$$V_{out} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$

Now $$\frac{R_f}{R_1} = \frac{R_f}{R_2} = \frac{R_f}{R_3} = \frac{1\ \text{k}\Omega}{3\ \text{k}\Omega} = \frac{1}{3}$$

∴ $$V_{out} = -\left(\frac{V_1 + V_2 + V_3}{3}\right)$$

Note that V_{out} is equal to the average of the three inputs. The negative sign shows the phase reversal.

2. As subtractor. A summing amplifier can be used to provide an output voltage that is equal to the difference of two voltages. Such a circuit is called a **subtractor** and is shown in Fig. 25.78. As we shall see, this circuit will provide an output voltage that is equal to the difference between V_1 and V_2.

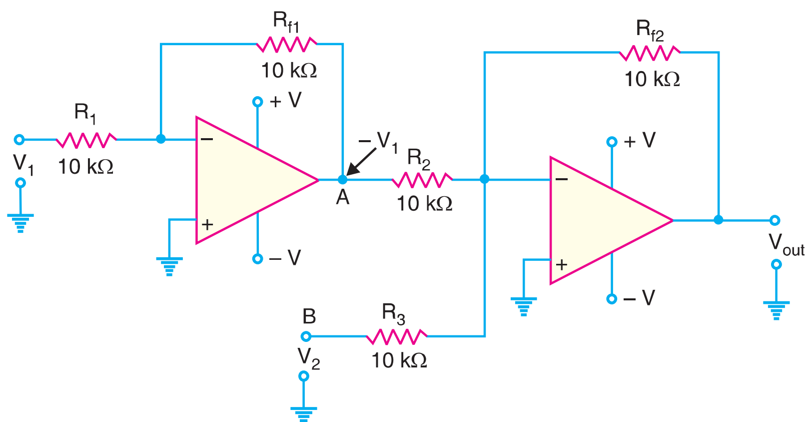


Fig. 25.78

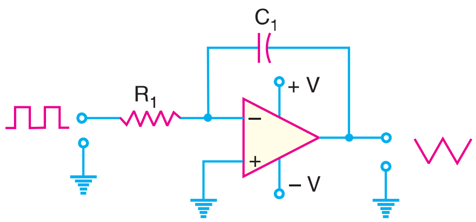
The voltage V_1 is applied to a standard inverting amplifier that has *unity gain.* Because of this, the output from the inverting amplifier will be equal to $-V_1$. This output is then applied to the summing amplifier (also having unity gain) along with V_2. Thus output from second *OP*-amp is given by;

$$V_{out} = -(V_A + V_B) = -(-V_1 + V_2) = V_1 - V_2$$

It may be noted that the gain of the second stage in the subtractor can be varied to provide an output that is proportional to (rather than equal to) the difference between the input voltages. However, if the circuit is to act as a subtractor, the input inverting amplifier *must* have unity gain. Otherwise, the output will not be proportional to the true difference between V_1 and V_2.

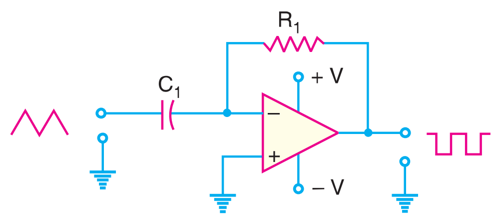
25.34 OP-Amp Integrators and Differentiators

A circuit that performs the mathematical integration of input signal is called an *integrator.* The output of an integrator is proportional to the area of the input waveform over a period of time. A circuit that performs the mathematical differentiation of input signal is called a *differentiator.* The output of a differentiator is proportional to the rate of change of its input signal. Note that the two operations are opposite.



OP-amp Integrator

(*i*)



OP-amp differentiator

(*ii*)

Fig. 25.79

Fig.25.79 shows *OP*-amp integrator and differentiator. As you can see, the two circuits are nearly identical in terms of their construction. Each contains a single *OP*-amp and an *RC* circuit. However, the difference in resistor/capacitor placement in the two circuits causes them to have input/output relationships that are exact opposites. For example, if the input to the integrator is a square wave, the output will be a triangular wave as shown in Fig. 25.79 (*i*). However, the differentiator will convert a triangular wave into square wave as shown in Fig. 25.79 (*ii*).

