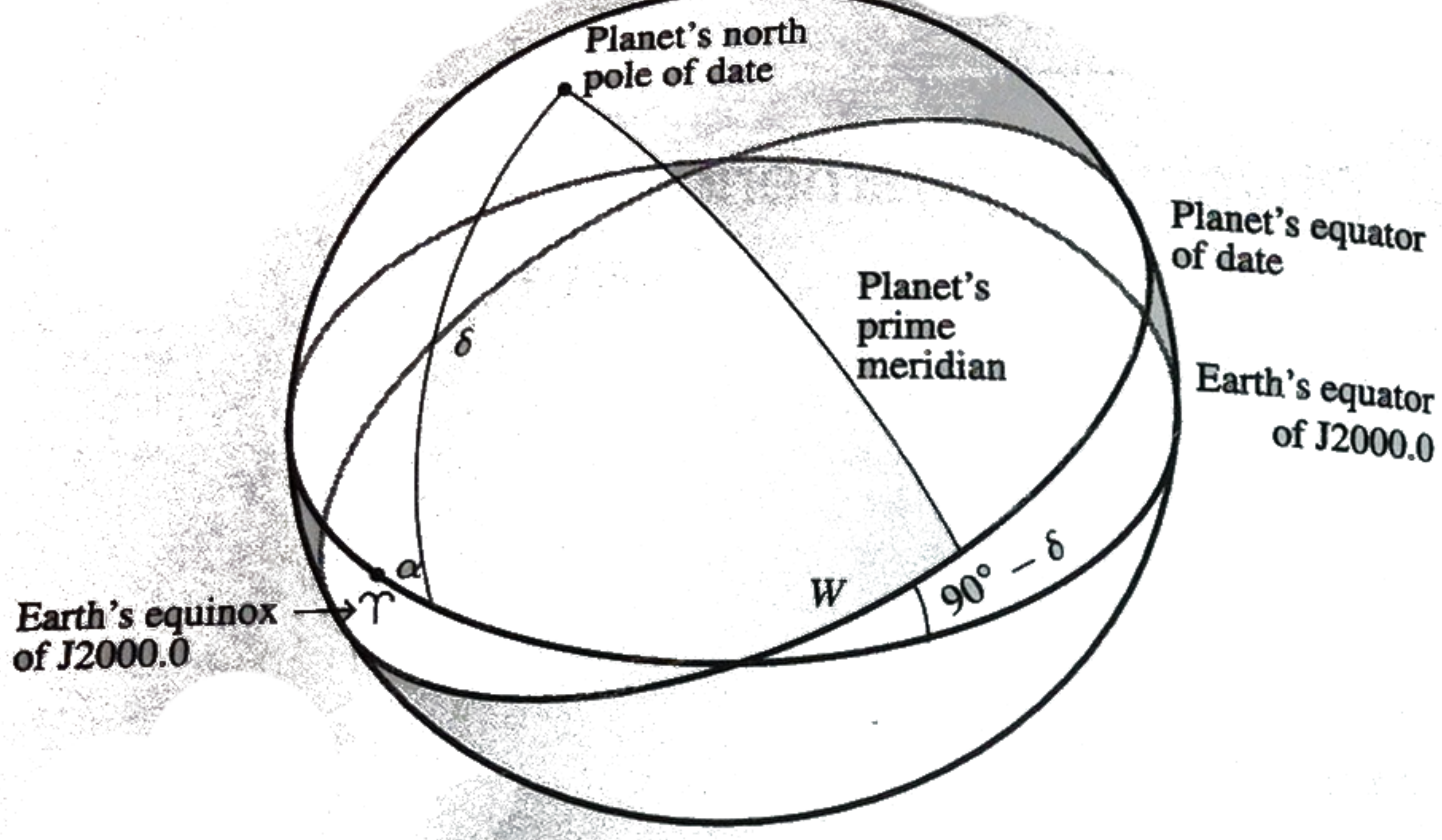

Figure 7.11.1
The position of the north pole and prime meridian of a planet in Earth equatorial coordinates

The quantities α_0 and δ_0 are the right ascension and declination of date of the object's north pole. The IAU definition of the north pole of a planet, from The IAU Working Group on Cartographic Coordinates and Rotational Elements of Planets and Satellites (Davies *et al.*, 1991), is the rotation axis that lies on the north side of the invariable plane of the solar system. α_0 and δ_0 may vary slowly with time because of the precession of the object about its polar axis. In the absence of other information, the axis of rotation is assumed to be normal to the mean orbital plane. This assumption is used for Mercury and most of the satellites.

The angle W is measured along the object's equator, in a counterclockwise direction when viewed from the north pole, from the ascending node of the object's equator on the Earth's mean equator of J2000.0 to the point where the prime meridian of the object crosses its equator. The right ascension of the node (Q) is at $6^h+\alpha_0$, and the inclination of the object's equator to the standard equator is $90°-\delta_0$. Generally, the prime meridian is assumed to rotate uniformly with the object, hence W varies linearly with time. For objects without observable, fixed surface features the adopted expression for W defines the prime meridian and is not subject to correction. However, if a cartographic position for the prime meridian is assigned to an object, that is, if the position of the prime meridian is defined by a suitable observable surface feature, the expression for W is chosen so that the ephemeris position follows the motion of the cartographic position. Although the definition of the ephemeris prime meridian is chosen to follow the cartographic prime meridian as closely as possible, there may be errors in the rotational elements of the object that will cause the cartographic position to drift away from the ephemeris position by a small amount ΔW. The angle ΔW is measured positively counterclockwise from the ephemeris position of W as viewed from above the north pole. For a prime meridian that has a cartographic definition, the ephemeris definition of W may be changed as more accurate information on the motion of the cartographic prime meridian is obtained.

