Lecture#15

Data Structures

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Faculty Profile

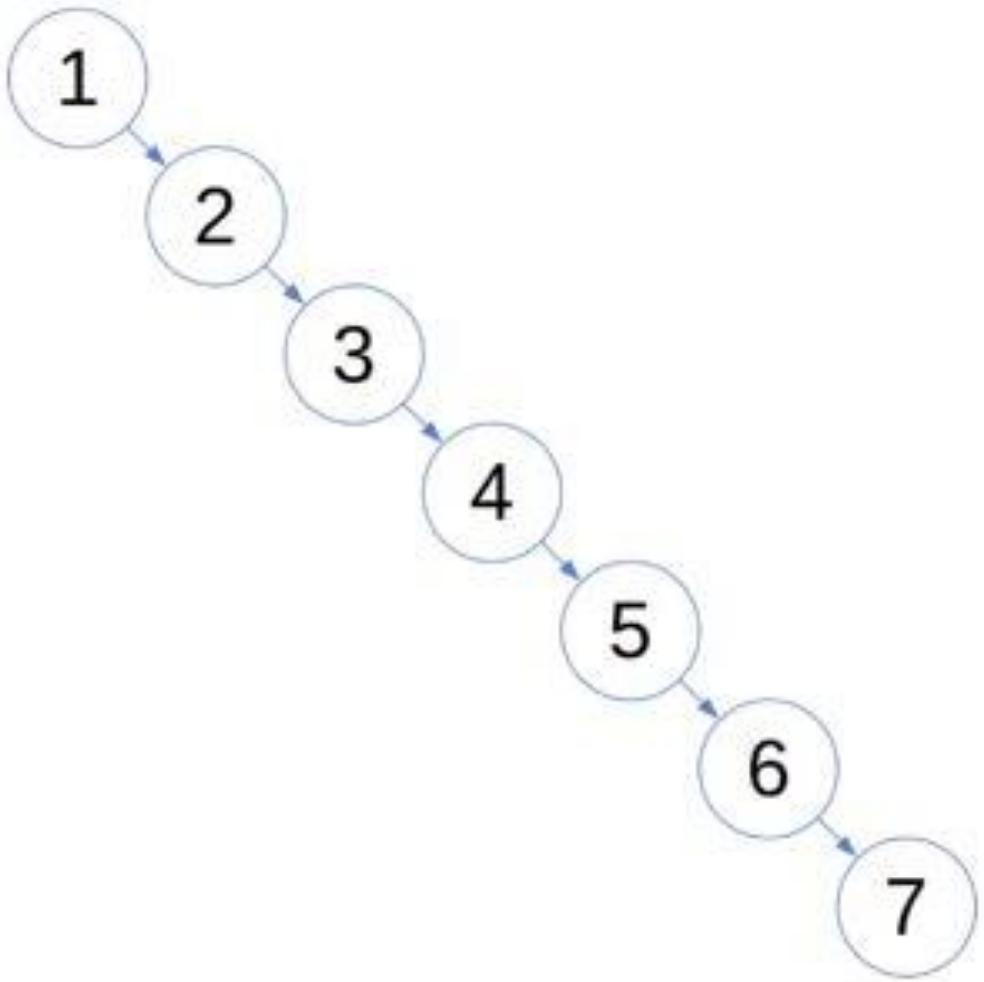
Tree

AVL Tree

An AVL tree defined as a self-balancing Binary Search Tree (BST) where the difference between heights of left and right subtrees for any node cannot be more than one.

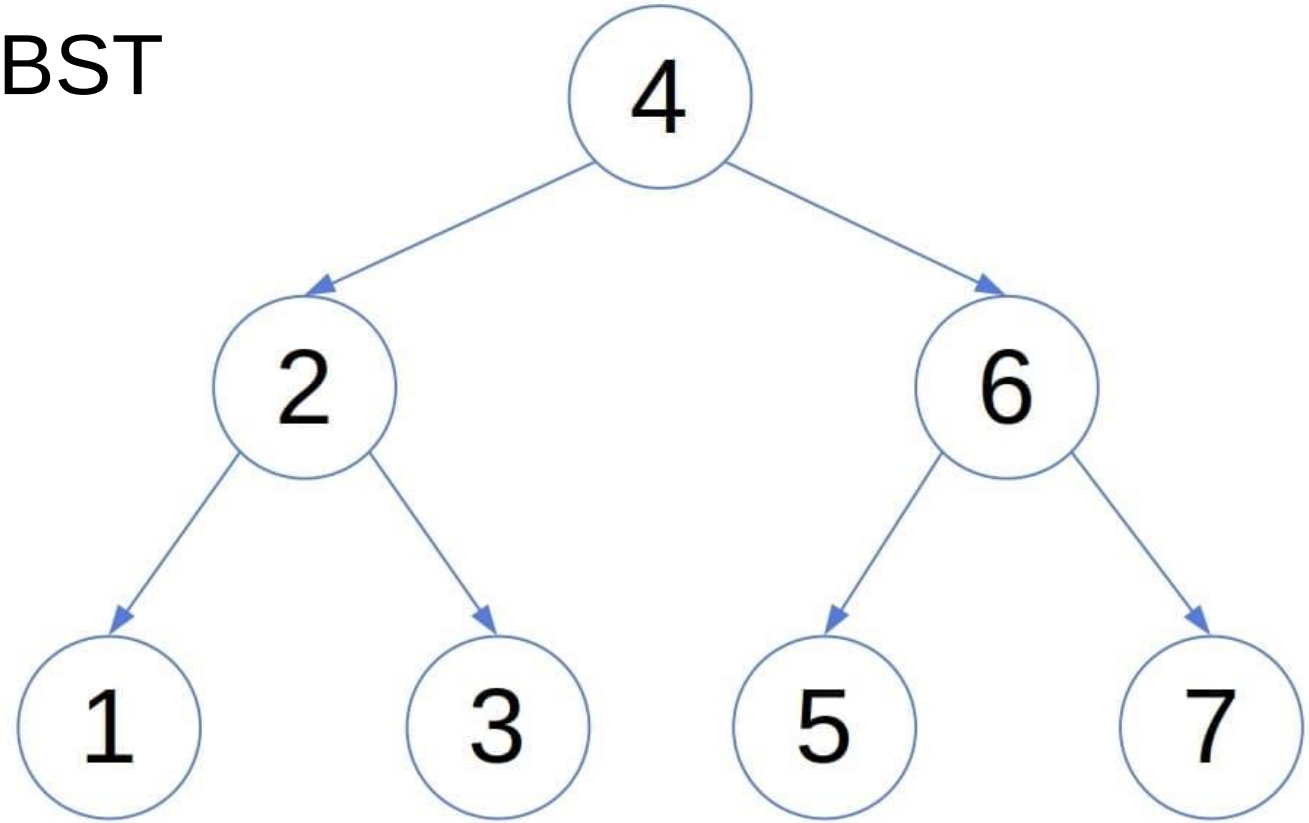
AVL Tree

Generate BST for –
1, 2, 3, 4, 5, 6, 7,



AVL Tree

Balanced BST



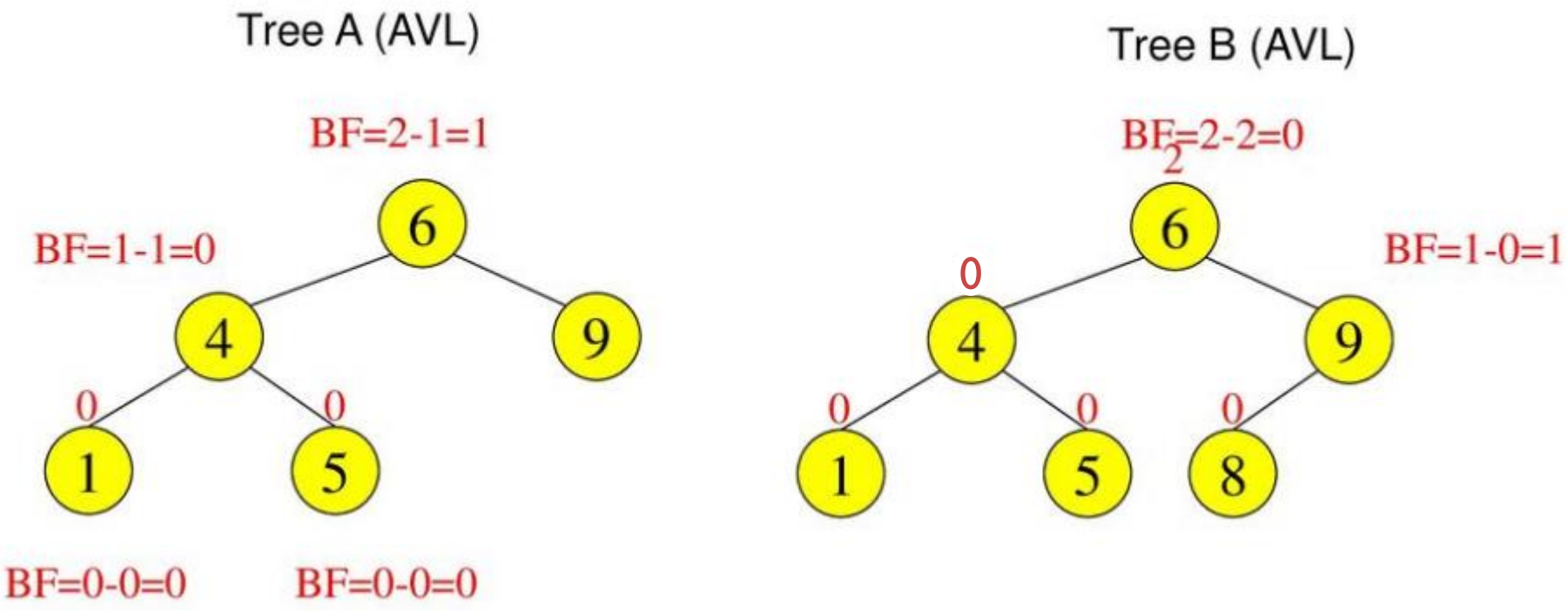
AVL Tree

- AVL trees are height-balanced binary search trees
- Balance factor of a node
  - › height(left subtree) - height(right subtree)
- An AVL tree has balance factor calculated at every node
  - › For every node, heights of left and right subtree can differ by no more than 1

AVL Tree

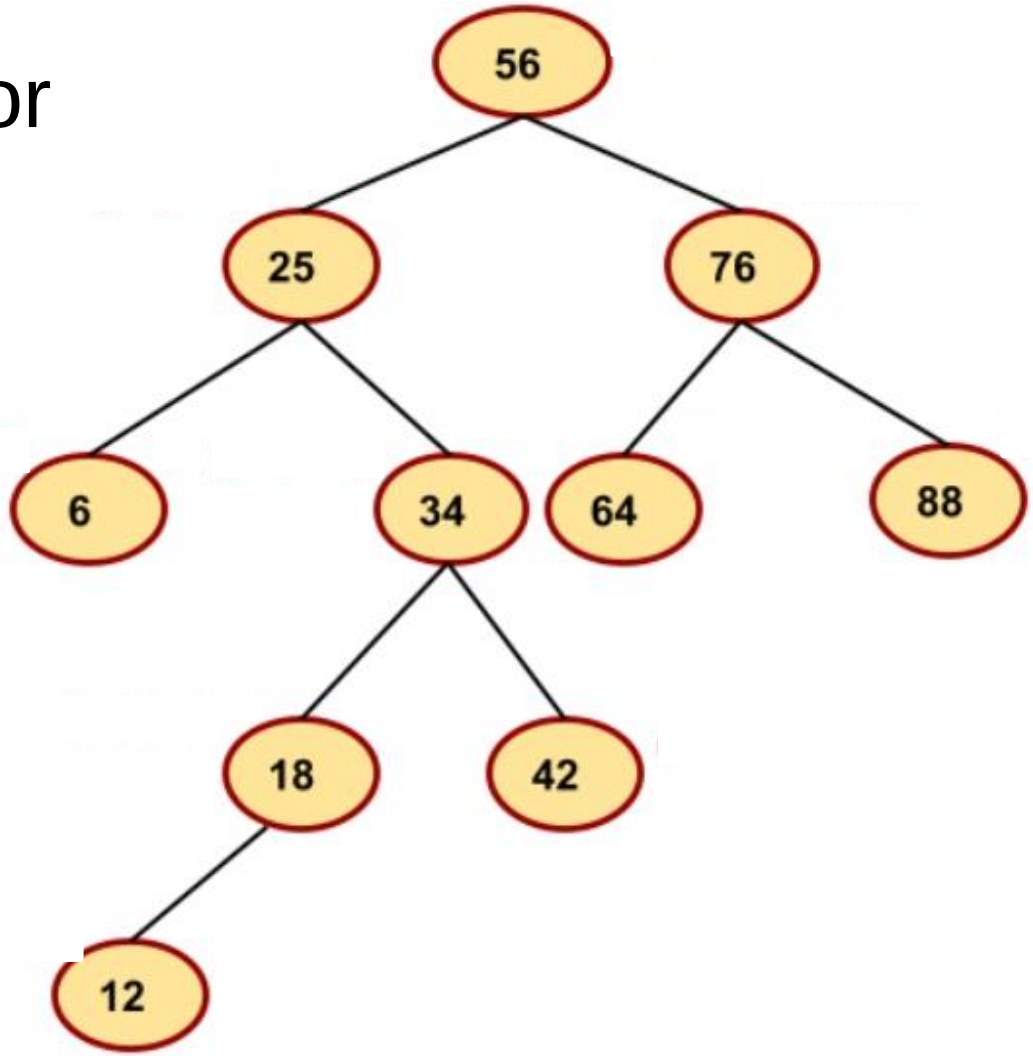
Balance Factor =

height (left sub-tree) – height (right sub-tree)



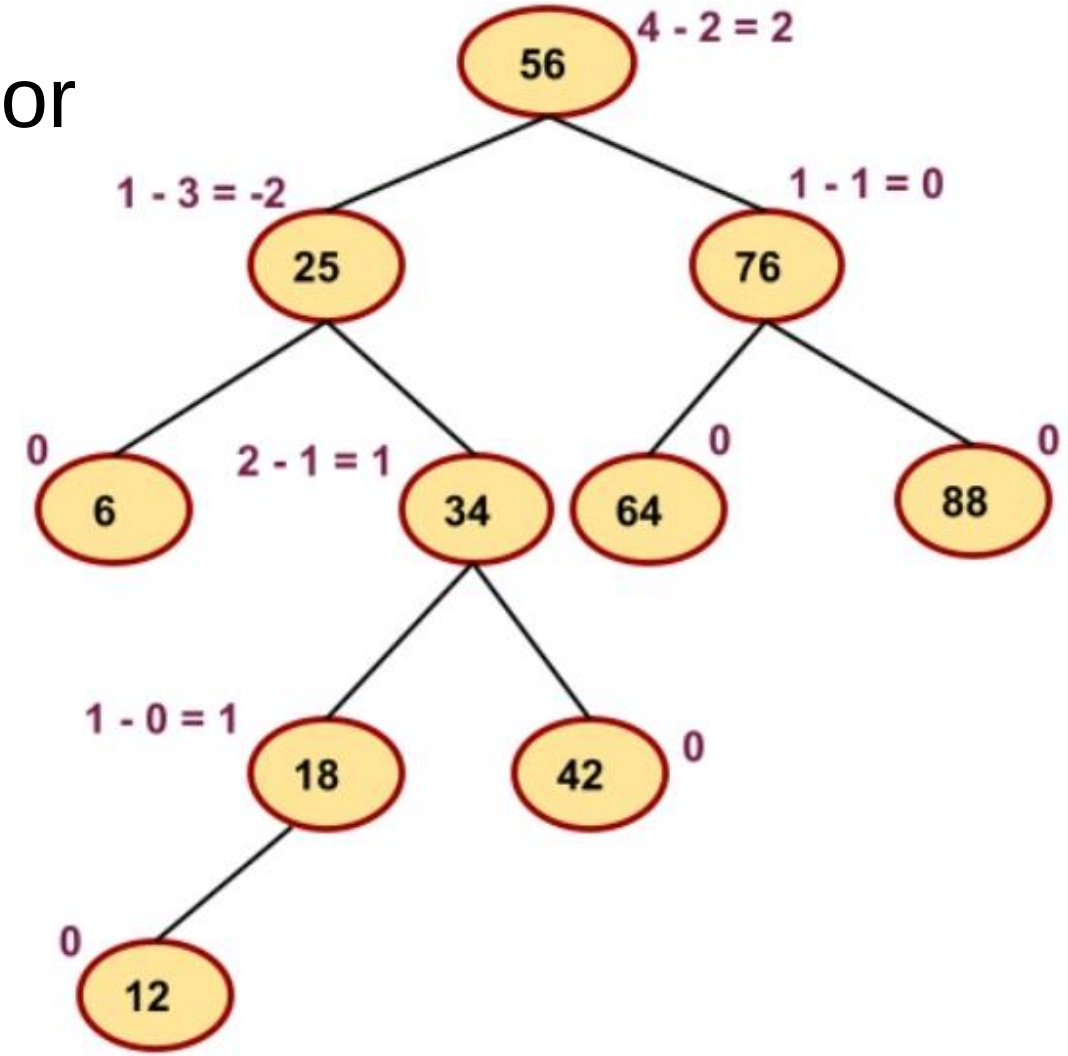
AVL Tree

Balance Factor



AVL Tree

Balance Factor



Insertions in AVL Trees

Let the node that needs rebalancing be α.