25.35 *OP*-Amp Integrator

As discussed above, an integrator is a circuit that performs integration of the input signal. The most popular application of an integrator is to produce a *ramp* output voltage (*i.e.* a linearly increasing or decreasing voltage). Fig. 25.80 shows the circuit of an *OP*-amp integrator. It consists of an *OP*-amp, input resistor *R* and feedback capacitor *C*. Note that the feedback component is a capacitor instead of a resistor.

As we shall see, when a signal is applied to the input of this circuit, the output-signal waveform will be the integration of input-signal waveform.

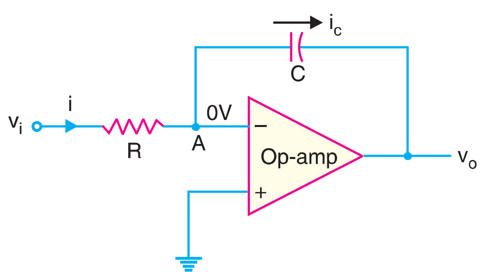


Fig. 25.80

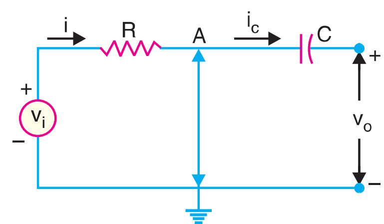


Fig. 25.81

Circuit Analysis. Since point *A* in Fig. 25.80 is at virtual ground, the *virtual-ground equivalent circuit of operational integrator will be as shown in Fig. 25.81. Because of virtual ground and infinite impedance of the *OP*-amp, all of the input current *i* flows through the capacitor *i.e.* $i = i_c$.

Now $$i = \frac{v_i - 0}{R} = \frac{v_i}{R} \quad ...(i)$$

Also voltage across capacitor is $v_c = 0 - v_o = - v_o$

$$\therefore \quad i_c = \frac{C\,dv_c}{dt} = -C\frac{dv_o}{dt} \quad ... (ii)$$

* Recall that virtual ground means that point *A* is 0V but it is not mechanically grounded. Therefore, no current flows from point *A* to ground.

From eqs. (*i*) and (*ii*),
$$\frac{v_i}{R} = -C\frac{dv_o}{dt}$$

or
$$\frac{dv_o}{dt} = -\frac{1}{RC}v_i \quad ...(iii)$$

To find the output voltage, we integrate both sides of eq. (*iii*) to get,

$$v_o = -\frac{1}{RC}\int_0^t v_i\,dt \quad ...(iv)$$

Eq. (*iv*) shows that the output is the integral of the input with an inversion and scale multiplier of $1/RC$.

Output voltage. If a fixed voltage is applied to the input of an integrator, eq. (*iv*) shows that the output voltage grows over a period of time, providing a ramp voltage. Eq. (*iv*) also shows that the output voltage ramp (for a fixed input voltage) is opposite in polarity to the input voltage and is multiplied by the factor $1/RC$. As an example, consider an input voltage v_1 = 1V to the integrator circuit of Fig. 25.82 (*i*). The scale factor of $1/RC$ is

$$-\frac{1}{RC} = -\frac{1}{(1\,\text{M}\Omega)\,(1\mu\text{F})} = -1$$

so that the output is a negative ramp voltage as shown in Fig. 25.82 (*ii*).

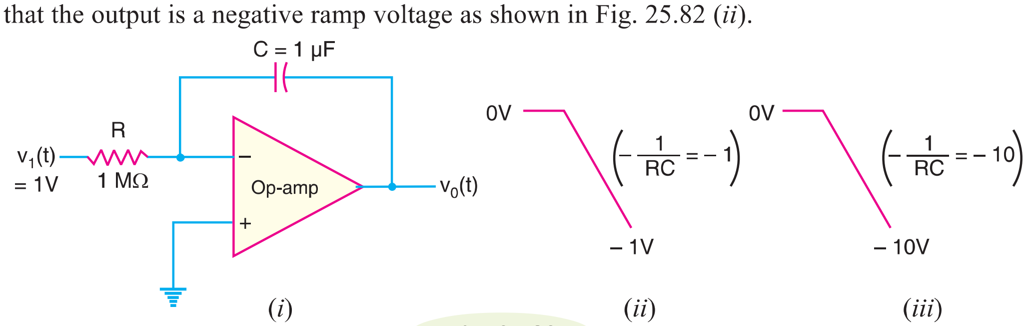


Fig. 25.82

If the scale factor is changed by making R = 100 kΩ, then,

$$-\frac{1}{RC} = -\frac{1}{(100\,\text{k}\Omega)\,(1\,\mu\text{F})} = -10$$

and output is then a steeper ramp voltage as shown in Fig. 25.82 (*iii*).