The definition of the north pole requires the rotation about the pole to be classified as either direct or retrograde. Direct rotation is counterclockwise as viewed from above the planet's north pole, while retrograde rotation is clockwise. For a planet with direct rotation the angle W increases with time. For most of the satellites it is assumed that the rotation period is equal to the mean orbital period.

Information on the point defining the prime meridian and pole for the Sun can be found in Carrington (1863) and Section 7.2 of this supplement. The definition for the prime meridian and pole of each planet can be found in Davies *et al.* (1991) and Sections 7.41–7.48 of this supplement. Information defining the prime meridian and pole of the primary planetary satellites can be found in Davies *et al.* (1991) and Sections 7.3 and 7.51 through 7.56 of this supplement. In general, the expressions defining α_0, δ_0, and W should be accurate to one tenth of a degree. Two decimal places are given, however, to assure consistency when changing coordinate systems. Zeros are added to the rate values ($\dot{W}$) for computational consistency and are not an indication of significant accuracy. Three significant digits beyond the decimal point are given in the expressions for the Moon and Mars, reflecting the greater confidence in their accuracy. The recommended coordinate system for the Moon is the mean Earth-polar axis system in contrast to the principal axis system. The Earth-polar axis system uses the pole of rotation of the Moon and the mean axis of the Moon, which points toward the center of the Earth. The principal axis system uses the axes of the rotation and the defined 0° meridian of the object.

Both planetocentric and planetographic systems of coordinates are used in the study of the planets and satellites. Both systems are based on the same fundamental axis of rotation but differ, as explained below, in their definitions of latitude, longitude, and range. Planetocentric coordinates are used for general purposes and are based on a right-handed system of axes with its origin at the center of mass of the object. Planetographic coordinates are used for cartographic purposes and depend on the adoption of a reference surface, usually a spheroid, that approximates an equipotential surface of the object. The latitude and ranges for both of these coordinate systems are shown in Figure 7.11.2.

The reference surfaces for most of the planets are spheroids for which the equatorial radius, a, is larger than the polar radius, b (see Figure 7.11.2). For some of the planets and most of the satellites the reference surface is a sphere. For some satellites, such as Phobos, Deimos, and Hyperion, the reference surface should be

