

# Lecture#12

## Data Structures

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[Faculty Profile](#)



# Tree



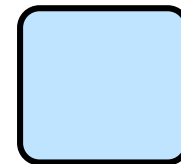


# Complete Binary Tree

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A heap is a certain kind of nearly **complete binary tree**.

- ▶ Every level except bottom is complete.
- ▶ On the bottom, nodes are placed as left as possible.



Root

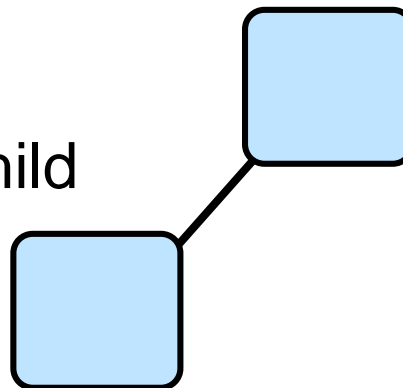
When a complete binary tree is built, its first node must be the root.



# Complete Binary Tree

Complete binary tree.

Left child  
of the  
root

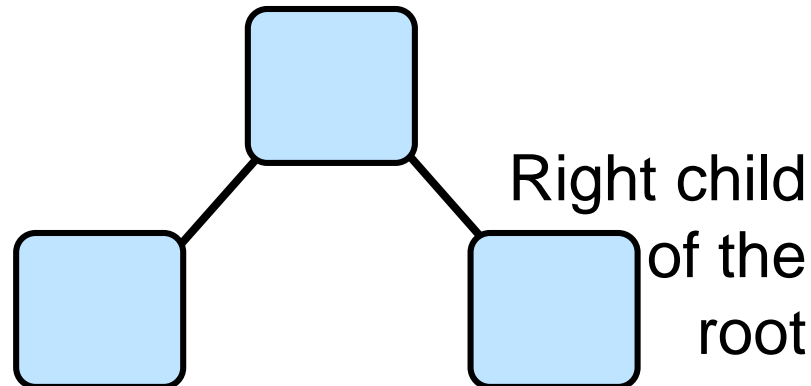


The second node is  
always the left child  
of the root.



# Complete Binary Tree

Complete binary tree.

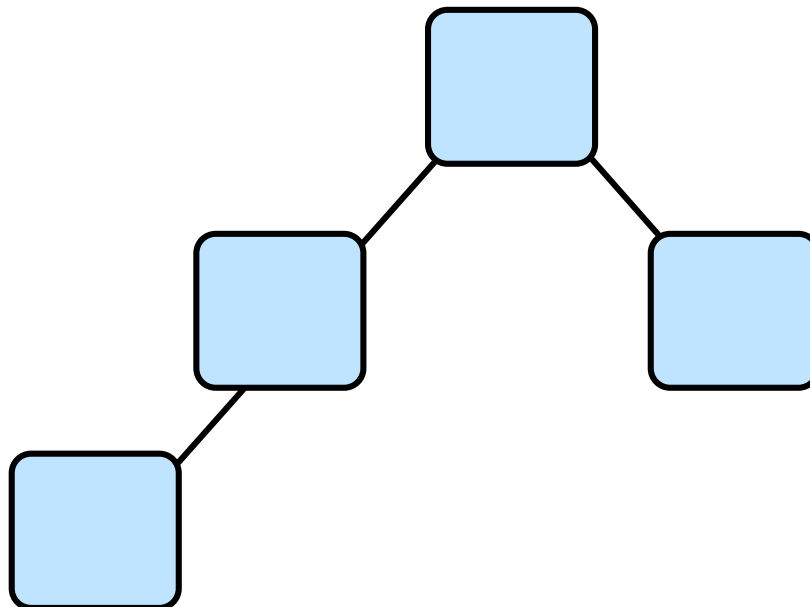


The third node is always the right child of the root.



# Complete Binary Tree

Complete binary tree.



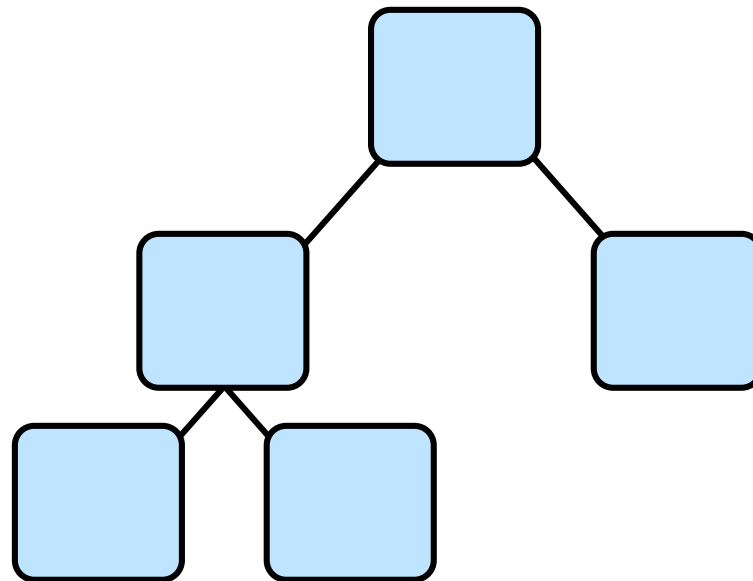
The next nodes  
always fill the next  
level from left-to-  
right.





# Complete Binary Tree

Complete binary tree.

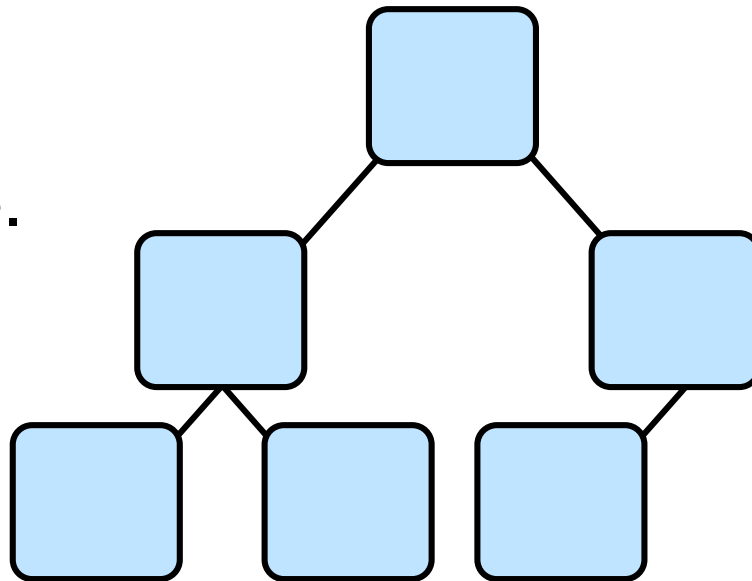


The next nodes  
always fill the next  
level from left-to-  
right.



# Complete Binary Tree

Complete binary tree.



The next nodes  
always fill the next  
level from left-to-  
right.

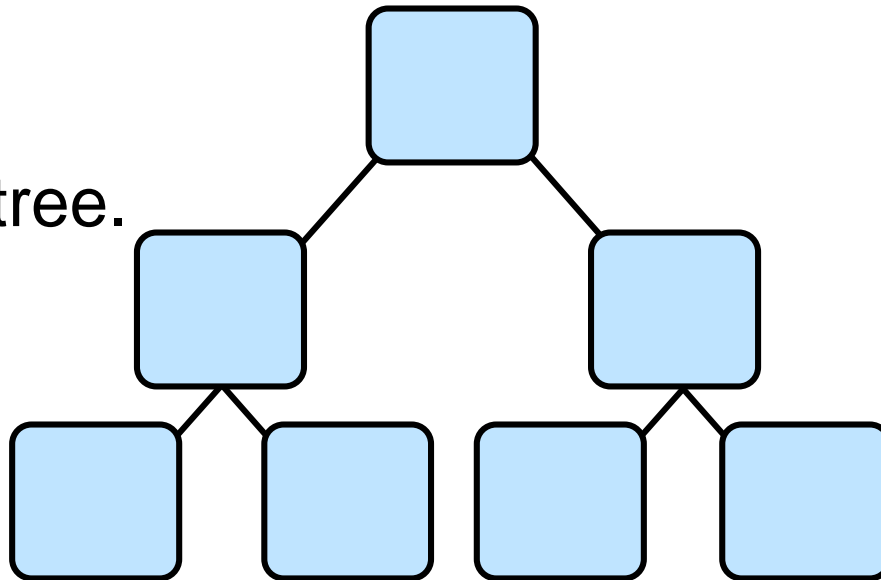






# Complete Binary Tree

Complete binary tree.