There are 4 cases:

Outside Cases (require single rotation) :

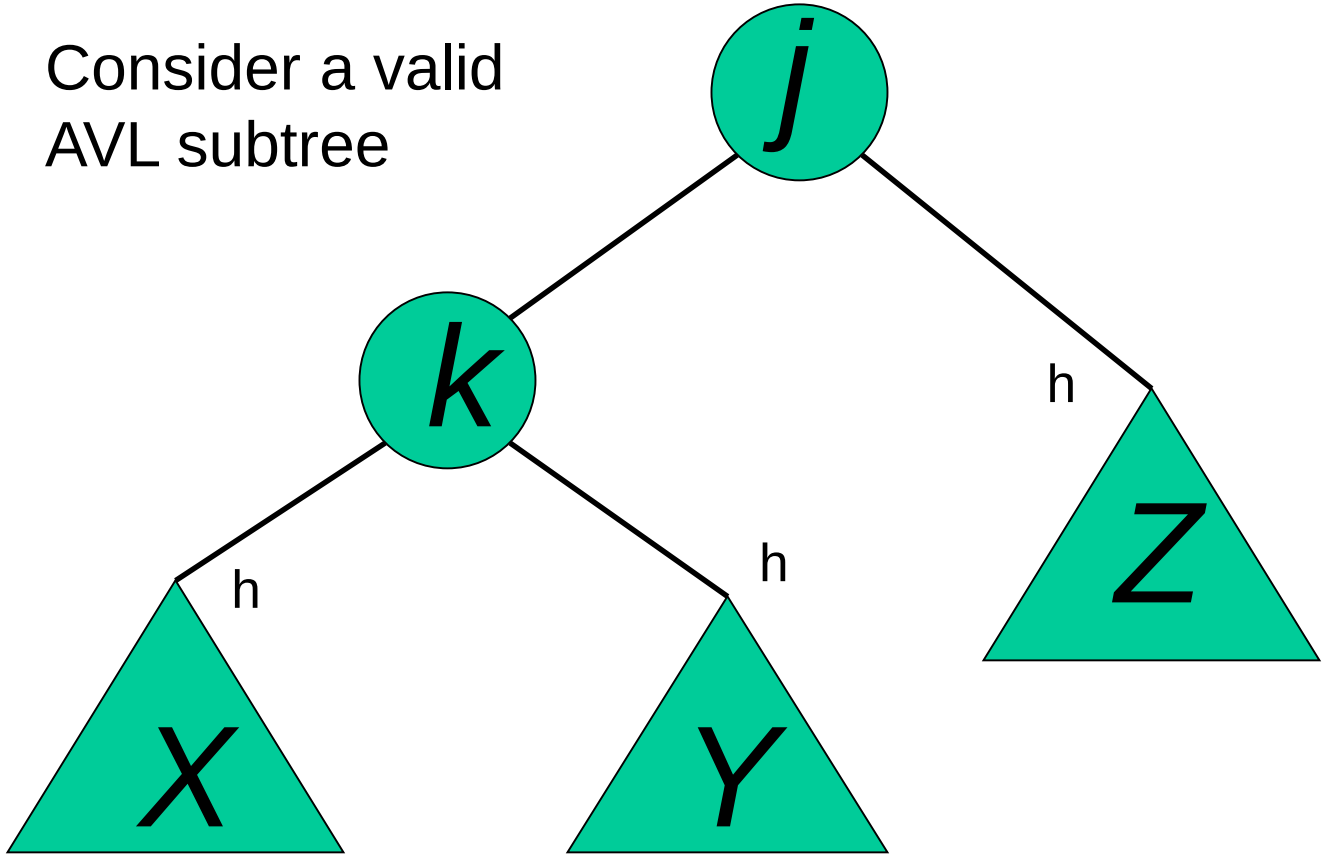
1. Insertion into left subtree of left child of α.
2. Insertion into right subtree of right child of α.

Inside Cases (require double rotation) :

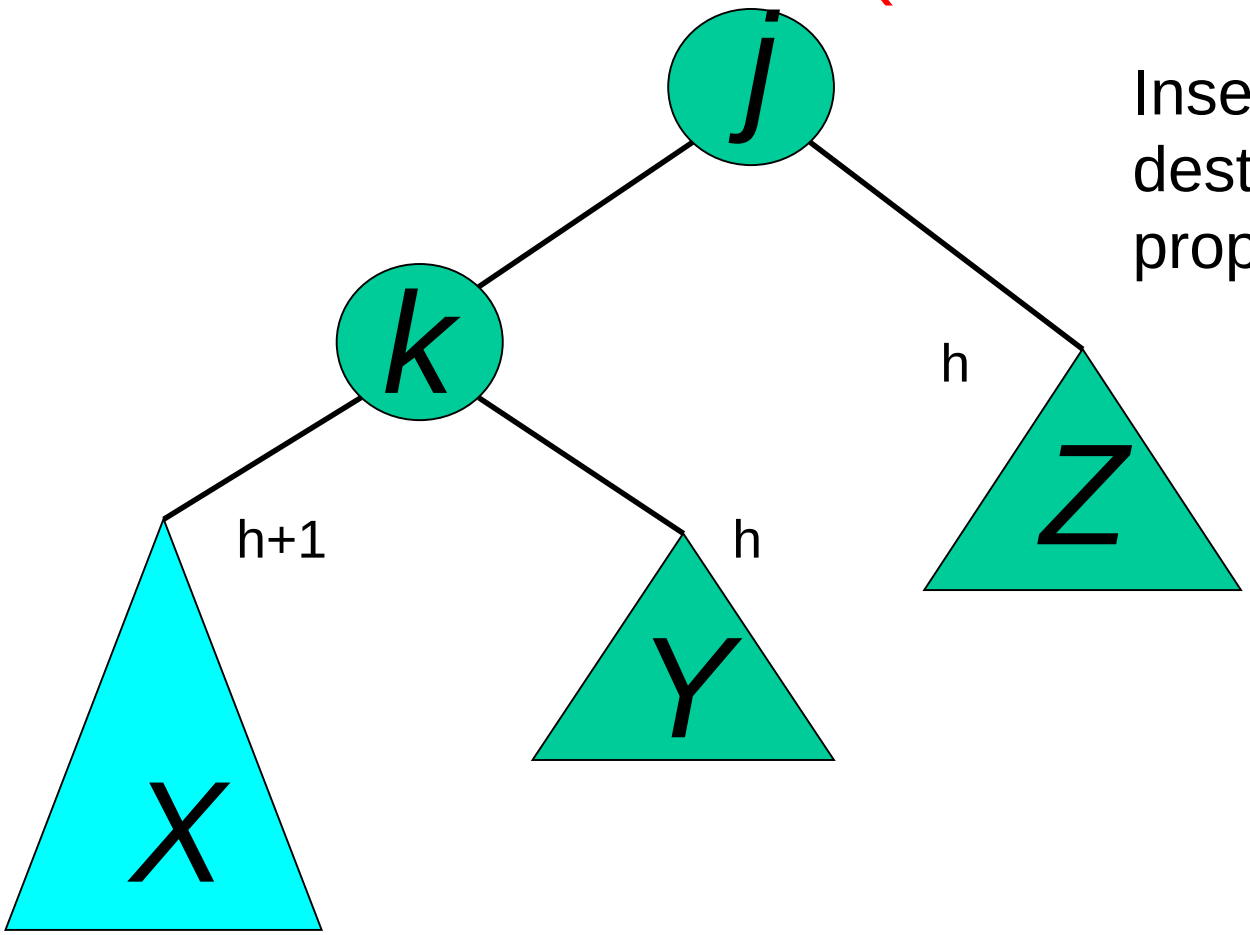
3. Insertion into right subtree of left child of α.
4. Insertion into left subtree of right child of α.

Insertions in AVL Trees (Outside Case)

Consider a valid AVL subtree

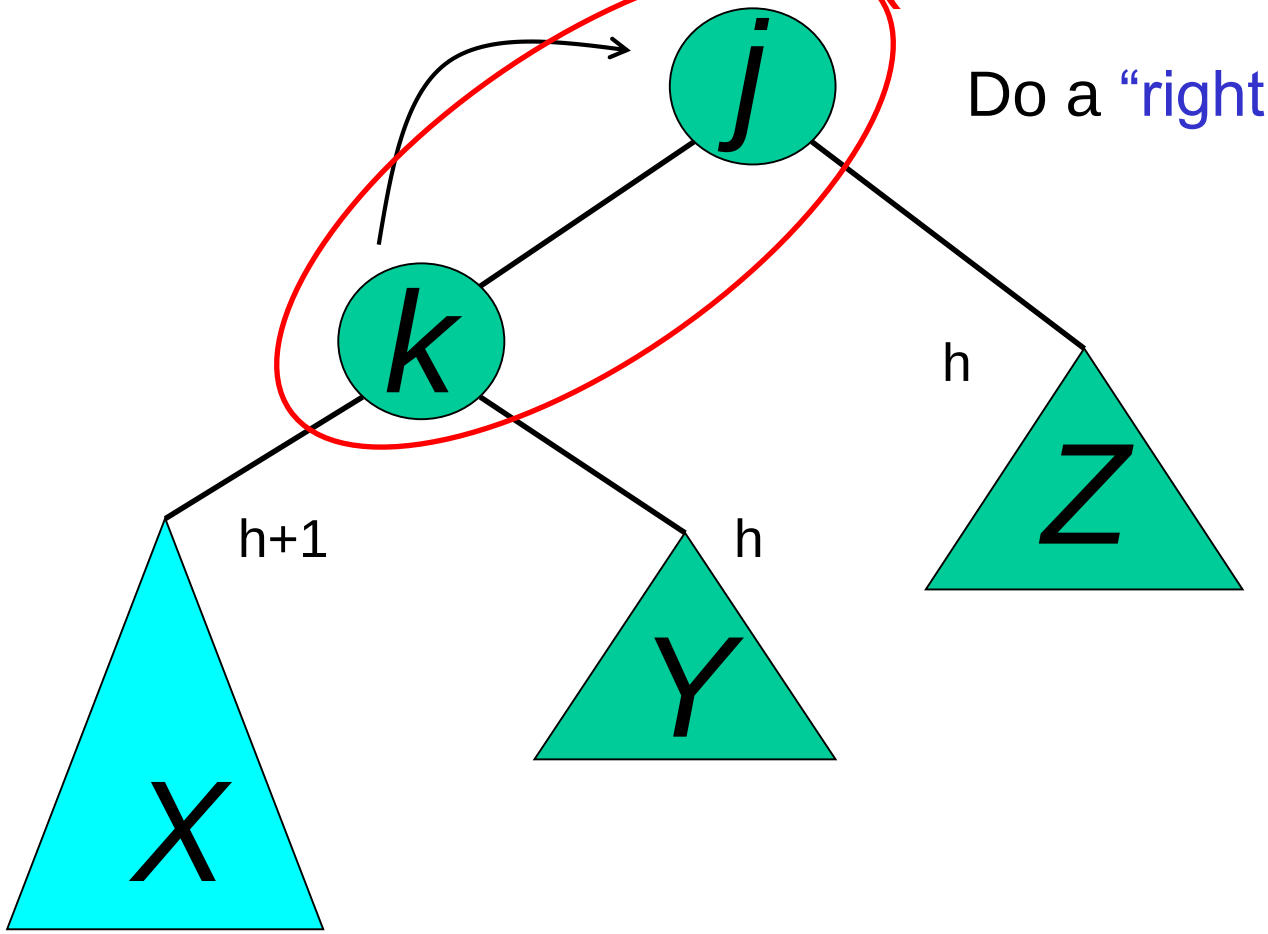


Insertions in AVL Trees (Outside Case)



