DISCRETE MATHEMATICS (CSE 2107)

FUNCTIONS

Course Teacher

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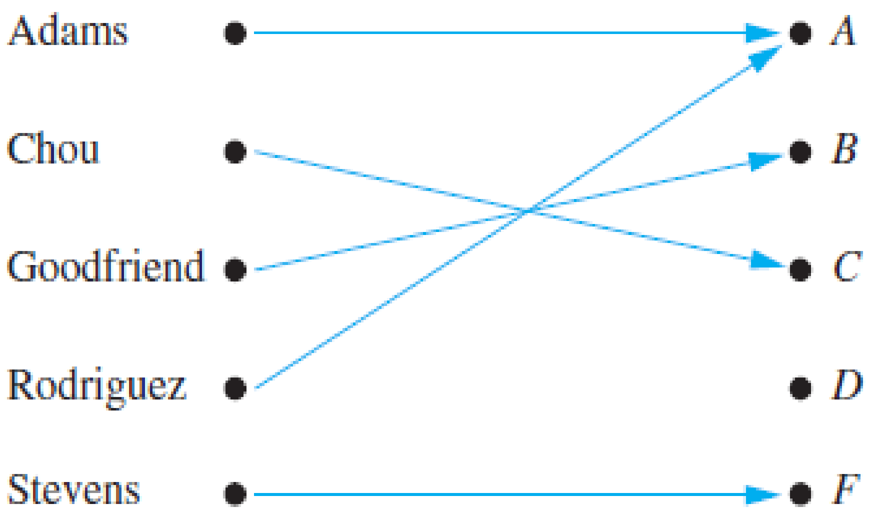
Function

- Let A and B be sets. A function f from *(map or "mapping") A to B* is an assignment of **exactly one** element of ***B*** to each element of ***A***.
- We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.
- If f is a function from A to B, we write $f : A \to B$.



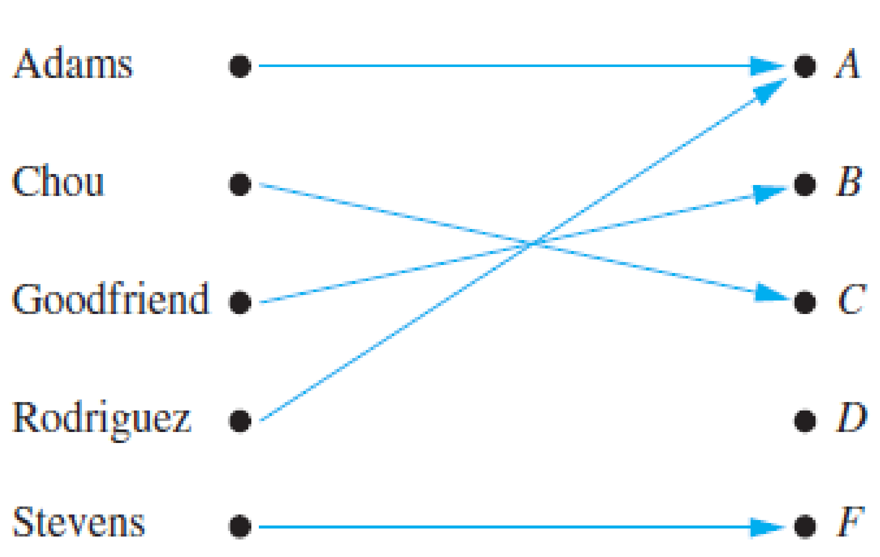
Assignment of grade

Functions (cont.)

- If f is a function from A to B, we say that A is the *domain* of f and B is the *codomain* of f.
- If $f(a) = b$, we say that b is the *image* of a and a is a *preimage* of b.
- The *range*, or *image*, of f is the set of all images of elements of A.
- Also, if f is a function from A to B, we say that f *maps* A to B.

Functions (cont.)

- What are the domain, codomain, and range of the function as shown in the figure?
- Let G be the function that assigns a grade to a student in our discrete mathematics class.
- Note that G(Adams) = A, for instance. The domain of G is the set {Adams, Chou, Goodfriend, Rodriguez, Stevens}, and the codomain is the set $\{A, B, C, D, F\}$.
- The range of G is the set $\{A, B, C, F\}$, because each grade except D is assigned to some student.