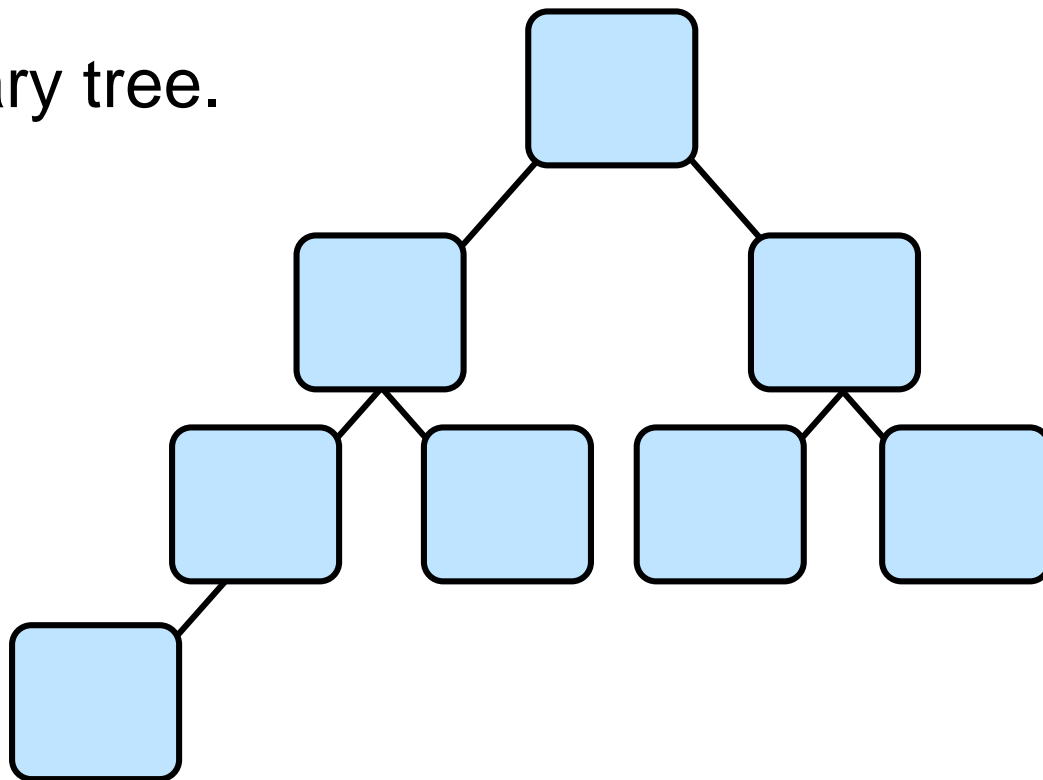


The next nodes  
always fill the next  
level from left-to-right.



# Complete Binary Tree

Complete binary tree.





# Binary Search Tree (BST)





# Binary Search Tree

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Construct a Binary Search Tree (BST) for the following sequence of numbers-