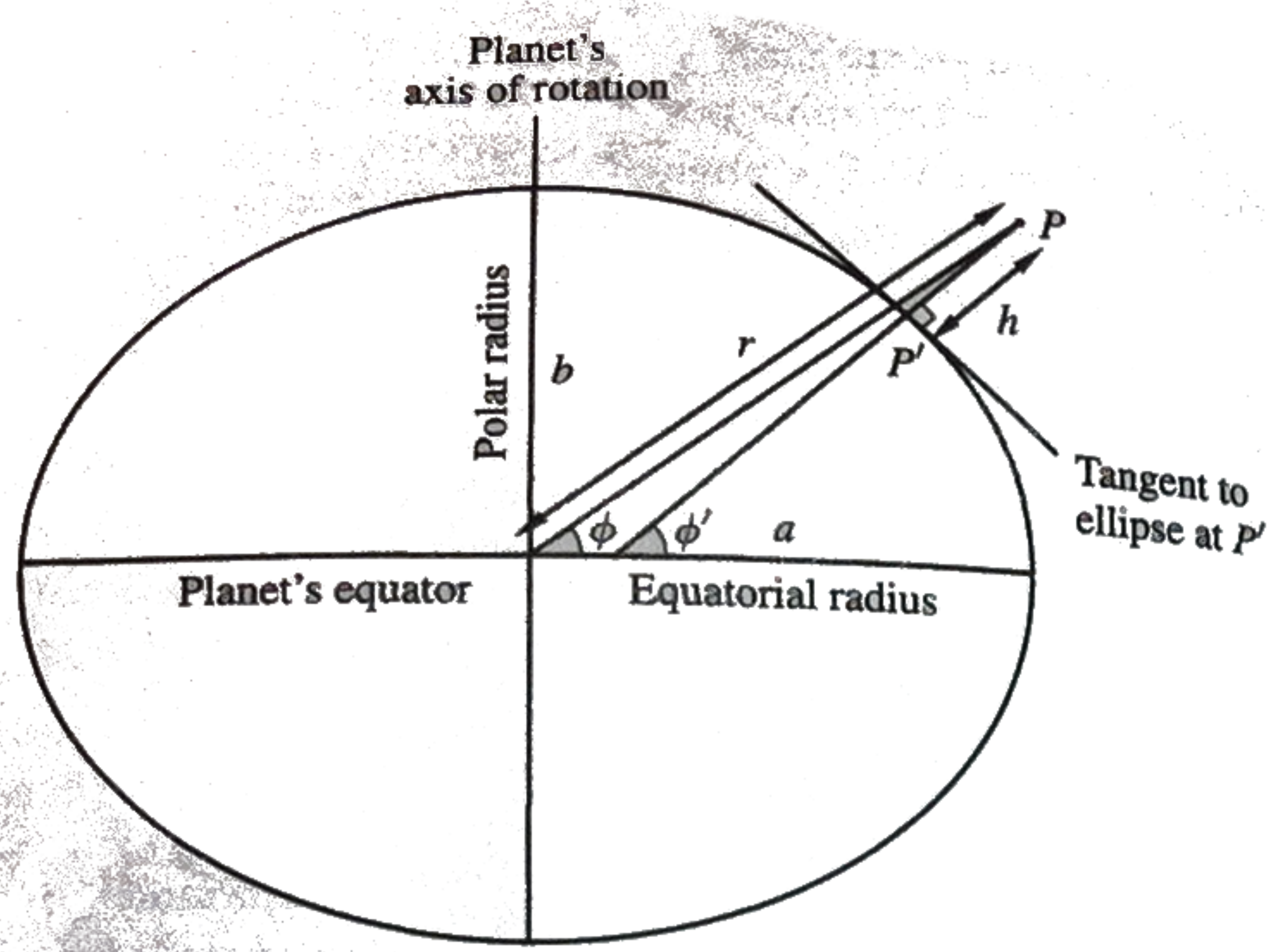

Figure 7.11.2
Planetocentric and planetographic coordinate systems. The quantities shown are: ϕ, the planetocentric latitude; r, the distance from the center of mass; ϕ' the planetographic latitude; and h, the height of a point above or below the planetographic reference surface. Also included are a, the equatorial radius of the object; and b, the polar radius.

triaxial. Triaxial ellipsoids would make many computations more complicated, especially those related to map projections. Therefore, spherical or spheroidal reference surfaces are usually used in mapping programs. The polar axis for each reference surface is assumed to be the mean axis of rotation as defined by the adopted rotational elements since the present accuracy of measurement does not reveal a deviation between the axis of rotation and the axis of figure for most of the planets. The radii and flattening of the planets are found in *The Astronomical Almanac* and in Section 7.4 of this supplement. The mean radii of the natural satellites of the planets are also found on page F3 of *The Astronomical Almanac*.

Many small bodies in the solar system (satellites, asteroids, and comet nuclei) have very irregular shapes. Sometimes spherical reference surfaces are used to preserve projection properties. Orthographic projections are often adopted for cartographic purposes because they preserve the irregular appearance of the body without artificial distortion.

In both systems, the fundamental reference z-axis is the mean axis of rotation with the positive direction being toward the north pole. The equator is the plane, passing through the center of mass of the planet, perpendicular to the z-axis. The x-axis is defined by the intersection of the plane of the equator and the plane containing the prime meridian. The choice of the prime meridian is arbitrary. The

y-axis is then orthogonal to the other two axes and makes a right-handed coordinate system. The angular coordinates used in these two systems are latitude and longitude. The range of the planetocentric coordinate system is the distance from the center of mass, and the range of the planetographic system is the height above or below the reference surface.

Latitude (see Figure 7.11.2) is measured north and south of the equator with the northern latitudes being designated as positive and the southern latitudes as negative. The *planetocentric latitude* of a point on the surface of a planet is the angle ϕ between the equatorial plane and the line segment connecting the point to the center of mass. The *planetographic latitude* of a point is the angle ϕ' between the line segment normal to the equipotential surface from the point in question to the equatorial plane. Since the equipotential surface is usually a spheroid, the shape of the spheroid is related to that of a sphere by its flattening, f,

$$f = (a - b) / a, \tag{7.11–1}$$

where a is the equatorial radius of the object and b is the polar radius of the object. For a spherical object, the planetocentric and planetographic latitudes are the same.

The values for the radii of the planets and axes given in Sections 7.41–7.48 and in *The Astronomical Almanac* are derived by different methods, and do not always refer to common definitions. Some radii use star or spacecraft occultation measurement, some use limb fitting, and some use control-network computations. For example, the spheroid for the Earth refers to mean sea level, a definition that can be used only for this planet. The radii and axes of the large gaseous planets, Jupiter, Saturn, Uranus, and Neptune refer to a one-bar pressure surface. The radii in the tables are not necessarily the appropriate values to be used in dynamical studies; the radii actually used to derive a value of J_2, for example, should always be used in conjunction with that value.

Longitude is measured along the equator from the prime meridian. *Planetocentric longitudes*, λ, are measured positively in the counterclockwise direction when viewed from the north pole. *Planetographic longitudes*, λ', are measured positively in the direction opposite to the rotation of the object. For an object with a direct rotation, the planetographic longitude increases in the clockwise direction when viewed from above the north pole. Planetocentric longitudes are measured from the ephemeris position of the prime meridian as defined by the rotational elements, but the planetographic longitudes are measured from the cartographic position of the prime meridian as defined by the adopted longitude of some clearly observable surface feature. For the Earth, the Sun, and the Moon, longitudes are measured from 0° to 180° east and west, and east longitude (counterclockwise when viewed from above the north pole) is commonly considered to be positive.

The surface brightness (SB) of an object is the average brightness value for the illuminated portion of the apparent disk. The units for the surface brightness are visual magnitudes per square arcsecond. This is represented mathematically by

$$SB = V + 2.5\log_{10}(k\pi ab'), \tag{7.12-3}$$

where a is the equatorial radius of the object, b' is the apparent polar radius of the object that will be given in equation 7.12–6, and k is the fraction of the object illuminated that will be given in Equation 7.12–32.