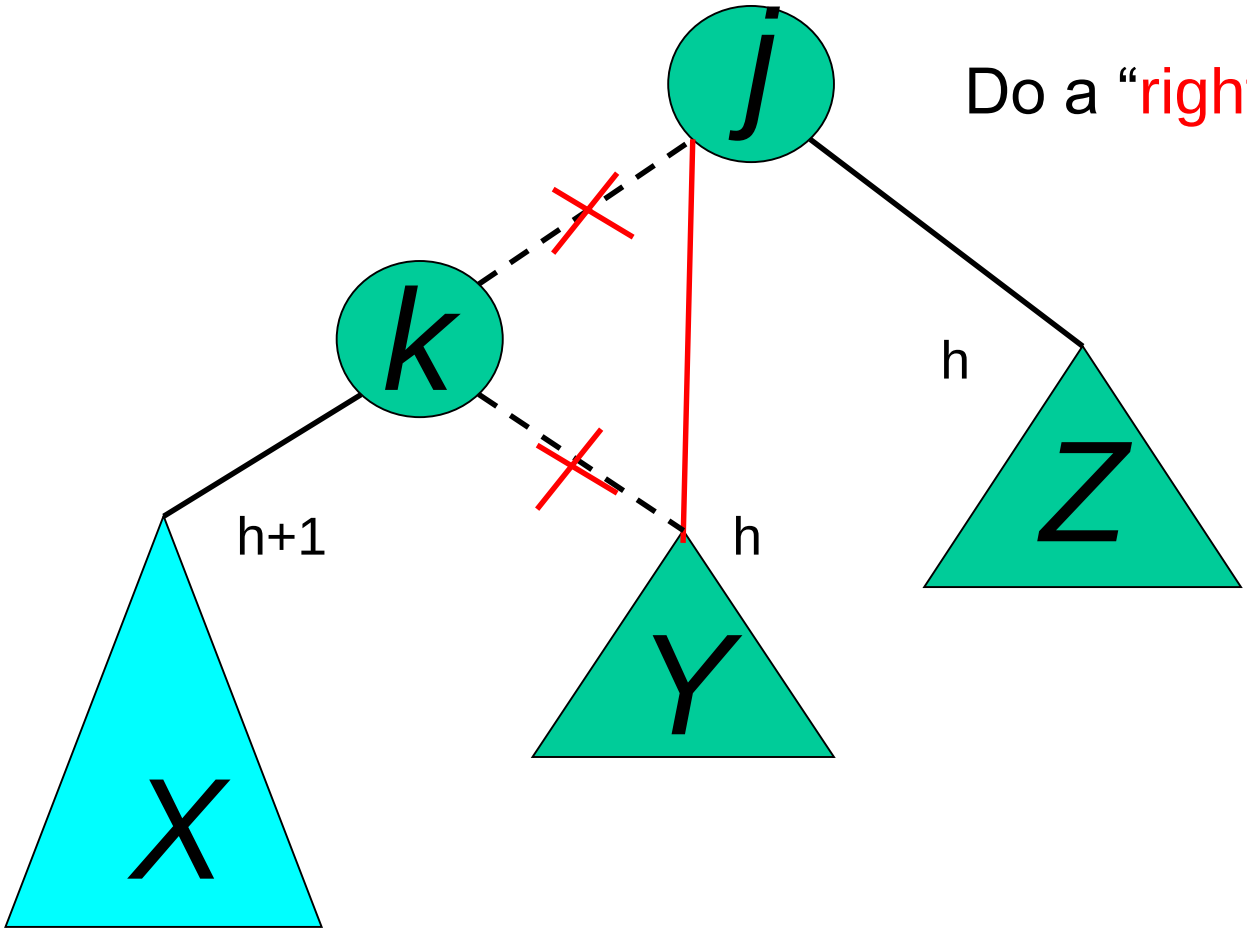
Inserting into X destroys the AVL property at node j

Insertions in AVL Trees (Outside Case)



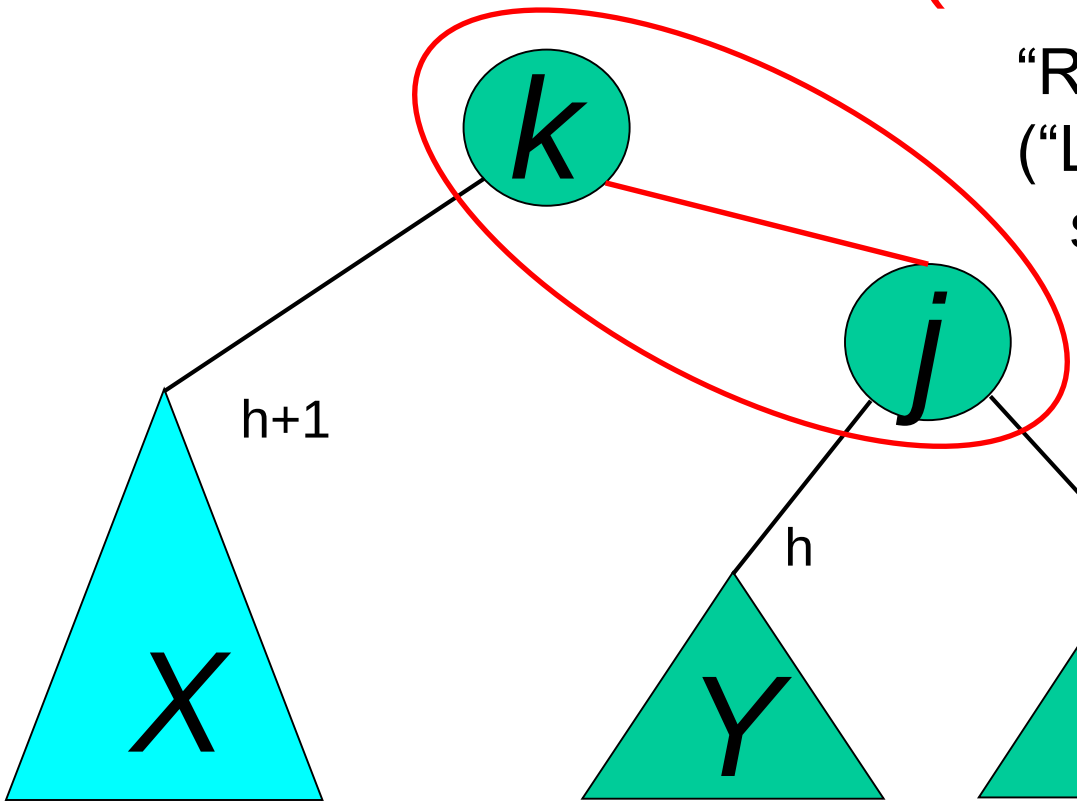
Do a "right rotation"

Insertions in AVL Trees (Outside Case)



Do a "right rotation"

Insertions in AVL Trees (Outside Case)

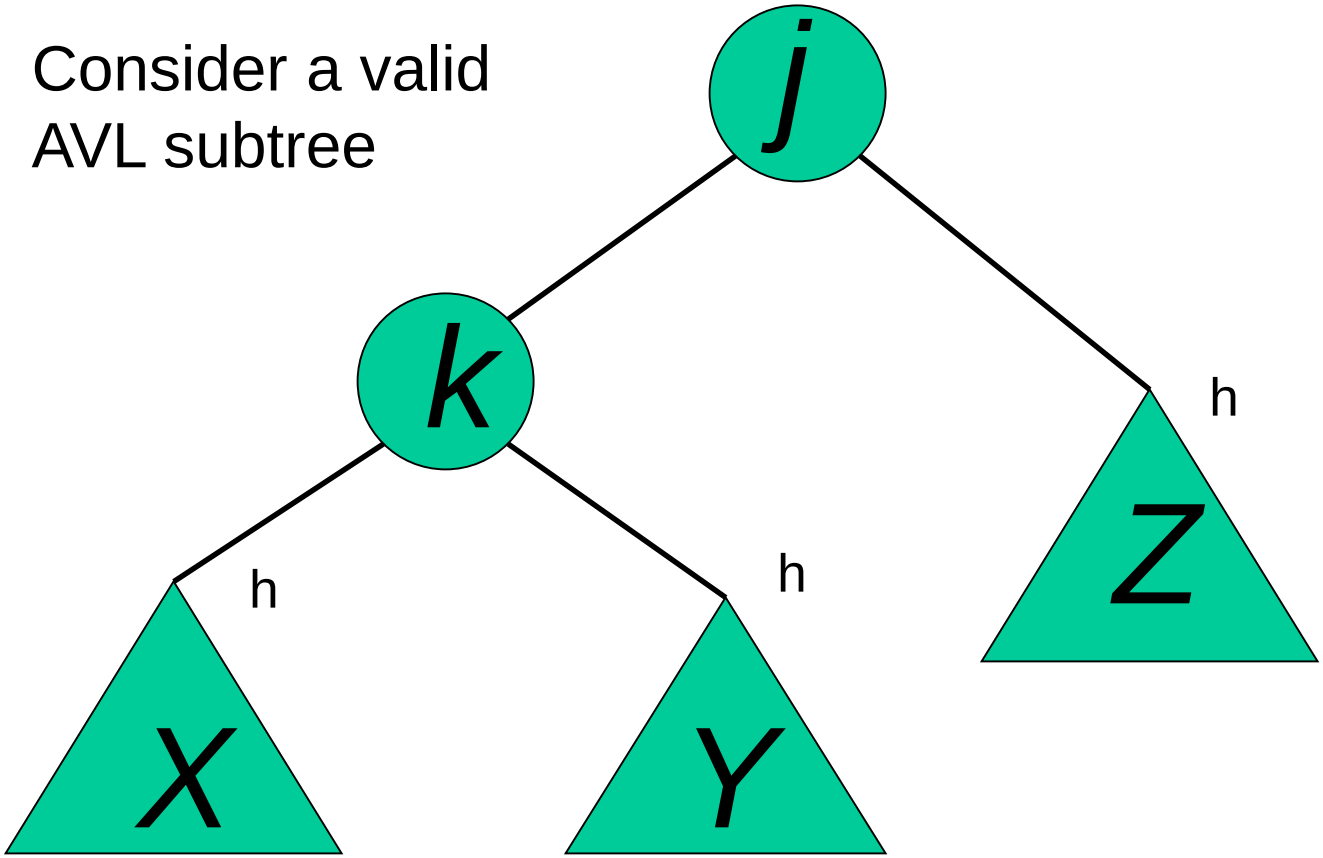


"Right rotation" done!
("Left rotation" is mirror symmetric)

AVL property has been restored!

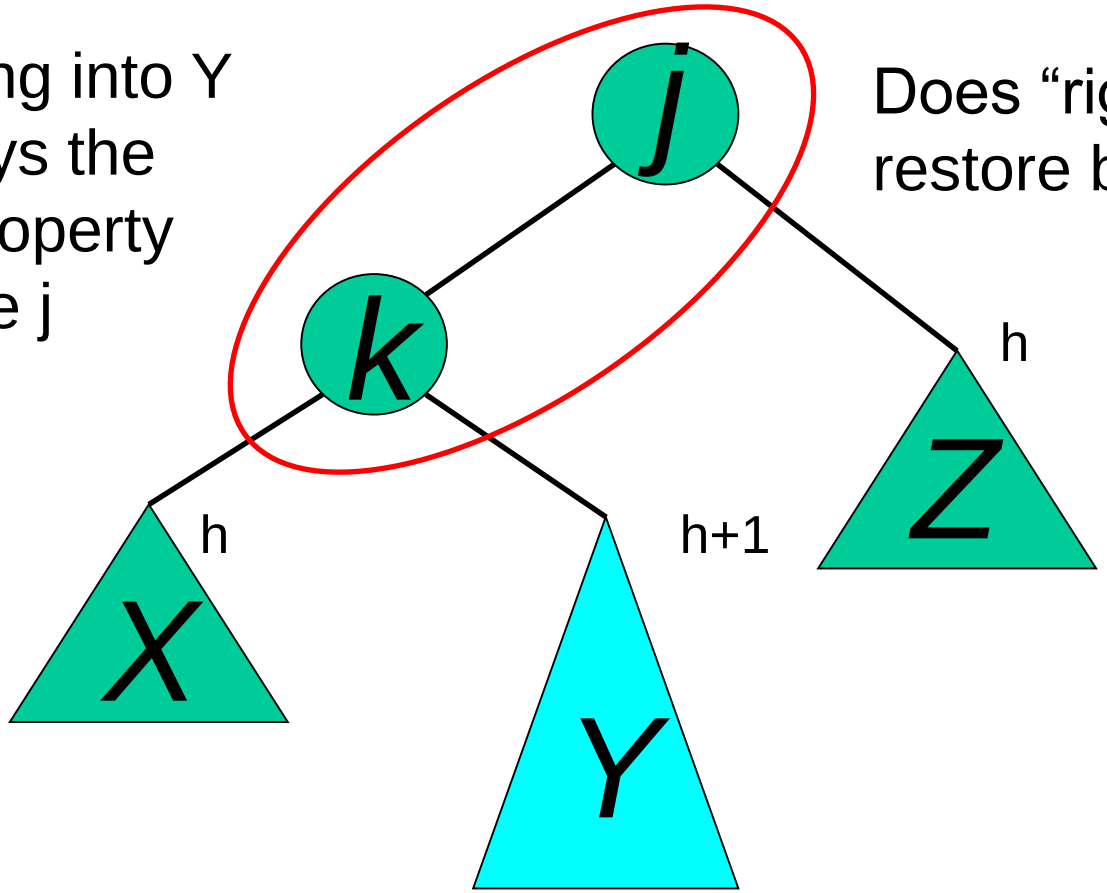
Insertions in AVL Trees (Inside Case)

Consider a valid AVL subtree



Insertions in AVL Trees (Inside Case)

Inserting into Y destroys the AVL property at node j



Does “right rotation” restore balance?

