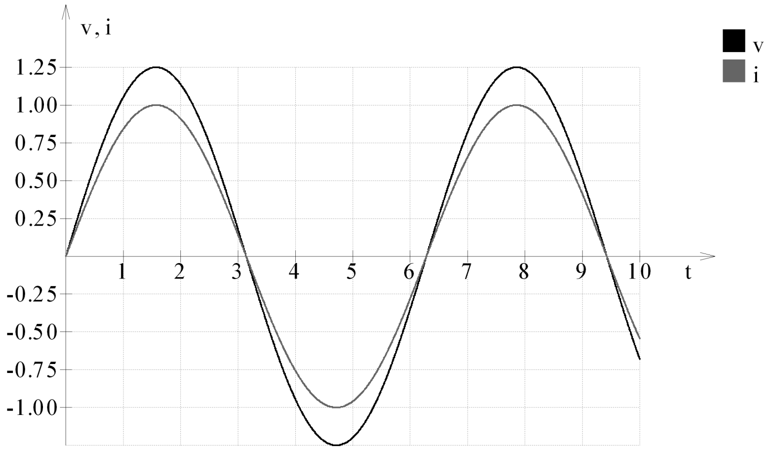

Fig. 1.24 Voltage and current of a resistor in AC network.

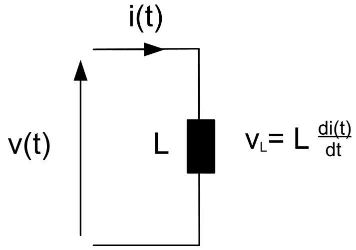

Fig. 1.25 Voltage across inductor in an AC circuit

$$v_L(t) = L\frac{di_L(t)}{dt} \tag{1.44}$$

For simplicity, the current waveform is assumed to be

$$i_L(t) = I_L sin(\omega t) \tag{1.45}$$

$$\frac{di_L(t)}{dt} = I_L \omega cos(\omega t) \tag{1.46}$$

Combining Equation (1.45) and Equation 1.45yields

$$Z_L = \frac{v_L(t)}{i_L(t)} = \frac{LI_L \omega cos(\omega t)}{I_L \omega sin(\omega t)} \tag{1.47}$$

Using the equality $cos(\omega t) = sin(\omega t + \frac{\pi}{2})$

$$Z_L = \frac{v_L(t)}{i_L(t)} = \frac{\omega L sin(\omega t + \frac{\pi}{2})}{sin(\omega t)} \tag{1.48}$$

Equation (1.48) shows that

- the ratio of AC voltage amplitude to AC current amplitude across an inductor is ωL
- the AC voltage **leads** the AC current across an inductor by $\frac{\pi}{2}$ radians or 90° as illustrated in Figure 1.26.

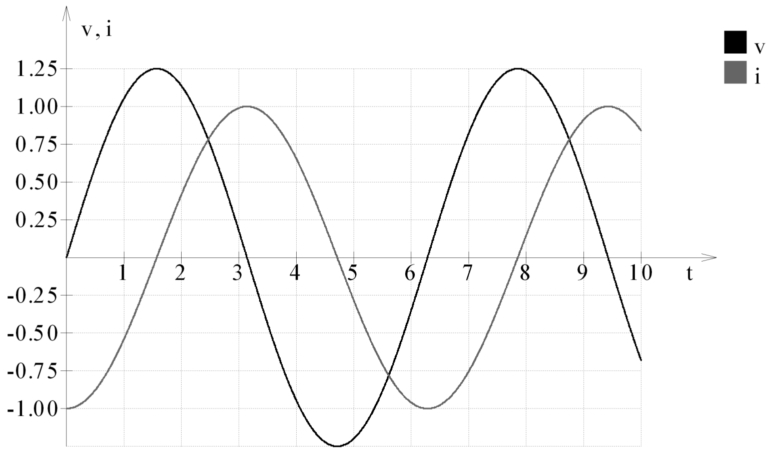

Fig. 1.26 Voltage and current on inductor in AC circuit

Equation (1.48) written in complex polar form is

$$\underline{Z}_L = \omega L e^{j\frac{\pi}{2}} \tag{1.49}$$

Using Euler's formula, equation (1.49) turns to be

$$\underline{Z}_L = j\omega L \tag{1.50}$$

Laplace transform can be used to calculate the inductor impedance in s-domain. Laplace transform of Equation 1.44gives

$$V_L(s) = L\left[sI_L(s) - I_{L0}\right] \tag{1.51}$$

where,
$V_L(s) = \mathcal{L}\{v_L(t)\}$,
$I_L(s) = \mathcal{L}\{i(t)\}$, and
$I_0 = i_L(t)\mid_{t=0}$
The definition of the complex impedance Z is the ratio of the complex voltage V divided by the complex current I while holding the initial state I_o

at zero:

$$Z_L(s) = \frac{V_L(s)}{I_L(s)}\,|_{I_{Lo}=0}$$

Using this definition and Equation (1.51), the inductor impedance is found to be:

$$Z_L(s) = sL \tag{1.52}$$

In case when $s = j\omega$ Equation (1.52) becomes Equation (1.50).

1.3.1.3 Complex Impedance of Capacitor

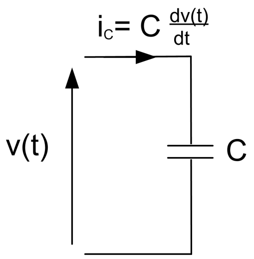

Fig. 1.27 Current through a capacitor in AC network

Relation of voltage and current of a capacitor is stated as:

$$i_c = C\frac{dv_c(t)}{dt} \tag{1.53}$$

Assuming the voltage waveform to be

$$v_c(t) = V_c sin(\omega t) \Rightarrow \frac{dv_c(t)}{dt} = \omega V_c cos(\omega t)$$

and applying to Equation (1.53), it follows that

$$\begin{aligned}\frac{v_c(t)}{t_c(t)} &= \frac{V_c sin(\omega t)}{\omega C V_c cos(\omega t)} \\ &= \frac{sin(\omega t)}{\omega C sin(\omega t + \frac{\pi}{2})}\end{aligned} \tag{1.54}$$

Equation (1.54) tells that:

- the ratio of AC voltage amplitude to AC current amplitude across a capacitor is $\frac{1}{\omega C}$
- the AC voltage **lags** compared to current across a capacitor by $\frac{\pi}{2}$ radians or 90° as illustrated in Figure 1.28.

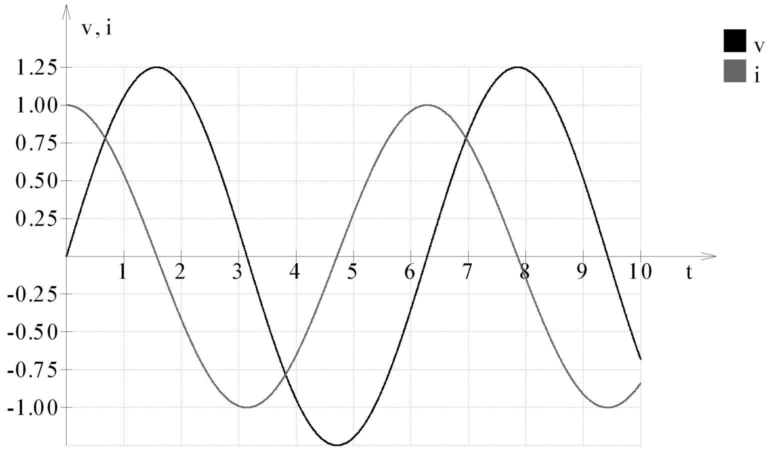

Fig. 1.28 Voltage and current on capacitor in AC circuit.

This result is commonly expressed in polar form as

$$Z_c = \frac{1}{\omega C} e^{-j\frac{\pi}{2}} \tag{1.55}$$

or after applying Euler's equation

$$\begin{aligned} Z_c &= -j\frac{1}{\omega C} \\ &= \frac{1}{j\omega C} \end{aligned} \tag{1.56}$$

Laplace transform to can again be used to calculate impedance of a capacitor in s-domain. Taking Laplace transform of Equation (1.53), it follows that

$$I_c(s) = C\left[sV_c(s) - V_{c0}\right] \tag{1.57}$$

where,
$I_c(s) = \mathcal{L}\{i(t)\}$,
$V_c(s) = \mathcal{L}\{v_c(t)\}$, and
$V_{c0} = v_c(t)\mid_{t=0}$

The definition of the complex impedance Z is the ratio of the complex voltage V divided by the complex current I while holding the initial state V_o at zero:

$$Z_c(s) = \frac{V_c(s)}{I_c(s)}\mid_{V_{co}=0}$$

Using this definition and Equation (1.57), we find:

$$Z_c(s) = \frac{1}{sC} \tag{1.58}$$

In case when $s = j\omega$, Equation (1.58) becomes Equation (1.56).

1.3.1.4 Representation of complex impedance

A complex impedance can be represented in two different formats in either the Euclidean or polar coordinate system (Figure 1.29):

- $z = x + jy$ - Euclidean representation, where x is the real (horizontal) and y is the imaginary (vertical) part of a complex variable;
- $z = |z|e^{j\angle z}$ - Polar representation where |z| is the complex number magnitude, and ∠z is the phase angle .

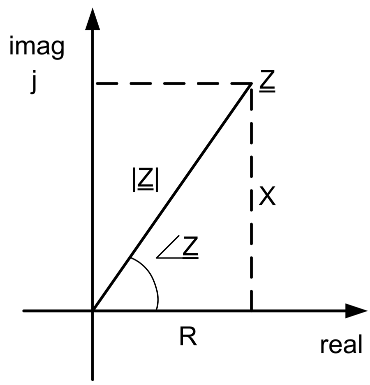

Fig. 1.29 Impedance in complex plane

Ohm's law for AC circuits in impedance and admittance form is given in Equation(1.59) and Equation(1.60) respectively:

$$\underline{V} = \underline{Z}\underline{I} = (R + jX)\underline{I} \tag{1.59}$$

$$\underline{I} = \underline{Y}\underline{V} = (G + jB)\underline{V} \tag{1.60}$$

$\underline{Y} = \frac{1}{\underline{Z}}$ is called admittance; X is reactance; B is susceptance. For RL circuit, the relations are:

$$\underline{V} = \underline{Z}\underline{I} = (R + j\omega L)\underline{I} = R\underline{I} + j\omega L\underline{I} \tag{1.61}$$

Phasor representation of Equation(1.61) is shown in Figure 1.30.

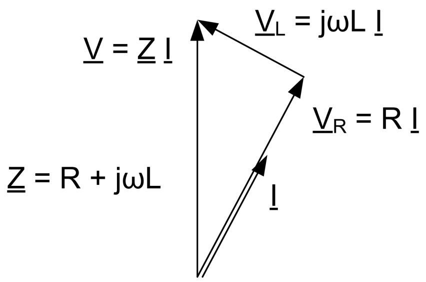

Fig. 1.30 Phasor diagram of a simple RL circuit

1.3.2 Power in AC Circuits

Consider a simple RLC circuit in Figure (1.31), supplied by an AC power source with maximum voltage V_{max} and maximum current I_{max}

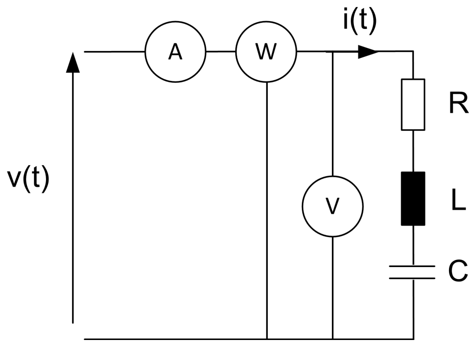

Fig. 1.31 RLC circuit supply by AC generator

The voltage and current at one point can be expressed by

$$v(t) = V_{max} sin(\omega t + \varphi_v)$$

and

$$i(t) = I_{max} sin(\omega t + \varphi_i)$$

as shown in Figure 1.32.

Instantaneous power delivered to the circuit is the product of the voltage $v(t)$ and the current$i(t)$ given by:

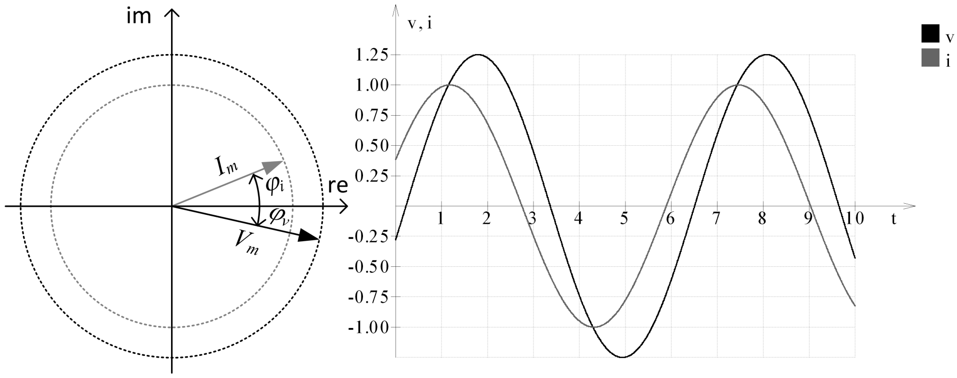

Fig. 1.32 RLC circuit powered by an AC generator

$$p(t) = v(t)i(t) \tag{1.62}$$

Inserting the voltage and current:

$$p(t) = V_m I_m sin(\omega t + \varphi_v) sin(\omega t + \varphi_i)$$

Using $sin\alpha sin\beta = \frac{1}{2}\left[cos(\alpha - \beta) - cos\,(\alpha + \beta)\right]$ the power can be rewritten to>

$$\begin{aligned} p(t) &= \frac{1}{2} V_m I_m \left[cos\,(\varphi_v - \varphi_i) - cos\,(2\omega t + \varphi_v + \varphi_i)\right] \\ &= \frac{1}{2} V_m I_m \left\{cos\,(\varphi_v - \varphi_i) - cos\,[2(\omega t + \varphi_v) - (\varphi_v - \varphi_i)]\right\} \end{aligned}$$

Let $\varphi = \varphi_v - \varphi_i = arg(\underline{Z})$. Then,

$$p(t) = \frac{1}{2} V_m I_m \left\{cos\,(\varphi) - cos\,[2(\omega t + \varphi_v) - (\varphi)]\right\}$$

applying $cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta$

$$\begin{aligned} p(t) &= \frac{1}{2} V_m I_m \left[cos\,(\varphi) - cos(2(\omega t + \varphi_v)) cos(\varphi) - sin(2(\omega t + \varphi_v)) sin(\varphi)\right] \\ &= \frac{1}{2} V_m I_m \left[1 - cos(2(\omega t + \varphi_v))\right] cos\varphi - \frac{1}{2} V_m I_m sin(2(\omega t + \varphi_v)) sin\varphi \end{aligned} \tag{1.63}$$

The average active power is defined as

$$P = \frac{1}{T} \int_0^T p(t) dt \tag{1.64}$$

From Equation(1.64) and Equation(1.63), it follows that the average active power delivered to the circuit is:

$$
\begin{aligned}
P &= \frac{1}{T}\int_0^T \left\{\frac{1}{2}V_m I_m\left[1-cos2(\omega t+\varphi_v)\right]cos\varphi-\frac{1}{2}V_m I_m sin2(\omega t+\varphi_v)sin\varphi\right\}dt \\
&= \frac{1}{2T}V_m I_m\left\{\left[t-\frac{1}{2\omega}sin(2\omega t+2\varphi_v)\right]cos\varphi+\frac{t}{2\omega}cos\left(2\omega t+2\varphi_v\right)sin\varphi\right\}\Big|_0^T \\
&= \frac{1}{2}V_m I_m cos\varphi
\end{aligned}
\tag{1.65}
$$

1.3.2.1 RMS Value

The value of an AC voltage is continually changing from zero up to the positive peak, through zero to the negative peak and back to zero again. Clearly, for most of the time it is less than the peak voltage, so this is not a good measure of its real effect. Instead, the root-mean-square or RMS voltage and current are used. They are $\frac{1}{\sqrt{2}}$ of the peak voltage and current, respectively. The RMS value is the effective value of a varying voltage or current. It is the equivalent steady DC (constant) value which gives the same effect.

$$
\begin{aligned}
P_{DC} &= P_{AC} \\
I^2R &= \frac{1}{T}\int_0^T i^2(t)Rdt \\
I^2 &= \frac{1}{T}\int_0^T i^2(t)dt \\
&= \frac{1}{T}\int_0^T I_m^2 Rsin^2(\omega t+\varphi_i)dt
\end{aligned}
\tag{1.66}
$$

$$
I^2 = \frac{1}{T}\int_0^T I_m^2 sin^2(\omega t+\varphi_i)dt \tag{1.67}
$$

In order to solve Equation (1.67) for I , it needs to be simplified using sin^2x expression:

$$
\begin{aligned}
sin^2x &= 1-1+sin^2x \\
&= 1-(sin^2x+cos^2x)+sin^2x \\
&= 1-sin^2x-(cos^2x-sin^2x)
\end{aligned}
\tag{1.68}
$$

the equality $cos(2x) = cos^2x - sin^2x$ is inserted into Equation (1.68),

$$\begin{aligned} sin^2x &= 1 - sin^2x - cos(2x) \\ 2sin^2x &= 1 - cos(2x) \\ sin^2x &= \frac{1}{2}(1 - cos(2x)) \end{aligned} \tag{1.69}$$

Applying simplification (1.69) in Equation (1.67)

$$\begin{aligned} I^2 &= \frac{I_m^2}{2T}\int_0^T [1 - cos(2\omega t + 2\varphi_i)]\, dt \\ &= \frac{I_m^2}{2T}\left[t - \frac{1}{2\omega} sin(2\omega t + 2\varphi_i)t\right]_0^T \\ &= \frac{I_m^2}{2T}[T - 0] \\ &= \frac{I_m^2}{2} \\ I &= \frac{I_m}{\sqrt{2}} \end{aligned} \tag{1.70}$$

AC voltmeters and ampere meters show the RMS value of the measured voltage or current. So in all complex calculations, RMS values are used. Time dependent quantity $i(t)$ given in Equation(1.36) can be represented in complex form as:

$$\underline{I} = \frac{I_m}{\sqrt{2}} e^{j\omega\varphi_v} = I e^{j\omega\varphi_v}$$

where $I = \frac{I_m}{\sqrt{2}}$ is the current effective or root mean square (RMS) value.

Example

Calculate the RMS value for the current shown in Figure 1.33.

Solution

The first quarter of the linear function can be written as

$$i(t) = \frac{I_{max}}{\frac{T}{4}} t = \frac{4I_{max}}{T} t$$

RMS value is calculated for the first quarter of the period. For other three quarters, the result is the same. According to Equation (1.67)

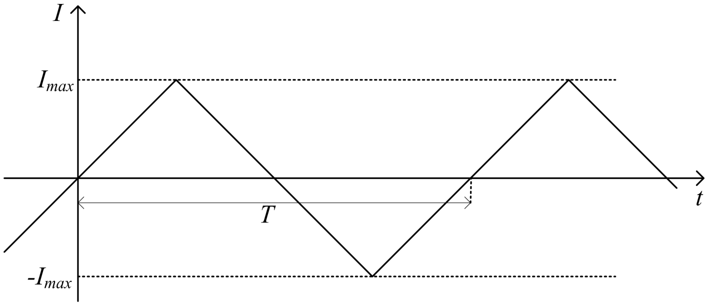

Fig. 1.33 Current with triangular waveform

$$
\begin{aligned}
I_{rms}^2 &= \frac{1}{\frac{T}{4}} \int_0^{\frac{T}{4}} \frac{4^2 I_{max}^2}{T^2} t^2 dt \\
&= \frac{4^3 I_{max}^2}{T^3} \int_0^{\frac{T}{4}} t^2 dt \\
&= \frac{4^3 I_{max}^2}{T^3} (\frac{1}{3} t^3) \mid_0^{\frac{T}{4}} \\
&= \frac{4^3 I_{max}^2}{3T^3} \left(\frac{T^3}{4^3} - 0 \right) \\
I_{rms}^2 &= \frac{I_{max}}{3} \\
I_{rms} &= \frac{I_{max}}{\sqrt{3}}
\end{aligned}
$$

The triangular waveform from Figure 1.33 has RMS current value of $\frac{I_{max}}{\sqrt{3}}$.

1.3.2.2 Active and reactive power

If voltage RMS value

$$V_{rms} = \frac{V_m}{\sqrt{2}} \tag{1.71}$$

and current RMS value

$$I_{rms} = \frac{I_m}{\sqrt{2}} \tag{1.72}$$

are inserted in Equation (1.65), the average active power is

$$P = V_{rms} I_{rms} cos\varphi \tag{1.73}$$

V_{rms} and I_{rms} are voltage and current root mean square (RMS) values. In case of load, term $cos\varphi$ is also called power factor. It is very important to keep its value close to 1 - the higher the power factor, the greater is the active power that can be generated, currents are lower, i.e. generation and transmission of energy is more efficient.

In power system analysis, Equation (1.63) is presented in the complex form. The complex power designated by S is given by:

$$\underline{S} = P + jQ \tag{1.74}$$

and is calculated using

$$\underline{S} = \underline{V}\underline{I}^* \tag{1.75}$$

where $\underline{I}*$ represents RMS conjugate complex current value.

Q is reactive power and from Equation1.74 it is defined as:

$$Q = Im\{\underline{V}\underline{I}^*\} = VIsin\varphi \tag{1.76}$$

where $\varphi = arg(\underline{V}) - arg(\underline{I})$.

Active power P is defined as:

$$P = Re\{\underline{V}\underline{I}^*\} = VIcos\varphi \tag{1.77}$$

Unit for S is VA, for P is W and for Q it is VAr.

1.3.2.3 Dissipated active power in resistor

According to Section 1.3.1.1, phase difference between voltage and current on resistor is equal to zero ($\varphi = 0$), as shown in Figure 1.34. After applying such φ into Equation (1.76) and (1.77) we obtain

$$\begin{aligned} Q_r &= 0 \\ P_r &= V_r I_r \end{aligned}$$

This shows that resistor *consumes* active power.

1.3.2.4 Reactive power in inductor

According to Section 1.3.1.2, phase difference between voltage and current on resistor is equal to $\frac{\pi}{2}$. ($\varphi = \frac{\pi}{2}$), as shown in Figure 1.35. After inserting this φ into Equation (1.76) and (1.77), the reactive and active power is

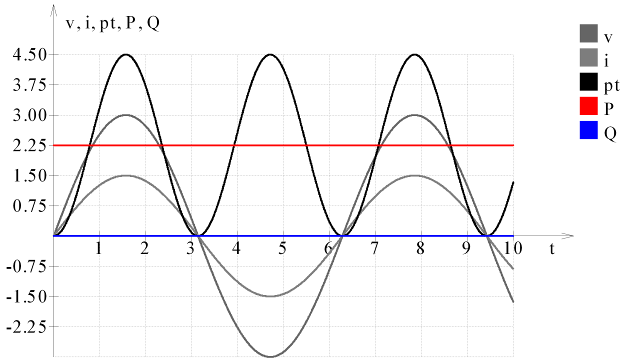

Fig. 1.34 AC current waveforms on resistor with average active and reactive power

$$Q_l = V_l I_l$$
$$P_r = 0$$

Thus, perfect inductor *consumes* reactive power.

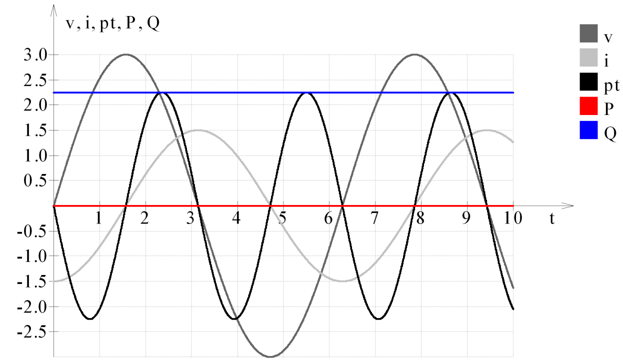

Fig. 1.35 C current waveforms on inductor with average active and reactive power

1.3.2.5 Reactive power in capacitor

According to Section 1.3.1.3, phase difference between voltage and current on resistor is equal to $-\frac{\pi}{2}$. ($\varphi = -\frac{\pi}{2}$), as shown in Figure 1.36. When this value is inserted as φ in Equation (1.76) and (1.77) the reactive and active power are

$$Q_l = -V_l I_l$$
$$P_r = 0$$

That is, capacitor *produces* reactive power.

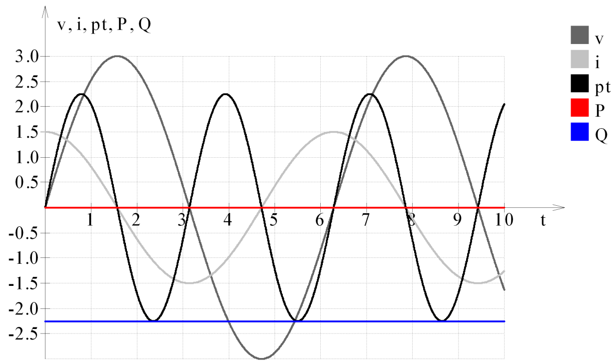

Fig. 1.36 C current waveforms on capacitor with average active and reactive power

1.3.2.6 Active and reactive power in complex plane

Similar to impedances, power components can also be shown in complex plane, as shown in Figure 1.37.

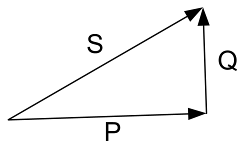

Fig. 1.37 Power components - active and reactive power in complex plane.

Example

Consider Figure 1.38. With $R = 1\Omega$, $X = 10\Omega$ and $\underline{V_1} = V_1 = 100kV$, calculate the complex voltage V_2 and the complex power on both sides for:
a) $\underline{I_1} = 2kA$,
b) $\underline{I_1} = j2kA$.

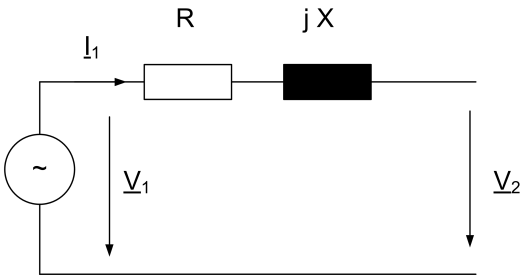

Fig. 1.38 Circuit with complex impedance R+jX

Solution

a) Kirchhoff's voltage law gives:

$$\underline{V}_1 - (R + jX)\underline{I}_1 - \underline{V}_2 = 0$$

$$\underline{V}_2 = \underline{V}_1 - (R + jX)\underline{I}_1$$

$$\underline{V}_2 = 100 \cdot 10^3 - (1 + j10) \cdot 2 \cdot 10^3$$

$$\underline{V}_2 = (98 - j20)kV$$

Complex power at side 1 is:

$$\underline{S}_1 = \underline{V}_1 \underline{I}_1^* = 200MVA$$

Complex power at side 2 is:

$$\underline{S}_2 = \underline{V}_2 \underline{I}_1^* = (196 - j40)MVA$$

Note that:

$$\underline{S}_1 - \underline{S}_2 = (4 + j40)MVA$$

which is equal to power delivered to impedance:

$$\triangle \underline{S} = \underline{Z} I_1^2 = (1 + j10) \cdot 4 \cdot 10^6 = (4 + j40)MVA$$

b) Same as above:

$$\underline{V}_2 = \underline{V}_1 - (R + jX)\underline{I}_1$$

$$\underline{V}_2 = 100 \cdot 10^3 - (1 + j10) \cdot j2 \cdot 10^3$$

$$\underline{V}_2 = (120 - j2)kV$$

Complex power at side 1 is:

$$\underline{S}_1 = \underline{V}_1 \underline{I}_1^* = -j200MVA$$

Complex power at side 2 is:

$$\underline{S}_2 = \underline{V}_2 \underline{I}_1^* = (-4 - j240)MVA$$

1.3.2.7 Node voltage method for AC networks

Node voltage method for AC networks is basically the same as for DC networks, as explained in subsection 1.2.3. The only modification is that admittances are used in stead of conductances. The application of node voltage method on AC networks is illustrated in the following example.

Example

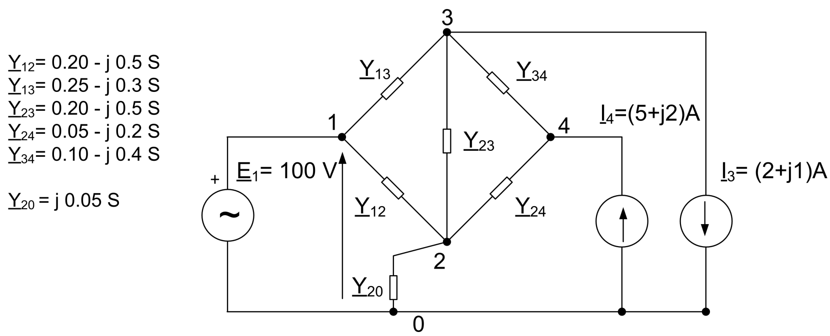

Fig. 1.39 AC electrical circuit

Obtain network currents for the network shown in Figure 1.39. Given is the following data: $\underline{Y}_{12} = (0.2-j0.5)S$, $\underline{Y}_{13} = (0.25-j0.3)S$, $\underline{Y}_{23} = (0.2-j0.5)S$, $\underline{Y}_{24} = (0.05 - j0.2)S$, $\underline{Y}_{34} = (0.1 - j0.4)S$, admittance of shunt capacitor $\underline{Y}_{20} = j0.05S$, $\underline{E}_1 = 100V$, $\underline{I}_4 = (5 + j2)A$ and $\underline{I}_3 = (2 + j1)A$.

Solution

Node voltage equations for the given network are:

$$\begin{bmatrix} \underline{E}_1 \\ \underline{I}_2 \\ \underline{I}_3 \\ \underline{I}_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\underline{Y}_{12} & \underline{Y}_{20}+\underline{Y}_{12}+\underline{Y}_{23}+\underline{Y}_{24} & -\underline{Y}_{23} & -\underline{Y}_{24} \\ -\underline{Y}_{13} & -\underline{Y}_{23} & \underline{Y}_{13}+\underline{Y}_{23}+\underline{Y}_{34} & -\underline{Y}_{34} \\ 0 & -\underline{Y}_{24} & -\underline{Y}_{34} & \underline{Y}_{24}+\underline{Y}_{34} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_3 \\ \underline{V}_4 \end{bmatrix}$$

$$\begin{bmatrix} 100 \\ 0 \\ -2-j1 \\ 5+j2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.2+j0.5 & 0.45-j1.25 & -0.2+j0.5 & -0.05+j0.2 \\ -0.25+j0.3 & -0.2+j0.5 & 0.55-j1.2 & -0.1+j0.4 \\ 0 & -0.05+j0.2 & -0.1+j0.4 & 0.15-j0.6 \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \underline{V}_3 \\ \underline{V}_4 \end{bmatrix}$$

From here, network voltages are obtained:

$$\begin{aligned} \underline{V}_1 &= 100V \\ \underline{V}_2 &= (94.88+j5.60)V \\ \underline{V}_3 &= (97.49+j5.25)V \\ \underline{V}_4 &= (95.44+j13.99)V \end{aligned}$$

Network currents are:

$$\underline{I}_{13} = (\underline{V}_1 - \underline{V}_3)\,\underline{Y}_{13} = (-0.95-j2.06)A$$

$$\underline{I}_{12} = (\underline{V}_1 - \underline{V}_2)\,\underline{Y}_{12} = (-1.78-j3.68)\,A$$

$$\underline{I}_{43} = (\underline{V}_4 - \underline{V}_3)\,\underline{Y}_{34} = (3.29+j1.69)\,A$$

$$\underline{I}_{42} = (\underline{V}_4 - \underline{V}_2)\,\underline{Y}_{24} = (1.71+j0.31)\,A$$

$$\underline{I}_{32} = (\underline{V}_3 - \underline{V}_2)\,\underline{Y}_{23} = (0.35-j1.37)\,A$$

$$\underline{I}_1 = \underline{I}_{12} + \underline{I}_{13} = (-2.72-j5.74)\,A$$

1.4 Conclusions

This chapter briefly recapitulates on the basics of circuit analysis, complex voltages, currents and power, their units, and components that are a part of a typical AC circuit. Some examples are given to illustrate the elementary methods which are used in power systems analysis and refresh knowledge of computer tools that can make this analysis simple. Appendix A is a manual to MATLAB giving sufficient instruction to develop a solver of such problems.

Chapter 2
Symmetrical Three-Phase Network

Common power systems of today are balanced three-phase systems designed to provide constant power delivery to the loads. Electric power is generated by the generator and then transmitted to the load over transmission system. If the network voltages of the three phases reach their peak values in the phase sequential order ABC, they are said to have a positive phase sequence, and if the phase order is ACB, they have a negative phase sequence. Assuming a positive phase sequence (phase order ABC) symmetrical voltage triangle is expressed as (Figure 2.1):

$$\begin{aligned} \underline{V}_A &= V_A \angle 0^o \\ \underline{V}_B &= V_A \angle -120^o \\ \underline{V}_C &= V_A \angle 120^o \end{aligned} \tag{2.1}$$

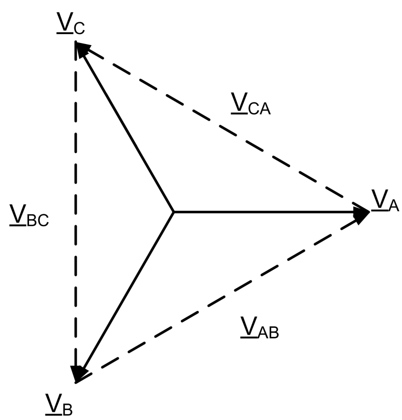

Fig. 2.1 Positive phase sequence

Introducing operator $\underline{a} = e^{j120^0}$ Equation(2.1) can be written as:

$$\begin{aligned} \underline{V}_A =& V_A \\ \underline{V}_B =& \underline{a}^2 V_A \\ \underline{V}_C =& \underline{a} V_A \end{aligned} \tag{2.2}$$

Operator $\underline{a}$ causes counterclockwise rotation of 120^o, thus:

$$\begin{aligned} \underline{a} = e^{j120^0} =& -0.5 + j\sqrt{3} \\ \underline{a}^2 = e^{j240^0} =& -0.5 - j\sqrt{3} \end{aligned} \tag{2.3}$$

It follows that:

$$1 + \underline{a} + \underline{a}^2 = 0 \tag{2.4}$$

Negative phase sequence of voltages is represented as:

$$\begin{aligned} \underline{V}_A =& V_A \\ \underline{V}_B =& \underline{a} V_A \\ \underline{V}_C =& \underline{a}^2 V_A \end{aligned} \tag{2.5}$$

In power system analysis, it is important to differentiate phase voltages from line-to-line voltages. Phase voltage is the potential difference between phase and the neutral, while line-to-line voltage is potential difference between two phases. Nominal voltages in symmetrical three phase networks are always expressed in terms of line-to-line voltages.

A power system has Y-connected generators and usually includes both Δ and Y-connected loads. Generators are rarely Δ-connected. Terminal voltages of the generator and terminal voltages of the load are also symmetrical. A Y-connected generator supplying one balanced Y-connected load is shown in Figure 2.2.

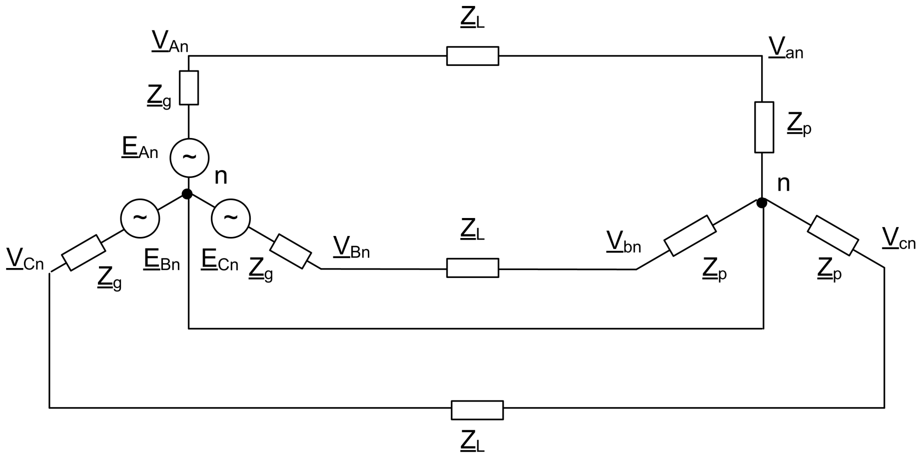

Fig. 2.2 Y-connected load

It can be seen that:

$$\underline{V}_{An} = \underline{E}_{An} - \underline{Z}_G \underline{I}_a \tag{2.6}$$

$$\underline{V}_{an} = \underline{V}_{An} - \underline{Z}_L \underline{I}_a \tag{2.7}$$

Using Kirchhoff's voltage law, the line voltages at the load terminals are:

$$\begin{aligned} \underline{V}_{ab} &= \underline{V}_{an} - \underline{V}_{bn} = V_p(1\angle 0^o - 1\angle -120^o) = \sqrt{3}V_p\angle 30^o \\ \underline{V}_{bc} &= \underline{V}_{bn} - \underline{V}_{cn} = V_p(1\angle 120^o - 1\angle 120^o) = \sqrt{3}V_p\angle -90^o \\ \underline{V}_{ca} &= \underline{V}_{cn} - \underline{V}_{an} = V_p(1\angle 120^o - 1\angle 0^o) = \sqrt{3}V_p\angle 150^o \end{aligned} \tag{2.8}$$

where V_p is the magnitude of phase voltage. From Equation(2.8) it can be concluded that in the case of Y-connected load, the magnitude of the line voltage is $\sqrt{3}$ times the magnitude of the phase voltage, and for the positive phase sequence, the set of line voltages leads the set of phase voltages by 30^o. The currents in lines are also the phase currents. It is obvious that:

$$\underline{I}_a + \underline{I}_b + \underline{I}_c = I_a(1 + \underline{a} + \underline{a}^2) = 0 \tag{2.9}$$

In case of balanced Δ-connected load (Figure (2.3)) the line voltages are the same as phase voltages. Line currents are:

$$\begin{aligned} \underline{I}_a &= \underline{I}_{ab} - \underline{I}_{ca} = \sqrt{3}I_p\angle -30^o \\ \underline{I}_b &= \underline{I}_{bc} - \underline{I}_{ab} = \sqrt{3}I_p\angle -150^o \\ \underline{I}_c &= \underline{I}_{ca} - \underline{I}_{bc} = \sqrt{3}I_p\angle 90^o \end{aligned} \tag{2.10}$$

$I_p$is the magnitude of phase current.

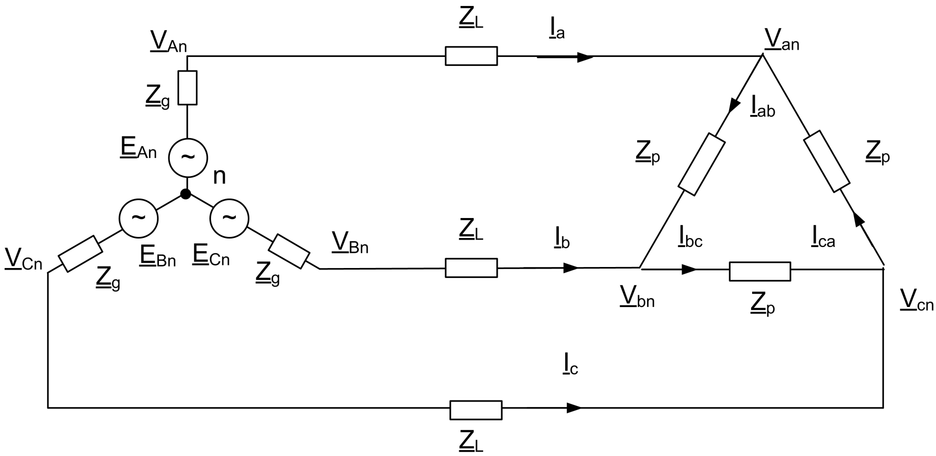

Fig. 2.3 △-connected load

It follows that the magnitude of line current is $\sqrt{3}$ times the magnitude of phase current, and with positive phase sequence, the set of line currents lags the set of phase currents by 30^o.

The three-phase active power is equal three times the real power in each phase:

$$P_{3\Phi} = 3V_pI_pcos\theta \tag{2.11}$$

The concept of complex power is extended to three phase systems by defining:

$$Q_{3\Phi} = 3V_pI_psin\theta \Rightarrow \underline{S}_{3\phi} = P_{3\Phi} + jQ_{3\Phi} \tag{2.12}$$

When the power is expressed in terms of line quantities for either Y or Δ-connection real power and reactive power are given by:

$$\begin{aligned} P_{3\Phi} &= \sqrt{3}V_LI_Lcos\theta \\ Q_{3\Phi} &= \sqrt{3}V_LI_Lsin\theta \end{aligned} \tag{2.13}$$

For analyzing network problems, it is sometimes convenient to replace the △-connected load with an equivalent Y-connected load (Figure 2.4).

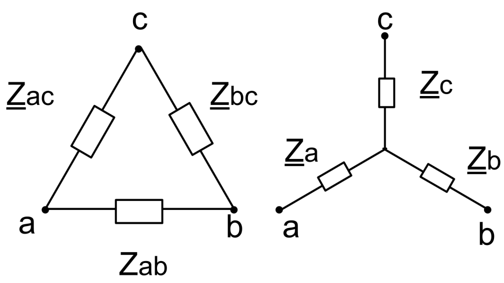

Fig. 2.4 △ and Y connected loads

Values $\underline{Z}_a$, $\underline{Z}_b$ and $\underline{Z}_c$ can be obtained from $\underline{Z}_{ab}$, $\underline{Z}_{bc}$ and $\underline{Z}_{ac}$:

$$\underline{Z}_a = \frac{\underline{Z}_{ab}\underline{Z}_{ac}}{\underline{Z}_{ab} + \underline{Z}_{ac} + \underline{Z}_{bc}}$$

$$\underline{Z}_b = \frac{\underline{Z}_{ab}\underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{ac} + \underline{Z}_{bc}}$$

$$\underline{Z}_c = \frac{\underline{Z}_{ac}\underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{ac} + \underline{Z}_{bc}} \tag{2.14}$$

It is also possible, but not usual to replace the Y-connected with △ -connected load:

$$\underline{Z}_{ab} = \underline{Z}_a + \underline{Z}_b + \frac{\underline{Z}_a\underline{Z}_b}{\underline{Z}_c}$$

$$\underline{Z}_{bc} = \underline{Z}_b + \underline{Z}_c + \frac{\underline{Z}_b\underline{Z}_c}{\underline{Z}_a}$$

$$\underline{Z}_{ac} = \underline{Z}_a + \underline{Z}_c + \frac{\underline{Z}_a\underline{Z}_c}{\underline{Z}_b} \tag{2.15}$$

Under balanced conditions, according to Equation(2.9) neutral current is equal zero:

$$\underline{I}_n = \underline{I}_a + \underline{I}_b + \underline{I}_c = 0 \tag{2.16}$$

Theoretically, this would make the neutral wire unnecessary. In practice, neutral wire must exist because of non-symmetry. When solving power system problems in steady state, in most cases the system is assumed balanced. Thus, the system is analyzed using the per-phase model.

Example

Consider three-phase load shown in Figure 2.5. Phase impedance of the load is $18 + j21\Omega$. Line impedance is $1 + j2\Omega$ per phase. The line is supplied from a three-phase source with a line-to-line voltage $190.53V$. Taking voltage of phase a as a reference, obtain current in phase a, line-to-line voltage at the load terminal, line losses and real and reactive power delivered to the load.

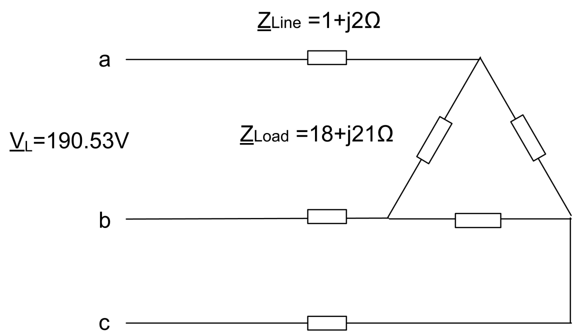

Fig. 2.5 Delta load

Solution

Since the given three-phase network is balanced, we can use per-phase model for its analysis. First we must transform Δ connected load into equivalent Y connected load:

$$\underline{Z}_Y = \frac{\underline{Z}_\Delta}{3} = \frac{18 + j21}{3} = 6 + j7\Omega$$

For the purpose of per-phase model line voltage must be converted to phase voltage. Since V_a phase voltage is taken as a reference, its value is:

$$\underline{V}_a = \frac{V_L}{\sqrt{3}}\angle 0^o = 110\angle 0^o V$$

The single phase equivalent circuit is shown in Figure 2.6.

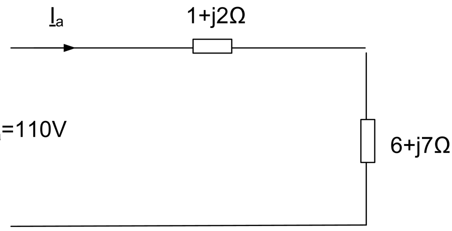

Fig. 2.6 Per-phase model

Current in phase a is:

$$\underline{I}_a = \frac{\underline{V}_a}{\underline{Z}_{Line} + \underline{Z}_{\triangle}} = \frac{110\angle 0^o}{7+j9} = \frac{110\angle 0^o}{11.4\angle 52.12^o} = 9.65\angle{-52.12^o}A = 5.92{-}j7.62A$$

To obtain line voltage at the load terminal, we must first obtain the phase voltage at that point:

$$\begin{aligned}\underline{V}_{p-L} = \underline{V}_a - \underline{I}_a \underline{Z}_{Line} = 110 - (1+j2)(5.92 - j7.62) &= 88,84 - j4,22V \\ &= 88.94\angle - 2.72^o\end{aligned}$$

Line voltage at the load terminal is:

$$\underline{V}_{L-L} = \sqrt{3}\underline{V_{p-L}}\angle 30^o = 154,05\angle 27.28^o V$$

Line losses can be calculated as:

$$\underline{S}_L = 3I_a^2\underline{Z}_{Line} = 3 \cdot (9.65)^2(1+j2) = 279.36 + j558.72VA$$

The total power supplied to the load is:

$$\underline{S}_{Load} = 3\underline{V}_{p-L}\underline{I}_a^* = 3 \cdot 88.94\angle - 2.72^o \cdot 9.65\angle 52.12^o = 2574.813\angle 49.4^o VA$$

Real power delivered to the load is:

$$P_{Load} = 1675.62W$$

Reactive power delivered to the load is:

$$Q_{Load} = 1954.98VAr$$

2.1 Symmetrical components

There are situations where power network is symmetric in nature but not balanced. One way to deal with such cases is the method of symmetrical components (SC) [1–3] where positive, negative and zero sequence network models are prepared and simultaneously solved. This reduces significantly the size of the problem. When applied to a π equivalent, the usage of symmetrical components allows to replace the original 6×6 matrix by three 2×2 diagonal matrices for positive, negative and zero sequence. This decoupling enables parallel computations as well [4].

The symmetrical components are used in [5] to decouple the network into three sequence components and solve them simultaneously and iteratively. The couplings between the admittance matrices are ignored resulting in loss of accuracy of the solution. These couplings between sequence admittance matrices due to the untransposed lines have been modeled as compensating current injections in positive, negative and zero sequence network and solved in parallel [6] and fast decoupled method [7]. This approach has been developed comprehensively to include transformer of various vector groups in [8, 9] for network of different R/X ratios. In [10] the positive sequence part is solved as power mismatch solution to keep the standard structure of balanced single phase power flow solution. [8] and [10] use combination of symmetrical and phase component depending on the type of the nodes.

In situation where the number of phases changes through single and two phase laterals, the usage of symmetrical component method does not offer much advantage because of the presence of the mutual coupling [11]. In fact, new couplings are observed between the parts with different sequence systems. The formulation and interface between three-phase and two-phase are not generic and straightforward. It is possible to use a hybrid approach such as phase approach to laterals and component approach to three phase mains [12, 13], or solve balanced parts with single phase approach and unbalanced parts with symmetrical component method with an iterative interface matrix of sequence component admittances [14]. Dealing with the laterals by forward/backward sweep algorithms can be found in e.g. [15–18]. An effective approach for solving unsymmetrical distribution networks is elaborated in [19].

2.1.1 Fortescue transformation theory

In a seminal paper [1], Fortescue proved that any unbalanced set of n phasors can be transformed into $n-1$ balanced n-phase system of different phase sequences and one zero phase sequence system by the following transformation (2.17).

$$\begin{bmatrix} \underline{v}_{a^0} \\ \underline{v}_{a^1} \\ \underline{v}_{a^2} \\ \vdots \\ \underline{v}_{a^n} \end{bmatrix} = \frac{1}{n} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \underline{a} & \underline{a}^2 & \cdots & \underline{a}^{n-1} \\ 1 & \underline{a}^2 & \underline{a}^4 & \cdots & \underline{a}^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \underline{a}^{n-1} & \underline{a}^{2(n-1)} & \cdots & \underline{a}^{(n-1)(n-1)} \end{bmatrix} \begin{bmatrix} \underline{v}_a \\ \underline{v}_b \\ \underline{v}_c \\ \vdots \\ \underline{v}_n \end{bmatrix} \tag{2.17}$$

where

$\underline{v}_{a^0}, \ldots, \underline{v}_{a^{n-1}}$: voltages in Fortescue coordinates

$\underline{v}_a, \ldots, \underline{v}_n$: voltages in phase coordinates

Because of the non-singularity of the transformation it is also proved that the quantities from Fortescue domain to phase domain can be recovered through an inverse $n \times n$ transformation given by (2.18):

$$\underline{T}_{F_n}^{ph_n} = (\underline{T}_{ph_n}^{F_n})^{-1} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \underline{a}^{n-1} & \underline{a}^{n-2} & \cdots & \underline{a} \\ 1 & \underline{a}^{n-2} & \underline{a}^{n-4} & \cdots & \underline{a}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \underline{a} & \underline{a}^2 & \cdots & \underline{a}^{(n-1)} \end{bmatrix} \tag{2.18}$$

In the following paragraphs, the transformation matrices for the special cases $n = 3$, $n = 2$ and $n = 1$ will be briefly reviewed.

The three phase system is transformed through $\underline{T}_{ph_3}^{F_3}$ into Fortescue coordinates using $\underline{a}_3 = e^{j2\pi/3}$. The transformed quantities (voltage, current) are recovered in phase domain through the inverse transformation $\underline{T}_{F_3}^{ph_3}$.

$$\underline{T}_{ph_3}^{F_3} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}_3 & \underline{a}_3^2 \\ 1 & \underline{a}_3^2 & \underline{a}_3 \end{bmatrix}, \quad \underline{T}_{F_3}^{ph_3} = \left(\underline{T}_{ph_3}^{F_3}\right)^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}_3^2 & \underline{a}_3 \\ 1 & \underline{a}_3 & \underline{a}_3^2 \end{bmatrix} \tag{2.19}$$

Electric current through a three phase element is given by:

$$\underline{I}_{ph_3} = \underline{Y}_{ph_3} \underline{V}_{ph_3} \tag{2.20}$$

Transforming voltage and current into Fortescue domain, equation (2.20) can be rewritten as:

$$\begin{aligned} \underline{T}_{F_3}^{ph_3} \underline{I}_{F_3} &= \underline{Y}_{ph_3} \underline{T}_{F_3}^{ph_3} \underline{V}_{F_3} \\ \underline{I}_{F_3} &= (\underline{T}_{F_3}^{ph_3})^{-1} \underline{Y}_{ph_3} \underline{T}_{F_3}^{ph_3} \underline{V}_{F_3} \\ \underline{I}_{F_3} &= \underline{Y}_{F_3} \underline{V}_{F_3} \end{aligned}$$

Three phase admittance matrix is transformed to Fortescue domain of order 3 as shown in equation (2.21).

$$\underline{Y}_{F_3} = \underline{T}_{ph_3}^{F_3} \underline{Y}_{ph_3} \underline{T}_{F_3}^{ph_3}, \; ph_3 \in \{abc\} \tag{2.21}$$

The admittance matrix for a symmetrical three phase element (stationary or rotating) is given by:

$$\underline{Y}_{ph_3} = \begin{bmatrix} \underline{y}_s & \underline{y}_m & \underline{y}_n \\ \underline{y}_n & \underline{y}_s & \underline{y}_m \\ \underline{y}_m & \underline{y}_n & \underline{y}_s \end{bmatrix} \tag{2.22}$$

The Fortescue transformation (2.21) when applied to (2.22) results in a 3×3 diagonal matrix, as follows:

$$\underline{Y}_{F_3} = \begin{bmatrix} \underline{y}_s+\underline{y}_m+\underline{y}_n & 0 & 0 \\ 0 & \underline{y}_s+\underline{a}_3^2\underline{y}_m+\underline{a}_3\underline{y}_n & 0 \\ 0 & 0 & \underline{y}_s+\underline{a}_3\underline{y}_m+\underline{a}_3^2\underline{y}_n \end{bmatrix}$$

2.1.2 Symmetrical transmission networks

For power transmission analysis, system of three phasors is used. Representation of power line in three phase symmetrical components is shown in Figure 2.7. In case of balanced conditions, only positive sequence system is used.

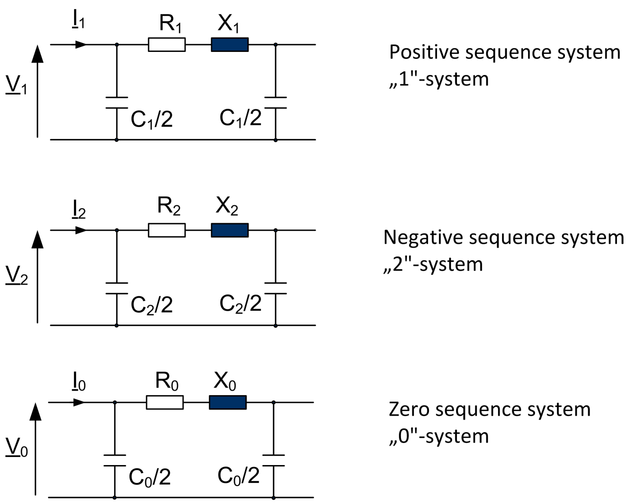

Fig. 2.7 Representation of symmetrical components

Positive, negative and zero phase sequence for three-phase unbalanced currents I_a, I_b and $I_c$is shown in Figure 2.8. These phasors are designated the positive phase sequence, the negative phase sequence and the zero phase sequence. The superscripts 1,2 and 0 are used to denote them respectively.

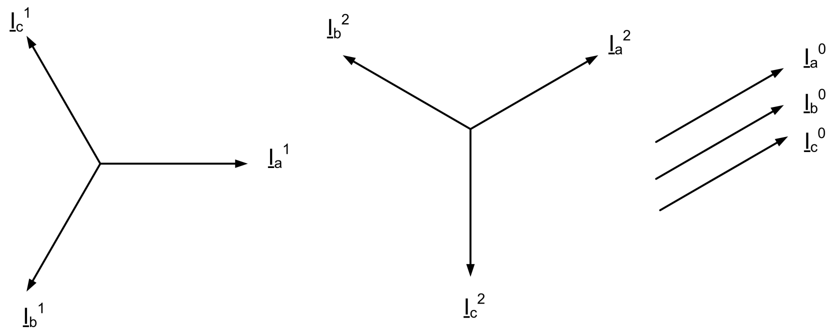

Fig. 2.8 Representation of phase currents symmetrical components

For positive phase sequence it can be written:

$$\begin{aligned} \underline{I}_b^{(1)} &= a^2 \underline{I}_a^{(1)} \\ \underline{I}_c^{(1)} &= a \underline{I}_a^{(1)} \end{aligned} \tag{2.23}$$

For negative phase sequence:

$$\begin{aligned} \underline{I}_b^{(2)} &= a \underline{I}_a^{(2)} \\ \underline{I}_c^{(2)} &= a^2 \underline{I}_a^{(2)} \end{aligned} \tag{2.24}$$

From the Figure 2.8 it is obvious that:

$$\underline{I}_a^{(0)} = \underline{I}_b^{(0)} = \underline{I}_c^{(0)} \tag{2.25}$$

Currents I_a, I_b and I_c can be written as:

$$\begin{aligned} \underline{I}_a &= \underline{I}_a^{(0)} + \underline{I}_a^{(1)} + \underline{I}_a^{(2)} \\ \underline{I}_b &= \underline{I}_a^{(0)} + a^2 \underline{I}_a^{(1)} + a \underline{I}_a^{(2)} \\ \underline{I}_c &= \underline{I}_a^{(0)} + a \underline{I}_a^{(1)} + a^2 \underline{I}_a^{(2)} \end{aligned} \tag{2.26}$$

We can express relations (2.26) in matrix form:

$$\begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{I}_a^{(0)} \\ \underline{I}_a^{(1)} \\ \underline{I}_a^{(2)} \end{bmatrix} \tag{2.27}$$

$$\underline{\mathbf{T}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \tag{2.28}$$

$\underline{T}$ is known as symmetrical component transformation matrix. The symmetrical component currents can be obtained from phasor currents:

$$\begin{bmatrix} \underline{I}_a^{(0)} \\ \underline{I}_a^{(1)} \\ \underline{I}_a^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} \tag{2.29}$$

From (2.29) the symmetrical components are:

$$\begin{aligned} \underline{I}_a^{(0)} &= \frac{1}{3}(\underline{I}_a + \underline{I}_b + \underline{I}_c) \\ \underline{I}_a^{(1)} &= \frac{1}{3}(\underline{I}_a + \underline{a}\underline{I}_b + \underline{a}^2\underline{I}_c) \\ \underline{I}_a^{(2)} &= \frac{1}{3}(\underline{I}_a + \underline{a}^2\underline{I}_b + \underline{a}\underline{I}_c) \end{aligned} \tag{2.30}$$

Voltages are expressed in similar way. Ohm’s law in system of symmetrical components is formulated starting from Ohm’s law formulation for phase quantities:

$$\underline{\mathbf{V}}_{\mathbf{abc}} = \underline{\mathbf{Z}}_{\mathbf{abc}}\underline{\mathbf{I}}_{\mathbf{abc}} \tag{2.31}$$

Writing $\underline{V}_{abc}$ and $\underline{I}_{abc}$ in terms of their symmetrical components and multiplying the obtained equation by $\underline{\mathbf{T}}^{-1}$, gives:

$$\underline{\mathbf{V}}_{\mathbf{012}} = \left(\underline{\mathbf{T}}^{-1}\underline{\mathbf{Z}}_{\mathbf{abc}}\underline{\mathbf{T}}\right)\underline{\mathbf{I}}_{\mathbf{012}} = \underline{\mathbf{Z}}_{\mathbf{012}}\underline{\mathbf{I}}_{\mathbf{012}} \tag{2.32}$$

Impedance matrix in system of symmetrical components is:

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \underline{\mathbf{T}}^{-1}\underline{\mathbf{Z}}_{\mathbf{abc}}\underline{\mathbf{T}} \tag{2.33}$$

Impedance matrix of power line shown in Figure 2.7 is:

$$\underline{\mathbf{Z}}_{\mathbf{abc}} = \begin{bmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{bmatrix} \tag{2.34}$$

From Equation(2.33) and Equation(2.34), impedance matrix of the three phase line in system of symmetrical components is:

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \frac{1}{3}\begin{bmatrix} \underline{Z}_{aa} + 2\underline{Z}_{ab} & 0 & 0 \\ 0 & \underline{Z}_{aa} - \underline{Z}_{ab} & 0 \\ 0 & 0 & \underline{Z}_{aa} - \underline{Z}_{ab} \end{bmatrix} = \begin{bmatrix} \underline{Z}_0 & 0 & 0 \\ 0 & \underline{Z}_1 & 0 \\ 0 & 0 & \underline{Z}_2 \end{bmatrix}$$

The impedance matrix is a diagonal matrix. Therefore the three sequences are independent. Currents of each phase sequence will produce voltage drops of the same phase sequence only. This permits the analysis of each sequence network on a per component basis.