25.36 Critical Frequency of Integrators

The integrator shown in Fig. 25.80 (Refer back) has no feedback at 0 Hz. This is a serious disadvantage in low-frequency applications. By connecting a feedback resistor R_f in parallel with the capacitor, precise closed-loop voltage gain is possible. The circuit shown in Fig. 25.83 is an integrator with a feedback resistor R_f to provide increased stability.

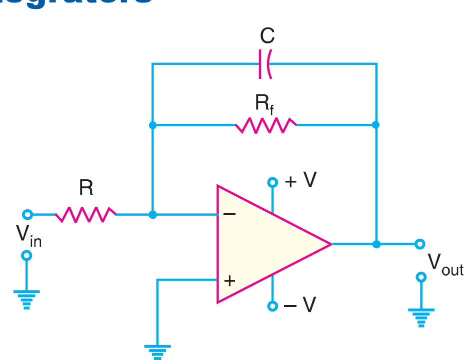
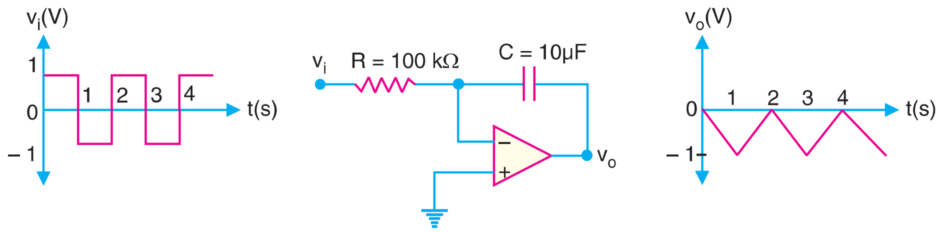


Fig. 25.83

All integrators have a critical frequency f_c below which they do not perform proper integration. If the input frequency is less than f_c, the circuit behaves like a simple inverting amplifier and no integration occurs. The following equation is used to calculate the critical frequency of an integrator:

$$f_c = \frac{1}{2\pi R_f C}$$

Example 25.50. *Fig. 25.84 (i) shows the OP-amp integrator and the square wave input. Find the output voltage.*



(*i*) (*ii*)

Fig. 25.84

Solution. The output voltage of this circuit is given by;

$$v_o = -\frac{1}{RC}\int_0^t v_i \, dt$$

Now $\qquad RC = (100\ \text{k}\Omega)\ (10\ \mu\text{F}) = (100 \times 10^3 \Omega)\ (10 \times 10^{-6}\ \text{F}) = 1\text{s}$

$\therefore$
$$v_o = -\int_0^t v_i \, dt$$

When we integrate a constant, we get a straight line. In other words, when input voltage to an integrator is constant, the output is a linear ramp. Therefore, the integration of the square wave results in the triangular wave as shown in Fig. 25.84 (*ii*). Since the input to the integrator is applied to the inverting input, the output of the circuit will be 180º out of phase with the input. Thus, when the input goes positive, the output will be a negative ramp. When the input is negative, the output will be a positive ramp. Fig. 25.84 (*ii*) shows this relationship.

Example 25.51. *Determine the lower frequency limit (critical frequency) for the integrator circuit shown in Fig. 25.85.*

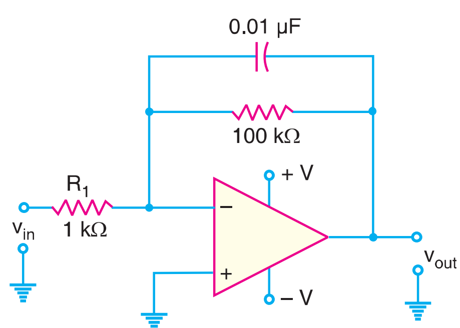


Fig. 25.85

Solution. The critical frequency for the integrator circuit shown in Fig. 25.85 is given by;