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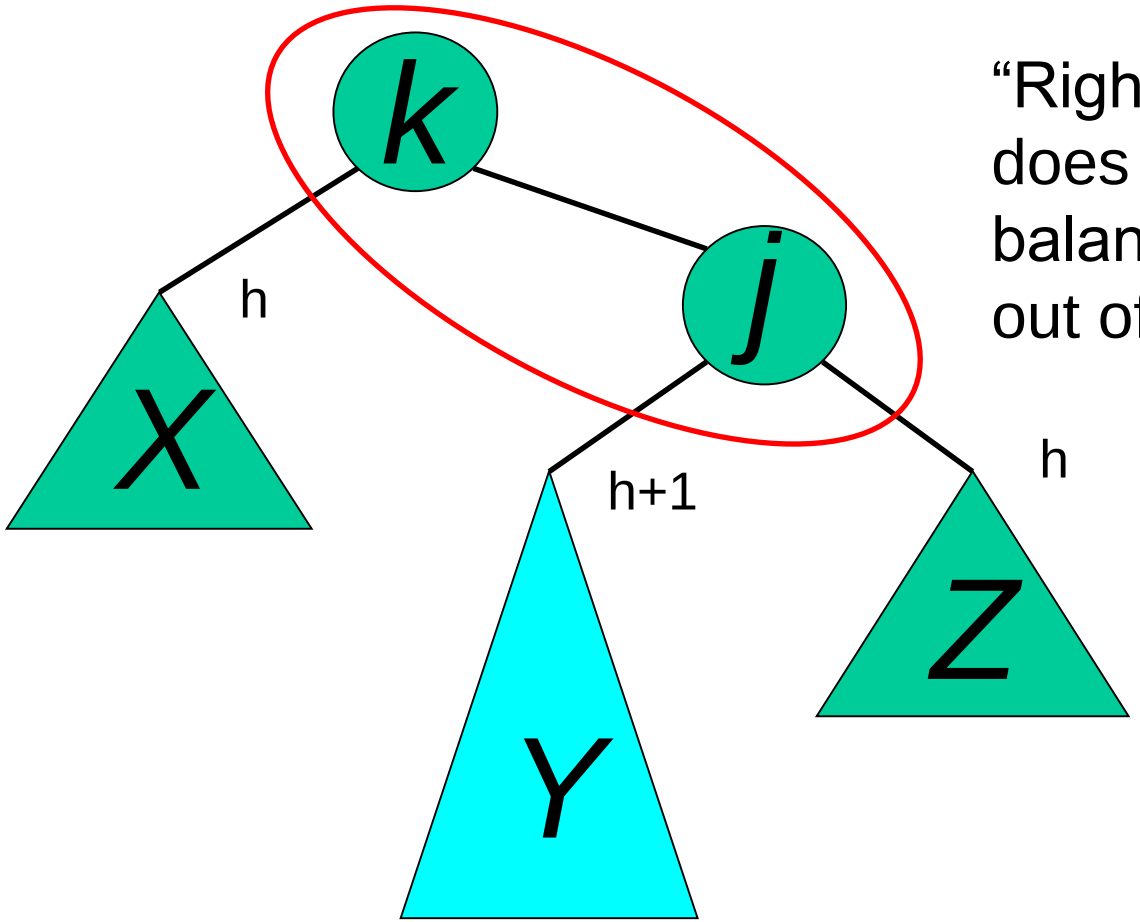
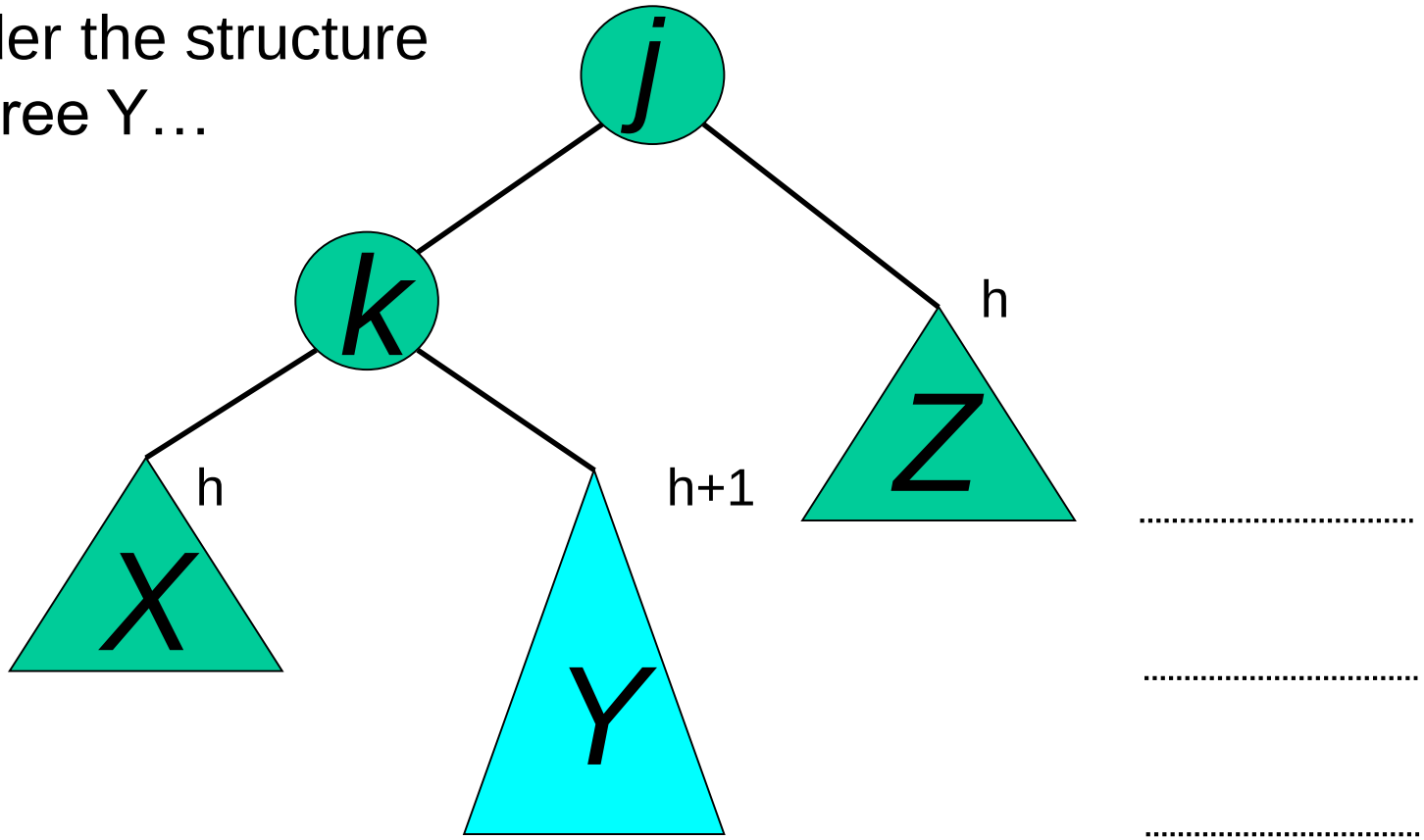
Insertions in AVL Trees (Inside Case)



"Right rotation" does not restore balance… now k is out of balance

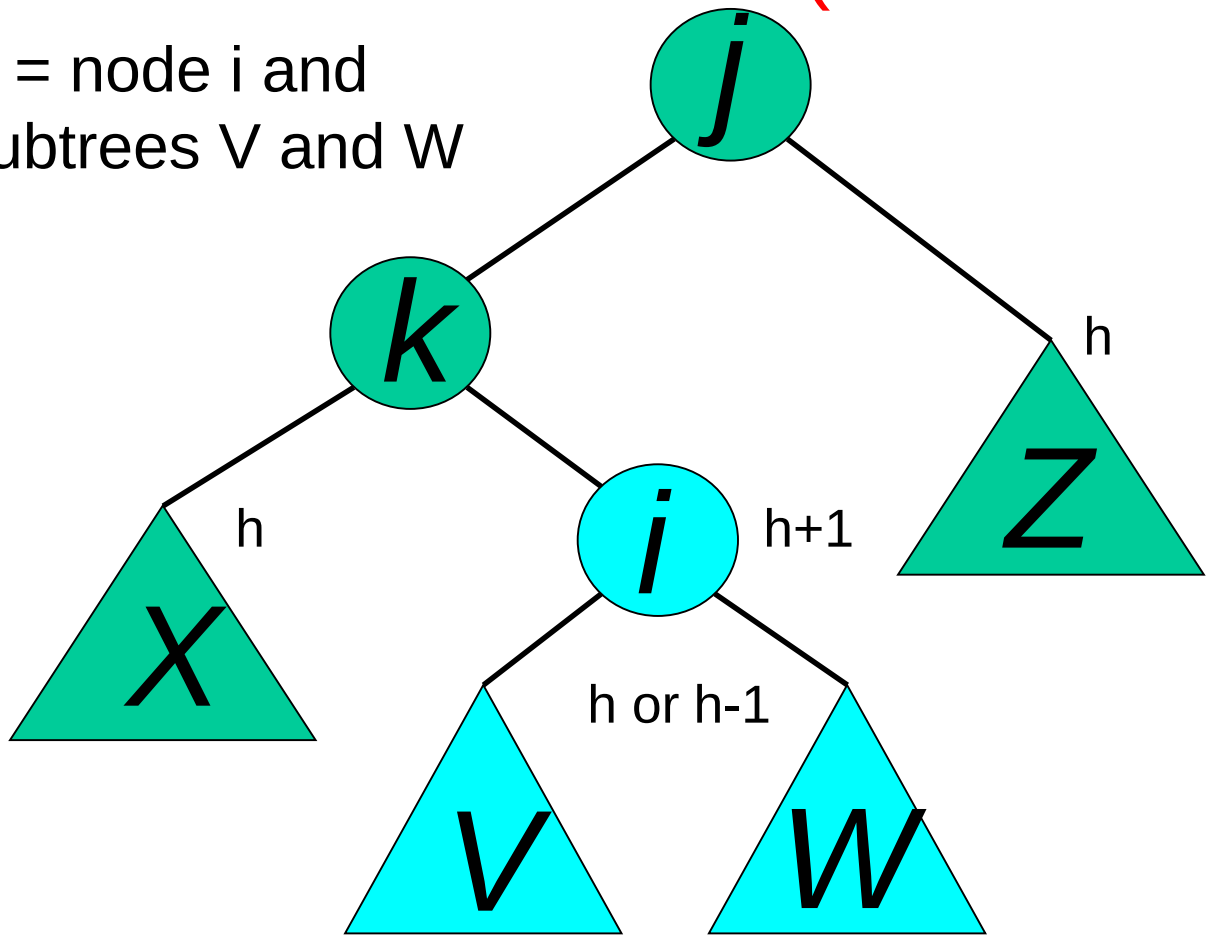
Insertions in AVL Trees (Inside Case)

Consider the structure of subtree Y…



Insertions in AVL Trees (Inside Case)

Y = node i and
subtrees V and W



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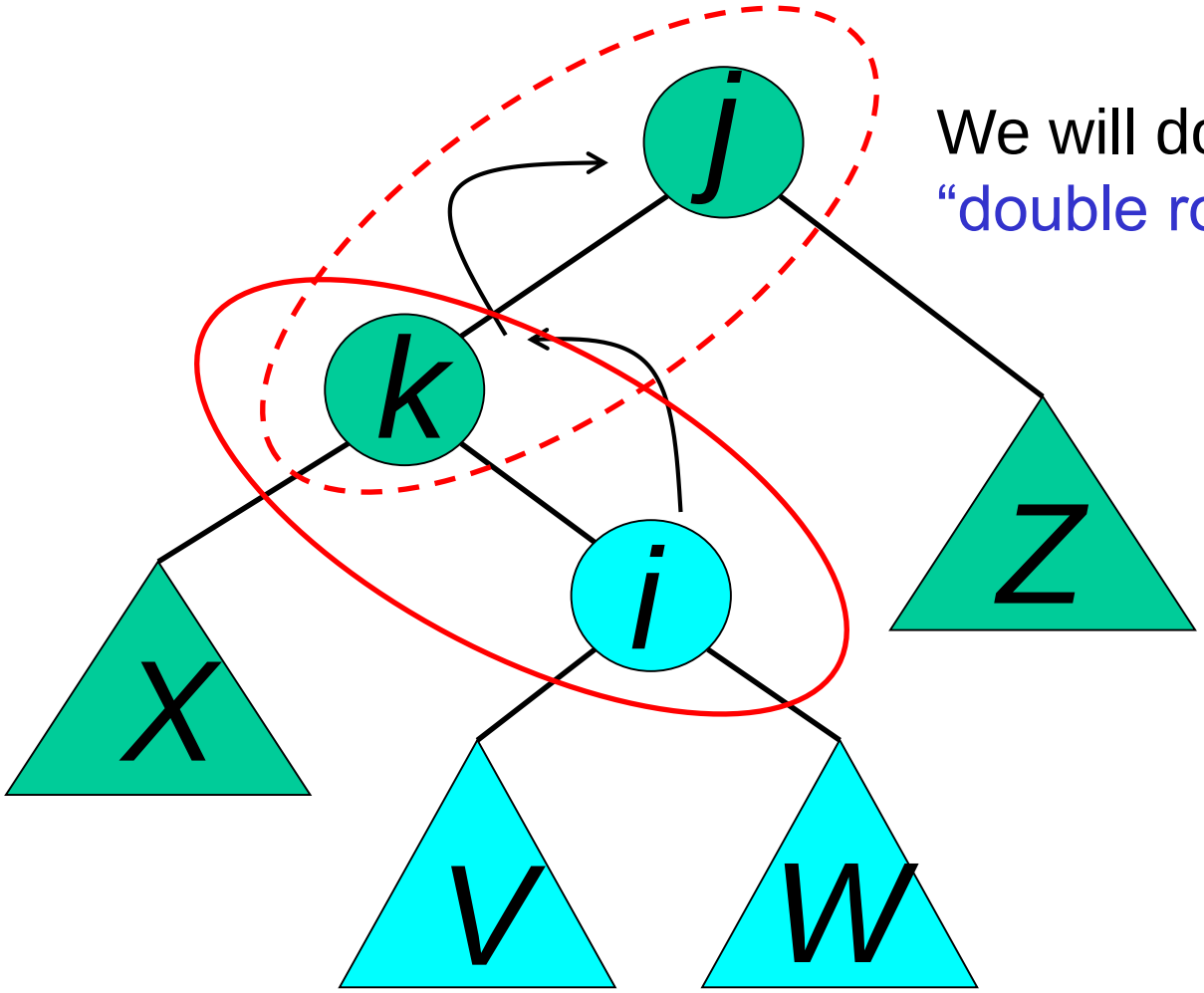
AVL Tree



We will do a left-right "double rotation" . . .

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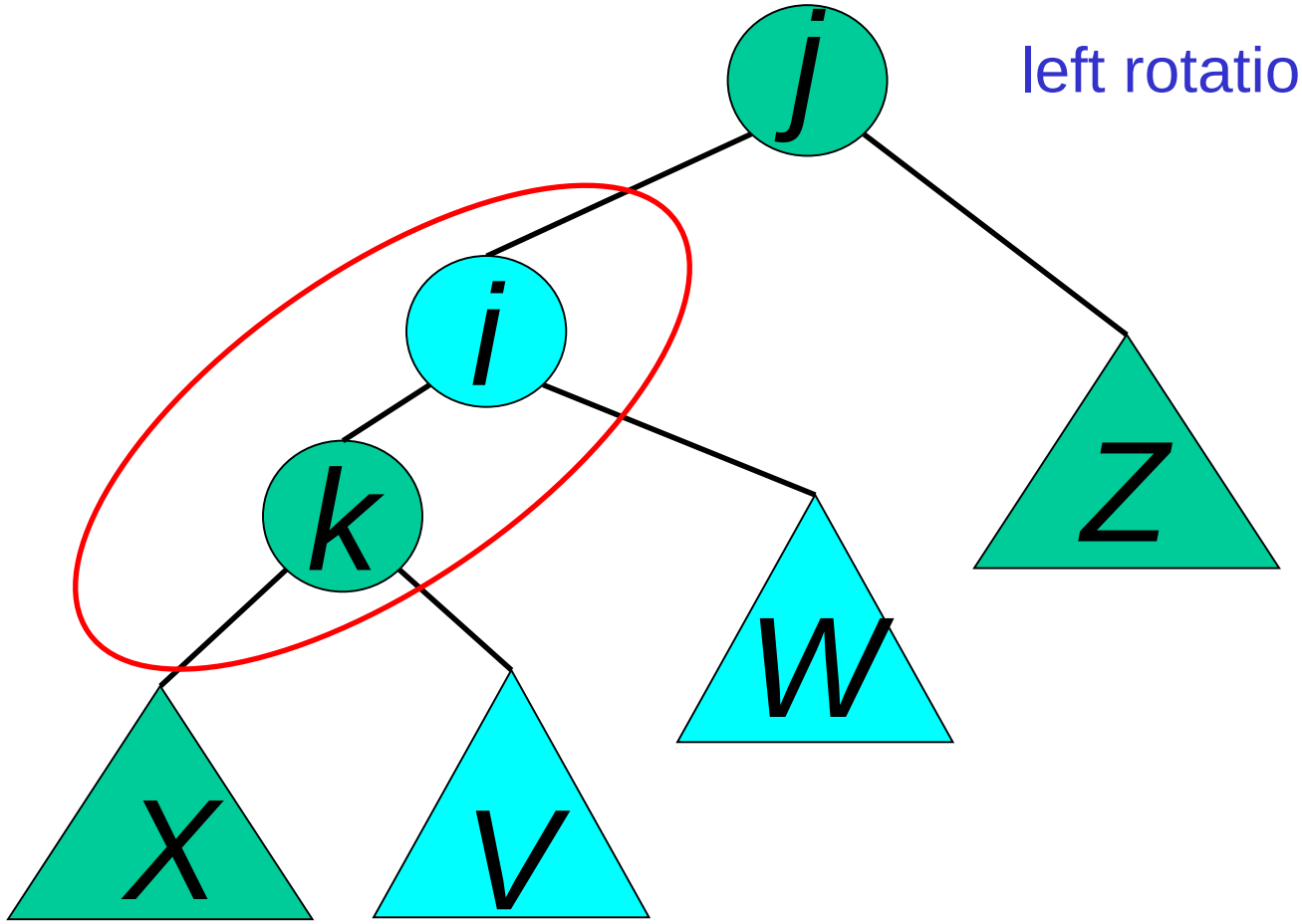
AVL Tree