**50, 70, 60, 20, 90, 10, 40, 100**





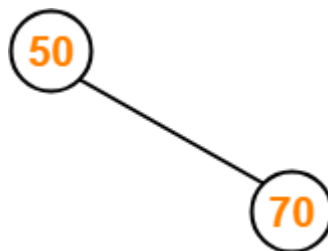
# Binary Search Tree

**50, 70, 60, 20, 90, 10, 40, 100**

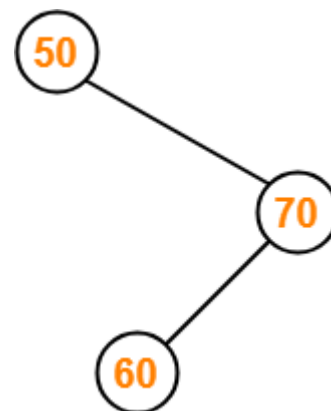
Insert 50-



Insert 70-



Insert 60-

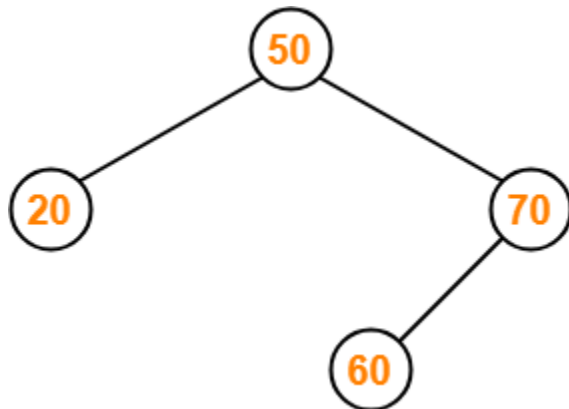




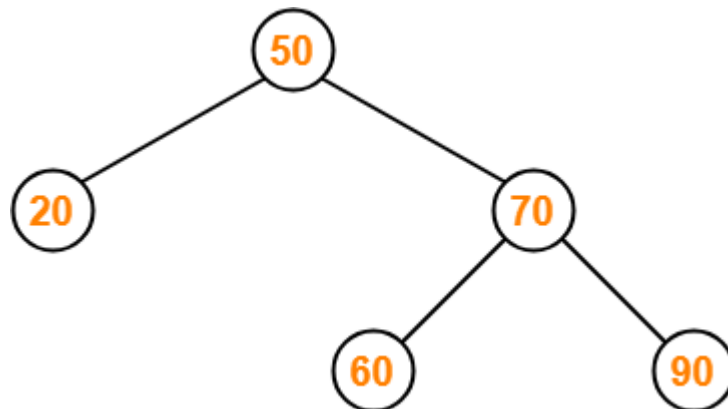
# Binary Search Tree

**50, 70, 60, 20, 90, 10, 40, 100**

Insert 20-



Insert 90-

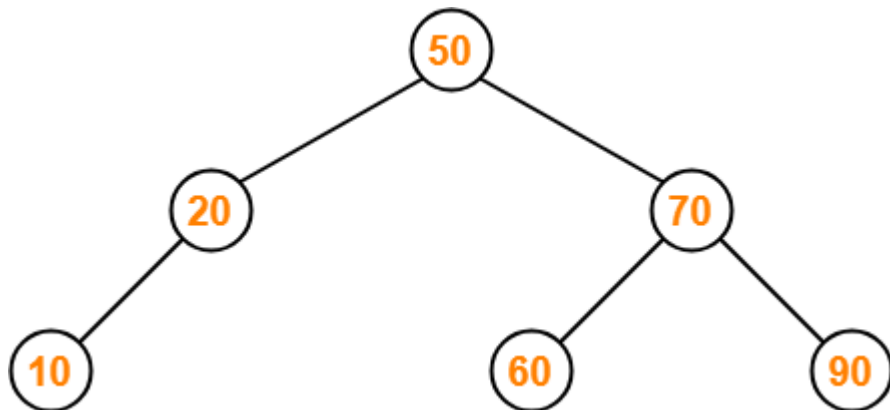




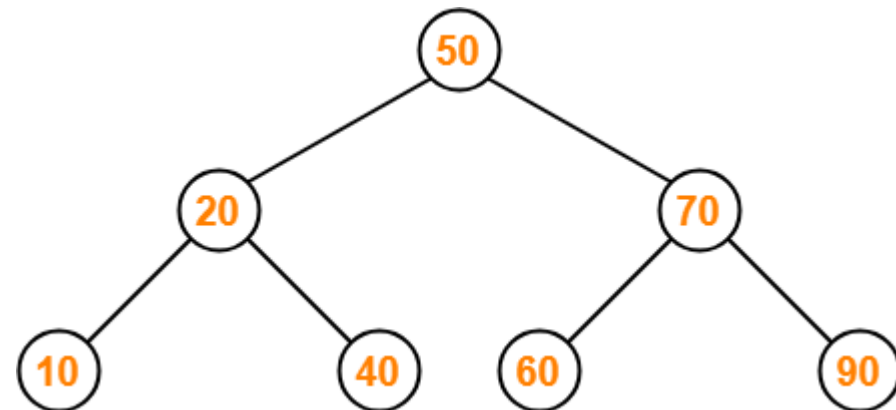
# Binary Search Tree

**50, 70, 60, 20, 90, 10, 40, 100**

Insert 10-



Insert 40-

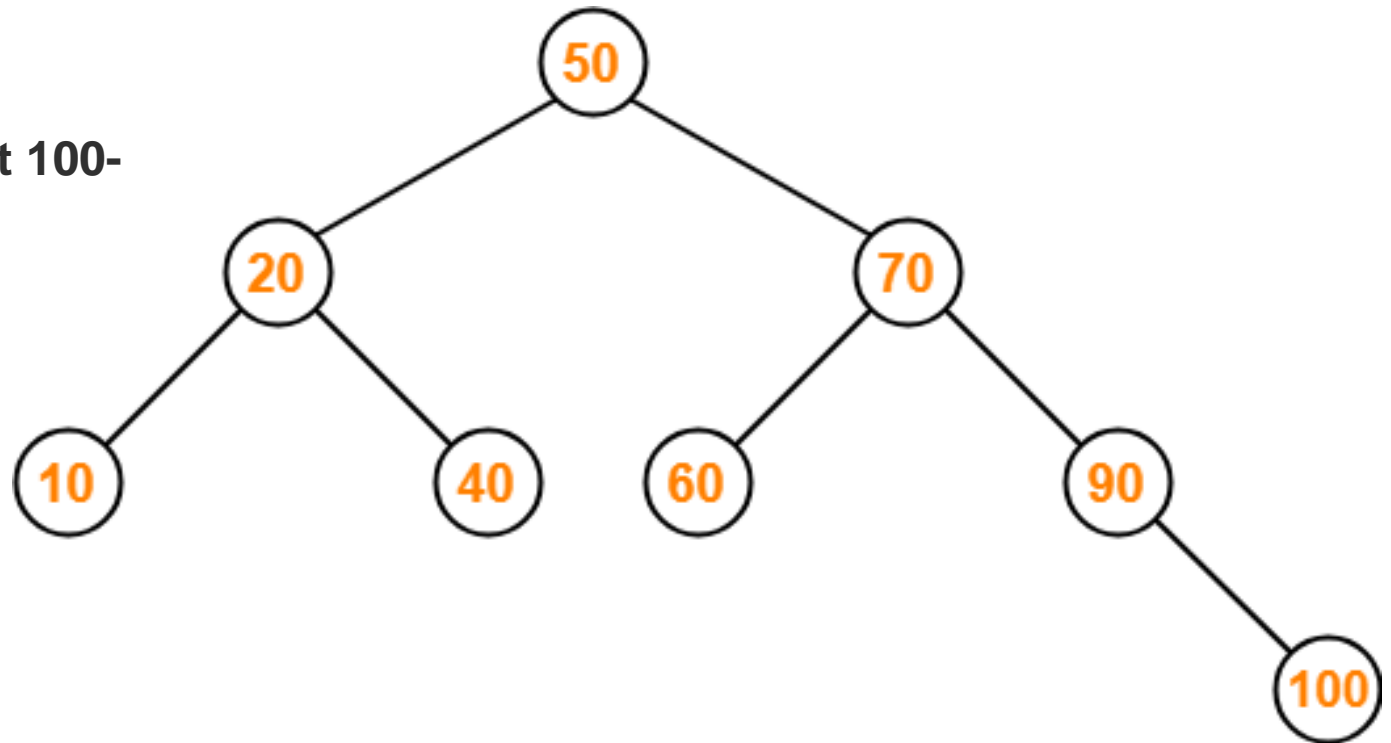




# Binary Search Tree

**50, 70, 60, 20, 90, 10, 40, 100**

Insert 100-



**Binary Search Tree**







# Binary Search Tree

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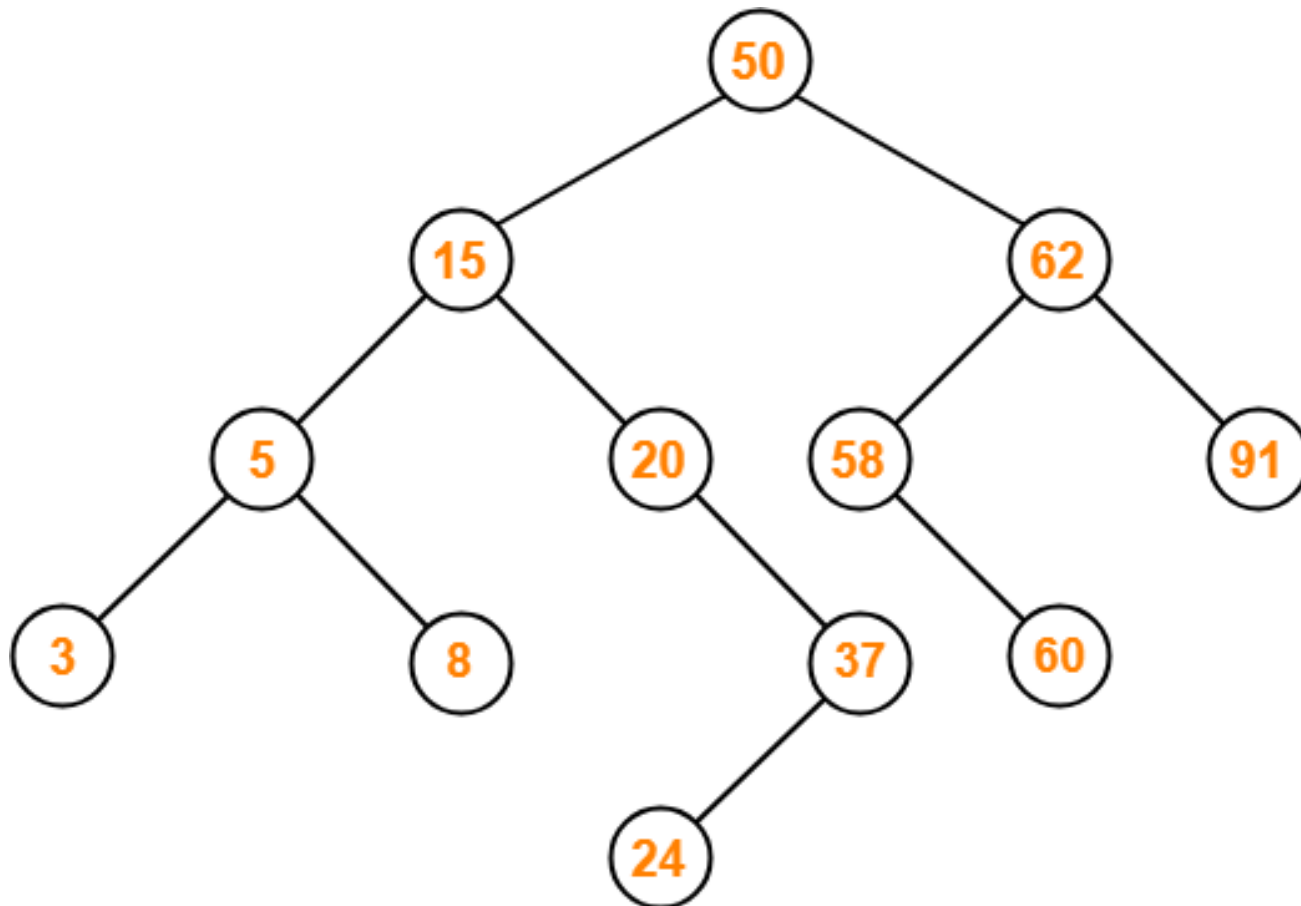
Construct a Binary Search Tree (BST) for the following sequence of numbers-

**50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24**





# Binary Search Tree



Binary Search Tree





# Binary Search Tree Practice

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Construct a Binary Search Tree (BST) for the following sequence of numbers-  
50, 76, 21, 4, 32, 64, 15, 52, 14, 100, 83, 2, 3, 70, 87, 80  
Then, delete the root node from the constructed Binary Search Tree.