The apparent disk of an object is always an ellipse. The apparent flattening is always less than or equal to the flattening of the object itself (depending on the apparent tilt of the objects axis). To derive the apparent polar radius and the apparent flattening of an object, start with the equation for an ellipse. The ellipse can be represented parametrically by

$$\begin{aligned} r &= (x^2 + y^2)^{1/2}, \\ x &= a\cos\gamma, \\ y &= b\sin\gamma, \end{aligned} \tag{7.12-4}$$

where x is the length of the projection of the point along the major axis from the center of the ellipse, y is the length of the projection of the point along the minor axis from the center, a is the length of the major axis, b is the length of the minor axis, and γ is the angle between the line segment connecting the point with the center of the ellipse and the semi-major axis (see Figure 7.12.2). For a planet, the apparent polar radius is the length of the radius, R, whose planetocentric latitude is the same as the planetocentric colatitude of the sub-Earth point, and γ is the latitude of the sub-Earth point. Using the definitions for x and y in the equation for R

$$R = (a^2\cos^2\gamma + b^2\sin^2\gamma)^{1/2}. \tag{7.12-5}$$

Rearranging the definition for flattening, Equation 7.11–1, gives

$$b = a(1 - f). \tag{7.12-6}$$

So

$$\begin{aligned} R &= (a^2\cos^2\gamma + a^2(1-f)^2\sin^2\gamma)^{1/2} \\ &= a(\cos^2\gamma + \sin^2\gamma - 2f\sin^2\gamma + f^2\sin^2\gamma)^{1/2} \\ &= a(1 - 2f\sin^2\gamma + f^2\sin^2\gamma)^{1/2} \end{aligned} \tag{7.12-7}$$

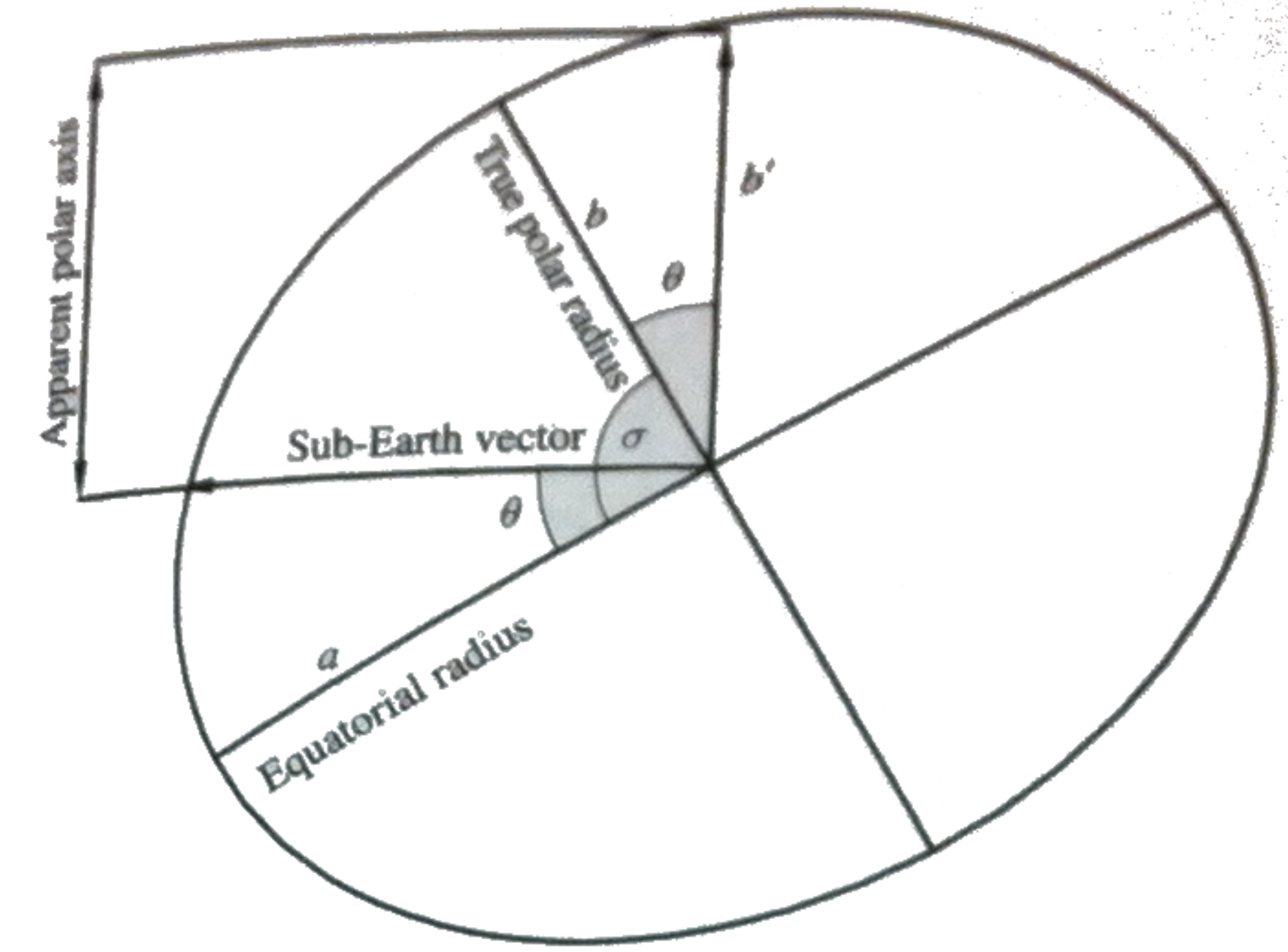

Figure 7.12.2
The geometric appearance of the apparent polar radius, b'

Using the binomial expansion:

$$b' = a[1 - \frac{1}{2}(2f\sin^2\gamma - f^2\sin^2\gamma) + \frac{1}{8}(2f\sin^2\gamma - f^2\sin^2\gamma)^2 - \cdots]. \tag{7.12-8}$$