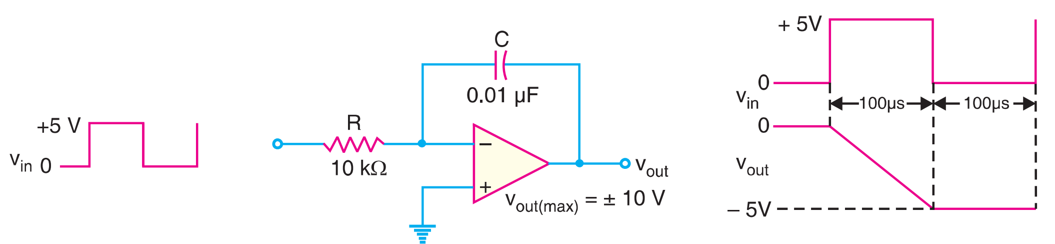
$$f_c = \frac{1}{2\pi R_f C}$$

Here $R_f = 100\ \text{k}\Omega = 10^5 \Omega$; $C = 0.01\ \mu\text{F} = 0.01 \times 10^{-6}$ F

$$\therefore \quad f_c = \frac{1}{2\pi \times (10^5) \times (0.01 \times 10^{-6})} = \mathbf{159\ Hz}$$

Example 25.52. (*i*) *Determine the rate of change of the output voltage in response to a single pulse input to the integrator circuit shown in Fig. 25.86* (*i*).

(*ii*) *Draw the output waveform.*



(*i*) (*ii*)

Fig. 25.86

Solution.

(*i*) Output voltage, $v_{out} = -\frac{1}{RC}\int_0^t v_{in}\, dt$

Therefore, the rate of change of output voltage is

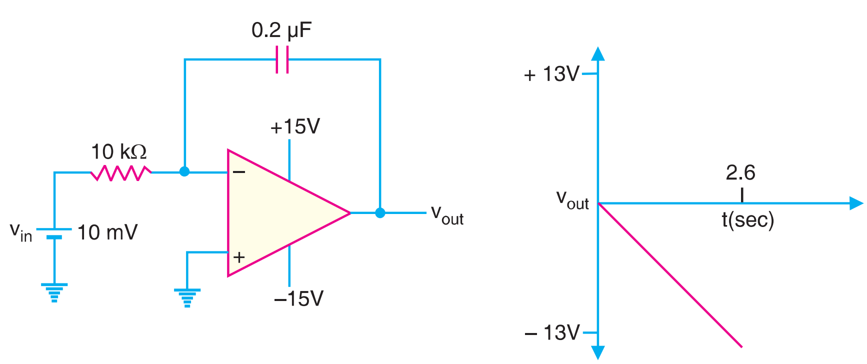
$$\frac{\Delta v_{out}}{dt} = -\frac{v_{in}}{RC} = -\frac{5\text{ V}}{(10\text{ k}\Omega)\,(0.01\,\mu\text{F})} = -50\text{ kV/s} = \mathbf{-50\ mV/\mu s}$$

(*ii*) The rate of change of output voltage is – 50 mV/μs. When the input is at + 5 V, the output is a negative-going ramp. When the input is at 0 V, the output is a constant level. In 100 μs, the output voltage decreases.

$$\therefore \quad \Delta v_{out} = \frac{\Delta v_{out}}{dt} \times dt = -\frac{50\text{ mV}}{\mu\text{s}} \times 100\,\mu\text{s} = -5\text{V}$$

Therefore, the negative-going ramp reaches – 5 V at the end of the pulse (*i.e.* after 100 μs from the initial condition). The output voltage then remains constant at – 5 V for the time the input is zero. Fig. 25.86 (*ii*) shows the output waveform.

Example 25.53. *For the integrator circuit shown in Fig. 25.87 (i), how long does it take for the output to reach saturation ?*



(*i*) (*ii*)

Fig. 25.87

Solution.