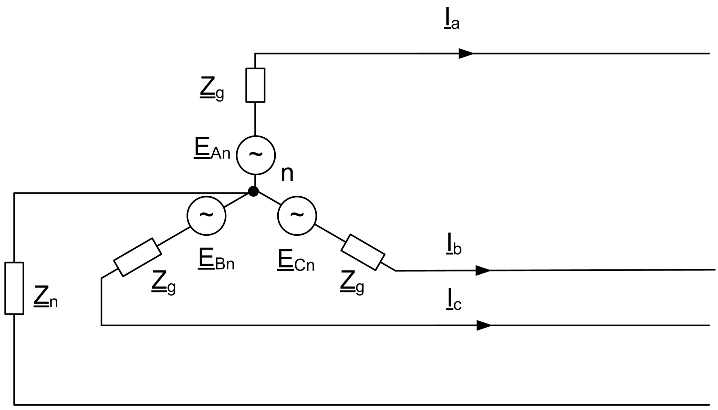

Fig. 2.9 Three-phase generator

Application of Kirchhoff's laws on a three phase synchronous generator with neutral grounded through an impedance Z_n, which supplies a three-phase balanced load (Figure 2.9) results in following equations:

$$\begin{aligned} \underline{V}_a =& \underline{E}_{An} - \underline{Z}_g \underline{I}_a - \underline{Z}_n \underline{I}_n \\ \underline{V}_b =& \underline{E}_{Bn} - \underline{Z}_g \underline{I}_b - \underline{Z}_n \underline{I}_n \\ \underline{V}_c =& \underline{E}_c - \underline{Z}_g \underline{I}_c - \underline{Z}_n \underline{I}_n \end{aligned} \tag{2.35}$$

$$I_n = I_a + I_b + I_c \tag{2.36}$$

Substituting I_n from Equation(2.35) for Equation(2.36), we get:

$$\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix} = \begin{bmatrix} \underline{E}_{An} \\ \underline{E}_{Bn} \\ \underline{E}_{Cn} \end{bmatrix} - \begin{bmatrix} \underline{Z}_g + \underline{Z}_n & \underline{Z}_n & \underline{Z}_n \\ \underline{Z}_n & \underline{Z}_g + \underline{Z}_n & \underline{Z}_n \\ \underline{Z}_n & \underline{Z}_n & \underline{Z}_g + \underline{Z}_n \end{bmatrix} \begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} \tag{2.37}$$

$$\underline{\mathbf{V}}_{\mathbf{abc}} = \underline{\mathbf{E}}_{\mathbf{abc}} - \underline{\mathbf{Z}}_{\mathbf{abc}} \underline{\mathbf{I}}_{\mathbf{abc}} \tag{2.38}$$

$$\underline{\mathbf{TV}}_{\mathbf{012}} = \underline{\mathbf{TE}}_{\mathbf{012}} - \underline{\mathbf{Z}}_{\mathbf{abc}}\underline{\mathbf{TI}}_{\mathbf{012}} \tag{2.39}$$

$$\underline{\mathbf{V}}_{\mathbf{012}} = \underline{\mathbf{E}}_{\mathbf{012}} - \underline{\mathbf{T}}^{-1}\underline{\mathbf{Z}}_{\mathbf{abc}}\underline{\mathbf{TI}}_{\mathbf{012}} = \underline{\mathbf{E}}_{\mathbf{012}} - \underline{\mathbf{Z}}_{\mathbf{012}}\underline{\mathbf{I}}_{\mathbf{012}} \tag{2.40}$$

where:

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \begin{bmatrix} \underline{Z}_g + 3\underline{Z}_n & 0 & 0 \\ 0 & \underline{Z}_g & 0 \\ 0 & 0 & \underline{Z}_g \end{bmatrix} = \begin{bmatrix} \underline{Z}_0 & 0 & 0 \\ 0 & \underline{Z}_1 & 0 \\ 0 & 0 & \underline{Z}_2 \end{bmatrix} \tag{2.41}$$

It is obvious that the grounding impedance is only reflected in zero sequence network. Since the generated EMF is balanced, there is only positive sequence voltage:

$$\underline{\mathbf{E}}_{\mathbf{012}} = \begin{bmatrix} 0 \\ \underline{E}_{An} \\ 0 \end{bmatrix} \tag{2.42}$$

Substituting $\underline{E}_{012}$ and $\underline{Z}_{012}$, in Equation2.40 we get:

$$\begin{aligned} \underline{V}_a^{(0)} &= 0 - \underline{Z}_0\underline{I}_a^{(0)} \\ \underline{V}_a^{(1)} &= \underline{E}_{An} - \underline{Z}_1\underline{I}_a^{(1)} \\ \underline{V}_a^{(2)} &= 0 - \underline{Z}_2\underline{I}_a^{(2)} \end{aligned} \tag{2.43}$$

These equations show that three sequences are independent and that only the positive sequence network has a voltage source (Figure 2.10). The zero sequence current can only flow if the circuit is grounded.

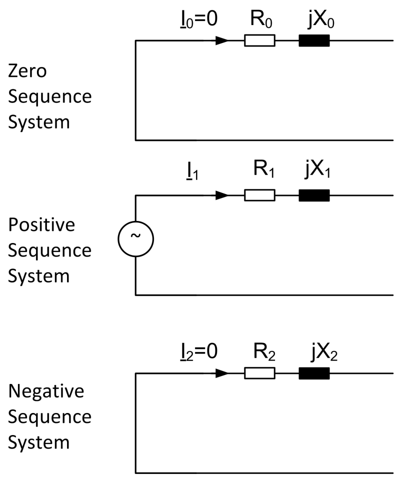

Fig. 2.10 Positive, negative and zero phase sequences

Complex power can be expressed in terms of symmetrical components as:

$$\underline{\mathbf{S}}_{\mathbf{3\Phi}} = (\underline{\mathbf{V}}_{\mathbf{abc}})^T \underline{\mathbf{I}}^*_{\mathbf{abc}} = (\underline{\mathbf{T}}\underline{\mathbf{V}}_{\mathbf{012}})^T (\underline{\mathbf{T}}\underline{\mathbf{I}}_{\mathbf{012}})^* = \underline{\mathbf{V}}_{\mathbf{012}}{}^T \underline{\mathbf{T}}^T \underline{\mathbf{T}}^* \underline{\mathbf{I}}_{\mathbf{012}}{}^* \tag{2.44}$$

From definition of $\underline{T}$ in Equation(2.28) it follows that:

$$\underline{\mathbf{T}}^T \underline{\mathbf{T}}^* = \underline{\mathbf{T}}\underline{\mathbf{T}}^* = \underline{\mathbf{T}} \cdot 3\underline{\mathbf{T}}^{-1} = 3 \tag{2.45}$$

From Equation(2.44) and Equation(2.45) it follows that complex power can be expressed as a sum of symmetrical component power:

$$\underline{\mathbf{S}}_{\mathbf{3\Phi}} = 3\left(\underline{\mathbf{V}}_{\mathbf{012}}{}^T \underline{\mathbf{I}}_{\mathbf{012}}{}^*\right) \tag{2.46}$$

The positive and negative sequence impedance of power transformer are the same. The zero sequence impedance can vary from an open circuit to some value, depending on the winding connection and whether or not the neutrals are grounded. Some of the most common transformer configurations relating to zero sequence impedance are shown in Figure 2.11.

In Y-Y connected transformer with both neutrals isolated, zero sequence circuit is open on both sides represent infinite impedance for zero sequence current flow. If both neutrals are grounded, there is a path for the zero sequence current to flow in the primary and secondary. When one of the neutrals is grounded, still no zero sequence currents can be transferred from the grounded side to the ungrounded side. Both neutrals must be grounded for the transfer of zero sequence. In Δ-Y or Y-Δ transformer with neutral grounded, zero sequence impedance is approximately equal to positive and negative sequence impedance viewed from Y-connection side. Zero sequence current circulates in the Δ-connected secondary, but no zero sequence current can leave the Δ terminals. So, the zero sequence impedance of the transformer viewed from Δ-connection side is an open circuit. If the Y winding neutral is isolated, zero sequence current cannot flow and the zero sequence impedance is infinite viewed from both sides. In case of Δ-Δ connection, zero sequence current circulates in the Δ-connected windings, but no current can leave the Δ terminals.

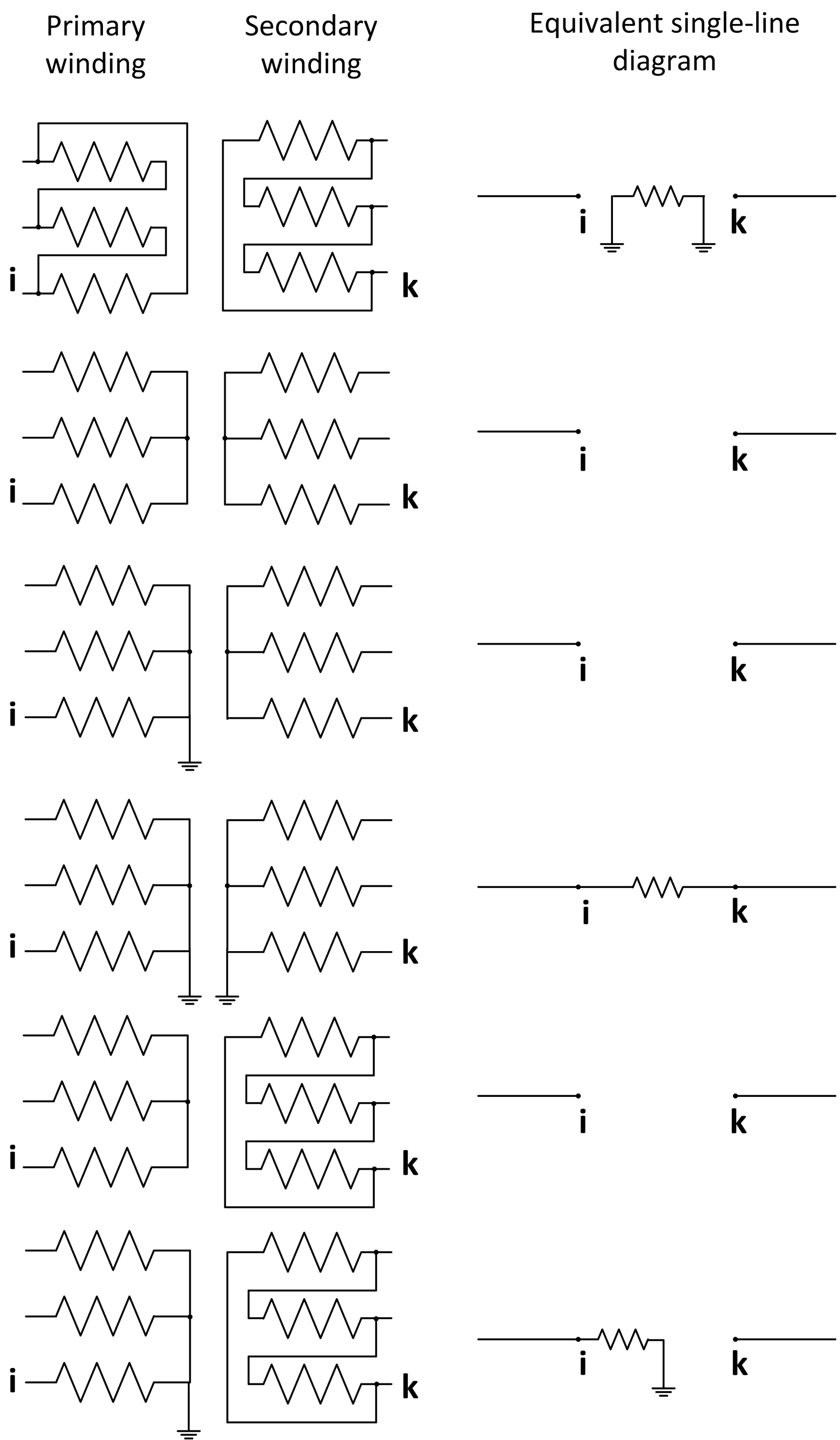

Fig. 2.11 Transformer zero sequence equivalent single-line circuits

2.2 Measuring of symmetrical components

It is possible to measure symmetrical components of system voltages, currents and impedances.

2.2.1 Measuring sequence impedances of solidly grounded Y load

Measurement system for positive and negative sequence impedances of solidly grounded Y load is shown in Figure 2.12. Black color denotes positive sequence quantities, grey color denotes negative sequence quantities.

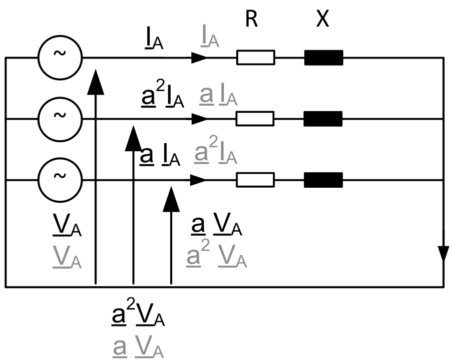

Fig. 2.12 Measurement system for positive and negative sequence impedances of solidly grounded Y load

$$\underline{Z}^1 = \frac{\underline{V}_A}{\underline{I}_A} = \underline{Z}^2 \tag{2.47}$$

Figure 2.13 represents measurement system for zero sequence impedance of solidly grounded Y load.

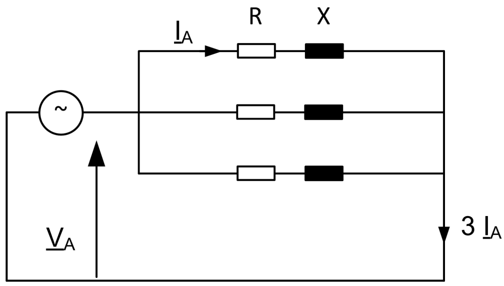

Fig. 2.13 Measurement system for zero sequence impedance of solidly grounded Y load

$$\underline{Z}^0 = \frac{\underline{V}_A}{\underline{I}_A} \tag{2.48}$$

Zero, positive and negative sequences are shown in Figure 2.14.

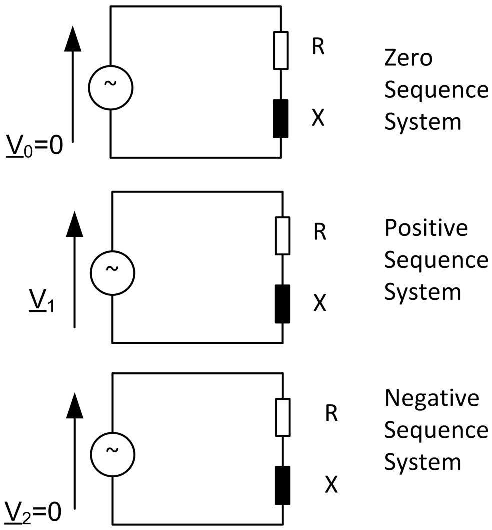

Fig. 2.14 Zero, positive and negative sequences of solidly grounded Y load

2.2.2 Measuring sequence impedances of isolated Y load and delta load

Measurement system for positive and negative sequence impedances of symmetrical isolated Y load is shown in Figure 2.15 (black denotes positive sequence quantities, grey denotes negative sequence quantities), while Figure 2.16 represents measurement system for positive and negative sequence impedances of symmetrical delta load.

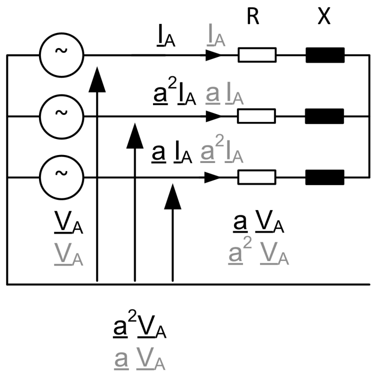

Fig. 2.15 Measurement system for positive and negative sequence impedances of isolated Y load

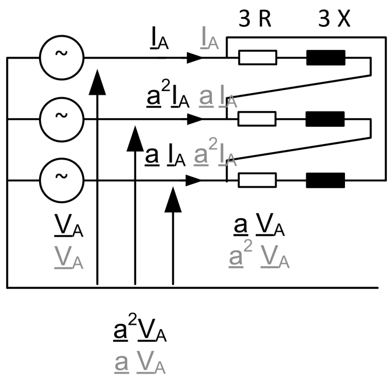

Fig. 2.16 Measurement system for positive and negative sequence impedances of delta load

In both cases:

$$\underline{Z}^1 = \frac{\underline{V}_A}{\underline{I}_A} = \underline{Z}^2 \tag{2.49}$$

Figure 2.17 and Figure 2.18 represent measurement systems for zero sequence impedances of isolated Y load and symmetrical delta load respectively.

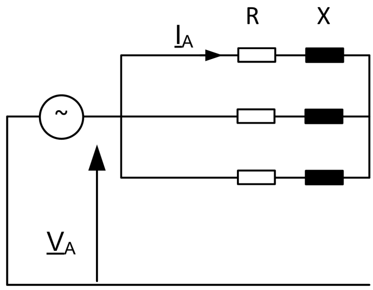

Fig. 2.17 Measurement system for zero sequence impedance of isolated Y load

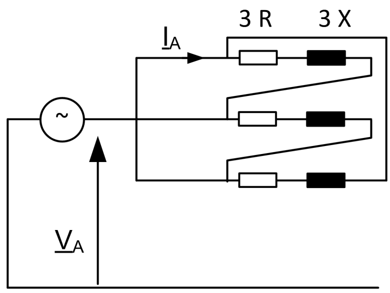

Fig. 2.18 Measurement system for zero sequence impedance of delta load

$$\underline{Z}^0 = \frac{\underline{V}_A}{\underline{I}_A} \tag{2.50}$$

For both cases zero, positive and negative sequences are shown in Figure 2.19.

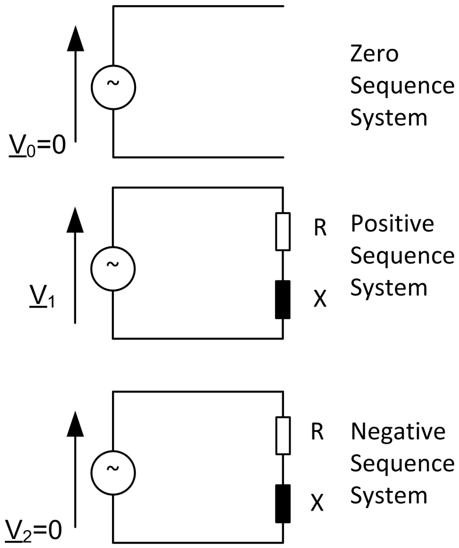

Fig. 2.19 Zero, positive and negative sequences of isolated Y load and delta load

2.2.3 Measuring sequence impedances of three phase elements with neutral wire

Measurement system for positive and negative sequence impedances of elements with neutral wire is shown in Figure 2.20. Black color denotes positive sequence quantities, grey color denotes negative sequence quantities.

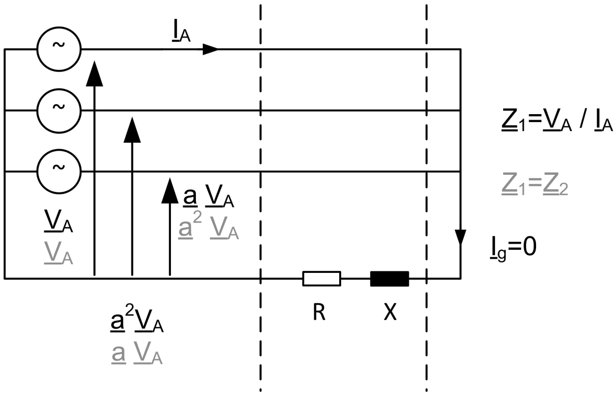

Fig. 2.20 Measurement system for positive and negative sequence impedances of neutral wire

$$\underline{Z}^{(1)} = \frac{\underline{V}_A}{\underline{I}_A} = \underline{Z}^{(2)} \tag{2.51}$$

Figure 2.21 represents measurement system for zero sequence of neutral wire.

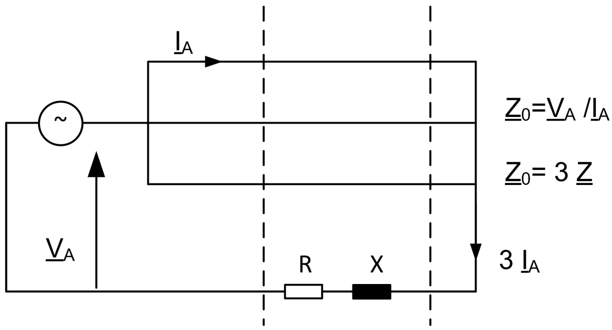

Fig. 2.21 Measurement system for zero sequence impedance of neutral wire

$$\underline{Z}^{(0)} = \frac{\underline{V}_A}{\underline{I}_A} \tag{2.52}$$

Zero, positive and negative sequences are shown in Figure 2.22.

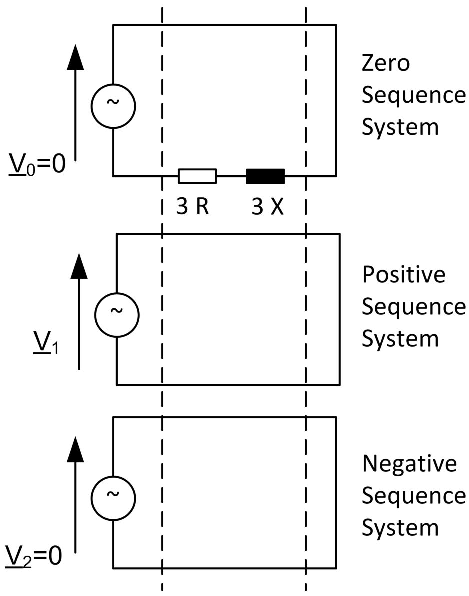

Fig. 2.22 Zero, positive and negative sequences of neutral wire

Example

If positive, negative and zero sequence voltage of phase voltage $\underline{V}_a$ are $\underline{V}_a^1 = 50\angle 0^o V$, $\underline{V}_a^2 = 20\angle 90^o V$ and $\underline{V}_a^0 = 10\angle 180^o V$ respectively, obtain phase voltages $\underline{V}_a$, $\underline{V}_b$ and $\underline{V}_c$.

Solution

Phase voltages can be expressed in terms of symmetrical components as:

$$\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{V}_a^0 \\ \underline{V}_a^1 \\ \underline{V}_a^2 \end{bmatrix}$$

From here:

$$\begin{aligned} \underline{V}_a &= \underline{V}_a^0 + \underline{V}_a^1 + \underline{V}_a^2 = -10 + 50 + j20 = (40 + j20)\, V = 44.72\angle 26.6^o V \\ \underline{V}_b &= \underline{V}_a^0 + \underline{a}^2 \underline{V}_a^1 + \underline{a}\underline{V}_a^2 = -10 + 50\angle 240^o + 20\angle 210^o = (-52.32 - j53.33)\, V \\ &= 7.47\angle - 134.4^o \\ \underline{V}_c &= \underline{V}_a^0 + \underline{a}\underline{V}_a^1 + \underline{a}^2 \underline{V}_a^2 = -10 + 50\angle 120^o + 20\angle 330^o = (-17.68 - j33.3)\, V \\ &= 37.7\angle 117^0 \end{aligned}$$

Example

Consider Y-connected balanced solidly grounded load with series impedance per phase $Z = 15 + j20\Omega$ and mutual impedance $Z_m = j5\Omega$ shown in Figure 2.23. Power to the load is delivered by three-phase unbalanced source with phase voltages:

$$\underline{\mathbf{V}}_{\mathbf{abc}} = \begin{bmatrix} 200\angle 180^o \\ 100\angle 0^o \\ 150\angle 90^o \end{bmatrix}$$

Obtain:
a) Symmetrical components of voltage.
b) Symmetrical components of current.
c) Phase currents.
d) Complex power delivered to the load.

Solution

a) Symmetrical components of phase voltages can be obtained as:

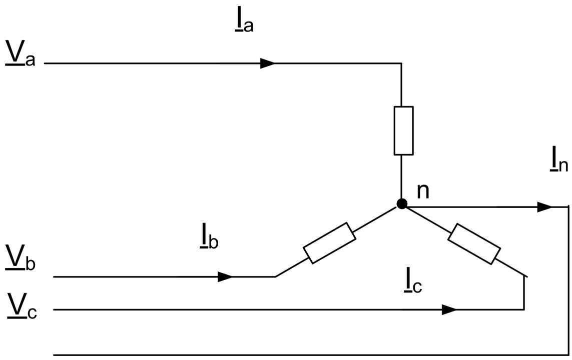

Fig. 2.23 Yg load

$$\underline{\mathbf{V}}_{\mathbf{012}} = \underline{\mathbf{T}}^{-1}\underline{\mathbf{V}}_{\mathbf{abc}} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix}\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix}$$

$$\underline{\mathbf{V}}_{\mathbf{012}} = \begin{bmatrix} -33.33 + j50 \\ -40.03 + j3.87 \\ -126.63 - j53.87 \end{bmatrix}$$

b) Symmetrical components of phase currents can be obtained using Ohm's law in system of symmetrical components:

$$\underline{\mathbf{V}}_{\mathbf{012}} = \underline{\mathbf{Z}}_{\mathbf{012}}\underline{\mathbf{I}}_{\mathbf{012}} \Rightarrow \underline{\mathbf{I}}_{\mathbf{012}} = \left(\underline{\mathbf{Z}}_{\mathbf{012}}\right)^{-1}\underline{\mathbf{I}}_{\mathbf{012}}$$

First, $\mathbf{Z^{012}}$is calculated as:

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \underline{\mathbf{T}}^{-1}\underline{\mathbf{Z}}_{\mathbf{abc}}\underline{\mathbf{T}}$$

Series impedance matrix of the load $\underline{Z}^{abc}$ is:

$$\underline{\mathbf{Z}}_{\mathbf{abc}} = \begin{bmatrix} 15 + j20 & j5 & j5 \\ j5 & 15 + j20 & j5 \\ j5 & j5 & 15 + j20 \end{bmatrix}$$

From here:

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix}\begin{bmatrix} 15 + j20 & j5 & j5 \\ j5 & 15 + j20 & j5 \\ j5 & j5 & 15 + j20 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$\underline{\mathbf{Z}}_{\mathbf{012}} = \begin{bmatrix} 15 + j30 & 0 & 0 \\ 0 & 15 + j15 & 0 \\ 0 & 0 & 15 + j15 \end{bmatrix}$$

Symmetrical components of phase currents are:

$$\underline{\mathbf{I}}_{\mathbf{012}} = \left(\underline{\mathbf{Z}}_{\mathbf{012}}\right)^{-1} \underline{\mathbf{V}}_{\mathbf{012}} = \begin{bmatrix} 0.89 + j1.56 \\ -1.20 + j1.46 \\ -6.02 + j2.43 \end{bmatrix}$$

Phase currents are obtained from symmetrical components:

$$\underline{\mathbf{I}}_{\mathbf{abc}} = \underline{\mathbf{T}}\underline{\mathbf{I}}_{\mathbf{012}} = \begin{bmatrix} -6.33 + j5.44 \\ 3.67 - j4.56 \\ 5.33 + j3.78 \end{bmatrix}$$

Complex power is:

$$\underline{\mathbf{S}}_{\mathbf{3\Phi}} = 3\left(\underline{\mathbf{V}}_{\mathbf{012}}{}^{T}\underline{\mathbf{I}}_{\mathbf{012}}{}^{*}\right) = \underline{\mathbf{V}}_{\mathbf{abc}}{}^{T}\underline{\mathbf{I}}_{\mathbf{abc}}{}^{*} = 2200 + j2344.44\, MVA$$

Chapter 3
Modeling of System Components

Modern power systems are designed with the purpose of generation, transmission and distribution of electrical energy to the customers. To provide the high quality of life, it is necessary to have continuous, abundant and economical supply of electrical energy. In today power systems, it is still necessary to generate and transmit electrical energy as it is consumed. Such electric system is dynamic and demands very complex analysis, monitoring, measurements and control, always having the economics in mind. In order to operate the system successfully and to fulfill all the required tasks, it is necessary to have the information about the circumstances on the network at any time. This makes network analysis the most demanding task in power systems management. Methods and procedures used for network analysis combined with the DMS (Distribution Management System) algorithms form theoretical base for energy system application development.

Electrical energy generation is a complex process of transforming different types of energy into mechanical energy that is then transformed into electrical energy. Generators such as steam-turbines and internal-combustion machines convert hydraulic or thermal energy into mechanical energy and then generators convert mechanical energy into electric power. Production centers (plants) generate electricity at voltages of several kilo-volts, typically from 6 to 20 kV. Standard frequency for Europe is 50Hz, and 60Hz for the USA. Transformers immediately transfer the voltage level to voltages of hundreds of kilo-volts (132, 220, 400, 500 and 700kV) because electrical energy transmission is much more economical at higher voltages, which results in lower currents i.e., lower power losses. Depending on voltage level network can be classified as transmission, distribution and low-voltage network. Voltage levels of transmission networks are 380kV, 220kV and 110kV and transmission circuits consist of overhead and/or underground cables. Lately the networks of 110kV are more often classified as distribution rather than transmission networks. Lower voltage levels of 35kV, 20 kV, 10 kV and 6,3kV are voltage levels of distribution network. Configuration of transmission network is usually interconnected, while for distribution networks it is mostly radial.

A radial network consists of lines that leave the station and pass through the network without connection to any other supply. An interconnected network has multiple connections to other points of supply. The transmission grid interconnects all the major production and consumption centers. The structure of transmission network guarantees high reliability. This is accomplished with alternative pathways which in case of failure ensure energy supply for important loads. These electric power transmission highways are interconnected at communication nodes known as electric substations. Transformers in successive substations step the voltage down in several phases. Consumers (loads) connect to the voltage level best suited to their power needs. Some loads like railway electrification, elevators etc. still requires DC power supply. In such cases it is common to use rectification of AC power rather than to generate DC power.

Power systems are mainly three phase networks. Under normal conditions elements of all three phases are symmetrical and balanced (values of currents and voltages have the same magnitude and angle of 120^o between phases), thus the network can be analyzed using the per-phase model. Each element of the network is represented with its impedance model. During the fault, phases of the network are not symmetrical anymore so the per-phase model cannot be used. Analyzing the network under these conditions includes the use of the symmetrical components. Four major elements of a power system are:

- Generators
- Lines
- Transformers
- Loads

Power system components can be generally classified it two groups: two pole components with one terminal (Figure 3.1) and four pole components with two terminals (Figure 3.2). One of the essential components of power system, the three phase AC generator, has two synchronously rotating fields, one produced by the rotor driven by synchronous speed and excited by DC current and the other produced in the stator windings. DC current is provided by excitation system. The resistance of generator is generally much smaller than the reactance and is often neglected:

$$Z_g = R_g + jX_g \approx jX_g \tag{3.1}$$

Load model in power system analysis is represented with the load impedance or as a constant current source.

3.1 Line Modeling

Due to the effects of magnetic and electric field created by currents flowing through them, all lines in power system exhibit the properties of inductance

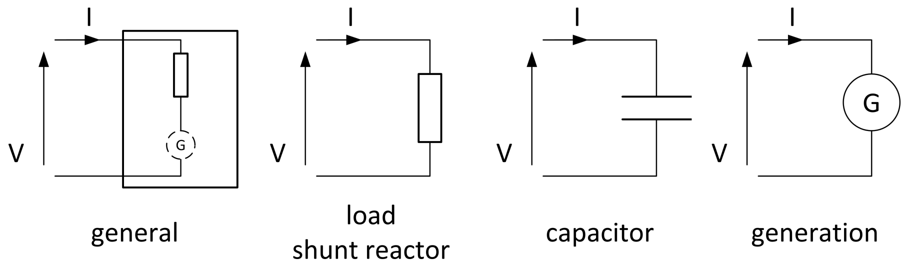

Fig. 3.1 Two pole components

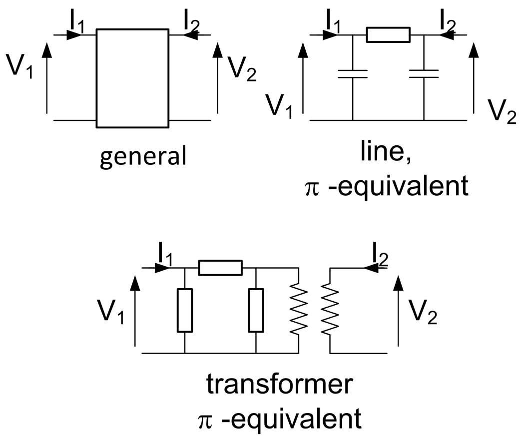

Fig. 3.2 Four pole components

and capacitance. Line resistance is a consequence of the conductor material opposition to the passage of current flow, while shunt conductance of the line occurs as a result of leakage currents in insulators and ionized air. There are two types of lines in power system: overhead lines and underground cables. Transmission lines are overhead lines, while distribution network lines can be both. Copper has the best conductive properties, but due to its price and limited quantity, aluminum is the most commonly used line material. At voltages below 230kV it is usual to have one conductor per phase, while above this voltage level it is preferable to have several conductors per phase placed around the edge of circumference. This is known as bundling of conductors. The advantages of bundling are increased line radius which reduces line inductance, decrease in electric field strength at the line surface which reduces corona losses, and better line cooling.

Corona is an electrical discharge that occurs when the electric field strength at the surface of the conductor is high enough to ionize the surrounding air. As a consequence the air becomes conductive and leakage currents occur. Corona causes power losess, noise, electromagnetic interference and insulation damage.

Power losses associated with corona are represented by shunt conductance. Under normal conditions these losses are usually neglected.

In case when the network can be considered balanced, per-phase line model is used. However, these assumptions are not always accurate, and correct results are obtained only if three-phase model is used. In this case, line model is described by series impedance and shunt admittance matrices. Series impedance matrix for three phase line can be defined as:

$$\mathbf{Z} = \begin{bmatrix} \underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac} \\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc} \\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc} \end{bmatrix} \tag{3.2}$$

Diagonal elements are line impedances of each phase, while off-diagonal elements represent mutual phase couplings. Series impedance matrix can also be written as:

$$\mathbf{Z} = \mathbf{R} + j\omega\mathbf{L} = \begin{bmatrix} R_{aa} & R_{ab} & R_{ac} \\ R_{ba} & R_{bb} & R_{bc} \\ R_{ca} & R_{cb} & R_{cc} \end{bmatrix} + j\omega \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix} \tag{3.3}$$

Line conductance is usually neglected, thus in shunt admittance matrix only self and mutual phase capacitance is considered:

$$\mathbf{Y} = j\omega\mathbf{C} = j\omega \begin{bmatrix} C_{aa} & C_{ab} & C_{ac} \\ C_{ba} & C_{bb} & C_{bc} \\ C_{ca} & C_{cb} & C_{cc} \end{bmatrix} \tag{3.4}$$

Diagonal elements are equal to the to the sum of corresponding phase-to-ground capacitance and phase-to-phase capacitance. Off-diagonal elements are negative values of the sum of corresponding phase-to-phase capacitance.

3.1.1 Line resistance

Conductor resistance to DC currents can be obtained as:

$$R = \rho \frac{l}{A} \tag{3.5}$$

where ρ is conductor resistivity, whose value is different for different conductor materials, l is conductor length and A is conductor cross-sectional area. Conductor resistance increases with temperature. If the surrounding temperature of the conductor T_i is different than $20^o C$ conductor resistance is changed according to:

$$R_{T_i} = R[1 + \alpha(T_1 - 20^o)] \tag{3.6}$$

where α is temperature coefficient of conductor material. Few things should be considered when obtaining conductor's resistance. First of all, high voltage conductors consist of several thin bounded (possibly different) conductors. Total conductor resistance is obtained as the resistance of their parallel connection. Second, these conductors are usually spiraled and they actual length is longer than the length of finished conductor which also has to be considered.

Due to skin effect, conductor resistance to AC currents is higher than to DC currents. At 50Hz, this effect can be neglected. It is important to make resistance correction when dealing with transients and high-frequency phenomena. DC resistance, temperature coefficient and resistance increase due to skin effects are always provided by manufacturer.

For three-phase models, resistance of each phase represents diagonal element of **R** matrix in Equation (3.3). Off-diagonal elements of **R** matrix can be considered equal to zero. In reality, they are different from zero due to earth effects. This is analyzed later.

3.1.2 Line inductance

Consequence of current flow is a magnetic field formed around the conductor. Effects of this field are causing the conductor to exhibit the property of inductance. In order to obtain conductor inductance, Ampere's law will be used:

$$\oint \overrightarrow{H} \cdot \overrightarrow{dl} = i \tag{3.7}$$

where $\overrightarrow{H}$ is a magnetic field strength, $\overrightarrow{dl}$ is elementary length and i is an instant current enclosed within the loop.

For long, round non-magnetic conductor of radius r, carrying uniformly distributed current i, magnetic field strength outside the conductor at distance $x > r$ can be obtained as:

$$H \cdot 2\pi x = i \Rightarrow H = \frac{i}{2\pi x},\ x > r \tag{3.8}$$

Magnetic field strength inside the conductor for $x < r$ is:

$$H = \frac{i_x}{2\pi x},\ x < r \tag{3.9}$$

where i_x is obtained as:

$$\frac{i_x}{x^2\pi} = \frac{i}{r^2\pi} \Rightarrow i_x = \frac{x^2}{r^2} i \tag{3.10}$$

From here, magnetic field inside the conductor is found to be:

$$H = \frac{1}{2\pi x} \cdot \frac{x^2}{r^2} i = \frac{i}{2\pi r^2} x \tag{3.11}$$

Linkage flux through the area of one meter length and dx thickness outside the conductor is equal to the flux because the entire current flow is linked outside the conductor:

$$d\lambda_o = d\phi \tag{3.12}$$

External flux linkage between conductor surface and distance D from conductor center is:

$$\begin{aligned} \lambda_o &= \int_r^D \mu_0 H \cdot 1 \cdot dx \\ &= \int_r^D \frac{\mu_0 i}{2\pi x} dx \\ &= \frac{\mu_0 i}{2\pi} ln \frac{D}{r} \end{aligned} \tag{3.13}$$

Considering that:

$$\lambda = Li \tag{3.14}$$

external conductor inductance per meter length is:

$$L_o = \frac{\mu_0}{2\pi} ln \frac{D}{r} \tag{3.15}$$

Magnetic flux inside the conductor through region of one meter length and dx thickness at distance x from the center of conductor is:

$$d\phi_x = \frac{\mu_0 i}{2\pi r^2} x dx \tag{3.16}$$

where $\mu_0 = 4\pi \cdot 10^{-7} H/m$ is magnetic permeability. Internal linkage flux can be obtained considering that only $\frac{x^2\pi}{r^2\pi}$ part of the conductor current is linked by $d\phi_x$ flux. Thus, internal linkage flux is:

$$\begin{aligned}\lambda_i &= \int_0^r \frac{x^2}{r^2}\frac{\mu_0 i}{2\pi r^2}xdx \\ &= \int_0^r \frac{\mu_0 i}{2\pi r^4}x^3 dx \\ &= \frac{\mu_0 i}{2\pi r^4}\cdot\frac{r^4}{4} \\ &= \frac{\mu_0 i}{8\pi}\end{aligned} \tag{3.17}$$

Internal conductor inductance from Equation (3.14) per meter length is:

$$L_i = \frac{\mu_0}{8\pi} \tag{3.18}$$

Total inductance of conductor with its return path per meter length is obtained from Equation (3.15) and Equation (3.18):

$$L = L_o + L_i = \frac{\mu_0}{2\pi}ln\frac{D}{r} + \frac{\mu_0}{8\pi} \tag{3.19}$$

This can be written as:

$$\begin{aligned}L &= \frac{\mu_0}{2\pi}(ln\frac{D}{r} + \frac{1}{4}) \\ &= \frac{\mu_0}{2\pi}(ln\frac{D}{r} + lne^{\frac{1}{4}}) \\ &= \frac{\mu_0}{2\pi}ln\frac{De^{\frac{1}{4}}}{r} \\ &= \frac{\mu_0}{2\pi}ln\frac{D}{re^{-\frac{1}{4}}} \\ &= \frac{\mu_0}{2\pi}ln\frac{D}{0.7788r} \\ &= \frac{\mu_0}{2\pi}ln\frac{1}{r'} + \frac{\mu_0}{2\pi}lnD\end{aligned} \tag{3.20}$$

$re^{-\frac{1}{4}}$ represents self-geometric mean distance (GMR) denoted by r'. $r'$is considered as a radius of conductor without internal flux, but with the inductance same as the inductance of considered conductor. The first term in obtained expression is conductor self-inductance and is a function of conductor's radius, while the second term is called spacing factor.

For three-phase line, diagonal elements of **L** matrix in Equation (3.3) represent self-inductance of each phase obtained from Equation (3.20) (first term). In order to obtain mutual phase inductance three conductors with their return paths will be considered (Figure 3.3).

Total linkage flux λ_2 from Equation (3.14) is:

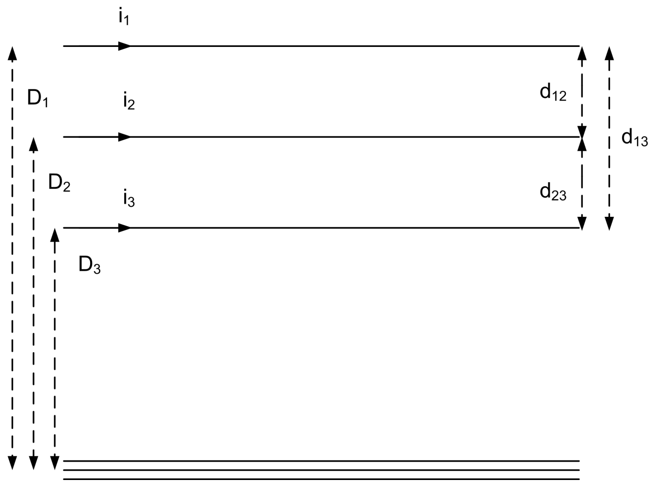

Fig. 3.3 Three-phase line

$$\lambda_2 = \lambda_{21} + \lambda_{22} + \lambda_{23} = L_{21}i_1 + L_{22}i_2 + L_{23}i_3 \tag{3.21}$$

Linkage flux λ_{21}created by current I_1 with the second conductor per meter length is:

$$\lambda_{21} = \frac{\mu_0 i_1}{2\pi} \int_{d_{12}}^{D_1} \frac{dx}{x} = \frac{\mu_0 i_1}{2\pi} ln \frac{D_1}{d_{12}} \tag{3.22}$$

Same way, linkage flux λ_{23} created by current I_3 with the second conductor is:

$$\lambda_{23} = \frac{\mu_0 i_3}{2\pi} \int_{d_{23}}^{D_3} \frac{dx}{x} = \frac{\mu_0 i_3}{2\pi} ln \frac{D_3}{d_{23}} \tag{3.23}$$

and from Equation (3.20):

$$\lambda_{22} = \frac{\mu_0 i_2}{2\pi} ln \frac{D_2}{0.7788 r_2} = \frac{\mu_0 i_2}{2\pi} ln \frac{D_2}{r_2'} \tag{3.24}$$

where r_2 is radius of the second conductor. Assuming that $D_1 \approx D_2 \approx D_3 \approx D$ Equation (3.21) can now be written as:

$$
\begin{aligned}
\lambda_2 &= \frac{\mu_0 i_1}{2\pi} ln \frac{D_1}{d_{12}} + \frac{\mu_0 i_2}{2\pi} ln \frac{D_2}{r_2'} + \frac{\mu_0 i_3}{2\pi} ln \frac{D_3}{d_{23}} \\
&= \frac{\mu_0}{2\pi}(i_1 ln \frac{D}{d_{12}} + i_2 ln \frac{D}{r_2'} + i_3 ln \frac{D}{d_{23}}) \\
&= \frac{\mu_0}{2\pi}[i_1 ln \frac{1}{d_{12}} + i_2 ln \frac{1}{r_2'} + i_3 ln \frac{1}{d_{23}} + lnD(i_1 + i_2 + i_3)] \\
&= \frac{\mu_0}{2\pi} ln \frac{1}{d_{12}} \cdot i_1 + \frac{\mu_0}{2\pi} ln \frac{1}{r_2'} \cdot i_2 + \frac{\mu_0}{2\pi} ln \frac{1}{d_{23}} \cdot i_3
\end{aligned}
\tag{3.25}
$$

Comparing the obtained result with Equation (3.21) yields:

$$
\begin{aligned}
L_{21} =& \frac{\mu_0}{2\pi} ln \frac{1}{d_{12}} \\
L_{22} =& \frac{\mu_0}{2\pi} ln \frac{1}{r_2'} \\
L_{23} =& \frac{\mu_0}{2\pi} ln \frac{1}{d_{23}}
\end{aligned}
\tag{3.26}
$$

In general, off-diagonal elements of inductance matrix are obtained as:

$$L_{ij} = \frac{\mu_0}{2\pi} ln \frac{1}{d_{ij}},\ i \neq j \tag{3.27}$$

while diagonal elements are obtained as:

$$L_{ii} = \frac{\mu_0}{2\pi} ln \frac{1}{r_i'} \tag{3.28}$$

3.1.3 Effects of earth on series impedance matrix

Effects of earth on line inductance and resistance are analyzed using Carson's method. Consider a conductor placed at height h from earth. Introducing conductor's image - fictitious conductor return path through earth at distance $D_e > h$ from the original, has the same effects as earth presence. Distance D_e is the function of network frequency, conductor geometry and earth parameters:

$$D_e = 659\sqrt{\frac{\rho}{f}} \tag{3.29}$$

where $\rho$is the specific resistivity of the earth and f is frequency. It is convenient to assume that $D_e = H_{ii} = 2h$ and to introduce correction coefficients for elements of series impedance matrix $\mathbf{Z}$. Diagonal elements of $\mathbf{Z}$ matrix will be obtained as:

$$Z_{ii} = (R_{ii} + \triangle R_{ii}) + j(\omega L_{ii} + \triangle X_{ii}) \tag{3.30}$$

where R_{ii} is the resistance of conductor i and L_{ii} is self inductance of conductor i in presence of its image conductor at distance H_{ii}:

$$L_{ii} = \frac{\mu_0}{2\pi} ln \frac{H_{ii}}{r'_i} \tag{3.31}$$

$\triangle R_{ii}$ and $\triangle X_{ii}$ are Carson correction coefficients. Off-diagonal elements of series impedance matrix with correction coefficients are:

$$Z_{ij} = \triangle R_{ij} + j(\omega L_{ij} + \triangle X_{ij}) \tag{3.32}$$

where L_{ij} is mutual inductance between conductor i and conductor j in presence of their fictitious conductors (Figure 3.4).

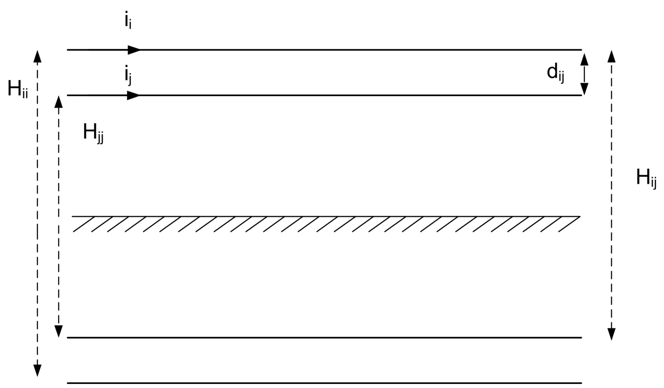

Fig. 3.4 Conductors i and j with their images

Mutual inductance L_{ij} is:

$$L_{ij} = \frac{\mu_0}{2\pi} ln \frac{H_{ij}}{d_{ij}} \tag{3.33}$$

Carson correction factors for normal frequency are:

$$\begin{aligned} \triangle R_{ij} =& 4\omega \cdot 10^{-4} (\frac{\pi}{8} - b \cdot a \cdot cos\varphi) \\ \triangle X_{ij} =& 4\omega \cdot 10^{-4} [\frac{1}{2}(0.6159315 - lna) + b \cdot a \cdot cos\varphi] \end{aligned} \tag{3.34}$$

where:
$b = \frac{\sqrt{2}}{6}$
$a = 0,00281 H_{ii} \sqrt{\frac{f}{\rho}}$, $cos\varphi = 1$ for diagonal elements, and
$a = 0,00281 H_{ij} \sqrt{\frac{f}{\rho}}$, $cos\varphi = \frac{H_{ii} + H_{jj}}{2H_{ij}}$ for off-diagonal elements.

3.1.4 Line capacitance

Gauss's law states that the entire flux through the closed surface is proportional to the electric charge enclosed inside the surface:

$$\oiint_A E \cdot dA = \frac{\sum Q}{\epsilon_0 \epsilon_r} \tag{3.35}$$

From Equation 4.10, electric field per one meter of conductor with uniform line charge $q[C/m]$ placed in the air at distance x outside the conductor is:

$$E \cdot 2\pi x \cdot 1 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{2\pi\epsilon_0 x}$$

Potential difference between two points placed at D_1 and D_2 is:

$$V = \int_{D_1}^{D_2} Edx = \frac{q}{2\pi\epsilon_0} ln\frac{D_2}{D_1} \tag{3.36}$$

In case of n parallel uniformly charged conductors potential difference between any two of them (for example j and k) is obtained by superposition of influence of all the conductors:

$$V_{jk} = \frac{1}{2\pi\epsilon_0} \sum_{i=1}^{n} q_i ln\frac{D_{ik}}{D_{ij}} \tag{3.37}$$

D_{jj} and D_{kk} represent r_j and r_k, radius of conductor j and radius of conductor k respectively. For the purpose of discussion, conductors 1 and 2 will be considered. Conductors are assumed to have radius r and their distance is D. Conductor 1 is uniformly charged by q and conductor 2 is uniformly charged by $-q$. Total V_{12} voltage is:

$$V_{12} = \frac{1}{2\pi\epsilon_0}\left(qln\frac{D}{r_1} - qln\frac{r_2}{D}\right) = \frac{q}{2\pi\epsilon_0} ln\frac{D^2}{r_1 r_2} = \frac{q}{2\pi\epsilon_0} ln\frac{D^2}{r^2} = \frac{q}{\pi\epsilon_0} ln\frac{D}{r} \tag{3.38}$$

Capacitance between conductors a and b is:

$$C_{12} = \frac{q}{V_{12}} = \frac{\pi\epsilon_0}{ln\frac{D}{r}} \tag{3.39}$$

Line capacitance can be taken as two equal line to ground (neutral) capacitances connected in series. Therefore:

$$C_1 = C_2 = 2C_{12} = \frac{2\pi\epsilon_0}{ln\frac{D}{r}} \tag{3.40}$$

Now capacitance of three-phase line with unsymmetrical phase spacing will be obtained. Phase conductors each of radius r are transposed and since the network is assumed balanced:

$$q_a + q_b + q_c = 0 \tag{3.41}$$

For the first transposed section V_{ab} is:

$$V_{ab1} = \frac{1}{2\pi\epsilon_0}\left(q_a ln\frac{D_{ab}}{r} + q_b ln\frac{r}{D_{ab}} + q_c ln\frac{D_{bc}}{D_{ac}}\right) \tag{3.42}$$

For the second transposed section V_{ab} is:

$$V_{ab2} = \frac{1}{2\pi\epsilon_0}\left(q_a ln\frac{D_{bc}}{r} + q_b ln\frac{r}{D_{bc}} + q_c ln\frac{D_{ac}}{D_{bc}}\right) \tag{3.43}$$

For the third transposed section V_{ab} is:

$$V_{ab3} = \frac{1}{2\pi\epsilon_0}\left(q_a ln\frac{D_{ac}}{r} + q_b ln\frac{r}{D_{ac}} + q_c ln\frac{D_{ab}}{D_{bc}}\right) \tag{3.44}$$

In order to simplify the solution average of these three voltages will be obtained:

$$\begin{aligned} V_{ab} &= \frac{V_{ab1} + V_{ab2} + V_{ab3}}{3} \\ &= \frac{1}{6\pi\epsilon_0}\left(q_a ln\frac{D_{ab}D_{bc}D_{ac}}{r^3} + q_b ln\frac{r^3}{D_{ab}D_{bc}D_{ac}} + q_c ln\frac{D_{ab}D_{bc}D_{ac}}{D_{ab}D_{bc}D_{ac}}\right) \\ &= \frac{1}{2\pi\epsilon_0}\left(q_a ln\frac{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}{r} + q_b ln\frac{r}{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}\right) \end{aligned} \tag{3.45}$$

Similarly:

$$V_{ac} == \frac{1}{2\pi\epsilon_0}\left(q_a ln\frac{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}{r} + q_c ln\frac{r}{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}\right) \tag{3.46}$$

Adding these two voltages and using Equation (3.41) gives:

$$V_{ab} + V_{ac} = \frac{3q_a}{2\pi\epsilon_0} ln\frac{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}{r} \tag{3.47}$$

Since $V_{ab} + V_{ac} = 3V_{an}$ Equation (3.47) becomes:

$$V_{an} = \frac{q_a}{2\pi\epsilon_0} ln\frac{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}{r} \tag{3.48}$$

Capacitance per phase to neutral:

$$C_{an} = \frac{q_a}{V_{an}} = \frac{2\pi\epsilon_0}{ln\frac{\sqrt[3]{D_{ab}D_{bc}D_{ac}}}{r}} \tag{3.49}$$

$\sqrt[3]{D_{ab}D_{bc}D_{ac}}$ is denoted as GMD (geometric mean distance).

3.1.5 Capacitance matrix of a three-phase overhead line

For precise and accurate three-phase model calculations capacitance matrix of a three-phase line is obtained using the potential coefficients matrix. Effects of earth are also taken into account. In general for system of n parallel conductors relation between their voltages and line charges can be written in matrix form as:

$$\mathbf{V} = \mathbf{Pq} \tag{3.50}$$

where **P** is potential coefficient matrix. Comparing Equation 3.50 with:

$$\mathbf{q} = \mathbf{CV} \tag{3.51}$$

it can be concluded that capacitance matrix is obtained as an inverse of **P** matrix:

$$\mathbf{C} = \mathbf{P}^{-1} \tag{3.52}$$

In order to obtain elements of potential coefficient matrix, two conductors, j and k will be considered. Since the earth surface is equipotential (its potential is zero) its influence on potential coefficients will be considered. This influence is obtained using the method of images. Method of images is based on the fact that line charge above earth will produce an electric field exactly the same as the same line charge and its image without earth in the half-space where the original charge is placed (Figure 3.5). The image line charge are always the negative of the original line charge.

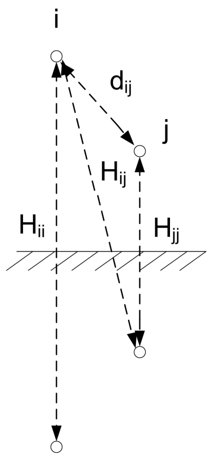

Fig. 3.5 Method of images

From Equation 3.37 potential difference between conductor j and its image is:

$$\begin{aligned} V_{jj'} &= \frac{1}{2\pi\epsilon_0}\left(q_j ln\frac{H_{jj}}{r_j} - q_j ln\frac{r_j}{H_{jj}} + q_k ln\frac{H_{jk}}{D_{jk}} - q_k ln\frac{D_{jk}}{H_{jk}}\right) \\ &= \frac{1}{2\pi\epsilon_0}\left(2q_j ln\frac{H_{jj}}{r_j} + 2q_k ln\frac{H_{jk}}{D_{jk}}\right) \end{aligned} \tag{3.53}$$

Potential difference between conductor j and the ground is one half of the potential difference between conductor j and its image:

$$V_j = \frac{1}{2\pi\epsilon_0}\left(q_j ln\frac{H_{jj}}{r_j} + q_k ln\frac{H_{jk}}{D_{jk}}\right) = p_{jj}q_j + p_{jk}q_k \tag{3.54}$$

From here, it can be concluded that self potential coefficients can be obtained as:

$$p_{jj} = \frac{1}{2\pi\epsilon_0} ln\frac{H_{jj}}{r_j} \tag{3.55}$$

Mutual potential coefficients are:

$$p_{jk} = \frac{1}{2\pi\epsilon_0} ln\frac{H_{jk}}{D_{jk}} \tag{3.56}$$

In case of a three-phase line with neutral wire, primitive coefficient matrix is:

$$\mathbf{P} = \begin{bmatrix} p_{aa} & p_{ab} & p_{ac} & p_{an} \\ p_{ba} & p_{bb} & p_{bc} & p_{bn} \\ p_{ca} & p_{cb} & p_{cc} & p_{cn} \\ p_{na} & p_{nb} & p_{nc} & p_{nn} \end{bmatrix} \tag{3.57}$$

In order to eliminate the influence of neutral wire, the matrix can be reduced using Kron reduction:

$$\mathbf{P}_{abc} = \begin{bmatrix} p_{aa} & p_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix} - \begin{bmatrix} p_{an} \\ p_{bn} \\ p_{cn} \end{bmatrix} [p_{nn}]^{-1} \begin{bmatrix} p_{na} & p_{nb} & p_{nc} \end{bmatrix} \tag{3.58}$$

$$\mathbf{C}_{abc} = \mathbf{P}_{abc}^{-1} \tag{3.59}$$

3.1.6 Transmission line modeling

Now that the line parameters are defined, transmission line model is to be obtained. Transmission system is considered balanced, thus transmission lines are represented in per-phase model. Model of short, medium and long transmission lines will be considered here. Short and medium lines are assumed to have lumped parameters, while for long lined effects of distributed parameters must be considered. Line modeling will be conducted in terms of four pole element as shown in Figure 3.6.

Fig. 3.6 Four pole transmission line representation

3.1.6.1 Short line model

With the lines less than 80km long, capacitance and conductance are usually neglected and the line is represented with series impedance only (Figure 3.7). Line impedance is obtained as:

$$\underline{Z}_L = (r + j\omega L) \cdot l \tag{3.60}$$

where r is line resistance per unit length, L is line inductance per unit length and l is line length.

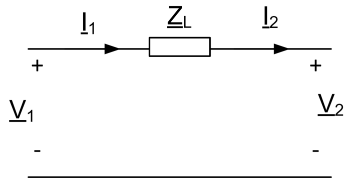

Fig. 3.7 Short line model

It is obvious that:

$$\begin{aligned} \underline{V}_1 =& \underline{V}_2 + \underline{Z}_L \underline{I}_2 \\ \underline{I}_1 =& \underline{I}_2 \end{aligned} \tag{3.61}$$

For four pole element relations between voltages and currents ca be written as:

$$\begin{bmatrix} \underline{V}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \underline{V}_2 \\ \underline{I}_2 \end{bmatrix} \tag{3.62}$$

From Equation (3.61) and Equation (3.62) it follows that:

$$\begin{aligned} A =& 1 \\ B =& \underline{Z}_L \\ C =& 0 \\ D =& 1 \end{aligned} \tag{3.63}$$

3.1.6.2 Medium line model

Lines between 80 km and 250 km are considered medium length lines. Shunt admittance of medium lines can not be neglected and the line is represented using T or π line model. In power system analysis π line model is commonly used. Medium power line model is shown in Figure (3.8).

Series impedance is given in Equation (3.60). $\underline{Y}_0$ is shunt admittance defined as:

$$\underline{Y}_0 = (g + j\omega C) \cdot l \tag{3.64}$$

where g is line conductance per unit length, C is line capacitance per unit length and l is line length. Under normal conditions g is assumed to be zero and shunt admittance is:

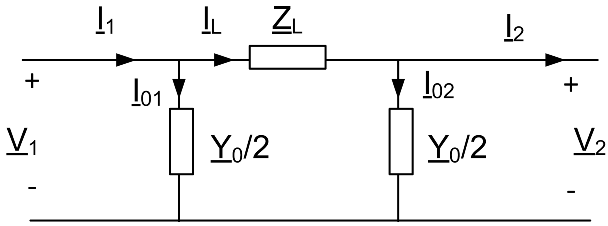

Fig. 3.8 Medium line model

$$\underline{Y}_0 = j\omega C \cdot l \tag{3.65}$$

From Figure (3.8) it is obvious:

$$\underline{I}_1 = \underline{I}_L + \underline{I}_{01} = \underline{I}_L + \frac{\underline{Y}_0}{2}\underline{V}_1 \tag{3.66}$$

$$\underline{V}_1 = \underline{V}_2 + \underline{Z}_L \underline{I}_L \tag{3.67}$$

$$\underline{I}_L = \underline{I}_2 + \frac{\underline{Y}_0}{2}\underline{V}_2 \tag{3.68}$$

Substituting $\underline{I}_L$ in Equation (3.67) for Equation (3.68) gives:

$$\underline{V}_1 = \underline{V}_2 + \underline{Z}_L\left(\underline{I}_2 + \frac{\underline{Y}_0}{2}\underline{V}_2\right) = \left(1 + \frac{\underline{Z}_L\underline{Y}_0}{2}\right)\underline{V}_2 + \underline{Z}_L\underline{I}_2 \tag{3.69}$$

Similarly, substituting $\underline{I}_L$ for Equation (3.68) and $\underline{V}_1$ for Equation (3.69) in Equation (3.66) gives:

$$\underline{I}_1 = \underline{Y}_0\left(1 + \frac{\underline{Z}_L\underline{Y}_0}{4}\right)\underline{V}_2 + \left(1 + \frac{\underline{Z}_L\underline{Y}_0}{2}\right)\underline{I}_2 \tag{3.70}$$

ABCD constants of four pole model are:

$$\begin{aligned} A =& 1 + \frac{\underline{Z}_L\underline{Y}_0}{2} \\ B =& \underline{Z}_L \\ C =& \underline{Y}_0\left(1 + \frac{\underline{Z}_L\underline{Y}_0}{4}\right) \\ D =& 1 + \frac{\underline{Z}_L\underline{Y}_0}{2} \end{aligned} \tag{3.71}$$

3.1.6.3 Long line model

Effects of distribution parameters should be considered when modeling long lines. In the following, ABCD parameters and π equivalent of long line will be obtained. Figure 3.10 shows part of a long line of $\triangle x$ length.