Functions (cont.)

- Let f be a function from A to B and let S be a subset of A. The *image* of S under the function f is the subset of B that consists of the images of the elements of S.
- We denote the image of S by $f(S)$, so

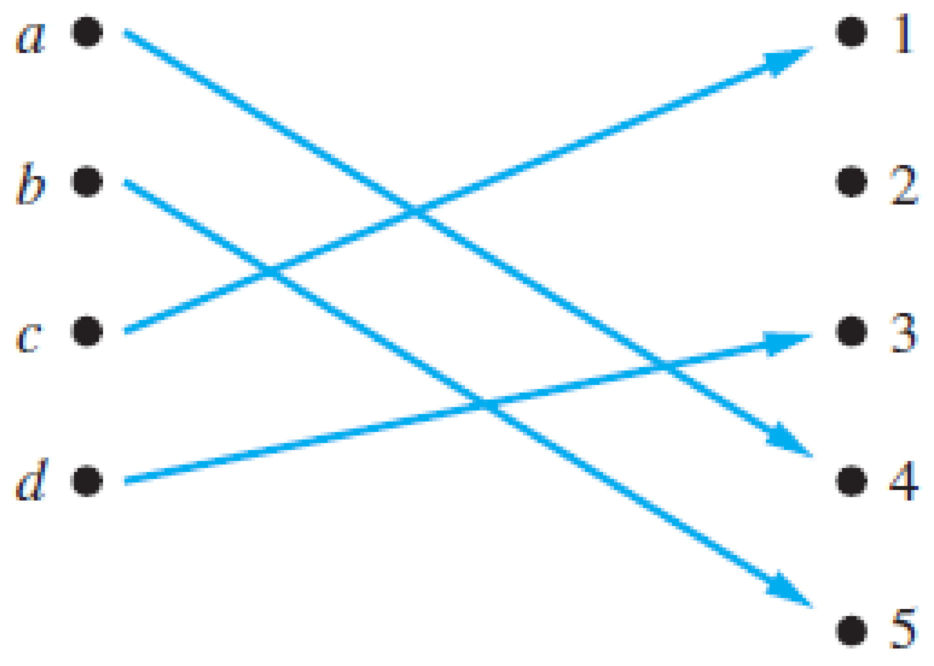
$$f(S) = \{t \mid \exists s \in S\ (t = f\ (s))\}.$$

Example: Let A = {a, b, c, d, e} and B = {1, 2, 3, 4} with f (a) = 2, f (b) = 1, f (c) = 4, f (d) = 1, and f (e) = 1.

- The image of the subset S = {b, c, d} is the set f (S) = {1, 4}.

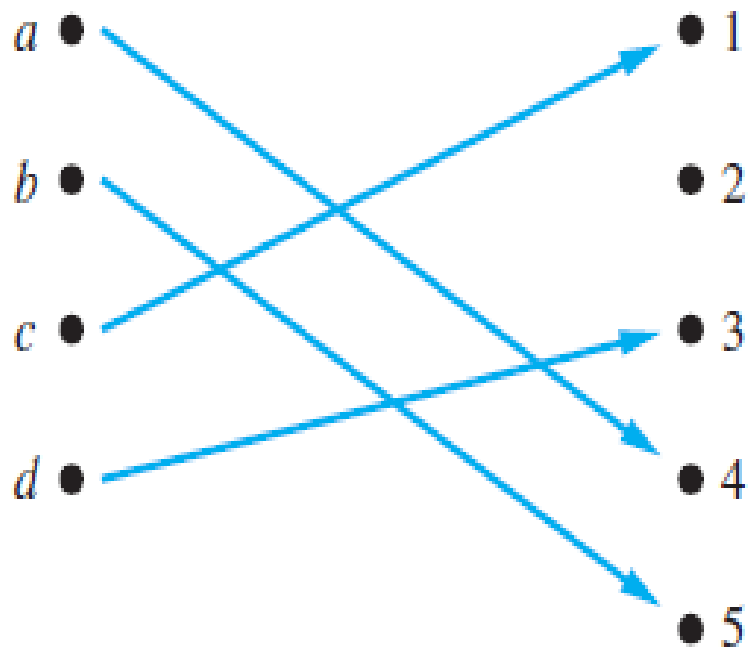
One-to-One Function

- A function f is said to be *one-to-one*, or an *injunction*, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.
- A function is said to be *injective* if it is one-to-one.