Output voltage, $v_{out} = -\frac{1}{RC}\int_0^t v_{in}\, dt$

Since the input voltage v_{in} (= 10 mV) is constant,

$\therefore \quad v_{out} = -\frac{1}{RC} v_{in} t = -\frac{1}{(10\, k\Omega)\,(0.2\, \mu F)} \times (10\, mV) \times t$

or $\quad v_{out} = -5t$ volts

Now $\quad$ Saturation voltage, $V_s = -V_{supply} + 2 = -15 + 2 = -13$ V

$\therefore \quad$ Time required, $t = \frac{V_s}{-5} = \frac{-13}{-5} =$ **2.6 seconds**

Fig. 25.87 (*ii*) shows the output waveform.

25.37 OP-Amp Differentiator

A differentiator is a circuit that performs differentiation of the input signal. In other words, a differentiator produces an output voltage that is proportional to the rate of change of the input voltage. Its important application is to produce a rectangular output from a ramp input. Fig. 25.88 shows the circuit of OP-amp differentiator. It consists of an OP-amp, an input capacitor C and feedback resistor R. Note how the placement of the capacitor and resistor differs from the integrator. The capacitor is now the input element.

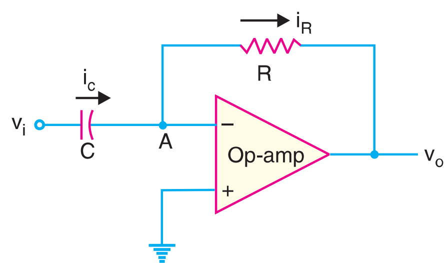


Fig. 25.88

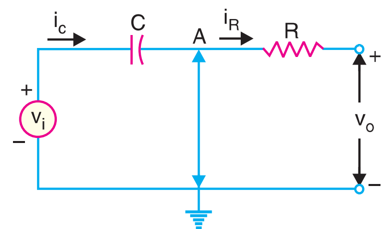


Fig. 25.89

Circuit analysis. Since point A in Fig. 25.88 is at virtual ground, the virtual-ground equivalent circuit of the operational differentiator will be as shown in Fig. 25.89. Because of virtual ground and infinite impedance of OP-amp, all the input current i_c flows through the feedback resistor R *i.e.* $i_c = i_R$.

$\therefore \quad i_R = \frac{0 - v_o}{R} = -\frac{v_o}{R} \quad$ and $\quad v_c = v_i - 0 = v_i$

Also $\quad i_c = C\frac{dv_c}{dt} = C\frac{dv_i}{dt}$

$\therefore \quad -\frac{v_o}{R} = C\frac{dv_i}{dt} \qquad (\because\ i_R = i_c)$

or $\quad v_o = -RC\frac{dv_i}{dt} \qquad$ ...(*i*)

Eq. (*i*) shows that output is the differentiation of the input with an inversion and scale multiplier of RC. If we examine eq. (*i*), we see that if the input voltage is constant, dv_i/dt is zero and the output voltage is zero. The faster the input voltage changes, the larger the magnitude of the output voltage.

Example 25.54. *Fig. 25.90 (i) shows the square wave input to a differentiator circuit. Find the output voltage if input goes from 0V to 5V in 0.1 ms.*

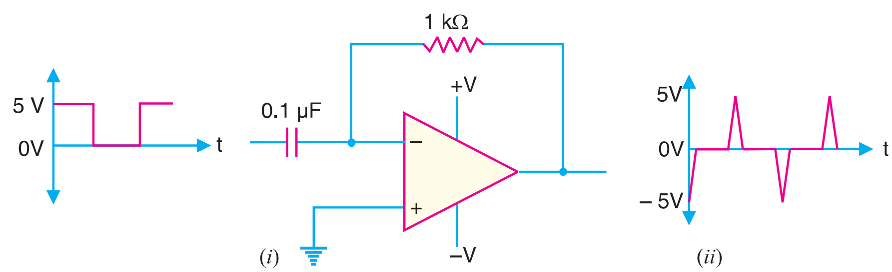


Fig. 25.90

Solution. Output voltage, $v_o = -RC\dfrac{dv_i}{dt}$

Now, $RC = (1\ k\Omega) \times (0.1\ \mu F) = (10^3\ \Omega)\,(0.1 \times 10^{-6}\ F) = 0.1 \times 10^{-3}\ s$

Also, $\dfrac{dv_i}{dt} = \dfrac{5V}{0.1\ ms} = \dfrac{5 \times 10^4\ V}{s} = 5 \times 10^4\ V/s$

$\therefore\quad v_o = -(0.1 \times 10^{-3})\,(5 \times 10^4) = \mathbf{-5V}$

The signal quickly returns to zero as the input signal becomes constant. The output will be as shown in Fig. 25.90 (*ii*).