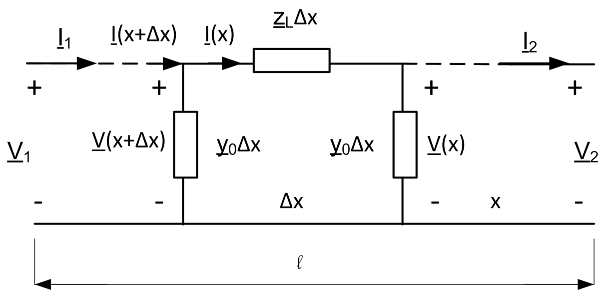

Fig. 3.9 Long line with distributed parameters

Series impedance per unit length is denoted by $\underline{z}_L$ and shunt admittance is denoted by $\underline{y}_0$:

$$\underline{z}_L = r + j\omega L$$
$$\underline{y}_0 = g + j\omega C$$

Voltages and currents are represented as functions of distance. From Figure 3.10 it is obvious:

$$\underline{V}(x+\triangle x) = \underline{V}(x) + \underline{z}_L \triangle x \underline{I}(x) \Rightarrow \underline{z}_L \underline{I}(x) = \frac{\underline{V}(x+\triangle x) - \underline{V}(x)}{\triangle x} \tag{3.72}$$

From here:

$$\frac{d\underline{V}(x)}{dx} = \underline{z}_L \underline{I}(x) \tag{3.73}$$

Similarly:

$$\underline{I}(x+\triangle x) = \underline{I}(x) + \underline{y}_0 \triangle x \underline{V}(x+\triangle x) \Rightarrow \frac{d\underline{I}(x)}{dx} = \underline{y}_0 \underline{V}(x) \tag{3.74}$$

Based on these equations the following second-order differential equation is obtained:

$$\frac{d^2\underline{V}(x)}{dx^2} - \underline{z}_L \underline{y}_0 \underline{V}(x) = 0 \tag{3.75}$$

$\underline{z}_L \underline{y}_0$ will be denoted as γ^2. γ is called *propagation constant.*

$$\gamma = \sqrt{\underline{z}_L \underline{y}_0} = \alpha + j\beta \tag{3.76}$$

α is *attenuation constant* and β is *phase constant.* Equation (3.75) now becomes:

$$\frac{d^2\underline{V}(x)}{dx^2} - \gamma^2 \underline{V}(x) = 0 \tag{3.77}$$

The general solution of Equation (3.77) is:

$$\underline{V}(x) = A_1 cosh\gamma x + A_2 sinh\gamma x \tag{3.78}$$

Constant A_1 is found using the fact that when $x = 0$, $\underline{V}(x) = \underline{V}_2$, $cosh\gamma x = 1$ and $sinh\gamma x = 0$. From here:

$$A_1 = \underline{V}_2 \tag{3.79}$$

Also, when $x = 0$, $\underline{I}(x) = \underline{I}_2$.From Equation (3.73) it follows:

$$\frac{d\underline{V}(0)}{dx} = \underline{z}_L \underline{I}_2 \tag{3.80}$$

Derivating Equation (3.78) yields:

$$\frac{d\underline{V}(x)}{dx} = A_1 \gamma sinh\gamma x + A_2 \gamma cosh\gamma x \tag{3.81}$$

For $x = 0$, from Equation (3.80) and Equation (3.81) follows:

$$\underline{z}_L \underline{I}_2 = A_2 \gamma \Rightarrow A_2 = \sqrt{\frac{\underline{z}_L}{\underline{y}_0}} \underline{I}_2 \tag{3.82}$$

$\sqrt{\frac{\underline{z}_L}{\underline{y}_0}}$ will be denoted as $\underline{Z}_c$. $\underline{Z}_c$ is called *characteristic impedance.* Now, $\underline{V}(x)$ is written as:

$$\underline{V}(x) = \underline{V}_2 cosh\gamma x + \underline{Z}_c \underline{I}_2 sinh\gamma x \tag{3.83}$$

$\underline{I}(x)$ can be expressed from Equation (3.73) as:

$$\underline{I}(x) = \frac{1}{\underline{z}_L} \frac{d\underline{V}(x)}{dx} \tag{3.84}$$

Substituting $\underline{V}(x)$ for Equation (3.83) it follows:

$$
\begin{aligned}
\underline{I}(x) &= \frac{1}{\underline{z}_L}\frac{d}{dx}(\underline{V}_2 cosh\gamma x + \underline{Z}_c\underline{I}_2 sinh\gamma x)\\
&= \frac{1}{\underline{z}_L}(\underline{V}_2\gamma sinh\gamma x + \underline{Z}_c\underline{I}_2\gamma cosh\gamma x)\\
&= \sqrt{\frac{\underline{y}_0}{\underline{z}_L}}\underline{V}_2 sinh\gamma x + \underline{I}_2 cosh\gamma x
\end{aligned}
$$

From here $\underline{I}(x)$ is written as:

$$
\underline{I}(x) = \frac{1}{\underline{Z}_c}\underline{V}_2 sinh\gamma x + \underline{I}_2 cosh\gamma x \tag{3.85}
$$

In case when $x = l$ Equation (3.83) and Equation (3.85) are:

$$
\begin{aligned}
\underline{V}_1 &= cosh\gamma l\underline{V}_2 + \underline{Z}_c sinh\gamma l\underline{I}_2\\
\underline{I}_1 &= \frac{1}{\underline{Z}_c} sinh\gamma l\underline{V}_2 + cosh\gamma l\underline{I}_2
\end{aligned} \tag{3.86}
$$

ABCD constants are:

$$
\begin{aligned}
A &= cosh\gamma l\\
B &= \underline{Z}_c sinh\gamma l\\
C &= \frac{1}{\underline{Z}_c} sinh\gamma l\\
D &= cosh\gamma l
\end{aligned} \tag{3.87}
$$

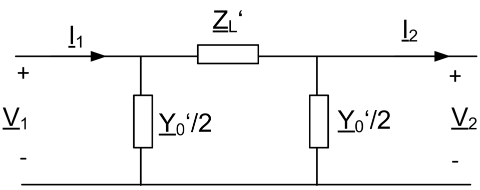

Fig. 3.10 Long line model

Now equivalent π model for long line shown in Figure 3.10 will be obtained from ABCD constants. Using Equation (3.69) and Equation (3.70) equations for equivalent π model shown in Figure 3.10 are:

$$
\begin{aligned}
\underline{V}_1 &= \left(1 + \frac{\underline{Z}'_L\underline{Y}'_0}{2}\right)\underline{V}_2 + \underline{Z}'_L\underline{I}_2\\
\underline{I}_1 &= \underline{Y}_0\left(1 + \frac{\underline{Z}'_L\underline{Y}'_0}{4}\right)\underline{V}_2 + \left(1 + \frac{\underline{Z}'_L\underline{Y}'_0}{2}\right)\underline{I}_2
\end{aligned} \tag{3.88}
$$

Comparing Equation (3.86) and Equation (3.88) parameters of equivalent π model are obtained:

$$
\begin{aligned}
\underline{Z}'_L &= \underline{Z}_L \frac{sinh\gamma l}{\gamma l} \\
\underline{Y}'_0 &= \frac{\underline{Y}_0}{2} \frac{tanh\frac{\gamma l}{2}}{\frac{\gamma l}{2}}
\end{aligned}
\tag{3.89}
$$

It can be concluded that due to the effects of distribution parameters, series impedance and shunt admittance of a long line should be multiplied by correction factors. These factors are functions of line length and for line lengths below 250 km they are approximately equal to one.

3.1.7 Distribution line modeling

Per-phase model used for transmission network analysis usually is not accurate enough when it comes to distribution network. Distribution of single-phase, two-phase and three-phase loads and non-equilateral conductor spacings of lines makes distribution network unbalanced. Distribution network analysis demands development of three-phase line model. Three-phase line model in phase coordinates will be considered here. The series impedance of a three-phase distribution line (overhead and underground) is represented by series impedance matrix consisted of phase resistances and the self and mutual inductive reactances. The shunt admittance of a line is represented by shunt admittance matrix consisted of phase conductances, which are usually ignored, and the capacitive susceptances. Phase capacitance is the result of the potential difference between the phase and the Earth and potential difference between phases. Formulation of series impedance matrix and shunt admittance matrix are already represented.

Model of a three-phase line with neglected conductances is shown in Figure 3.11.

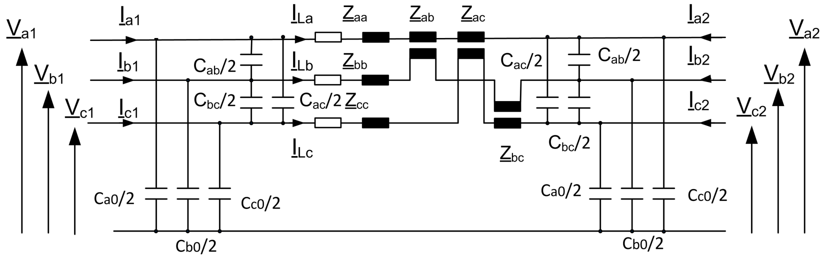

Fig. 3.11 Model of a three-phase line

It is obvious that:

$$\begin{aligned}\begin{bmatrix}\underline{I}_{La}\\ \underline{I}_{Lb}\\ \underline{I}_{Lc}\end{bmatrix} &= \begin{bmatrix}\underline{I}_{a2}\\ \underline{I}_{b2}\\ \underline{I}_{c2}\end{bmatrix} + \tfrac{j\omega}{2}\begin{bmatrix}C_{aa} & C_{ab} & C_{ac}\\ C_{ba} & C_{bb} & C_{bc}\\ C_{ca} & C_{cb} & C_{cc}\end{bmatrix}\begin{bmatrix}\underline{V}_{a2}\\ \underline{V}_{b2}\\ \underline{V}_{c2}\end{bmatrix}\\ &= \begin{bmatrix}\underline{I}_{a2}\\ \underline{I}_{b2}\\ \underline{I}_{c2}\end{bmatrix} + \tfrac{1}{2}\begin{bmatrix}\underline{Y}_{aa} & \underline{Y}_{ab} & \underline{Y}_{ac}\\ \underline{Y}_{ba} & \underline{Y}_{bb} & \underline{Y}_{bc}\\ \underline{Y}_{ca} & \underline{Y}_{cb} & \underline{Y}_{cc}\end{bmatrix}\begin{bmatrix}\underline{V}_{a2}\\ \underline{V}_{b2}\\ \underline{V}_{c2}\end{bmatrix}\end{aligned} \tag{3.90}$$

Diagonal elements of $[C]$ matrix are equal to the sum of corresponding phase-to-ground capacitance and phase-to-phase capacitances.

$$\begin{bmatrix}\underline{V}_{a1}\\ \underline{V}_{b1}\\ \underline{V}_{c1}\end{bmatrix} = \begin{bmatrix}\underline{V}_{a2}\\ \underline{V}_{b2}\\ \underline{V}_{c2}\end{bmatrix} + \begin{bmatrix}\underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac}\\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc}\\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc}\end{bmatrix}\begin{bmatrix}\underline{I}_{La}\\ \underline{I}_{Lb}\\ \underline{I}_{Lc}\end{bmatrix} \tag{3.91}$$

$$\begin{bmatrix}\underline{I}_{a1}\\ \underline{I}_{b1}\\ \underline{I}_{c1}\end{bmatrix} = \begin{bmatrix}\underline{I}_{La}\\ \underline{I}_{Lb}\\ \underline{I}_{Lc}\end{bmatrix} + \frac{1}{2}\begin{bmatrix}\underline{Y}_{aa} & \underline{Y}_{ab} & \underline{Y}_{ac}\\ \underline{Y}_{ba} & \underline{Y}_{bb} & \underline{Y}_{bc}\\ \underline{Y}_{ca} & \underline{Y}_{cb} & \underline{Y}_{cc}\end{bmatrix}\begin{bmatrix}\underline{V}_{a1}\\ \underline{V}_{b1}\\ \underline{V}_{c1}\end{bmatrix} \tag{3.92}$$

Substituting $[\underline{I}_L]_{abc}$ in Equation (3.91) for Equation(3.90) gives:

$$\begin{aligned}\begin{bmatrix}\underline{V}_{a1}\\ \underline{V}_{b1}\\ \underline{V}_{c1}\end{bmatrix} = &\left(\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}\underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac}\\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc}\\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc}\end{bmatrix}\begin{bmatrix}\underline{Y}_{aa} & \underline{Y}_{ab} & \underline{Y}_{ac}\\ \underline{Y}_{ba} & \underline{Y}_{bb} & \underline{Y}_{bc}\\ \underline{Y}_{ca} & \underline{Y}_{cb} & \underline{Y}_{cc}\end{bmatrix}\right)\cdot\begin{bmatrix}\underline{V}_{a2}\\ \underline{V}_{b2}\\ \underline{V}_{c2}\end{bmatrix}\\ &+\begin{bmatrix}\underline{Z}_{aa} & \underline{Z}_{ab} & \underline{Z}_{ac}\\ \underline{Z}_{ba} & \underline{Z}_{bb} & \underline{Z}_{bc}\\ \underline{Z}_{ca} & \underline{Z}_{cb} & \underline{Z}_{cc}\end{bmatrix}\begin{bmatrix}\underline{I}_{a2}\\ \underline{I}_{b2}\\ \underline{I}_{c2}\end{bmatrix}\end{aligned} \tag{3.93}$$

or:

$$[\underline{V}_1]_{abc} = \left([E] + \frac{1}{2}[\underline{Z}]_{abc}[\underline{Y}]_{abc}\right)[\underline{V}_2]_{abc} + [\underline{Z}]_{abc}[\underline{I}_2]_{abc} \tag{3.94}$$

$[E]$ is identity matrix.

Substituting $[\underline{I}_L]_{abc}$ in Equation (3.92) for Equation (3.90) gives:

$$[\underline{I}_1]_{abc} = [\underline{I}_2]_{abc} + \frac{1}{2}[\underline{Y}]_{abc}[\underline{V}_2]_{abc} + \frac{1}{2}[\underline{Y}]_{abc}[\underline{V}_1]_{abc} \tag{3.95}$$

Substituting $[\underline{V}_1]_{abc}$ from Equation (3.94) gives:

$$[\underline{I}_1]_{abc} = \left([\underline{Y}]_{abc} + \frac{1}{4}[\underline{Y}]_{abc}[\underline{Z}]_{abc}[\underline{Y}]_{abc}\right)[\underline{V}_2]_{abc} + \left([E] + \frac{1}{2}[\underline{Y}]_{abc}[\underline{Z}]_{abc}\right)[\underline{I}_2]_{abc} \tag{3.96}$$

ABCD constants for four pole model are now obtained from Equation (3.94) and Equation (3.96):

$$\begin{aligned}
A &= [E] + \frac{1}{2}\,[\underline{Z}]_{abc}\,[\underline{Y}]_{abc} \\
B &= [\underline{Z}]_{abc} \\
C &= [\underline{Y}]_{abc} + \frac{1}{4}\,[\underline{Y}]_{abc}\,[\underline{Z}]_{abc}\,[\underline{Y}]_{abc} \\
D &= [E] + \frac{1}{2}\,[\underline{Y}]_{abc}\,[\underline{Z}]_{abc}
\end{aligned} \tag{3.97}$$

In most cases, except for long, lightly loaded or underground lines, shunt admittance matrix can be neglected in distribution line modeling. ABCD constants in that case are:

$$\begin{aligned}
A &= [E] \\
B &= [\underline{Z}]_{abc} \\
C &= [0] \\
D &= [E]
\end{aligned} \tag{3.98}$$

In case when the line is symmetrical, the only known line data is positive and zero sequence impedances. In that case series impedance matrix is obtained using symmetrical components theory:

$$[\underline{Z}]_{abc} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{Z}_0 & 0 & 0 \\ 0 & \underline{Z}_1 & 0 \\ 0 & 0 & \underline{Z}_1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}^{-1} \tag{3.99}$$

$$[\underline{Z}]_{abc} = \frac{1}{3} \begin{bmatrix} 2\underline{Z}_1 + \underline{Z}_0 & \underline{Z}_0 - \underline{Z}_1 & \underline{Z}_0 - \underline{Z}_1 \\ \underline{Z}_0 - \underline{Z}_1 & 2\underline{Z}_1 + \underline{Z}_0 & \underline{Z}_0 - \underline{Z}_1 \\ \underline{Z}_0 - \underline{Z}_1 & \underline{Z}_0 - \underline{Z}_1 & 2\underline{Z}_1 + \underline{Z}_0 \end{bmatrix} \tag{3.100}$$

In this case self as well as mutual impedances in approximate series impedance matrix are equal. This is the case when three-phase distributed line is assumed to be transposed.

3.2 Transformer Modeling

Models of transformers have been presented in a large variety of publications [17,20–26] where [17] presents an excellent tutorial for symmetrically and non-symmetrically constructed transformers in three phase distribution networks. Especially for transformer banks, symmetry of the phases is generally not given, e.g. if one of the three transformers is additionally related to a large one phase load. The large number of different vector groups also influences the model for an individual transformer. The following issues are to be considered when a transformer is to be modeled in an unbalanced and un-symmetrically constructed distribution network:

- impedances of the transformer/transformer bank may be different between the phases
- transformer ratio may be different between the phases for a voltage control individually per phase
- 3 phase transformers may be un-complete (Open Wye -Open Delta transformers)
- Large number of different vector groups.
- Different grounding of the transformer neutral connectors
- Singularity of the admittance matrices of a transformer with delta windings [21, 23, 24].

Taking all of these issues into account for a general model for three phase transformers, this model requires many different representations, at least one for each vector group. In [20], some individual connection matrices for different transformer vector groups are displayed. Methods for grounding of transformers are discussed in [27]. The handling of the singularity via additional shunts and injections is found in [21–23].

In [28–30], modeling of transformers is derived for the phase as well as for symmetrical components, This approach uses 3 by 3 block matrices in the phase components, which are to be defined in advance for each vector group.

3.2.1 Model in phase coordinates

The transformer model used for single phase network applications is normally given in the admittance form according to

$$\begin{bmatrix} \underline{y}_{Cu} + \dfrac{\underline{y}_{Fe}}{2} & -\underline{t}\underline{y}_{Cu} \\ -\underline{t}^{*}\underline{y}_{Cu} & t^2\left(\underline{y}_{Cu} + \dfrac{\underline{y}_{Fe}}{2}\right) \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix} = \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$

For 3-phase-transformers, the corresponding equation looks as follows in phase coordinates

$$\begin{bmatrix} \underline{Y}_{11}^{(ABC)} & \underline{Y}_{12}^{(ABC)} T^{(ABC)} \\ T^{(ABC)t}\underline{Y}_{21}^{(ABC)} & T^{(ABC)t}\underline{Y}_{22}^{(ABC)} T^{(ABC)} \end{bmatrix} \begin{bmatrix} \underline{V}_1^{(ABC)} \\ \underline{V}_2^{(ABC)} \end{bmatrix} = \begin{bmatrix} \underline{I}_1^{(ABC)} \\ \underline{I}_2^{(ABC)} \end{bmatrix} \quad (3.101)$$

$$\begin{bmatrix} 1 & 0 \\ 0 & T^{(ABC)t} \end{bmatrix} \begin{bmatrix} \underline{Y}_{11}^{(ABC)} & \underline{Y}_{12}^{(ABC)} \\ \underline{Y}_{21}^{(ABC)} & \underline{Y}_{22}^{(ABC)} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & T^{(ABC)} \end{bmatrix} \begin{bmatrix} \underline{V}_1^{(ABC)} \\ \underline{V}_2^{(ABC)} \end{bmatrix} = \begin{bmatrix} \underline{I}_1^{(ABC)} \\ \underline{I}_2^{(ABC)} \end{bmatrix} \tag{3.102}$$

It is important to mention that the admittance matrices $\underline{Y}_{ij}^{(ABC)}$ do not depend on the vector group. With an open-circuit and a short circuit test the admittances can be evaluated in a similar way as it is done for single phase transformers.

3.2.2 Voltage transformation

The voltage ratio matrix $T^{(ABC)}$ is strongly dependent on the vector group of the transformer. For YY- and DD-transformers, this ratio matrix has a diagonal form. For all other transformer vector groups, this matrix is responsible for the phase- and the magnitude shift of the transformer ratio, which means that it also includes the connectivity between the phases at both sides of the transformer. For a YD5 transformer, which shifts the voltage by $150° = 5 \cdot 30°$, one possible relationship between the phase to phase voltages at side 1 (Y-winding) and the phase to neutral voltages at the side 2 (D-winding) is given by (3.103). Another representation is possible:

$$\begin{bmatrix} \underline{V}_A \\ \underline{V}_B \\ \underline{V}_C \end{bmatrix} = -t \begin{bmatrix} \underline{V}_{ab} \\ \underline{V}_{bc} \\ \underline{V}_{ca} \end{bmatrix} = -\frac{t}{3} \begin{bmatrix} 4 & -2 & 1 \\ 1 & 4 & -2 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} \underline{V}_{na} \\ \underline{V}_{nb} \\ \underline{V}_{nc} \end{bmatrix} \tag{3.103}$$

For a DY7 transformer with a phase shift of $-150°$, the corresponding relationship is given by

$$\begin{bmatrix} \underline{V}_A \\ \underline{V}_B \\ \underline{V}_C \end{bmatrix} = -t \begin{bmatrix} \underline{V}_{ab} \\ \underline{V}_{bc} \\ \underline{V}_{ca} \end{bmatrix} = -\frac{t}{3} \begin{bmatrix} 4 & 1 & -2 \\ -2 & 4 & 1 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} \underline{V}_{na} \\ \underline{V}_{nb} \\ \underline{V}_{nc} \end{bmatrix}$$

The same ratio for the three phases is assumed in these equations.

3.2.3 Admittances of the transformer

The admittance matrices $\underline{Y}_{ij}^{(ABC)}$ in (3.102) include the couplings between the three phases which are not caused by the vector group. For a transformer bank, these couplings do not exist for constructional reasons. The admittance matrices for the transformer can be easily created in phase coordinates. All the four block matrices $\underline{Y}_{ij}^{(ABC)}$ of (3.102) have a diagonal form. However, the electrical characteristics may differ significantly between the 3 two-phase-transformers, e.q. different rated power. For transformer banks, the admittance matrices in (3.102) can easily be defined in the phase coordinates.

For 'real' 3 phase transformers (one core transformer) the transformer impedances are normally given by the impedance in the positive sequence system and the ratio between the reactance and resistance in the positive sequence and the zero sequence system. For these transformers, the admittance matrices can easily be defined in the symmetrical components by the diagonal matrices.

$$\underline{Y}_{ij}^{(012)} = \begin{bmatrix} \underline{Y}_{ij}^{(0)} & 0 & 0 \\ 0 & \underline{Y}_{ij}^{(1)} & 0 \\ 0 & 0 & \underline{Y}_{ij}^{(2)} \end{bmatrix}$$

3.2.4 Transformer model in 012-components

In 012-components, also called symmetrical components, the angle shift caused by the different vector groups can be modeled much easier than using phase coordinates. The impedances are given very often in symmetrical components for 'real' 3-phase transformers. For transformer banks, the admittance matrices have a diagonal form and are very easily transformed to the symmetrical components by the following equation:

$$\underline{Y}^{(012)} = \underline{T}_{p2s}\underline{Y}^{(ABC)}\underline{T}_{p2s}^{-1}$$

Using 012-components, (3.101) turns to

$$\begin{bmatrix} \underline{Y}_{11}^{(012)} & \underline{Y}_{12}^{(012)}T^{(012)} \\ T^{(012)*}\underline{Y}_{21}^{(012)} & T^{(012)*}\underline{Y}_{22}^{(012)}T^{(012)} \end{bmatrix} \begin{bmatrix} \underline{V}_1^{(012)} \\ \underline{V}_2^{(012)} \end{bmatrix} = \begin{bmatrix} \underline{I}_1^{(012)} \\ \underline{I}_2^{(012)} \end{bmatrix} \tag{3.104}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & T^{(012)*} \end{bmatrix} \begin{bmatrix} \underline{Y}_{11}^{(012)} & \underline{Y}_{12}^{(012)} \\ \underline{Y}_{21}^{(012)} & \underline{Y}_{22}^{(012)} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & T^{(012)} \end{bmatrix} \begin{bmatrix} \underline{V}_1^{(012)} \\ \underline{V}_2^{(012)} \end{bmatrix} = \begin{bmatrix} \underline{I}_1^{(012)} \\ \underline{I}_2^{(012)} \end{bmatrix} \tag{3.105}$$

The difference between (3.101) and (3.104) is, that voltages, currents and block matrices are converted to the 012-system. The transformer ratio $T^{(ABC)}$ turns to a complex diagonal matrix.

$$\underline{T}^{(012)} = \begin{bmatrix} \underline{t} & 0 & 0 \\ 0 & \underline{t} & 0 \\ 0 & 0 & \underline{t}^* \end{bmatrix}, \ \underline{t} = t \cdot e^{j\alpha}$$

It includes the magnitude shift as well as the angle shift a caused by the vector group. This is the only parameter, which is needed to describe the vector group of the transformer.

Let us assume that there is a transformer bank of 3 different transformers, the impedances/admittances for each of them be known. Then, vector group of the transformer bank will also be known, e.g. YD5.

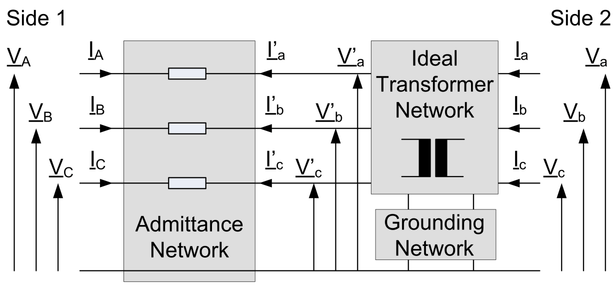

Fig. 3.12 3-phase transformer in modeled in phase coordinates

According to our assumptions for a transformer bank, there are no couplings in the transformer admittance matrix in the phase coordinates. The model consists of the three transfer impedances, related to side 1 on the left, a network of ideal transformers and a grounding network. No-load losses are ignored in this derivation.

The block admittance matrices for the transformer admittance network are given by

$$\underline{Y}_{11}^{(ABC)} = \underline{Y}_{22}^{(ABC)} = \begin{bmatrix} \underline{Y}_A & 0 & 0 \\ 0 & \underline{Y}_B & 0 \\ 0 & 0 & \underline{Y}_C \end{bmatrix} = -\underline{Y}_{12}^{(ABC)} = -\underline{Y}_{21}^{(ABC)}$$

Concerning the voltage ratio matrix $T^{(ABC)}$, its exact representation is not of interest. The only information we need from this matrix is, that it is to turn the phase to ground voltages by $5 \cdot 30°$ for a YD5-vector group.

In the next step the admittance matrix $\underline{Y}_{ij}^{(ABC)}$ are transformed to 012-components:

$$\underline{Y}_{11}^{(012)} = \underline{T}_{p2s}\underline{Y}_{11}^{(ABC)}\underline{T}_{p2s}^{-1}$$

$$\underline{Y}_{11}^{(012)} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{Y}_A & 0 & 0 \\ 0 & \underline{Y}_B & 0 \\ 0 & 0 & \underline{Y}_C \end{bmatrix} \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} \underline{Y}^{(d)} & \underline{Y}^{(1)} & \underline{Y}^{(2)} \\ \underline{Y}^{(2)} & \underline{Y}^{(d)} & \underline{Y}^{(1)} \\ \underline{Y}^{(1)} & \underline{Y}^{(2)} & \underline{Y}^{(d)} \end{bmatrix}$$

where

$$\underline{Y}^{(d)} = \frac{1}{3}\left(\underline{Y}_A + \underline{Y}_B + \underline{Y}_C\right)$$

$$\underline{Y}^{(1)} = \frac{1}{3}\left(\underline{Y}_A + \underline{a}^2\underline{Y}_B + \underline{a}\,\underline{Y}_C\right)$$

$$\underline{Y}^{(2)} = \frac{1}{3}\left(\underline{Y}_A + \underline{a}\,\underline{Y}_B + \underline{a}^2\underline{Y}_C\right)$$

In 012-components the block matrices are only diagonal, if the admittances in the three phases are the same However, the matrix in 012-components is cyclically symmetric and only three elements have to be calculated.

In 01-components, the transformer model is displayed in the following figures for a YY and a YD transformer, each Y winding connected to ground via an impedance:

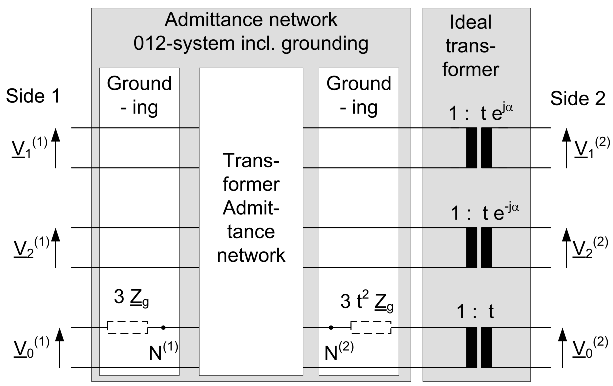

Fig. 3.13 3-phase transformer (vector group YY) modeled in 012-components

The grounding impedances displayed in these figure are connected from additional nodes in the zero sequence system. These nodes must be eliminated in the Admittance network of Fig. 3.13 and Fig. 3.14.

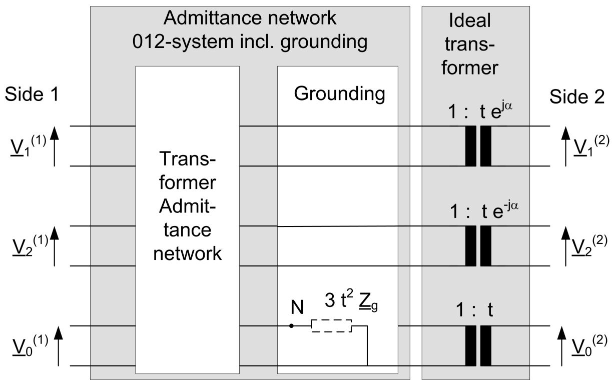

Fig. 3.14 3-phase transformer (vector group YD) modeled in 012-components

3.2.4.1 Elimination of the neutral node

When analyzing Fig. 3.13 and Fig. 3.14 it is obvious, that there exist one or two additional nodes only needed for the grounding. These nodes, however, are not of interest for the subsequent steps and should be eliminated.

The matrix $\underline{Y}_{TrGr}^{(012)}$ according to (3.106) represents the complete admittance matrix (still without the ideal transformers, but with the internal neutral nodes in the zero-sequence system):

$$\underline{Y}_{TrGr}^{(012)} = \begin{bmatrix} \underline{Y}_{11}^{(012)} & \underline{Y}_{12}^{(012)} & \underline{Y}_{1N}^{(012)} \\ \underline{Y}_{21}^{(012)} & \underline{Y}_{22}^{(012)} & \underline{Y}_{2N}^{(012)} \\ \underline{Y}_{N1}^{(012)} & \underline{Y}_{N2}^{(012)} & \underline{Y}_{NN}^{(012)} \end{bmatrix} \tag{3.106}$$

The additional neutral nodes can be eliminated by the following formal equation, well known as the Kron-elimination:

$$\underline{Y}_{g-ij}^{(012)} = \underline{Y}_{ij}^{(012)} - \underline{Y}_{iN}^{(012)} \underline{Y}_{NN}^{(012)^{-1}} \underline{Y}_{Nj}^{(012)} \tag{3.107}$$

where $\underline{Y}_{g-ij}^{(012)}$ represents the admittance block matrix at position ij, where the grounding is considered (with neutral nodes eliminated). The final admittance matrix

$$\underline{Y}_{g}^{(012)} = \begin{bmatrix} \underline{Y}_{g-11}^{(012)} & \underline{Y}_{g-12}^{(012)} \\ \underline{Y}_{g-21}^{(012)} & \underline{Y}_{g-22}^{(012)} \end{bmatrix}$$

has the desired dimension of 6×6.

3.2.4.2 YY-Transformer

In case that the grounding impedance is not zero or the neutral node is not grounded at all, the corresponding node must be eliminated. This principle approach for the treatment of grounding impedances is proposed for YY transformers and transformer banks with identical 2 winding transformers for calculations in the phase domain. In the following approach, the internal connectivity nodes, where the grounding impedances are connected to, are eliminated in the zero system of the transformer.

Fig. 3.15 displays the symmetrical components of a YY transformer before and after the elimination of the node. This general representation considers couplings between the symmetrical components of the transformer.

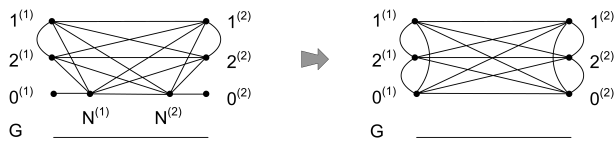

Fig. 3.15 Elimination of the internal nodes N_{ABC} and N_{abc} for a YY- transformer with not solidly grounded neutral node

If a YY- transformer is solidly grounded at side 1 and/or 2, then the corresponding grounding impedance in Fig. 3.15 is zero. The internal node N_1 and/or N_2 and the ports 0_1 and/or 0_2 are to be reduced to one node at side 1 and/or 2. There is no elimination of the corresponding internal node necessary in this case.

Fig. 3.16 displays the symmetrical components before and after the elimination of the neutral nodes, when N_1 is solidly grounded, but N_2 is not grounded or grounded via an impedance.

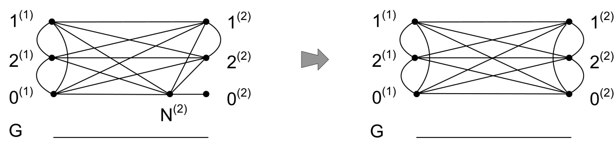

Fig. 3.16 Elimination of the neutral node N_2 for a YY- transformer with not solidly grounded neutral node N_1

In case, that both neutral nodes are solidly grounded, no elimination of these nodes (N_1 and N_2) is necessary.

Shunt admittances of the transformer can be quite large in the zero sequence system and need to be considered in general. In cases, where these shunt admittances can be ignored or where no couplings between the 012-system of the transformer exist, the grounding impedances of both sides may be added to one effective grounding impedance. Otherwise, both impedances should be modeled as individual grounding impedances.

3.2.4.3 YD and DY transformer

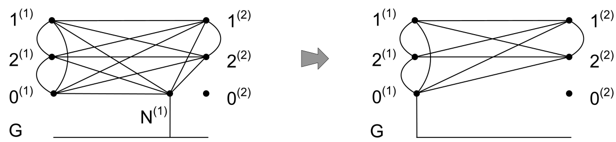

Fig. 3.17 Elimination of the neutral node N_1 for a YD- transformer

For YD transformer, the neutral node is to be eliminated in any case, no matter whether this transformer is grounded, solidly grounded or not grounded.

Fig. 3.17 displays the symmetrical components of a YD transformer before and after the elimination of the neutral node. Affected connections between the nodes are displayed with dashed lines. The treatment of DY transformers is equivalent.

3.2.4.4 Consideration of the ideal transformer network

Using the matrix according to (3.107), the final admittance matrix of the transformer can be calculated in 012- components:

$$\underline{Y}_{gT}^{(012)} = \begin{bmatrix} 1 & 0 \\ 0 & \underline{T}^{(012)^*} \end{bmatrix} \begin{bmatrix} \underline{Y}_{g-11}^{(012)} & \underline{Y}_{g-12}^{(012)} \\ \underline{Y}_{g-21}^{(012)} & \underline{Y}_{g-22}^{(012)} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \underline{T}^{(012)} \end{bmatrix}$$

$$\underline{Y}_{gT}^{(012)} = \begin{bmatrix} \underline{Y}_{g-11}^{(012)} & \underline{Y}_{g-12}^{(012)}\underline{T}^{(012)} \\ \underline{T}^{(012)^*}\underline{Y}_{g-21}^{(012)} & \underline{T}^{(012)^*}\underline{Y}_{g-22}^{(012)}\underline{T}^{(012)} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{gT-11}^{(012)} & \underline{Y}_{gT-12}^{(012)} \\ \underline{Y}_{gT-21}^{(012)} & \underline{Y}_{gT-22}^{(012)} \end{bmatrix} \quad (3.108)$$

where $\underline{Y}_{gT}^{(012)}$ represents the final admittance matrix including the grounding and the voltage ratio matrix. For transformers with tap changers at both sides, (3.108) turns to

$$\underline{Y}_{gT}^{(012)} = \begin{bmatrix} \underline{T}_1^{(012)^*} & 0 \\ 0 & \underline{T}_2^{(012)^*} \end{bmatrix} \begin{bmatrix} \underline{Y}_{g-11}^{(012)} & \underline{Y}_{g-12}^{(012)} \\ \underline{Y}_{g-21}^{(012)} & \underline{Y}_{g-22}^{(012)} \end{bmatrix} \begin{bmatrix} \underline{T}_1^{(012)} & 0 \\ 0 & \underline{T}_2^{(012)} \end{bmatrix}$$

where $\underline{T}_1^{(012)}$ and $\underline{T}_2^{(012)}$ represent the transformer ratio matrix for side 1 and side 2 in 012-components. The matrix

$$\begin{bmatrix} \underline{T}_1^{(012)} & 0 \\ 0 & \underline{T}_2^{(012)} \end{bmatrix}$$

with $\underline{T}_{1|2}^{(012)} = diag(t_{1|2}, t_{1|2}, t_{1|2}^*)$ is the equivalent to the connection matrix known from the phase coordinates.

3.2.4.5 Transfer to phase coordinates

In case that the admittance matrix of the transformer is needed in phase coordinates, then each of the block matrices in (3.108) is to be transferred to the phase coordinates by the following equation:

$$\underline{Y}_{g-ij}^{(ABC)} = \underline{T}_{p2s}^{-1} \underline{Y}_{g-ij}^{(012)} \underline{T}_{p2s}$$

This step is not necessary for calculations which are executed in symmetrical components.

3.2.4.6 Modeling of a real tree-phase-transformer

The presented approach has been derived for a transformer bank. For transformers with different electrical characteristics, there are no couplings between the different phases in the admittance matrix $\underline{Y}^{(ABC)}$.

For a real three phase transformer, the impedances of the positive- and zero-sequence system are mostly given directly or they can be easily calculated from the name plate data.

In these cases, it is sufficient to create the transformer block matrices directly in the 012-system.

3.2.5 Special case

In the previous sections, the admittance matrices for a YD and YY transformer have been derived. In the following, the remaining transformer vector groups are explained.

3.2.5.1 DY transformer

A DY transformer with a phase shift of α a can be derived from the YD model by the following steps:

- exchange the counting schema of the transformer. use the Y side instead of the D-side as side 1 and vice versa
- the angle of voltage shift is to be multiplied with -1
- the transformer ratio turns to its inverse in 012-system

The new model of a DY-transformer is reduced to the known model of a YD-transformer. There is no change of the principle model necessary.

3.2.5.2 DD- transformer

While a DD and a YY transformer bank with identical data can be modeled like a YY transformers, this approach needs additional considerations, when the electrical data are different. The following approach transforms a DD transformer in such a way that the transformer models in the previous sections can directly be used and the individual characteristics still exists.

Transformer can be modeled as a pi-branch with 2 ideal transformers with a solidly grounded Y at their internal terminals. Equivalently, a DD transformer can be reduced to the model of a YD transformer by a combination of a solidly grounded DY winding at side 1 and a solidly grounded YD winding at side 2, one of them with a positive angle shift of α, the other with a negative angle shift of $-\alpha$. The transformer matrix in 012-system from (3.108) converts to:

$$\underline{Y}_{gT}^{(012)} = \begin{bmatrix} \underline{T}^{(012)^*}\underline{Y}_{g-11}^{(012)}\underline{T}^{(012)} & \underline{T}^{(012)^*}\underline{Y}_{g-12}^{(012)}\underline{T}^{(012)} \\ \underline{T}^{(012)^*}\underline{Y}_{g-21}^{(012)}\underline{T}^{(012)} & \underline{T}^{(012)^*}\underline{Y}_{g-22}^{(012)}\underline{T}^{(012)} \end{bmatrix} \quad (3.109)$$

This representation is equivalent to a serial of two ideal DY transformers.

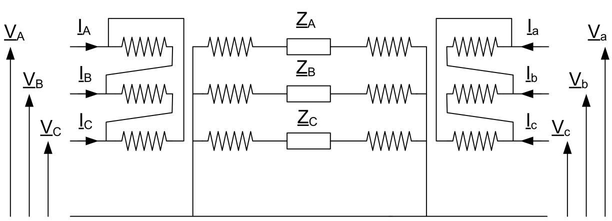

Fig. 3.18 DD-transformer, modeled by a DY- and a YD-transformer

Using this model for a DD- transformer has some advantages when this transformer is also shifting the voltage angle by $n \cdot 30°$ with n an element from

$\{2, 4, 6, 8, 10\}$. As such transformers are very seldom. They are mentioned here for completeness reasons.

3.2.5.3 Open Wye-Open Delta transformer (oYoD)

Non-complete 3 phase transformers are often used for creating a symmetrical voltage from a two phase network.

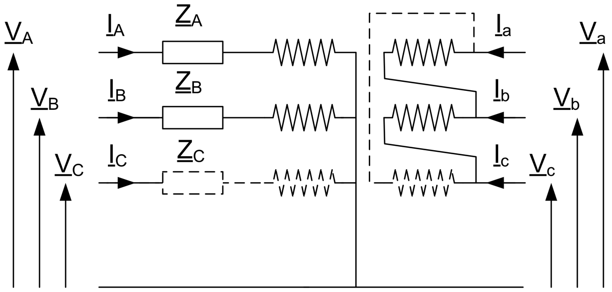

Fig. 3.19 oYoD-transformer, modeled as a complete YD transformer. The missing, virtual transformer is displayed in dashed lines

This kind of transformer is extended to a complete YD transformer by adding a virtual transformer with an infinite impedance (admittance is equal to zero). The angle shift of the oYoD transformer and the YD transformer is to be the same.

For this transformer (here un-complete in phase C), the admittance block matrices are as follows:

$$\begin{aligned}\underline{Y}_{11}^{(ABC)} = \underline{Y}_{22}^{(ABC)} &= \begin{bmatrix} \underline{Y}_A & 0 & 0 \\ 0 & \underline{Y}_B & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= -\underline{Y}_{12}^{(ABC)} = -\underline{Y}_{21}^{(ABC)}\end{aligned}$$

Using these block matrices, the oYoD-transformer is to be treated like an ordinary YD transformer.

3.2.5.4 Open Delta-Open Delta transformer (oDoD)

oDoD-transformers are handled in the same way as a oYoD-transformer: by adding a virtual transformer the uncomplete 3 phase transformer is completed.

The impedance of this virtual transformer is to be infinite (or, in other words, the admittance must be 0).

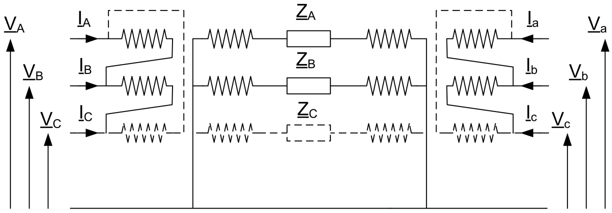

Fig. 3.20 oDoD-transformer, modeled by a DY- and a YDtransformer. The missing, virtual transformer is displayed in dashed lines

3.2.5.5 Grounding of both sides using the same impedance

In section 3.2.4.1, the elimination of the neutral stars is discussed for a direct connection from the neutral star to ground. In Fig. 3.21 a more common grounding with an impedance $\underline{Z}_G$ used for both sides of a YY-transformer. As the current through the impedance $\underline{Z}_G$ can not be assigned to one of the transformer sides. The grounding model previously discussed cannot be used here. This also concerns grounded auto-transformers, with mutual grounding impedance $\underline{Z}_G$. For these transformers, the impedances $\underline{Z}_{G1}$ and $\underline{Z}_{G2}$ are zero. By construction the neutral nodes for both sides is degenerates to one single neutral node.

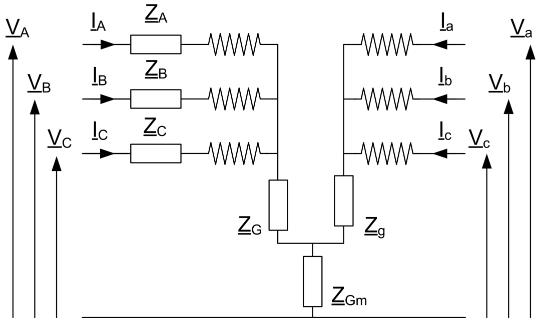

Fig. 3.21 YY-transformer, grounded at both sides via a mutual impedance Z_{Gm}

Splitting this transformer representation into two networks, one representing a YY transformer solidly grounded at both sides, and one representing the grounding impedance network $\underline{Z}_G$, the final impedance matrix of the transformer can be calculated by

$$\underline{Z}^{(ABC)} = \underline{Z}_T^{(ABC)} + \underline{Z}_G^{(ABC)}$$

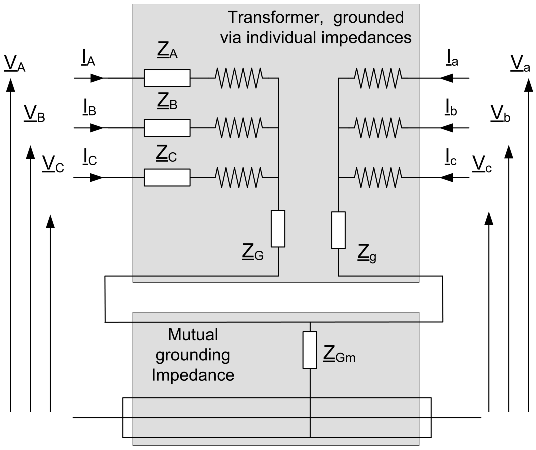

Fig. 3.22 Transformer grounding via a mutual grounding impedance

In distribution networks, this type of grounding is very rare. The additional effort caused by the inversion of 6×6 matrices is acceptable for a few number of transformers, which have such mutual grounding impedances.

Example

Calculate three phase equivalent for a YD-Transformer with 30° leading (vector group YD1).

Solution

For a YD-Transformer with 30° leading (vector group YD1) the admittance matrix related to the admittance Y of the transformer is calculated using the proposed method. No-load losses are ignored.

$$\frac{\underline{Y}_{YD1}}{\underline{Y}} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ 0 & 1 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \hline \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{3} & \frac{2}{3} & \frac{-1}{3} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 & \frac{-1}{3} & \frac{-1}{3} & \frac{2}{3} \end{array}\right] \tag{3.110}$$

Figure Fig. 3.23 displays the connectivity and the phasor diagram of this transformer.

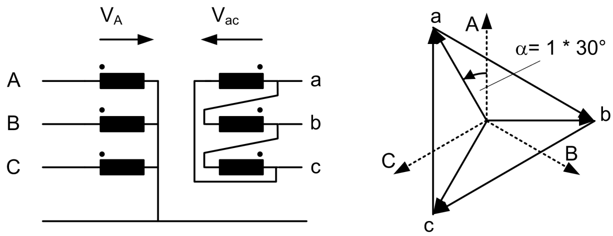

Fig. 3.23 Connectivity and phasor diagram of a YD1 transformer

For a oYoD transformer with the same vector group (phase C missing in the Y-side), the admittance matrix is calculated. It is identical with the representation in [21] with the transformer ratio at both sides of $\alpha = \beta = 1$.

$$\frac{\underline{Y}_{oYoD1}}{\underline{Y}} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ 0 & 1 & 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 & \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 0 & \frac{-1}{\sqrt{3}} & 0 & \frac{-1}{3} & \frac{1}{3} & 0 \\ \frac{1}{\sqrt{3}} & 0 & 0 & \frac{-1}{3} & 0 & \frac{1}{3} \end{array}\right] \tag{3.111}$$

All elements in the column and raw which belong to the non existing phase C on the Y-side are zero.

This representation of the oYoD-transformer also proves that the proposed model handles transformers with different admittances per phase. In this case, the admittance of the non-existing, virtual transformer is set to zero.

Chapter 4
Power Flow Analysis

Under normal conditions, electrical transmission systems operate in their steady-state mode and the basic calculation required to determine the characteristics of this state is termed power flow (or load flow). The object of load-flow calculations is to determine the steady-state operating characteristics of the power generation/transmission system for a given set of loads . Active power generation is normally specified according to economic dispatching practice and the generator voltage magnitude is normally maintained at a specified level. Loads are normally specified by their constant active and reactive power requirement, assumed unaffected by the small variations of voltage and frequency expected during normal steady-state operation. The power flow solution is consisted of two steps. The first one is to find the complex voltage at all buses. The second step consist of computing currents, active and reactive power flows, ohmic losses, etc. T power flow solver is used as a stand-alone tool or as a subroutine within more complex processes like stability analysis, optimization problems, etc. The power and voltage constraints make the problem nonlinear thus the iterative numerical solution must be applied. Problems faced in the development of load flow are an increasing size of systems, on-line applications for automatic control, and system optimization. Main properties required of a load-flow solution method are [31]:

- High computational speed is important when dealing with large systems, real time applications, multiple case load flow such as in system security assessment, and also in interactive applications.
- Low computer storage is important for large systems and in the use of computers with small core storage availability.
- The solution has to be obtained for ill-conditioned problems, in outage studies and for real time applications.
- An ability on the part of load flow to handle conventional and special features (e.g. the adjustment of tap ratios on transformers; different representations of power system apparatus), and its suitability for incorporation into more complicated processes.

- The coding of a computer program of the load-flow algorithm has to be simple .

In solving the power flow problem the system is assumed to be operating under balanced conditions and four quantities are associated with each system bus: voltage magnitude and phase angle, active and reactive power. According to known and unknown quantities system buses are generally classified into three types: Slack, Load and Regulated bus.

- Slack bus is taken as a reference and the magnitude and phase angle of the voltage are specified.
- Load or P-Q bus is a bus with specified active and reactive power. The magnitude and phase angle of the voltage are to be determined.
- Regulated or P-V bus is a generator bus and its active power and voltage magnitude are specified. The phase angle and reactive power are to be determined.

The mathematical formulation of power flow problem results in a system of algebraic nonlinear equations which must be solved by iterative techniques.

4.1 The Per-Unit System

The solution of an interconnected power system having several different voltage levels requires the transformation of all impedances to a single voltage level. The basic idea of the per-unit system is to express various electrical quantities such as power, voltage, current and impedance in terms of base quantities. Such approach would make the different voltage levels disappear,meaning that power network involving generators, transformers and lines of different voltage levels would be reduced to a system of simple impedances. Also, the comparison of results from different voltage levels becomes easy and transparent. Any quantity in per-unit system can be defined as:

$$\textit{Quantity in per-unit} = \textit{actual quantity/base value of quantity}$$

Actual values are phasor quantities or complex numbers. Base values are always real numbers. To completely define a per-unit system: base volt-ampere, voltage, current and impedance are needed. Usually, three-phase base volt-ampere S_b and line-to-line voltage V_b are selected and then base current and base impedance are calculated:

$$I_b = \frac{S_b}{\sqrt{3}V_b} \tag{4.1}$$

$$Z_b = \frac{V_b}{\sqrt{3}I_b} = \frac{V_b^2}{S_b} \tag{4.2}$$

Base voltage of a certain voltage level of the system is selected and all the other base voltages depend on that voltage base and various transformer turns ratios. For example, if on a low-voltage side of a $34.5/115kV$ transformer the base voltage of $36kV$ is selected, then the base voltage on high-voltage side must be $36 \cdot (115/34.5)kV = 120kV$. An arbitrary base for apparent power of the system is selected and all per unit impedances and currents will be expressed based on this common system base. The most important characteristic of the per-unit system is that the phase and line quantities are the same and all circuit laws are valid. Once selected base apparent power remains the same for all the parts of the system, which can be demonstrated:

$$S_{b1} = \frac{V_{b1}^2}{Z_{b1}}, \quad S_{b2} = \frac{V_{b2}^2}{Z_{b2}}$$

$$\frac{S_{b1}}{S_{b2}} = \frac{Z_{b1}}{Z_{b2}} \left(\frac{V_{b1}}{V_{b2}} \right)^2 = \frac{n^2}{n^2} = 1 \Rightarrow S_{b1} = S_{b2} \tag{4.3}$$

Example

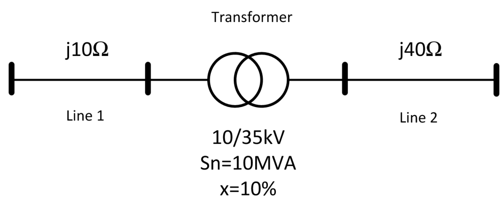

Fig. 4.1 Part of AC network

For part of a network shown in Figure 4.1 obtain equivalent impedances for voltage reference:

a) 10kV
b) 35kV
c) 110kV
d) in per unit.

Solution

a) For 10kV reference impedance of Line 1 is:

$$z_{L1} = j10\Omega$$

Impedance of Line 2 is:

$$z_{L2} = j40 \cdot \left(\frac{10}{35}\right)^2 = j3.26\Omega$$

x is given by the manufacturer and it indicates the percentage of nominal impedance equal to element impedance. Transformer resistance is neglected, thus the impedance of the transformer for 10kV reference is:

$$z_T = j\frac{x}{100}\frac{V_n^2}{S_n} = j\frac{10}{100}\frac{10^2}{10} = j1\Omega$$

b) For 35kV reference impedance of Line 1 is:

$$z_{L1} = j10 \cdot \left(\frac{35}{10}\right)^2 = j122.5\Omega$$

Impedance of Line 2 is:

$$z_{L2} = j40\Omega$$

Impedance of the transformer for 35kV reference is:

$$z_T = j\frac{10}{100}\frac{35^2}{10} = j12.25\Omega$$

c) For 110kV reference impedance of Line 1 is:

$$z_{L1} = j10 \cdot \left(\frac{110}{10}\right)^2 = j1210\Omega$$

Impedance of Line 2 is:

$$z_{L2} = j40 \cdot \left(\frac{110}{35}\right)^2 = j355,1\Omega$$

Impedance of the transformer for 110kV reference is:

$$z_T = j\frac{10}{100}\frac{110^2}{10} = j121\Omega$$

d) Since the base power is not specified, $S_b = 10MVA$ will be taken. Base voltage on the left side of the transformer is $V_{b1} = 10kV$, and the base voltage of the right side of the transformer:

$$V_{b2} = V_{b1}\frac{35}{10} = 35kV$$

Thus, base impedances are:

$$Z_{b1} = \frac{V_{b1}^2}{S_b} = \frac{10^2}{100} = 10\Omega$$

$$Z_{b2} = \frac{V_{b2}^2}{S_b} = \frac{35^2}{100} = 122.5\Omega$$

Impedance of Line 1 is:

$$z_{L1}^{pu} = \frac{z_{L1}}{Z_{b1}} = j1\Omega$$

Impedance of Line 2 is:

$$z_{L2}^{pu} = \frac{z_{L2}}{Z_{b2}} = j0.326\Omega$$

Impedance of the transformer:

$$z_T = \frac{jX_{T1}}{Z_{b1}} = \frac{j\frac{x}{100}\frac{V_{n1}^2}{S_n}}{Z_{b1}} = j\frac{\frac{10}{100}\frac{10^2}{10}}{10} = j0.1pu$$

Note that also:

$$z_T = \frac{jX_{T2}}{Z_{b2}} = \frac{j\frac{x}{100}\frac{V_{n2}^2}{S_n}}{Z_{b2}} = j\frac{\frac{10}{100}\frac{35^2}{10}}{122.5} = j0.1pu$$

In per unit transformer impedance is independent of voltage level.

Example

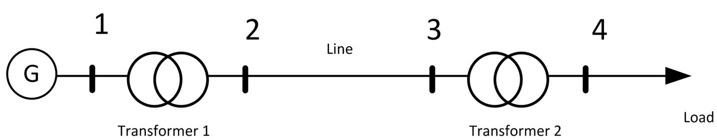

Fig. 4.2 Power network with two transformers

Consider the electric power system shown in Figure 4.2. Draw an impedance diagram for the system showing all impedances in per unit on common 100MVA base. Given is the following data:

Generator: $S_n = 80MVA \quad V_n = 20kV \quad x = 9\%$

Transformer 1: $S_n = 50MVA \quad V_1/V_2 = 20/220kV \quad x = 10\%$

Transformer 2: $S_n = 50MVA \quad V_3/V_4 = 220/11kV \quad x = 10\%$

Line: $Z = 120 + j200\Omega$

Load: $V_n = 10kV \quad \underline{S} = 35 + j45MVA$

x is given by the manufacturer and it indicates the percentage of nominal impedance equal to element impedance.

Solution

Base voltage for bus 1 - the generator bus will be taken equal to generator nominal voltage $V_{b1} = 20kV$. Now we must determine base voltages for all sections of the network in accordance to the transformer turn ratios. V_{b2} is base voltage on the HV side of Transformer 1 and it is also base voltage for the line:

$$V_{b2} = V_{b1}\frac{220}{20} = 220kV$$

V_{b3} is base voltage on the LV side of Transformer 2 and it is also base voltage for the load bus:

$$V_{b3} = V_{b2}\frac{11}{220} = 11kV$$

Now that all base voltages as well as 100 MVA base apparent power are specified, we can obtain pet unit impedances for all system elements. Since the per unit impedance is equal to the ratio of actual impedance and base impedance it is convenient to obtain base impedances for all voltage levels of the network. Base impedance for 20kV voltage level is:

$$Z_{b1} = \frac{V_{b1}^2}{S_b} = \frac{20^2}{100} = 4\Omega$$

Base impedance for 220kV voltage level is:

$$Z_{b2} = \frac{V_{b2}^2}{S_b} = \frac{220^2}{100} = 484\Omega$$

Base impedance for 11kV voltage level is:

$$Z_{b3} = \frac{V_{b3}^2}{S_b} = \frac{11^2}{100} = 1.21\Omega$$

Resistances and shunt branches of the generator and transformers are neglected. Per unit internal impedance of the generator is:

$$\underline{Z}_g^{pu} = \frac{jX_g}{Z_{b1}} = \frac{j\frac{x}{100}\frac{V_n^2}{S_n}}{Z_{b1}} = j\frac{\frac{9}{100}\frac{20^2}{80}}{4} = j0.11pu$$

Per unit impedances of Transformer 1 and Transformer 2 are:

$$\underline{Z}_{T1}^{pu} = \frac{jX_{T1}}{Z_{b1}} = \frac{j\frac{x}{100}\frac{V_n^2}{S_n}}{Z_{b1}} = j\frac{\frac{10}{100}\frac{20^2}{50}}{4} = j0.2pu$$

$$\underline{Z}_{T2}^{pu} = \frac{jX_{T2}}{Z_{b2}} = \frac{j\frac{x}{100}\frac{V_n^2}{S_n}}{Z_{b2}} = j\frac{\frac{10}{100}\frac{220^2}{50}}{484} = j0.2pu$$

Per unit impedance of the line is:

$$\underline{Z}_{Line}^{pu} = \frac{\underline{Z}_{Line}}{Z_{b2}} = \frac{120 + j200}{484} = 0,24 + j0,41pu$$

For the load bus, real and reactive power and nominal voltage of the load are specified. First load impedance must be obtained:

$$\underline{Z}_{Load} = \frac{V_n^2}{\underline{S}} = \frac{10^2}{35 - j45} = \frac{10^2}{35 - j45}\frac{35 + j45}{35 + j45} = \frac{3500 + j4500}{35^2 + 45^2} = 1,07 + j1,38\Omega$$

Per unit load impedance is:

$$\underline{Z}_{Load}^{pu} = \frac{\underline{Z}_{Load}}{Z_{b3}} = \frac{1,07 + j1,38}{1.21} = 0,88 + j1,14pu$$

The impedance diagram is shown in Figure

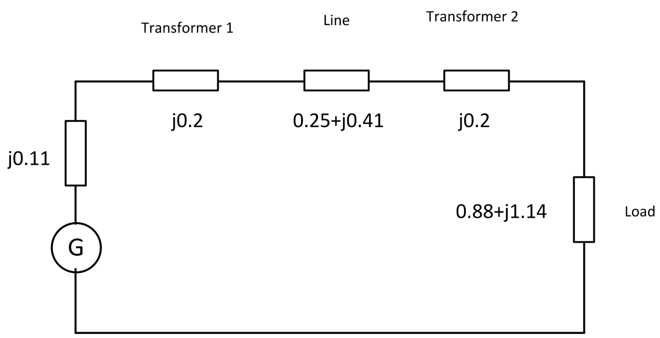

Fig. 4.3 Impedance diagram

4.2 Gauss-Seidel Method

Gauss–Seidel method for solution of system of non-linear equations is also known the method of successive displacement [32–35]. It is very similar to Jacobi method. Convergence of this method is only guaranteed in case that the system matrix is diagonally dominant, or symmetric and positive definite. Gauss-Seidel method will be explained on the general example of system of N equations:

$$\begin{aligned} f_1(x_1, x_2, \cdots, x_N) &= c_1 \\ f_2(x_1, x_2, \cdots, x_N) &= c_2 \\ &\vdots \\ f_N(x_1, x_2, \cdots, x_N) &= c_N \end{aligned} \tag{4.4}$$

One variable is expressed in each equation, which results in following expressions:

$$\begin{aligned} x_1 &= c_1 + g_1(x_1, x_2, \cdots, x_N) \\ x_2 &= c_2 + g_2(x_1, x_2, \cdots, x_N) \\ &\vdots \\ x_N &= c_N + g_n(x_1, x_2, \cdots, x_N) \end{aligned} \tag{4.5}$$

Initial estimate $[x_1^0, x_2^0, \cdots, x_N^0]$ is chosen and the first iteration yields approximate solution $[x_1^1, x_2^1, \cdots, x_N^1]$. In each iteration, values of the variables calculated in the preceding equations are used in the solution of the subsequent equations:

$$\begin{aligned} x_1^{n+1} =& c_1 + g_1(x_1^n, x_2^n, \cdots, x_{N-1}^n, x_N^n) \\ x_2^{n+1} =& c_2 + g_2(x_1^{n+1}, x_2^n, \cdots, x_{N-1}^n, x_N^n) \\ &\vdots \\ x_N^{n+1} =& c_N + g_N(x_1^{n+1}, x_2^{n+1}, \cdots, x_{N-1}^{n+1}, x_N^n) \end{aligned} \tag{4.6}$$

Iteration process ends when the change in all variables in two iterations is less than specifies accuracy.