left rotation complete

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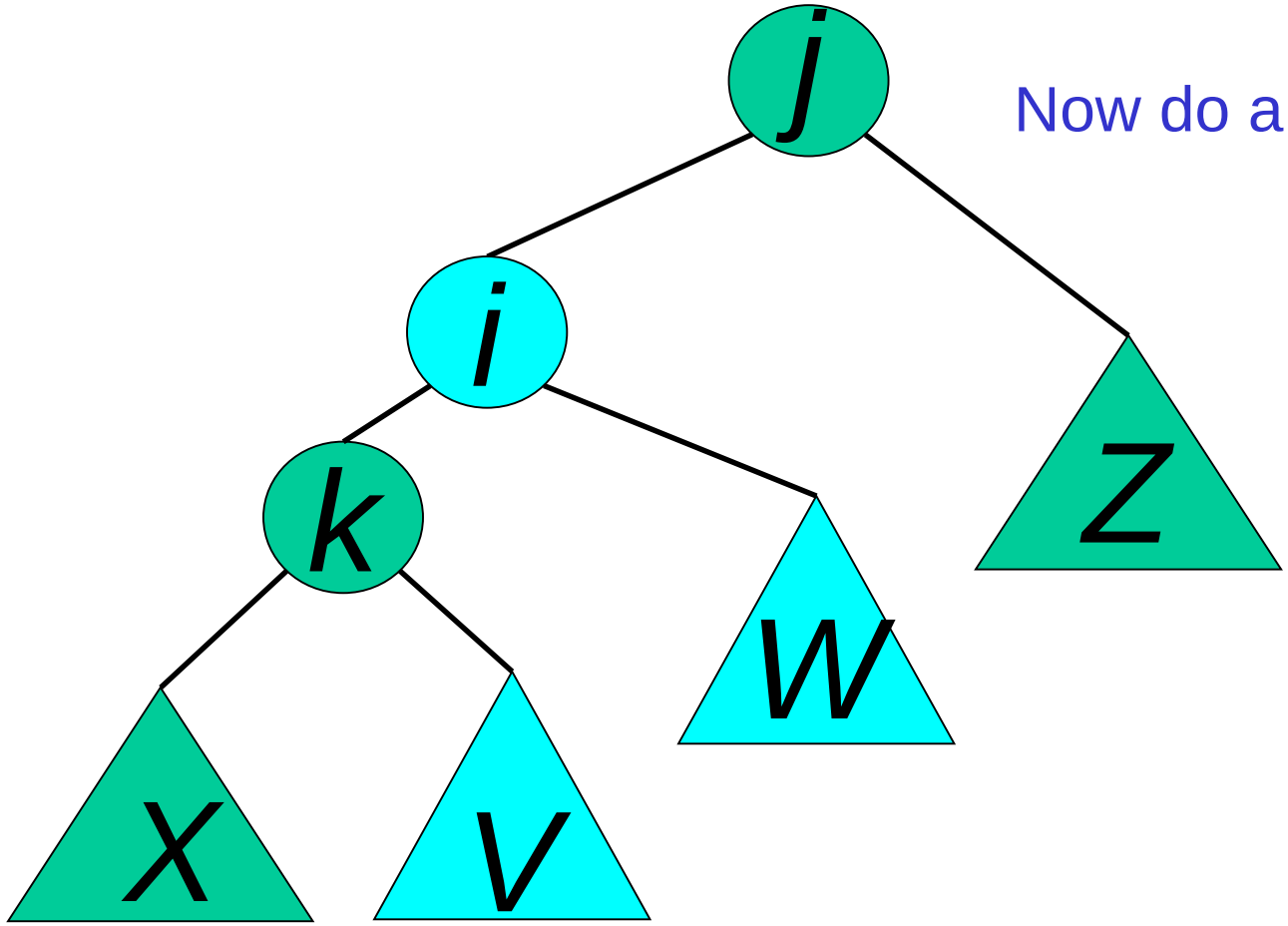
AVL Tree



Now do a right rotation

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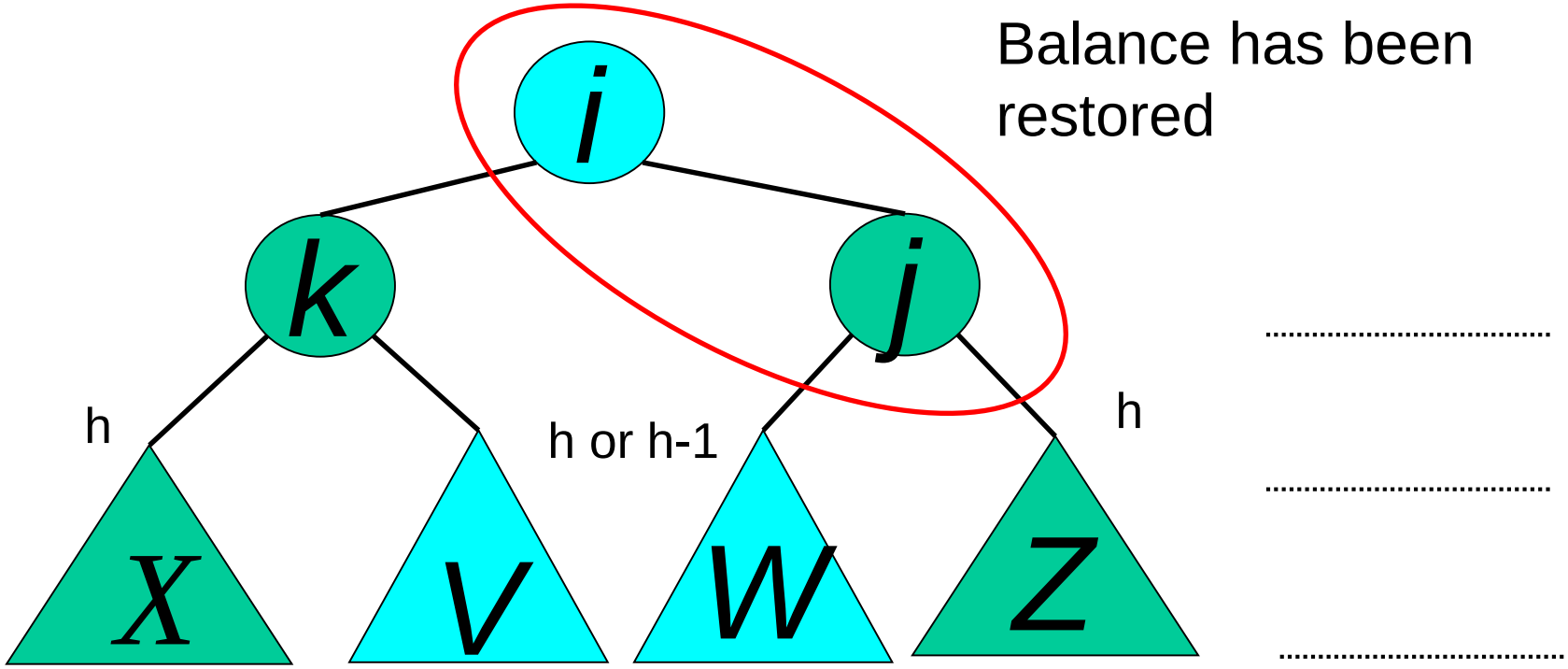
AVL Tree

right rotation complete



Balance has been restored

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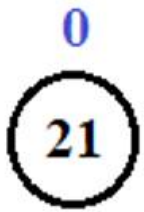
AVL Tree

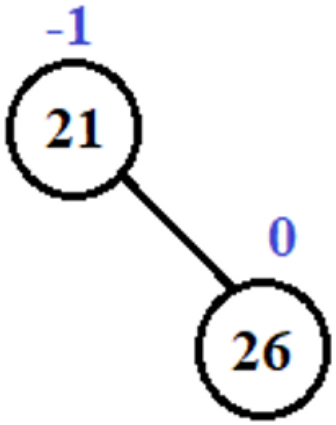
Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

Step 1 - Insert 21



Tree is Balanced

Step 2 - Insert 26

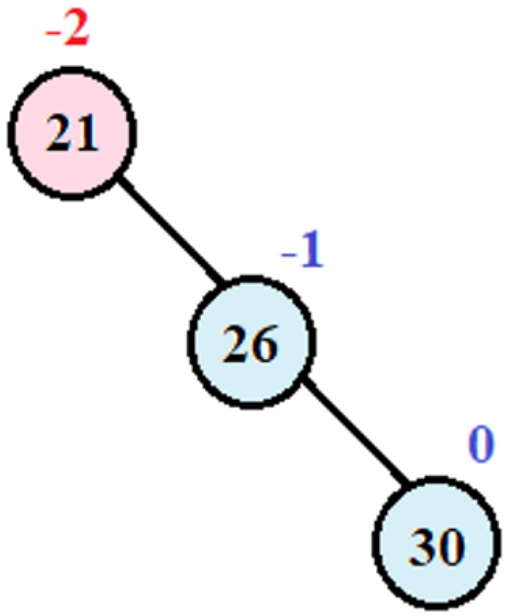


Tree is Balanced

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

Step 3 - Insert 30

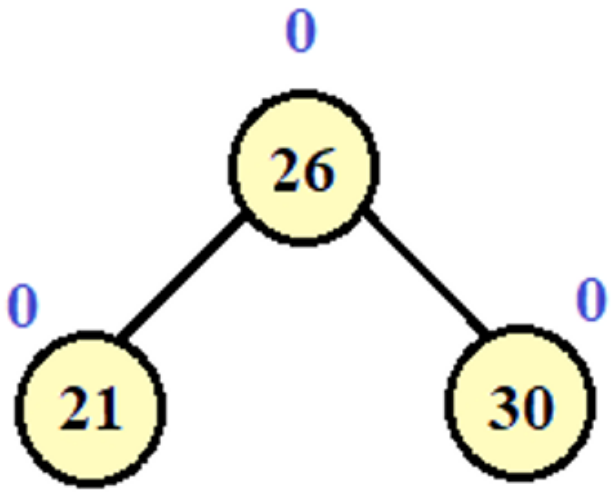


LL Rotation

Tree is Not Balanced, Need a Rotation

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

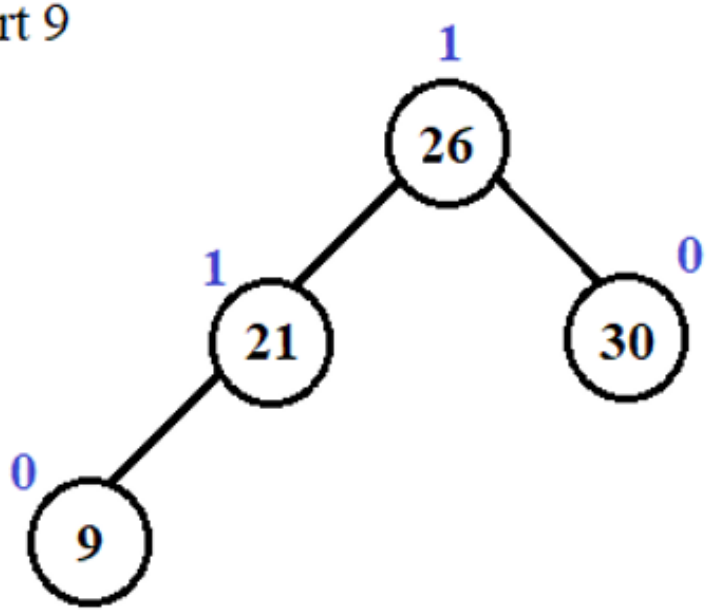


Tree is Balanced

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

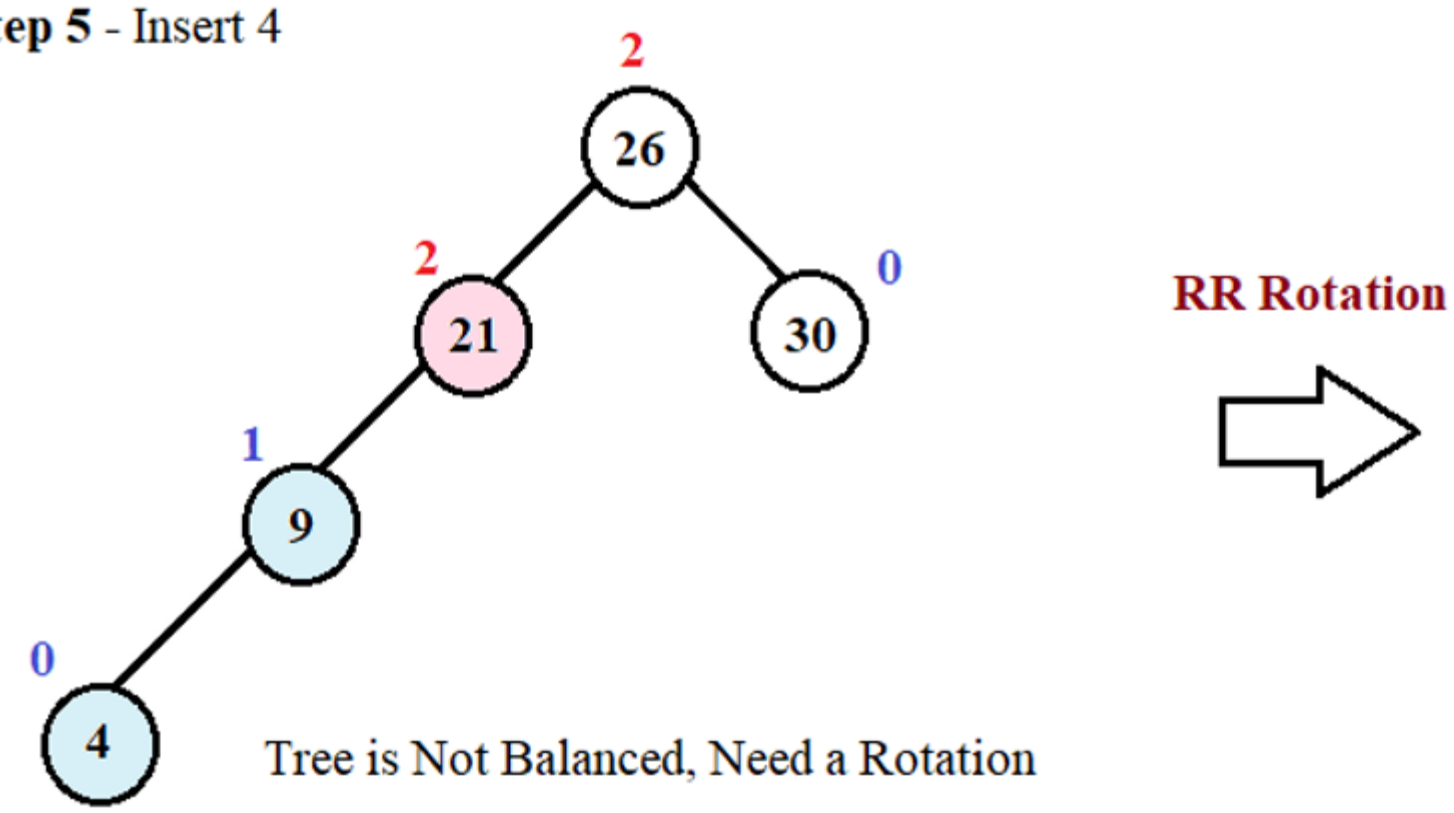
Step 4 - Insert 9



Tree is Balanced

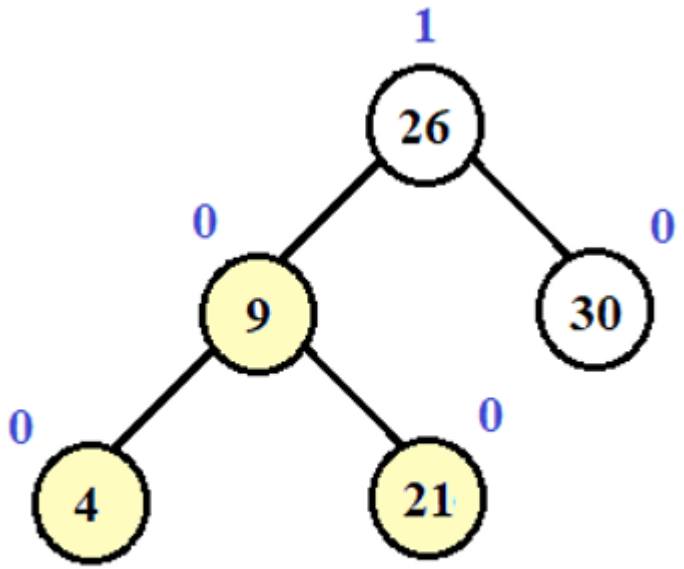
AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7



AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

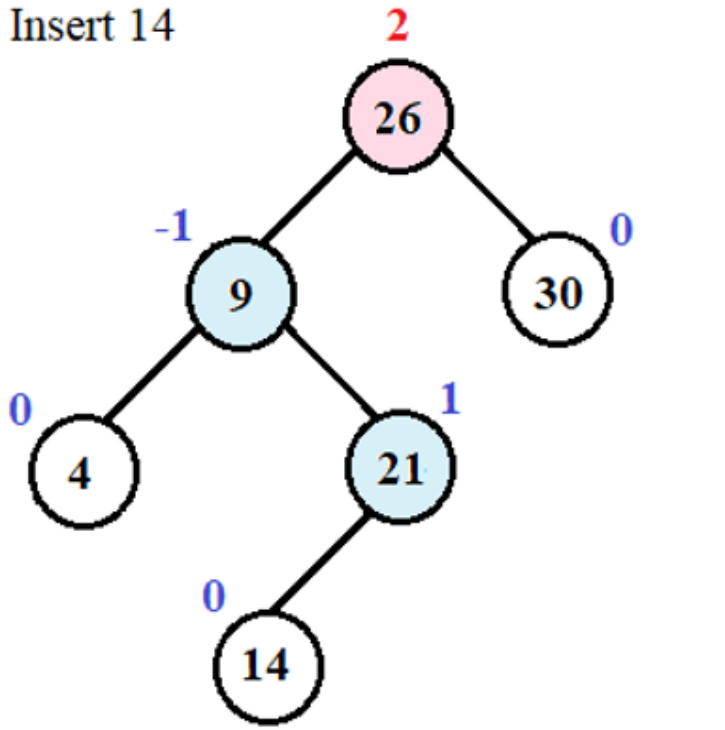


Tree is Balanced

AVL Tree

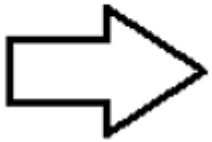
Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

Step 6 - Insert 14



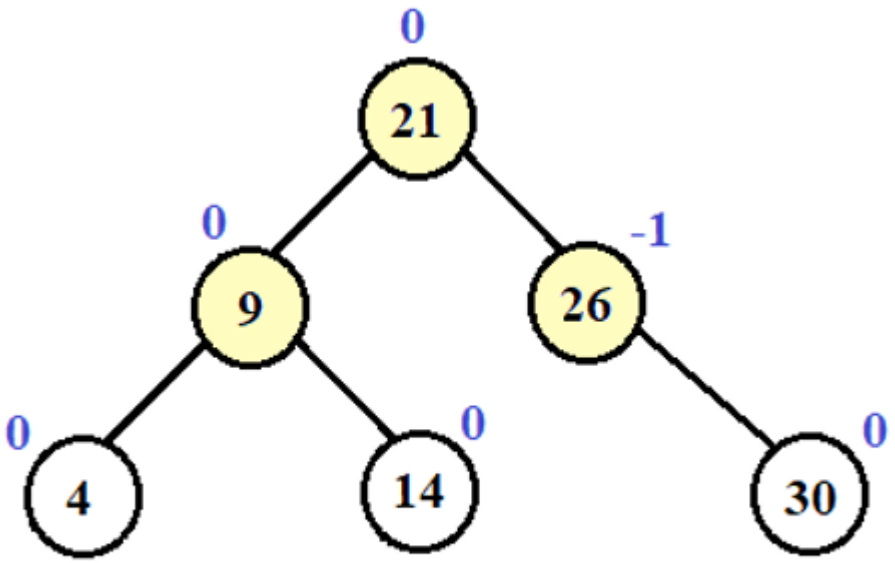
Tree is Not Balanced, Need a Rotation

LR Rotation



AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

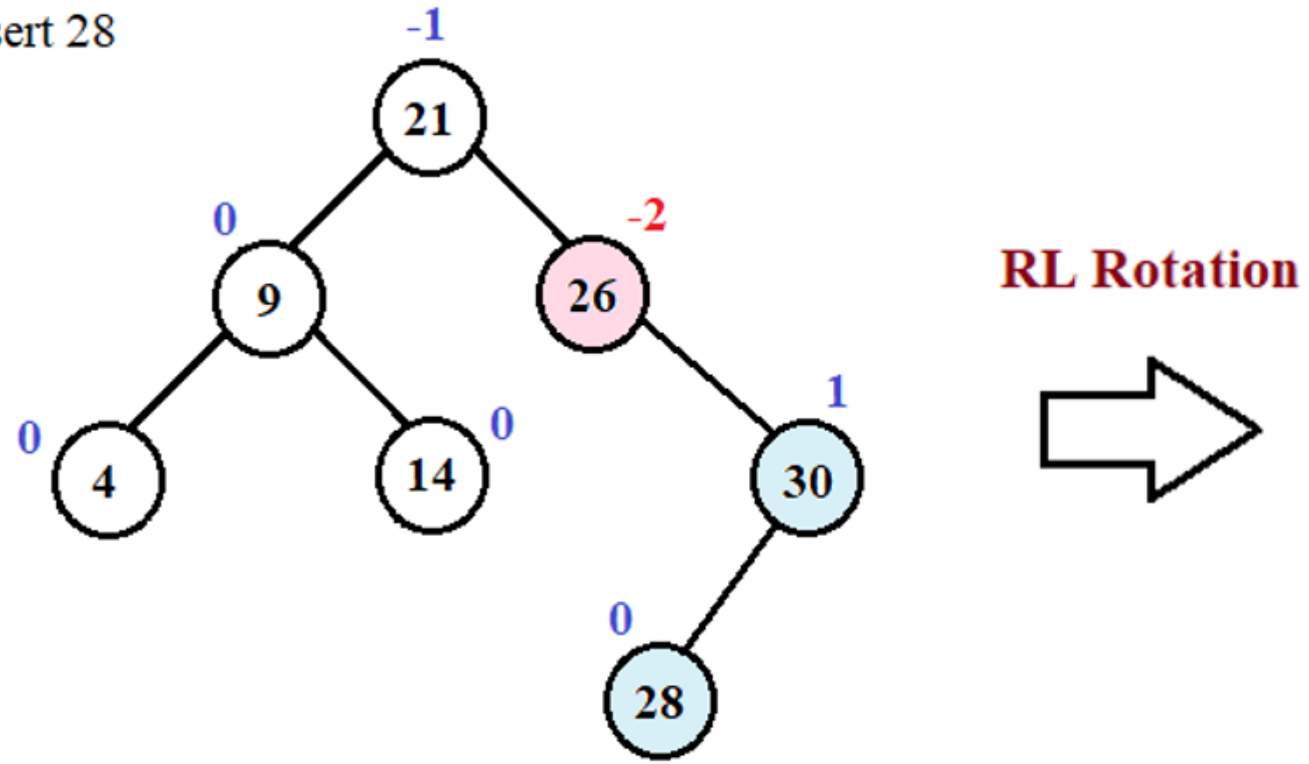


Tree is Balanced

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

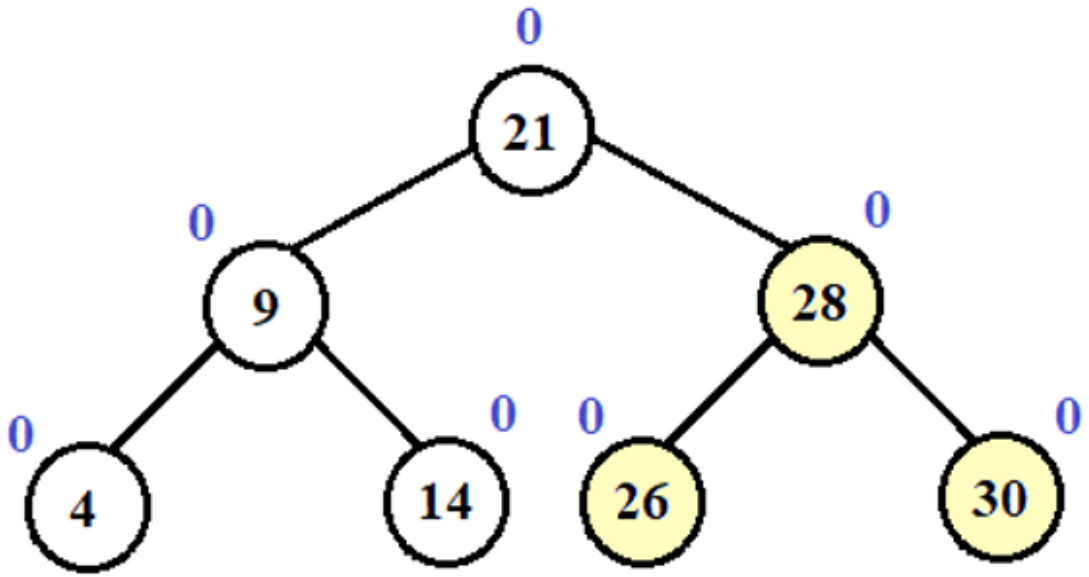
Step 7 - Insert 28



Tree is Not Balanced, Need a Rotation

AVL Tree

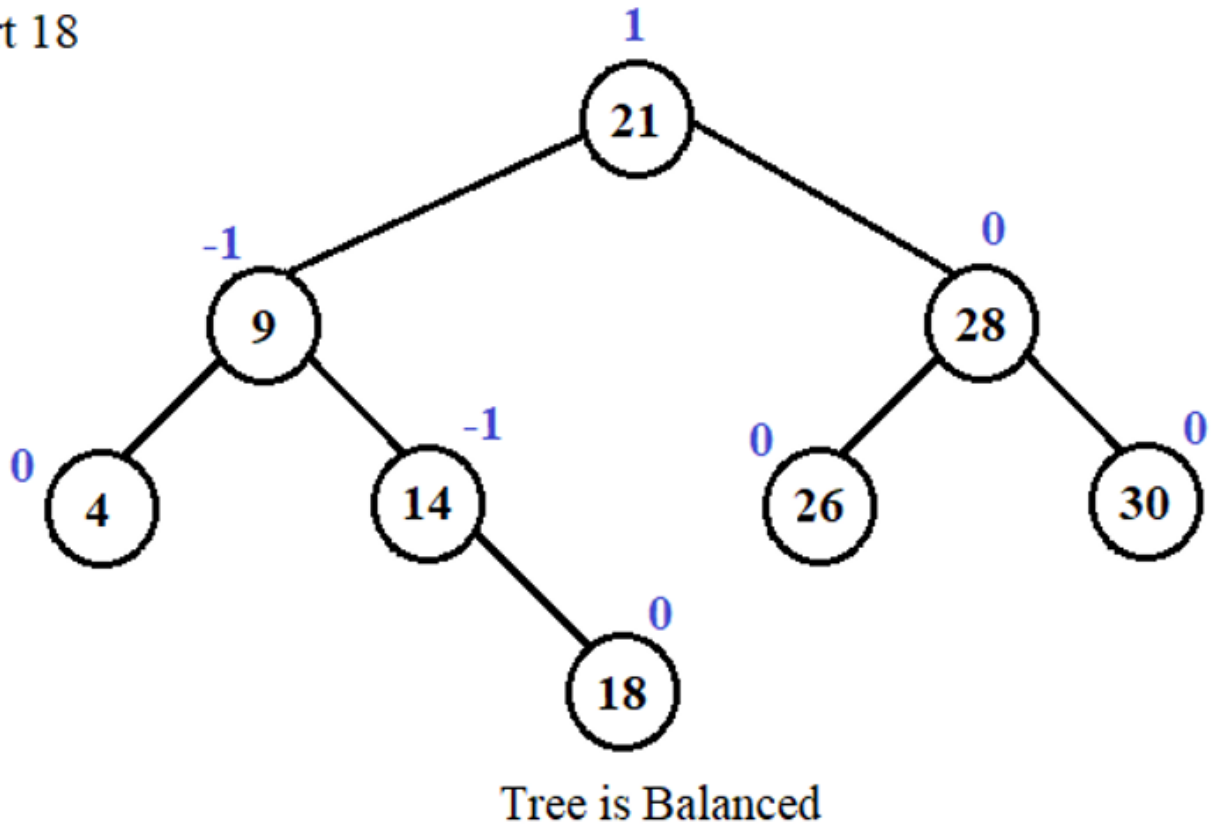
Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7