Example 25.55. *For the differentiator circuit shown in Fig. 25.91, determine the output voltage if the input goes from 0V to 10V in 0.4s. Assume the input voltage changes at constant rate.*

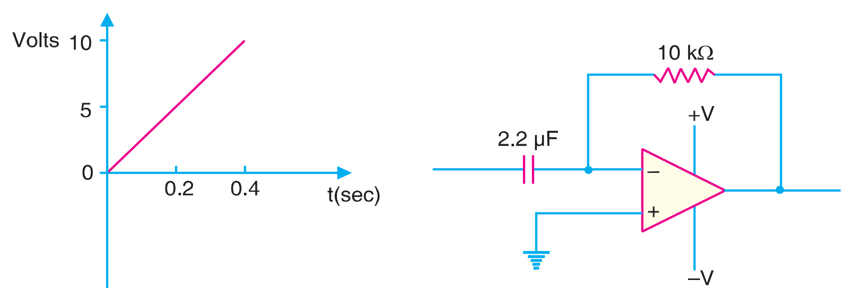


Fig. 25.91

Solution. Output voltage, $v_o = -RC\dfrac{dv_i}{dt}$

Now, $RC = (10\ k\Omega) \times (2.2\ \mu F) = (10^4\ \Omega)\ (2.2 \times 10^{-6}\ F) = 2.2 \times 10^{-2}\ s$

Also, $\dfrac{dv_i}{dt} = \dfrac{(10-0)\,V}{0.4s} = \dfrac{10V}{0.4s} = 25\ V/s$

$\therefore\quad v_o = -(2.2 \times 10^{-2}) \times 25 = \mathbf{-0.55\ V}$

The output voltage stays constant at – 0.55 V.

Example 25.56. *For the differentiator circuit shown in Fig. 25.92(i), determine (i) the expression for the output voltage (ii) the output voltage for the given input.*

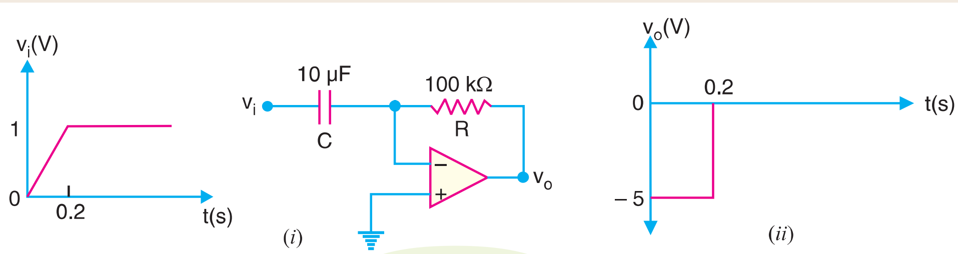


Fig. 25.92

Solution.

(*i*) For the differentiator shown in Fig. 25.92 (*i*), the output voltage is given by;

$$v_o = -RC\frac{dv_i}{dt} = -(100\,\text{k}\Omega)\times(10\,\mu\text{F})\frac{dv_i}{dt}$$

$$= -(100\times10^3\,\Omega)\times(10\times10^{-6}\,\text{F})\frac{dv_i}{dt} = -\frac{dv_i}{dt}$$

(*ii*) Since the input voltage is a straight line between 0 and 0.2*s*, the output voltage is

$$v_o = -\frac{dv_i}{dt} = -\frac{(1-0)}{0.2} = -5\text{V}$$

Therefore, between 0 to 0.2*s*, the output voltage is constant at – 5 V. For $t > 0.2s$, the input is constant so that output voltage is zero. Fig. 25.92 (*ii*) shows the output waveform.

25.38. Comparators

Often we want to compare one voltage to another to see which is larger. In this situation, a *comparator* may be used. *A comparator is an OP-amp circuit without negative feedback* and takes advantage of very high open-loop voltage gain of OP-amp. A comparator has two input voltages (noninverting and inverting) and one output voltage. Because of the high open-loop voltage gain of an *OP*-amp, a very small difference voltage between the two inputs drives the amplifier to saturation. For example, consider an *OP*-amp having A_{OL} = 100,000. A voltage difference of only 0.25 mV between the inputs will produce an output voltage of (0.25 mV) (100,000) = 25V. However, most of OP-amps have output voltages of less than ± 15V because of their d.c. supply voltages. Therefore, a very small differential input voltage will drive the OP-amp to saturation. This is the key point in the working of comparator.