If the convergence is slow, it can be accelerated using the method of successive over-relaxation (SOR). At the end of each iteration, each variable is modified as follows:

$$x_{i,modified}^{n+1} = x_i^n + \alpha \left(x_i^{n+1} - x_i^n \right)$$

Values of $\alpha > 1$ are used to speedup convergence of a slow-converging process, while values of $\alpha < 1$ are often used to help establish convergence of a diverging iterative process. For power flow solution α usually has values between 1.3 and 1.7.

Power flow solution using Gauss-Seidel method starts with system node voltage equation:

$$\begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} & \cdots & \underline{Y}_{1N} \\ \underline{Y}_{21} & \underline{Y}_{22} & \cdots & \underline{Y}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{Y}_{N1} & \underline{Y}_{N2} & \cdots & \underline{Y}_{NN} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \\ \vdots \\ \underline{V}_N \end{bmatrix} = \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \vdots \\ \underline{I}_N \end{bmatrix} \tag{4.7}$$

All the quantities are expressed in per unit. Network currents are usually unknown, but power is not. Thus the current at bus i can be expressed as:

$$P_i + jQ_i = \underline{V}_i \underline{I}_i^* \Rightarrow \underline{I}_i = \frac{P_i - jQ_i}{\underline{V}_i^*} \tag{4.8}$$

P_i and Q_i are total real and reactive power at bus i expressed in per unit. For load buses P_i and Q_i have negative values, while for regulated (generator) buses they have positive values. Substituting currents in Equation(4.7) for Equation(4.8) and expressing bus voltages from Equation(4.7) results in system of non-linear equations:

$$\begin{aligned}
\underline{V}_1 &= \frac{\frac{P_1 - jQ_1}{\underline{V}_1^*} - \underline{Y}_{12}\underline{V}_2 - \underline{Y}_{13}\underline{V}_3 - ... - \underline{Y}_{1N}\underline{V}_N}{Y_{11}} \\
\underline{V}_2 &= \frac{\frac{P_2 - jQ_2}{\underline{V}_2^*} - \underline{Y}_{21}\underline{V}_1 - \underline{Y}_{23}\underline{V}_3 - ... - \underline{Y}_{2N}\underline{V}_N}{Y_{22}} \\
&\vdots \\
\underline{V}_N &= \frac{\frac{P_N - jQ_N}{\underline{V}_N^*} - \underline{Y}_{N1}\underline{V}_1 - \underline{Y}_{N2}\underline{V}_2 - ... - \underline{Y}_{NN-1}\underline{V}_{NN-1}}{Y_{NN}}
\end{aligned} \tag{4.9}$$

Applying Gauss-Seidel method to Equation(4.9) gives:

$$\begin{aligned}
\underline{V}_1^{n+1} &= \frac{\frac{P_1 - jQ_1}{(\underline{V}_1^n)^*} - \underline{Y}_{12}\underline{V}_2^n - \underline{Y}_{13}\underline{V}_3^n - ... - \underline{Y}_{1N}\underline{V}_N^n}{\underline{Y}_{11}} \\
\underline{V}_2^{n+1} &= \frac{\frac{P_2 - jQ_2}{(\underline{V}_2^n)^*} - \underline{Y}_{21}\underline{V}_1^{n+1} - \underline{Y}_{23}\underline{V}_3^n - ... - \underline{Y}_{2N}\underline{V}_N^n}{\underline{Y}_{22}} \\
&\vdots \\
\underline{V}_N^{n+1} &= \frac{\frac{P_N - jQ_N}{(\underline{V}_N^n)^*} - \underline{Y}_{N1}\underline{V}_1^{n+1} - \underline{Y}_{N2}\underline{V}_2^{n+1} - ... - \underline{Y}_{NN-1}\underline{V}_{N-1}^n}{\underline{Y}_{NN}}
\end{aligned} \tag{4.10}$$

Initial estimate for all bus voltages is usually $1 + j0$ and the equation corresponding to slack bus is neglected.

For load buses real and reactive power are known and the Equation(4.10) can be solved, while regulated buses with known real power and voltage magnitude demand special treatment. If i assumed to be regulated bus, its reactive power can be expressed from corresponding node voltage equation from Equation(4.7) as:

$$Q_i = -Im\left\{\underline{V}_i^*(\underline{Y}_{i1}\underline{V}_1 + \underline{Y}_{i2}\underline{V}_2 + ... + \underline{Y}_{iN}\underline{V}_N)\right\} \tag{4.11}$$

At the beginning of each iteration reactive power for each regulated bus is obtained using the following expression:

$$Q_i^{n+1} = -Im\left\{(\underline{V}_i^n)^*(\underline{Y}_{i1}\underline{V}_1^n + \underline{Y}_{i2}\underline{V}_2^n + ... + \underline{Y}_{iN}\underline{V}_N^n)\right\} \tag{4.12}$$

Then approximate voltage values are obtained from Equation(4.10). Since voltage magnitude is specified, obtained voltages for regulated buses are modified in a way that only its imaginary part is retained and the real part is calculated as:

$$(\underline{V}_i^{re})^{n+1} = \sqrt{V_i^2 - (\underline{V}_i^{im})^{n+1^2}} \tag{4.13}$$

The iterations are continued until the following conditions are satisfied:

$$\left|(\underline{V}_i^{re})^{n+1} - (\underline{V}_i^{re})^n\right| \leq \epsilon_1$$

$$\left|(\underline{V}_i^{im})^{n+1} - (\underline{V}_i^{im})^n\right| \leq \epsilon_1$$

or the power mismatch at each bus is less than the specified accuracy:

$$|P_i^{sch} - Re\left\{(\underline{V}_i^{n+1})^*(\underline{Y}_{i1}\underline{V}_1^{n+1} + \underline{Y}_{i2}\underline{V}_2^{n+1} + ... + \underline{Y}_{iN}\underline{V}_N^{n+1})\right\}| < \epsilon_2$$

$$|Q_i^{sch} + Im\left\{(\underline{V}_i^{n+1})^*(\underline{Y}_{i1}\underline{V}_1^{n+1} + \underline{Y}_{i2}\underline{V}_2^{n+1} + ... + \underline{Y}_{iN}\underline{V}_N^{n+1})\right\}| < \epsilon_2$$

Example

Using Gaiss-Seidel method obtain bus voltages and slack bus real and reactive power for network shown in Figure 4.4 with power precision 10^{-6}. Given is the following data in per unit:

Bus 1: $\underline{V}_1 = 1.05 + j0$
Bus 2: $P_2 = 1.1, \quad Q_2 = 0.4$
Bus 3: $P_3 = 0.7, \quad Q_3 = 0.3$
Line 1-2: $R = 0.01, \quad X = 0.03$
Line 2-3:$R = 0.0125, \quad X = 0.025$
Line 1-3:$R = 0.02, \quad X = 0.04$

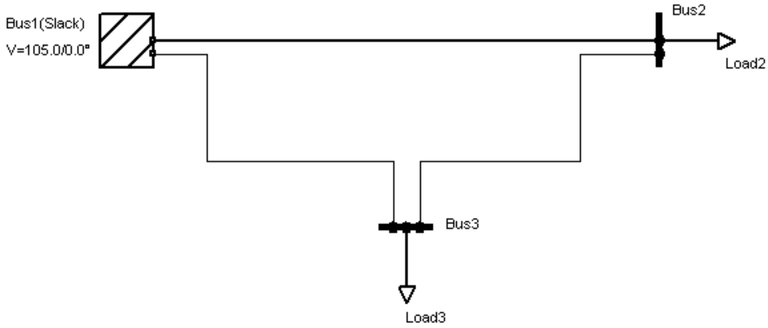

Fig. 4.4 Meshed power network

Solution

First step is to obtain bus admittance matrix. Line admittances are:

$$\begin{aligned}
\underline{y}_{12} =& \frac{1}{0.01 + j0.03} &&= 10 - j30 \\
\underline{y}_{23} =& \frac{1}{0.0125 + j0.025} &&= 16 - j32 \\
\underline{y}_{13} =& \frac{1}{0.02 + j0.04} &&= 10 - j20
\end{aligned}$$

Bus admittance matrix is:

$$\mathbf{Y} = \begin{bmatrix} 20 - j50 & -10 + j30 & -10 + j20 \\ -10 + j30 & 26 - j62 & -16 + j32 \\ -10 + j20 & -16 + j32 & 26 - j52 \end{bmatrix}$$

Bus 1 is slack bus, thus only voltages $\underline{V}_2$ and $\underline{V}_3$ are unknown. Initial estimates are:

$$\begin{aligned}
\underline{V}_2 =& 1 + j0 \\
\underline{V}_3 =& 1 + j0
\end{aligned}$$

Voltages $\underline{V}_2$ and $\underline{V}_3$ are:

$$
\begin{aligned}
\underline{V}_2^1 &= \frac{\frac{P_2 - jQ_2}{(\underline{V}_2^0)^*} - \underline{Y}_{21}\underline{V}_1^0 - \underline{Y}_{23}\underline{V}_3^0}{\underline{Y}_{22}} \\
&= \frac{\frac{-1.1+j0.4}{1} + (10 - j30)\cdot 1 + (16 - j32)\cdot 1}{26 - j62} \\
&= 0.9881858 - j0.0127876 \\
\underline{V}_3^1 &= \frac{\frac{P_3 - jQ_3}{(\underline{V}_2^0)^*} - \underline{Y}_{31}\underline{V}_1^0 - \underline{Y}_{32}\underline{V}_2^1}{\underline{Y}_{33}} \\
&= \frac{\frac{-0.7+j0.3}{1} + (10 - j20)\cdot 1 + (16 - j32)\cdot (0.9881858 - 0.0127876)}{26 - j52} \\
&= 0.9827297 - j0.0163308
\end{aligned}
$$

The process is continued and the final solution with 10^{-6} accuracy is:

$$
\begin{aligned}
\underline{V}_2 &= -97.476711 - j1.5415934 = 97.4889\angle - 1.58661^o \\
\underline{V}_3 &= -96.94353 - j9.5766438 = 97.4154\angle - 1.47233^o
\end{aligned}
$$

Slack bus power is obtained from:

$$P_1 - jQ_1 = \underline{V}_1^* \left(\underline{Y}_{11}\underline{V}_1 + \underline{Y}_{12}\underline{V}_2 + \underline{Y}_{13}\underline{V}_3\right)$$

4.3 Newton-Raphson Method

Among many techniques for iterative solution of nonlinear algebraic equations in power system analysis, Newton-Raphson method is the most widely used power flow algorithm [31,36–44]. The easiest way to understand the principle of Newton-Raphson method is to introduce single nonlinear equation solution and then to extend the method to n-dimensional problem. Consider the equation given by:

$$f(x) = 0 \tag{4.14}$$

The initial estimate of the variable x is assumed to be near the solution value:

$$f(x^0 + \triangle x^0) = 0 \tag{4.15}$$

Taylor series of the function given in Equation(4.15) is:

$$f(x^0) + \triangle x^0 f^{'}(x^0) + \frac{(\triangle x^0)^2}{2!} f^{''}(x^0) + \cdots = 0 \tag{4.16}$$

$\triangle x^0$ is be relatively small and all terms of higher powers in Equation(4.16) can be neglected:

$$f(x^0) + \triangle x^0 f^{'}(x^0) = 0 \tag{4.17}$$

From here $\triangle x^0$ is:

$$\triangle x^0 = -\frac{f(x^0)}{f^{'}(x^0)} \tag{4.18}$$

Then for the next iteration:

$$x^1 = x^0 + \triangle x^0 = x^0 - \frac{f(x^0)}{f^{'}(x^0)} \tag{4.19}$$

In general:

$$x^{n+1} = x^n + \triangle x^n = x^n - \frac{f(x^n)}{f^{'}(x^n)} \tag{4.20}$$

For each iteration of the Newton-Raphson method, the nonlinear equation is approximated by the linear one. Equation (4.17) may be rewritten as:

$$f(x^n) = -J^n \triangle x^n \tag{4.21}$$

In order to extend the procedure to n-dimensional problem, system of N equations and N variables is considered:

$$\begin{aligned} f_1(x_1, x_2, \cdots, x_N) &= 0 \\ f_2(x_1, x_2, \cdots, x_N) &= 0 \\ &\vdots \\ f_N(x_1, x_2, \cdots, x_N) &= 0 \end{aligned} \tag{4.22}$$

Expanding the left-hand side of these equations in the Taylor's series about the initial estimates and neglecting all higher order terms gives:

$$\begin{aligned} f_1 + \frac{\partial f_1}{\partial x_1}\triangle x_1 + \frac{\partial f_1}{\partial x_2}\triangle x_2 + \cdots \frac{\partial f_1}{\partial x_n}\triangle x_N &= 0 \\ f_2 + \frac{\partial f_2}{\partial x_1}\triangle x_1 + \frac{\partial f_2}{\partial x_2}\triangle x_2 + \cdots \frac{\partial f_2}{\partial x_n}\triangle x_N &= 0 \\ &\vdots \\ f_N + \frac{\partial f_N}{\partial x_1}\triangle x_1 + \frac{\partial f_N}{\partial x_2}\triangle x_2 + \cdots \frac{\partial f_N}{\partial x_n}\triangle x_N &= 0 \end{aligned} \tag{4.23}$$

In matrix form for iteration k the system is presented as:

$$-\begin{bmatrix} f_1^{(n)} \\ f_2^{(n)} \\ \vdots \\ f_N^{(n)} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(n)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(n)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(n)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(n)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(n)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial f_N}{\partial x_1}\right)^{(n)} & \left(\frac{\partial f_N}{\partial x_2}\right)^{(n)} & \cdots & \left(\frac{\partial f_N}{\partial x_n}\right)^{(n)} \end{bmatrix} \begin{bmatrix} \triangle x_1^{(n)} \\ \triangle x_2^{(n)} \\ \vdots \\ \triangle x_N^{(n)} \end{bmatrix} \tag{4.24}$$

$$-\mathbf{f}^{(n)} = \mathbf{J}^{(n)} \triangle \mathbf{X}^{(n)} \Rightarrow \triangle \mathbf{X}^{(n)} = -\left[\mathbf{J}^{(n)}\right]^{-1} \mathbf{f}^{(n)} \tag{4.25}$$

The Newton-Raphson algorithm for the n-dimensional case then becomes:

$$\mathbf{X}^{(n+1)} = \mathbf{X}^{(n)} + \triangle \mathbf{X}^{(n)} = \mathbf{X}^{(n)} - \left[\mathbf{J}^{(n)}\right]^{-1} \mathbf{f}^{(n)} \tag{4.26}$$

$J^{(k)}$ is called the Jacobian matrix. It is assumed that $J^{(n)}$ has an inverse in each iteration. The Newton-Raphson algorithm will converge quadratically if the functions have continuous first derivatives in the neighborhood of the solution, the Jacobian matrix is non-singular, and the initial approximations of x are close to the actual solutions. However, the method is sensitive to the behavior of the functions f_k and hence to their formulation. The more linear they are, the more rapidly and reliably Newton's method converges. Non-smoothness in any of the functions in the region of interest can cause convergence delays or misdirection to non useful solution.

Solution of power flow equations for transmission network using Newton's method is mathematically superior to any other iterative method because of its quadratic convergence. Another advantage of this method is its independence of number of iterations of the system size. It is important to emphasize that power flow equations for Newton-Raphson method are formulated in polar form. Newton-Raphson method can be found in several modified forms. Most popular is decoupled formulation [45–55].

In Figure (4.5)the net current $\underline{I}i$, injected to bus i obeys Kirchhoff's current law:

$$\underline{I}_i = \underline{y}_{i0}\underline{V}_i + \underline{y}_{i1}(\underline{V}_i - \underline{V}_1) + \cdots + \underline{y}_{iN}(\underline{V}_i - \underline{V}_N) \tag{4.27}$$

$$\underline{I}_i = \underline{V}_i \sum_{j=0\neq k}^{N} \underline{y}_{ij} - \sum_{j=0\neq k}^{N} \underline{y}_{ij}\underline{V}_j \tag{4.28}$$

In terms of the bus admittance matrix, Equation4.28 can be rewritten as:

$$\underline{I}_i = \sum_{j=1}^{N} \underline{Y}_{ij}\underline{V}_j \tag{4.29}$$

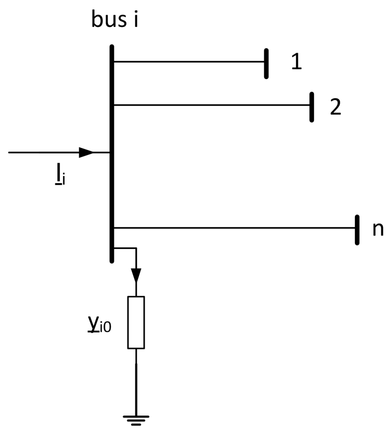

Fig. 4.5 Bus connected to a subset of n buses.

Y_{ij} denotes element of the bus admittance matrix. The complex power at bus i can be expressed as:

$$P_i + jQ_i = \underline{V}_i \underline{I}_i^* \Rightarrow \underline{I}_i = \frac{P_i - jQ_i}{\underline{V}_i^*} \tag{4.30}$$

Expressing Equation(4.29) in polar form and substituting I_i from Equation(4.30) results in a set of nonlinear algebraic equations in terms of voltage magnitude in per unit, and phase angle in radians:

$$P_i - jQ_i = V_i \angle - \delta_i \sum_{j=1}^{n} Y_{ij} V_j \angle(\theta_{ij} + \delta_j) \tag{4.31}$$

Real and imaginary part of Equation(4.31) are:

$$P_i = \sum_{j=1}^{n} V_i V_j Y_{ij} cos(\theta_{ij} - \delta_i + \delta_j) \tag{4.32}$$

$$Q_i = -\sum_{j=1}^{n} V_i V_j Y_{ij} sin(\theta_{ij} - \delta_i + \delta_j) \tag{4.33}$$

For voltage magnitudes the reference is ground, and for voltage angles the reference is chosen as one of the bus voltage angles, which is fixed at the value zero (usually). Expanding these equations in Taylor's series about the initial estimate, assuming bus 1 to be the slack bus and neglecting all higher order terms results in the set of linear equations:

$$\left[\begin{array}{ccc|ccc} \left(\frac{\partial P_2}{\partial \delta_2}\right)^n & \cdots & \left(\frac{\partial P_2}{\partial \delta_n}\right)^n & \left(\frac{\partial P_2}{\partial V_2}\right)^n & \cdots & \left(\frac{\partial P_2}{\partial V_n}\right)^n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial P_N}{\partial \delta_2}\right)^n & \cdots & \left(\frac{\partial P_N}{\partial \delta_n}\right)^n & \left(\frac{\partial P_N}{\partial V_2}\right)^n & \cdots & \left(\frac{\partial P_N}{\partial V_n}\right)^n \\ \hline \left(\frac{\partial Q_2}{\partial \delta_2}\right)^n & \cdots & \left(\frac{\partial Q_2}{\partial \delta_n}\right)^n & \left(\frac{\partial Q_2}{\partial V_2}\right)^n & \cdots & \left(\frac{\partial Q_2}{\partial V_n}\right)^n \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \left(\frac{\partial Q_N}{\partial \delta_2}\right)^n & \cdots & \left(\frac{\partial Q_N}{\partial \delta_n}\right)^n & \left(\frac{\partial Q_N}{\partial V_2}\right)^n & \cdots & \left(\frac{\partial Q_N}{\partial V_n}\right)^n \end{array}\right] \begin{bmatrix} \triangle\delta_2^n \\ \vdots \\ \triangle\delta_N^n \\ -- \\ \triangle {V_2}^n \\ \vdots \\ \triangle {V_N}^n \end{bmatrix} = \begin{bmatrix} \triangle P_2^n \\ \vdots \\ \triangle P_N^n \\ -- \\ \triangle Q_2^n \\ \vdots \\ \triangle Q_N^n \end{bmatrix} \tag{4.34}$$

$$\begin{bmatrix} \triangle P^n \\ \triangle Q^n \end{bmatrix} = \begin{bmatrix} \mathbf{J_1^n} & \mathbf{J_2^n} \\ \mathbf{J_3^n} & \mathbf{J_4^n} \end{bmatrix} \begin{bmatrix} \triangle\delta^n \\ \triangle V^n \end{bmatrix} \tag{4.35}$$

The terms ΔPi and ΔQi are the difference between the scheduled values and values calculated in each iteration:

$$\triangle P_i^n = P_i^{sch} - P_i^n \tag{4.36}$$

$$\triangle Q_i^n = Q_i^{sch} - Q_i^n \tag{4.37}$$

Bus 1 is assumed to be the slack bus and it is not included in calculations. Elements of the Jacobian matrix are partial derivatives of Equation(4.32) and Equation(4.33) evaluated at $\triangle\delta_i$ and $\triangle V_i$. For each load bus there are two equations given by Equation(4.32) and Equation(4.33). For regulated buses, the voltage magnitudes and active power are known, so there is only one equation for each regulated bus and that is Equation(4.32). Therefore, if m buses of the system are regulated, m equations involving $\triangle Q$ and $\triangle V$ and the corresponding columns of the Jacobian matrix from Equation(4.34) are eliminated. The diagonal and the off-diagonal elements of $\mathbf{J_1}$ are:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j\neq i} V_i V_j Y_{ij} sin(\theta_{ij} - \delta_i + \delta_j) \tag{4.38}$$

$$\frac{\partial P_i}{\partial \delta_j} = -V_i V_j Y_{ij} sin(\theta_{ij} - \delta_i + \delta_j), \quad j \neq i \tag{4.39}$$

The diagonal and the off-diagonal elements of $\mathbf{J_2}$ are:

$$\frac{\partial P_i}{\partial V_i} = 2V_i Y_{ii} cos\theta_{ii} + \sum_{j\neq i} V_j Y_{ij} cos(\theta_{ij} - \delta_i + \delta_j) \tag{4.40}$$

$$\frac{\partial P_i}{\partial V_j} = V_i Y_{ij} cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i \tag{4.41}$$

The diagonal and the off-diagonal elements of $\mathbf{J_3}$ are:

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} V_i V_j Y_{ij} cos(\theta_{ij} - \delta_i + \delta_j) \tag{4.42}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -V_i V_j Y_{ij} cos(\theta_{ij} - \delta_i + \delta_j), \quad j \neq i \tag{4.43}$$

The diagonal and the off-diagonal elements of $\mathbf{J_4}$ are:

$$\frac{\partial Q_i}{\partial V_i} = -2V_i Y_{ii} sin\theta_{ii} - \sum_{j \neq i} V_j Y_{ij} sin(\theta_{ij} - \delta_i + \delta_j) \tag{4.44}$$

$$\frac{\partial Q_i}{\partial V_j} = -V_i Y_{ij} sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i \tag{4.45}$$

The new estimates for voltage magnitudes and phase angles are computed from:

$$\delta_i^{n+1} = \delta_i^n + \triangle \delta_i^n \tag{4.46}$$

$$V_i^{n+1} = V_i^n + \triangle V_i^n \tag{4.47}$$

To sum up, when the power flow equations for the system and the bus admittance matrix are written the procedure for Newton-Raphson method is as follows:

1. In case of load buses, initial estimates for voltage magnitude and phase angle are set equal to the slack bus values or 1 and 0 respectively. In case of regulated buses phase angle is set equal to the slack bus phase angle or 0. All values in calculations are expressed in polar form.
2. Using power flow equations active and reactive power for load buses and active power for regulated bus are obtained.
3. $\triangle P_i$ and $\triangle Q_i$ for load and $\triangle P_i$ for regulated bus are obtained.
4. The elements of Jacobian matrix are obtained.
5. $\triangle \delta$ and $\triangle V$ for load and $\triangle \delta$ for regulated buses are then obtained from Equation (4.34).
6. The new phase angles and voltage magnitudes are computed from Equation (4.46) and Equation (4.47).
7. This process is repeated from step two until $|\triangle P_i^n|$ and $|\triangle Q_i^n|$ are less than specified accuracy ϵ:

$$|P_i^{sch} - P_i^n| < \epsilon$$

$$|\boldsymbol{Q_i^{sch} - Q_i^n}| < \boldsymbol{\epsilon}$$

Example

Using Newton-Raphson method obtain bus voltages for network given in the previous example.

Solution

As already obtained, bus admittance matrix is:

$$\mathbf{Y} = \begin{bmatrix} 20 - j50 & -10 + j30 & -10 + j20 \\ -10 + j30 & 26 - j62 & -16 + j32 \\ -10 + j20 & -16 + j32 & 26 - j52 \end{bmatrix}$$

In Newton-Raphson method all quantities are expressed in polar form with angles in radians. Thus elements of $\mathbf{Y}$ matrix have to be converted to polar form:

$$20 - j50 = \sqrt{20^2 + 50^2}\angle arctan\frac{-50}{20} = 53.85165\angle - 1.9029$$

$$-10 + j30 = \sqrt{10^2 + 30^2}\angle arctan\frac{30}{-10} + 3.14 = 31.62278\angle 1.8909$$

$$\vdots$$

Polar form of $\mathbf{Y}$ matrix is:

$$\mathbf{Y} = \begin{bmatrix} 53.85165\angle - 1.9029 & 31.62278\angle 1.8909 & 22.36068\angle 2.0328 \\ 31.62278\angle 1.8909 & 67.23095\angle - 1.1737 & 35.77709\angle 2.0328 \\ 22.36068\angle 2.0328 & 35.77709\angle 2.0328 & 58.13777\angle - 1.1071 \end{bmatrix}$$

Since bus 1 is slack bus, and bus 2 and bus 3 are load buses, unknown variables are voltage magnitudes and voltage angles for bus 2 and bus 3. Initial estimates of unknown variables are:

$$\begin{aligned} V_2 &= 1.0 \\ V_3 &= 1.0 \\ \delta_2 &= 0 \\ \delta_3 &= 0 \end{aligned}$$

Expressions for real and reactive power at bus 2 and bus 3 are:

$$P_2 = V_2 V_1 Y_{21} cos(\theta_{21} - \delta_2 + \delta_1) + V_2^2 Y_{22} cos\theta_{22} + V_2 V_3 Y_{23} cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = V_3V_1Y_{31}cos(\theta_{31} - \delta_3 + \delta_1) + V_3V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2) + V_3^2Y_{33}cos\theta_{33}$$

$$Q_2 = -V_2V_1Y_{21}sin(\theta_{21} - \delta_2 + \delta_1) - V_2^2Y_{22}sin\theta_{22} - V_2V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3)$$

$$Q_3 = -V_3V_1Y_{31}sin(\theta_{31} - \delta_3 + \delta_1) - V_3V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2) - V_3^2Y_{33}sin\theta_{33}$$

Expressions for elements of Jacobian matrix are:

$$\frac{\partial P_2}{\partial \delta_2} = V_2V_1Y_{21}sin(\theta_{21} - \delta_2 + \delta_1) + V_2V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -V_2V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial V_2} = 2V_2Y_{22}cos\theta_{22} + V_1Y_{21}cos(\theta_{21} - \delta_2 + \delta_1) + V_3Y_{23}cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial V_3} = V_2Y_{23}cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -V_3V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = V_3V_1Y_{31}sin(\theta_{31} - \delta_3 + \delta_1) + V_3V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial V_2} = V_3Y_{32}cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial V_3} = 2V_3Y_{33}cos\theta_{33} + V_1Y_{31}cos(\theta_{31} - \delta_3 + \delta_1) + V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = V_2V_1Y_{21}cos(\theta_{21} - \delta_2 + \delta_1) + V_2V_3Y_{23}cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -V_2V_3Y_{23}cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial V_2} = -2V_2Y_{22}sin\theta_{22} - V_1Y_{21}sin(\theta_{21} - \delta_2 + \delta_1) - V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial V_3} = -V_2Y_{23}sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_3}{\partial \delta_2} = -V_3V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial \delta_3} = V_3V_1Y_{31}cos(\theta_{31} - \delta_3 + \delta_1) + V_3V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial V_2} = -V_3Y_{32}sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_3}{\partial V_3} = -2V_3Y_{33}sin\theta_{33} - V_1Y_{31}sin(\theta_{31} - \delta_3 + \delta_1) - V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2)$$

Power mismatches for initial estimates are:

$$\triangle P_2^0 = P_2 - P_2^0 = -1.1 + 1.113927 = 0.013927$$

$$\triangle P_3^0 = P_3 - P_3^0 = -0.7 + 0.700121 = 0.000121$$

$$\triangle Q_2^0 = Q_2 - Q_2^0 = -0.4 + 0.357375 = -0.042625$$

$$\triangle Q_3^0 = Q_3 - Q_3^0 = -0.3 + 0.275435 = -0.024565$$

Power mismatches do not satisfy set precision and the process is continued. Jacobian matrix elements for initial estimates are:

$$\frac{\partial P_2}{\partial \delta_2} = V_2V_1Y_{21}sin(\theta_{21} - \delta_2 + \delta_1) + V_2V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3) = 63.5$$

$$\frac{\partial P_2}{\partial \delta_3} = -V_2V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3) = -32$$

$$\frac{\partial P_2}{\partial V_2} = 2V_2Y_{22}cos\theta_{22} + V_1Y_{21}cos(\theta_{21} - \delta_2 + \delta_1) + V_3Y_{23}cos(\theta_{23} - \delta_2 + \delta_3) = 25.5$$

$$\frac{\partial P_2}{\partial V_3} = V_2Y_{23}cos(\theta_{23} - \delta_2 + \delta_3) = -16$$

$$\frac{\partial P_3}{\partial \delta_2} = -V_3V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2) = -32$$

$$\frac{\partial P_3}{\partial \delta_3} = V_3V_1Y_{31}sin(\theta_{31} - \delta_3 + \delta_1) + V_3V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2) = 53$$

$$\frac{\partial P_3}{\partial V_2} = V_3Y_{32}cos(\theta_{32} - \delta_3 + \delta_2) = -16$$

$$\frac{\partial P_3}{\partial V_3} = 2V_3Y_{33}cos\theta_{33} + V_1Y_{31}cos(\theta_{31} - \delta_3 + \delta_1) + V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2) = 25.5$$

$$\frac{\partial Q_2}{\partial \delta_2} = V_2V_1Y_{21}cos(\theta_{21} - \delta_2 + \delta_1) + V_2V_3Y_{23}cos(\theta_{23} - \delta_2 + \delta_3) = -26.5$$

$$\frac{\partial Q_2}{\partial \delta_3} = -V_2V_3Y_{23}cos(\theta_{23} - \delta_2 + \delta_3) = 16$$

$$\frac{\partial Q_2}{\partial V_2} = -2V_2Y_{22}sin\theta_{22} - V_1Y_{21}sin(\theta_{21} - \delta_2 + \delta_1) - V_3Y_{23}sin(\theta_{23} - \delta_2 + \delta_3) = 60.5$$

$$\frac{\partial Q_2}{\partial V_3} = -V_2Y_{23}sin(\theta_{23} - \delta_2 + \delta_3) = -32$$

$$\frac{\partial Q_3}{\partial \delta_2} = -V_3V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2) = 16$$

$$\frac{\partial Q_3}{\partial \delta_3} = V_3V_1Y_{31}cos(\theta_{31} - \delta_3 + \delta_1) + V_3V_2Y_{32}cos(\theta_{32} - \delta_3 + \delta_2) = -26.5$$

$$\frac{\partial Q_3}{\partial V_2} = -V_3Y_{32}sin(\theta_{32} - \delta_3 + \delta_2) = -32$$

$$\frac{\partial Q_3}{\partial V_3} = -2V_3Y_{33}sin\theta_{33} - V_1Y_{31}sin(\theta_{31} - \delta_3 + \delta_1) - V_2Y_{32}sin(\theta_{32} - \delta_3 + \delta_2) = 51$$

Power flow equation for first iteration is:

$$\begin{bmatrix} 63.5 & -32 & 25.5 & -16 \\ -32 & 53 & -16 & 25.5 \\ -26.5 & 16 & 60.5 & -32 \\ 16 & -26.5 & -32 & 51 \end{bmatrix} \begin{bmatrix} \triangle\delta_2^0 \\ \triangle\delta_3^0 \\ \triangle V_2^0 \\ \triangle V_3^0 \end{bmatrix} = \begin{bmatrix} 0.013927 \\ 0.000121 \\ -0.042625 \\ -0.024565 \end{bmatrix}$$

From here:

$$\begin{bmatrix} \triangle\delta_2^0 \\ \triangle\delta_3^0 \\ \triangle V_2^0 \\ \triangle V_3^0 \end{bmatrix} = \begin{bmatrix} 63.5 & -32 & 25.5 & -16 \\ -32 & 53 & -16 & 25.5 \\ -26.5 & 16 & 60.5 & -32 \\ 16 & -26.5 & -32 & 51 \end{bmatrix}^{-1} \begin{bmatrix} 0.013927 \\ 0.000121 \\ -0.042625 \\ -0.024565 \end{bmatrix} = \begin{bmatrix} -0.025717 \\ -0.023829 \\ 0.027236 \\ 0.0265013 \end{bmatrix}$$

Voltage magnitudes and voltage angles are:

$$\begin{aligned} \delta_2^1 &= \delta_2^0 + \triangle\delta_2^0 = -0.025717 \\ \delta_3^1 &= \delta_3^0 + \triangle\delta_3^0 = -0.023829 \\ V_2^1 &= V_2^0 + \triangle V_2^0 = 1.027236 \\ V_3^1 &= V_3^0 + \triangle V_3^0 = 1.026501 \end{aligned}$$

The solution with precision of 10^{-6} converges after three iterations and voltage magnitudes and angles are:

$$\begin{aligned} \delta_2 &= -0.025053 \\ \delta_3 &= -0.023251 \\ V_2 &= 1.026177 \\ V_3 &= 1.025478 \end{aligned}$$

Results can be verified using [56].

4.4 Transformers with load tap changers (LTC)

Load tap changing transformers (LTCs) are used for regulation of voltages and reactive power. Transformer can have LTCs on both primary and secondary sides. Usually only one of them is used for regulation. Equivalent circuit of transformer connected between buses i and k is shown in Figure 4.6. π model of transformer is connected to ideal transformers on both sides. Tap ratio on i side is $t_i : 1$ and tap ratio on k side is $1 : t_k$. Admittance matrix of the transformer must be modify to include these tap ratios.

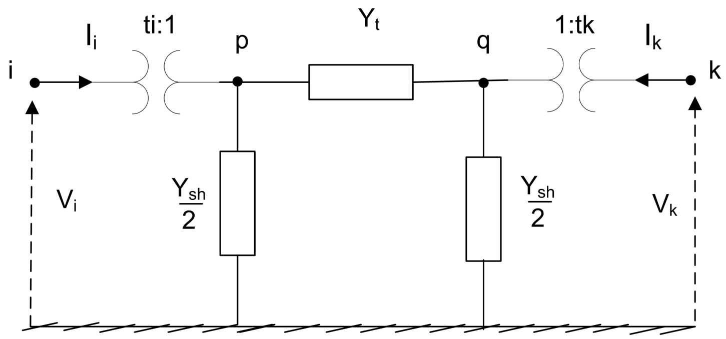

Fig. 4.6 LTC π model

Node voltage equation for LTC shown in Figure 4.6is:

$$\begin{bmatrix} \underline{Y}_{ii} & \underline{Y}_{ik} \\ \underline{Y}_{ki} & \underline{Y}_{kk} \end{bmatrix} \begin{bmatrix} \underline{V}_p \\ \underline{V}_q \end{bmatrix} = \begin{bmatrix} \underline{I}_p \\ \underline{I}_q \end{bmatrix} \tag{4.48}$$

Setting the relations between voltages and currents of i, p and q, k buses gives:

$$\frac{\underline{V}_i}{\underline{V}_p} = t_i \Rightarrow \underline{V}_p = \frac{\underline{V}_i}{t_i}$$

$$\frac{\underline{i}_p}{\underline{i}_i} = t_i \Rightarrow \underline{i}_p = \underline{i}_i t_i$$

$$\frac{\underline{V}_k}{\underline{V}_q} = t_k \Rightarrow \underline{V}_q = \frac{\underline{V}_k}{t_k}$$

$$\frac{\underline{i}_q}{\underline{i}_k} = t_k \Rightarrow \underline{i}_q = \underline{i}_k t_k$$

From here:

$$\begin{bmatrix} \underline{Y}_{ii} & \underline{Y}_{ik} \\ \underline{Y}_{ki} & \underline{Y}_{kk} \end{bmatrix} \begin{bmatrix} \frac{\underline{V}_i}{t_i} \\ \frac{\underline{V}_k}{t_k} \end{bmatrix} = \begin{bmatrix} t_i \underline{I}_i \\ t_k \underline{I}_k \end{bmatrix} \tag{4.49}$$

$$\begin{bmatrix} \frac{\underline{Y}_{ii}}{t_i} & \frac{\underline{Y}_{ik}}{t_k} \\ \frac{\underline{Y}_{ki}}{t_k} & \frac{\underline{Y}_{kk}}{t_i} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} t_i \underline{I}_i \\ t_k \underline{I}_k \end{bmatrix} \tag{4.50}$$

Finally:

$$\begin{bmatrix} \frac{\underline{Y}_{ii}}{t_i^2} & \frac{\underline{Y}_{ik}}{t_i t_k} \\ \frac{\underline{Y}_{ki}}{t_i t_k} & \frac{\underline{Y}_{kk}}{t_k^2} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} \tag{4.51}$$

Basic power flow algorithm must be improved extended to include voltage regulation using LTCs. Modification of Newton-Raphson power flow method for LTC voltage regulation will be presented.

4.5 Newton-Raphson Power Flow with LTCs

Consider a power transformer connected between buses i and k with LTC on i side that regulates V_k voltage of bus k. From Equation(4.51) it follows that transformer node voltage equation is:

$$\begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} = \begin{bmatrix} \frac{\underline{Y}_{ii}}{t_i^2} & \frac{\underline{Y}_{ik}}{t_i} \\ \frac{\underline{Y}_{ki}}{t_i} & \underline{Y}_{kk} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} \tag{4.52}$$

where $\underline{Y}_{ii}$, $\underline{Y}_{ik}$, $\underline{Y}_{ki}$ and $\underline{Y}_{kk}$ are members of $[Y]$ matrix and t_i is tap ratio. From here:

$$\begin{aligned} \underline{I}_i &= \tfrac{\underline{Y}_{ii}}{t_i^2}\underline{V}_i + \tfrac{\underline{Y}_{ik}}{t_i}\underline{V}_k \\ \underline{I}_k &= \tfrac{\underline{Y}_{ki}}{t_i}\underline{V}_i + \underline{Y}_{kk}\underline{V}_k \end{aligned} \tag{4.53}$$

Apparent power at bus i and apparent power at bus k are:

$$\begin{aligned} S_i = P_i + jQ_i = \underline{V}_i\underline{I}_i^* &= \frac{\underline{Y}_{ii}^*}{t_i^2}V_i^2 + \frac{\underline{Y}_{ik}^*}{t_i}\underline{V}_k^*\underline{V}_i \\ S_k = P_k + jQ_k = \underline{V}_k\underline{I}_k^* &= \frac{\underline{Y}_{ki}^*}{t_i}\underline{V}_i^*\underline{V}_k + \underline{Y}_{kk}^*V_k^2 \end{aligned} \tag{4.54}$$

Separating real and imaginary part of these expressions gives real and reactive power of buses i and k:

$$\begin{aligned} P_i = Re\left\{S_i\right\} &= Re\left\{\frac{V_i^2Y_{ii}\angle-\theta_{ii}}{t_i^2} + \frac{Y_{ik}V_kV_i\angle(\delta_i-\delta_k-\theta_{ik})}{t_i}\right\} \\ &= \frac{V_i^2Y_{ii}}{t_i^2}cos\theta_{ii} + \frac{Y_{ik}V_kV_i}{t_i}cos(\delta_i-\delta_k-\theta_{ik}) \end{aligned} \tag{4.55}$$

$$\begin{aligned} Q_i = Im\left\{S_i\right\} &= Im\left\{\frac{V_i^2Y_{ii}\angle-\theta_{ii}}{t_i^2} + \frac{Y_{ik}V_kV_i\angle(\delta_i-\delta_k-\theta_{ik})}{t_i}\right\} \\ &= -\frac{V_i^2Y_{ii}}{t_i^2}sin\theta_{ii} + \frac{Y_{ik}V_kV_i}{t_i}sin(\delta_i-\delta_k-\theta_{ik}) \end{aligned} \tag{4.56}$$

$$\begin{aligned} P_k = Re\{S_k\} &= Re\left\{\frac{Y_{ki}V_kV_i\angle(\delta_k-\delta_i-\theta_{ki})}{t_i}+V_k^2Y_{kk}\angle-\theta_{kk}\right\} \\ &= V_k^2Y_{kk}cos\theta_{kk}+\frac{Y_{ki}V_kV_i}{t_i}cos(\delta_k-\delta_i-\theta_{ki}) \end{aligned} \tag{4.57}$$

$$\begin{aligned} Q_k = Im\{S_k\} &= Im\left\{\frac{Y_{ki}V_kV_i\angle(\delta_k-\delta_i-\theta_{ki})}{t_i}+V_k^2Y_{kk}\angle-\theta_{kk}\right\} \\ &= -V_k^2Y_{kk}sin\theta_{kk}+\frac{Y_{ki}V_kV_i}{t_i}sin(\delta_k-\delta_i-\theta_{ki}) \end{aligned} \tag{4.58}$$

Power flow equations for buses i and k for iteration n are:

$$\begin{aligned} \triangle P_i^n &= P_{i,sch}-P_i^n \\ \triangle Q_i^n &= Q_{i,sch}-Q_i^n \\ \triangle P_k^n &= P_{k,sch}-P_k^n \\ \triangle Q_k^n &= Q_{k,sch}-Q_k^n \end{aligned} \tag{4.59}$$

Since the V_k voltage is regulated voltage, its magnitude is set and unknown variables $V_i, \delta_i, \delta_k, t_i$. Solution of power flow equations is:

$$\begin{bmatrix} \triangle P_i^n \\ \triangle P_k^n \\ \triangle Q_i^n \\ \triangle Q_k^n \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P_i}{\partial \delta_i}\right)^n & \left(\frac{\partial P_i}{\partial \delta_k}\right)^n & \left(\frac{\partial P_i}{\partial V_i}\right)^n & \left(\frac{\partial P_i}{\partial t_i}\right)^n \\ \left(\frac{\partial P_k}{\partial \delta_i}\right)^n & \left(\frac{\partial P_k}{\partial \delta_k}\right)^n & \left(\frac{\partial P_k}{\partial V_i}\right)^n & \left(\frac{\partial P_k}{\partial t_i}\right)^n \\ \left(\frac{\partial Q_i}{\partial \delta_i}\right)^n & \left(\frac{\partial Q_i}{\partial \delta_k}\right)^n & \left(\frac{\partial Q_i}{\partial V_i}\right)^n & \left(\frac{\partial Q_i}{\partial t_i}\right)^n \\ \left(\frac{\partial Q_k}{\partial \delta_i}\right)^n & \left(\frac{\partial Q_k}{\partial \delta_k}\right)^n & \left(\frac{\partial Q_k}{\partial V_i}\right)^n & \left(\frac{\partial Q_k}{\partial t_i}\right)^n \end{bmatrix} \begin{bmatrix} \triangle \delta_i^n \\ \triangle \delta_k^n \\ \triangle V_i^n \\ \triangle t_i^n \end{bmatrix} \tag{4.60}$$

Members of Jacobian matrix in general are:

$$\left(\frac{\partial P_i}{\partial \delta_i}\right)^n = -\frac{Y_{ik}V_kV_i^{n-1}}{t_i^{n-1}}sin(\delta_i^{n-1}-\delta_k^{n-1}-\theta_{ik})$$

$$\left(\frac{\partial P_i}{\partial \delta_k}\right)^n = \frac{Y_{ik}V_kV_i^{n-1}}{t_i^{n-1}}sin(\delta_i^{n-1}-\delta_k^{n-1}-\theta_{ik})$$

$$\left(\frac{\partial P_i}{\partial V_i}\right)^n = \frac{2V_i^{n-1}Y_{ii}}{(t_i^{n-1})^2}cos\theta_{ii}+\frac{Y_{ik}V_k}{t_i^{n-1}}cos(\delta_i^{n-1}-\delta_k^{n-1}-\theta_{ik})$$

$$\left(\frac{\partial P_i}{\partial t_i}\right)^n = -\frac{2(V_i^{n-1})^2Y_{ii}}{(t_i^{n-1})^3}cos\theta_{ii}-\frac{Y_{ik}V_kV_i^{n-1}}{(t_i^{n-1})^2}cos(\delta_i^{n-1}-\delta_k^{n-1}-\theta_{ik})$$

$$\left(\frac{\partial P_k}{\partial \delta_i}\right)^n = \frac{Y_{ki}V_kV_i^{n-1}}{t_i^{n-1}}sin(\delta_i^{n-1}-\delta_k^{n-1}-\theta_{ki})$$

$$\left(\frac{\partial P_k}{\partial \delta_k}\right)^n = -\frac{Y_{ki}V_kV_i^{n-1}}{t_i^{n-1}}sin(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

$$\left(\frac{\partial P_k}{\partial V_i}\right)^n = \frac{Y_{ki}V_k}{t_i^{n-1}}cos(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

$$\left(\frac{\partial P_k}{\partial t_i}\right)^n = -\frac{Y_{ki}V_kV_i^{n-1}}{(t_i^{n-1})^2}cos(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

$$\left(\frac{\partial Q_i}{\partial \delta_i}\right)^n = \frac{Y_{ik}V_kV_i^{n-1}}{t_i^{n-1}}cos(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ik})$$

$$\left(\frac{\partial Q_i}{\partial \delta_k}\right)^n = -\frac{Y_{ik}V_kV_i^{n-1}}{t_i^{n-1}}cos(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ik})$$

$$\left(\frac{\partial Q_i}{\partial V_i}\right)^n = -\frac{2V_i^{n-1}Y_{ii}}{(t_i^{n-1})^2}sin\theta_{ii} + \frac{Y_{ik}V_k}{t_i^{n-1}}sin(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ik})$$

$$\left(\frac{\partial Q_i}{\partial t_i}\right)^n = \frac{2(V_i^{n-1})^2Y_{ii}}{(t_i^{n-1})^3}sin\theta_{ii} - \frac{Y_{ik}V_kV_i^{n-1}}{(t_i^{n-1})^2}sin(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ik})$$

$$\left(\frac{\partial Q_k}{\partial \delta_i}\right)^n = -\frac{Y_{ki}V_kV_i^{n-1}}{t_i^{n-1}}cos(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

$$\left(\frac{\partial Q_k}{\partial \delta_k}\right)^n = \frac{Y_{ki}V_kV_i^{n-1}}{t_i^{n-1}}cos(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

$$\left(\frac{\partial Q_k}{\partial V_i}\right)^n = \frac{Y_{ki}V_k}{t_i^{n-1}}sin(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

$$\left(\frac{\partial Q_k}{\partial t_i}\right)^n = -\frac{Y_{ki}V_kV_i^{n-1}}{(t_i^{n-1})^2}sin(\delta_i^{n-1} - \delta_k^{n-1} - \theta_{ki})$$

After solving Equation(4.60) unknown variables are obtained as:

$$\begin{bmatrix} \delta_i^n \\ \delta_k^n \\ V_i^n \\ t_i^n \end{bmatrix} = \begin{bmatrix} \delta_i^{n-1} \\ \delta_k^{n-1} \\ V_i^{n-1} \\ t_i^{n-1} \end{bmatrix} + \begin{bmatrix} \triangle\delta_i^n \\ \triangle\delta_k^n \\ \triangle V_i^n \\ \triangle t_i^n \end{bmatrix}$$

4.6 Current Iteration Method for Power Flow

In the past most of the models and analysis techniques for power systems were developed for large interconnected transmission networks because of their importance and the effects of possible faults at this part of the system. Very little attention was devoted to the distribution networks. As a consequence, most power flow algorithms were designed for transmission networks, while they were useless for distribution network analysis due to its wide range of R/X ratio and radial structure. These algorithms were modified for distribution network problems which usually did not result in ideal solutions. In recent years the idea of smart grids has put distribution networks in focus of research and analysis. Some authors designed unified power flow algorithms which observe transmission and distribution network. Requirements of today's on-line DMS applications have set very strict demands for the algorithms from the aspect of time, precision and accuracy which poses a great challenge given the fact that the algorithms that are not based on Newton-Raphson method have significantly increased number of iterations. Y-bus or Z-bus methods are applicable on meshed networks, but same as the backward/forward method they need compensation techniques for the PV buses.

Until recently distributed generation (DG) has been quite limited for economic reasons. Increased environmental awareness, market deregulation and need for security, as well as constant decline in price of wind and PV energy generation, are increasing the penetration of DG lately and the impact on the grid is becoming more tangible. This new situation leads to significant changes in network planning, operation and architecture and it demands new approach enabling the coexistence of radial and meshed operational mode. Special concern for the network operators are the effects of DG on bus voltages which have to be within the specified limits. This is a particular problem for renewable resources because of their intermittent nature.

Current iteration (CI) is a method for solving power flow problems with constant $[Y]$ matrix and modeling automatic local controls. Changes in reactive power injections and tap positions are modeled through the constant current sources and have effects only on current injection vector.

CI is based on the fundamental matrix equation:

$$[Y][V] = [I] \tag{4.61}$$

where $[Y]$ is complex, symmetric, NxN nodal admittance matrix, $[V]$ is an N-vector of complex node voltages, $[I]$ is an N-vector of complex node currents. For all problems of distribution systems CI is better than currently used FD and NR methods. Distribution networks are distinguished from transmission networks because they are consisted of all load (PQ) nodes and have lines with large R/X ratio. The CI algorithm is organized around the factorized nodal admittance matrix. Each iteration consists mainly of a solution of Equation(4.61) for $[V]$, given $[I]$:

$$[V] = [Y]^{-1}[I] \tag{4.62}$$

where $[Y]^{-1}$ symbolizes the solution by sparse triangular factors. At each node i complex power $\underline{S}_i$ is specified as:

$$\underline{S}_i = P_i + jQ_i, i = 1, \ldots, N \tag{4.63}$$

The problem is solved when vectors $[V]$ and $[I]$ in Equation(4.61) also satisfy Equation(4.64):

$$\underline{V}_i \underline{I}_i^* = \underline{S}_i \pm \epsilon, i = 1, \ldots, N \tag{4.64}$$

where ε is mismatch tolerance. In brief, CI consists of systematically changing $[I]$ and solving for $[V]$ in Equation(4.62) until elements of $[I]$, $[V]$ are found to satisfy Equation(4.64). $[I]$ is a vector of total node currents formed in each iteration so that the factorize matrix $[Y]$ remains constant. From the aspects of memory and time resources, this is one of the most important characteristics of CI method. Unless indicated otherwise, all quantities are complex in rectangular coordinates and expressed in per unit.

4.6.1 Basic CI Algorithm

Basic block diagram for CI method is shown in Figure 4.7. It is consisted of the following steps:
Step 1: Formulate $[Y]$ matrix.
Step 2: Factorize $[Y]$.
Step 3: Make initial estimate of bus voltages: $\underline{V}_i = \underline{V}_{slack}$.
Step 4: Set iteration constant $n = 0$.
Step 5: $n = n + 1$
Step 6: Form bus current vector $[I^n]$:

$$\underline{I}_i^n = \left(\frac{\underline{S}_i}{\underline{V}_i^{n-1}} \right)^* \tag{4.65}$$

Step 7: Solve for $[V^n]$ in Equation(4.61). Forward/backward.
Step 8: Check for solution, i.e. check if the power mismatches are within the limits of mismatch tolerance.
If solved, stop; else go to Step 5.
In Equation(4.65) when i node is connected to slack node, $\underline{I}_i^n$ must be augmented by the constant Norton current from the slack node:

$$\underline{I}_i^n = \left(\frac{\underline{S}_i}{\underline{V}_i^{n-1}} \right)^* - \underline{Y}_{i,slack} \underline{V}_{slack} \tag{4.66}$$

In CI method slack bus affects $[Y]$ (and consequently $[Z]$) matrix in a way that all the elements of corresponding row are equal to zero, while diagonal element is equal to one:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ \underline{\gamma}_{21} & \underline{Z}_{22} & \cdots & \underline{Z}_{2i} & \cdots & \underline{Z}_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{\gamma}_{i1} & \underline{Z}_{i2} & \cdots & \underline{Z}_{ii} & \cdots & \underline{Z}_{iN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{\gamma}_{N1} & \underline{Z}_{N2} & \cdots & \underline{Z}_{Ni} & \cdots & \underline{Z}_{NN} \end{bmatrix} \begin{bmatrix} \underline{V}_{slack} \\ \underline{I}_2 \\ \vdots \\ \underline{I}_i \\ \vdots \\ \underline{I}_N \end{bmatrix} = \begin{bmatrix} 1 \\ \underline{V}_2 \\ \vdots \\ \underline{V}_i \\ \vdots \\ \underline{V}_N \end{bmatrix} \tag{4.67}$$

This kind of treatment does not require some special operation, nor time because γ coefficients in Equation(4.67) are easily obtained by factorization of $[Y]$ matrix.

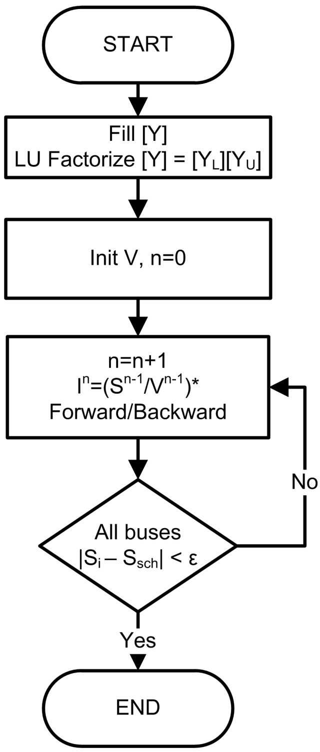

Fig. 4.7 Basic CI Algorithm

4.6.2 Special Property of CI Method

Unlike FD and NR, CI gives the exact solution for the no-load problem in one step. If there is no load, the independent vector $[I]$ is null except for entries at nodes connected to the slack node. The no-load case is of no practical importance, but the property turns out to be important in solving general problems. Constant shunt admittance loads are the same as the no-load case and this is important for distribution systems. In any problem solved by CI method the power at passive nodes (no-load) is zero in every iteration. This is because the node current from Equation(4.65) is always zero. If $\underline{S}_i = 0$ and $\underline{I}_i^n = 0$, this makes $\underline{S}_i^n = 0$. This does not mean that the node voltages are at their solved values, only that the branch currents at passive nodes always sum to zero in CI. If somehow the solution voltage magnitudes could be known in advance, then all loads could be converted to constant shunt admittances at these node voltage magnitudes and the problem could be solved as a no-load case in one step. At node i where $\underline{V}_i$ is known the scheduled load $\underline{S}_i = P_i + jQ_i$ can be converted to shunt admittance:

$$\underline{Y}_{is} = G_{is} + jB_{is}$$

$$\underline{Y}_{is} = \frac{\underline{S}_i^*}{{V_i}^2} \tag{4.68}$$

Although the node voltage magnitudes of the solution are not known precisely before the beginning, they are known approximately. Therefore by using nominal values of voltage magnitudes a large fraction of P_i and Q_i load can be converted to shunt admittances which appear in the diagonals of $[Y]$. Different approach can be used - constant power, constant current or constant impedance part of P_i and Q_i or the combination of any two or all of them can be included in $[Y]$ matrix. In general load active power and load reactive power are given as:

$$\begin{aligned} P_i =& P_n(k_{p0} + k_{p1}V_i + k_{p2}{V_i}^2) \\ Q_i =& Q_n(k_{q0} + k_{q1}V_i + k_{q2}{V_i}^2) \end{aligned} \tag{4.69}$$

V_i is load bus voltage magnitude. In case of including the constant power part in $[Y]$, shunt admittance added to diagonal element will be:

$$\underline{Y}_{is} = \frac{P_n k_{p0} - jQ_n k_{q0}}{V_i^2} \tag{4.70}$$

V_i is the magnitude of nominal voltage. Similarly, shunt admittance in case of including constant current part, will be:

$$\underline{Y}_{is} = \frac{P_n k_{p1} V_i - jQ_n k_{q1} V_i}{V_i^2} \tag{4.71}$$

and for constant impedance part:

$$\underline{Y}_{is} = \frac{P_n k_{p2} V_i^2 - jQ_n k_{q2} V_i^2}{V_i^2} \tag{4.72}$$

If combining any of these cases, the shunt admittance will be equal to the sum of the corresponding shunt admittances. The rest of P_i and Q_i that is not included in $[Y]$ matrix will be represented as constant current source and included in $[I]$ vector in each iteration as:

$$\underline{I}_i^n = \left(\frac{\underline{S}_i^*}{(\underline{V}_i^n)^2} - \underline{Y}_{is} \right) \underline{V}_i^n \tag{4.73}$$

$\underline{S}_i = P_i + jQ_i$ is the total load apparent power, where P_i and $Q_i$are given in Equations(4.69). The admittances of shunt loads are relatively small compared to admittances of transmission lines. Therefore, the changes in the diagonals of $[Y]$ are relatively small. There is no danger of affecting its numerical properties in factorization.

To compute P_i and Q_i mismatches for completed iteration n, it is necessary to compute the actual complex power $\underline{S}_i^n$ at this stage first.

$$\underline{S}_i^n = (P_i + jQ_i)^n \tag{4.74}$$

In order to satisfy I Kirchhoff's law for each bus, the currents in the shunt admittances must then be taken into account in computing the actual power when calculating power mismatch. At iteration n the current $\underline{I}_{is}^n$ into the shunt admittance load at node i is:

$$\underline{I}_{is}^n = \underline{V}_i^n \underline{Y}_{is}, \quad i = 1, \ldots, N \tag{4.75}$$

The actual power $\underline{S}_i^n$ is:

$$\underline{S}_i^n = \underline{V}_i^n (\underline{I}_i^n + \underline{I}_{is}^n)^* \quad i = 1, \ldots, N \tag{4.76}$$

Subtracting the scheduled power $\underline{S}_i$ from $\underline{S}_i^n$ gives the mismatch $\triangle \underline{S}_i^n = \triangle P_i^n + j \triangle Q_i^n$ at the end of iteration n.

$$\begin{aligned} \triangle \underline{S}_i^n &= \underline{S}_i^n - \underline{S}_i \\ \triangle P_i^n &= P_i^n - P_i \\ \triangle Q_i^n &= Q_i^n - Q_i \end{aligned} \tag{4.77}$$

The results of including shunt loads in $[Y]$ matrix and the possible net benefit will be investigated on a simple network shown in Figure 4.8.