However, for the planets $f \ll 1$, so terms of order f^2 or larger will be negligible, and

$$\begin{aligned} R &\approx a(1 - f\sin^2\gamma) \\ &= a(1 - f\cos^2\theta) \\ &= a\{1 - f[(\mathbf{b}'\cdot\mathbf{b})/(|\mathbf{b}'||\mathbf{b}|)]^2\}, \end{aligned} \tag{7.12-9}$$

where $\mathbf{b}'$ is the vector from the planetocenter to the apparent polar axis, $\mathbf{b}$ is the vector from the planetocenter to the true polar axis, and θ is the planetocentric colatitude of $\mathbf{b}'$. The apparent flattening of the planet will then be

$$f' = (a - b')/a. \tag{7.12-10}$$

The remaining tabulated quantities are most easily understood from Figure 7.12.3, which shows the apparent disk of an object as it is seen from the Earth. At the center of the apparent disk is the sub-Earth point, e. Other points of reference are the

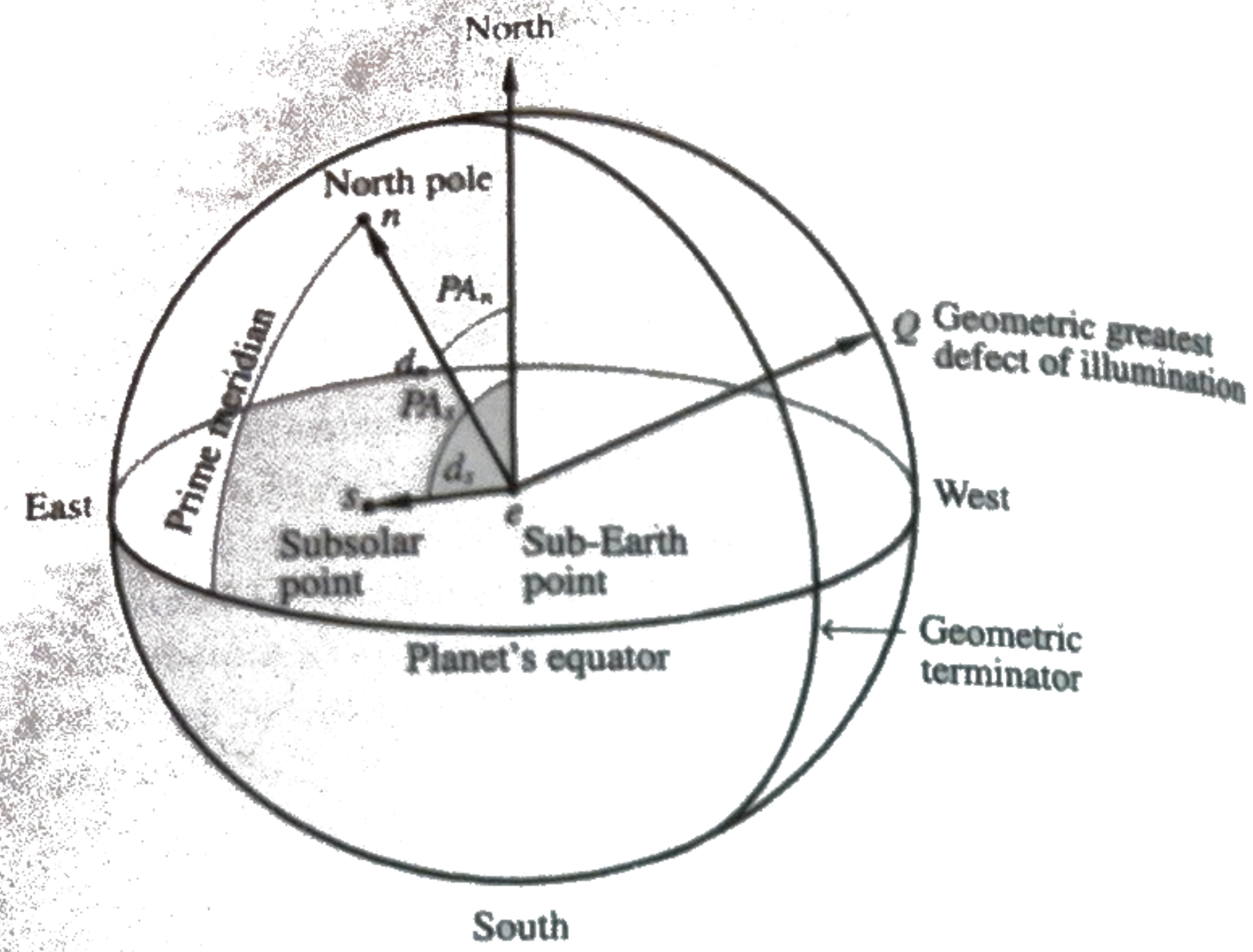

Figure 7.12.3
The disk of a planet as seen by an observer on the Earth. The quantities shown are: e, the sub-Earth point; s, the subsolar point; d_s the apparent distance from the sub-Earth point to the subsolar point; PA_s the position angle of the subsolar point; n, the north pole of the planet; d_n the apparent distance from the north pole to the sub-Earth point; PA_n, the position angle of the north pole; and Q, the geometric maximum defect of illumination.