Fig. 25.93 illustrates the action of a comparator. The input voltages are v_1 (signal) and v_2 (*reference voltage). If the differential input is positive, the circuit is driven to saturation and output goes to maximum positive value (**$+V_{sat}$ = + 13V). Reverse happens when the differential input goes negative *i.e.* now output is maximum negative ($-V_{sat}$ = – 13V). This circuit is called comparator because it compares v_1 to v_2 to produce a saturated positive or negative output voltage. Note that output voltage rapidly changes from – 13V to + 13V and *vice-versa*.

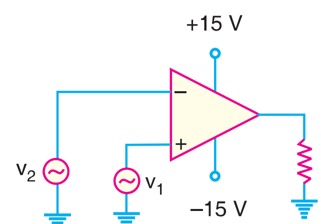


Fig. 25.93

25.39 Comparator Circuits

A comparator circuit has the following two characteristics :

(*i*) It uses no feedback so that the voltage gain is equal to the open-loop voltage gain (A_{OL}) of *OP*-amp.

(*ii*) It is operated in a non-linear mode.

These properties of a comparator permit it to perform many useful functions.

Two comparator integrated circuits.

* If this terminal is grounded, v_2 = 0V.

** Since in our case supply voltages are ± 15V,

$$+V_{sat} = +V_{supply} - 2 = 15 - 2 = +13\text{V}$$
$$-V_{sat} = -V_{supply} + 2 = -15 + 2 = -13\text{V}$$

1. As a square wave generator. A comparator can be used to produce a square wave output from a sine wave input. Fig. 25.94 shows the circuit of a comparator to produce square wave output. Note that inverting terminal (–) is grounded and signal (v_{in}) is applied to the noninverting terminal (+). Since the gain of a comparator is equal to A_{OL}, virtually any difference voltage at the inputs will cause the output to go to one of the voltage extremes ($+V_{sat}$ or $-V_{sat}$) and stay there until the voltage difference is removed. The polarity of the input difference voltage will determine to which extreme ($+V_{sat}$ or $-V_{sat}$) the output of the comparator goes.

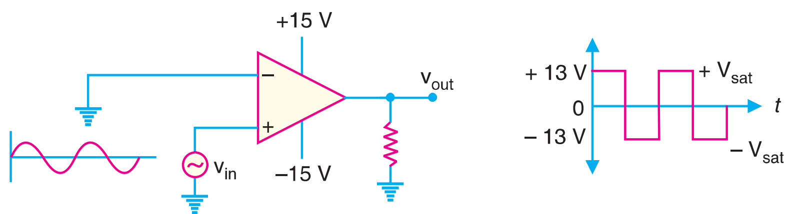


Fig. 25.94

When the input signal goes positive, the output jumps to about + 13 V. When the input goes negative, the output jumps to about – 13 V. The output changes rapidly from – 13 V to + 13 V and *vice-versa*. This change is so rapid that we get a square wave output for a sine wave input.

2. As a zero-crossing detector. When one input of a comparator is connected to ground, it is known as zero-crossing detector because the output changes when the input crosses 0 V. The zero-crossing circuit is shown in Fig. 25.95. The input and output waveforms are also shown. When the input signal is positive-going, the output is driven to positive maximum value (*i.e.* $+V_{sat} = +13$ V). When the input crosses the zero axis and begins to go negative, the output is driven to negative maximum value (*i.e.* $-V_{sat} = -13$ V).