Example

Use CI method to obtain bus voltages with power mismatch precision of 10^{-6} for power system shown in Figure 4.8:
a) if loads are not included in $[Y]$ matrix,
b) if constant Z part of the loads are included in $[Y]$ matrix. Given is the following data:

Bus 1: $\underline{V}_1 = 1.05\angle 0^o$
Bus 2: $P_2 = 1.0{V_2}^2, \quad Q_2 = 0.8{V_2}^2$
Bus 3: $P_3 = 0.5{V_3}^2, \quad Q_3 = 0.2{V_3}^2$
Line 1-2: $R = 0.05, \quad X = 0.09$
Line 2-3: $R = 0.0125, \quad X = 0.025$
Line 1-3: $R = 0.03, \quad X = 0.09$

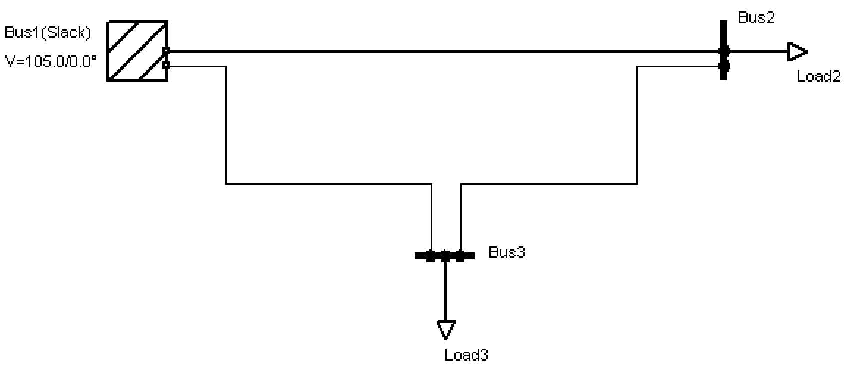

Fig. 4.8 Simple network with load buses only

Solution

a) First step is to obtain bus admittance matrix **Y**. In this case loads are not included in **Y** matrix. Bus 1 is slack bus, thus first element of the matrix will be equal to one and all the other elements of the first row are equal to zero. Line admittances are:

$$
\begin{aligned}
\underline{y}_{12} =& \frac{1}{0.05 + j0.09} = & 4.71698 - j8.49057 \\
\underline{y}_{23} =& \frac{1}{0.0125 + j0.025} = & 16.0 - j32.0 \\
\underline{y}_{13} =& \frac{1}{0.03 + j0.09} = & 3.33333 - j10.0
\end{aligned}
$$

From here bus admittance matrix $\mathbf{Y}$ is:

$$
\mathbf{Y} = \begin{bmatrix} 1 & 0 & 0 \\ -4.71698 + j8.49057 & 20.717 - j40.4906 & -16 + j32 \\ -3.33333 + j10 & -16 + j32 & 19.3333 - j42 \end{bmatrix}
$$

Initial voltage estimates are equal to slack bus voltage:

$$
\begin{aligned}
\underline{V}_2 =& 1.05 + j0 \\
\underline{V}_3 =& 1.05 + j0
\end{aligned}
$$

Now the vector of injection currents is formulated having in mind that the loads are not included in $\mathbf{Y}$ matrix. Load currents are:

$$
\begin{aligned}
\underline{I}^1_{load,2} &= \left(\frac{-\underline{S}^0_{load,2}}{\underline{V}^0_2} \right)^* \\
&= \left(\frac{-1.0(\underline{V}^0_2)^2 - j0.8(\underline{V}^0_2)^2}{\underline{V}^0_2} \right)^* \\
&= \frac{-1.0 \cdot 1.1025 + j0.8 \cdot 1.1025}{1.05} \\
&= -1.05 + j0.84
\end{aligned}
$$

$$
\begin{aligned}
\underline{I}^1_{load,3} &= \left(\frac{-\underline{S}^0_{load,3}}{\underline{V}^0_3} \right)^* \\
&= \frac{-0.5 \cdot 1.1025 + j0.2 \cdot 1.1025}{1.05} \\
&= -0.52 + j0.21
\end{aligned}
$$

The element corresponding to slack bus is equal to slack bus voltage. Vector of injection currents is:

$$
\mathbf{I}^1 = \begin{bmatrix} 1.05 \\ -1.05 + j0.84 \\ -0.52 + j0.21 \end{bmatrix}
$$

Network voltages are obtained as:

$$\mathbf{V} = \mathbf{Y}^{-1}\mathbf{I} = \mathbf{ZI}$$

$\mathbf{Z}$ matrix is obtained by factorization of $\mathbf{Y}$ matrix. Since $\mathbf{Y}$ matrix is constant, factorization is enforced only once. Voltages in the first iteration are:

$$\mathbf{V}^1 = \begin{bmatrix} 1.05 \\ 0.96497 - j0.05079 \\ 0.97663 - j0.04981 \end{bmatrix} = \begin{bmatrix} 1.05 \\ 0.96631\angle - 3.01291^o \\ 0.97790\angle - 2.92143^o \end{bmatrix}$$

Power mismatches are:

$$\triangle P_2^1 = P_2 - P_2^1 = 1.0 \cdot (0.96631)^2 - Re\left\{\underline{V}_2^1(\underline{I}_{load,2}^1)^*\right\} = 0.122132$$

$$\triangle P_3^1 = P_3 - P_3^1 = 0.5 \cdot (0.97790)^2 - Re\left\{\underline{V}_3^1(\underline{I}_{load,3}^1)^*\right\} = 0.045047$$

$$\triangle Q_2^1 = Q_2 - Q_2^1 = 0.8 \cdot (0.96631)^2 - Im\left\{\underline{V}_2^1(\underline{I}_{load,2}^1)^*\right\} = 0.010250$$

$$\triangle Q_3^1 = Q_3 - Q_3^1 = 0.2 \cdot (0.97790)^2 - Im\left\{\underline{V}_3^1(\underline{I}_{load,3}^1)^*\right\} = -0.012319$$

Power mismatches do not satisfy set precision and the process is continued. Vector of current injections is formulated with new voltage values and the process is repeated. The process converges after 6 iterations with with precision of 10^{-6} and voltages are:

$$\underline{V}_2 = 0.970211\angle - 2.57206^o$$
$$\underline{V}_3 = 0.980945\angle - 2.52272^o$$

Figure 4.9 shows convergence process of network voltages V_2 and V_3 in case that the loads are not included in $[Y]$ matrix.

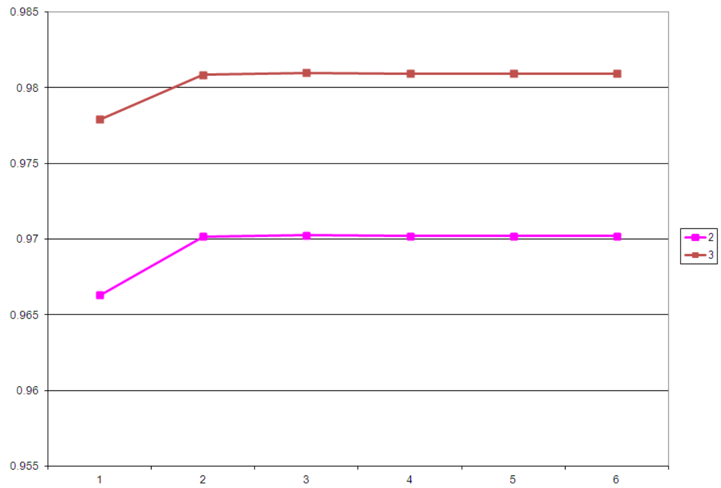

Fig. 4.9 Network voltages for case that the loads are not included in [Y] matrix

b) In this case constant Z parts of the loads are included in **Y** matrix. Line admittances are the same as above and the same rule is applied for corresponding row of the slack bus. Constant Z part of load connected to bus i is added in a form of shunt admittance to the corresponding diagonal element of the **Y** matrix:

$$\underline{y}_{shunt,i} = \frac{P_{n,i} \cdot k_{p2,i} {V_n}^2 - jQ_{n,i} \cdot k_{q2,i} {V_n}^2}{V_n^2}$$

where $V_n = 1.0$. From here bus admittance matrix **Y** is:

$$\mathbf{Y} = \begin{bmatrix} 1 & 0 & 0 \\ -4.71698 + j8.49057 & 21.717 - j41.2906 & -16 + j32 \\ -3.33333 + j10 & -16 + j32 & 19.8333 - j42.2 \end{bmatrix}$$

Initial voltage estimates are equal to slack bus voltage:

$$\underline{V}_2 = 1.05 + j0$$
$$\underline{V}_3 = 1.05 + j0$$

Now the injection current for load connected at bus i is formulated having in mind the part that is already included in **Y** matrix:

$$\underline{I}^1_{load,i} = -\left(\frac{(\underline{S}^0_{load,i})^*}{(V^0_i)^2} - \underline{y}_{shunt,i} \right) \underline{V}^0_i$$

For the purpose of power mismatches calculation total bus current is obtained as:

$$\underline{I}_i^1 = \left(\frac{-\underline{S}_{load,i}^0}{\underline{V}_i^0} \right)^*$$

The element corresponding to slack bus is equal to slack bus voltage. Since both loads consist of constant Z part only which is included in **Y** matrix, their injection currents are equal to zero. Vector of injection currents is:

$$\mathbf{I}^1 = \begin{bmatrix} 1.05 \\ 0 \\ 0 \end{bmatrix}$$

Vector of total bus currents is:

$$\mathbf{I}_T^1 = \begin{bmatrix} 1.05 \\ -1.05 + j0.84 \\ -0.52 + j0.21 \end{bmatrix}$$

Voltages in the first iteration are:

$$\mathbf{V}^1 = \begin{bmatrix} 1.05 \\ 0.96923 - j0.04354 \\ 0.98 - j0.04318 \end{bmatrix} = \begin{bmatrix} 1.05 \\ 0.970211\angle - 2.57206^o \\ 0.980945\angle - 2.52272^o \end{bmatrix}$$

Power mismatches are calculated using total bus currents:

$$\triangle P_2^1 = P_2 - P_2^1 = 1.0 \cdot (0.96631)^2 - Re\left\{\underline{V}_2^1(\underline{I}_2^1)^*\right\} = 0.112958$$

$$\triangle P_3^1 = P_3 - P_3^1 = 0.5 \cdot (0.97790)^2 - Re\left\{\underline{V}_3^1(\underline{I}_3^1)^*\right\} = 0.042438$$

$$\triangle Q_2^1 = Q_2 - Q_2^1 = 0.8 \cdot (0.96631)^2 - Im\left\{\underline{V}_2^1(\underline{I}_2^1)^*\right\} = 0.015393$$

$$\triangle Q_3^1 = Q_3 - Q_3^1 = 0.2 \cdot (0.97790)^2 - Im\left\{\underline{V}_3^1(\underline{I}_3^1)^*\right\} = -0.00932$$

Power mismatches do not satisfy set precision and the process is continued. Vector of current injections is the same as in previous iteration. From here it follows that the vector of voltages is also the same. Vector of total currents formulated with new voltage values is:

$$\mathbf{I}_T^2 = \begin{bmatrix} 1.05 \\ -0.934402 + j0.818926 \\ -0.481361 + j0.217587 \end{bmatrix}$$

Power mismatches are:

$$\triangle P_2^2 = 1.0 \cdot (0.96631)^2 - Re\left\{\underline{V}_2^2(\underline{I}_2^2)^*\right\} = -10^{-16}$$

$$\triangle P_3^2 = 0.5 \cdot (0.97790)^2 - Re\left\{\underline{V}_3^2(\underline{I}_3^2)^*\right\} = -10^{-16}$$

$$\triangle Q_2^2 = 0.8 \cdot (0.96631)^2 - Im\left\{\underline{V}_2^2(\underline{I}_2^2)^*\right\} = -10^{-16}$$

$$\triangle Q_3^2 = 0.2 \cdot (0.97790)^2 - Im\left\{\underline{V}_3^2(\underline{I}_3^2)^*\right\} = -10^{-16}$$

Figure 4.10 shows convergence process of network voltages in case that the constant Z of the loads are included in $[Y]$ matrix.

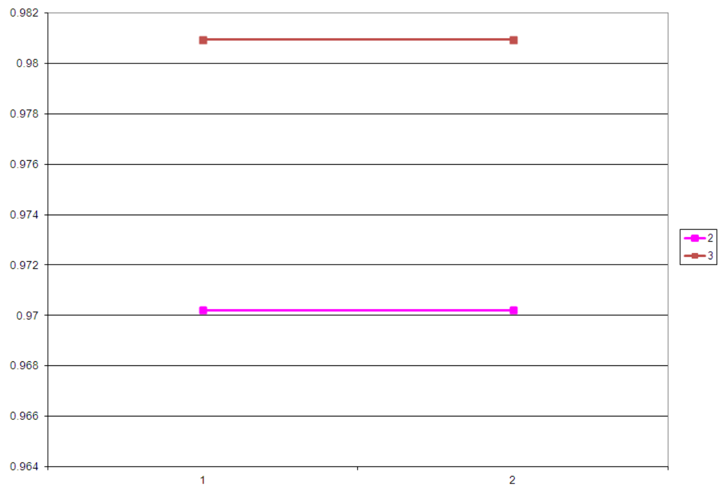

Fig. 4.10 Network voltages for constant Z part of the load included in [Y] matrix

The advantages of load inclusion in $[Y]$ matrix are obvious. In case of the constant Z load included in $[Y]$ matrix, final bus voltages are attained in one iteration only which can be seen in Figure 4.10.

Example

Solve power flow problem for the network given in the previous example if constant Z part and constant I part of the loads are included in $[Y]$ matrix. Loads are given as:

$P_2 = 0.3 + 0.3V_2 + 0.4V_2^2, \quad Q_2 = 0.3 + 0.3V_2 + 0.4V_2^2$

$$P_3 = 0.3 + 0.3V_3 + 0.4V_3^2, \quad Q_3 = 0.3 + 0.3V_3 + 0.4V_3^2$$

Solution

Constant Z and constant I parts of the loads are included in **Y** matrix. Line admittances are the same as in the previous example and the same rule is applied for corresponding row of the slack bus. Constant Z part and constant I part of load connected to bus i is added in a form of shunt admittance to the corresponding diagonal element of the **Y** matrix:

$$\underline{y}_{shunt,i} = \frac{P_{n,i}(k_{p1,i}V_n + k_{p2,i}{V_n}^2) - jQ_{n,i}(k_{q1,i}V_n + k_{q2,i}{V_n}^2)}{V_n^2}$$

where $V_n = 1.0$. From here bus admittance matrix **Y** is:

$$\mathbf{Y} = \begin{bmatrix} 1 & 0 & 0 \\ -4.71698 + j8.49057 & 21.417 - j41.1906 & -16 + j32 \\ -3.33333 + j10 & -16 + j32 & 20.0333 - j42.7 \end{bmatrix}$$

Initial voltage estimates are equal to slack bus voltage:

$$\underline{V}_2 = 1.05 + j0$$
$$\underline{V}_3 = 1.05 + j0$$

Injection current for load connected at bus i is:

$$\underline{I}^1_{load,i} = -\left(\frac{(\underline{S}^0_{load,i})^*}{(V_i^0)^2} - \underline{y}_{shunt,i}\right)\underline{V}_i^0$$

For the purpose of power mismatches calculation total bus current is obtained as:

$$\underline{I}_i^1 = \left(\frac{-\underline{S}^0_{load,i}}{\underline{V}_i^0}\right)^*$$

The element corresponding to slack bus is equal to slack bus voltage. Vector of injection currents is:

$$\mathbf{I}^1 = \begin{bmatrix} 1.05 \\ -0.27 + j0.27 \\ -0.27 + j0.27 \end{bmatrix}$$

Vector of total bus currents is:

$$\mathbf{I}_T^1 = \begin{bmatrix} 1.05 \\ -1.0057 + j1.0057 \\ -1.0057 + j1.0057 \end{bmatrix}$$

Voltages in the first iteration are:

$$\mathbf{V}^1 = \begin{bmatrix} 1.05 \\ 0.92733 - j0.04196 \\ 0.92941 - j0.04439 \end{bmatrix} = \begin{bmatrix} 1.05 \\ 0.928276\angle - 2.59187^o \\ 0.930471\angle - 2.73561^o \end{bmatrix}$$

Power mismatches are calculated using total bus currents:

$$\triangle P_2^1 = P_2 - P_2^1 = 1.0 \cdot (0.96631)^2 - Re\left\{\underline{V}_2^1(\underline{I}_2^1)^*\right\} = 0.051661$$

$$\triangle P_3^1 = P_3 - P_3^1 = 0.5 \cdot (0.97790)^2 - Re\left\{\underline{V}_3^1(\underline{I}_3^1)^*\right\} = 0.053911$$

$$\triangle Q_2^1 = Q_2 - Q_2^1 = 0.8 \cdot (0.96631)^2 - Im\left\{\underline{V}_2^1(\underline{I}_2^1)^*\right\} = -0.032731$$

$$\triangle Q_3^1 = Q_3 - Q_3^1 = 0.2 \cdot (0.97790)^2 - Im\left\{\underline{V}_3^1(\underline{I}_3^1)^*\right\} = -0.035369$$

Power mismatches do not satisfy set precision and the process is continued. The process converges after six iterations with with precision of 10^{-6}. The results are shown in Figure 4.11. Bus voltages are:

$$\underline{V}_2 = 0.917921\angle - 2.69838^o$$
$$\underline{V}_3 = 0.920275\angle - 2.85566^o$$

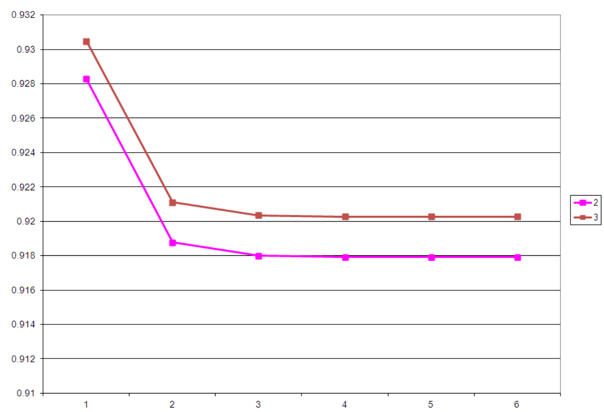

Fig. 4.11 Network voltages for constant Z part of the load included in [Y] matrix

The tests showed that the best results in general are obtained when constant Z and constant I part of the load are included in $[Y]$ matrix. Thus, this approach will be used in the examples that follow.

4.6.3 Comparisons of CI with FD and NR

In short, the results of comparisons of CI method with Newton-Raphson and Fast-Decoupled methods are:
• FD and NR solve Equation (4.61) in order to compute $[I^n]$ while CI already has $[I^n]$ to start with.
• CI requires no polar-rectangular conversion because it is performed entirely in rectangular coordinates. FD and NR require polar-rectangular conversion.
• The matrix in CI is NxN complex. In FD there are two NxN real matrices. The floating point arithmetic operations for the matrix solution in CI are twice those of FD per iteration. This loss is offset by not having to compute the node currents in each iteration. Therefore, the speed per iteration of CI and FD should be about equal.
• CI is fully coupled. It is unaffected by R/X ratios that cause ill conditioning for FD. CI and FD both have linear convergence, but the rate of convergence for CI should be greater, leading to fever iterations for a given accuracy. CI will solve problems on which FD fails, but not vice versa.
• There are several promising possibilities for improving the speed of CI. The possibilities for FD and NR seem to be exhausted.

4.6.4 Automatic Local Controls

The problem of voltage control is related to manipulation of load tap changers, capacitive compensators, reactive power injections and loss reduction [57–71]. The problem can be explained on the example of small network given in Fig 4.12 includes two transformers which are used to keep the voltages V_8 and V_{12} in a given range and also two PV generators controlling the voltages V_4 and V_5. It is obvious that changes in reactive power of the generators influence voltages V_8 and V_{12}. Also, changes of the tap positions of the transformers will influence other network voltages as well.

The question is how to measure the influence of these changes to network voltages. Current solution is a kind "try and check" method. $\triangle Q$ and $\triangle T$ are obtained and power flow is used to get corresponding voltages. Serious disadvantage of this approach is that multiple generators have to be treated separately (one by one) which results in many runs of power flow. The aim is to estimate total changes of all regulated bus voltages depending on changes

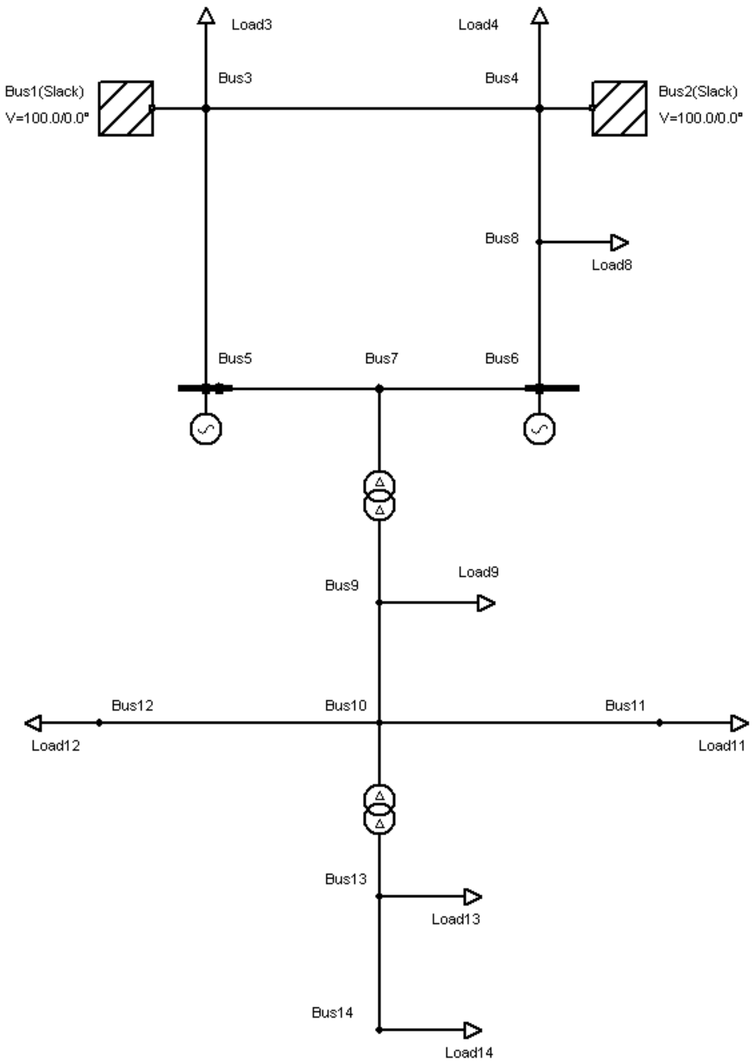

Fig. 4.12 Simple network

in tap positions and reactive power injections. The proposed method will be called *Sensitivity Analysis.*

4.6.5 Sensitivity Formulation

Sensitivity analysis based on $[Z]$ matrix is used to obtain voltage dependencies with respect to change in reactive power of PV buses and changes of LTC's positions. In order to attain set regulated voltages in the network, calculated voltage dependencies and the differences between actual and set regulated voltages, are use to estimate needed reactive power injections and LTC's positions. In general, sensitivity equation for a network containing n_g PV buses and n_t LTCs is:

$$\begin{bmatrix} \frac{\partial V_1}{\partial Q_1} & \frac{\partial V_1}{\partial Q_2} & \cdots & \frac{\partial V_1}{\partial Q_{ng}} & \frac{\partial V_1}{\partial T_1} & \frac{\partial V_1}{\partial T_2} & \cdots & \frac{\partial V_1}{\partial T_{nt}} \\ \frac{\partial V_2}{\partial Q_1} & \frac{\partial V_2}{\partial Q_2} & \cdots & \frac{\partial V_2}{\partial Q_{ng}} & \frac{\partial V_2}{\partial T_1} & \frac{\partial V_2}{\partial T_2} & \cdots & \frac{\partial V_2}{\partial T_{nt}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial V_k}{\partial Q_1} & \frac{\partial V_k}{\partial Q_2} & \cdots & \frac{\partial V_k}{\partial Q_{ng}} & \frac{\partial V_k}{\partial T_1} & \frac{\partial V_k}{\partial T_2} & \cdots & \frac{\partial V_k}{\partial T_{nt}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial V_m}{\partial Q_1} & \frac{\partial V_m}{\partial Q_2} & \cdots & \frac{\partial V_m}{\partial Q_{ng}} & \frac{\partial V_m}{\partial T_1} & \frac{\partial V_m}{\partial T_2} & \cdots & \frac{\partial V_m}{\partial T_{nt}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial V_n}{\partial Q_1} & \frac{\partial V_n}{\partial Q_2} & \cdots & \frac{\partial V_n}{\partial Q_{ng}} & \frac{\partial V_n}{\partial T_1} & \frac{\partial V_n}{\partial T_2} & \cdots & \frac{\partial V_n}{\partial T_{nt}} \end{bmatrix} \begin{bmatrix} \triangle Q_1 \\ \triangle Q_2 \\ \vdots \\ \triangle Q_{ng} \\ \triangle T_1 \\ \triangle T_2 \\ \vdots \\ \triangle T_{nt} \end{bmatrix} = \begin{bmatrix} \triangle V_1 \\ \triangle V_2 \\ \vdots \\ \triangle V_k \\ \vdots \\ \triangle V_m \\ \vdots \\ \triangle V_n \end{bmatrix}, n = n_g + n_t \tag{4.78}$$

Derivatives in sensitivity matrix are calculated in each iteration of CI method using the expressions obtained previously. Also $[\triangle V]$ is obtained in each iteration as a difference between calculated voltages and corresponding set voltages for regulated buses. After solving the Equation(4.78) in iteration n using forward/backward method, calculated $\triangle Q_i^n$ and $\triangle T_i^n$ values are added to Q_i^{n-1} and T_i^{n-1} values from previous iteration. New Q_i^n and T_i^n are used for calculation of current injections in the next iteration:

$$\begin{aligned} Q_i^n =& Q_i^{n-1} + \triangle Q_i^n \\ T_i^n =& T_i^{n-1} + \triangle T_i^n \end{aligned} \tag{4.79}$$

After this it is necessary to check whether Q_i^n and $t_i^n = t_{initial} + T_i^n$ are within the given limits. Conditions:

$$Q_{imin} < Q_i^n < Q_{imax}$$

$$t_{imin} < t_i^n < t_{imax}$$

have to be fulfilled for each PV bus or LTC in each iteration. If these conditions are violated, bus i is no longer regulated bus and is no longer the subject of sensitivity analysis. Q_i^n and T_i^n are then set to relevant limits and kept constant. Sensitivity analysis will be introduced separately for PV buses and LTCs and then the general conclusions will be made.

4.6.5.1 PV Buses

As mentioned previously, the influence of PV bus k in iteration n is reflected through the kth member of $[I]$ vector as:

$$\underline{I}_k^n = \left(\frac{\underline{S}_k^{n-1}}{\underline{V}_k^{n-1}}\right)^* = \frac{\left(\underline{S}_k^{n-1}\right)^*}{\left(\underline{V}_k^{n-1}\right)^*} = \frac{P_{k,sch} - jQ_k^{n-1}}{\left(\underline{V}_k^{n-1}\right)^*} \tag{4.80}$$

In each iteration n of the CI algorithm with local controls, $\triangle Q_k^n$ is calculated depending on all network controls using sensitivity analysis. Then it is added to Q_k^{n-1} to get Q_k^n for next iteration:

$$Q_k^n = Q_k^{n-1} + \triangle Q_k^n \tag{4.81}$$

PV node has scheduled active power $P_{k,sch}$ and scheduled voltage magnitude V_k. V_k must be enforced as long as $Q_{kmin} < Q_k < Q_{kmax}$. If enforcing V_k causes a reactive power violation, bus k is no longer regulated bus and Q_k is set equal to the relevant limit and it is no longer the subject of sensitivity analysis.

Sensitivity analysis for PV buses will be presented on the example of a network with n_g PV generators and one slack bus. The relation between voltage variations $\triangle V$ and reactive power changes $\triangle Q$ is given in Equation(4.82).

$$\begin{bmatrix} \frac{\partial V_1}{\partial Q_1} & \frac{\partial V_1}{\partial Q_2} & \cdots & \frac{\partial V_1}{\partial Q_k} & \cdots & \frac{\partial V_1}{\partial Q_m} & \cdots & \frac{\partial V_1}{\partial Q_n} \\ \frac{\partial V_2}{\partial Q_1} & \frac{\partial V_2}{\partial Q_2} & \cdots & \frac{\partial V_2}{\partial Q_k} & \cdots & \frac{\partial V_2}{\partial Q_m} & \cdots & \frac{\partial V_2}{\partial Q_n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \frac{\partial V_k}{\partial Q_1} & \frac{\partial V_k}{\partial Q_2} & \cdots & \frac{\partial V_k}{\partial Q_k} & \cdots & \frac{\partial V_k}{\partial Q_m} & \cdots & \frac{\partial V_k}{\partial Q_n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \frac{\partial V_m}{\partial Q_1} & \frac{\partial V_m}{\partial Q_2} & \cdots & \frac{\partial V_m}{\partial Q_k} & \cdots & \frac{\partial V_m}{\partial Q_m} & \cdots & \frac{\partial V_m}{\partial Q_n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \frac{\partial V_n}{\partial Q_1} & \frac{\partial V_n}{\partial Q_2} & \cdots & \frac{\partial V_n}{\partial Q_k} & \cdots & \frac{\partial V_n}{\partial Q_m} & \cdots & \frac{\partial V_n}{\partial Q_n} \end{bmatrix} \begin{bmatrix} \triangle Q_1 \\ \triangle Q_2 \\ \vdots \\ \triangle Q_k \\ \vdots \\ \triangle Q_m \\ \vdots \\ \triangle Q_n \end{bmatrix} = \begin{bmatrix} \triangle V_1 \\ \triangle V_2 \\ \vdots \\ \triangle V_k \\ \vdots \\ \triangle V_m \\ \vdots \\ \triangle V_n \end{bmatrix} \tag{4.82}$$

Voltage vector is obtained from scheduled voltages and power flow results. Sensitivity matrix needs to be calculated in each iteration in order to get $[\triangle Q]$ vector. Analytical expressions for derivations in sensitivity matrix can not be obtained using the standard $[Y]$ matrix approach. This however can be achieved using sensitivity analysis based on $[Z]$ (impedance) matrix approach.

$[Z]$ matrix equation for the discussed network is:

$$[Z]\,[I] = [V] \tag{4.83}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_{21} & Z_{22} & \dots & Z_{2k} & \dots & Z_{2m} & \dots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{k1} & Z_{k2} & \dots & Z_{kk} & \dots & Z_{km} & \dots & Z_{kn} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{m1} & Z_{m2} & \dots & Z_{mk} & \dots & Z_{mm} & \dots & Z_{mn} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{n1} & Z_{n2} & \dots & Z_{nk} & \dots & Z_{nm} & \dots & Z_{nn} \end{bmatrix} \begin{bmatrix} V_{slack} \\ I_2 \\ \vdots \\ I_k \\ \vdots \\ I_m \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} 1 \\ V_2 \\ \vdots \\ V_k \\ \vdots \\ V_m \\ \vdots \\ V_n \end{bmatrix} \tag{4.84}$$

For the purpose of sensitivity analysis all currents except I_k will be set equal to zero. Voltage V_k can be obtained by multiplying kth row of the matrix $[Z]$ with the current vector:

$$\underline{\gamma}_{k1}\underline{V}_{slack} + \underline{Z}_{kk}\underline{I}_k = \underline{V}_k \tag{4.85}$$

$$\underline{I}_k = \left(\frac{\underline{S}k}{\underline{V}k}\right)^* = \frac{P_k - jQ_k}{V_k^{re} - jV_k^{im}}, \quad \underline{V}_{\gamma k} = \underline{\gamma}_{k1}\underline{V}_{slack} \tag{4.86}$$

$$\underline{V}_{\gamma k} + \underline{Z}_{kk}\underline{I}_k = \underline{V}_k \tag{4.87}$$

Equation(4.87) can be rewritten in terms of real and imaginary components as:

$$(V_{\gamma k}^{re} + jV_{\gamma k}^{im})(V_k^{re} - jV_k^{im}) + (R_{kk} + jX_{kk})(P_k - jQ_k) = V_k^2 \tag{4.88}$$

Real part of the Equation((4.88)) is:

$$V_{\gamma k}^{re}V_k^{re} + V_{\gamma k}^{im}V_k^{im} + R_{kk}P_k + X_{kk}Q_k = V_k^{re^2} + V_k^{im^2} \tag{4.89}$$

Imaginary part of the Equation((4.88)) is:

$$V_{\gamma k}^{im}V_k^{re} - V_{\gamma k}^{re}V_k^{im} + X_{kk}P_k - R_{kk}Q_k = 0 \tag{4.90}$$

In order to obtain changes in V_k depending on reactive power injection change at bus k, Equation(4.89) and Equation(4.90) will be derived by Q_k:

$$V_{\gamma k}^{re}\frac{\partial V_k^{re}}{\partial Q_k} + V_{\gamma k}^{im}\frac{\partial V_k^{im}}{\partial Q_k} + X_{kk} = 2V_k^{re}\frac{\partial V_k^{re}}{\partial Q_k} + 2V_k^{im}\frac{\partial V_k^{im}}{\partial Q_k} \tag{4.91}$$

$$V_{\gamma k}^{im}\frac{\partial V_k^{re}}{\partial Q_k} - V_{\gamma k}^{re}\frac{\partial V_k^{im}}{\partial Q_k} - R_{kk} = 0 \Rightarrow \frac{\partial V_k^{im}}{\partial Q_k} = \frac{V_{\gamma k}^{im}\frac{\partial V_k^{re}}{\partial Q_k} - R_{kk}}{V_{\gamma k}^{re}} \tag{4.92}$$

Substituting $\frac{\partial V_k^{im}}{\partial Q_k}$from Equation(4.91) for Equation(4.92) gives:

$$V_{\gamma k}^{re}\frac{\partial V_k^{re}}{\partial Q_k} + V_{\gamma k}^{im}\frac{V_{\gamma k}^{im}\frac{\partial V_k^{re}}{\partial Q_k} - R_{kk}}{V_{\gamma k}^{re}} + X_{kk} = 2V_k^{re}\frac{\partial V_k^{re}}{\partial Q_k} + 2V_k^{im}\frac{V_{\gamma k}^{im}\frac{\partial V_k^{re}}{\partial Q_k} - R_{kk}}{V_{\gamma k}^{re}} \tag{4.93}$$

$$V_{\gamma k}^{re}\frac{\partial V_k^{re}}{\partial Q_k} + \frac{V_{\gamma k}^{im^2}}{V_{\gamma k}^{re}}\frac{\partial V_k^{re}}{\partial Q_k} - \frac{V_{\gamma k}^{im}}{V_{\gamma k}^{re}}R_{kk} + X_{kk} = 2V_k^{re}\frac{\partial V_k^{re}}{\partial Q_k} + \frac{2V_k^{im}}{V_{\gamma k}^{re}}V_{\gamma k}^{im}\frac{\partial V_k^{re}}{\partial Q_k} - \frac{2V_k^{im}}{V_{\gamma k}^{re}}R_{kk} \tag{4.94}$$

$$\frac{\partial V_k^{re}}{\partial Q_k} = \frac{\frac{V_{\gamma k}^{im}}{V_{\gamma k}^{re}}R_{kk} - X_{kk} - \frac{2V_k^{im}}{V_{\gamma k}^{re}}R_{kk}}{V_{\gamma k}^{re} + \frac{V_{\gamma k}^{im2}}{V_{\gamma k}^{re}} - 2V_k^{re} - \frac{2V_k^{im}}{V_{\gamma k}^{re}}V_{\gamma k}^{im}} = \frac{V_{\gamma k}^{im}R_{kk} - V_{\gamma k}^{re}X_{kk} - 2V_k^{im}R_{kk}}{V_{\gamma k}^{re^2} + V_{\gamma k}^{im^2} - 2V_k^{re}V_{\gamma k}^{re} - 2V_k^{im}V_{\gamma k}^{im}} \tag{4.95}$$

Then $\frac{\partial V_k^{im}}{\partial Q_k}$ is:

$$\frac{\partial V_k^{im}}{\partial Q_k} = \frac{V_{\gamma k}^{im}\frac{V_{\gamma k}^{im}R_{kk} - V_{\gamma k}^{re}X_{kk} - 2V_k^{im}R_{kk}}{V_{\gamma k}^{re2} + V_{\gamma k}^{im2} - 2V_k^{re}V_{\gamma k}^{re} - 2V_k^{im}V_{\gamma k}^{im}} - R_{kk}}{V_{\gamma k}^{re}} =$$

$$= \frac{V_{\gamma k}^{im}}{V_{\gamma k}^{re}}\frac{V_{\gamma k}^{im}R_{kk} - V_{\gamma k}^{re}X_{kk} - 2V_k^{im}R_{kk}}{V_{\gamma k}^{re2} + V_{\gamma k}^{im2} - 2V_k^{re}V_{\gamma k}^{re} - 2V_k^{im}V_{\gamma k}^{im}} - \frac{R_{kk}}{V_{\gamma k}^{re}} \tag{4.96}$$

Now that the $\frac{\partial V_k^{ire}}{\partial Q_k}$ and $\frac{\partial V_k^{im}}{\partial Q_k}$ are obtained, total change $\frac{\partial V_k}{\partial Q_k}$ will be calculated from:

$$V_k^2 = V_k^{re^2} + V_k^{im^2} / \frac{\partial}{\partial Q_k} \tag{4.97}$$

$$2V_k\frac{\partial V_k}{\partial Q_k} = 2V_k^{re}\frac{\partial V_k^{re}}{\partial Q_k} + 2V_k^{im}\frac{\partial V_k^{im}}{\partial Q_k} \tag{4.98}$$

$$\frac{\partial V_k}{\partial Q_k} = \frac{V_k^{re}\frac{\partial V_k^{re}}{\partial Q_k} + V_k^{im}\frac{\partial V_k^{im}}{\partial Q_k}}{V_k} \tag{4.99}$$

For the purpose of discussing the influence of reactive power variation $\triangle Q_m$ on V_k, all the currents except I_m and $I_k$in Equation(4.84) will be assumed equal to zero:

$$\underline{\gamma}_{k1}\underline{V}_{slack} + \underline{Z}_{km}\underline{I}_m = \underline{V}_k \tag{4.100}$$

$$(V_{\gamma k}^{re}+jV_{\gamma k}^{im})(V_m^{re}-jV_m^{im})+(R_{km}+jX_{km})(P_m-jQ_m) = (V_k^{re}+jV_k^{im})(V_m^{re}-jV_m^{im}) \tag{4.101}$$

Real part of Equation(4.101) is:

$$V_{\gamma k}^{re}V_m^{re} + V_{\gamma k}^{im}V_m^{im} + R_{km}P_m + X_{km}Q_m = V_k^{re}V_m^{re} + V_k^{im}V_m^{im} \tag{4.102}$$

Imaginary part of Equation(4.101) is:

$$V_{\gamma k}^{im}V_m^{re} - V_{\gamma k}^{re}V_m^{im} + X_{km}P_m - R_{km}Q_m = V_k^{im}V_m^{re} - V_k^{re}V_m^{im} \tag{4.103}$$

Derivation of real part by Q_m gives:

$$V_{\gamma k}^{re}\frac{\partial V_m^{re}}{\partial Q_m}+V_{\gamma k}^{im}\frac{\partial V_m^{im}}{\partial Q_m}+X_{km} = V_k^{re}\frac{\partial V_m^{re}}{\partial Q_m}+V_m^{re}\frac{\partial V_k^{re}}{\partial Q_m}+V_k^{im}\frac{\partial V_m^{im}}{\partial Q_m}+V_m^{im}\frac{\partial V_k^{im}}{\partial Q_m} \tag{4.104}$$

Derivation of imaginary part by Q_m gives:

$$V_{\gamma k}^{im}\frac{\partial V_m^{re}}{\partial Q_m}-V_{\gamma k}^{re}\frac{\partial V_m^{im}}{\partial Q_m}-R_{km} = V_k^{im}\frac{\partial V_m^{re}}{\partial Q_m}+V_m^{re}\frac{\partial V_k^{im}}{\partial Q_m}-V_k^{re}\frac{\partial V_m^{im}}{\partial Q_m}-V_m^{im}\frac{\partial V_k^{re}}{\partial Q_m} \tag{4.105}$$

Introducing variables f_1 and f_2 yields:

$$f_1 = V_{\gamma k}^{re}\frac{\partial V_m^{re}}{\partial Q_m} + V_{\gamma k}^{im}\frac{\partial V_m^{im}}{\partial Q_m} + X_{km} - V_k^{re}\frac{\partial V_m^{re}}{\partial Q_m} - V_k^{im}\frac{\partial V_m^{im}}{\partial Q_m} \tag{4.106}$$

$$f_2 = V_{\gamma k}^{im}\frac{\partial V_m^{re}}{\partial Q_m} - V_{\gamma k}^{re}\frac{\partial V_m^{im}}{\partial Q_m} - R_{km} - V_k^{im}\frac{\partial V_m^{re}}{\partial Q_m} + V_k^{re}\frac{\partial V_m^{im}}{\partial Q_m} \tag{4.107}$$

Now the system of two unknown variables is solved:

$$f_1 = V_m^{re}\frac{\partial V_k^{re}}{\partial Q_m} + V_m^{im}\frac{\partial V_k^{im}}{\partial Q_m} \tag{4.108}$$

$$f_2 = V_m^{re}\frac{\partial V_k^{im}}{\partial Q_m} - V_m^{im}\frac{\partial V_k^{re}}{\partial Q_m} \Rightarrow \frac{\partial V_k^{im}}{\partial Q_m} = \frac{f_2 + V_m^{im}\frac{\partial V_k^{re}}{\partial Q_m}}{V_m^{re}} \tag{4.109}$$

$$f_1 = V_m^{re}\frac{\partial V_k^{re}}{\partial Q_m} + V_m^{im}\frac{f_2 + V_m^{im}\frac{\partial V_k^{re}}{\partial Q_m}}{V_m^{re}} \tag{4.110}$$

$$V_m^{re} f_1 - V_m^{im} f_2 = V_m^{re^2}\frac{\partial V_k^{re}}{\partial Q_m} + V_m^{im^2}\frac{\partial V_k^{re}}{\partial Q_m} \tag{4.111}$$

$$\frac{\partial V_k^{re}}{\partial Q_m} = \frac{V_m^{re} f_1 - V_m^{im} f_2}{V_m} \tag{4.112}$$

$$\frac{\partial V_k^{im}}{\partial Q_m} = \frac{f_2 + V_m^{im}\frac{\partial V_k^{re}}{\partial Q_m}}{V_m^{re}} \tag{4.113}$$

Finally $\frac{\partial V_k^{re}}{\partial Q_m}$ and $\frac{\partial V_k^{im}}{\partial Q_m}$ are:

$$\frac{\partial V_k^{re}}{\partial Q_m} = \frac{V_m^{re}\left(V_{\gamma k}^{re}\frac{\partial V_m^{re}}{\partial Q_m} + V_{\gamma k}^{im}\frac{\partial V_m^{im}}{\partial Q_m} + X_{km} - V_k^{re}\frac{\partial V_m^{re}}{\partial Q_m} - V_k^{im}\frac{\partial V_m^{im}}{\partial Q_m}\right)}{V_m} -$$

$$- \frac{V_m^{im}\left(V_{\gamma k}^{im}\frac{\partial V_m^{re}}{\partial Q_m} - V_{\gamma k}^{re}\frac{\partial V_m^{im}}{\partial Q_m} - R_{km} - V_k^{im}\frac{\partial V_m^{re}}{\partial Q_m} + V_k^{re}\frac{\partial V_m^{im}}{\partial Q_m}\right)}{V_m} \tag{4.114}$$

$$\frac{\partial V_k^{im}}{\partial Q_m} = \frac{\left(V_{\gamma k}^{im}\frac{\partial V_m^{re}}{\partial Q_m} - V_{\gamma k}^{re}\frac{\partial V_m^{im}}{\partial Q_m} - R_{km} - V_k^{im}\frac{\partial V_m^{re}}{\partial Q_m} + V_k^{re}\frac{\partial V_m^{im}}{\partial Q_m}\right) + V_m^{im}\frac{\partial V_k^{re}}{\partial Q_m}}{V_m^{re}} \tag{4.115}$$

$\frac{\partial V_k}{\partial Q_m}$ will be calculated using the Equation(4.99).

In general, it is not uncommon to have more than one slack bus in the network. Assuming that buses from 1 to h are slack buses and all the currents except I_k are equal to zero, V_k would be:

$$\sum_{i=1}^{h} \gamma_{ki} V_{slacki} + Z_{kk} I_k = V_k \tag{4.116}$$

$$\sum_{i=1}^{h} (V_{\gamma ki}^{re} + jV_{\gamma ki}^{im}) + Z_{kk} I_k = V_k \tag{4.117}$$

$$\sum_{i=1}^{h} V_{\gamma ki}^{re} + j\sum_{i=1}^{h} V_{\gamma ki}^{im} + Z_{kk} I_k = V_k \tag{4.118}$$

$$V_{\gamma k}^{re} = \sum_{i=1}^{h} V_{\gamma ki}^{re}$$

$$V_{\gamma k}^{im} = \sum_{i=1}^{h} V_{\gamma ki}^{im}$$

$$(V_{\gamma k}^{re} + jV_{\gamma k}^{im}) + Z_{kk}I_k = V_k \tag{4.119}$$

$$(V_{\gamma k}^{re} + jV_{\gamma k}^{im})(V_k^{re} - jV_k^{im}) + (R_{kk} + jX_{kk})(Pk - jQ_k) = V_k^2 \tag{4.120}$$

$$\vdots$$

$$\frac{\partial V_k^{re}}{\partial Q_k} = \frac{V_{\gamma k}^{im} R_{kk} - V_{\gamma k}^{re} X_{kk} - 2V_k^{im} R_{kk}}{V_{\gamma k}^{re^2} + V_{\gamma k}^{im^2} - 2V_k^{re} V_{\gamma k}^{re} - 2V_k^{im} V_{\gamma k}^{im}} \tag{4.121}$$

$$\frac{\partial V_k^{im}}{\partial Q_k} = \frac{V_{\gamma k}^{im} \frac{\partial V_k^{re}}{\partial Q_k} - R_{kk}}{V_{\gamma k}^{re}} \tag{4.122}$$

CI method block diagram for a network with PV buses is shown in Figure 4.13.

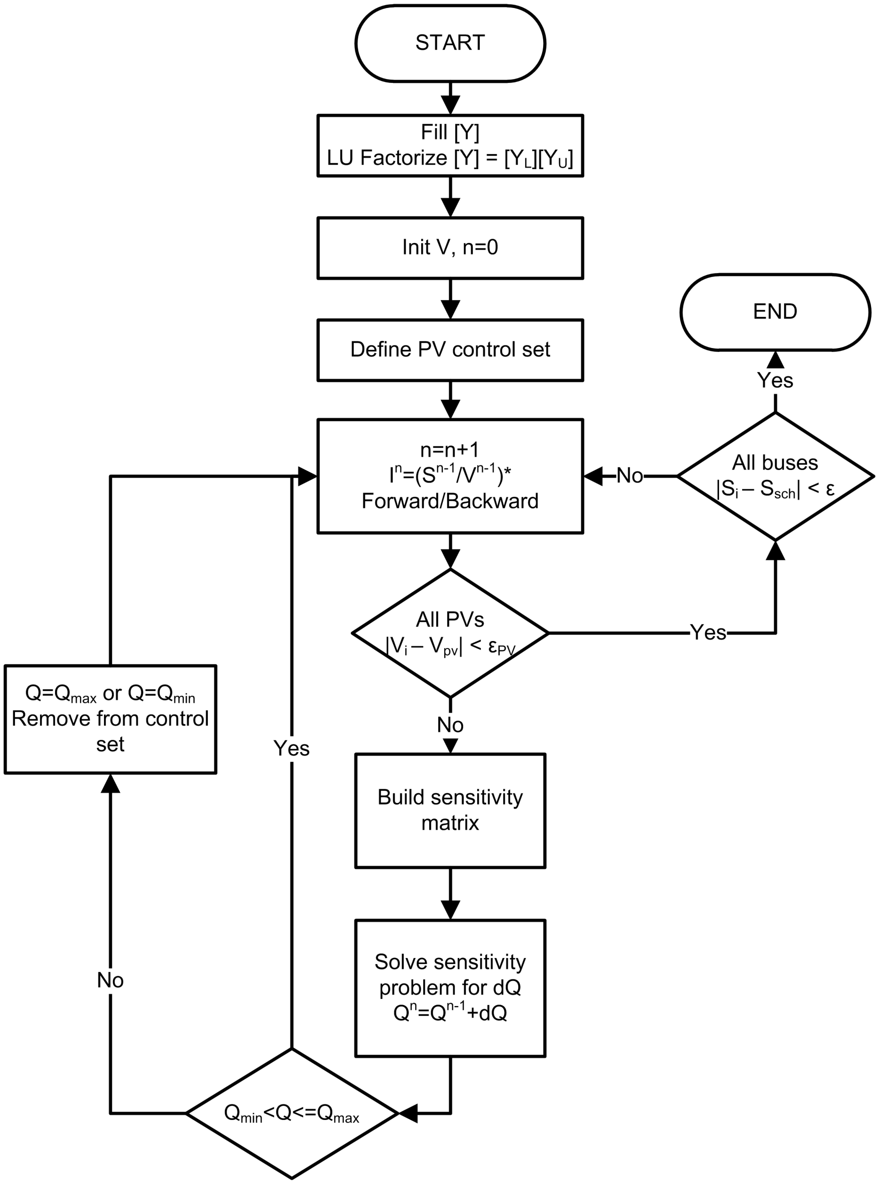

Fig. 4.13 Block diagram for a network with PV buses

In order to illustrate the behavior of CI method for network with PV nodes, Figure 4.14 will be considered.

Example

Use CI method to obtain bus voltages for power system shown in Figure 4.14 with power mismatch precision of 10^{-6} and voltage precision of 10^{-4}. Given is the following data in per-unit:

Bus 1: $\underline{V}_1 = 1\angle 0^o$
Bus 2: $\underline{V}_2 = 1.05\angle 0^o$
Bus 3: $P_3 = 1.0, \quad Q_3 = 0.8$
Bus 4: $P_4 = 0.4 + 0.3V_4 + 0.3V_4^2, \quad Q_4 = 0.5$
Bus 5: $P_5 = 0.1, \quad V_5 = 0.97$
Bus 6: $P_6 = 2.4 \cdot (0.4 + 0.3V_6 + 0.3V_6^2), \quad Q_6 = 1.45 \cdot (0.4 + 0.3V_6 + 0.3V_6^2)$
Bus 7: $P_7 = 0.2, \quad V_7 = 0.975$
Line 1-3: $R = 0.013, \quad X = 0.039$
Line 2-4:$R = 0.007, \quad X = 0.021$
Line 3-4:$R = 0.01, \quad X = 0.03$
Line 3-5:$R = 0.008, \quad X = 0.024$
Line 4-7:$R = 0.001, \quad X = 0.03$
Line 5-6:$R = 0.01, \quad X = 0.03$
Line 6-7:$R = 0.01, \quad X = 0.03$

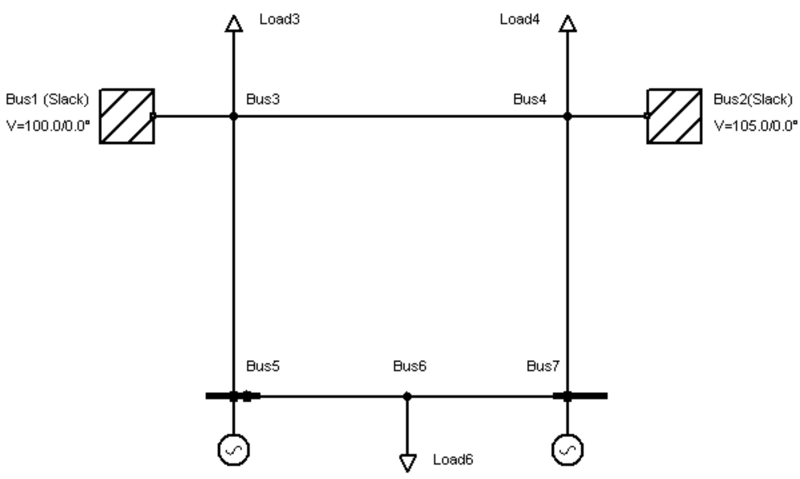

Fig. 4.14 Simple network with PQ and PV nodes

Solution

This example will be solved including constant Z and constant I parts of the loads in **Y** matrix. Constant Z part and constant I part of the load connected to bus i is added in a form of shunt admittance to the corresponding diagonal element of the **Y** matrix:

$$\underline{y}_{shunt,i} = \frac{P_{n,i}(k_{p1,i}V_n + k_{p2,i}{V_n}^2) - jQ_{n,i}(k_{q1,i}V_n + k_{q2,i}{V_n}^2)}{V_n^2}$$

where $V_n = 1.0$. Bus admittance matrix **Y** is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -7.7{+}j23.08 & 0 & 30.19{-}j90.58 & -10{+}j30 & -12.5{+}j37.5 & 0 & 0 \\ 0 & -14.3{+}j42.86 & -10{+}j30 & 26{-}j106.15 & 0 & 0 & -1.11{+}j33.3 \\ 0 & 0 & -12.5{+}j37.5 & 0 & 22.5{-}j67.5 & -10{+}j30 & 0 \\ 0 & 0 & 0 & 0 & -10{+}j30 & 21.4{-}j60.9 & -10{+}j30 \\ 0 & 0 & 0 & -1.11{+}j33.3 & 0 & -10{+}j30 & 11.11{-}j63.3 \end{bmatrix}$$

Initial voltage estimates are equal to the voltage of the first slack bus (Bus 1):

$$\underline{V}_i = 1.0 + j0, \quad i = 3, 4, 5, 6, 7$$

Injection current for load connected at bus i is:

$$\underline{I}^1_{load,i} = -\left(\frac{(\underline{S}^0_{load,i})^*}{(V^0_i)^2} - \underline{y}_{shunt,i}\right)\underline{V}^0_i$$

Injection current for regulated bus is obtained from:

$$\underline{I}^1_{PV,i} = \frac{(\underline{S}^0_{PV,i})^*}{(\underline{V}^0_i)^*} = \frac{P_{sch,i} - jQ^0_i}{(\underline{V}^0_i)^*}$$

At the begging reactive power of each regulated bus (Bus 5 and Bus 7 in this example) is set to zero, $Q^0_5 = 0,\ Q^0_7 = 0$. For the purpose of power mismatches calculation total bus current for load bus i is obtained as:

$$\underline{I}^1_i = \left(\frac{-\underline{S}^0_{load,i}}{\underline{V}^0_i}\right)^*$$

For regulated buses $\underline{I}^n_i = \underline{I}^n_{PV,i}$. Vector of injection currents is:

$$\mathbf{I}^1 = \begin{bmatrix} 1.0 \\ 1.05 \\ -1.0 + j0.8 \\ -0.4 + j0.5 \\ 1.0 \\ -0.96 + j0.58 \\ 0.2 \end{bmatrix}$$

Vector of total bus currents is:

$$\mathbf{I}_T^1 = \begin{bmatrix} 1.0 \\ 1.05 \\ -1.0 + j0.8 \\ -1.0 + j0.5 \\ 1.0 \\ -2.4 + j1.45 \\ 0.2 \end{bmatrix}$$

Voltages in the first iteration are:

$$\mathbf{V}^1 = \begin{bmatrix} 1.0 \\ 1.05 \\ 0.9587 - j0.0462 \\ 0.9856 - j0.037 \\ 0.939 - j0.0644 \\ 0.9134 - j0.0901 \\ 0.9549 - j0.065 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.05 \\ 0.9598\angle - 0.0481^o \\ 0.9863\angle - 0.0375^o \\ 0.9412\angle - 0.0685^o \\ 0.9178\angle - 0.0984^o \\ 0.9571\angle - 0.068^o \end{bmatrix}$$

Now for regulated buses it is checked whether their voltage magnitudes are within the set limits of voltage precision:

$$\left|V_{sch,5} - V_5^1\right| = 0.0288$$
$$\left|V_{sch,7} - V_7^1\right| = 0.0179$$

Voltage mismatch matrix:

$$\triangle\mathbf{V} = \begin{bmatrix} 0.0288 \\ 0.0179 \end{bmatrix}$$

Since both voltages are outside the limits sensitivity analysis is conducted. Sensitivity matrix dimensions are 2x2:

$$\mathbf{S} = \begin{bmatrix} \frac{\partial V_5}{\partial Q_5} & \frac{\partial V_5}{\partial Q_7} \\ \frac{\partial V_7}{\partial Q_5} & \frac{\partial V_7}{\partial Q_7} \end{bmatrix}$$

First diagonal element is obtained.

$$\frac{\partial V_5^{re}}{\partial Q_5} = \frac{V_{\gamma 5}^{im} R_{55} - V_{\gamma 5}^{re} X_{55} - 2V_5^{im} R_{55}}{V_{\gamma 5}^{re^2} + V_{\gamma 5}^{im^2} - 2V_5^{re} V_{\gamma 5}^{re} - 2V_5^{im} V_{\gamma 5}^{im}} = 0.0375$$

$$\frac{\partial V_5^{im}}{\partial Q_5} = \frac{V_{\gamma 5}^{im}}{V_{\gamma 5}^{re}} \cdot \frac{V_{\gamma 5}^{im} R_{55} - V_{\gamma 5}^{re} X_{55} - 2V_5^{im} R_{55}}{V_{\gamma 5}^{re^2} + V_{\gamma 5}^{im^2} - 2V_5^{re} V_{\gamma 5}^{re} - 2V_5^{im} V_{\gamma 5}^{im}} - \frac{R_{55}}{V_{\gamma 5}^{re}} = -0.0134$$

R_{55} is real part of Z_{55}. $V_{\gamma 5}^{re}$ and $V_{\gamma 5}^{im}$ are obtained as:

$$\underline{V}_{\gamma 5} = \sum_{i=1}^{2} \gamma_{5i} V_{slack,i} = \sum_{i=1}^{2} (V_{\gamma ki}^{re} + jV_{\gamma ki}^{im})$$

$$V_{\gamma 5}^{re} = \sum_{i=1}^{2} V_{\gamma 5i}^{re}$$

$$V_{\gamma 5}^{im} = \sum_{i=1}^{2} V_{\gamma 5i}^{im}$$

V_5^{re} and V_5^{im} are real and imaginary part of current Bus 5 voltage.

$$\frac{\partial V_5}{\partial Q_5} = \frac{V_5^{re} \frac{\partial V_5^{re}}{\partial Q_5} + V_5^{im} \frac{\partial V_5^{im}}{\partial Q_5}}{V_5} = 0.0384$$

Similar:

$$\frac{\partial V_7}{\partial Q_7} = 0.038$$

Off diagonal elements are obtained as:

$$\frac{\partial V_5^{re}}{\partial Q_7} = \frac{V_7^{re} \left(V_{\gamma 5}^{re} \frac{\partial V_7^{re}}{\partial Q_7} + V_{\gamma 5}^{im} \frac{\partial V_7^{im}}{\partial Q_7} + X_{57} - V_5^{re} \frac{\partial V_7^{re}}{\partial Q_7} - V_5^{im} \frac{\partial V_7^{im}}{\partial Q_7} \right)}{V_7} -$$

$$- \frac{V_7^{im} \left(V_{\gamma 5}^{im} \frac{\partial V_7^{re}}{\partial Q_7} - V_{\gamma 5}^{re} \frac{\partial V_7^{im}}{\partial Q_7} - R_{57} - V_5^{im} \frac{\partial V_7^{re}}{\partial Q_7} + V_5^{re} \frac{\partial V_7^{im}}{\partial Q_7} \right)}{V_7} = 0.0195$$

$$\frac{\partial V_5^{im}}{\partial Q_7} = \frac{\left(V_{\gamma 5}^{im} \frac{\partial V_7^{re}}{\partial Q_7} - V_{\gamma 5}^{re} \frac{\partial V_7^{im}}{\partial Q_7} - R_{57} - V_5^{im} \frac{\partial V_7^{re}}{\partial Q_7} + V_5^{re} \frac{\partial V_7^{im}}{\partial Q_7} \right) + V_7^{im} \frac{\partial V_5^{re}}{\partial Q_7}}{V_7^{re}}$$
$$= -0.0053$$

$$\frac{\partial V_5}{\partial Q_7} = 0.0198$$

Similarly:

$$\frac{\partial V_7}{\partial Q_5} = 0.0195$$

Sensitivity equation is:

$$\mathbf{S} \cdot \triangle \mathbf{Q} = \triangle \mathbf{V}$$

$$\begin{bmatrix} 0.0384 & 0.0198 \\ 0.0195 & 0.038 \end{bmatrix} \begin{bmatrix} \triangle Q_5 \\ \triangle Q_7 \end{bmatrix} = \begin{bmatrix} 0.0288 \\ 0.0179 \end{bmatrix}$$

$$\begin{bmatrix} \triangle Q_5 \\ \triangle Q_7 \end{bmatrix} = \begin{bmatrix} 0.0384 & 0.0198 \\ 0.0195 & 0.038 \end{bmatrix}^{-1} \begin{bmatrix} 0.0288 \\ 0.0179 \end{bmatrix} = \begin{bmatrix} 0.6883 \\ 0.1183 \end{bmatrix}$$

Now reactive powers are:

$$\begin{aligned} Q_5^1 =& Q_5^0 + \triangle Q_5 &= 0 + 0.6883 = 0.6883 \\ Q_7^1 =& Q_7^0 + \triangle Q_7 &= 0 + 0.1183 = 0.1183 \end{aligned}$$

Since no limits for reactive power on regulated buses are specified in this example, the process continues. New vector of injection currents is formulated, new voltages are obtained and voltage magnitudes for regulated buses are checked. When voltage magnitude mismatch on regulated bus is less than specified, that bus is no longer subject of sensitivity analysis - Q is fixed. Once all regulated voltages are within specified limits, power mismatches are calculated. If the power mismatches are less than specified precision, the process is finished. Otherwise, the process starts again from injection currents vector formulation.

In this example the process converges after seven iterations with with precision of 10^{-6}. Diagram showing network voltages through iterations is given in Figure 4.15. Final bus voltages are:

$$\begin{aligned} \underline{V}_3 &= 0.973942\angle - 3.08427^o \\ \underline{V}_4 &= 0.995289\angle - 2.38244^o \\ \underline{V}_5 &= 0.97\angle - 4.54884^o \\ \underline{V}_6 &= 0.938076\angle - 6.09361^o \\ \underline{V}_7 &= 0.975\angle - 4.22459^o \end{aligned}$$

.

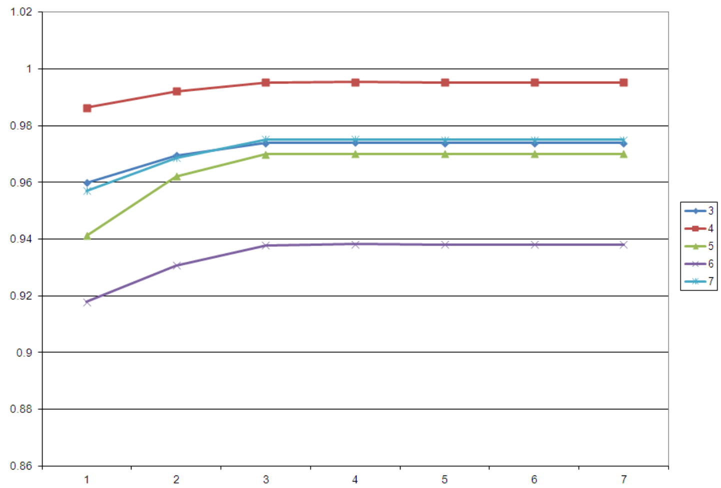

Fig. 4.15 Network voltages through iterations

4.6.5.2 Voltage control with LTC

At the begging of CI method, initial tap positions (if different than neutral) for all network transformers (regulated or not) are considered in $[Y]$ matrix formulation. Additional changes $\triangle T$ for LTCs obtained through the solution are reflected through $[I]$ vector in order to keep$[Y]$ matrix constant. It is also necessary to take these current sources into account when computing the actual power of the buses for power mismatch calculation.