subsolar point, s, and the north pole, n. For most of the planets, the planetographic
longitude and latitude of the points e and s are tabulated. The apparent positions
of the subsolar point, s, and the north pole, n, with respect to the apparent center
of the disk (point e) are tabulated by the apparent distances, d_n and d_s, and posi-
tion angles, PA_n and PA_s for n and s, respectively. The position angle is measured
eastward from north, N, on the celestial sphere. The direction north being defined
by the great circle on the celestial sphere passing through the center of the object's
apparent disk and the true celestial north pole of date. The apparent distance of
a tabulated point is listed as positive if the point lies on the visible hemisphere of
the object and negative if the point is on the far side of the object. Thus, as point
n or s passes from the visible hemisphere to the far side, or vice versa, the sign of
the distance changes abruptly, but the position angle varies continuously. However,
when the point passes close to the center of the disk, the sign of the distance re-
mains unchanged, but both the distance and position angle may vary rapidly and
may appear to be discontinuous in the fixed interval tabulations.

The planetographic coordinates of the sub-Earth and subsolar points are calcu-
lated in the following manner. First, the planetary ephemerides are used to calculate

the heliocentric ecliptic rectangular coordinates for the planet, $\mathbf{r} = (x, y, z)$, and the Earth, $\mathbf{r}_e = (x_e, y_e, z_e)$ at a given Terrestrial Julian Date (TJD). The vector from the Earth to the object is then determined by subtracting $\mathbf{r}_e$ from $\mathbf{r}$:

$$\begin{aligned} \mathbf{d} &= \mathbf{r} - \mathbf{r}_e \\ &= (x - x_e, y - y_e, z - z_e) \\ &= (x_d, y_d, z_d). \end{aligned} \tag{7.12–11}$$

Next, the light-time correction is found by determining the length of $\mathbf{d}$ and dividing by the speed of light, c, to obtain the time:

$$\begin{aligned} |\mathbf{d}| &= (\mathbf{d} \cdot \mathbf{d})^{1/2} = (x_d^2 + y_d^2 + z_d^2)^{1/2}, \\ t &= |\mathbf{d}| / c. \end{aligned} \tag{7.12–12}$$

The difference $TJD - t$ is calculated to approximate the time at which the image of the planet visible at the Earth at TJD left the planet. This time, $TJD - t$, is used to determine the approximate position of the planet, $\mathbf{r}_1 = (x_1, y_1, z_1)$, as seen from the Earth at time TJD. If desired, the previous process can be iterated to produce a more accurate planetary position; however, planetary motions are slow enough that to the accuracy of *The Astronomical Almanac* only one iteration is required. The position of the Earth, $\mathbf{r}_e$, and the apparent position of the planet, $\mathbf{r}_1$, are then transformed from the mean equator and ecliptic of J2000.0 to the equator and ecliptic of date using the precession routines described in Section 3.21 and the nutation routines described in Section 3.22. The precessed and nutated position vectors are designated $\mathbf{r}_{ed}$ for the Earth and $\mathbf{r}_d$ for the planet. A new vector $\mathbf{d}$ is computed by subtracting $\mathbf{r}_{ed}$ from $\mathbf{r}_d$ as in Equation 7.12–11 and then correcting for aberration in position given in Section 3.25. The unit vector along the planet–Sun line, $\mathbf{j}$, is a vector of unit length denoting the direction of the Sun from the planet. The vector $\mathbf{j}$ is found by dividing $-\mathbf{r}_d$ by its length, $|\mathbf{r}_d|$. The unit vector along the planet–Earth line, $\mathbf{j}_d$, is found in a similar manner.

The position of the north pole of the planet, (α_0, δ_0), is then found from Davies et al. (1991). These coordinates are then precessed and nutated to the ecliptic and equinox of date, (α_d, δ_d). A unit vector, $\mathbf{\tilde{n}}$, in the ecliptic rectangular coordinates is then calculated:

$$\mathbf{\tilde{n}} = (\cos\delta_d \cos\alpha_d, \cos\delta_d \sin\alpha_d, \sin\delta_d). \tag{7.12–13}$$

This vector gives the planetocentric direction of the north pole of the planet in the coordinates of the equator and ecliptic of date.

Finally, the ecliptic rectangular coordinates of the unit vector pointing from the planet's center to the intersection of the planet's prime meridian and the equator,

