MANDATORY HOMEWORK 1 STAT 211 - H21

Teaching assistant: Robert Clay Glastad

JANUARY 27, FEBRUARY 03: 0815 - 1000

VERSION-A-210127 2021-02-01-23-11-22

DUE AT THE END OF FEBRUARY 08

PROBLEM 1.1
[BD, Exercise 1.4, page 35]

Let $\{Z_t,\, t \in \mathbb{Z}\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2, and let a, b, and c be constants. Which, if any, of the following processes are stationary? For each stationary process specify the mean and autocovariance function.

a) $X_t = a + bZ_t + cZ_{t-2}$
b) $X_t = a + bZ_1 \cos(ct) + bZ_2 \sin(ct)$
c) $X_t = a + bZ_t \cos(ct) + bZ_{t-1} \sin(ct)$
d) $X_t = a + bZ_0$
e) $X_t = Z_0 \cos(ct)$
f) $X_t = Z_t\, Z_{t-1}$

PROBLEM 1.2
[BD, Exercise 1.5, page 35]

Let $\{X_t\}$ be the moving-average process of order 2 given by $X_t = Z_t + \theta Z_{t-2}$, where $\{Z_t\}$ is $\mathrm{WN}(0,1)$.

a) Find the autocovariance and autocorrelation functions for this process when $\theta = 0.8$.
b) Compute the variance of the sample mean $(X_1 + X_2 + X_3 + X_4)/4$ when $\theta = 0.8$.
c) Repeat (a) when $\theta = -0.8$ and compare your answer with the result obtained in (a).

PROBLEM 1.3
[BD, Exercise 1.6, page 35]

Let $\{X_t\}$ be the AR(1) process defined in Example 1.4.5.

a) Compute the variance of the sample mean $(X_1 + X_2 + X_3 + X_4)/4$ when $\phi = 0.9$ and $\sigma^2 = 1$.
b) Repeat (a) when $\phi = -0.9$ and compare your answer with the result obtained in (a).

PROBLEM 1.4
[BD, Exercise 1.7, page 35]

If $\{X_t\}$ and $\{Y_t\}$ are uncorrelated stationary sequences, i.e., if X_r and Y_s are uncorrelated for every r and s, show that $\{X_t + Y_t\}$ is stationary with autocovariance function equal to the sum of the autocovariance functions of $\{X_t\}$ and $\{Y_t\}$.

Problem 1.5
[BD, Exercise 1.17, page 36]

Load the dataset **deaths** in R using the **read.table** function. Plot the data. Also create a histogram of the data using the R function **hist** . Plot the sample autocorrelation function using the **acf** function. The presence of a strong seasonal component with period 12 is evident in the graph of the data and in the sample autocorrelation function.

Problem 1.6
[BD, Exercise 1.18, page 37]

We are still studying the dataset deaths. In this exercise, you are supposed to reproduce the figures 1-24 and 1-25 in [BD, pp. 27-28] . In 1.17, we found a period of length 12. Fit a seasonal component using the procedure described in section 1.5.2.1 on page 26. You may use the following functions or write your own:

```
# Function for calculating a moving average when d is even
ma <- function(x,n=12){filter(x,c(.5,rep(1,n-1),.5)/n, sides=2)}
# Function for finding the seasonal component
seasonal.component <- function(x){
  # First step: detrending
detrended <- deaths-ma(deaths)
# Second step: Calculating sesonal component from detrended data
wt<-rowMeans(matrix(detrended[!is.na(detrended)],
nrow=12,byrow=FALSE))
st<-(wt-mean(wt))[c(7:12,1:6)] #seasonal component
return(st)
}
```

Plot the deseasonalized data (as in figure 1-24). Fit a quadratic trend (polynomial of order two) to the deseasonalized data and add the curve to the plot you just created. The trend should be $\widehat{m} = 9952 - 71.82t + 0.8260t2$ for $1 \leq t \leq 72$. This can be done using the following code:

```
 M <- poly(1:72, degree=2, raw=TRUE)
trend<-lm(detrended ~ M) # Re-estimating trend of the detrended data
```

Plot the sample autocorrelation function of Yt. Forecast the data for the next 24 months without allowing for this dependence, based on the assumption that the estimated seasonal and trend components are true values and that Yt is a white noise sequence with zero mean. Calculate $\widehat{s}_{72+k}$ for $k = 1, \ldots, 24$ and do the forecasting by

$$\widehat{X}_t = \widehat{m}_{72+k} + \widehat{s}_{72+k}, \qquad k = 1, \ldots, 24.$$

Plot the original data with the forecasts appended. Later we shall see how to improve on these forecasts by taking into account the dependence in the series Yt. *Hint: To calculate* $\widehat{m}_{72+k}$ *the following code may be useful:*

```
 M <- poly(72 + 1:24, 2, raw=TRUE)
m.ha<- predict(trend, newdata= M)
```

References

Peter J Brockwell and Richard A Davis. *Introduction to time series and forecasting; 3rd ed.* Springer texts in statistics. Springer, Cham, 2016. doi: 10.1007/978-3-319-29854-2. URL http://cds.cern.ch/record/2213342.