4.6.5.3 LTC Modeling

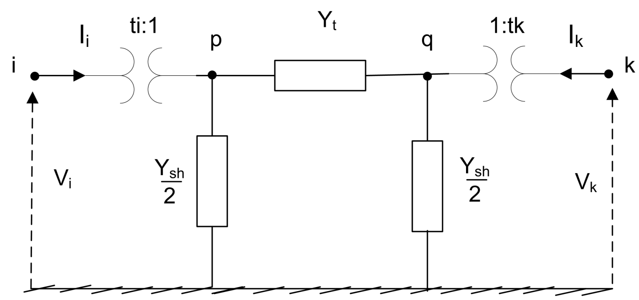

Fig. 4.16 LTC π model

It is already shown in Section 6.3 that for transformer in Figure 4.16 node voltage equation is:

$$\begin{bmatrix} \frac{\underline{Y}_{ii}}{t_i^2} & \frac{\underline{Y}_{ik}}{t_i t_k} \\ \frac{\underline{Y}_{ki}}{t_i t_k} & \frac{\underline{Y}_{kk}}{t_k^2} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} \tag{4.123}$$

Equation (4.123) can be written as:

$$\begin{bmatrix} \underline{Y}_{ii}^{'} & \underline{Y}_{ik}^{'} \\ \underline{Y}_{ki}^{'} & \underline{Y}_{kk}^{'} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} \tag{4.124}$$

Equation (4.123) shows the effects of tap positions on elements of transformer's admittance matrix. These effects are considered in the process of $[Y]$ matrix formulation in CI method. Additional tap changes, obtained during the solution in order to attain set voltages on regulated sides of LTCs, should be represented as current sources if $[Y]$ matrix is to remain constant. The effects of the change of tap position (on side i) for T_i are:

$$\begin{bmatrix} \frac{\underline{Y}_{ii}^{'}}{(1+T_i)^2} & \frac{\underline{Y}_{ik}^{'}}{1+T_i} \\ \frac{\underline{Y}_{ki}^{'}}{1+T_i} & \underline{Y}_{kk}^{'} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} \tag{4.125}$$

The aim is to get the equation with the admittance matrix identical to one in Equation (4.124), while the effects of tap position change would be reflected on the right side of the equation through additional current sources $[\triangle I]$:

$$\begin{aligned} \frac{\underline{Y}_{ii}^{'}}{(1+T_i)^2}\underline{V}_i + \frac{\underline{Y}_{ik}^{'}}{1+T_i}\underline{V}_k &= \underline{I}_i/(1+T_i)^2 \\ \frac{\underline{Y}_{ki}^{'}}{1+T_i}\underline{V}_i + \underline{Y}_{kk}^{'}V_k &= \underline{I}_k/(1+T_i) \end{aligned} \tag{4.126}$$

$$\begin{aligned} \underline{Y}_{ii}^{'}\underline{V}_i + \underline{Y}_{ik}^{'}(1+T_i)\underline{V}_k &= \underline{I}_i(1+T_i)^2 \\ \underline{Y}_{ki}^{'}\underline{V}_i + \underline{Y}_{kk}^{'}(1+T_i)\underline{V}_k &= \underline{I}_k(1+T_i) \end{aligned} \tag{4.127}$$

Equating the right sides of Equations (4.127) with the right side of the Equation (4.124) gives:

$$\begin{aligned} \underline{Y}_{ii}^{'}\underline{V}_i + \underline{Y}_{ik}^{'}\underline{V}_k &= \underline{I}_i + \underline{I}_i T_i(2+T) - \underline{Y}_{ik}^{'}T_i\underline{V}_k \\ \underline{Y}_{ki}^{'}\underline{V}_i + \underline{Y}_{kk}^{'}\underline{V}_k &= \underline{I}_k + T_i\underline{I}_k - \underline{Y}_{kk}^{'}T_i\underline{V}_k \end{aligned} \tag{4.128}$$

Leaving free members $\underline{I}_i$ and $\underline{I}_k$ and substituting those with free coefficients with expression from Equations (4.126) gives:

$$\underline{Y}_{ii}^{'}\underline{V}_i + \underline{Y}_{ik}^{'}\underline{V}_k = \underline{I}_i + \left[\frac{\underline{Y}_{ii}^{'}}{(1+T_i)^2}\underline{V}_i + \frac{\underline{Y}_{ik}^{'}}{1+T_i}\underline{V}_k\right] T_i(2+T_i) - \underline{Y}_{ik}^{'}T_i\underline{V}_k$$

$$\underline{Y}_{ki}^{'}\underline{V}_i + \underline{Y}_{kk}^{'}\underline{V}_k = \underline{I}_k + \left[\frac{\underline{Y}_{ki}^{'}}{1+T_i}\underline{V}_i + \underline{Y}_{kk}^{'}\underline{V}_k\right]T_i - \underline{Y}_{kk}^{'}T_i\underline{V}_k \tag{4.129}$$

$$\underline{Y}_{ii}^{'}\underline{V}_i + \underline{Y}_{ik}^{'}\underline{V}_k = \underline{I}_i +$$

$$+\left[\frac{\underline{Y}_{ii}^{'}}{(1+T_i)^2}\underline{V}_i + \frac{\underline{Y}_{ik}^{'}}{1+T_i}\underline{V}_k\right](1+T_i)^2 - \underline{Y}_{ik}^{'}T_i\underline{V}_k - \frac{\underline{Y}_{ii}^{'}}{(1+T_i)^2}\underline{V}_i - \frac{\underline{Y}_{ik}^{'}}{1+T_i}\underline{V}_k$$

$$\underline{Y}_{ki}^{'}\underline{V}_i + \underline{Y}_{kk}^{'}\underline{V}_k = \underline{I}_k + \frac{T_i}{1+T_i}\underline{Y}_{ki}^{'}\underline{V}_i + \underline{Y}_{kk}^{'}T_i\underline{V}_k - \underline{Y}_{kk}^{'}T_i\underline{V}_k \tag{4.130}$$

$$\underline{Y}_{ii}^{'}\underline{V}_i + \underline{Y}_{ik}^{'}\underline{V}_k = \underline{I}_i + \underline{Y}_{ii}^{'}\underline{V}_i + (1+T_i)\underline{Y}_{ik}^{'}\underline{V}_k - T_i\underline{Y}_{ik}^{'}\underline{V}_k - \frac{1}{(1+T_i)^2}\underline{Y}_{ii}^{'}\underline{V}_i - \frac{1}{1+T_i}\underline{Y}_{ik}^{'}\underline{V}_k$$

$$\underline{Y}_{ki}^{'}\underline{V}_i + \underline{Y}_{kk}^{'}\underline{V}_k = \underline{I}_k + \frac{T_i}{1+T_i}\underline{Y}_{ki}^{'}\underline{V}_i \tag{4.131}$$

Injection currents are:

$$\begin{bmatrix} \underline{Y}_{ii}^{'} & \underline{Y}_{ik}^{'} \\ \underline{Y}_{ki}^{'} & \underline{Y}_{kk}^{'} \end{bmatrix}\begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} + \begin{bmatrix} \underline{Y}_{ii}^{'}\underline{V}_i - \frac{1}{(1+T_i)^2}\underline{Y}_{ii}^{'}\underline{V}_i + \frac{T_i}{1+T_i}\underline{Y}_{ik}^{'}\underline{V}_k \\ \frac{T_i}{1+T_i}\underline{Y}_{ki}^{'}\underline{V}_i \end{bmatrix} \tag{4.132}$$

$$\begin{bmatrix} \underline{Y}_{ii}^{'} & \underline{Y}_{ik}^{'} \\ \underline{Y}_{ki}^{'} & \underline{Y}_{kk}^{'} \end{bmatrix}\begin{bmatrix} \underline{V}_i \\ \underline{V}_k \end{bmatrix} = \begin{bmatrix} \underline{I}_i \\ \underline{I}_k \end{bmatrix} + \begin{bmatrix} \triangle\underline{I}_i \\ \triangle\underline{I}_k \end{bmatrix} \tag{4.133}$$

Changes in tap position T_i are modeled through current sources on side i and side k as shown in Equation (4.132)

4.6.5.4 T-V dependencies

In order to obtain LTC voltages dependencies on T_i a transformer with tap side i and regulated side k will be considered. Voltage V_i will be assumed constant:

$$\frac{\partial V_i}{\partial T_i} = 0 \tag{4.134}$$

For the purpose of further analysis, all currents will be assumed zero except $\triangle I_i$ and $\triangle I_k$ which reflect tap change T_i .

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_{21} & Z_{22} & ... & Z_{2i} & ... & Z_{2j} & .. & Z_{2k} & ... & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{i1} & Z_{i2} & ... & Z_{ii} & ... & Z_{ij} & .. & Z_{ik} & ... & Z_{in} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{j1} & Z_{j2} & ... & Z_{ji} & ... & Z_{jj} & .. & Z_{jk} & ... & Z_{jn} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{k1} & Z_{k2} & ... & Z_{ki} & ... & Z_{kj} & .. & Z_{kk} & ... & Z_{kn} \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ \gamma_{n1} & Z_{n2} & ... & Z_{ni} & ... & Z_{nj} & .. & Z_{nk} & ... & Z_{nn} \end{bmatrix} \begin{bmatrix} V_{slack} \\ 0 \\ \vdots \\ \triangle I_i \\ \vdots \\ 0 \\ \vdots \\ \triangle I_k \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_j \\ \vdots \\ V_k \\ \vdots \\ V_n \end{bmatrix} \tag{4.135}$$

To estimate $\frac{\partial V_k}{\partial T_i}$ k row of the Equation ((4.135)) will be considered, adding constant current source I_k to $\triangle I_k$:

$$\underline{\gamma}_{k1}\underline{V}_{slack} + \underline{Z}_{ki}\triangle\underline{I}_i + \underline{Z}_{kk}\left(\triangle\underline{I}_k + \frac{P_k - jQ_k}{\underline{V}_k^*}\right) = \underline{V}_k \tag{4.136}$$

where

$$P_k = P_{k,generated} - P_{k,constant\,load}$$

$$Q_k = Q_{k,generated} - Q_{k,constant\,load}$$

Optimal CI method results were attained when constant current and constant impedance part of the load were included in $[Y]$ matrix. In sensitivity analysis these were considered through $[Z]$ matrix elements, while the constant power part is considered through P_k and Q_k in Equation (4.136). Substituting $\triangle\underline{I}_i$ and $\triangle\underline{I}_k$ in Equation (4.136) for the expressions obtained in Equation (4.132) gives:

$$\begin{aligned} \underline{V}_{\gamma k} + \underline{Z}_{ki}[\underline{Y}^{'}_{ii}\underline{V}_i - \frac{1}{(1+T_i)^2}\underline{Y}^{'}_{ii}\underline{V}_i + \frac{T_i}{1+T_i}\underline{Y}^{'}_{ik}\underline{V}_k] + \\ + \underline{Z}_{kk}[\frac{T_i}{1+T_i}\underline{Y}^{'}_{ki}\underline{V}_i + \frac{P_k - jQ_k}{\underline{V}_k^*}] = \underline{V}_k/\underline{V}_k^* \end{aligned} \tag{4.137}$$

$$\begin{aligned} \underline{V}_{\gamma k}\underline{V}_k^* + \underline{Z}_{ki}\underline{Y}^{'}_{ii}\underline{V}_i\underline{V}_k^* - \frac{1}{(1+T_i)^2}\underline{Z}_{ki}\underline{Y}^{'}_{ii}\underline{V}_i\underline{V}_k^* + \\ + \frac{T_i}{1+T_i}\underline{Z}_{ki}\underline{Y}^{'}_{ik}V_k^2 + \frac{T_i}{1+T_i}\underline{Z}_{kk}\underline{Y}^{'}_{ki}\underline{V}_i\underline{V}_k^* + \underline{Z}_{kk}(P_k - jQ_k) = V_k^2 \end{aligned} \tag{4.138}$$

$$\underline{g}_1 = \underline{Z}_{ki}\underline{Y}_{ii}$$

$$\underline{g}_2 = \underline{Z}_{kk}\underline{Y}_{ki}$$

$$\underline{g}_3 = \underline{Z}_{ki}\underline{Y}_{ik}$$

$$\underline{V}_{\gamma k}\underline{V}_k^* + \left[1 - \frac{1}{(1+T_i)^2}\right]\underline{g}_1\underline{V}_i\underline{V}_k^* + \frac{T_i}{1+T_i}\underline{g}_2\underline{V}_i\underline{V}_k^* +$$

$$+ \frac{T_i}{1+T_i}\underline{g}_3 V_k^2 + (R_{kk} + jX_{kk})(P_k - jQ_k) = V_k^2 \tag{4.139}$$

Separating real part of the Equation (4.139) gives:

$$V_{\gamma k}^{re}V_k^{re} + V_{\gamma k}^{im}V_k^{im} + \left[1 - \frac{1}{(1+T_i)^2}\right] g_1^{re}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im})$$

$$- \left[1 - \frac{1}{(1+T_i)^2}\right] g_1^{im}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im}) + \frac{T_i}{1+T_i} g_2^{re}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im})$$

$$- \frac{T_i}{1+T_i} g_2^{im}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im}) + \frac{T_i}{1+T_i} g_3^{re} V_k^2 + (R_{kk}P_k + X_{kk}Q_k) = V_k^2 / \frac{\partial}{\partial T_i} \tag{4.140}$$

Separating imaginary part of the Equation (4.139) gives:

$$V_{\gamma k}^{im}V_k^{re} - V_{\gamma k}^{re}V_k^{im} + \left[1 - \frac{1}{(1+T_i)^2}\right] g_1^{im}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im}) +$$

$$+ \left[1 - \frac{1}{(1+T_i)^2}\right] g_1^{re}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im}) + \frac{T_i}{1+T_i} g_2^{im}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im})$$

$$+ \frac{T_i}{1+T_i} g_2^{re}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im}) + \frac{T_i}{1+T_i} g_3^{im} V_k^2 + (X_{kk}P_k - R_{kk}Q_k) = 0 / \frac{\partial}{\partial T_i} \tag{4.141}$$

In order to get changes in V_k voltage magnitude depending on Ti, Equation (4.140) and Equation (4.141) will be derived by T_i. Equation (4.140) derivation by T_i:

$$V_{\gamma k}^{re}\frac{\partial V_k^{re}}{\partial T_i} + V_{\gamma k}^{im}\frac{\partial V_k^{im}}{\partial T_i} + \left[1 - \frac{1}{(1+T_i)^2}\right] g_1^{re}\left(V_i^{re}\frac{\partial V_k^{re}}{\partial T_i} + V_i^{im}\frac{\partial V_k^{im}}{\partial T_i}\right) +$$

$$+\frac{2}{(1+T_i)^3}g_1^{re}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})-\left[1-\frac{1}{(1+T_i)^2}\right]g_1^{im}(V_i^{im}\frac{\partial V_k^{re}}{\partial T_i}-V_i^{re}\frac{\partial V_k^{im}}{\partial T_i})$$

$$-\frac{2}{(1+T_i)^3}g_1^{im}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})$$

$$+\frac{T_i}{1+T_i}g_2^{re}(V_i^{re}\frac{\partial V_k^{re}}{\partial T_i}+V_i^{im}\frac{\partial V_k^{im}}{\partial T_i})+\frac{1}{(1+T_i)^2}g_2^{re}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})$$

$$-\frac{T_i}{1+T_i}g_2^{im}(V_i^{im}\frac{\partial V_k^{re}}{\partial T_i}-V_i^{re}\frac{\partial V_k^{im}}{\partial T_i})-\frac{1}{(1+T_i)^2}g_2^{im}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})$$

$$+\frac{1}{(1+T_i)^2}g_3^{re}V_k^2+\frac{T_i}{1+T_i}g_3^{re}(2V_k^{re}\frac{\partial V_k^{re}}{\partial T_i}+2V_k^{im}\frac{\partial V_k^{im}}{\partial T_i})=2V_k^{re}\frac{\partial V_k^{re}}{\partial T_i}+2V_k^{im}\frac{\partial V_k^{im}}{\partial T_i} \tag{4.142}$$

Equation (4.141) derivation by Ti:

$$V_{\gamma k}^{im}\frac{\partial V_k^{re}}{\partial T_i}-V_{\gamma k}^{re}\frac{\partial V_k^{im}}{\partial T_i}+\left[1-\frac{1}{(1+T_i)^2}\right]g_1^{im}(V_i^{re}\frac{\partial V_k^{re}}{\partial T_i}+V_i^{im}\frac{\partial V_k^{im}}{\partial T_i})+$$

$$+\frac{2}{(1+T_i)^3}g_1^{im}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})+\left[1-\frac{1}{(1+T_i)^2}\right]1_4^{re}(V_i^{im}\frac{\partial V_k^{re}}{\partial T_i}-V_i^{re}\frac{\partial V_k^{im}}{\partial T_i})$$

$$+\frac{2}{(1+T_i)^3}g_1^{re}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})$$

$$+\frac{T_i}{1+T_i}g_2^{im}(V_i^{re}\frac{\partial V_k^{re}}{\partial T_i}+V_i^{im}\frac{\partial V_k^{im}}{\partial T_i})+\frac{1}{(1+T_i)^2}g_2^{im}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})$$

$$+\frac{T_i}{1+T_i}g_2^{re}(V_i^{im}\frac{\partial V_k^{re}}{\partial T_i}-V_i^{re}\frac{\partial V_k^{im}}{\partial T_i})+\frac{1}{(1+T_i)^2}g_2^{re}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})+$$

$$+\frac{1}{(1+T_i)^2}g_3^{im}V_k^2+\frac{T_i}{1+T_i}g_3^{im}(2V_k^{re}\frac{\partial V_k^{re}}{\partial T_i}+2V_k^{im}\frac{\partial V_k^{im}}{\partial T_i})=0 \tag{4.143}$$

Grouping of constant coefficients in Equation (4.142) gives:

$$\frac{\partial V_k^{re}}{\partial T_i}\{V_{\gamma k}^{re}+\left[1-\frac{1}{(1+T_i)^2}\right]\left(g_1^{re}V_i^{re}-g_1^{im}V_i^{im}\right)+\frac{T_i}{1+T_i}\left(g_2^{re}V_i^{re}-g_2^{im}V_i^{im}\right)$$

$$+\frac{2T_i}{1+T_i}g_3^{re}V_k^{re}-2V_k^{re}\}+\frac{\partial V_k^{im}}{\partial T_i}\{V_{\gamma k}^{im}+\left[1-\frac{1}{(1+T_i)^2}\right]\left(g_1^{re}V_i^{im}+g_1^{im}V_i^{re}\right)+$$

$$+\frac{T_i}{1+T_i}\left(g_2^{re}V_i^{im}+g_2^{im}V_i^{re}\right)+\frac{2T_i}{1+T_i}g_3^{re}V_k^{im}-2V_k^{im}\}$$

$$=\frac{2}{(1+T_i)^3}\left[g_1^{im}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})-g_1^{re}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})\right]+$$

$$+\frac{1}{(1+T_i)^2}\left[g_2^{im}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})-g_2^{re}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})-g_3^{re}V_k^2\right] \tag{4.144}$$

Grouping of constant coefficients in Equation (4.143) gives:

$$\frac{\partial V_k^{re}}{\partial T_i}\{V_{\gamma k}^{im}+\left[1-\frac{1}{(1+T_i)^2}\right]\left(g_1^{im}V_i^{re}+g_1^{re}V_i^{im}\right)+\frac{T_i}{1+T_i}\left(g_2^{im}V_i^{re}+g_2^{re}V_i^{im}\right)+$$

$$+\frac{2T_i}{1+T_i}g_3^{im}V_k^{re}\}+\frac{\partial V_k^{im}}{\partial T_i}\{-V_{\gamma k}^{re}+\left[1-\frac{1}{(1+T_i)^2}\right]\left(g_1^{im}V_i^{im}-g_1^{re}V_i^{re}\right)+$$

$$+\frac{T_i}{1+T_i}\left(g_2^{im}V_i^{im}-g_2^{re}V_i^{re}\right)+\frac{2T_i}{1+T_i}g_3^{im}V_k^{im}\}$$

$$=-\frac{2}{(1+T_i)^3}\left[g_1^{im}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})+g_1^{re}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})\right]$$

$$-\frac{1}{(1+T_i)^2}\left[g_2^{im}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})+g_2^{re}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})+g_3^{im}V_k^2\right] \tag{4.145}$$

The result is a system of two equations with two unknown variables:

$$f_1\frac{\partial V_k^{re}}{\partial T_i}+f_2\frac{\partial V_k^{im}}{\partial T_i}=f_3 \tag{4.146}$$

$$f_4\frac{\partial V_k^{re}}{\partial T_i}+f_5\frac{\partial V_k^{im}}{\partial T_i}=f_6 \tag{4.147}$$

where:

$$f_1 = V_{\gamma k}^{re} + \left[1 - \frac{1}{(1+T_i)^2}\right]\left(g_1^{re}V_i^{re} - g_1^{im}V_i^{im}\right) +$$

$$+\frac{T_i}{1+T_i}\left(g_2^{re}V_i^{re} - g_2^{im}V_i^{im}\right) + \frac{2T_i}{1+T_i}g_3^{re}V_k^{re} - 2V_k^{re}$$

$$f_2 = V_{\gamma k}^{im} + \left[1 - \frac{1}{(1+T_i)^2}\right]\left(g_1^{re}V_i^{im} + g_1^{im}V_i^{re}\right) +$$

$$+\frac{T_i}{1+T_i}\left(g_2^{re}V_i^{im} + g_2^{im}V_i^{re}\right) + \frac{2T_i}{1+T_i}g_3^{re}V_k^{im} - 2V_k^{im}$$

$$f3 = \frac{2}{(1+T_i)^3}\left[g_1^{im}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im}) - g_1^{re}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im})\right] +$$

$$+\frac{1}{(1+T_i)^2}\left[g_2^{im}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im}) - g_2^{re}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im}) - g_3^{re}V_k^2\right]$$

$$f_4 = V_{\gamma k}^{im} + \left[1 - \frac{1}{(1+T_i)^2}\right]\left(g_1^{im}V_i^{re} + g_1^{re}V_i^{im}\right) +$$

$$+\frac{T_i}{1+T_i}\left(g_2^{im}V_i^{re} + g_2^{re}V_i^{im}\right) + \frac{2T_i}{1+T_i}g_3^{im}V_k^{re}$$

$$f_5 = -V_{\gamma k}^{re} + \left[1 - \frac{1}{(1+T_i)^2}\right]\left(g_1^{im}V_i^{im} - g_1^{re}V_i^{re}\right) +$$

$$+\frac{T_i}{1+T_i}\left(g_2^{im}V_i^{im} - g_2^{re}V_i^{re}\right) + \frac{2T_i}{1+T_i}g_3^{im}V_k^{im}$$

$$f_6 = -\frac{2}{(1+T_i)^3}\left[g_1^{im}(V_i^{re}V_k^{re} + V_i^{im}V_k^{im}) + g_1^{re}(V_i^{im}V_k^{re} - V_i^{re}V_k^{im})\right]$$

$$-\frac{1}{(1+T_i)^2}\left[g_2^{im}(V_i^{re}V_k^{re}+V_i^{im}V_k^{im})+g_2^{re}(V_i^{im}V_k^{re}-V_i^{re}V_k^{im})+g_6^{im}V_k^2\right]$$

Applying Cramer's rule on system of Equation (4.146) and Equation (4.147) gives:

$$\frac{\partial V_k^{re}}{\partial T_i}=\frac{f_3f_5-f_2f_6}{f_1f_5-f_2f_4} \tag{4.148}$$

$$\frac{\partial V_k^{im}}{\partial T_i}=\frac{f_1f_6-f_3f_4}{f_1f_5-f_2f_4} \tag{4.149}$$

$\frac{\partial V_k}{\partial T_i}$ will be calculated using the Equation ((4.99)). In order to estimate $\frac{\partial V_j}{\partial T_i}$ (j is some arbitrary regulated bus of the network), jth row of the Equation ((4.135)) will be considered. Note that $\frac{\partial V_i}{\partial T_i}=0$:

$$\underline{\gamma}_{j1}\underline{V}_{slack}+\underline{Z}_{ji}\triangle\underline{I}_i+\underline{Z}_{jk}\triangle\underline{I}_k=\underline{V}_j \tag{4.150}$$

$$\underline{V}_{\gamma j}+\underline{Z}_{ji}[\underline{Y}'_{ii}\underline{V}_i-\frac{1}{(1+T_i)^2}\underline{Y}'_{ii}\underline{V}_i+\frac{T_i}{1+T_i}\underline{Y}'_{ik}\underline{V}_k]+\underline{Z}_{jk}[\frac{T_i}{1+T_i}\underline{Y}'_{ki}\underline{V}_i]=\underline{V}_j \tag{4.151}$$

$$\underline{g}_4=\underline{Z}_{ji}\underline{Y}_{ii}$$

$$\underline{g}_5=\underline{Z}_{jk}\underline{Y}_{ki}$$

$$\underline{g}_6=\underline{Z}_{ji}\underline{Y}_{ik}$$

$$\underline{V}_{\gamma j}+\underline{g}_4\underline{V}_i-\underline{g}_1\frac{1}{(1+T_i)^2}\underline{V}_i+\underline{g}_5\frac{T_i}{1+T_i}\underline{V}_i+\underline{g}_6\frac{T_i}{1+T_i}\underline{V}_k=\underline{V}_j \tag{4.152}$$

$$V_{\gamma j}^{re}+g_4^{re}V_i^{re}-g_4^{im}V_i^{im}-\frac{1}{(1+T_i)^2}(g_4^{re}V_i^{re}-g_4^{im}V_i^{im})+\frac{T_i}{1+T_i}(g_5^{re}V_i^{re}-g_5^{im}V_i^{im})+$$

$$+\frac{T_i}{1+T_i}(g_6^{re}V_k^{re}-g_6^{im}V_k^{im})=V_j^{re}/\frac{\partial}{\partial T_i} \tag{4.153}$$

$$V_{\gamma j}^{im}+g_4^{re}V_i^{im}+g_4^{im}V_i^{re}-\frac{1}{(1+T_i)^2}(g_4^{re}V_i^{im}+g_4^{im}V_i^{re})+\frac{T_i}{1+T_i}(g_5^{re}V_i^{im}+g_5^{im}V_i^{re})+$$

$$+\frac{T_i}{1+T_i}(g_6^{re}V_k^{im}+g_6^{im}V_k^{re})=V_j^{im}/\frac{\partial}{\partial T_i} \tag{4.154}$$

From here the expression for $\frac{\partial V_j^{re}}{\partial T_i}$ is:

$$\frac{\partial V_j^{re}}{\partial T_i}=\frac{2}{(1+T_i)^3}(g_4^{re}V_i^{re}-g_4^{im}V_i^{im})+\frac{1}{(1+T_i)^2}(g_5^{re}V_i^{re}-g_5^{im}V_i^{im})+$$

$$+\frac{1}{(1+T_i)^2}(g_6^{re}V_k^{re}-g_6^{im}V_k^{im})+\frac{T_i}{1+T_i}(g_6^{re}\frac{\partial V_k^{re}}{\partial T_i}-g_6^{im}\frac{\partial V_k^{im}}{\partial T_i}) \tag{4.155}$$

The expression for $\frac{\partial V_j^{im}}{\partial T_i}$ is:

$$\frac{\partial V_j^{im}}{\partial T_i}=\frac{2}{(1+T_i)^3}(g_4^{re}V_i^{im}+g_4^{im}V_i^{re})+\frac{1}{(1+T_i)^2}(g_5^{re}V_i^{im}+g_5^{im}V_i^{re})+$$

$$+\frac{1}{(1+T_i)^2}(g_6^{re}V_k^{im}+g_6^{im}V_k^{re})+\frac{T_i}{1+T_i}(g_6^{re}\frac{\partial V_k^{im}}{\partial T_i}+g_6^{im}\frac{\partial V_k^{re}}{\partial T_i}) \tag{4.156}$$

$\frac{\partial V_i}{\partial T_i}$ and $\frac{\partial V_j}{\partial T_i}$ should also be obtained in case when side i is regulated and tap is placed on i side too. It will be assumed that $\frac{\partial V_k}{\partial T_i}$=0. Now i-th row of the Equation ((4.135)) will be considered, adding constant current source $\underline{I}_i$ to $\triangle\underline{I}_i$:

$$\underline{\gamma}_{i1}\underline{V}_{slack}+\underline{Z}_{ii}\left(\triangle\underline{I}_i+\frac{P_i-jQ_i}{\underline{V}_i^*}\right)+\underline{Z}_{ik}\triangle\underline{I}_k=\underline{V}_i \tag{4.157}$$

The rest of the process is the same as before. Substituting $\triangle\underline{I}_i$ and $\triangle\underline{I}_k$ in Equation (4.157) for the expressions obtained in Equation (4.132) gives:

$$\underline{V}_{\gamma i}+\underline{Z}_{ii}[\underline{Y}_{ii}^{'}\underline{V}_i-\frac{1}{(1+T_i)^2}\underline{Y}_{ii}^{'}\underline{V}_i+\frac{T_i}{1+T_i}\underline{Y}_{ik}^{'}\underline{V}_k+\frac{P_i-jQ_i}{\underline{V}_i^*}]+$$

$$+\underline{Z}_{ik}\frac{T_i}{1+T_i}\underline{Y}_{ki}^{'}\underline{V}_i=\underline{V}_i/\underline{V}_i^* \tag{4.158}$$

$$\underline{g}_7=\underline{Z}_{ii}\underline{Y}_{ii}$$

$$\underline{g}_8=\underline{Z}_{ik}\underline{Y}_{ki}$$

$$\underline{g}_9 = \underline{Z}_{ii}\underline{Y}_{ik}$$

$$\underline{V}_{\gamma i}\underline{V}_i^* + \underline{g}_7\left[1 - \frac{1}{(1+T_i)^2}\right]V_i^2 + \underline{g}_8\frac{T_i}{1+T_i}V_i^2$$
$$+\underline{g}_9\frac{T_i}{1+T_i}\underline{V}_k\underline{V}_i^* + (R_{ii}+jX_{ii})(P_i - jQ_i) = V_i^2 \tag{4.159}$$

Real part of Equation (4.159) is:

$$V_{\gamma i}^{re}V_i^{re} + V_{\gamma i}^{im}V_i^{im} + g_7^{re}\left[1 - \frac{1}{(1+T_i)^2}\right]V_i^2 + g_8^{re}\frac{T_i}{1+T_i}V_i^2 + (R_{ii}P_i + X_{ii}Q_i)$$
$$+\frac{T_i}{1+T_i}g_9^{re}(V_k^{re}V_i^{re} + V_k^{im}V_i^{im}) - \frac{T_i}{1+T_i}g_9^{im}(V_k^{im}V_i^{re} - V_k^{re}V_i^{im}) = V_i^2/\frac{\partial}{\partial T_i} \tag{4.160}$$

Imaginary part of Equation (4.159) is:

$$V_{\gamma i}^{im}V_i^{re} - V_{\gamma i}^{re}V_i^{im} + g_7^{im}\left[1 - \frac{1}{(1+T_i)^2}\right]V_i^2 + g_8^{im}\frac{T_i}{1+T_i}V_i^2 + (X_{ii}P_i - R_{ii}Q_i)$$
$$+\frac{T_i}{1+T_i}g_9^{im}(V_k^{re}V_i^{re} + V_k^{im}V_i^{im}) + \frac{T_i}{1+T_i}g_9^{re}(V_k^{im}V_i^{re} - V_k^{re}V_i^{im}) = 0/\frac{\partial}{\partial T_i} \tag{4.161}$$

Derivation of Equation (4.160) gives:

$$V_{\gamma i}^{re}\frac{\partial V_i^{re}}{\partial T_i} + V_{\gamma i}^{im}\frac{\partial V_i^{im}}{\partial T_i} + g_7^{re}\left[1 - \frac{1}{(1+T_i)^2}\right](2V_i^{re}\frac{\partial V_i^{re}}{\partial T_i} + 2V_i^{im}\frac{\partial V_i^{im}}{\partial T_i})+$$

$$+g_7^{re}\frac{2}{(1+T_i)^3}V_i^2 + g_8^{re}\frac{T_i}{1+T_i}(2V_i^{re}\frac{\partial V_i^{re}}{\partial T_i} + 2V_i^{im}\frac{\partial V_i^{im}}{\partial T_i}) + g_8^{re}\frac{1}{(1+T_i)^2}V_i^2 +$$

$$+\frac{T_i}{1+T_i}g_9^{re}(V_k^{re}\frac{\partial V_i^{re}}{\partial T_i} + V_k^{im}\frac{\partial V_i^{im}}{\partial T_i}) + \frac{1}{(1+T_i)^2}g_9^{re}(V_k^{re}V_i^{re} + V_k^{im}V_i^{im})$$

$$-\frac{T_i}{1+T_i}g_9^{im}(V_k^{im}\frac{\partial V_i^{re}}{\partial T_i} - V_k^{re}\frac{\partial V_i^{im}}{\partial T_i}) - \frac{1}{(1+T_i)^2}g_9^{im}(V_k^{im}V_i^{re} - V_k^{re}V_i^{im}) =$$

$$= 2V_i^{re}\frac{\partial V_i^{re}}{\partial T_i} + 2V_i^{im}\frac{\partial V_i^{im}}{\partial T_i} \tag{4.162}$$

Derivation of Equation (4.161) gives:

$$V_{\gamma i}^{im}\frac{\partial V_i^{re}}{\partial T_i} - V_{\gamma i}^{re}\frac{\partial V_i^{im}}{\partial T_i} + g_7^{im}\left[1 - \frac{1}{(1+T_i)^2}\right](2V_i^{re}\frac{\partial V_i^{re}}{\partial T_i} + 2V_i^{im}\frac{\partial V_i^{im}}{\partial T_i})$$

$$+g_7^{im}\frac{2}{(1+T_i)^3}V_i^2 + g_8^{im}\frac{T_i}{1+T_i}(2V_i^{re}\frac{\partial V_i^{re}}{\partial T_i} + 2V_i^{im}\frac{\partial V_i^{im}}{\partial T_i}) + g_8^{im}\frac{1}{(1+T_i)^2}V_i^2$$

$$+\frac{T_i}{1+T_i}g_9^{im}(V_k^{re}\frac{\partial V_i^{re}}{\partial T_i} + V_k^{im}\frac{\partial V_i^{im}}{\partial T_i}) + \frac{1}{(1+T_i)^2}g_9^{im}(V_k^{re}V_i^{re} + V_k^{im}V_i^{im}) +$$

$$+\frac{T_i}{1+T_i}g_9^{re}(V_k^{im}\frac{\partial V_i^{re}}{\partial T_i} - V_k^{re}\frac{\partial V_i^{im}}{\partial T_i})$$

$$+\frac{1}{(1+T_i)^2}g_9^{re}(V_k^{im}V_i^{re} - V_k^{re}V_i^{im}) = 0 \tag{4.163}$$

Grouping of constant coefficients results in the system of two equations with two unknown variables:

$$\frac{\partial V_i^{re}}{\partial T_i}\{V_{\gamma i}^{re} + 2V_i^{re}g_7^{re}\left[1 - \frac{1}{(1+T_i)^2}\right] + 2V_i^{re}g_8^{re}\frac{T_i}{1+T_i} + g_9^{re}V_k^{re}\frac{T_i}{1+T_i} - g_9^{im}V_k^{im}\frac{T_i}{1+T_i} - 2V_i^{re}\}$$

$$\frac{\partial V_i^{im}}{\partial T_i}\{V_{\gamma i}^{im} + 2V_i^{im}g_7^{re}\left[1 - \frac{1}{(1+T_i)^2}\right] + 2V_i^{im}g_8^{re}\frac{T_i}{1+T_i} + g_9^{re}V_k^{im}\frac{T_i}{1+T_i} + g_9^{im}V_k^{re}\frac{T_i}{1+T_i} - 2V_i^{im}\}$$

$$= -g_7^{re}\frac{2}{(1+T_i)^3}V_i^2 - g_8^{re}\frac{1}{(1+T_i)^2}V_i^2 +$$

$$+\frac{1}{(1+T_i)^2}g_9^{im}(V_k^{im}V_i^{re} - V_k^{re}V_i^{im}) - \frac{1}{(1+T_i)^2}g_9^{re}(V_k^{re}V_i^{re} + V_k^{im}V_i^{im}) \tag{4.164}$$

$$\frac{\partial V_i^{re}}{\partial T_i}\{V_{\gamma i}^{im} + 2V_i^{re}g_7^{im}\left[1 - \frac{1}{(1+T_i)^2}\right] + 2V_i^{re}g_8^{im}\frac{T_i}{1+T_i} + g_9^{im}V_k^{re}\frac{T_i}{1+T_i} + g_9^{re}V_k^{im}\frac{T_i}{1+T_i}\}$$

$$\frac{\partial V_i^{im}}{\partial T_i}\{-V_{\gamma i}^{re} + 2V_i^{im}g_7^{im}\left[1 - \frac{1}{(1+T_i)^2}\right] + 2V_i^{im}g_8^{im}\frac{T_i}{1+T_i} + g_9^{im}V_k^{im}\frac{T_i}{1+T_i} - g_9^{re}V_k^{re}\frac{T_i}{1+T_i}\}$$

$$= -g_7^{im}\frac{2}{(1+T_i)^3}V_i^2 - g_8^{im}\frac{1}{(1+T_i)^2}V_i^2 +$$

$$-g_9^{re}\frac{1}{(1+T_i)^2}(V_k^{im}V_i^{re}-V_k^{re}V_i^{im})-g_9^{im}\frac{1}{(1+T_i)^2}(V_k^{re}V_i^{re}+V_k^{im}V_i^{im}) \quad (4.165)$$

$$f_7\frac{\partial V_i^{re}}{\partial T_i}+f_8\frac{\partial V_i^{im}}{\partial T_i}=f_9 \quad (4.166)$$

$$f_{10}\frac{\partial V_i^{re}}{\partial T_i}+f_{11}\frac{\partial V_i^{im}}{\partial T_i}=f_{12} \quad (4.167)$$

The solution of the system:

$$\frac{\partial V_i^{re}}{\partial T_i}=\frac{f_9f_{11}-f_8f_{12}}{f_7f_{11}-f_8f_{10}} \quad (4.168)$$

$$\frac{\partial V_i^{im}}{\partial T_i}=\frac{f_7f_{12}-f_9f_{10}}{f_7f_{11}-f_8f_{10}} \quad (4.169)$$

where:

$$f_7=V_{\gamma i}^{re}+2V_i^{re}g_7^{re}\left[1-\frac{1}{(1+T_i)^2}\right]+2V_i^{re}g_8^{re}\frac{T_i}{1+T_i}+$$

$$+g_9^{re}V_k^{re}\frac{T_i}{1+T_i}-g_9^{im}V_k^{im}\frac{T_i}{1+T_i}-2V_i^{re}$$

$$f_8=V_{\gamma i}^{im}+2V_i^{im}g_7^{re}\left[1-\frac{1}{(1+T_i)^2}\right]+2V_i^{im}g_8^{re}\frac{T_i}{1+T_i}+$$

$$+g_9^{re}V_k^{im}\frac{T_i}{1+T_i}+g_9^{im}V_k^{re}\frac{T_i}{1+T_i}-2V_i^{im}$$

$$f_9=-g_7^{re}\frac{2}{(1+T_i)^3}V_i^2-g_8^{re}\frac{1}{(1+T_i)^2}V_i^2+$$

$$+\frac{1}{(1+T_i)^2}g_9^{im}(V_k^{im}V_i^{re}-V_k^{re}V_i^{im})-\frac{1}{(1+T_i)^2}g_9^{re}(V_k^{re}V_i^{re}+V_k^{im}V_i^{im})$$

$$f_{10}=V_{\gamma i}^{im}+2V_i^{re}g_7^{im}\left[1-\frac{1}{(1+T_i)^2}\right]+2V_i^{re}g_8^{im}\frac{T_i}{1+T_i}+$$

$$+g_9^{im}V_k^{re}\frac{T_i}{1+T_i}+g_9^{re}V_k^{im}\frac{T_i}{1+T_i}$$

$$f_{11}=-V_{\gamma i}^{re}+2V_i^{im}g_7^{im}\left[1-\frac{1}{(1+T_i)^2}\right]+2V_i^{im}g_8^{im}\frac{T_i}{1+T_i}+$$

$$+g_9^{im}V_k^{im}\frac{T_i}{1+T_i}-g_9^{re}V_k^{re}\frac{T_i}{1+T_i}$$

$$f_{12} = -g_7^{im}\frac{2}{(1+T_i)^3}V_i^2 - g_8^{im}\frac{1}{(1+T_i)^2}V_i^2 +$$

$$-g_9^{re}\frac{1}{(1+T_i)^2}(V_k^{im}V_i^{re} - V_k^{re}V_i^{im}) - g_9^{im}\frac{1}{(1+T_i)^2}(V_k^{re}V_i^{re} + V_k^{im}V_i^{im})$$

$\frac{\partial V_i}{\partial T_i}$ will be calculated using the Equation (4.99).

In order to estimate $\frac{\partial V_j}{\partial T_i}$ in case that i is tap and regulated side at the same time (j is some arbitrary bus of the network) all the currents except $\triangle \underline{I}_i$ and $\triangle \underline{I}_k$ are assumed zero in Equation (4.135). Voltage $\underline{V}_j$ is equal to:

$$\underline{\gamma}_{j1}\underline{V}_{slack} + \underline{Z}_{ji}\triangle \underline{I}_i + \underline{Z}_{jk}\triangle \underline{I}_k = \underline{V}_j \tag{4.170}$$

$$\underline{V}_{\gamma j} + \underline{Z}_{ji}[\underline{Y}'_{ii}\underline{V}_i - \frac{1}{(1+T_i)^2}\underline{Y}'_{ii}\underline{V}_i + \frac{T_i}{1+T_i}\underline{Y}'_{ik}\underline{V}_k] + \underline{Z}_{jk}[\frac{T_i}{1+T_i}\underline{Y}'_{ki}\underline{V}_i] = \underline{V}_j \tag{4.171}$$

$$\underline{g}_{10} = \underline{Z}_{ji}\underline{Y}_{ii}$$

$$\underline{g}_{11} = \underline{Z}_{jk}\underline{Y}_{ki}$$

$$\underline{g}_{12} = \underline{Z}_{ji}\underline{Y}_{ik}$$

$$\underline{V}_{\gamma j} + \underline{g}_{10}\underline{V}_i - \underline{g}_{10}\frac{1}{(1+T_i)^2}\underline{V}_i + \underline{g}_{11}\frac{T_i}{1+T_i}\underline{V}_i + \underline{g}_{12}\frac{T_i}{1+T_i}\underline{V}_k = \underline{V}_j \tag{4.172}$$

Real part of Equation (4.172) is:

$$V_{\gamma j}^{re} + g_{10}^{re}V_i^{re} - g_{10}^{im}V_i^{im} - \frac{1}{(1+T_i)^2}(g_{10}^{re}V_i^{re} - g_{10}^{im}V_i^{im}) + \frac{T_i}{1+T_i}(g_{11}^{re}V_i^{re} - g_{11}^{im}V_i^{im}) +$$

$$+\frac{T_i}{1+T_i}(g_{12}^{re}V_k^{re} - g_{12}^{im}V_k^{im}) = V_j^{re}/\frac{\partial}{\partial T_i} \tag{4.173}$$

Imaginary part:

$$V_{\gamma j}^{im} + g_{10}^{re}V_i^{im} + g_{10}^{im}V_i^{re} - \frac{1}{(1+T_i)^2}(g_{10}^{re}V_i^{im} + g_{10}^{im}V_i^{re}) + \frac{T_i}{1+T_i}(g_{11}^{re}V_i^{im} + g_{11}^{im}V_i^{re}) +$$

$$+\frac{T_i}{1+T_i}(g_{12}^{re}V_k^{im} + g_{12}^{im}V_k^{re}) = V_j^{im}/\frac{\partial}{\partial T_i} \tag{4.174}$$

From here expressions for $\frac{\partial V_j^{re}}{\partial T_i}$ and $\frac{\partial V_j^{im}}{\partial T_i}$ are:

$$\frac{\partial V_j^{re}}{\partial T_i} = \left[1 - \frac{1}{(1+T_i)^2}\right](g_{10}^{re}\frac{\partial V_i^{re}}{\partial T_i} - g_{10}^{im}\frac{\partial V_i^{im}}{\partial T_i}) + \frac{2}{(1+T_i)^3}(g_{10}^{re}V_i^{re} - g_{10}^{im}V_i^{im}) +$$

$$+\frac{T_i}{1+T_i}(g_{11}^{re}\frac{\partial V_i^{re}}{\partial T_i} - g_{11}^{im}\frac{\partial V_i^{im}}{\partial T_i}) + \frac{1}{(1+T_i)^2}(g_{11}^{re}V_i^{re} - g_{11}^{im}V_i^{im}) +$$

$$+\frac{1}{(1+T_i)^2}(g_{12}^{re}V_k^{re} - g_{12}^{im}V_k^{im}) \tag{4.175}$$

$$\frac{\partial V_j^{im}}{\partial T_i} = \left[1 - \frac{1}{(1+T_i)^2}\right](g_{10}^{re}\frac{\partial V_i^{im}}{\partial T_i} + g_{10}^{im}\frac{\partial V_i^{re}}{\partial T_i}) + \frac{2}{(1+T_i)^3}(g_{10}^{re}V_i^{im} + g_{10}^{im}V_i^{re}) +$$

$$+\frac{T_i}{1+T_i}(g_{11}^{re}\frac{\partial V_i^{im}}{\partial T_i} + g_{11}^{im}\frac{\partial V_i^{re}}{\partial T_i}) + \frac{1}{(1+T_i)^2}(g_{11}^{re}V_i^{im} + g_{11}^{im}V_i^{re}) +$$

$$+\frac{1}{(1+T_i)^2}(g_{12}^{re}V_k^{im} + g_{12}^{im}V_k^{re}) \tag{4.176}$$

Example

Use CI method to obtain bus voltages for power system shown in Figure 4.17 with power mismatch precision of 10^{-6} and regulated voltage precision of 10^{-4}. Given is the following data in per-unit:

Bus 1: $\underline{V}_1 = 1\angle 0^o$
Bus 2: $P_2 = 0, \quad Q_2 = 0$
Bus 3: $P_3 = 0.5, \quad Q_3 = 0.2$
Bus 4: $P_4 = 0.5, \quad Q_4 = 0.2$
Line 1-2: $R = 0.003, \quad X = 0.09$
Line 3-4: $R = 0.003, \quad X = 0.09$
Transformer: $R = 0.1, \quad X = 0.3$, $V_{min} = 0.99995, \quad V_{max} = 1.00005$, step size 0.5, initial tap positions on both sides are neutral, $t_{from} = 1$, $t_{to} = 1$.

Obtain the solution in two cases:
a) regulated side of the transformer is side connected to Bus 3, tap changer is placed on the side connected to Bus 2.
b) regulated side of the transformer is side connected to Bus 3, tap changer is placed on that side.

Solution

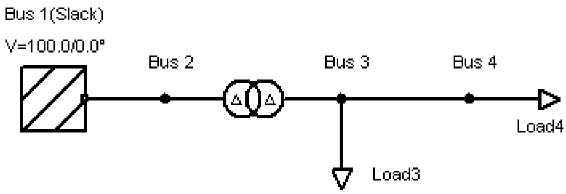

Fig. 4.17 Network with voltage regulating transformer

a)Constant Z and constant I parts of the loads will be included in **Y** matrix. Constant Z part and constant I part of the load connected to bus i is added in a form of shunt admittance to the corresponding diagonal element of the **Y** matrix:

$$\underline{Y}_{shunt,i} = \frac{P_{n,i}(k_{p1,i}V_n + k_{p2,i}{V_n}^2) - jQ_{n,i}(k_{q1,i}V_n + k_{q2,i}{V_n}^2)}{V_n^2}$$

Since both loads have constant power, in both cases $\underline{Y}_{shunt,i} = 0$. Transformer admittance is:

$$\underline{Y}_t = \frac{1}{\underline{Z}_t} = \frac{1}{0.1 + j0.3} = 1 - j3 \tag{4.177}$$

Transformer shunt admittance is neglected, $\underline{Y}_{t,shunt} = 0$. Transformer π model admittance matrix is:

$$\mathbf{Y_t} = \begin{bmatrix} \underline{Y}'_{22} & \underline{Y}'_{23} \\ \underline{Y}'_{32} & \underline{Y}'_{33} \end{bmatrix} = \begin{bmatrix} \frac{\underline{Y}_t + \underline{Y}_{t,shunt}}{t_{from} * t_{from}} & -\frac{\underline{Y}_t}{t_{from} * t_{to}} \\ -\frac{\underline{Y}_t}{t_{from} * t_{to}} & \frac{\underline{Y}_t + \underline{Y}_{t,shunt}}{t_{to} * t_{to}} \end{bmatrix} = \begin{bmatrix} 1 - j3 & -1 + j3 \\ -1 + j3 & 1 - j3 \end{bmatrix}$$

Transformer initial tap position $T_2^0 = 0$. Network bus admittance matrix **Y** is:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -0.37 + j11.0988 & 1.37 - j14.0988 & -1 + j3 & 0 \\ 0 & -1 + j3 & 1.37 - j14.0988 & -0.37 + j11.0988 \\ 0 & 0 & -0.37 + j11.0988 & 0.37 - j11.0988 \end{bmatrix}$$

Initial voltage estimates are equal to the voltage of the slack bus:

$$\underline{V}_i = 1.0 + j0, \quad i = 2, 3, 4$$

Injection current for load connected at bus i is:

$$\underline{I}^1_{load,i} = -\left(\frac{(\underline{S}^0_{load,i})^*}{(V^0_i)^2} - \underline{Y}_{shunt,i}\right)\underline{V}^0_i = -\frac{(\underline{S}^0_{load,i})^*}{(V^0_i)^*}$$

Injection current for transformer tap bus, Bus 2 is obtained as:

$$\underline{I}^1_{t,2} = \underline{Y}'_{22}\underline{V}^0_2 - \frac{1}{(1+T^0_2)^2}\underline{Y}'_{22}\underline{V}^0_2 + \frac{T^0_2}{1+T^0_2}\underline{Y}'_{23}\underline{V}^0_3 = 0$$

Injection current for transformer regulated bus, Bus 3 is obtained as:

$$\underline{I}^1_{t,3} = \frac{T^0_2}{1+T^0_2}\underline{Y}'_{32}\underline{V}^0_2 = 0$$

Vector of injection currents is:

$$\mathbf{I}^1 = \begin{bmatrix} 1.0 \\ 0 \\ -0.5 + j0.2 \\ -0.5 + j0.2 \end{bmatrix}$$

Vector of total bus currents is:

$$\mathbf{I}^1_T = \begin{bmatrix} 1.0 \\ 0 \\ -0.5 + j0.2 \\ -0.5 + j0.2 \end{bmatrix}$$

Voltages in the first iteration are:

$$\mathbf{V}^1 = \begin{bmatrix} 1.0 \\ 0.961 - j0.0888 \\ 0.741 - j0.3488 \\ 0.7215 - j0.3932 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.9651\angle - 5.2793^o \\ 0.819\angle - 25.2071^o \\ 0.8217\angle - 28.5893^o \end{bmatrix}$$

Now for regulated bus, Bus 3 it is checked whether the voltage magnitude is within the set limits of voltage precision:

$$V^1_3 > V_{min,3} \; and \; V^1_3 < V_{max,3}$$

Since these conditions are not satisfied, sensitivity analysis is conducted. Voltage mismatch is:

$$\triangle V = \left|\frac{V_{min,3} + V_{max,3}}{2} - V^1_3\right| = 0.181$$

Voltage mismatch is:

$$\triangle V = 0.181$$

Since the regulated voltage is outside the precision limits, sensitivity analysis is conducted. Sensitivity matrix dimensions are 1x1:

$$S = \frac{\partial V_3}{\partial T_2}$$

$\frac{\partial V_3^{re}}{\partial T_2}$ is obtained using Equation (4.148):

$$\frac{\partial V_3^{re}}{\partial T_2} = -1.9223$$

$\frac{\partial V_3^{im}}{\partial T_2}$ is obtained using Equation (4.149):

$$\frac{\partial V_3^{im}}{\partial T_2} = -0.202$$

From here:

$$\frac{\partial V_3}{\partial T_2} = \frac{V_3^{re}\frac{\partial V_3^{re}}{\partial T_2} + V_3^{im}\frac{\partial V_3^{im}}{\partial T_2}}{V_3} = -1.6532$$

Sensitivity equation is:

$$S \cdot \triangle T = \triangle V$$

$$-1.6532 \cdot \triangle T = 0.181$$

$$\triangle T = \frac{\triangle V}{S} = -0.1905$$

$$T_2^1 = T_2^0 + \triangle T_2 \quad = -\,0.1905$$

Since no limits for tap position is specified in this example, the process continues. New vector of injection currents is formulated, new voltages are obtained and voltage magnitudes for regulated bus is checked. When voltage magnitude mismatch on regulated bus is less than specified power mismatches are calculated. If the power mismatches are less than specified precision, the process is finished. Otherwise, the process starts again from injection currents vector formulation. At the end of iteration process transformer final tap position is obtained and rounded:

$$t_2 = \frac{T_2 \cdot 100}{stepSize} \approx -15$$

Final bus voltages are:

$$\underline{V}_2 = 0.9484\angle - 5.4304^o$$
$$\underline{V}_3 = 1.0022\angle - 6.86657^o$$
$$\underline{V}_4 = 0.9813\angle - 9.45399^o$$

b)Same as in previous case both loads have constant power and in both cases $\underline{Y}_{shunt,i} = 0$. Transformer admittance is the same $\underline{Y}_t = 1 - j3$ and transformer shunt admittance is neglected. Transformer π model admittance matrix $\mathbf{Y}_t$ and network bus admittance matrix $\mathbf{Y}$ are also the same. Transformer initial tap position $T_3^0 = 0$. Initial voltage estimates are equal to the voltage of the slack bus:

$$\underline{V}_i = 1.0 + j0, \quad i = 2, 3, 4$$

Injection current for loads are obtained in the same way. Injection current for non regulated transformer bus, Bus 2 is obtained as:

$$\underline{I}^1_{t,2} = \frac{T_3^0}{1 + T_3^0}\underline{Y}'_{23}\underline{V}_3^0 = 0$$

Injection current for transformer regulated bus, Bus 3 is obtained as:

$$\underline{I}^1_{t,3} = \underline{Y}'_{33}\underline{V}_3^0 - \frac{1}{(1 + T_3^0)^2}\underline{Y}'_{33}\underline{V}_3^0 + \frac{T_3^0}{1 + T_3^0}\underline{Y}'_{32}\underline{V}_2^0 = 0$$

Vector of injection currents is equal to vector of total bus currents:

$$\mathbf{I}^1 = \begin{bmatrix} 1.0 \\ 0 \\ -0.5 + j0.2 \\ -0.5 + j0.2 \end{bmatrix}$$

Voltages in the first iteration are:

$$\mathbf{V}^1 = \begin{bmatrix} 1.0 \\ 0.961 - j0.0888 \\ 0.741 - j0.3488 \\ 0.7215 - j0.3932 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.9651\angle - 5.2793^o \\ 0.819\angle - 25.2071^o \\ 0.8217\angle - 28.5893^o \end{bmatrix}$$

Now for regulated bus, Bus 3 it is checked whether the voltage magnitude is within the set limits of voltage precision:

$$V_3^1 > V_{min,3} \; and \; V_3^1 < V_{max,3}$$

Since these conditions are not satisfied, sensitivity analysis is conducted.Voltage mismatch is:

$$\triangle V = \left| \frac{V_{min,3} + V_{max,3}}{2} - V_3^1 \right| = 0.181$$

Since the regulated voltage is outside the precision limits, sensitivity analysis is conducted. Sensitivity matrix dimensions are 1x1:

$$S = \frac{\partial V_3}{\partial T_3}$$

$\frac{\partial V_3^{re}}{\partial T_3}$ is obtained using Equation (4.168):

$$\frac{\partial V_3^{re}}{\partial T_3} = 1.4575$$

$\frac{\partial V_3^{im}}{\partial T_3}$ is obtained using Equation (4.169):

$$\frac{\partial V_3^{im}}{\partial T_3} = -0.3368 \tag{4.178}$$

From here:

$$\frac{\partial V_3}{\partial T_2} = \frac{V_3^{re}\frac{\partial V_3^{re}}{\partial T_3} + V_3^{im}\frac{\partial V_3^{im}}{\partial T_3}}{V_3} = 1.4621$$

Sensitivity equation is:

$$S \cdot \triangle T = \triangle V$$

$$1.4621 \cdot \triangle T = 0.181$$

$$\triangle T = \frac{\triangle V}{S} = 0.1238$$

$$T_3^1 = T_3^0 + \triangle T_3 \quad = 0.1238$$

And then the procedure is the same as in previous case. Final bus voltages are:

$$\begin{aligned} \underline{V}_2 &= 0.9484\angle - 5.431^o \\ \underline{V}_3 &= 0.9988\angle - 6.8768^o \\ \underline{V}_4 &= 0.9778\angle - 9.4825^o \end{aligned}$$

Transformer final tap position is:

$$t_3 = 8$$

4.6.6 PF Algorithm with Local Control

Figure 4.18 shows block diagram for CI method with local voltage controls algorithm. First steps are basic CI method steps: $[Y]$ matrix formation and factorization, voltage vector and iteration counter initialization. Controlled bus set is also defined. Then in each iteration injection currents are calculated and network voltages are obtained from $[I]$ vector of injection currents and constant $[Y]$ bus admittance matrix using forward/backward method.

Conditions for LTCs are checked and if not satisfied for any regulated LTC bus, sensitivity analysis is conducted. In case that conditions for LTCs are satisfied, voltages of PV buses are checked. If the set voltage for any of PV buses is not attained, again sensitivity analysis is conducted. When the voltages of all regulated buses (PV and LTC) are within specified limits, tap positions are rounded and sensitivity analysis is conducted once more for PV buses only to check whether some of the conditions were violated due to tap positions rounding. If not, power mismatches are checked.

In each iteration it is necessary to check whether Q_i^n and $t_i^n = t_{initial} + T_i^n$ are within the set limits. Conditions:

$$\begin{aligned} Q_{imin} < Q_i^n < Q_{imax} \\ t_{imin} < t_i^n < t_{imax} \end{aligned} \tag{4.179}$$

have to be fulfilled for each PV bus or LTC. If these conditions are violated, Q_i^n and $T_i^n$are set to relevant limits, bus i is no longer regulated bus and no longer is the subject of sensitivity analysis. CI power flow with local voltage controls algorithm is consisted of the following steps:

Step 1: Formulate and factorize $[Y]$ matrix.
Step 2: Make initial estimate of bus voltages: $\underline{V}_i = \underline{V}_{slack}$, set iteration constant $n = 0$, set LTCSensitivity flag ON.
Step 3: Define LTC and PV control set.
Step 4: Increase iteration number: $n = n + 1$. Form bus current vector $[I^n]$. In case that bus i is PQ buses use Equation (4.65) in order to calculate $\underline{I}_i^n$. For PV bus use Equation (4.80). For buses of regulation transformers additional current sources must be taken into account. Obtain $[V^n]$ using forward/backward.
Step 5: Check if each regulation transformer voltage is within set limits.
If yes, go to Step 11; else go to Step 6.
Step 6: Set LTCSensitivity flag ON.
Step 7: Build sensitivity matrix and calculate voltage mismatches for LTCs and PV buses.
Step 8: Obtain $[dQ^n]$ and $[dT^n]$ using forward/backward. Calculate Q^n and T^n using Equation (4.79).
Step 9: Check if Q_i^n and T_i^n satisfy conditions given in (4.179).
If yes, go to Step 4; else go to Step 7.
Step 10: Set Q_i^n and T_i^n to relevant limits and remove bus i from control set. Go to Step 11.