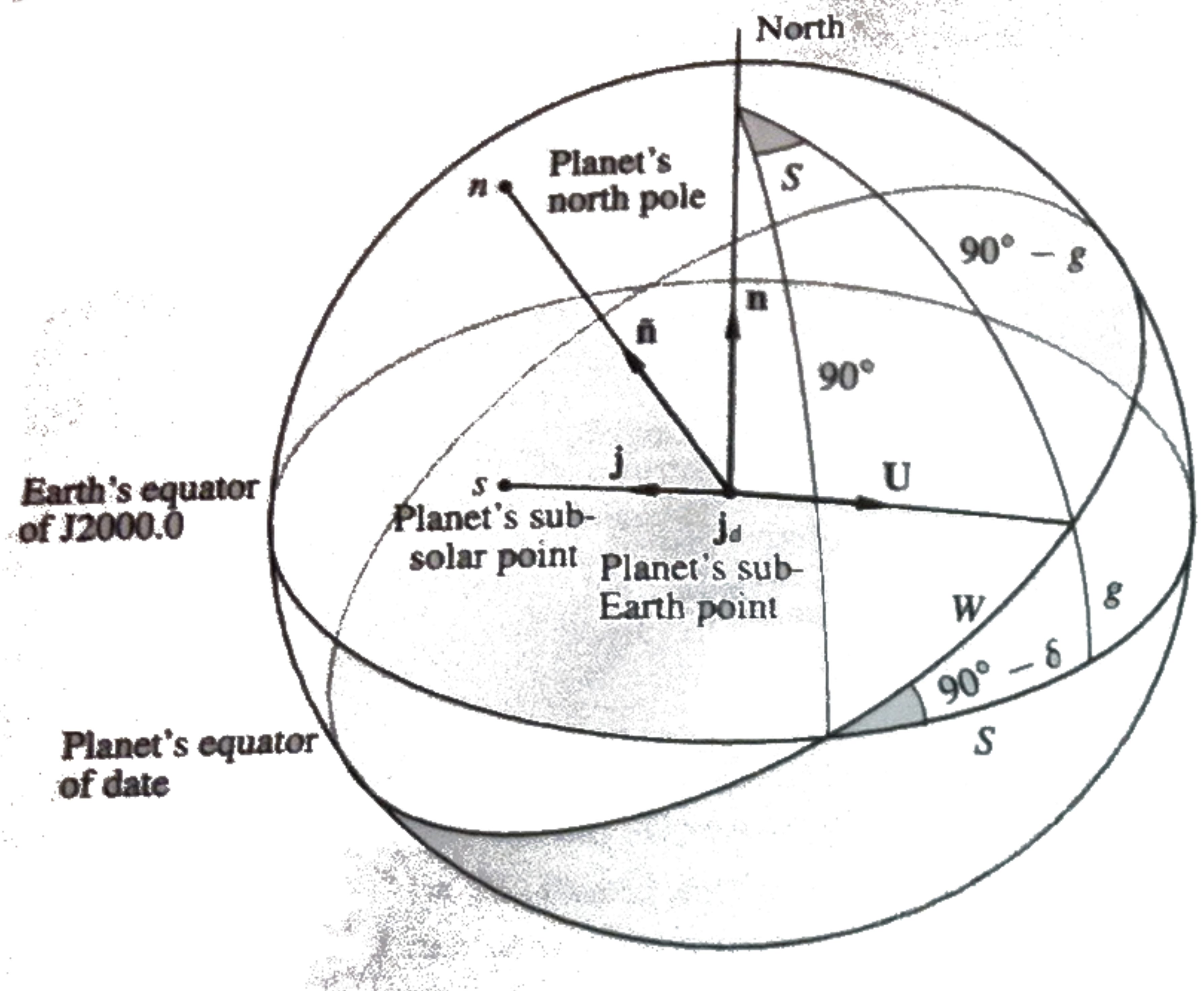

Figure 7.12.4
Planetocentric unit vectors for points of interest on the disk of the planet, and the angles between the Earth's equator of J2000.0 and the planet's equator of date. The quantities depicted are: *n*, the north pole of the planet; **n**, the north pole of the Earth's equator and equinox of J2000.0; **ñ**, the planetocentric unit vector for the north pole; *s*, the subsolar point; **j**, the planetocentric unit vector for the subsolar point; $\mathbf{j}_d$, the planetocentric unit vector for the sub-Earth point; **U**, the planetocentric unit vector pointing to the intersection of the object's prime meridian and its equator; *W*, the angle along the object's equator from the ascending node on the Earth's equator of date to the object's prime meridian; *g*, the angle along the great circle from the Earth's equator of J2000.0 to the Earth's north pole of J2000.0, which goes from the Earth's equator to the intersection of the planet's prime meridian and its equator of date; and, *S*, the angle along the Earth's equator from the ascending node of the planet's equator to the point where the angle *g* reaches the equator. The planetocentric unit vector for the sub-Earth point, $\mathbf{j}_d$ points directly at the observer.

ν, is needed. To obtain this vector, first determine the intermediate angles *g* and *S* (Figure 7.12.4):

$$\sin g = \sin W \cos \delta_d, \tag{7.12–14}$$

$$\sin S = \sin W \sin \delta_d / \cos g, \tag{7.12–15}$$

$$\cos S = \cos W / \cos g. \tag{7.12–16}$$

Since *g* runs from +90° to −90° (see Figure 7.12.3), there is no loss of generality in not directly determining cos *g*. The right ascension and declination of **U** are then

$$\alpha_U = \alpha_d + 90° + \tan^{-1}(\sin S / \cos S), \tag{7.12–17}$$

$$\delta_U = \tan^{-1}(\sin g / \cos g). \tag{7.12–18}$$

The coordinates for the sub-Earth and subsolar points are then computed by using the vectors for the north pole of the planet and **U** to determine the rotation from the equatorial ecliptic coordinate system to give the planetocentric latitude and the planetographic longitude. (The planetographic longitude is supplied from the definition of *W*, and the planetocentric latitude is determined because the angles are measured with respect to the axes of the coordinate system at the center of mass of the planet and not the local normal on the equipotential surface.) The flattening of the planet, f, is used to convert the planetocentric latitude to the planetographic latitude. Using **ñ** as the *z*-axis, **U** as the *x*-axis, and defining **y** as the *y*-axis of the coordinate system,

$$\mathbf{y} = \mathbf{U} \times \tilde{\mathbf{n}}. \tag{7.12–19}$$

The unit vector for the sub-Earth point is

$$\mathbf{j}_d = (\mathbf{j}_d \cdot \mathbf{U}, \mathbf{j}_d, \cdot \mathbf{y}, \mathbf{j}_d \cdot \tilde{\mathbf{n}}). \tag{7.12–20}$$

The vector, $\mathbf{j}_d = (j_{dx}, j_{dy}, j_{dz})$ is converted into the latitude and longitude of the sub-Earth point:

$$\phi_e = \sin^{-1}(j_{dz}), \tag{7.12–21}$$

$$\phi'_e = \tan^{-1}[\tan \phi / (1 - f)^2], \tag{7.12–22}$$

$$\lambda'_e = \tan^{-1}(j_{dy} / j_{dx}). \tag{7.12–23}$$

The planetographic latitude and longitude of the subsolar point are found by using the unit vector for the Sun, **j**, instead of the unit vector for the Earth, in Equations 7.12–20 through 7.12–23.