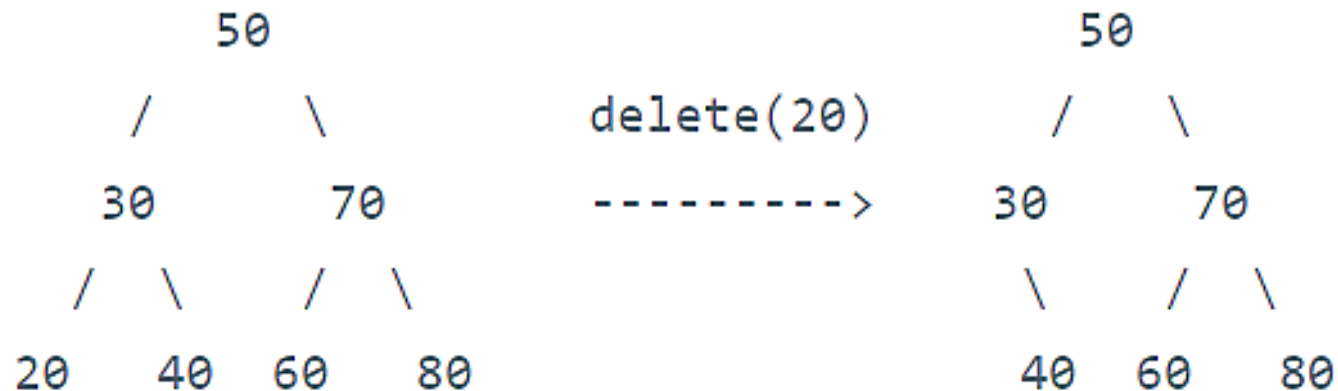




# Deletion in Binary Search Tree

The node to be deleted is a leaf node

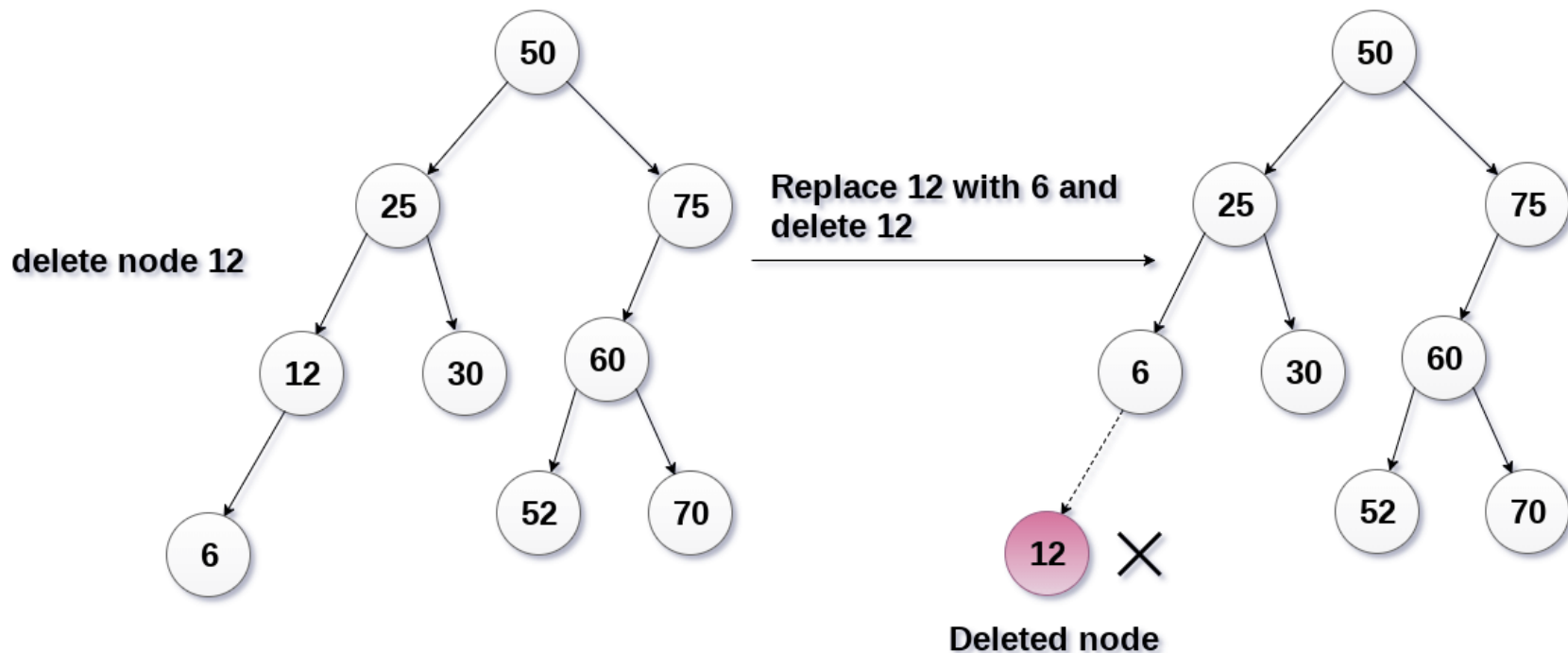
Simply remove the node from the tree.





# Deletion in Binary Search Tree

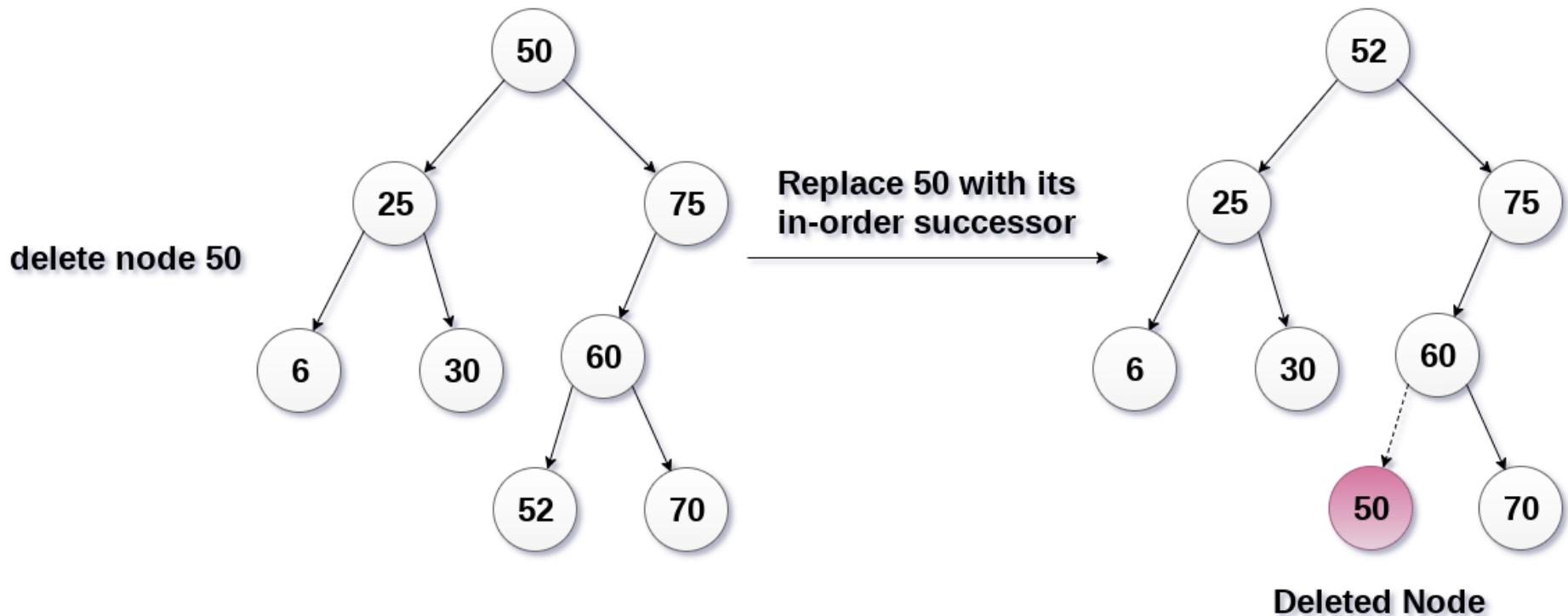
The node to be deleted has only one child.