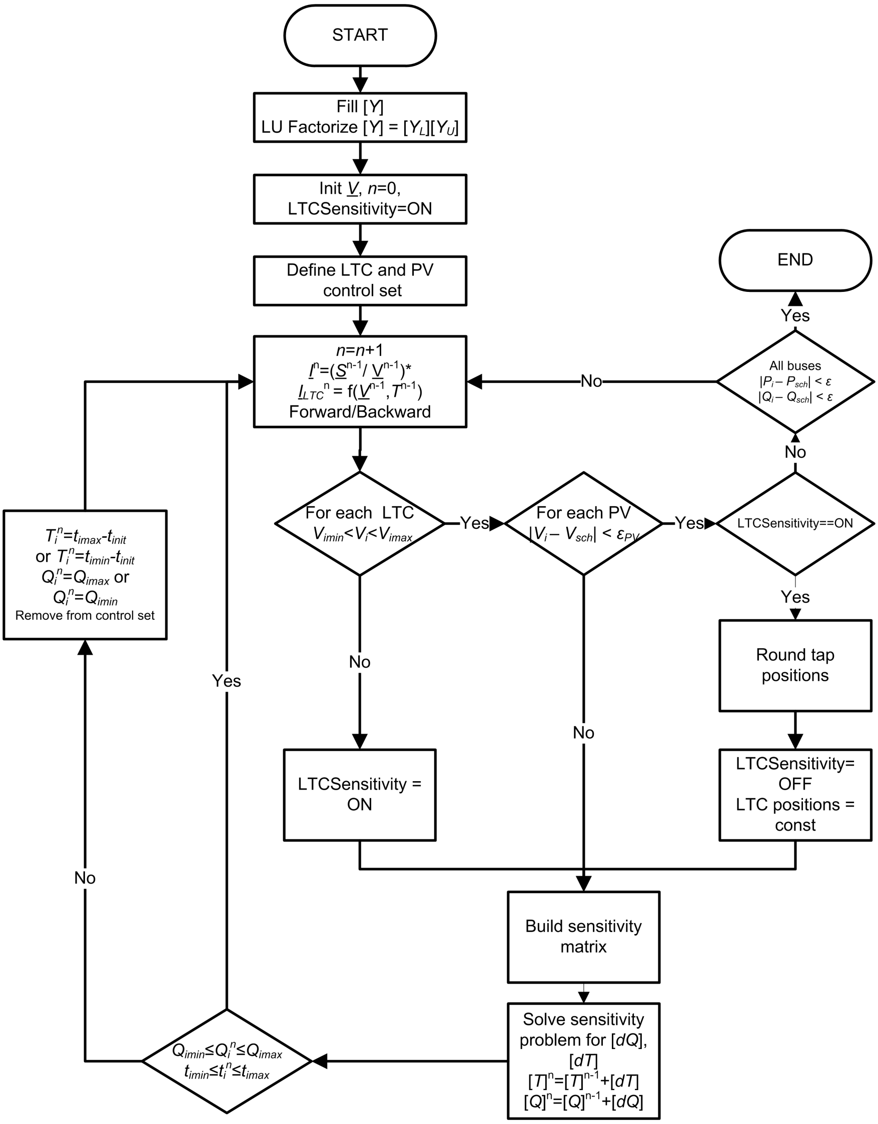

Fig. 4.18 CI Algorithm

Step 11: For each PV bus check if $|V_i - V_{sch}| < \epsilon_{PV}$.
If yes, go to Step 12; else go to Step 7.
Step 12: Check the status of LTCSensitivity flag.
If LTCSensitivity is ON, go to Step 13; else go to Step 15.
Step 13: For each LTC round tap position.
Step 14: Set LTCSensitivity flag OFF and all LTC positions constant. Go to Step 7.
Step 15: For each bus check if $|P_i - P_{sch}| < \epsilon$ and $|Q_i - Q_{sch}| < \epsilon$.
If yes, stop; else go to Step 4.
Complete CI method with local voltage controls is considered on the example of the network shown in Figure (4.12) with the following data in per-unit:

Bus 1: $\underline{V}_1 = 1\angle 0^o$
Bus 2: $\underline{V}_2 = 1\angle 0^o$
Bus 3: $P_3 = 0.4 + 0.3V_3 + 0.3V_3^2, \quad Q_3 = 0.4 + 0.3V_3 + 0.3V_3^2$
Bus 4: $P_4 = 0.4 + 0.3V_4 + 0.3V_4^2, \quad Q_4 = 0.4 + 0.3V_4 + 0.3V_4^2$
Bus 5: $P_5 = 0.1, \quad V_5 = 0.97$
Bus 6: $P_6 = 0.1, \quad V_6 = 0.975$
Bus 8: $P_8 = 0.2, \quad Q_8 = 0.05$
Bus 9: $P_9 = 0.12, \quad Q_9 = 0.04$
Bus 11: $P_{11} = 0.2, \quad Q_{11} = 0.05$
Bus 12: $P_{12} = 0.1, \quad Q_{12} = 0.03$
Bus 13: $P_{13} = 0.12, \quad Q_{13} = 0.03$
Bus 14: $P_{14} = 0.1, \quad Q_{14} = 0.04$
Line 1-3: $R = 0.013, \quad X = 0.039$
Line 2-4:$R = 0.007, \quad X = 0.021$
Line 3-4:$R = 0.01, \quad X = 0.03$
Line 3-5:$R = 0.008, \quad X = 0.024$
Line 4-8:$R = 0.001, \quad X = 0.03$
Line 6-8:$R = 0.01, \quad X = 0.03$
Line 5-7:$R = 0.01, \quad X = 0.03$
Line 6-7:$R = 0.01, \quad X = 0.03$
Line 9-10:$R = 0.001, \quad X = 0.05$
Line 10-11:$R = 0.001, \quad X = 0.03$
Line 10-12:$R = 0.001, \quad X = 0.03$
Line 13-14:$R = 0.001, \quad X = 0.03$

Transformer 7-9: $R = 0.05, \quad X = 0.15$, initial tap position is neutral, tap side 7, regulated side 9, $V_{min} = 0.99765, \quad V_{max} = 1.01635$

Transformer 10-13: $R = 0.07, \quad X = 0.21$, initial tap position is neutral, tap side 10, regulated side 13, $V_{min} = 1.0028, \quad V_{max} = 1.0112$

Figure 4.19 shows network voltages through iterations. The solution converges after twelve iterations and final network voltages are:

$$\underline{V}_3 = 0.96355\angle -1.95264^o$$
$$\underline{V}_4 = 0.968096\angle -1.59231^o$$
$$\underline{V}_5 = 0.97\angle -2.52981^o$$
$$\underline{V}_6 = 0.975\angle -2.7899^o$$
$$\underline{V}_7 = 0.964587\angle -3.18364^o$$
$$\underline{V}_8 = 0.971551\angle -2.3253^o$$
$$\underline{V}_9 = 1.00735\angle -7.72216^o$$
$$\underline{V}_{10} = 0.998325\angle -9.20503^o$$
$$\underline{V}_{11} = 0.996601\angle -9.54768^o$$
$$\underline{V}_{12} = 0.997318\angle -9.37594^o$$
$$\underline{V}_{13} = 1.00676\angle -11.4698^o$$
$$\underline{V}_{14} = 1.00547\angle -11.6373^o$$

Tap position for transformer 7-9 is -11 and for transformer 10-13 tap position is -9.

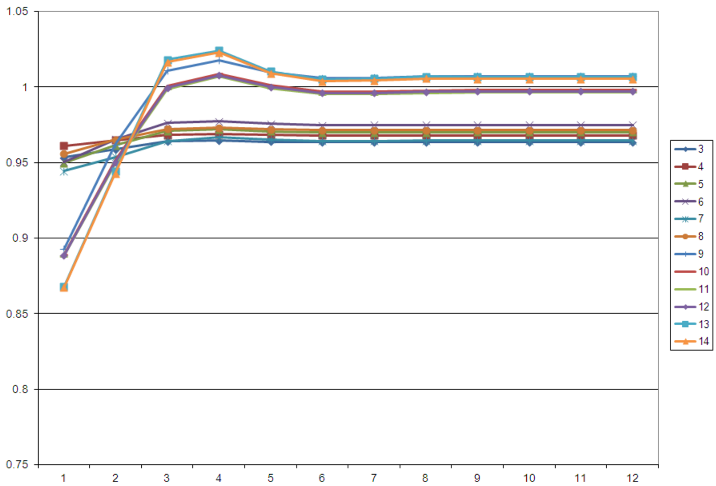

Fig. 4.19 Bus voltages for the network shown in Figure 4.12

4.7 Line Losses

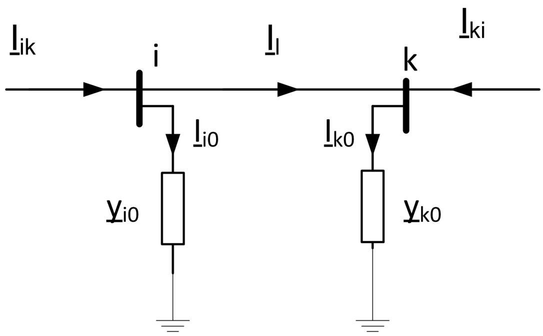

Fig. 4.20 Line connecting bus i and bus k

After calculating bus voltages, line flows and losses can be obtained. Consider the line shown in Figure 4.20. The current $\underline{I}_{ik}$ is defined positive in the direction $i \to k$ and the current $\underline{I}_{ki}$ is defined positive in the direction $k \to i$. These currents are:

$$\underline{I}_{ik} = \underline{I}_l + \underline{I}_{i0} = \underline{y}_{i0}\underline{V}_i + \underline{y}_{ik}(\underline{V}_i - \underline{V}_k) \tag{4.180}$$

$$\underline{I}_{ki} = -\underline{I}_l + \underline{I}_{k0} = \underline{y}_{k0}\underline{V}_i + \underline{y}_{ik}(\underline{V}_k - \underline{V}_i) \tag{4.181}$$

Complex power flow from bus i to bus k is:

$$\underline{S}_{ik} = \underline{V}_i\underline{I}_{ik}^* \tag{4.182}$$

Complex power flow from bus k to bus i is:

$$\underline{S}_{ki} = \underline{V}_k\underline{I}_{ki}^* \tag{4.183}$$

The power loss in line i-k is equal:

$$\underline{S}_{Lik} = \underline{S}_{ik} + \underline{S}_{ki} \tag{4.184}$$

Power loss in line must be expressed in MW and MVAr.

Chapter 5
Faults

Faults occur in power systems and cause damaging electrodynamic and thermal stresses. When fault occurs it is the task of switching devices to isolate the fault in the shortest period of time to prevent any serious damage. The main characteristic of switching devices is that they operate under fault conditions without suffering any damage. Fault analysis includes calculation of short-circuit currents, rating structure of switching devices, effects of short-circuit currents, limitation of short-circuit currents, etc. During the fault in most cases the impedances of static elements (lines and transformers) are assumed to be time invariant. Impedances of synchronous and inductive motors cannot be considered time invariant. The reactance of the generator under short-circuit conditions is a time-varying quantity. There are three reactances defined: the sub-transient reactance (first few cycles of the fault current), the transient reactance (approximately next 30 cycles of the fault current) and the synchronous reactance (thereafter). Which reactance to use, depends on the scope of calculation. For the purpose of analyzing the nature of fault current an ideal sinusoidal voltage source is assumed to be connected to a line of impedance $\underline{Z}_L$. If the fault occurs on the line terminal and the voltage source impedance is neglected, differential equation is:

$$L\frac{di}{dz} + Ri = Esin(\omega t + \theta) \tag{5.1}$$

θ is the angle of voltage at which the fault occurred. Solving the equation gives:

$$i = Isin(\omega t + \theta - \phi) - Isin(\theta - \phi)e^{-\frac{t}{T}}, \quad T = \frac{L}{R}, \quad I = \frac{E}{Z_L}, \quad tan\theta = \frac{\omega L}{R} \tag{5.2}$$

Obviously, there are two components of a short circuit current. One of them is the non-decaying AC component, and the other is the decaying DC component. T is termed the time constant. The transient current is reduced to 36,8% of its initial value after an elapsed time equal to T, and below 1% of its initial value after $5T$. If a short circuit occurs when $\theta = 0$ (when the voltage wave is crossing through zero amplitude), then the instantaneous value of short-circuit current is $2I$. If a short circuit occurs when $\theta = \pi/2$ (when the voltage wave peaks), then the DC component of short-circuit current is zero and there is no transient component.

Explanation for this is the fact that the current of an inductive circuit cannot change suddenly. Thus, when the fault occurs at the instant when the voltage is at peak, the current is at zero value and no DC component current is needed to satisfy the mentioned condition. However, when the fault occurs at instant when $\theta = 0$, the current is at peak and there has to be a transient current.

In this chapter, calculation of currents and voltages during balanced and unbalanced faults is represented [72–78]. Generally, fault represents a structural network change equivalent to that caused by the addition of impedance at the faulted bus. A bolted fault means that fault impedance $\underline{Z}_f$ is zero. Balanced fault is defined as the simultaneous short-circuit across all three phases. Bolted three-phase fault generally gives the maximum short-circuit currents, and although it is very uncommon in real life, it is the main parameter for setting switching devices. During the three-phase fault, network remains balanced and can be analyzed using per-phase model.

Faults involving one, or two phases and possibly ground are called unbalanced faults. There are two types of unbalanced faults: shunt and series faults. Shunt fault occurs when one or two phases and ground are connected and can be:

- line-to-ground fault,
- line-to-line fault,
- double line-to-ground fault.

Series fault occurs when one or two phases are opened due to storm or fuses, isolators and circuit breakers operation. Series fault can be:

- one conductor open
- two conductors open

Under certain conditions, the line-to-ground fault or double line-to ground fault can result in currents that are even higher than three-phase symmetrical fault currents. Unbalanced faults are analyzed using the symmetrical components. Shunt fault analysis will be conducted ignoring all static load currents as they do not contribute to short-circuit currents.

5.1 Shunt faults analysis

5.1.1 Balanced Fault

Balanced fault is a three phase short-circuit, and when it occurs, it results in a balanced network conditions with identical magnitudes of phase currents and voltages, with phase shift of 120^o. In that case, the network can be solved on a per-phase basis. One of the methods for solving faulted network is Thevenin's method. The first step of the procedure is to obtain the pre-fault bus voltages using power flow method. If the fault analysis is conducted for some empirical calculations, the load currents can be ignored because the short-circuit currents are much higher. When this is not the case, the load currents must be considered as constant admittances obtained for pre-fault bus voltages:

$$\underline{Y}_{load,i} = \frac{S^*_{load,i}}{|V_i(0)|^2} \tag{5.3}$$

The next step is to obtain bus voltages of Thevenin's circuit. In order to obtain bus voltages during the fault, these values are then added to the corresponding pre-fault bus voltages. Then all fault network currents are calculated. Since Thevenin's method is not the best choice for solving the large networks because of its inefficiency, nodal method is usually applied. Nodal method will be demonstrated on the example of balanced power system with N buses in case when three phase fault occurs at bus i through fault impedance $\underline{Z}_f$.

Network lines are represented by their equivalent π model and all impedances are expressed in per unit. Pre-fault bus voltages are obtained using power flow analysis and are represent as voltage vector $\mathbf{V(0)}$. In order to obtain Thevenin's circuit, bus loads are represented by constant impedances, voltage source $V_i(0)$ (pre-fault voltage at faulted bus i) is added to bus i and all other voltage sources are short-circuited as shown in Figure 5.1.

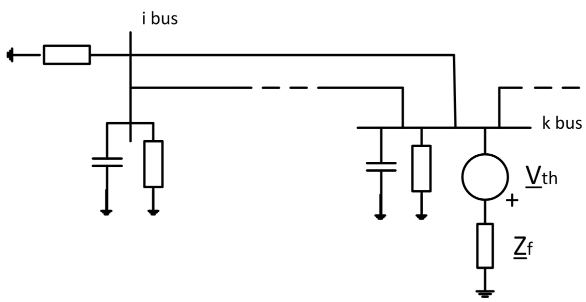

Fig. 5.1 Thevenin's circuit for the n-bus power system with the fault at bus k

As already said, Thevenin's theorem states that bus voltages during the fault are obtained by superposition of the pre-fault bus voltages and the changes in the bus voltages caused by the fault $\triangle\mathbf{V}$:

$$\mathbf{V(F)} = \mathbf{V(0)} + \triangle\mathbf{V} \tag{5.4}$$

The nodal equation applied to the Thevenin's circuit shown in Figure 5.1 is:

$$\begin{bmatrix} 0 \\ \vdots \\ -\underline{I}_i(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{Y}_{11} & \cdots & \underline{Y}_{1i} & \cdots & \underline{Y}_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{Y}_{i1} & \cdots & \underline{Y}_{ii} & \cdots & \underline{Y}_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{Y}_{N1} & \cdots & \underline{Y}_{Ni} & \cdots & \underline{Y}_{NN} \end{bmatrix} \begin{bmatrix} \triangle\underline{V}_1 \\ \vdots \\ \triangle\underline{V}_i \\ \vdots \\ \triangle\underline{V}_N \end{bmatrix} \tag{5.5}$$

Negative sign of current at bus i denotes that the current is flowing from the bus to the ground. Equation (5.5) can be written in a lumped form as:

$$\mathbf{I(F)} = \mathbf{Y}\triangle\mathbf{V} \Rightarrow \triangle\mathbf{V} = \mathbf{Y}^{-1}\mathbf{I(F)} = \mathbf{ZI(F)} \tag{5.6}$$

$$\mathbf{V(F)} = \mathbf{V(0)} + \mathbf{ZI(F)} \tag{5.7}$$

From (5.7), the i-th equation is:

$$\underline{V}_i(F) = \underline{V}_i(0) + \underline{Z}_{ii}\underline{I}_i(F) \tag{5.8}$$

$\underline{Z}_{ii}$ is obtained from network impedance matrix. From Figure 5.1:

$$\underline{V}_i(F) = \underline{Z}_f\underline{I}_i(F) \tag{5.9}$$

Substituting $\underline{V}_i(F)$ in Equation (5.8) for Equation (5.9) results in fault current:

$$\underline{I}_i(F) = \frac{\underline{V}_i(0)}{\underline{Z}_{ii} + \underline{Z}_f} \tag{5.10}$$

Diagonal element $\underline{Z}_{ii}$ is equal to the Thevenin impedance to the faulted bus. From (5.7) for bus some arbitrary network bus j fault voltage is:

$$\underline{V}_j(F) = \underline{V}_j(0) - \underline{Z}_{ji}\underline{I}_i(F) = \underline{V}_j(0) - \underline{Z}_{ji} \cdot \frac{\underline{V}_i(0)}{\underline{Z}_{ii} + \underline{Z}_f} \tag{5.11}$$

Fault current in the line between buses j and k is:

$$\underline{I}_{jk}(F) = \frac{\underline{V}_j(F) - \underline{V}_k(F)}{\underline{z}_{jk}} \tag{5.12}$$

where $\underline{z}_{jk}$ is the line impedance.

5.1.2 Line-to-ground Fault

Line-to-ground fault is an unbalanced fault that occurs when one of the phases is grounded through fault impedance $\underline{Z}_f$. It is the most common type of a fault in power system with 70%. Calculation of network currents during the line-to-ground fault is demonstrated on a grounded generator, with load current neglected. It is assumed that single line-to-ground fault occurs at phase a (Figure (5.2)). If the neutral was isolated, fault current would be zero because it would not have a return path.

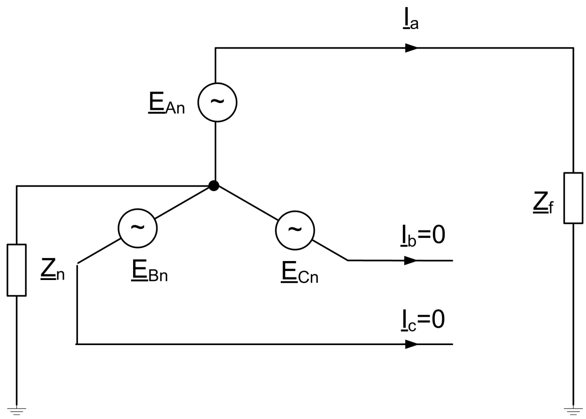

Fig. 5.2 Single-line-to ground fault

Currents in phase b and phase c are zero and the voltage of phase a at the fault point is:

$$\underline{V}_a = \underline{Z}_f \underline{I}_a \tag{5.13}$$

The sequence components of the current in phase a are equal, which is obvious from:

$$\begin{bmatrix} \underline{I}_a^0 \\ \underline{I}_a^1 \\ \underline{I}_a^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{I}_a \\ 0 \\ 0 \end{bmatrix} \tag{5.14}$$

$$\underline{I}_a^0 = \underline{I}_a^1 = \underline{I}_a^2 = \frac{1}{3} \underline{I}_a \tag{5.15}$$

In terms of symmetrical components $\underline{V}_a$ is written as:

$$\underline{V}_a = \underline{V}_a^0 + \underline{V}_a^1 + \underline{V}_a^2 \tag{5.16}$$

From (2.43) given in symmetrical components chapter:

$$\begin{aligned}\underline{V}_a^0 &= 0 - \underline{Z}_0\underline{I}_a^0\\ \underline{V}_a^1 &= \underline{E}_{An} - \underline{Z}_1\underline{I}_a^1\\ \underline{V}_a^2 &= 0 - \underline{Z}_2\underline{I}_a^2\end{aligned}$$

Now:

$$\underline{Z}_f(\underline{I}_a^0 + \underline{I}_a^1 + \underline{I}_a^2) = -\underline{Z}_0\underline{I}_a^0 + (\underline{E}_{An} - \underline{Z}_1\underline{I}_a^1) - \underline{Z}_2\underline{I}_a^2 \tag{5.17}$$

$$3\underline{Z}_f\underline{I}_a^0 = \underline{E}_{An} - (\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2)\underline{I}_a^0 \tag{5.18}$$

From here:

$$\underline{I}_a^0 = \frac{\underline{E}_{An}}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\underline{Z}_f} \Rightarrow \underline{I}_a = 3\underline{I}_a^0 = \frac{3\underline{E}_{An}}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\underline{Z}_f} \tag{5.19}$$

This shows that the equivalent fault circuit can be constructed as a series connection of the positive, negative and zero sequence, along with the $3\underline{Z}_f$ impedance. The voltage of phase b to ground is:

$$\underline{V}_b = \underline{V}_a^0 + \underline{a}^2\underline{V}_a^1 + \underline{a}\underline{V}_a^2 = \frac{\underline{E}_{An}}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\underline{Z}_f}\left[\underline{Z}_0(\underline{a}^2 - 1) + \underline{Z}_2(\underline{a}^2 - \underline{a}) + 3\underline{a}^2\underline{Z}_f\right] \tag{5.20}$$

$\underline{V}_c$ is calculated similarly:

$$\underline{V}_c = \underline{V}_a^0 + \underline{a}^2\underline{V}_a^1 + \underline{a}\underline{V}_a^2 = \frac{\underline{E}_{An}}{\underline{Z}_0 + \underline{Z}_1 + \underline{Z}_2 + 3\underline{Z}_f}\left[\underline{Z}_0(\underline{a} - 1) + \underline{Z}_2(\underline{a} - \underline{a}^2) + 3\underline{a}\underline{Z}_f\right] \tag{5.21}$$

In order to obtain the solution for the unbalanced fault at bus i in N bus network, bus impedance matrix for each sequence network is obtained separately, then the sequence impedances $\underline{Z}_{ii}^0$, $\underline{Z}_{ii}^1$ and $\underline{Z}_{ii}^2$ are used to obtain fault currents. In case of single line-to-ground fault at bus i, symmetrical components of fault current are:

$$\underline{I}_{i,a}^0 = \underline{I}_{i,a}^1 = \underline{I}_{i,a}^2 = \frac{\underline{V}_i(0)}{\underline{Z}_{ii}^0 + \underline{Z}_{ii}^1 + \underline{Z}_{ii}^2 + 3\underline{Z}_f} \tag{5.22}$$

$\underline{V}_i(0)$ is the pre-fault voltage at bus i, where the fault occurred. The fault phase currents at bus i are then obtained using the equation:

$$\begin{bmatrix}\underline{I}_{i,a}\\ \underline{I}_{i,b}\\ \underline{I}_{i,c}\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & \underline{a}^2 & \underline{a}\\ 1 & \underline{a} & \underline{a}^2\end{bmatrix}\begin{bmatrix}\underline{I}_{i,a}^0\\ \underline{I}_{i,a}^1\\ \underline{I}_{i,a}^2\end{bmatrix} \tag{5.23}$$

Symmetrical components of some arbitrary bus j voltage during the fault at bus i are obtained as:

$$\begin{aligned}\underline{V}_{j,a}^{0} =& 0 - \underline{Z}_{ji}^{0}\underline{I}_{i,a}^{0}\\ \underline{V}_{j,a}^{1} =& \underline{V}_{j,a}^{1}(0) - \underline{Z}_{ji}^{1}\underline{I}_{i,a}^{1}\\ \underline{V}_{j,a}^{2} =& 0 - \underline{Z}_{ji}^{2}\underline{I}_{i,a}^{2}\end{aligned} \tag{5.24}$$

Phase voltages are:

$$\begin{bmatrix}\underline{V}_{j,a}\\ \underline{V}_{j,b}\\ \underline{V}_{j,c}\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & \underline{a}^2 & \underline{a}\\ 1 & \underline{a} & \underline{a}^2\end{bmatrix}\begin{bmatrix}\underline{V}_{j,a}^{0}\\ \underline{V}_{j,a}^{1}\\ \underline{V}_{j,a}^{2}\end{bmatrix} \tag{5.25}$$

5.1.3 Line-to-line Fault

Line-to-line fault is an unbalanced fault that occurs when two phases are connected through fault impedance $\underline{Z}_f$ (Figure (5.3)).

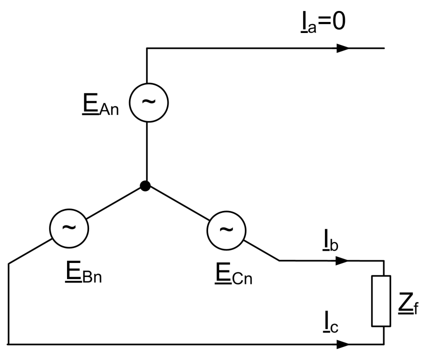

Fig. 5.3 Line-to-line fault

Generator with load current neglected and connected phases are b and c will be considered. Fault current circulates between phases b and c, flowing back to source through phase b and returning through phase c. It is obvious that:

$$I_a = 0, \quad I_b = -I_c \tag{5.26}$$

Symmetrical components are:

$$\begin{bmatrix} \underline{I}_a^0 \\ \underline{I}_a^1 \\ \underline{I}_a^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} 0 \\ -\underline{I}_c \\ \underline{I}_c \end{bmatrix} \tag{5.27}$$

From here:

$$\begin{aligned} \underline{I}_a^0 &= 0 \\ \underline{I}_a^1 &= -\underline{I}_a^2 = \frac{1}{3}\left(a^2 - a\right)\underline{I}_c \end{aligned} \tag{5.28}$$

Using (5.28) the following can be written:

$$\begin{bmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} 0 \\ \underline{I}_a^1 \\ -\underline{I}_a^1 \end{bmatrix} \Rightarrow \underline{I}_b = -\underline{I}_c = (\underline{a}^2 - \underline{a})\underline{I}_a^1 \tag{5.29}$$

Also,

$$\begin{aligned} \underline{V}_b - \underline{V}_c &= \underline{I}_b \underline{Z}_f \\ &= (\underline{a}^2 - \underline{a})\underline{I}_a^1 \underline{Z}_f \\ &= (\underline{a}^2 - \underline{a})(\underline{V}_a^1 - \underline{V}_a^2) \\ &= (\underline{a}^2 - \underline{a})[\underline{E}_{An} - (\underline{Z}_1 + \underline{Z}_2)\underline{I}_a^1] \end{aligned} \tag{5.30}$$

From here:

$$\underline{E}_{An} - (\underline{Z}_1 + \underline{Z}_2)\underline{I}_a^1 = \frac{(\underline{a}^2 - \underline{a})\underline{I}_a^1 \underline{Z}_f}{\underline{a}^2 - \underline{a}} \Rightarrow \underline{I}_a^1 = \frac{\underline{E}_{An}}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_f} \tag{5.31}$$

The fault currents are:

$$\underline{I}_b = -\underline{I}_c = (\underline{a}^2 - \underline{a})\underline{I}_a^1 = -\frac{j\sqrt{3}\underline{E}_{An}}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_f} \tag{5.32}$$

This equation can be represented by connecting positive and negative sequence networks in opposite. Using bus impedance matrix symmetrical current components in case of a fault at network bus i can be obtained as:

$$I_{i,a}^0 = 0 \tag{5.33}$$

$$I_{i,a}^1 = -I_{i,a}^2 = \frac{V_i(0)}{\underline{Z}_{ii}^1 + \underline{Z}_{ii}^2 + \underline{Z}_f} \tag{5.34}$$

The phase currents and voltages are obtained using Equation (5.23), Equation (5.24) and Equation (5.25).

5.1.4 Double Line-to-Ground Fault

Double line-to-ground fault is an unbalanced fault that occurs when two phases are connected to ground through a fault impedance $\underline{Z}_f$.

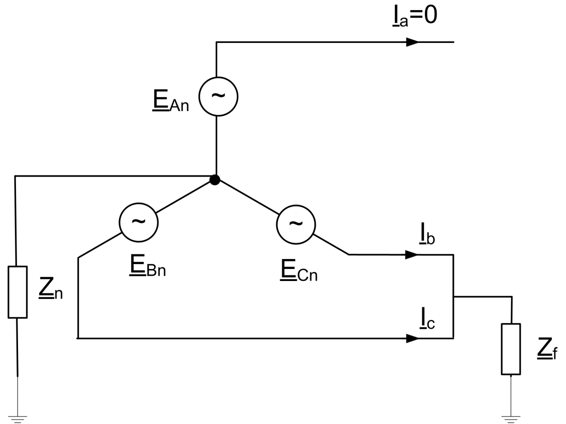

Fig. 5.4 Line-to-line fault

Since in Figure 5.4 b and c are faulted phases then:

$$\underline{I}_a = 0 \Rightarrow \underline{I}_a^0 = \underline{I}_a^1 = \underline{I}_a^2 = 0 \tag{5.35}$$

$$\underline{V}_b = \underline{V}_c \tag{5.36}$$

Thus,

$$\begin{bmatrix} \underline{V}_a^0 \\ \underline{V}_a^1 \\ \underline{V}_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix}\begin{bmatrix} \underline{V}_a \\ \underline{V}_b \\ \underline{V}_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \underline{V}_a + 2\underline{V}_b \\ \underline{V}_a + (\underline{a}^2 + \underline{a})\underline{V}_b \\ \underline{V}_a + (\underline{a}^2 + \underline{a})\underline{V}_b \end{bmatrix} \Rightarrow \underline{V}_a^1 = \underline{V}_a^2 \tag{5.37}$$

Since $\underline{V}_b = \underline{V}_c$, it follows:

$$\underline{V}_b = \underline{V}_c = (\underline{I}_b + \underline{I}_c)\underline{Z}_f = (\underline{I}_a^0 + \underline{a}^2\underline{I}_a^1 + \underline{a}\underline{I}_a^2 + \underline{I}_a^0 + \underline{a}\underline{I}_a^1 + \underline{a}^2\underline{I}_a^2)\underline{Z}_f = (2\underline{I}_a^0 - \underline{I}_a^1 - \underline{I}_a^2)\underline{Z}_f = 3\underline{I}_a^0\underline{Z}_f \tag{5.38}$$

Writing $\underline{V}_b$ in terms of symmetrical components gives:

$$\underline{V}_a^0 + (\underline{a}^2 + \underline{a})\underline{V}_a^1 = \underline{V}_a^0 - \underline{V}_a^1 = 3\underline{I}_a^0\underline{Z}_f \tag{5.39}$$

Also, $\underline{V}_a^1 = \underline{V}_a^2$. Substituting for the symmetrical components of voltage and solving $\underline{I}_a^0$ and $\underline{I}_a^2$ in last two equations gives:

$$\underline{I}_a^0 = -\frac{\underline{E}_{An} - \underline{Z}_1 \underline{I}_a^1}{\underline{Z}_0 + \underline{Z}_f} \tag{5.40}$$

$$\underline{I}_a^2 = -\frac{\underline{E}_{An} - \underline{Z}_1 \underline{I}_a^1}{\underline{Z}_2} \tag{5.41}$$

Substituting $\underline{I}_a^0$ and $\underline{I}_a^2$ into $\underline{I}_a^0 + \underline{I}_a^1 + \underline{I}_a^2 = 0$, gives:

$$\underline{I}_a^1 = \frac{\underline{E}_{An}}{\underline{Z}_1 + \frac{\underline{Z}_2(\underline{Z}_0 + 3\underline{Z}_f)}{\underline{Z}_2 + \underline{Z}_0 + 3\underline{Z}_f}} \tag{5.42}$$

The fault current is obtained from:

$$\underline{I}_f = \underline{I}_b + \underline{I}_c = 3\underline{I}_a^0 \tag{5.43}$$

These equations can be represented by connecting the positive sequence impedance in series with the parallel connection of the negative sequence and zero sequence networks. Using bus impedance matrix, the symmetrical current components in case of the fault at bus i can be obtained:

$$I_{i,a}^1 = \frac{V_i(0)}{\underline{Z}_{ii}^1 + \frac{\underline{Z}_{ii}^2(\underline{Z}_{ii}^0 + 3\underline{Z}_f)}{\underline{Z}_{ii}^2 + \underline{Z}_{ii}^0 + 3\underline{Z}_f}} \tag{5.44}$$

$$I_{i,a}^0 = -\frac{V_i(0) - \underline{Z}_{ii}^1 I_{i,a}^1}{\underline{Z}_{ii}^0 + 3\underline{Z}_f} \tag{5.45}$$

$$I_{i,a}^2 = -\frac{V_i(0) - \underline{Z}_{ii}^1 I_{i,a}^1}{\underline{Z}_{ii}^2} \tag{5.46}$$

The phase currents and voltages are obtained using Equation (5.23), Equation (5.24) and Equation (5.25).

5.2 Series fault analysis

As already said series fault occurs when one or two phases are opened. Two sides of the fault point will be denoted by f and f.

5.2.1 One-Conductor Open Fault

One-conductor open fault occurs when conductor of one phase is broken. Figure 5.5 shows a section between buses i and k with phase a conductor broken.

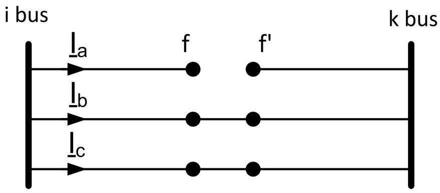

Fig. 5.5 One-conductor open fault

Current in phase a is equal to zero, hence:

$$\underline{I}_a = \underline{I}_a^0 + \underline{I}_a^1 + \underline{I}_a^2 = 0 \tag{5.47}$$

Phase b and phase c are closed and corresponding voltage drops are:

$$\begin{aligned} \underline{V}_{ff',b} &= 0 \\ \underline{V}_{ff',c} &= 0 \end{aligned} \tag{5.48}$$

Symmetrical components of voltage drops across the fault point are:

$$\begin{bmatrix} \underline{V}_{ff'}^0 \\ \underline{V}_{ff'}^1 \\ \underline{V}_{ff'}^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{V}_{ff',a} \\ 0 \\ 0 \end{bmatrix} \tag{5.49}$$

From here:

$$\underline{V}_{ff'}^0 = \underline{V}_{ff'}^1 = \underline{V}_{ff'}^2 = \frac{1}{3}\underline{V}_{ff',a} \tag{5.50}$$

This means that a consequences of an open conductor in one phase are equal voltages across the fault point in each sequence network. It can be concluded that zero, positive and negative sequence networks are connected in parallel across the fault point.

5.2.2 Two-Conductor Open Fault

Two-conductor open fault occurs when conductors of two phases are broken. Figure 5.6 shows a section between buses i and k with phase b and phase c conductors broken.

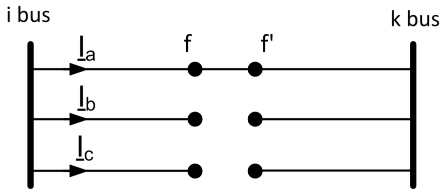

Fig. 5.6 Two-conductor open fault

It is obvious that:

$$\begin{aligned} \underline{I}_b &= 0 \\ \underline{I}_c &= 0 \\ \underline{V}_{ff',a} &= 0 = \underline{V}^0_{ff'} + \underline{V}^1_{ff'} + \underline{V}^2_{ff'} \end{aligned} \tag{5.51}$$

Sequence currents are:

$$\begin{bmatrix} \underline{I}^0_a \\ \underline{I}^1_a \\ \underline{I}^2_a \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{I}_a \\ 0 \\ 0 \end{bmatrix} \tag{5.52}$$

From here:

$$\underline{I}^0_a = \underline{I}^1_a = \underline{I}^2_a = \frac{1}{3}\underline{I}_a \tag{5.53}$$

This shows that zero, positive and negative sequence networks are in series closed circuit. Zero, positive and negative sequence networks between f and f' will be obtained in the following.

5.2.3 Sequence networks

The sequence networks as seen between f and f' can be schematically represented as shown in Figure 5.7. As already concluded, their connection depends on the type of open conductor fault. Parameters $\underline{Z}^0_{ff'}$, $\underline{Z}^1_{ff'}$, $\underline{Z}^2_{ff'}$ and $\underline{E}^1_{ff'}$ are to be obtained.

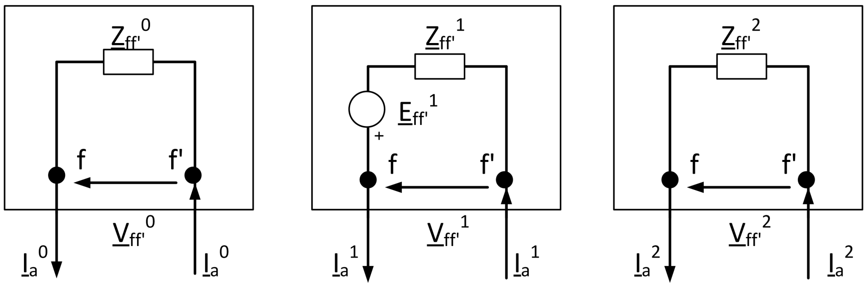

Fig. 5.7 Sequence networks as seen between f and f'

Complete removal of the line between buses i and k can be represented by adding $-\underline{z}_{ik}^{0}$, $-\underline{z}_{ik}^{1}$, and $-\underline{z}_{ik}^{2}$ between buses i and k to corresponding sequence networks. $\underline{z}_{ik}^{0}$, $\underline{z}_{ik}^{1}$, and $\underline{z}_{ik}^{2}$ zero, positive and negative sequence impedances of the line connecting buses i and k. Equivalent zero, positive and negative sequences of a broken conductor is shown in Figure 5.8. x denotes the length from bus i to the break point f .

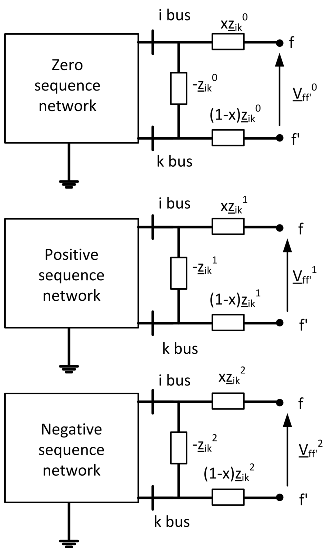

Fig. 5.8 Zero, positive and negative sequences of a broken conductor

Further analysis will be conducted on positive sequence and then extended to the zero and negative sequences. From Figure 5.8 positive sequence equivalent impedance $\underline{Z}_{ff'}^{1}$ is:

$$\underline{Z}_{ff'}^1 = x\underline{z}_{ik}^1 - \frac{\underline{Z}_{th,ik}^1 \underline{z}_{ik}^1}{\underline{Z}_{th,ik}^1 - \underline{z}_{ik}^1} + (1-x)\underline{z}_{ik}^1 = -\frac{(\underline{z}_{ik}^1)^2}{\underline{Z}_{th,ik}^1 - \underline{z}_{ik}^1} \tag{5.54}$$

Similarly for zero and negative sequences:

$$\underline{Z}_{ff'}^2 = -\frac{(\underline{z}_{ik}^2)^2}{\underline{Z}_{th,ik}^2 - \underline{z}_{ik}^2}, \quad \underline{Z}_{ff'}^0 = -\frac{(\underline{z}_{ik}^0)^2}{\underline{Z}_{th,ik}^0 - \underline{z}_{ik}^0}$$

$\underline{Z}_{th,ik}^0$, $\underline{Z}_{th,ik}^1$, $\underline{Z}_{th,ik}^2$ are zero, positive and negative sequence Thevenin's impedances between buses i and k of the network. For shunt faults, the situation was simpler because one point of the network (faulted bus) was of interest and the parameters of Thevenin's circuit were easy to obtain. For series faults, there are two points of interest (buses i and k) and Thevenin's impedance between these two points is needed. Series faults are also analyzed using $\mathbf{Z}$ matrix, but relationship between Thevenin's circuit parameters and $\mathbf{Z}$ matrix must be established. This will be done using N bus network.

If $\mathbf{V(0)}$ is the vector of pre-fault bus voltages, $\mathbf{I(0)}$ is the vector of pre-fault bus currents and $\mathbf{Z}$ is the NxN network impedance matrix, then the following relation is established:

$$\mathbf{V(0)} = \mathbf{I(0)Z} \tag{5.55}$$

Changes in bus currents for $\triangle\mathbf{I}$, result in changes of bus voltages for $\triangle\mathbf{V}$:

$$\mathbf{V} = \mathbf{IZ} = (\mathbf{I}(0) + \triangle\mathbf{I})\mathbf{Z} = \mathbf{I}(0)\mathbf{Z} + \triangle\mathbf{IZ} = \mathbf{V(0)} + \triangle\mathbf{V} \tag{5.56}$$

Now in order to estimate the changes of bus voltages, constant current sources $\triangle\mathbf{I}_i$ and $\triangle\mathbf{I}_k$ are connected to buses i and k respectively. In that case voltage changes can be expressed as:

$$\begin{bmatrix} \triangle\underline{V}_1 \\ \triangle\underline{V}_2 \\ \vdots \\ \triangle\underline{V}_i \\ \vdots \\ \triangle\underline{V}_k \\ \vdots \\ \triangle\underline{V}_N \end{bmatrix} = \begin{bmatrix} \underline{Z}_{11} & \underline{Z}_{12} & \cdots & \underline{Z}_{1i} & \cdots & \underline{Z}_{1k} & \cdots & \underline{Z}_{1N} \\ \underline{Z}_{21} & \underline{Z}_{22} & \cdots & \underline{Z}_{2i} & \cdots & \underline{Z}_{2k} & \cdots & \underline{Z}_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{Z}_{i1} & \underline{Z}_{i2} & \cdots & \underline{Z}_{ii} & \cdots & \underline{Z}_{ik} & \cdots & \underline{Z}_{iN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{Z}_{k1} & \underline{Z}_{k2} & \cdots & \underline{Z}_{ki} & \cdots & \underline{Z}_{kk} & \cdots & \underline{Z}_{kN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \underline{Z}_{N1} & \underline{Z}_{N2} & \cdots & \underline{Z}_{Ni} & \cdots & \underline{Z}_{Nk} & \cdots & \underline{Z}_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \triangle\underline{I}_i \\ \vdots \\ \triangle\underline{I}_k \\ \vdots \\ 0 \end{bmatrix} \tag{5.57}$$

For Equation (5.56) and Equation (5.57), bus voltages i and k are :

$$\begin{aligned} \underline{V}_i &= \underline{V}_i(0) + \underline{Z}_{ii}\triangle\underline{I}_i + \underline{Z}_{ik}\triangle\underline{I}_k \\ \underline{V}_k &= \underline{V}_k(0) + \underline{Z}_{ki}\triangle\underline{I}_i + \underline{Z}_{kk}\triangle\underline{I}_k \end{aligned} \tag{5.58}$$

These equations can be rewritten as:

$$\begin{aligned} \underline{V}_i =& \underline{V}_i(0) + (\underline{Z}_{ii} - \underline{Z}_{ik})\triangle\underline{I}_i + \underline{Z}_{ik}(\triangle\underline{I}_i + \triangle\underline{I}_k) \\ \underline{V}_k =& \underline{V}_k(0) + (\underline{Z}_{kk} - \underline{Z}_{ki})\triangle\underline{I}_k + \underline{Z}_{ki}(\triangle\underline{I}_i + \triangle\underline{I}_k) \end{aligned} \tag{5.59}$$

From these equations equivalent Thevenin's circuit of the network seen from buses i and k can be represented as shown in Figure 5.9.

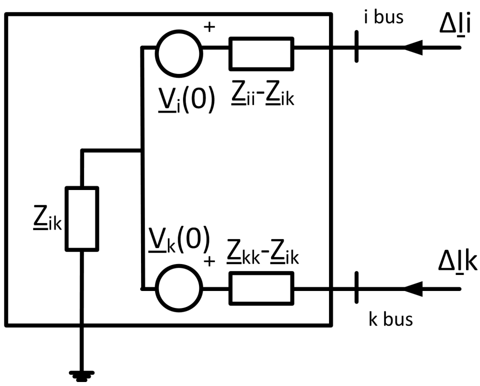

Fig. 5.9 Thevenin's circuit of the network seen from buses i and k

In order to obtain Thevenin impedance initial voltages are set to zero and ideal current source $\triangle\underline{I}$ is connected between buses i and k such that $\triangle\underline{I}_i = \triangle\underline{I}$ and $\triangle\underline{I}_j = -\triangle\underline{I}$. Thevenin's impedance between buses i and k is:

$$\underline{Z}_{th,ik} = \frac{\underline{V}_i - \underline{V}_k}{\triangle\underline{I}} = \frac{(\underline{Z}_{ii} - \underline{Z}_{ik})\triangle\underline{I} + (\underline{Z}_{kk} - \underline{Z}_{ik})\triangle\underline{I}}{\triangle\underline{I}} = \underline{Z}_{ii} + \underline{Z}_{kk} - 2\underline{Z}_{ik} \tag{5.60}$$

Now in order to obtain Thevenin's voltage between f and f' impedance $x\underline{z}^1_{ik} + (1-x)\underline{z}^1_{ik} = \underline{z}^1_{ik}$ and voltage $\underline{V}^1_{ff'}$ shown in Figure 5.8 are replaced by constant current source $\underline{I}^1_{ff'} = \frac{\underline{V}^1_{ff'}}{\underline{z}^1_{ik}}$ (Figure 5.10).

Parallel combination of $\underline{z}^1_{ik}$ and $-\underline{z}^1_{ik}$ is replaced with open circuit. Then the current injected to bus i is $\frac{\underline{V}^1_{ff'}}{\underline{z}^1_{ik}}$ and the current injected to bus k is $-\frac{\underline{V}^1_{ff'}}{\underline{z}^1_{ik}}$. From here zero, positive and negative sequence voltage drops for arbitrary bus j due to the current injections at buses i and k can be obtained using Equation 5.57.

Voltage $\underline{V}^1_{th,ff'}$ can be expressed as:

$$\underline{V}^1_{th,ff'} = -\frac{\underline{z}^1_{ik}}{\underline{Z}^1_{th,ik} - \underline{z}^1_{ik}}(\underline{V}^1_i(0) - \underline{V}^1_k(0)) \tag{5.61}$$

From Equation (5.54) and Equation (5.61), $\underline{V}^1_{th,ff'}$ is:

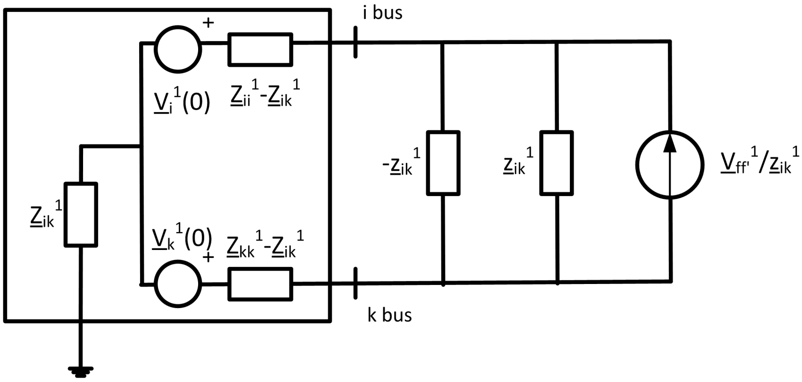

Fig. 5.10 Positive sequence of a broken conductor

$$\underline{V}^1_{th,ff'} = \frac{\underline{Z}^1_{ff'}}{\underline{z}^1_{ik}}(\underline{V}^1_i(0) - \underline{V}^1_k(0)) = \underline{Z}^1_{ff'}\underline{I}^1_{ik}(0) \tag{5.62}$$

where $\underline{I}^1_{ik}(0)$ is the current that was flowing through phase a conductor prior to the occurrence of the fault. Obtained Thevenin voltage is equal to $\underline{E}^1_{ff'}$ shown in Figure (5.7).

Chapter 6
Power Electronics

Power electronics is the area of semiconductor electronics that refers to change of electrical power characteristics using electronic switches, capacitive and inductive elements as well as control systems. Development of power electronics began in late fifties of the last century with the investigation and application of thyristor switching mechanism. Among other members of thyristor family, silicon controlled rectifier (SCR) is the most widely used. Today, power electronics is an interdisciplinary technology related to electronics, solid state physics, control, power systems, signal processing, etc. Typical devices of power electronics are:

- Two terminal devices - PiN diodes, Schottky diodes
- Three terminal devices – switches, bipolar junction transistor (BJT) which is not much in use, insulated-gate bipolar transistor (IGBT) which is a hybrid of BJT and MOSFET and thyristor.

There are numerous of applications of power electronics. Power electronics equipment is used as a substitute for electromechanical equipment enabling faster and more efficient operation. First power electronic controllers based on thyristors were used in HVDC transmission and then the technology was extended to FACTS in order to enhance system stability and power transfer. The concept of FACTS was introduced by Dr. Hingorani in 1988. FACTS is an acronym for Flexible Alternating Current Transmission Systems. Focus of this chapter are inverters and rectifiers as key elements of HVDC link with thyristors as core component and FACTS.

Inverter or DC-AC converter is used for conversion of DC input to AC output with specified magnitude and frequency. Opposite to that rectifier or AC-DC converter converts input AC to output DC with specified magnitude. Both of these converters are important part of HVDC substations. HVDC power transmission uses direct current transmission for long distances unlike commonly used alternating current systems. Connection of two alternating current systems is only possible if they have the same frequency. The problem are also losses in the long underwater AC cables caused by their capacitance.

Solution for all the problems of long high voltage lines is HVDC transmission that offers number of advantages: power loss reduction, reduction of heavy capacitive currents flowing through underwater cables, possibility of connecting two unsynchronised networks, lower cost of the HVDC conductors which justifies the investment in bit more expensive power electronic components, etc.

The thyristor is a semi-conductive component similar to the diode, with three terminals: anode, cathode and gate. The ability to switch large currents at very high voltages in a very short time made it one of the most commonly used components of power electronics circuits. Thyristor operates in three basic states that can be described in comparison with diode states. When thyristor is reverse biased, same as a diode, it blocks current flow. This state is called reverse blocking. Forward biased thyristor unlike diode does not conduct current. This state is called forward blocking. Forward biased thyristor remains in this state until it is triggered by a current pulse through the gate terminal when it starts conducting. This state is called forward conducting. Figure 6.1 shows thyristor symbol (a), simplified construction of thyristor (b) and two-transistor thyristor model (c).

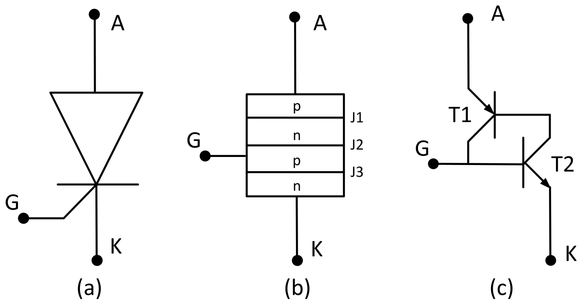

Fig. 6.1 Thyristor symbol (a), simplified construction (b), two-transistor model (c)

In reality, level of doping and thickness are different for all four layers. Most heavily doped is cathode layer followed by the gate and anode. Central n type layer has the lowest doping level and is thicker than the other layers which enables a large blocking voltage to be supported. With absence of any gate signal and with the voltage of anode less than the voltage of cathode, junction J1 and junction J3 are reverse biased the thyristor blocks current flow. In case when the anode voltage is larger than the cathode voltage, junctions J1 and J3 are forward biased, but junction J2 is still reverse biased meaning that the thyristor still does not conduct. Gate current pulse makes junction J2 forward biased causing large current flow between anode and cathode. Note that this current continues to flow even when the pulse is gone. In terms of two-transistor model this process can be explained as following. Thyristor is represented as a connection of one *npn* and one *pnp* transistor. If the thyristor is reverse biased, then both transistors are reverse biased and no current flow is established. In case of forward biased thyristor, current flows only when the gate is made positive with respect to T2 emitter. As a consequence, this turns on T2 and causes its collector voltage to decrease rapidly. T1 turns on because base emitter junction becomes forward biased. Current flowing through the thyristor at this point is limited by the external thyristor circuit only, and continues to flow until it falls below a threshold value or if the thyristor is reverse biased.

FACTS controllers can be defined as electronic based components that enable control of power flow and voltage in the network. They are designed to manage system changes allowing flexible network operation and maintaining system stability. FACTS controllers for steady-state system operation are: thyristor-controlled phase shifter, load tap changer controlled by thyristor switches, thyristor-controlled reactor, thyristor-controlled series capacitor, inter-phase power controller, static compensator, solid-state series controller, unified power flow controller. Constant need for improvements in HVDC transmission technologies, loss reduction, power quality and system stability are the driving force of power electronics development in the future [79–97].

6.1 Rectifiers

Rectifiers or AC/DC converters are used for conversion AC to DC. In general, rectifiers can be classified as controlled and uncontrolled devices depending whether they are built on diodes or thyristors. Rectifiers built on thyristors have the ability to control DC output. Based on their design and output, rectifiers can be bridge or midpoint rectifiers, single or three phase rectifiers, half or full wave rectifiers. Six pulse three-phase rectifier shown in Figure 6.2 is considered in the following.

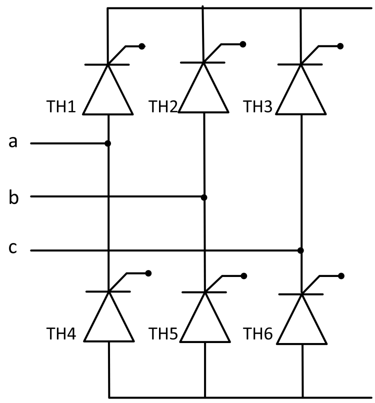

Fig. 6.2 Six pulse rectifier

In most cases in each phase there is a single phase transformer before the rectifier used to isolate rectifier from AC supply and to adjust the desired voltage level. Also, different three phase transformers can be used to shift voltages and currents (depending on transformer connection). This way it is possible to remove some low order harmonics. Six pulse rectifier is built of six thyristors. Cathodes of first three resistors are connected to one point as well as the anodes of the remaining three. At any instant, only two diodes conduct: one from the cathode-connected group, and one from the anode-connected group. Resulting output voltage consists of six pulses per period. Combinations of open thyristors from Figure 6.2 in one period are: TH1 and TH6, TH6 and TH2, TH2 and TH4, TH4 and TH3, TH3 and TH5, TH5 and TH1. Output voltage varies as the delay angle for the thyristors is changed. Zero delay angle means the thyristors are operating like diodes and DC output voltage can be calculated as:

$$V_{out} = \frac{3\sqrt{3}\sqrt{2}V_{in,rms}}{\pi} \tag{6.1}$$

Average DC voltage with delay angle α is:

$$V_{out} = \frac{3\sqrt{3}\sqrt{2}V_{in,rms}}{\pi} cos\alpha \tag{6.2}$$

During the commutation process, it is possible to have more than two thyristors open.

Different filters are additionally used to filter remaining harmonics. Usually the output of the rectifier is connected to shunt capacitor, series inductor, LC or CLC filter in order to improve the quality of the output DC voltage by eliminating ripple components. Shunt capacitor filter represents a large capacitor connected between rectifier output terminals. The capacitor of that size represents a small reactance for AC currents which are conducted through the capacitor. Stored energy in the capacitor disables fast fluctuations of voltage and minimizes ripple effects. Series inductor filter represents an inductor connected in series to the rectifier output. The inductor opposes variation in current, i.e. it opposes AC current components, while at the same time it represents zero resistance for DC component. LC filter is a combination of inductor and capacitor as shown in Figure 6.3. It reduces output ripple by combining the advantages of the two previously mentioned filters. CLC filter represents a connection of capacitor shunt filter with LC filter and is also known as π filter.

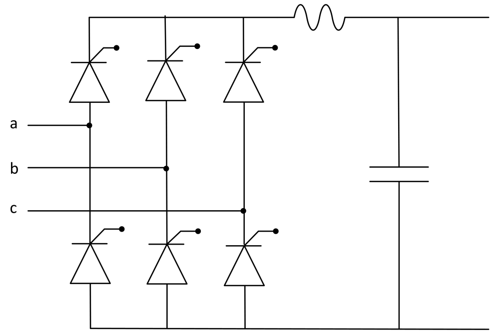

Fig. 6.3 LC filter

In order to reduce harmonics twelve pulse rectifier shown in Figure 6.4 is used. It is designed as a parallel connection of two six-pulse rectifiers. In most applications of twelve-pulse rectifier first six-pulse rectifier is connected to why-why transformer and second six-pulse rectifier is connected to delta-why transformer. This connection enables elimination of 5th and 7th harmonics, while 1st, 11th, 13th, 23rd and 25th are still present in output signal.

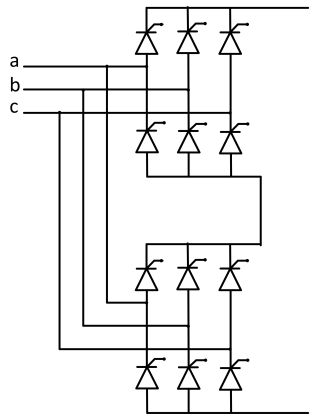

Fig. 6.4 Twelve pulse rectifier

Improvements in rectifiers design, harmonic elimination, voltage and current control in the future are expected by introducing rectifiers built on transistors as fully controlled components.

6.2 Inverters

Inverters are essentially built similarly to rectifiers. Their function is to convert DC supply to single-phase or three-phase AC output of specified frequency and magnitude. Inverters are actually oscillators. They can be classified as voltage source inverters and current source inverters. Voltage source inverters form the voltage of specified characteristics and they are commonly used. They are usually used in combination with large capacitor connected across voltage source terminals to prevent the fluctuations in the input voltage. Current source inverters are the source of current of specified characteristics. There is almost always an inductor connected between the source and the inverter in order to keep supply current constant.

Three-phase inverters are used in motor drives, active filters and power transmission. Early transistors did not have sufficient power ratings for power electronics applications and thyristors were used as switching component. The main problem with this design is commutation of thyristors. When the thyristor is supplied from AC voltage source it is turned off when the voltage polarity is changed. On the other hand, in case of inverters the thyristor is supplied from DC voltage source and requires forced commutation. Lately, development of transistors enabled their use in inverter design. Standard configuration of three-phase inverter is shown in Figure 6.5. Transistors are IGBT and the function of diodes is to protect transistors from reverse currents that occur during switching. Usually three capacitors in delta connection are connected to three-phase output in order to filter current harmonics and to make the output sinusoidal.

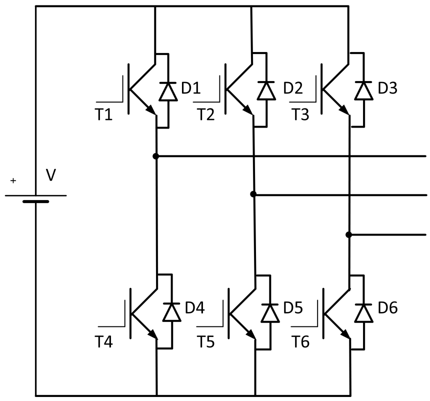

Fig. 6.5 Three-phase inverter

Each of the switches operates after 60 degrees of the output waveform, thus the output voltages consist of discrete voltage levels: negative voltage source value, zero voltage and source value. Note that two switches placed in vertical branch can not be both open at the same time because that would short-circuit the voltage source. The output waveform has a zero voltage value between the positive and negative sections. For higher power ratings two inverters can be connected in parallel or in series. These connections also improve the quality of the output waveform. Switching control module is used to control the switching sequence of IGBTs, such that it results in sinusoidal output voltage waveform.

6.3 FACTS

AC transmission network is meshed and highly interconnected with other systems. Some of the benefits of system interconnections are generation reserves, economic benefits and reliability, but on the other hand huge problem is system stability and security as the disturbances from one part of the system are easily spread over the entire system. In general, system power flow is inversely proportional to transmission line reactance. Low line reactance enables larger power flows, while it is the opposite with high line reactance. Obviously, the longer the power line is, its reactance increases causing the decrease in power flow. Traditionally tap-changing and phase-shifting transformers and capacitor batteries are used for system power flow and voltage control. Series capacitors are used to decrease line reactances in such way that they are switched on and off depending on power flow conditions. However, daily load variations (which can be predictable and unpredictable) and generation in deregulated environment can result in overloading some lines. The network of today in general is additionally stressed by load and generation growth which leads to network operation closer to stability limits. This problem can be overcome by upgrade of transmission facilities in order to preserve system stability and control power flows, but such solution takes time and money to implement. Also it always faces some social and ecological demands that have to be fulfilled.