One-to-One Function (Cont..)

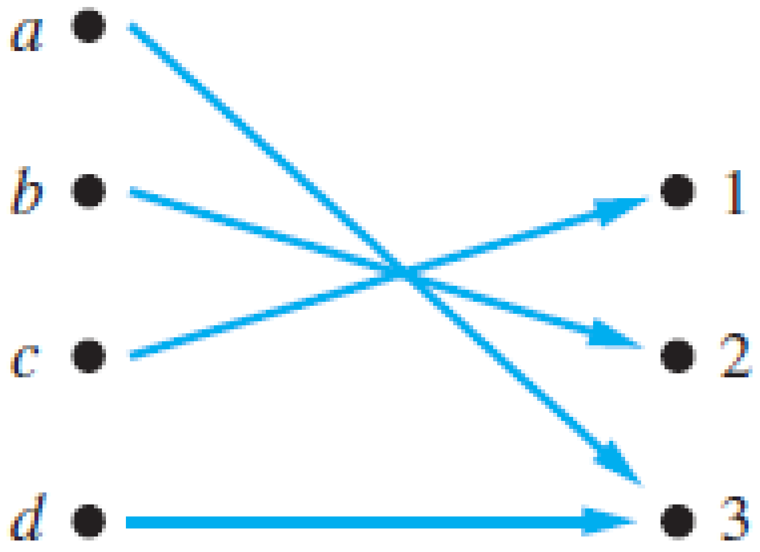
- Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$ is one-to-one.



- The function f is one-to-one because f takes on different values at the four elements of its domain.

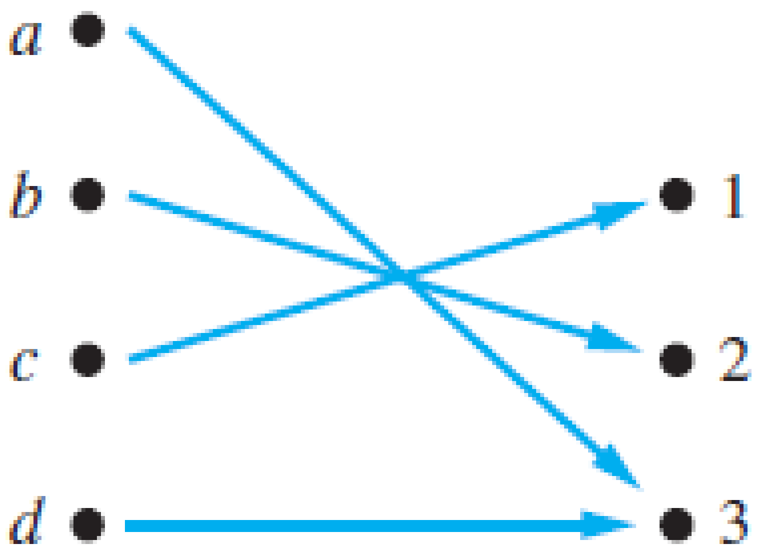
Onto Function

- A function f from A to B is called *onto*, or a *surjection*, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.
- A function f is called *surjective* if it is onto.



Onto Function (Cont..)