The position angle and apparent distance for the subsolar point and north pole are computed by determining the vectors specifying a coordinate system in which the *yz*-plane is the plane of the sky, the *x*-axis points toward the Earth, and the *y*-axis points toward the west as seen on the plane of the sky. In the equatorial coordinates of date, the direction for the north pole of the Earth is $\mathbf{j}_n = (0, 0, 1)$. So the directions for axes, (u_i, u_j, u_k), of the coordinate system in the ecliptic coordinates of date are

$$x\text{-axis,}\quad \mathbf{u}_i = \mathbf{j}_d; \tag{7.12–24}$$

$$y\text{-axis,}\quad \mathbf{u}_j = \mathbf{j}_d \times \mathbf{j}_n; \tag{7.12–25}$$

$$z\text{-axis,}\quad \mathbf{u}_k = \mathbf{u}_j \times \mathbf{j}_d. \tag{7.12–26}$$

The coordinates for the north pole vector in the preceding coordinate system are found from the equatorial coordinates by

$$\mathbf{n}_u = p_p(\tilde{\mathbf{n}} \cdot \mathbf{u}_i, \tilde{\mathbf{n}} \cdot \mathbf{u}_j, \tilde{\mathbf{n}} \cdot \mathbf{u}_k), \tag{7.12–27}$$

where p_p is the polar diameter of the planet and $\tilde{\mathbf{n}}$ is the unit vector pointing in the direction of the north pole found in Equation 7.12–13. The position angle of the north pole is then

$$PA_n = \tan^{-1}(n_{uy} / n_{uz}). \tag{7.12–28}$$

The length of the projection from the center of mass to the north pole on the planet is

$$d = (n_{ux}^2 + n_{uz}^2)^{1/2}. \tag{7.12–29}$$

Finally, the apparent distance in arcseconds is

$$d_n = 206264.8062[d / (|\mathbf{d}| - n_{ux})]. \tag{7.12–30}$$

Similarly, the position angle and apparent distance of the subsolar point is calculated using the unit vector for the Sun, $\mathbf{j}$, and the apparent radius of the planet at the latitude of the subsolar point rather than $\tilde{\mathbf{n}}$ and the apparent polar radius. The apparent radius of the planet at the subsolar point is derived using Equation 7.12–9 and substituting the planetocentric latitude of the subsolar point, ϕ_s.

The phase angle, i, of a planet is the planetocentric elongation of the Earth from the Sun. This can be found from the dot product of $\mathbf{j}$ and $\mathbf{j}_d$:

$$\cos i = \mathbf{j} \cdot \mathbf{j}_d. \tag{7.12–31}$$

The phase, k, is the ratio of the illuminated area of the disk to the total area of the disk, as seen from the Earth:

$$k = 0.5(1 + \cos i). \tag{7.12–32}$$

The geometric terminator is used in computing the phase and the defect of illumination. The geometric terminator is the plane orthogonal to the direction of the Sun passing through the center of mass of the object (see Figure 7.12.1). The maximum geometric defect of illumination, Q, is the length of the portion of the line segment across the face of the planet passing through the points s and e that is not illuminated (see Figure 7.12.3). The defect calculated does *not* include non-geometric effects such as the refraction of light by the planet's atmosphere. The apparent equatorial semi-diameter of the planet in seconds of arc is

$$s_{eq} = 206264.8062a / |\mathbf{d}| \tag{7.12–33}$$

The ratio of the apparent polar diameter to the equatorial diameter that depends on the latitude of the sub-Earth point (see Figure 7.12.2)

$$b' / a = 1 - f(1 - \sin^2 \gamma), \tag{7.12–34}$$

where f is the flattening of the planet and γ is the latitude of the sub-Earth point. The defect of illumination is then

$$Q = 2s_{eq}[1 - (1 - b' / a)\sin^2(PA_s - PA_n + 90°)](1 - k). \tag{7.12–35}$$

The position angle for the defect of illumination, measured from celestial north, is:

$$\begin{aligned} PA_Q &= PA_s + 180° \qquad PA_s < 180°, \\ &= PA_s - 180° \qquad PA_s \geq 180° \end{aligned} \tag{7.12–36}$$

7.2 PHYSICAL EPHEMERIS OF THE SUN

The elements used to calculate the physical ephemeris of the Sun are from Carrington (1863). The values of the physical constants for the ephemeris are:

(1) mean sidereal period of rotation, $P_s = 25.38$ days;
(2) inclination of solar equator to the ecliptic, $I = 7.°25$;
(3) longitude of the ascending node of the solar equator on the ecliptic, $\Omega = 75.°76 + 0.°01397\,T$;
(4) the equatorial diameter of the Sun, $d_s = 696000$ km.

The flattening of the Sun is thought to be extremely small ($f = 0$). Because the Sun is a gaseous body, it does not rotate rigidly. The sidereal period given is for the mean rotation rate of the Sun.