# Deletion in Binary Search Tree

The node to be deleted has two children.

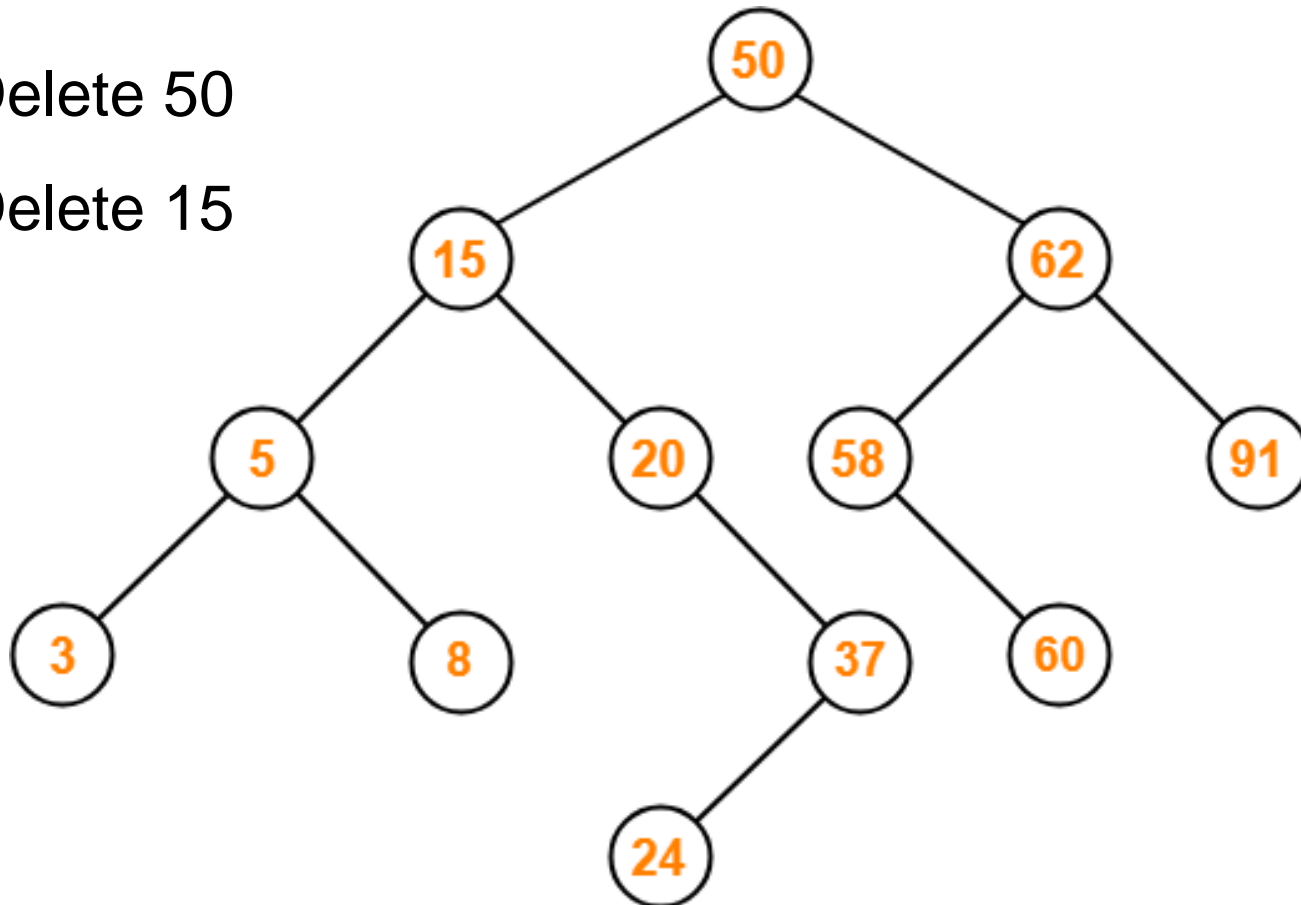




# Deletion in Binary Search Tree

Delete 50

Delete 15



**Binary Search Tree**





# Huffman Tree







# Huffman Coding – Huffman Tree

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The steps involved in the construction of Huffman Tree are as follows-

## **Step-01:**

- Create a leaf node for each character of the text.
- Leaf node of a character contains the occurring frequency of that character.

## **Step-02:**

- Arrange all the nodes in increasing order of their frequency value.





# Huffman Coding – Huffman Tree

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## **Step-03:**

Considering the first two nodes having minimum frequency,

- Create a new internal node.
- The frequency of this new node is the sum of frequency of those two nodes.
- Make the first node as a left child and the other node as a right child of the newly created node.

## **Step-04:**

- Keep repeating Step-02 and Step-03 until all the nodes form a single tree.
- The tree finally obtained is the desired Huffman Tree.





# Huffman Coding – Huffman Tree

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$$\text{Average code length per character} = \frac{\sum (\text{frequency}_i \times \text{code length}_i)}{\sum \text{frequency}_i}$$

Total number of bits in Huffman encoded message

= Total number of characters in the message x  
Average code length per character





# Huffman Coding – Huffman Tree

## Problem-

A file contains the following characters with the frequencies as shown. If Huffman Coding is used for data compression, determine-

- Huffman Code for each character
- Average code length
- Length of Huffman encoded message (in bits)

Characters	Frequencies
a	10
e	15
i	12
o	3
u	4
s	13
t	1



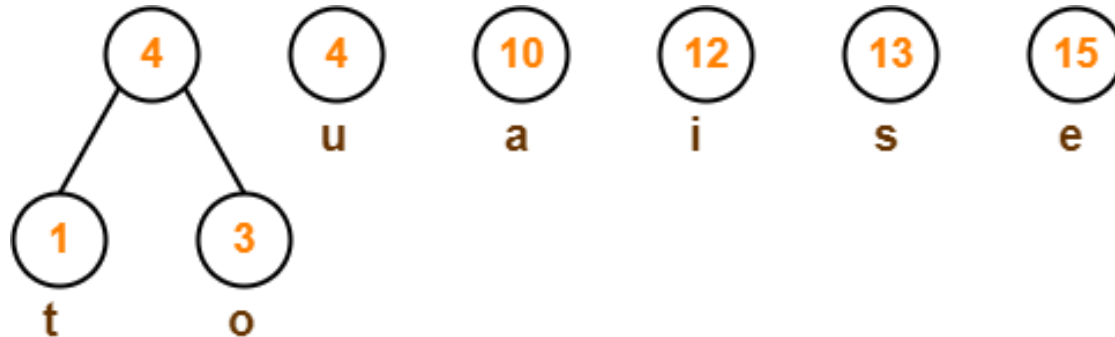


# Huffman Coding – Huffman Tree

## Step-01:



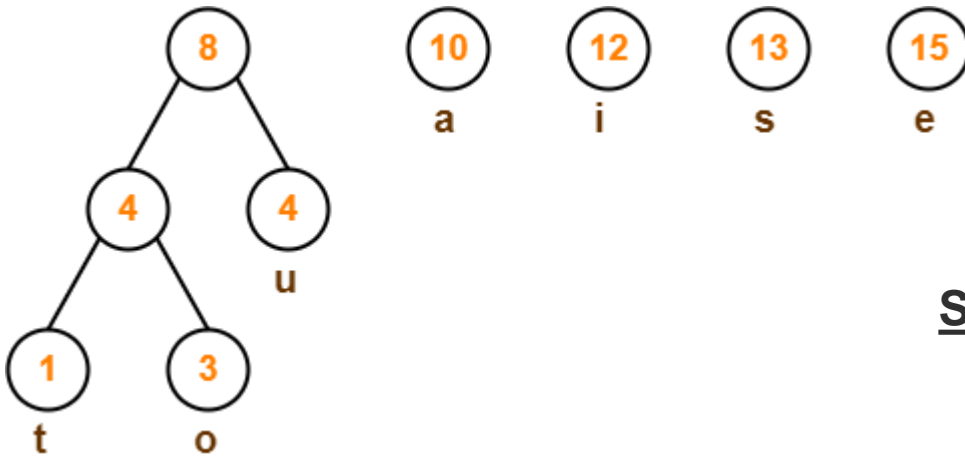
## Step-02:



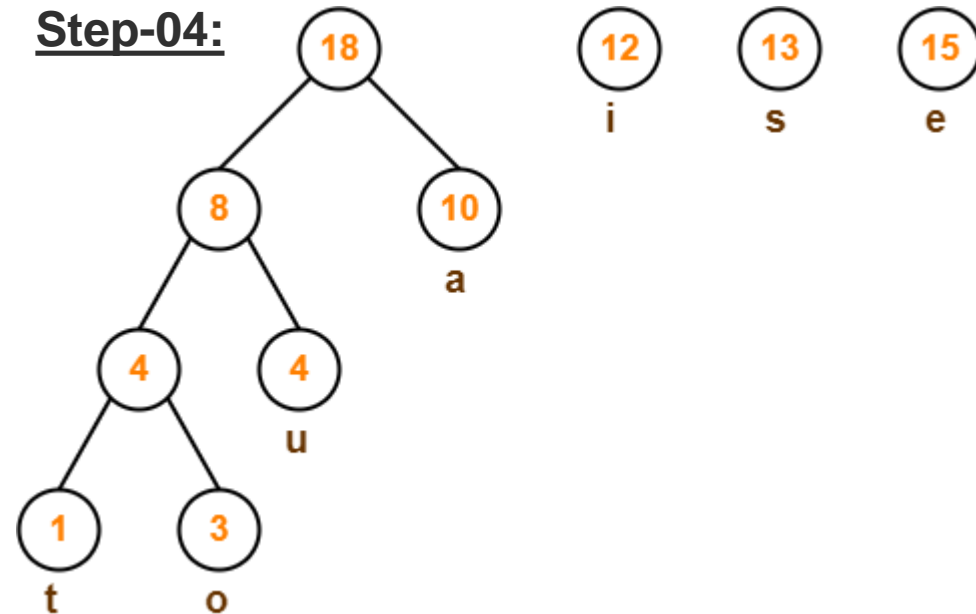


# Huffman Coding – Huffman Tree

## Step-03:



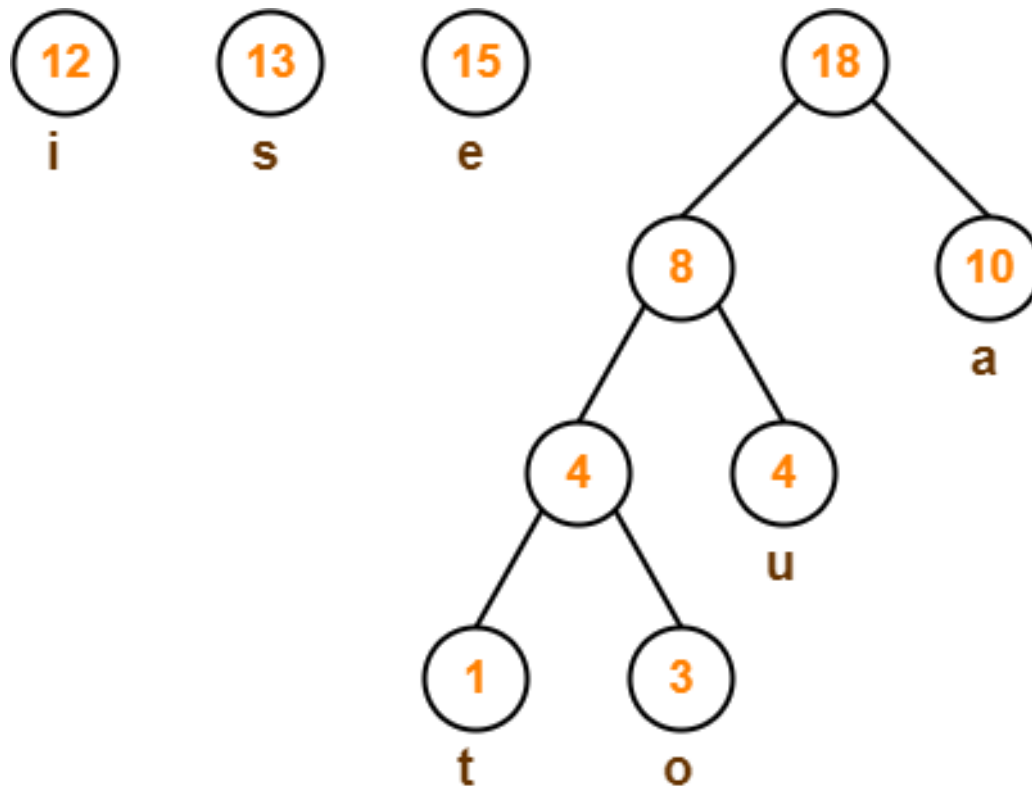
## Step-04:





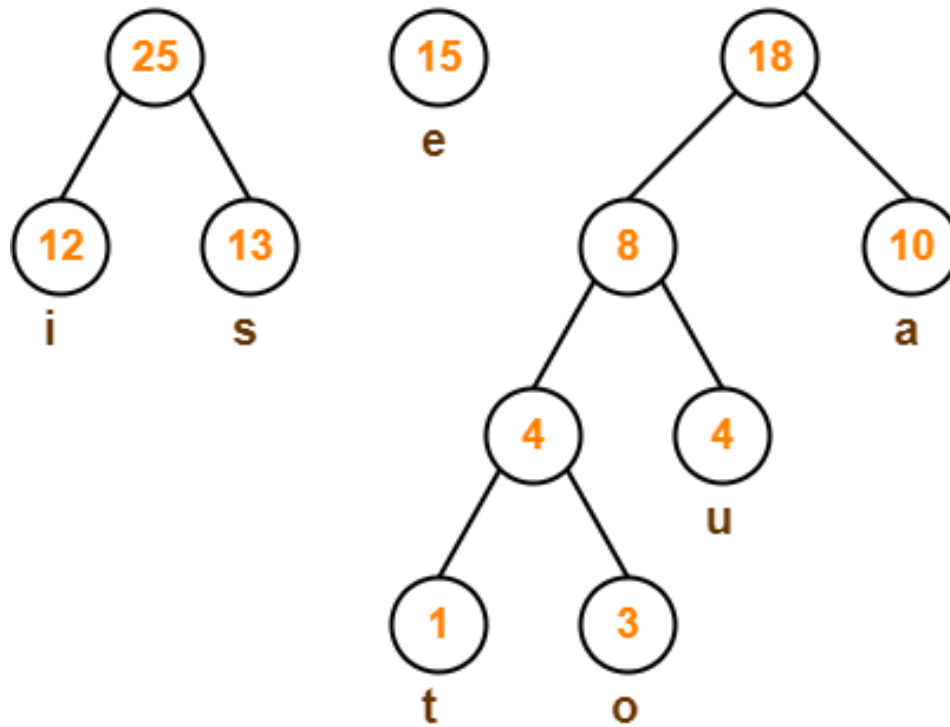
# Huffman Coding – Huffman Tree

Step-05:





# Huffman Coding – Huffman Tree

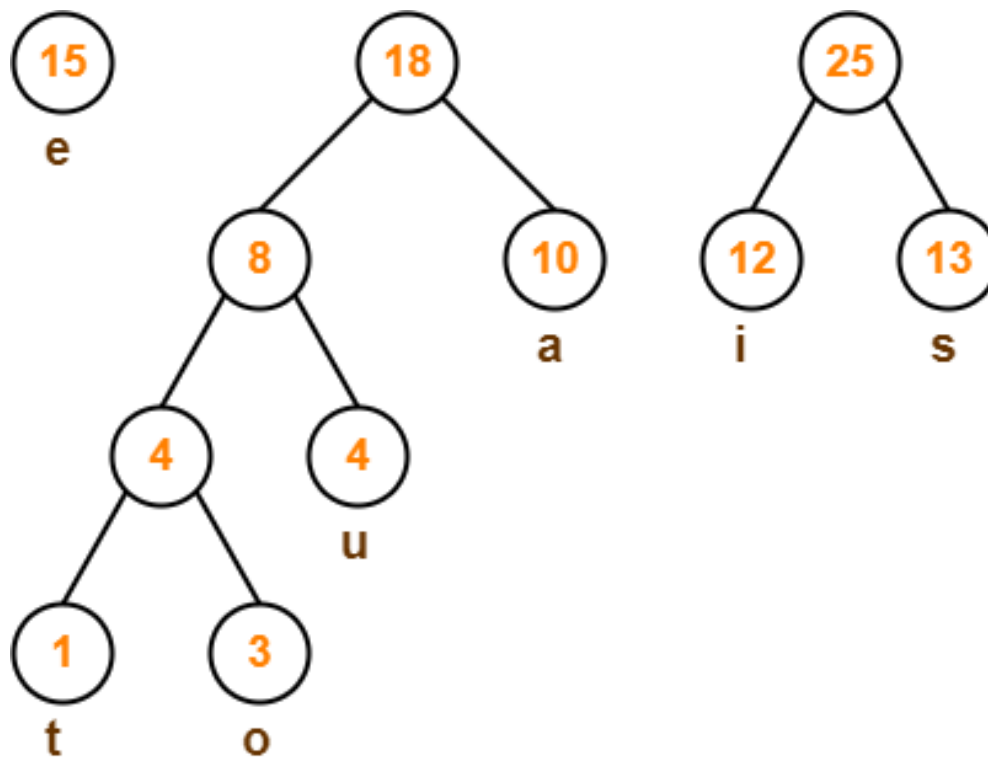






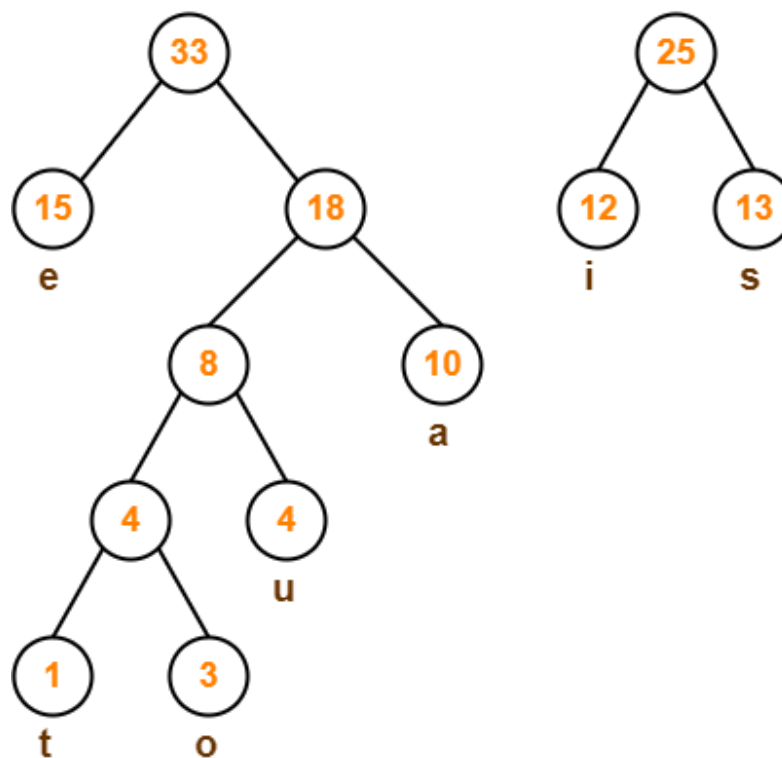
# Huffman Coding – Huffman Tree

Step-06:





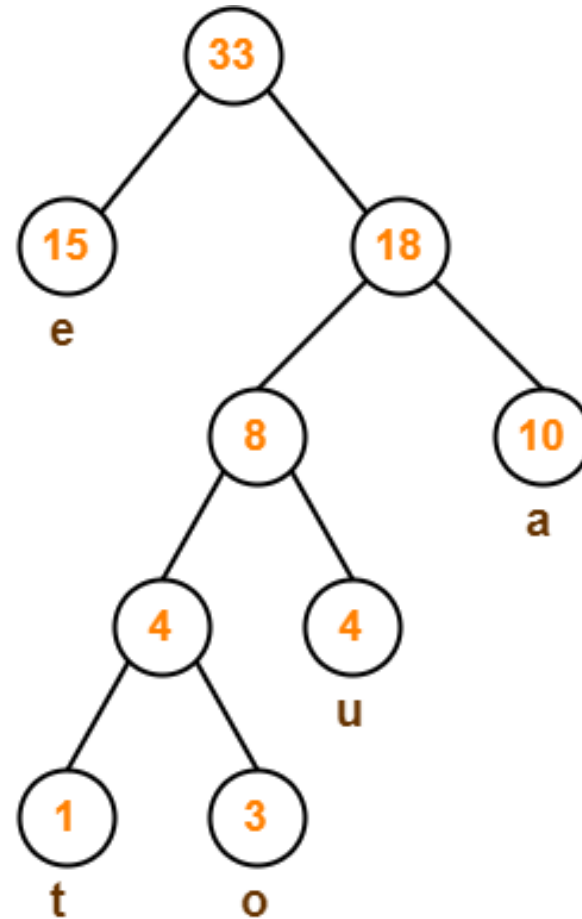
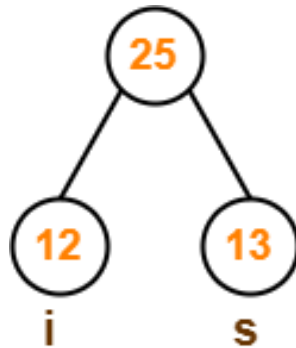
# Huffman Coding – Huffman Tree





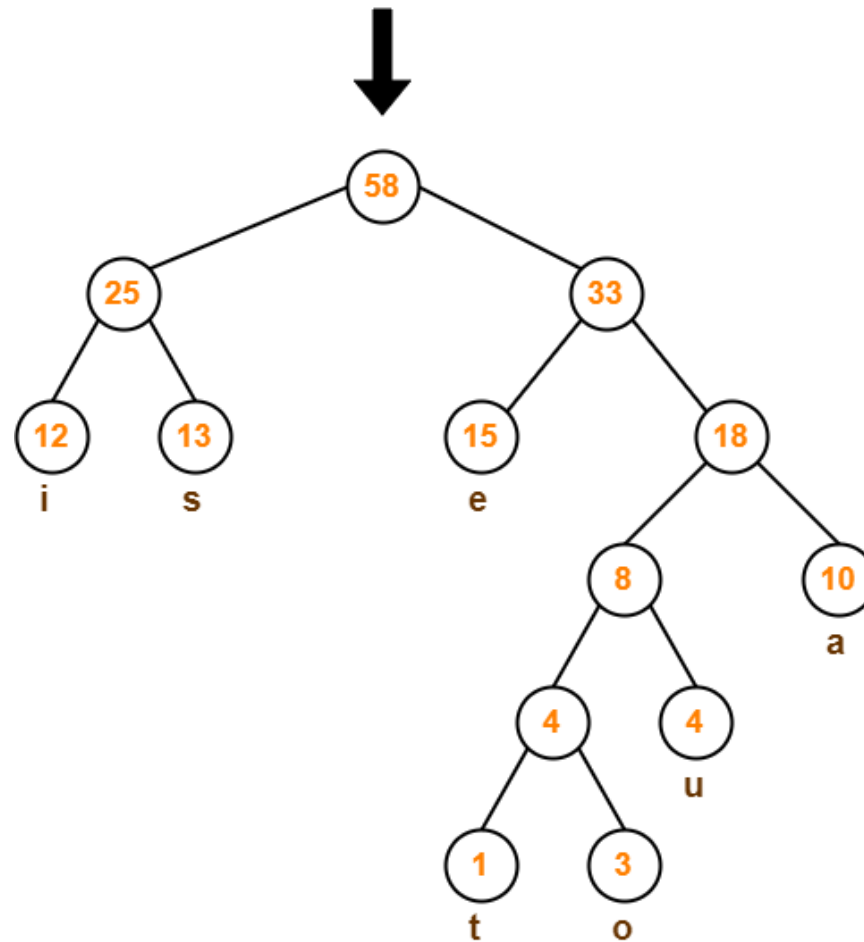
# Huffman Coding – Huffman Tree

Step-07:





# Huffman Coding – Huffman Tree



Huffman Tree





# Huffman Coding – Huffman Tree

After assigning weight to all the edges, the modified Huffman Tree is:-

To write Huffman Code for any character, traverse the Huffman Tree from root node to the leaf node of that character.

a = 111

e = 10

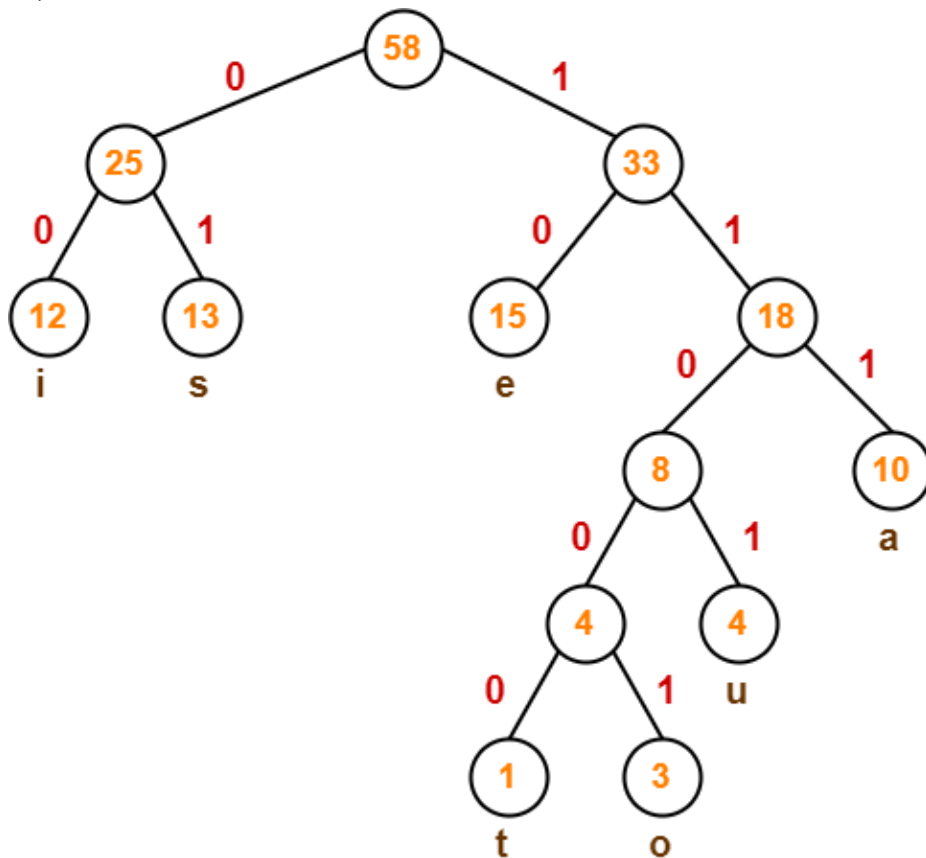
i = 00

o = 11001

u = 1101

s = 01

t = 11000



Huffman Tree





# Huffman Coding – Huffman Tree

Characters	Frequencies	Code	Code Length	Total Length
a	10	111	3	30
e	15	10	2	30
i	12	00	2	24
o	3	11001	5	15
u	4	1101	4	16
s	13	01	2	26
t	1	11000	5	5

146 bits





# Huffman Coding – Huffman Tree

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**Average code length:**

$$= \sum ( \text{frequency}_i \times \text{code length}_i ) / \sum ( \text{frequency}_i )$$

$$= \{ (10 \times 3) + (15 \times 2) + (12 \times 2) + (3 \times 5) + (4 \times 4) + (13 \times 2) + (1 \times 5) \} / (10 + 15 + 12 + 3 + 4 + 13 + 1)$$

$$= 2.52$$





# Huffman Coding – Huffman Tree

**Problem-** A file contains the following characters with the frequencies as shown. determine-

- Huffman Code for each character
- Average code length
- Length of Huffman encoded message (in bits)

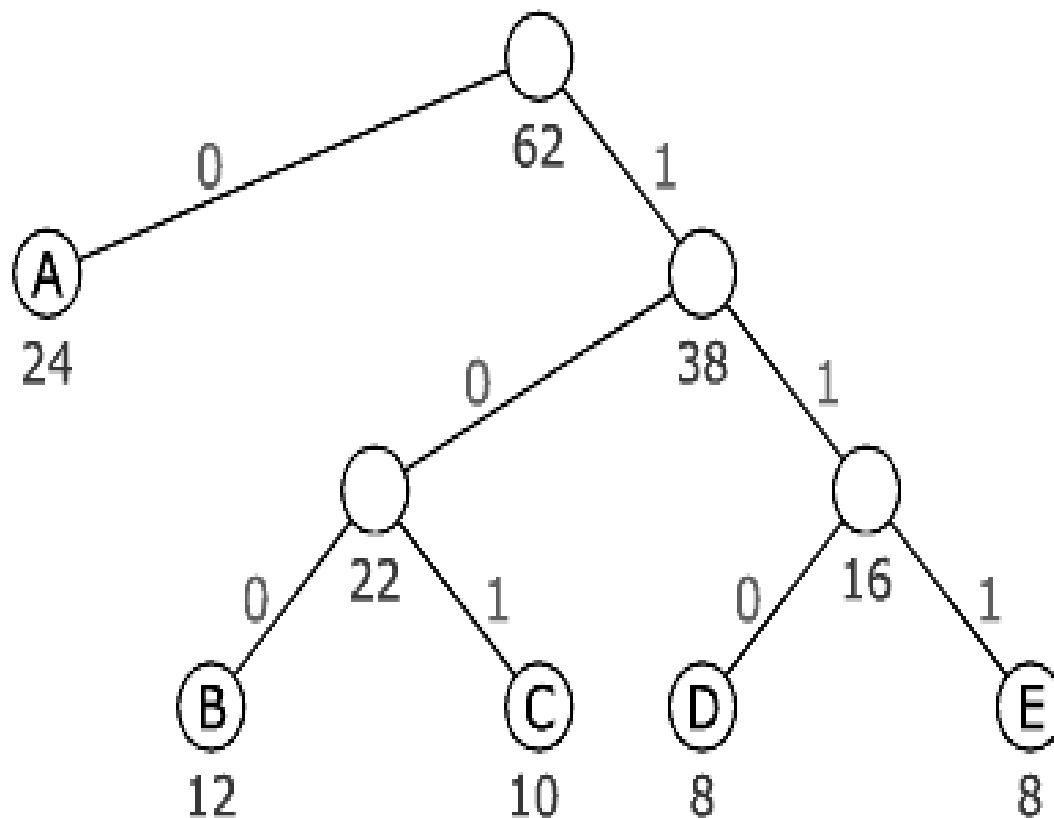
Symbol	Frequency
A	24
B	12
C	10
D	8
E	8







# Huffman Coding – Huffman Tree





# Huffman Coding – Huffman Tree

Symbol	Frequency	Code	Code Length	Total Length
A	24	0	1	24
B	12	100	3	36
C	10	101	3	30
D	8	110	3	24
E	8	111	3	24

Total : 138 bits