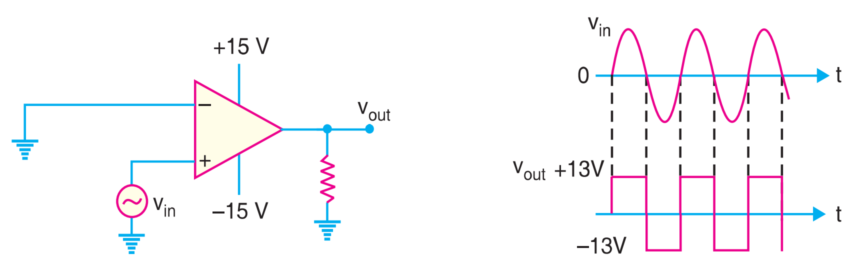


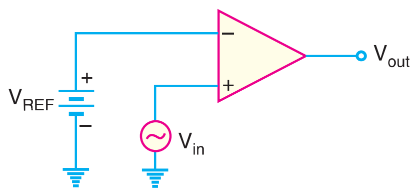
Fig. 25.95

From the input/output waveforms, you can see that every time the input crosses 0 V going positive, the output jumps to + 13 V. Similarly, every time the input crosses 0 V going negative, the output jumps to – 13 V. Since the change (+ 13 V or – 13 V) occurs every time the input crosses 0 V, we can tell when the input signal has crossed 0 V. Hence the name zero-crossing detector.

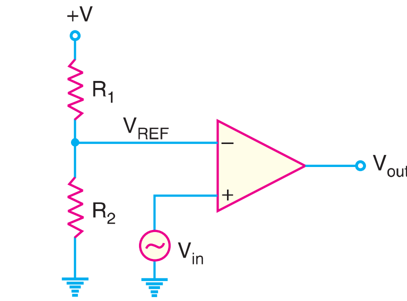
3. As a level detector. When a comparator is used to compare a signal amplitude to a fixed d.c. level (reference voltage), the circuit is referred to as a level detector. We can modify zero-crossing detector circuit to construct level detector. This can be done by connecting a fixed reference voltage V_{REF} to the inverting input as shown in Fig. 25.96 (*i*). A more practical arrangement is shown in Fig. 25.96 (*ii*) using a voltage divider to set the reference voltage as follows :

$$V_{REF} = \frac{R_2}{R_1 + R_2}(+V)$$

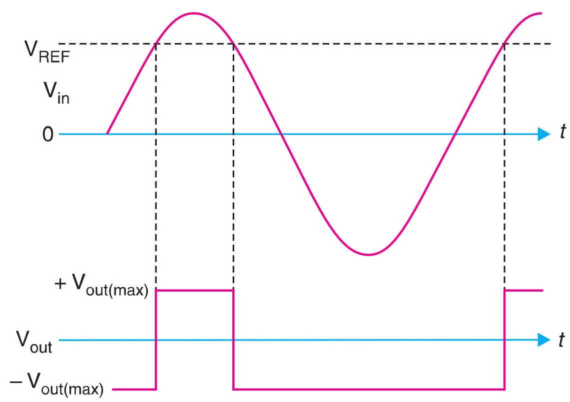
where + V is the positive *OP*-amp d.c. supply voltage.



(*i*) Battery reference



(*ii*) Voltage-divider reference



(*iii*) Waveforms

Fig. 25.96

The circuit action is as follows. Suppose the input signal v_{in} is a sine wave. When the input voltage is less than the reference voltage (*i.e.* $V_{in} < V_{REF}$), the output goes to maximum negative level. It remains here until V_{in} increases above V_{REF}. When the input voltage exceeds the reference voltage (*i.e.* $V_{in} > V_{REF}$), the output goes to its maximum positive state. It remains here until V_{in} decreases below V_{REF}. Fig. 25.96 (*iii*) shows the input/output waveforms. Note that this circuit is used for non zero-level detection.

MULTIPLE-CHOICE QUESTIONS

1. A differential amplifier ....
   (*i*) is a part of an OP-amp
   (*ii*) has one input and one output
   (*iii*) has two outputs
   (*iv*) answers (*i*) and (*iii*)
2. When a differential amplifier is operated single-ended, ...
   (*i*) the output is grounded
   (*ii*) one input is grounded and signal is applied to the other
   (*iii*) both inputs are connected together
   (*iv*) the output is not inverted
3. In differential-mode, ...
   (*i*) opposite polarity signals are applied to the inputs
   (*ii*) the gain is one
   (*iii*) the outputs are of different amplitudes
   (*iv*) only one supply voltage is used
4. In the common-mode, ...
   (*i*) both inputs are grounded
   (*ii*) the outputs are connected together
   (*iii*) an identical signal appears on both inputs
   (*iv*) the output signals are in-phase