All the issues of AC transmission system are successfully solved using FACTS controllers without the need to invest in transmission and generation facilities. They enable AC transmission system control aiming to increase power transfer capacity, system security and reliability, to reduce power losses, to enhance the quality of the electric energy and to change power flow under dynamic conditions. FACTS controllers can be classified as variable impedance type controllers and voltage source converter based controllers (VSC). Some of them are presented below:

- Variable impedance type controllers
  - Thyristor-Controlled Reactor
  - Static Var Compensator
  - Thyristor Controlled Series Compensator
- Voltage source converter based controllers
  - Static Synchronous Compensator
  - Static Synchronous Series Compensator
  - Unified Power Flow Controller

FACTS controllers were primarily developed and used in transmission network control aiming to enhance system security and transmission capabilities through fast control of line impedances, bus voltages and power flow paths. The other application of FACTS controllers in power system is focused on power quality in low-voltage distribution. Electrical market deregulation, life standard improvement and use of sensitive electronic equipment and non-linear loads resulted in very strict power quality demands. Beside the need for continuous power supply and voltage and frequency level control, it has become unnecessary to eliminate transient disturbances in voltage magnitude, waveform and frequency. Generally, power quality problems can be classified as: transients, voltage and frequency variations, voltage unbalance, waveform distortion, flickers. Development of FACTS controllers for distribution network is still under progress and they are not the focus of this chapter. Some used in regulation of power quality in distribution network are:

- Distribution STATCOM (DSTATCOM)
- Dynamic Voltage Restorer (DVR)
- Unified Power Quality Conditioner (UPQC)

6.3.1 Modeling of FACTS controllers

Power flow analysis requires modeling of all system components including FACTS controllers. Design of thyristor-controlled reactor, static VAR compensator and thyristor-controlled series compensator is thyristor based while static compensator, solid state series compensator and unified power flow controller incorporate voltage source converter as their basic component. Thyristor controllers are used for real-time electrical line length control instead of mechanically controlled capacitors used in the past. Controllers based on voltage source converter like solid state series compensator and unified power flow controller, regulate either nodal voltage magnitude or injection of reactive power and active power flow through the controller. Design characteristics of unified power flow controller enable regulation of bus voltage magnitude and reactive power injection. All power electronics components produce harmonic distortion. There are a number of ways to eliminate this side-effect which is note the focus of this chapter; hence all harmonic distortion in the following power flow analysis is considered eliminated.

6.3.1.1 The Thyristor-Controlled Reactor (TCR)

TCR represents a series connection of an inductor and anti parallel thyristor pair. It acts as a controllable susceptance controlled by firing angle of the employed thyristors. Basic design of TCR is represented in Figure . Unless the TCR is in condition of full conduction, it generates significant harmonic distortion which are eliminated using filters.

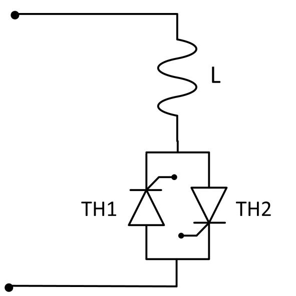

Fig. 6.6 Thyristor-controlled reactor

Let the thyristors firing angle be denoted by α and the conduction angle by σ. In the case of full conduction, thyristors are short-circuited and TCR acts as a reactor. Neglecting reactor resistance, it can be assumed that the current is sinusoidal and lagging the voltage by $\frac{\pi}{2}$. Firing angle α corresponds to current zero-crossing and it is equal to $\frac{\pi}{2}$ with the voltage as a reference. In the case when α is increased above $\frac{\pi}{2}$, reactor inductance is increased due to sinusoidal current waveform distortion. Expression for TCR current is:

$$\begin{aligned} i(t) &= \frac{1}{L}\int_{\alpha}^{\omega t} v(t) &&= \frac{1}{L}\int_{\alpha}^{\omega t} \sqrt{2}V sin\omega t = \frac{\sqrt{2}V}{\omega L}(cos\alpha - cos\omega t), \alpha \leq \omega t \leq \alpha + \sigma \\ i(t) &= 0, && \alpha \geq \omega t \wedge \alpha + \sigma \leq \omega t \end{aligned} \tag{6.3}$$

If harmonic distortions are considered eliminated, then fundamental frequency current (obtained using Fourier transform) is:

$$\underline{I} = \frac{1}{j\omega L} \cdot \frac{V}{\pi}\left[2\left(\pi - \alpha\right) + sin2\alpha\right] = -jB_{TCR}V \tag{6.4}$$

From here:

$$B_{TCR} = \frac{2\left(\pi - \alpha\right) + sin2\alpha}{\omega L\pi} \tag{6.5}$$

Equation(6.5) represents equivalent susceptance of TCR defined as a function of thyristor firing angle.

6.3.1.2 The Static VAR Compensator (SVC)

SVC is designed as a parallel connection of a TCR and a bank of capacitors. It acts like the shunt variable reactance which absorbs or injects reactive power in order to regulate bus voltage at the connection point. In reality, three-phase three-winding transformer is used to connect SVC to network bus. Secondary winding is connected to Y connected capacitor bank, while tertiary winding is connected to three $\triangle$ connected TCRs. Assuming that capacitors have equal capacitance and that TCRs have equal reactances SVC current is:

$$\underline{I} = jB_{SVC}\underline{V} \tag{6.6}$$

where

$$B_{SVC} = \omega C - \frac{2\left(\pi - \alpha\right) + sin2\alpha}{\omega L\pi} \tag{6.7}$$

6.3.1.3 The Thyristor-Controlled Series Compensator (TCSC)

TCSC is used for active power flow regulation as it controls transmission line reactance. TCSC comprises one or more modules consisted of a TCR in parallel with a fix capacitor (Figure 6.7) and it is connected in parallel with phase conductor. Due to the low capacitance impedance, harmonic currents generated by TCR thyristors are trapped inside the TCSC.

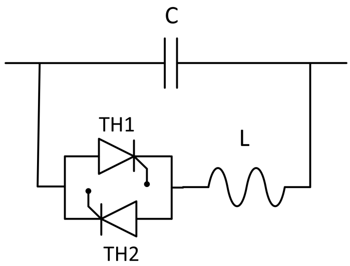

Fig. 6.7 TCSC module

Electrical parameters of TCSC are obtained from relatively complicated procedure that involves Laplace transformation and are not presented here. Resulting equivalent reactance of TCSC based on the assumption that all harmonic distortions are eliminated is:

$$\begin{aligned} X_{TCSC} = &- X_C + \frac{X_C + X_{LC}}{\pi} \left\{ 2\left(\pi - \alpha\right) + sin\left[2\left(\pi - \alpha\right)\right]\right\} \\ &- \frac{4X_{LC}^2}{\pi X_L} cos^2\left(\pi - \alpha\right) \left\{\varpi tan\left[\varpi\left(\pi - \alpha\right)\right] - tan\left(\pi - \alpha\right)\right\} \end{aligned} \tag{6.8}$$

where:

$$X_{LC} = \frac{X_C X_L}{X_C - X_L}$$

$$\varpi = \frac{\omega_0}{\omega}$$

ω_0 is natural frequency, X_L is inductive reactence, X_C is capacitive reactance and α is thyristor firing angle. Assuming that TCSC is connected between buses i and m nodal transfer admittance matrix for the purpose of power flow calculations is:

$$\begin{bmatrix} \underline{I}_i \\ \underline{I}_j \end{bmatrix} = \begin{bmatrix} -\frac{1}{jX_{TCSC}} & \frac{1}{jX_{TCSC}} \\ \frac{1}{jX_{TCSC}} & -\frac{1}{jX_{TCSC}} \end{bmatrix} \begin{bmatrix} \underline{V}_i \\ \underline{V}_j \end{bmatrix} \tag{6.9}$$

6.3.1.4 The Voltage Source Converter (VSC)

VSC is power electronic controller based on fully controlled semiconductor devices like insulated gate bipolar transistor (IGBT) and the gate turn-off thyristor (GTO). VSC is actually inverter and its topology is shown in Figure 6.5. There are also some other design solutions, but this is conventional one.

VSC is an DC–AC voltage converter based on fully controlled semiconductors and it is a basic component of many controllers like static compensator, solid state series controllers and unified power flow controller.

6.3.1.5 The Static Compensator (STATCOM)

In power network STATCOM has the same function as the SVC but its performance is more robust. It absorbs or injects reactive power in order to regulate the voltage of the bus it is connected to. STATCOM consists of VSC and shunt-connected transformer as shown in Figure 6.8.

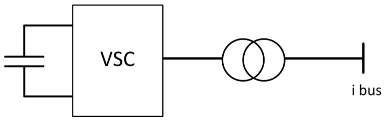

Fig. 6.8 Static compensator

Equivalent STATCOM circuit can be represented as a Thevenin impedance viewed from bus i and variable voltage source:

$$\underline{E} = V(cos\delta + jsin\delta) \tag{6.10}$$

Voltage magnitude V and phase angle δ are adjusted to satisfy a specified voltage magnitude at the bus i. For V maximum and minimum limits are a function of STATCOM capacitor rating, while δ may vary from zero to 2π. Figure 6.9 shows STATCOM equivalent circuit.

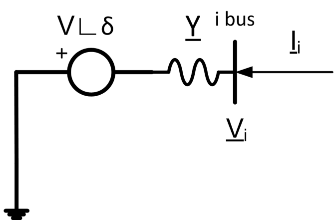

Fig. 6.9 STATCOM equivalent circuit

From Figure 6.9 is is obvious that:

$$\underline{I}_i = \underline{YV}_i - \underline{Y}V\angle\delta \tag{6.11}$$

6.3.1.6 The Solid State Series Compensator (SSSC)

SSSC representation and equivalent circuit are shown in Figure 6.10. It performs a function of a phase shifter without drawing reactive power from the network, thanks to its own capacitor. Thus, SSSC regulates active and reactive power flow and voltage.

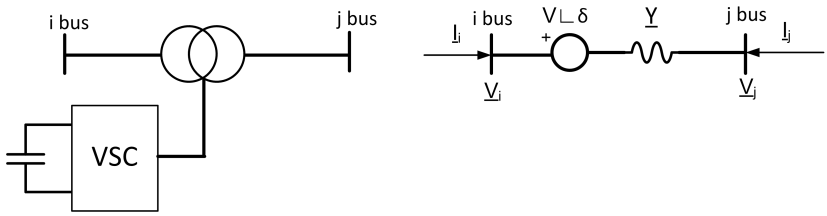

Fig. 6.10 Solid state series compensator

Voltage source in Figure 6.10 is represented as:

$$\underline{E} = V(cos\delta + jsin\delta) \tag{6.12}$$

Voltage magnitude and phase angle are functions of specified active and reactive power flow across the SSSC. Voltage magnitude has to be within specified limits. From equivalent circuit it can be concluded that:

$$\begin{aligned} \underline{I}_i &= \underline{YV}_i - \underline{YV}_j - \underline{Y}V\angle\delta \\ \underline{I}_j &= \underline{YV}_j - \underline{YV}_i + \underline{Y}V\angle\delta \end{aligned} \tag{6.13}$$

6.3.1.7 The Unified Power Flow Controller (UPFC)

UPFC can be represented as a connection of two VSCs with unified control sharing a capacitor on their input side. Schematic representation of UPFC and equivalent circuit are shown in Figure 6.11. UPFC enables control of active and reactive and voltage magnitude at its terminals.

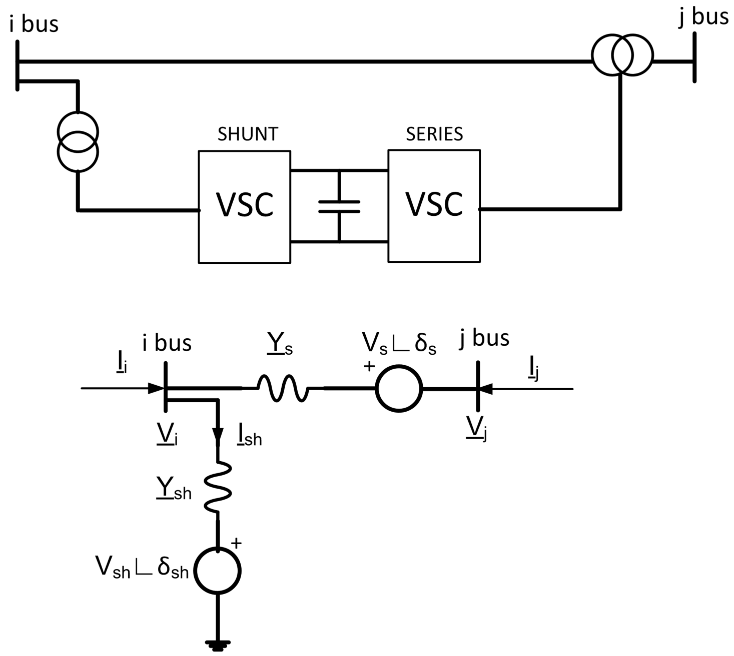

Fig. 6.11 Unified power flow controller

Equivalent circuit consists of one shunt voltage source and one voltage source connected in series:

$$
\begin{aligned}
\underline{E}_{sh} &= V_{sh}(cos\delta_{sh} + jsin\delta_{sh}) \\
\underline{E}_{s} &= V_{s}(cos\delta_{s} + jsin\delta_{s})
\end{aligned}
\tag{6.14}
$$

These voltage sources are related by constraint equation:

$$
Re\left\{\underline{E}_{sh}\underline{I}_{sh}^{*} + \underline{E}_{s}\underline{I}_{j}\right\} = 0
\tag{6.15}
$$

Similar to STATCOM and SSSC voltage sources also have voltage limits. Relations between bus currents and voltages are:

$$
\begin{aligned}
\underline{I}_i &= \left(\underline{Y}_s + \underline{Y}_{sh}\right)\underline{V}_i - \underline{Y}_s\underline{V}_j - \underline{Y}_s V_s \angle\delta_s - \underline{Y}_{sh} V_{sh} \angle\delta_{sh} \\
\underline{I}_j &= \underline{Y}_s\underline{V}_j - \underline{Y}_s\underline{V}_i + \underline{Y}_s V_s \angle\delta_s
\end{aligned}
\tag{6.16}
$$

Appendix A
Introduction to Matlab

MATLAB is a very convenient and easy-to-learn tool for solving most of exercises and engineering problems that are encountered in the field of electrical engineering in general. With its many packages, it can tackle problems involving control systems, digital electronics, signal processing, but this short MATLAB manual is prepared to demonstrate by a few examples how to model and solve simple exercises typical in the course dealing with power systems, giving the base for further independent work. No special packages are used in the following.

MATLAB is in the context of the course on power systems particularly convenient for solving systems of linear equations. Dealing with complex numbers is also easy and intuitive in MATLAB, making it ideal to express e.g. power flows in AC circuits containing capacitive and inductive elements.

Another important and powerful feature of MATLAB is the plotting that illustrates solutions and tendencies of the involved functions. The plots can be used to optimize a system and discover the influence of various effects on the final result.

MATLAB is primarily derived from FORTRAN subroutines for linear and eigenvalue systems, and is an interpreted language, like e.g. Java and is still efficient in execution. Thus, MATLAB can be used as a programming language where commands are executed line by line, or simply as a calculator! When using MATLAB as a calculator, desired operations are typed in the command window. A **script** is written for more complicated simulations and modeling.

A.1 Scripts

MATLAB scripts are collections of commands to be executed in sequence. They are written in the MATLAB editor and saved as MATLAB files with .m extension. Within the script, variables are defined, decisions, loops and consecutive commands are stated.

A.1.1 Variables

It is important to stress that, unlike many program languages, in MATLAB there is no need to initialize variables. There are various types of variables, e.g. 16-bit chars, 16-bit integers, 64-bit doubles, strings etc., as well as vectors and matrices made up of these. To create a variable, it is sufficient just to assign a value:

```
wow = 2.15;
wuf = 'wuf';
```

The semicolon is used to prevent the result of intermediate calculation or variable declaration to be shown as output in command window. As mentioned earlier, all numbers can be real or complex. The imaginary part is created by the built-in variable `i` or `j`. The following declarations are equivalent:

```
cpxwow = 3 + 4i;
cpxwow = 3 + 4*i;
cpxwow = 3 + 4j;
cpxwow = 5*exp(asin(4/5)*j);
```

Variable names are case sensitive. Therefore, `wuf_1='one';` and `Wuf_1='two';` are different variables. The first character of a variable name must be a letter. After the first character, any letter and/or number and/or underscore can follow. However, some names are reserved and should not be used in scripts. Those are:

`i` and `j` are used to create complex numbers,
the number `pi` has the value 3.1415926...,
`ans` (short of answer) stores the last unassigned value (like on a calculator),
`Inf` and `-Inf` are positive and negative infinity, and
`NaN` stands for 'Not a Number' (e.g. answer to `0/0`).

MATLAB is a MATRIX Laboratory. Operations with matrices are the crucial part of MATLAB. Matrices are created pretty much like variables. There are 1-dimensional arrays – rows and columns. Row arrays or vectors are created as values between brackets separated with comma or space.

```
row = [1 2 3.4 -5 6]; or
row = [1, 2, 3.4, -5, 6];
```

The command `length(row)` returns value 5 while the dimension is given by `size(row)` and is `1 5`. First the rows and then the columns are stated. Column array/vector is created using semicolon to separate values between brackets. That is

```
col = [1;2;3;4];
```

The command `length(col)` still returns value 5 while `size(row)` is `5 1`. A 2-dimensional array can be created by assigning values for rows and columns:

```
twoD = [1 2 3; 4 5 6; 7 8 9; 10 11 12];
```

The `size(twoD)` returns 4 3, and `length(twoD)` is 4! The number of rows is taken as the length for 2-D matrix. Note that arithmetic operations can be carried out on matrices in two ways – element-by-element and matrix-wise operations! This is one of the common sources of errors that inexperienced MATLAB users can do. To explain, the following example can be used:

```
matr=[1 2 3; 4 5 6; 7 8 9]
```

```
1 2 3
4 5 6
7 8 9
```

The result of multiplication
`matr * matr` is

```
 30  36  42
 66  81  96
102 126 150
```

wheras the multiplication element-by-element is achieved using the operator `.*`

`matr .* matr` gives

```
 1  4  9
16 25 36
49 64 81
```

Besides element-by-element multiplication `.*`, there are also element-by-element division `./` and exponentiation `.^` . In order to perform the operations, the dimensions of variables must be appropriate, i.e. it is not possible to multiply element-by-element a 4x3 array with a 3x5 array, but a matrix multiplication is possible since the number of columns of the first array is equal to the number of rows of the second.

Very often, a time or space variable is needed when a function is to be evaluated in many consecutive points. Say, a hundred points are needed to evaluate a sine function with certain frequency and phase, within five seconds. For creation of time variable with points evenly distributed in a certain interval, linspace is usually used. For example, 100 points between 0 and 5 are created to represent time for a sine wave

```
t=linspace(0,5,100); freq=2; phase=0.5; Amp=2;
wave=Amp*sin(freq*t+phase);
plot(t,wave)
```

Besides linspace, there is also logspace where the distribution of points has logarithmic dependence. This can be used to evaluate and plot with even resolution functions in e.g. double logarithmic plots.

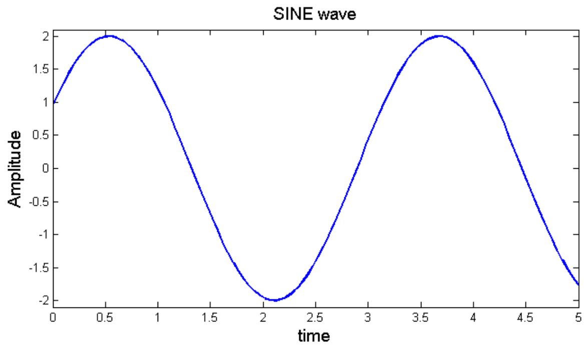

Fig. A.1 Plot of the sine wave with the defined frequency, phase and amplitude.

A.1.2 Loops

As in all program languages, loops exist in MATLAB as well. Execution of loops is usually slow, so whenever possible, they are omitted. For loop in MATLAB

```
for f=1:10
var_a(f)=var_a(f)+1;
end
can be omitted by saying
var_a=var_a+1;
```

The addition applies for all elements of var_a.

Another loop configuration in MATLAB is the standard while loop. Here, statements within while-end are executed as long as a certain condition is satisfied. As an example, one can say

```
learningMATLAB = 0;
while logical(floor(learningMATLAB)) ~= true
learningMATLAB = learningMATLAB + 0.1;
end
```

A.1.3 Decisions

Programming languages need a statement of decision to conduct envisaged algorithms where sequences are executed based on certain conditions. MATLAB has the standard if – (elseif – else) – end construction and can be explained by the following example:

```
blackbeltMATLAB = 0;
if blackbeltMATLAB^0 == learningMATLAB
blackbeltMATLAB = 1
elseif learningMATLAB >= 0.5
blackbeltMATLAB = 0.25 else
blackbeltMATLAB = 0.1
end
```

Notice that the relational operator `==` used to compare values is not the same as the sign of equality `=` which is used to assign values to variables.

A.1.4 System of Linear Equations

A very useful application of MATLAB and its ability to handle matrices is solving systems of linear equations. This is directly relevant and applicable in power systems and circuit theory in general for e.g. finding voltages and currents in a specific network.

The problem is usually expressed in the simple form

$Y \cdot \mathbf{v} = \mathbf{i}_n$

where Y is a square *nxn* matrix, $\mathbf{i}_n$ is a column vector and $\mathbf{v}$ is the unknown column vector, all of them having the same size n - the number of linearly independent equations must be equal (or greater than) the number of unknowns. Y can represent complex impedances or conductances in a network, $\mathbf{i}_n$ would be the currents and $\mathbf{v}$ the node voltages. Consider example 1.4 and the system of linear equations:

The coefficient matrix `Y` is defined, as previously explained, as a 2-D matrix

```
Y=[1 0 0 0 ; -0.2+0.5j 0.45-1.25j -0.2+0.5j -0.05+0.2j; -0.25+0.3j
-0.2+0.5j 0.55-1.2j -0.1+0.4j; 0 -0.05+0.2j -0.1+0.4j 0.15-0.6j];
```

And the column vector as `i_n =[100; 0; -2-j; 5+2j];`

The solution to the vector $\mathbf{v}$ is $\mathbf{v}= Y^{-1}$ in, or in MATLAB `v=inv(Y)*i_n`

Note the multiplication is not element-by-element but matrix multiplication, and i_n is used to denote currents – not the complex number i. As an efficient alternative, to get the same result it is also possible to use the backslash operator and type `v = Y\i_n`

Anyhow, it is important to keep in mind that the coefficient matrix, after Gauss elimination, and current and voltage vectors must have the correct dimensions, to use the correct operators in matrix manipulation.

A.1.5 Comments and Help

For programs longer than some 50 lines, it is worthwhile to comment thoroughly to avoid wasting time when analyzing the code later. In MATLAB, any character following a `%` is seen as a comment (and does not execute if it were some commands). The first comment becomes also the script's help file. That is, `help myfile` written in the command window returns the first comment in myfile.m script. Finally, MATLAB must be explored and used for one to comprehend and appreciate its full capabilities. The command `help` is always handy for discovering the many functions of MATLAB on your own. There is help for each and every command, but there is also an overview of help topics – obtained by writing just `help` in the command window. For help on general purpose commands (the ones used most frequently), `help general` is written in the command window. If there is a really tough problem, error messages or the output is not as expected, there is always command `why` – it may explain the reason for code malfunctioning.

Appendix B
Introduction to Power System Simulator

Power System Simulator (PSS) is a software tool designed to enable simulation of different power system states. Extension of PSS files is *pf*. PSS window and power flow analysis will be presented here.

After opening PSS window, Tools palette, Color Map Legend and Message pop-ups are displayed. B.1 shows PSS window with Tools palette opened. Since no figures are opened yet, tool buttons are disabled. Color Map Legend and Message windows are closed for better visibility of the workspace. Their function will be explained later.

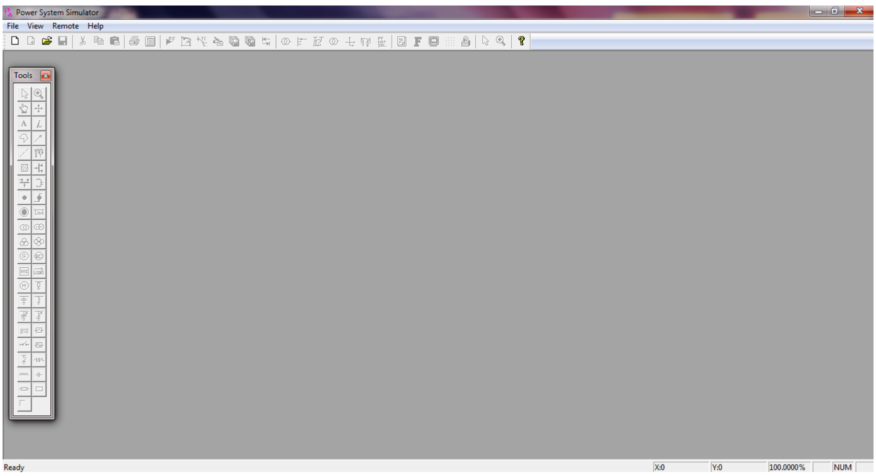

Fig. B.1 PSS window

To get started, existing file can be opened or the new one can be created. A click on *New* button on the toolbar (first one from the left) or a click on *File* >> *New* opens a new file and a dialog that contains a line where the name of the initial figure should be entered. This is optional, but is desirable because one file can contain several figures. Once the file is opened, click on *New Figure* button (second from the left) on the toolbar or *File* >> *Add Figure* opens new figure. Existing figures are listed in *Window* menu. Figure B.2 shows maximized *Network* figure in *Example.pf* file.

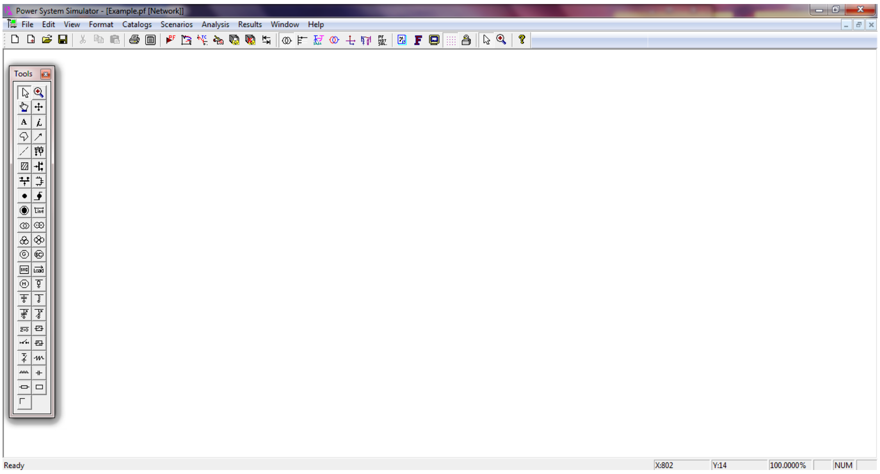

Fig. B.2 *Network* figure in *Example.pf*

Tools palette contains network elements and tools required for network design (Figure B.3). A tool-tip is displayed when the mouse is over any of the palette's button.

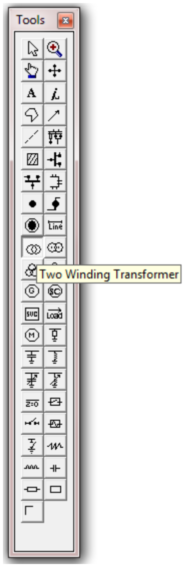

Fig. B.3 Tools palette

Power flow solution in PSS will be presented on the example of the network shown in Figure (B.4) with the following data given in per-unit:

Bus 1: $\underline{V}_1 = 1\angle 0^o$
Bus 2: $\underline{V}_2 = 1\angle 0^o$
Bus 3: $P_3 = 0.4 + 0.3V_3 + 0.3V_3^2, \quad Q_3 = 0.4 + 0.3V_3 + 0.3V_3^2$
Bus 4: $P_4 = 0.4 + 0.3V_4 + 0.3V_4^2, \quad Q_4 = 0.4 + 0.3V_4 + 0.3V_4^2$
Bus 5: $P_5 = 0.1, \quad V_5 = 0.97$
Bus 6: $P_6 = 0.1, \quad V_6 = 0.975$
Bus 8: $P_8 = 0.2, \quad Q_8 = 0.05$
Bus 9: $P_9 = 0.12, \quad Q_9 = 0.04$
Bus 11: $P_{11} = 0.2, \quad Q_{11} = 0.05$
Bus 12: $P_{12} = 0.1, \quad Q_{12} = 0.03$
Bus 13: $P_{13} = 0.12, \quad Q_{13} = 0.03$
Bus 14: $P_{14} = 0.1, \quad Q_{14} = 0.04$
Line 1-3: $R = 0.013, \quad X = 0.039$
Line 2-4:$R = 0.007, \quad X = 0.021$
Line 3-4:$R = 0.01, \quad X = 0.03$
Line 3-5:$R = 0.008, \quad X = 0.024$
Line 4-8:$R = 0.001, \quad X = 0.03$
Line 6-8:$R = 0.01, \quad X = 0.03$
Line 5-7:$R = 0.01, \quad X = 0.03$
Line 6-7:$R = 0.01, \quad X = 0.03$
Line 9-10:$R = 0.001, \quad X = 0.05$
Line 10-11:$R = 0.001, \quad X = 0.03$

Line 10-12: $R = 0.001, \quad X = 0.03$
Line 13-14: $R = 0.001, \quad X = 0.03$

Transformer 7-9: $R = 0.05, \quad X = 0.15$, initial tap position is neutral, tap side 7, regulated side 9, $V_{min} = 0.99765, \quad V_{max} = 1.01635$, step size 0.935.

Transformer 10-13: $R = 0.07, \quad X = 0.21$, initial tap position is neutral, tap side 10, regulated side 13, $V_{min} = 1.0028, \quad V_{max} = 1.0112$, step size 0.42.

Base voltage is 100kV and base power 100MVA. This information is important because quantities (except impedance) in PSS are not expressed in per unit and have be converted to actual values.

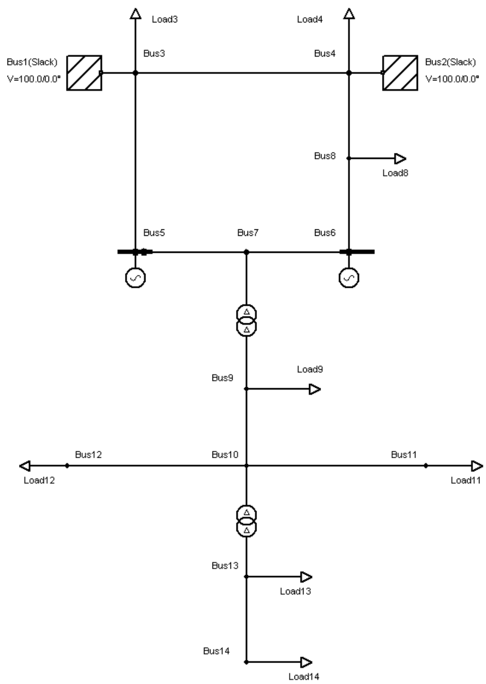

Fig. B.4 Network

Network elements that will be used are: equivalent source or slack bus (sixth button in the first column), connectivity node (eight button in the first column), line (ninth button in the second column), load (thirteenth button in the second column), horizontal busbar section (seventh button in the first column), generating unit (twelfth button in the first column) and two winding transformer (tenth button in the first column).

To begin, click equivalent source on tools palette. Move the cursor on the workspace where you want to place the slack bus and click. Then click *Selection* button on tolls palette (at the top left) to proceed. Double click on element or click *Properties* in the right click menu to edit element properties. Figure B.5 shows slack bus and properties dialog. Change the bus name into "Bus1(Slack)", set the desired voltage to $100kV$ and select all three check-boxex in the *Appearance* group to show bus name, voltage, real and reactive power.

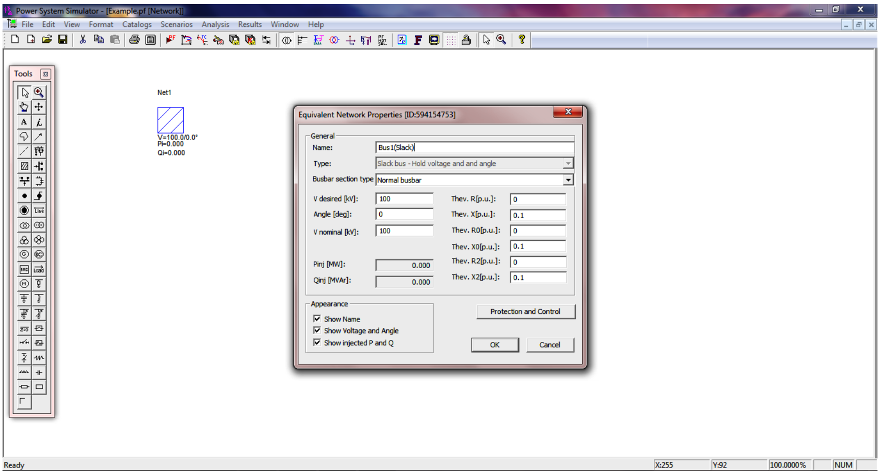

Fig. B.5 Slack bus

In the same way add connectivity node, name it "Bus3" and select check-boxes to show name and voltage. Then connect *Bus1* and *Bus3*. Select the line on tolls palette and click on the element on the *from side* of the line (in this case Bus1), then draw the line to the element connected to *to side* and click again. Click on selection button on the tools palette and then double click on the line to set its properties - name, resistance, reactance, conductance, susceptance, length, current limits and zero sequence quantities. Set $R = 0.013, \quad X = 0.039$ as given and line length to $1km$. Connect the load to *Bus3*. Select the load and rotate its arrow end to vertical position or choose *Rotate +90* from the right click menu. Set the load name, real and reactive power as shown in Figure B.6.

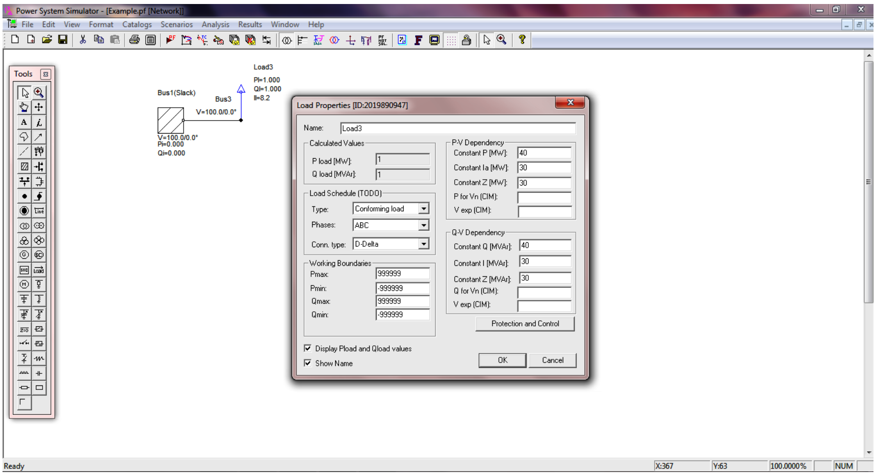

Fig. B.6 Load

Then add *Bus5* (horizontal busbar section) and connect it with *Bus3*. In the properties dialog of *Bus5* choose bus type and set its name, desired voltage and voltage limits as shown in Figure B.7. Connect the generation unit to *Bus5* and set its active power to $10MW$. You can also choose to display generator active and reactive power.

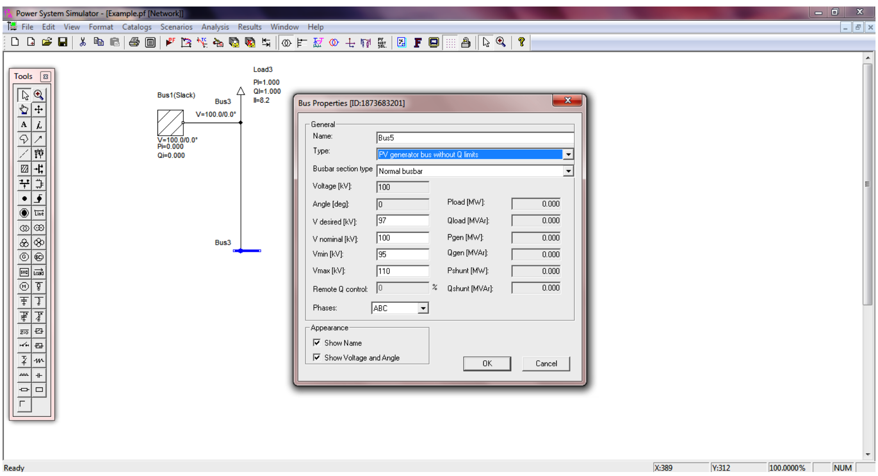

Fig. B.7 Bus

Network transformers are control two winding transformers. Since tap changer of the first transformer is placed on *Bus7* tap position is changed on *from side*. Desired voltage can be obtained as $\frac{V_{min}+V_{max}}{2}$ and corresponding tap position should be determined. In this case with the given step size of 0.935 it is-11 (Figure B.8). Set $R = 0.05, \quad X = 0.15$ as given.

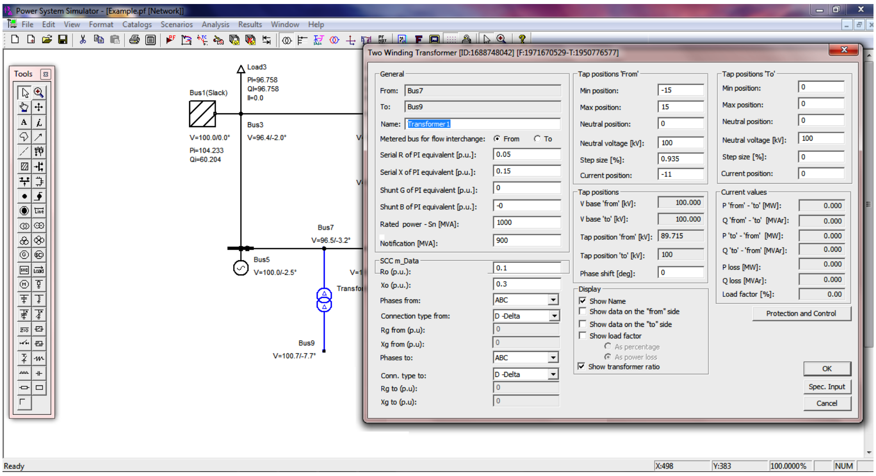

Fig. B.8 Transformer

Draw the rest of the network and click *Analysis >> Solve Power Flow* in the main menu or click *PF* button on the toolbar. Power flow analysis in PSS is conducted using Newton-Raphson method. Obtained results are shown in Figure B.9. Red and blue arrows indicate directions of real and reactive power flows respectively. Bus voltages, real and reactive power of slack bus and load are displayed. Line losses and line flows can be viewed in line properties dialog and displayed by selecting corresponding checkboxes. Red element color indicates violated voltage or current constrains.

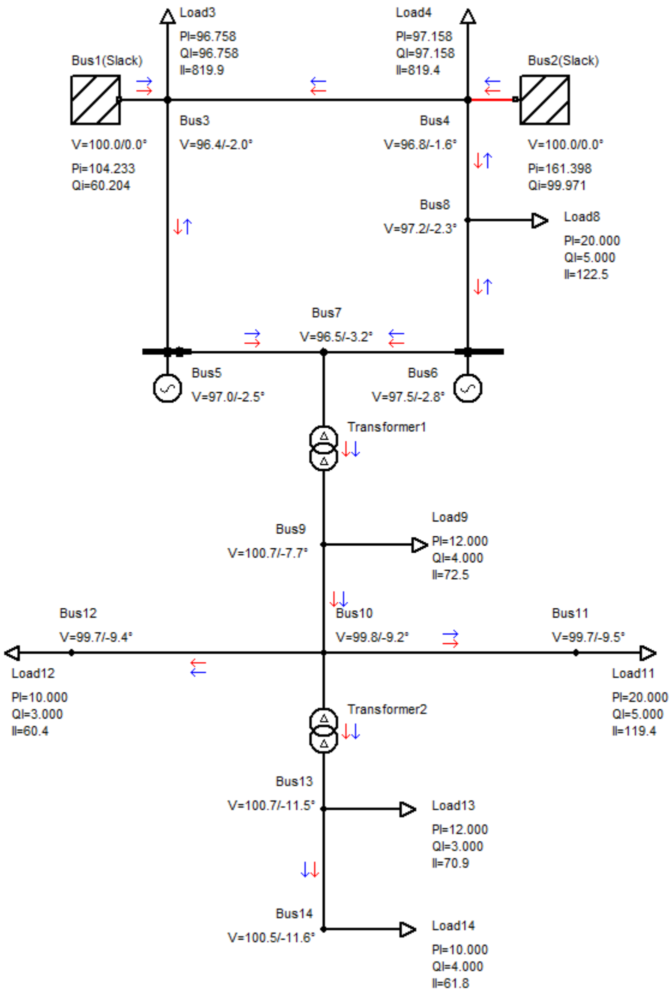

Fig. B.9 Power flow solution

Simulation details like simulation time, voltage and current violations and number of iterations are shown in *Message* pop-up shown in Figure B.10 (closed at the beginning). It can be displayed by clicking *View >> Messages* in the main menu. Additionally for each element line color, line width and font properties can be set. Different voltage levels, energization and loading can be visually differentiated by colors using the actions offered in *View* menu.

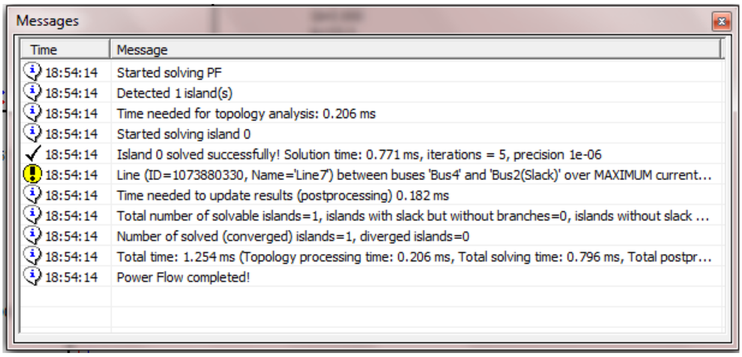

Fig. B.10 Messages

Results menu enables presentation of simulation results in the form of graph, table and view, export of the results to database and their further analysis.

References

1. C. L. Fortescue, "Method of symmetrical co-ordinates applied to the solution of polyphase networks," *Trans. of the American Institute of Electrical Engineers*, vol. 37, no. 2, pp. 1027–1140, 1918.
2. P. M. Anderson, *Analysis of faulted power systems.* IEEE press New York, 1995.
3. J. L. Blackburn, *Symmetrical components for power systems engineering.* CRC Press, 1993, vol. 85.
4. I. Dzafic, H. Neisius, and D. Ablakovic, "Multi process real-time network applications in Distribution Management System," *IPEC, 2010 Conference Proceedings*, pp. 340 –345, oct. 2010.
5. K. L. Lo and C. Zhang, "Decomposed three-phase power flow solution using the sequence component frame," *IEE Proc. - Generation, Transmission and Distribution*, vol. 140, no. 3, pp. 181–188, 1993.
6. X. P. Zhang and H. Chen, "Asymmetrical three-phase load-flow study based on symmetrical component theory," *IEE Proc. - Generation, Transmission and Distribution*, vol. 141, no. 3, pp. 248–252, 1994.
7. X. P. Zhang, "Fast three phase load flow methods," *IEEE Trans. on Power Systems*, vol. 11, no. 3, pp. 1547–1554, 1996.
8. M. A. Akher, K. M. Nor, and A. H. A. Rashid, "Development of unbalanced three-phase distribution power flow analysis using sequence and phase components," *12th International Middle-East Power System Conference*, pp. 406–411, 2008.
9. H. T. Neisius and I. Dzafic, "Three-phase transformer modeling using symmetrical components," *Innovative Smart Grid Technologies (ISGT), 2011 IEEE PES*, pp. 1–6, Jan. 2011.
10. B. C. Smith and J. Arrillaga, "Improved three-phase load flow using phase and sequence components," *IEE Proc. - Generation, Transmission and Distribution*, vol. 145, no. 3, pp. 245–250, 1998.
11. I. Dzafic and H. T. Neisius, "Generic three-phase power flow methods using symmetrical components for symmetrical and unsymmetrical power system networks," *Innovative Smart Grid Technologies (ISGT), 2011 IEEE PES*, pp. 1 –6, jan. 2011.
12. M. A. Akher, K. M. Nor, and A. H. A. Rashid, "Improved three-phase power-flow methods using sequence components," *IEEE Trans. on Power Systems*, vol. 20, no. 3, pp. 1389–1397, 2005.
13. S. Ghazali, K. M. Nor, and M. A. Akher, "Parallel sequence decoupled full newton-raphson three phase power flow," *IEEE Region 10 Conference TENCON 2009*, pp. 1–6, 2009.
14. J. M. T. Marinho and G. N. Taranto, "A hybrid three-phase single-phase power flow formulation," *IEEE Trans. on Power Systems*, vol. 23, no. 3, pp. 1063–1070, 2008.
15. C. S. Cheng and D. Shirmohammadi, "A three-phase power flow method for real-time distribution system analysis," *IEEE Trans. on Power Systems*, vol. 10, no. 2, pp. 671–679, 1995.
16. D. Shirmohammadi, H. W. Hong, A. Semlyen, and G. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Trans. on Power Systems*, vol. 3, no. 2, pp. 753–762, 1988.
17. W. H. Kersting, *Distribution system modeling and analysis.* CRC press, 2002.
18. E. R. Ramos, A. G. Expósito, and G. A. Cordero, "Quasi-coupled three-phase radial load flow," *IEEE Trans. on Power Systems*, vol. 19, no. 2, pp. 776–781, 2004.
19. I. Dzafic, H.-T. Neisius, M. Gilles, S. Henselmeyer, and V. Landerberger, "Three-phase power flow in distribution networks using fortescue transformation," *Power Systems, IEEE Transactions on*, vol. PP, no. 99, p. 1, 2012.

20. A. Tan, W.-H. E. Liu, and D. Shirmohammadi, "Transformer and load modeling in short circuit analysis for distribution systems," *IEEE Transactions on Power Systems*, vol. 12, no. 3, 1997.
21. Z. Wang, F. Chen, and J. Li, "Implementing transformer nodal admittance matrices into backward/forward sweep-based power flow analysis for unbalanced radial distribution systems," *IEEE Transactions on Power Systems*, vol. 18, no. 4, 2004.
22. S. S. Moorthy and D. Doadley, "A new phase coordinate transformer model for ybus analysis," *IEEE Transactions on Power Systems*, vol. 17, no. 4, 2002.
23. D. M. Anderson and B. F. Wollenberg, "Solving for three phase conductive isolated busbar voltages using phase component analysis," *IEEE Transactions on Power Systems*, vol. 12, no. 1, 1995.
24. P. Xiao and D. C. Yu, "A unified three-phase transformer model for distribution load flow calculations," *IEEE Transactions on Power Systems*, vol. 12, no. 2, 1997.
25. M. E. Baran and E. A. Staton, "Distribution transformer models for branch current based feeder analysis," *IEEE Transactions on Power Systems*, vol. 21, no. 1, 2006.
26. M. Todorovsky and D. Rajicic, "Handling of three-winding transformers and loads in short circuit analysis by the admittance summation method," *IEEE Transactions on Power Systems*, vol. 18, no. 3, 2003.
27. M. R. Irving and A. K. Al-Othman, "Admittance matrix of three-phase transformers with various neutral grounding configurations," *IEEE Transactions on Power Systems*, vol. 18, no. 3, 2003.
28. W. E. Dillon and M. S. Chen, "Transformer modeling in unbalanced three-phase networks," *IEEE Summer Power Meeting, Vancouver, Canada*, 1972.
29. ——, "Power system modeling," *Proc. IEEE*, vol. 62, no. 7, pp. 901–915, 1974.
30. T. H. Chen, M. S. Chen, T. Inoue, and E. A. Chebl, "Three-phase cogenerator and transformer models for distribution system analysis," *IEEE Transactions on Power Delivery*, vol. 6, no. 4, 1991.
31. B. Stott, "Review of load-flow calculation methods," *Proceedings of the IEEE*, vol. 62, no. 7, pp. 916 – 929, july 1974.
32. J. Treece, "Bootstrap gauss-seidel load flow," *Electrical Engineers, Proceedings of the Institution of*, vol. 116, no. 5, pp. 866 –870, may 1969.
33. A. Semlyen, "Fundamental concepts of a krylov subspace power flow methodology," *Power Systems, IEEE Transactions on*, vol. 11, no. 3, pp. 1528 –1537, aug 1996.
34. A. Hasan, T. Krishnaparandhama, and A. Howaidi, "Load flow analysis by g-s reduction and restoration method," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 139, no. 5, pp. 383 –386, sep 1992.
35. G. Huang and W. Ongsakul, "Managing the bottlenecks in parallel gauss-seidel type algorithms for power flow analysis," *Power Systems, IEEE Transactions on*, vol. 9, no. 2, pp. 677 –684, may 1994.
36. B. Stott, "Effective starting process for newton-raphson load flows," *Electrical Engineers, Proceedings of the Institution of*, vol. 118, no. 11, p. 1634, november 1971.
37. B. Stott, J. Jardim, and O. Alsac, "Dc power flow revisited," *Power Systems, IEEE Transactions on*, vol. 24, no. 3, pp. 1290 –1300, aug. 2009.
38. J. Britton, "Improved area interchange control for newton's method load flows," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-88, no. 10, pp. 1577 –1581, oct. 1969.
39. M. Irving and M. Sterling, "Efficient newton-raphson algorithm for load-flow calculation in transmission and distribution networks," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 134, no. 5, pp. 325 –330, september 1987.
40. A. Laha, K. Bollinger, R. Billinton, and S. Dhar, "Modified form of newton's method for faster load-flow solutions," *Electrical Engineers, Proceedings of the Institution of*, vol. 121, no. 8, pp. 849 –853, august 1974.

41. H. Le Nguyen, "Newton-raphson method in complex form [power system load flow analysis]," *Power Systems, IEEE Transactions on*, vol. 12, no. 3, pp. 1355 –1359, aug 1997.
42. S. Tripathy, G. Prasad, O. Malik, and G. Hope, "Load-flow solutions for ill-conditioned power systems by a newton-like method," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-101, no. 10, pp. 3648 –3657, oct. 1982.
43. Y. Wallach and R. Even, "Application of newton's method to load-flow calculations," *Electrical Engineers, Proceedings of the Institution of*, vol. 114, no. 3, pp. 372 –374, march 1967.
44. R. Wasley and M. Shlash, "Newton-raphson algorithm for 3-phase load flow," *Electrical Engineers, Proceedings of the Institution of*, vol. 121, no. 7, pp. 630 –638, july 1974.
45. B. Stott and O. Alsac, "Fast decoupled load flow," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-93, no. 3, pp. 859 –869, may 1974.
46. B. Stott, "Decoupled newton load flow," *Power Apparatus and Systems, IEEE Transactions on*, vol. PAS-91, no. 5, pp. 1955 –1959, sept. 1972.
47. P. Bodger, "Modified and standard back-substitution fast-decoupled power flows compared to a newton-like method for ill-conditioned systems," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 131, no. 1, pp. 23 –24, january 1984.
48. A. Garcia and M. Zago, "Three-phase fast decoupled power flow for distribution networks," *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 143, no. 2, pp. 188 –192, mar 1996.
49. A. Keyhani, A. Abur, and S. Hao, "Evaluation of power flow techniques for personal computers," *Power Systems, IEEE Transactions on*, vol. 4, no. 2, pp. 817 –826, may 1989.
50. J. Nanda, P. Bijwe, J. Henry, and V. Bapi Raju, "General purpose fast decoupled power flow," *Generation, Transmission and Distribution, IEE Proceedings C*, vol. 139, no. 2, pp. 87 –92, mar 1992.
51. D. Rajicic and A. Bose, "A modification to the fast decoupled power flow for networks with high r/x ratios," *Power Systems, IEEE Transactions on*, vol. 3, no. 2, pp. 743 –746, may 1988.
52. O. Saavedra, A. Garcia, and A. Monticelli, "The representation of shunt elements in fast decoupled power flows," *Power Systems, IEEE Transactions on*, vol. 9, no. 3, pp. 1434 –1440, aug 1994.
53. S. Tripathy, N. Rao, and A. Kumar, "Real-time monitoring of power systems using fast-decoupled load flow," *Electrical Engineers, Proceedings of the Institution of*, vol. 124, no. 7, pp. 602 –606, july 1977.
54. L. Wang and X. Lin, "Robust fast decoupled power flow," *Power Systems, IEEE Transactions on*, vol. 15, no. 1, pp. 208 –215, feb 2000.
55. R. Zimmerman and H.-D. Chiang, "Fast decoupled power flow for unbalanced radial distribution systems," *Power Systems, IEEE Transactions on*, vol. 10, no. 4, pp. 2045 –2052, nov. 1995.
56. I. Dzafic, "An object-oriented graphical framework for power system analysis," *International Conference on Power Engineering, Energy and Electrical Drives, 2007. POWERENG 2007*, pp. 255 –264, april 2007.
57. A. G. Madureira and J. A. Pecas Lopes, "Coordinated voltage support in distribution networks with distributed generation and microgrids," *IET Renewable Power Generation*, vol. 3, no. 4, pp. 439–454, December 2009.
58. P. M. S. Carvalho, P. F. Correia, and L. A. F. Ferreira, "Distributed reactive power generation control for voltage rise mitigation in distribution networks," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 766–772, May 2008.
59. I. Roytelman and V. Ganesan, "Modeling of local controllers in distribution network applications," *IEEE Transactions on Power Delivery*, vol. 15, no. 4, pp. 1232–1237, October 2000.

60. W. H. Kersting, "Distribution feeder voltage regulation control," *IEEE Transactions on Industry Applications*, vol. 46, no. 2, pp. 620–626, March-April 2010.
61. T. Senjyu, Y. Miyazato, A. Yona, N. Urasaki, and T. Funabashi, "Optimal distribution voltage control and coordination with distributed generation," *IEEE Transactions on Power Delivery*, vol. 23, no. 2, pp. 1236–1242, April 2008.
62. W. Kidd and J. L. Carr, "The application of automatic voltage and switch control to electrical distribution systems," *Journal of the Institution of Electrical Engineers*, vol. 74, no. 448, pp. 285–297, April 1934.
63. J. J. Grainger and S. Civanlar, "Volt/var control on distribution systems with lateral branches using shunt capacitors and voltage regulators Part I: The overall problem," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, no. 11, pp. 3278–3283, November 1985.
64. P. A. N. Garcia, J. L. R. Pereira, and J. Carneiro, S, "Voltage control devices models for distribution power flow analysis," *IEEE Transactions on Power Systems*, vol. 16, no. 4, pp. 586–594, November 2001.
65. K. P. Schneider and J. C. Fuller, "Voltage control devices on the IEEE 8500 node test feeder," April 2010, pp. 1–6.
66. R. Lawrence, "Voltage optimization," *IEEE Industry Applications Magazine*, vol. 12, no. 5, pp. 28–33, September-October 2006.
67. I. Dzafic, P. Mohapatra, and H.-T. Neisius, "Modeling of automatic local controllers in three phase load flow calculation," jan. 2011, pp. 1 –5.
68. G. H. Landis, "Voltage regulation and control in the development of a rural distribution system," *Transactions of the American Institute of Electrical Engineers*, vol. 57, no. 9, pp. 541–547, September 1938.
69. S. Conti, S. Raiti, and G. Vagliasindi, "Voltage sensitivity analysis in radial MV distribution networks using constant current models," July 2010, pp. 2548–2554.
70. R. Yan and T. K. Saha, "Voltage variation sensitivity analysis for unbalanced distribution networks due to photovoltaic power fluctuations," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 1078–1089, May 2012.
71. I. Roytelman and V. Ganesan, "Modeling of local controllers in distribution network applications," *IEEE Transactions on Power Delivery*, vol. 15, no. 4, pp. 1232–1237, October 2000.
72. R. Wamkeue, I. Kamwa, and M. Chacha, "Line-to-line short-circuit-based finite-element performance and parameter predictions of large hydrogenerators," *Energy Conversion, IEEE Transactions on*, vol. 18, no. 3, pp. 370 – 378, sept. 2003.
73. Z. Sun, J. Wang, D. Howe, and G. Jewell, "Analytical prediction of the short-circuit current in fault-tolerant permanent-magnet machines," *Industrial Electronics, IEEE Transactions on*, vol. 55, no. 12, pp. 4210 –4217, dec. 2008.
74. R. Chokhawala, J. Catt, and L. Kiraly, "A discussion on igbt short-circuit behavior and fault protection schemes," *Industry Applications, IEEE Transactions on*, vol. 31, no. 2, pp. 256 –263, mar/apr 1995.
75. J. Morren and S. de Haan, "Short-circuit current of wind turbines with doubly fed induction generator," *Energy Conversion, IEEE Transactions on*, vol. 22, no. 1, pp. 174 –180, march 2007.
76. O. R. Schurig, "Experimental determination of short-circuit currents in electric power networks," *American Institute of Electrical Engineers, Transactions of the*, vol. XLII, pp. 10 –23, jan. 1923.
77. J. Dunki-Jacobs, B. Lam, and R. Stratford, "A comparison of ansi-based and dynamically rigorous short-circuit current calculation procedures," *Industry Applications, IEEE Transactions on*, vol. 24, no. 6, pp. 1180 –1194, nov/dec 1988.
78. Z. Posedel, "Inspection of stator cores in large machines with a low yoke induction method-measurement and analysis of interlamination short-circuits," *Energy Conversion, IEEE Transactions on*, vol. 16, no. 1, pp. 81 –86, mar 2001.

79. J. Mutale and G. Strbac, "Transmission network reinforcement versus facts: an economic assessment," *Power Systems, IEEE Transactions on*, vol. 15, no. 3, pp. 961 –967, aug 2000.
80. Y. Li, C. Rehtanz, S. Ruberg, L. Luo, and Y. Cao, "Wide-area robust coordination approach of hvdc and facts controllers for damping multiple interarea oscillations," *Power Delivery, IEEE Transactions on*, vol. 27, no. 3, pp. 1096 –1105, july 2012.
81. Y. Xiao, Y. Song, and Y. Sun, "Power flow control approach to power systems with embedded facts devices," *Power Systems, IEEE Transactions on*, vol. 17, no. 4, pp. 943 – 950, nov 2002.
82. T. Orfanogianni and R. Bacher, "Steady-state optimization in power systems with series facts devices," *Power Systems, IEEE Transactions on*, vol. 18, no. 1, pp. 19 – 26, feb 2003.
83. Y. Xiao, Y. Song, C.-C. Liu, and Y. Sun, "Available transfer capability enhancement using facts devices," *Power Systems, IEEE Transactions on*, vol. 18, no. 1, pp. 305 – 312, feb 2003.
84. L. Dong, M. Crow, Z. Yang, C. Shen, L. Zhang, and S. Atcitty, "A reconfigurable facts system for university laboratories," *Power Systems, IEEE Transactions on*, vol. 19, no. 1, pp. 120 – 128, feb. 2004.
85. P. Yan and A. Sekar, "Steady-state analysis of power system having multiple facts devices using line-flow-based equations," *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 152, no. 1, pp. 31 –39, jan. 2005.
86. T. Sidhu, R. Varma, P. Gangadharan, F. Albasri, and G. Ortiz, "Performance of distance relays on shunt-facts compensated transmission lines," *Power Delivery, IEEE Transactions on*, vol. 20, no. 3, pp. 1837 – 1845, july 2005.
87. P. Yan and A. Sekar, "Analysis of radial distribution systems with embedded series facts devices using a fast line flow-based algorithm," *Power Systems, IEEE Transactions on*, vol. 20, no. 4, pp. 1775 – 1782, nov. 2005.
88. U. Mhaskar and A. Kulkarni, "Power oscillation damping using facts devices: modal controllability, observability in local signals, and location of transfer function zeros," *Power Systems, IEEE Transactions on*, vol. 21, no. 1, pp. 285 – 294, feb. 2006.
89. B. Kumar, S. Singh, and S. Srivastava, "Placement of facts controllers using modal controllability indices to damp out power system oscillations," *Generation, Transmission Distribution, IET*, vol. 1, no. 2, pp. 209 –217, march 2007.
90. H. Johal and D. Divan, "Design considerations for series-connected distributed facts converters," *Industry Applications, IEEE Transactions on*, vol. 43, no. 6, pp. 1609 –1618, nov.-dec. 2007.
91. M. Haque, "Evaluation of first swing stability of a large power system with various facts devices," *Power Systems, IEEE Transactions on*, vol. 23, no. 3, pp. 1144 –1151, aug. 2008.
92. R. Wibowo, N. Yorino, M. Eghbal, Y. Zoka, and Y. Sasaki, "Facts devices allocation with control coordination considering congestion relief and voltage stability," *Power Systems, IEEE Transactions on*, vol. 26, no. 4, pp. 2302 –2310, nov. 2011.
93. A. Lashkar Ara, A. Kazemi, and S. Nabavi Niaki, "Multiobjective optimal location of facts shunt-series controllers for power system operation planning," *Power Delivery, IEEE Transactions on*, vol. 27, no. 2, pp. 481 –490, april 2012.
94. D. Gotham and G. Heydt, "Power flow control and power flow studies for systems with facts devices," *Power Systems, IEEE Transactions on*, vol. 13, no. 1, pp. 60 –65, feb 1998.
95. A. Edris, "Facts technology development: an update," *Power Engineering Review, IEEE*, vol. 20, no. 3, pp. 4 –9, mar 2000.
96. N. Li, Y. Xu, and H. Chen, "Facts-based power flow control in interconnected power system," *Power Systems, IEEE Transactions on*, vol. 15, no. 1, pp. 257 –262, feb 2000.

97. S. Gerbex, R. Cherkaoui, and A. Germond, "Optimal location of multi-type facts devices in a power system by means of genetic algorithms," *Power Systems, IEEE Transactions on*, vol. 16, no. 3, pp. 537 –544, aug 2001.

Index