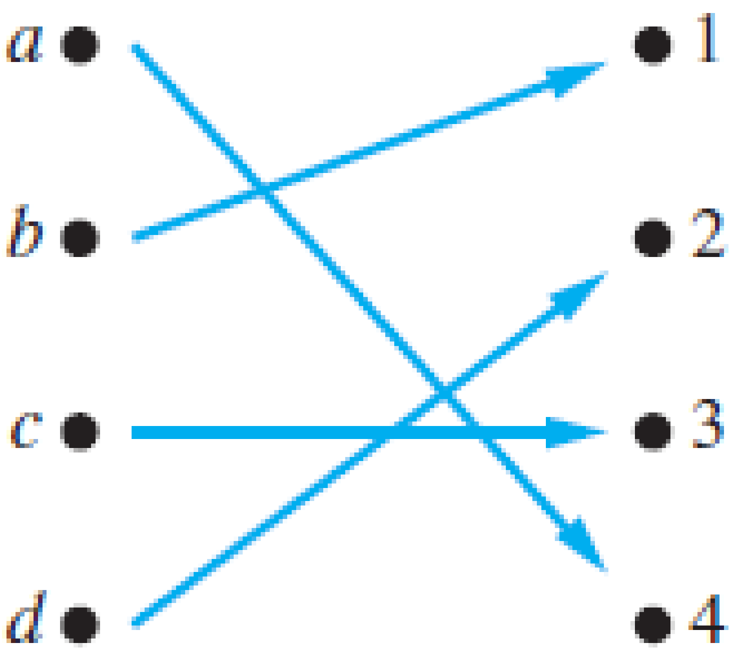
- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f an onto function?.



- Because all three elements of the codomain are images of elements in the domain, we see that f is onto.
- Note that if the codomain were $\{1, 2, 3, 4\}$, then f would not be onto.

One-to-One correspondence

- The function *f* is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto.
- We also say that such a function is *bijective*.

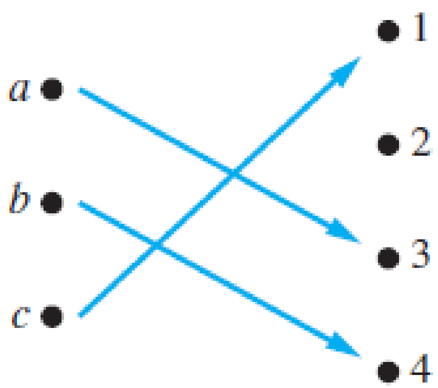


One-to-One correspondence (Cont..)

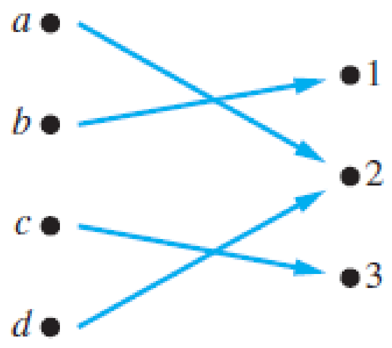
- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$.
- Is f a bijection?

- The function f is one-to-one and onto. It is one-to-one because no two values in the domain are assigned the same function value.
- It is onto because all four elements of the codomain are images of elements in the domain.
- Hence, f is a bijection.

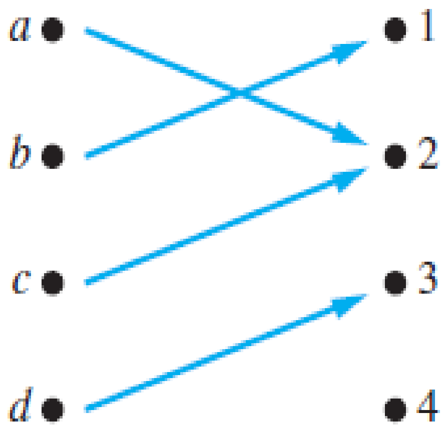
Different Types Functions



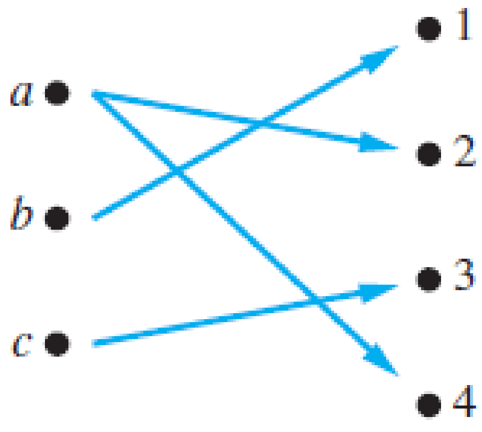
One-to-one,
not onto



Onto,
not one-to-one



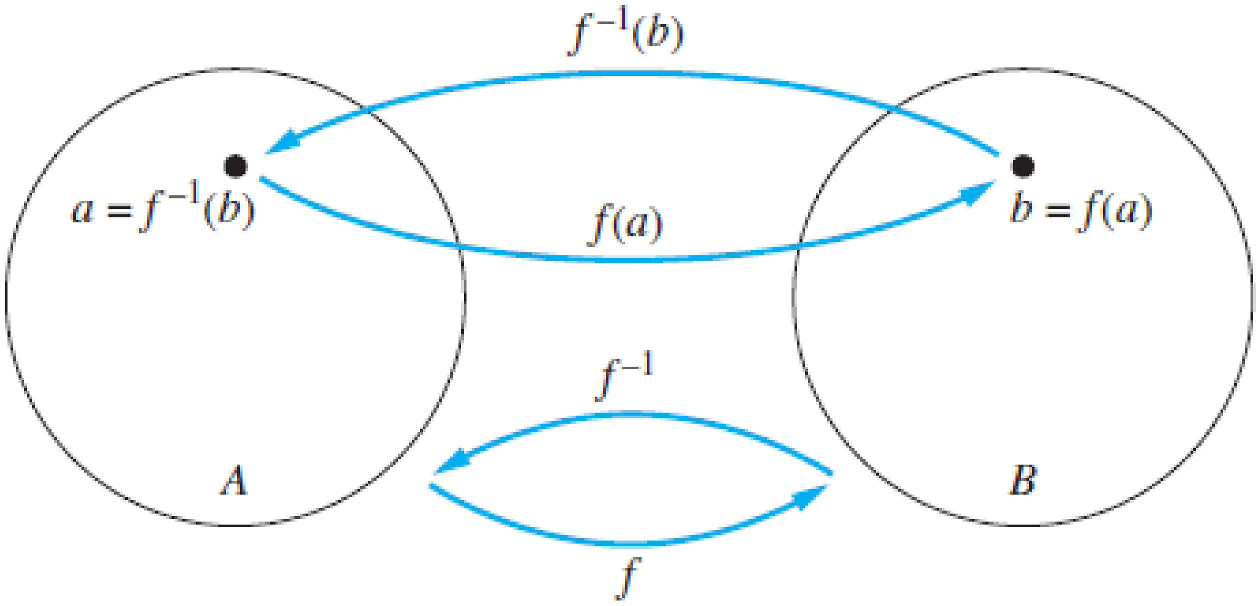
Neither one-to-one
nor onto



Not a function

Inverse Functions

- Let f be a one-to-one correspondence from the set A to the set B. The inverse function of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$.
- The inverse function of f is denoted by f^{-1}. Hence, $f^{-1}(b) = a$ when $f(a) = b$.



Inverse Functions (Cont..)

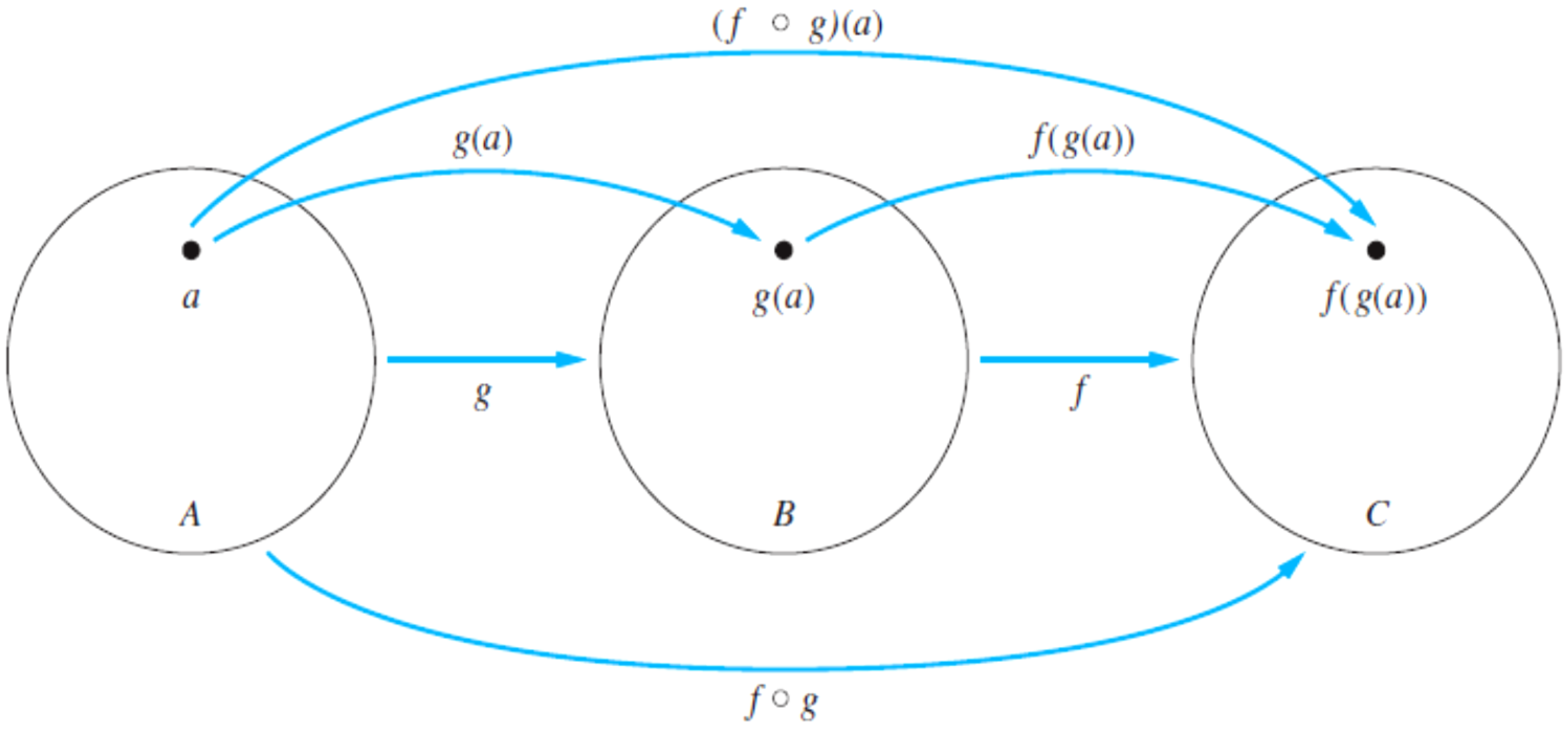
- A one-to-one correspondence is called **invertible** because we can define an inverse of this function.
- A function is **not invertible** if it is not a one-to-one correspondence, because the inverse of such a function does not exist.

- Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$.
- Is f invertible, and if it is, what is its inverse?

- The function f is invertible because it is a one-to-one correspondence.
- The inverse function f^{-1} reverses the correspondence given by f , so $f^{-1}(1) = c$, $f^{-1}(2) = a$, and $f^{-1}(3) = b$.

Composition of Functions

- Let g be a function from the set A to the set B and let f be a function from the set B to the set C. The *composition* of the functions f and g, denoted for all $a \in A$ by $f \circ g$, is defined by $(f \circ g)(a) = f(g(a))$.



Composition of Functions (Cont..)

- Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$.
- Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$.
- What is the composition of f and g, and what is the composition of g and f ?

- The composition $f \circ g$ is defined by $(f \circ g)(a) = f(g(a)) = f(b) = 2$, $(f \circ g)(b) = f(g(b)) = f(c) = 1$, and $(f \circ g)(c) = f(g(c)) = f(a) = 3$.
- Note that $g \circ f$ is not defined, because the range of f is not a subset of the domain of g.

Composition of Functions (Cont..)

- Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$.
- What is the composition of f and g? What is the composition of g and f ?

- Both the compositions $f \circ g$ and $g \circ f$ are defined.
- Moreover, $(f \circ g)(x) = f(g(x)) = f(3x + 2) = 2(3x + 2) + 3 = 6x + 7$

  and

  $(g \circ f)(x) = g(f(x)) = g(2x + 3) = 3(2x + 3) + 2 = 6x + 11.$

Thanks for your Attention