Tree is Balanced

AVL Tree

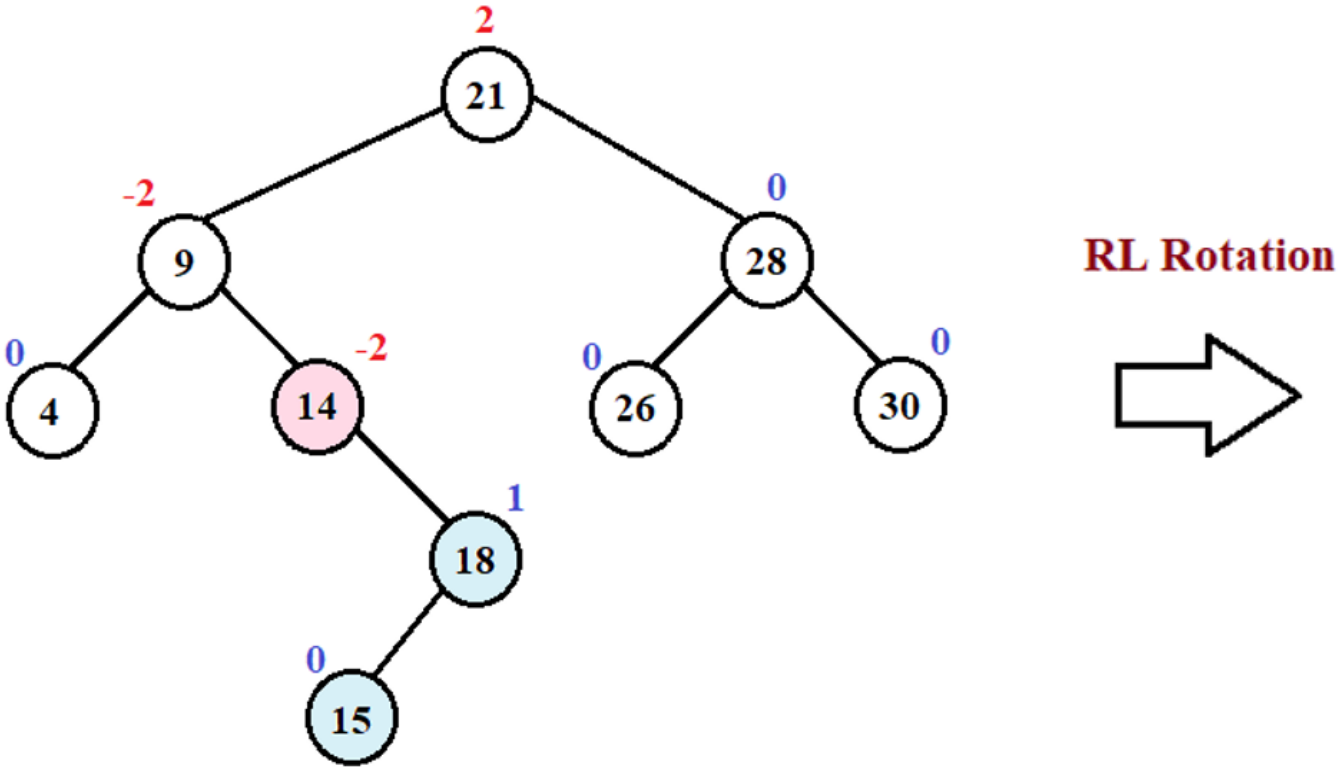
Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7



AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

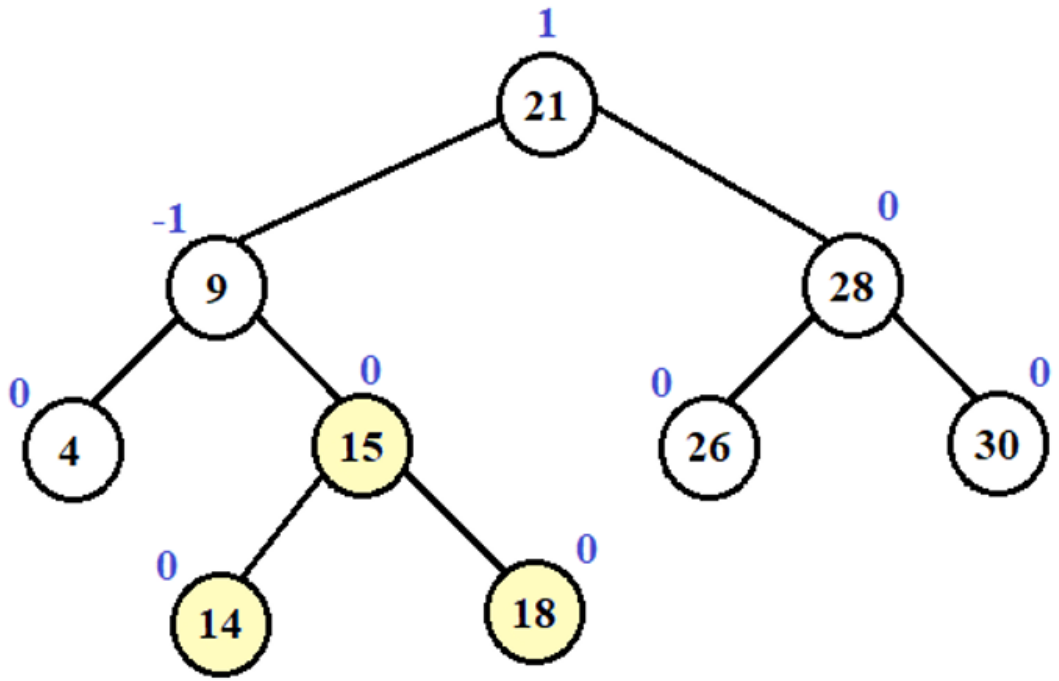
Step 9 - Insert 15



Tree is Not Balanced, Need a Rotation

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

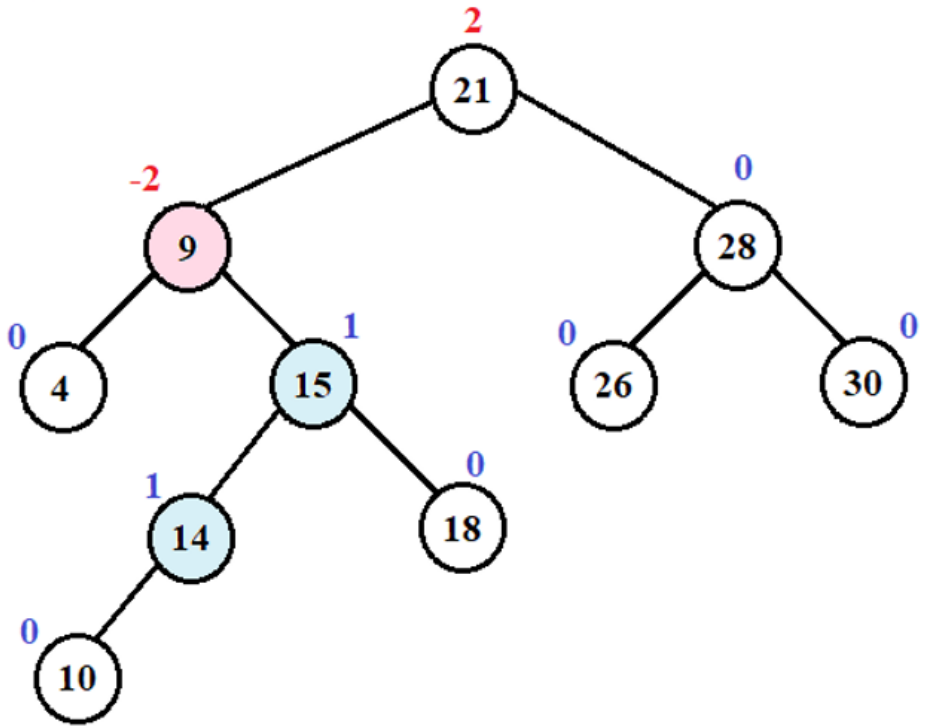


Tree is Balanced

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

Step 10 - Insert 10

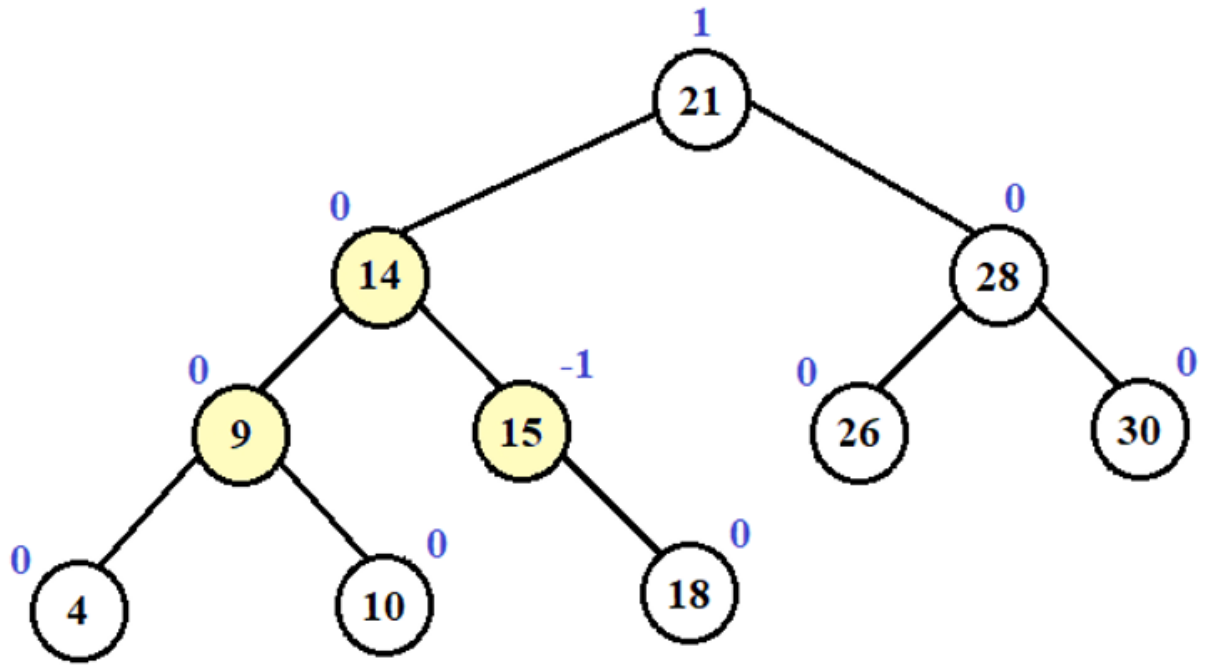


Tree is Not Balanced, Need a Rotation

RL Rotation

AVL Tree

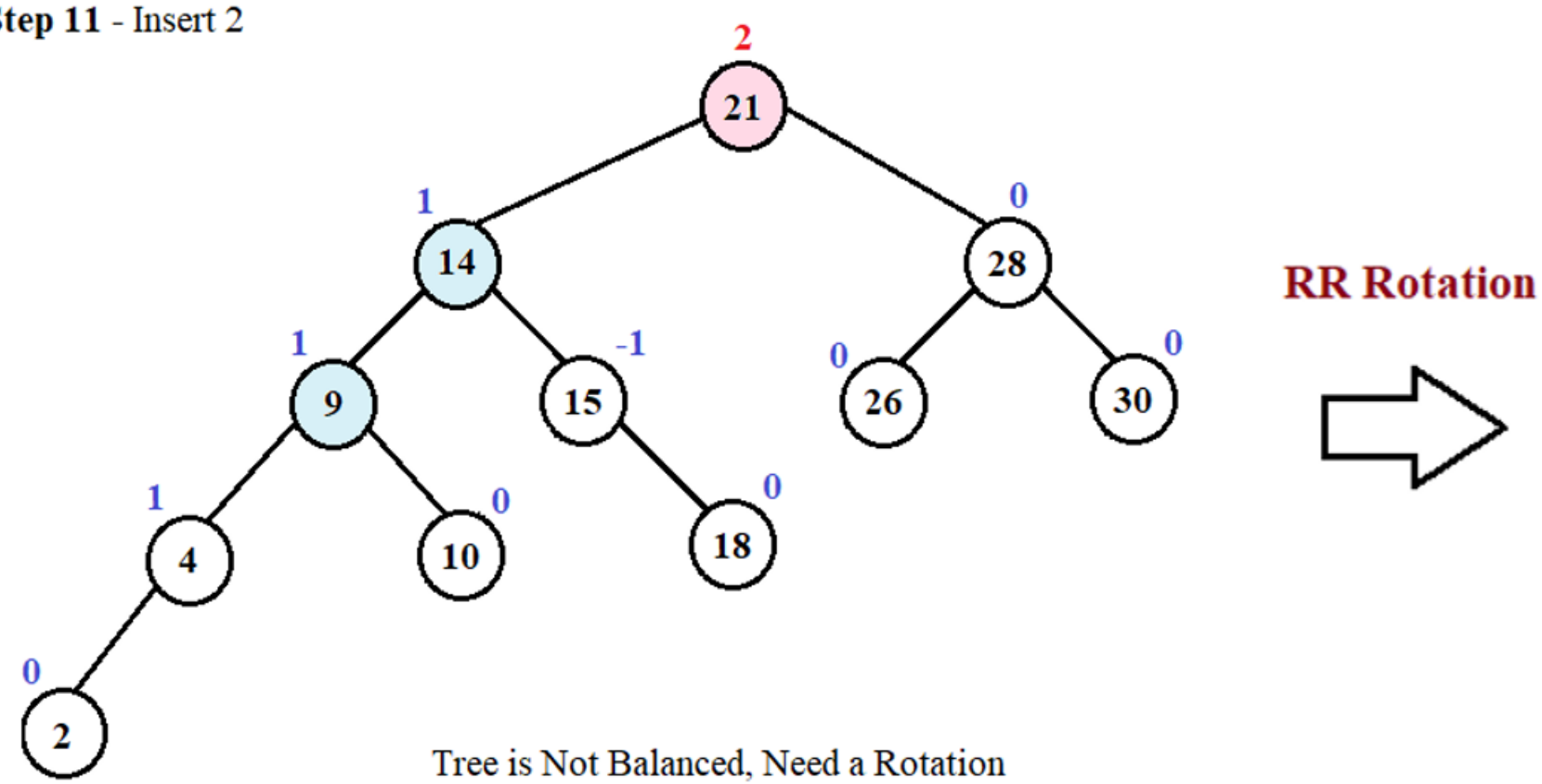
Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7



Tree is Balanced

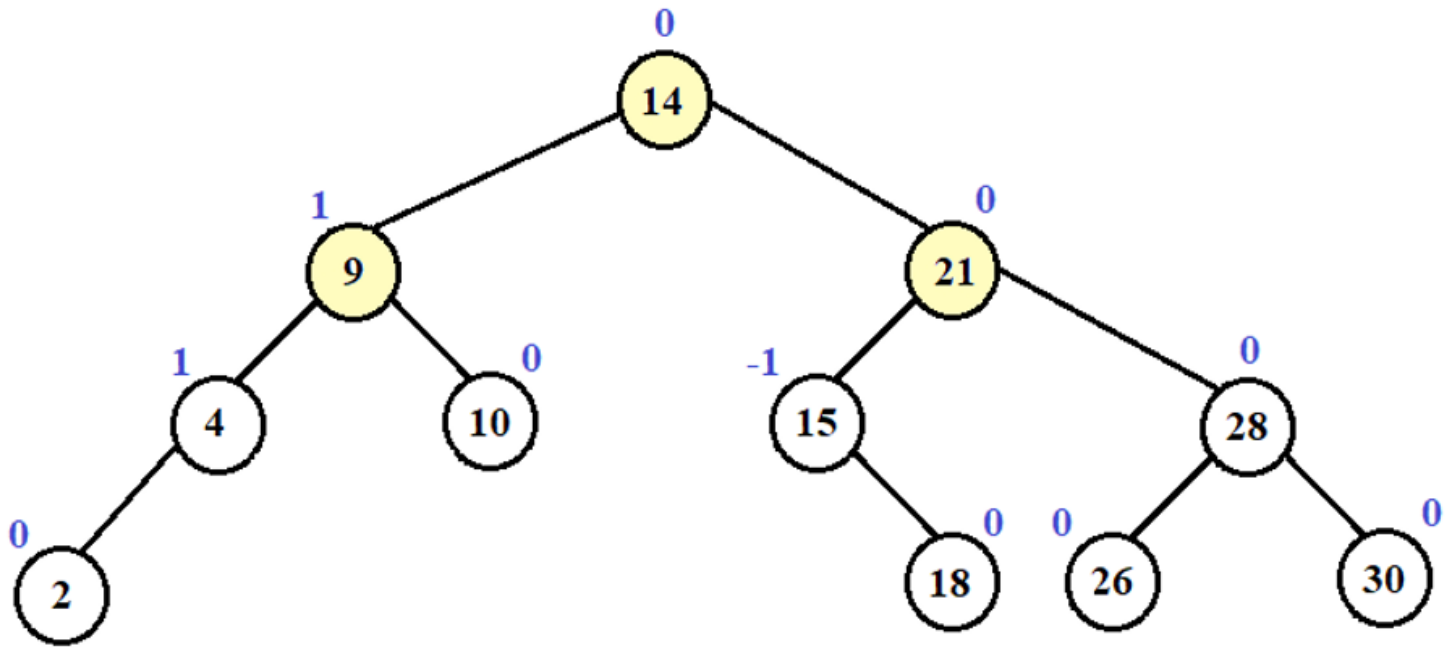
AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7



AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

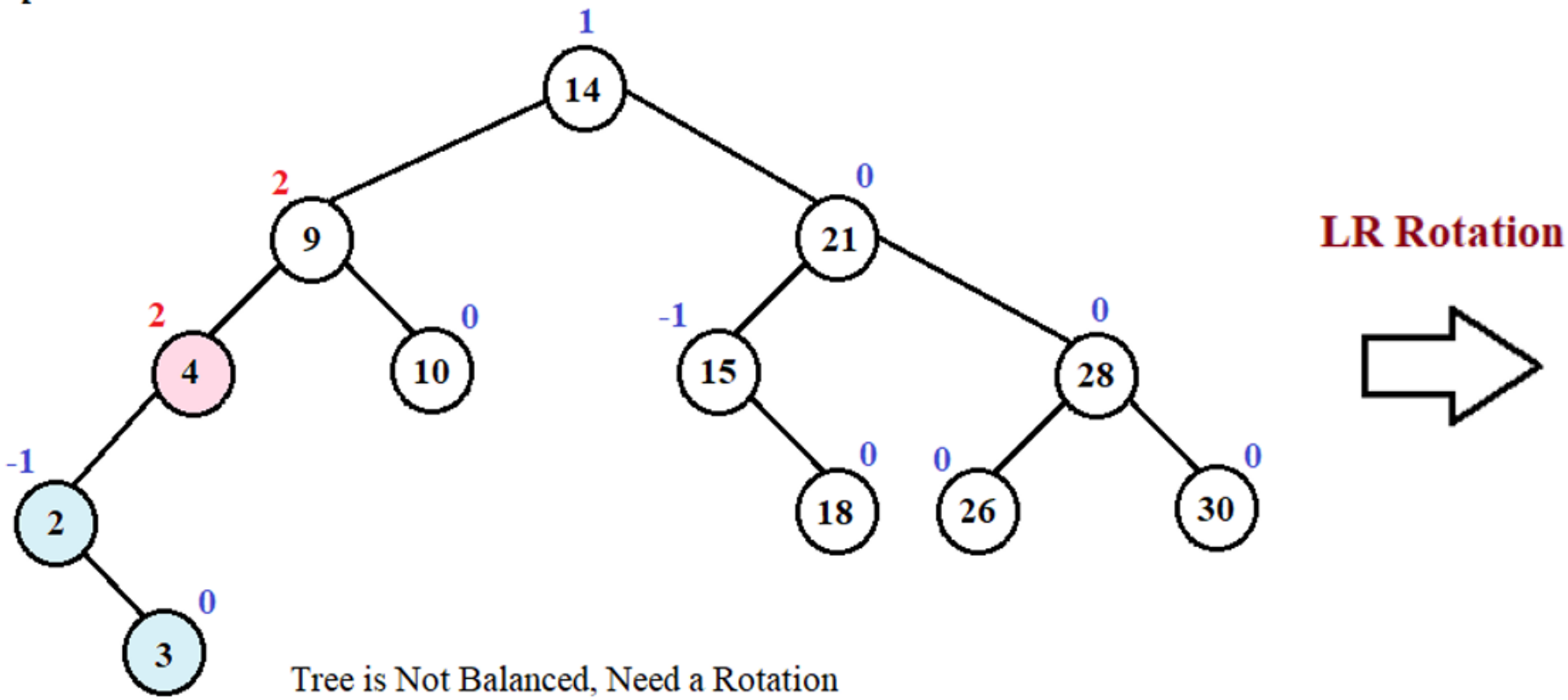


Tree is Balanced

AVL Tree

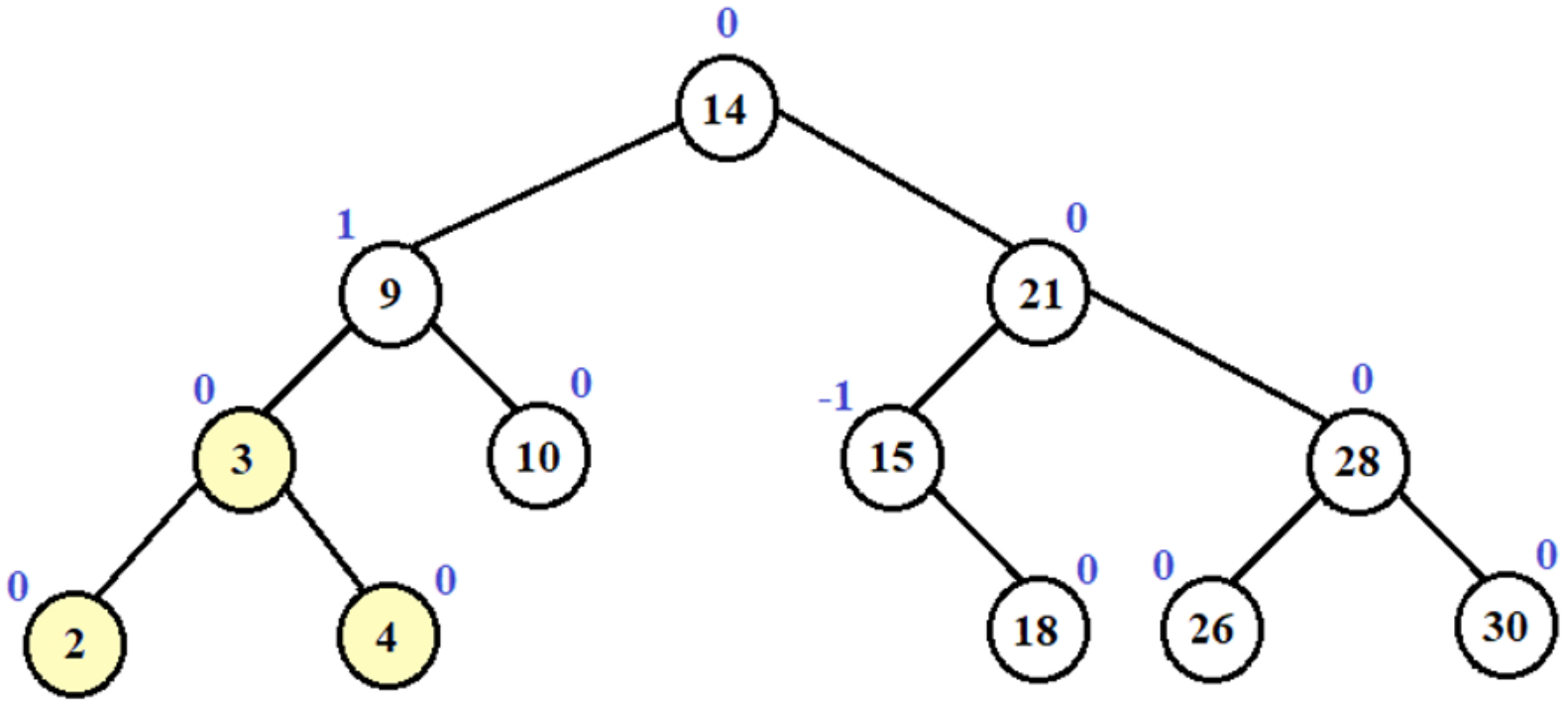
Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

Step 12 - Insert 3



AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

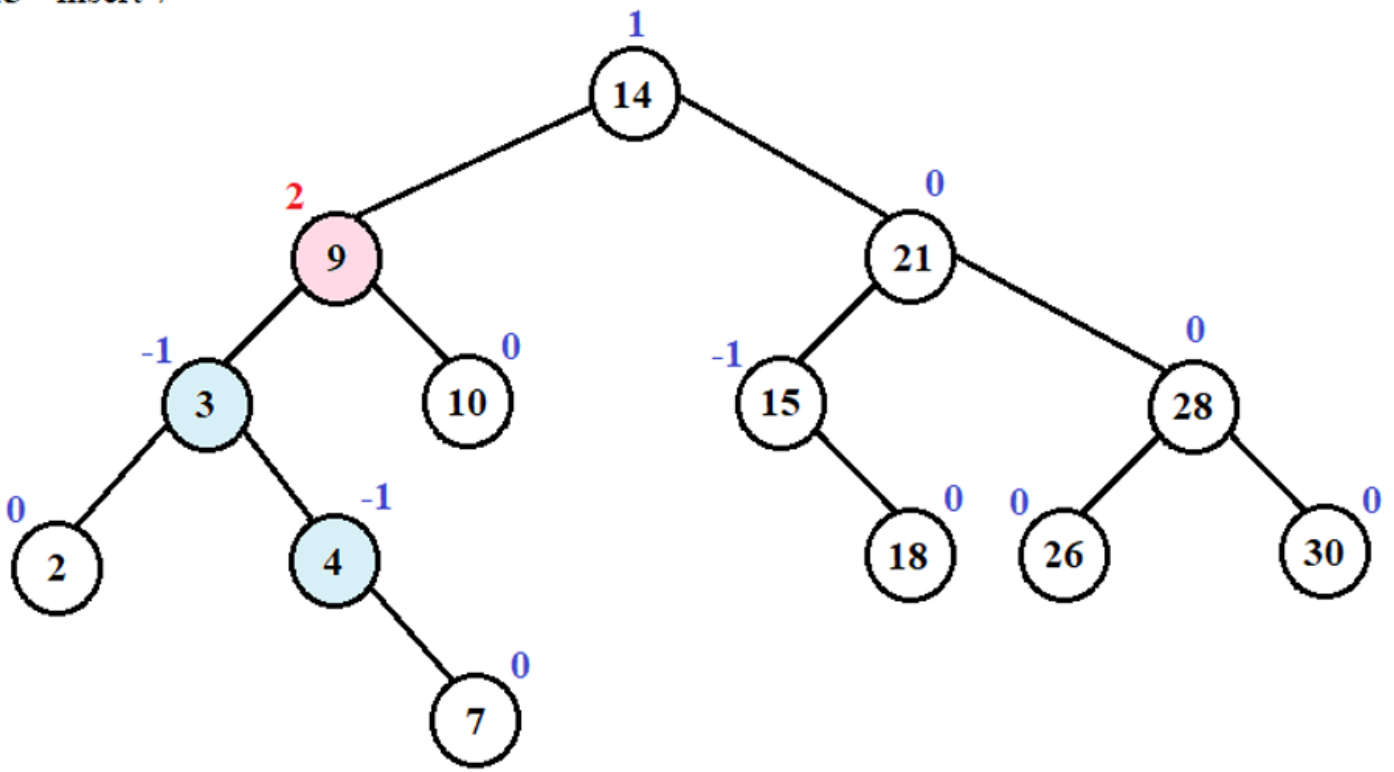


Tree is Balanced

AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7

Step 13 - Insert 7

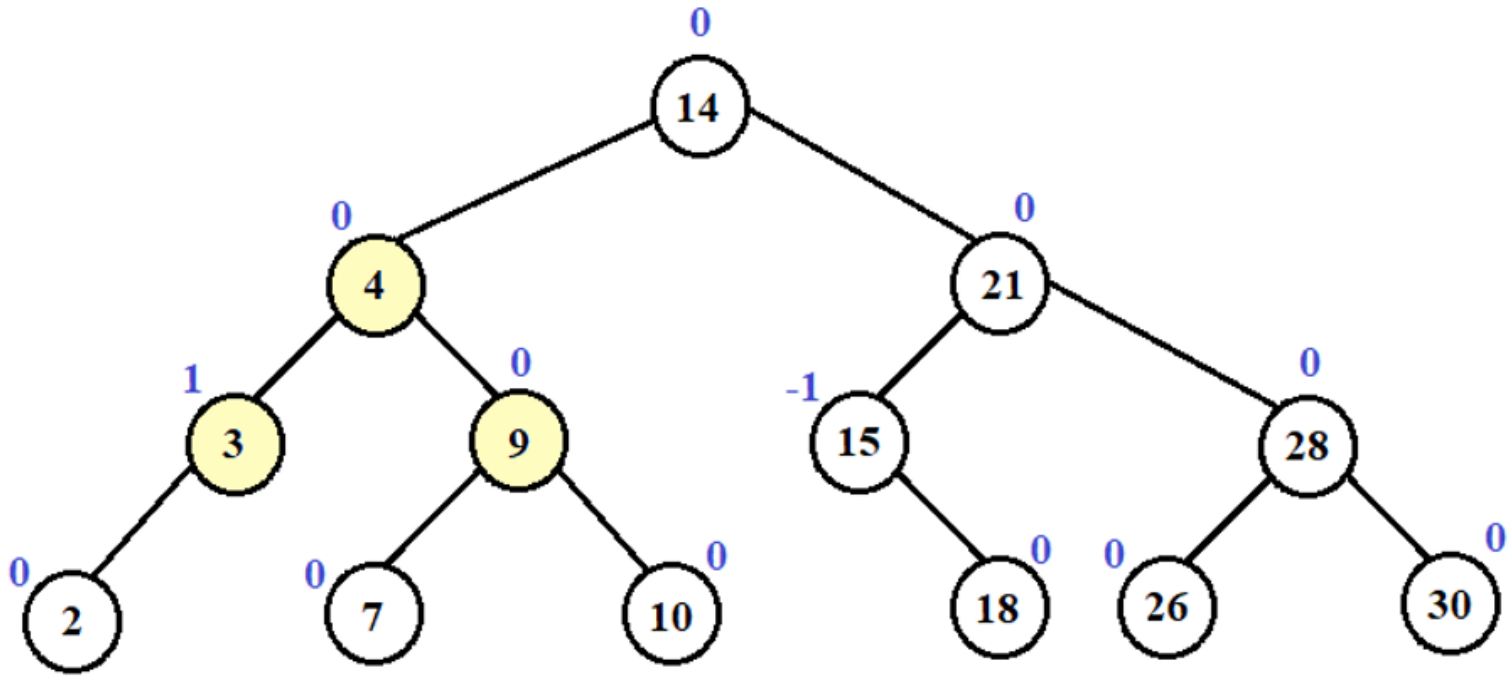


Tree is Not Balanced, Need a Rotation

LR Rotation

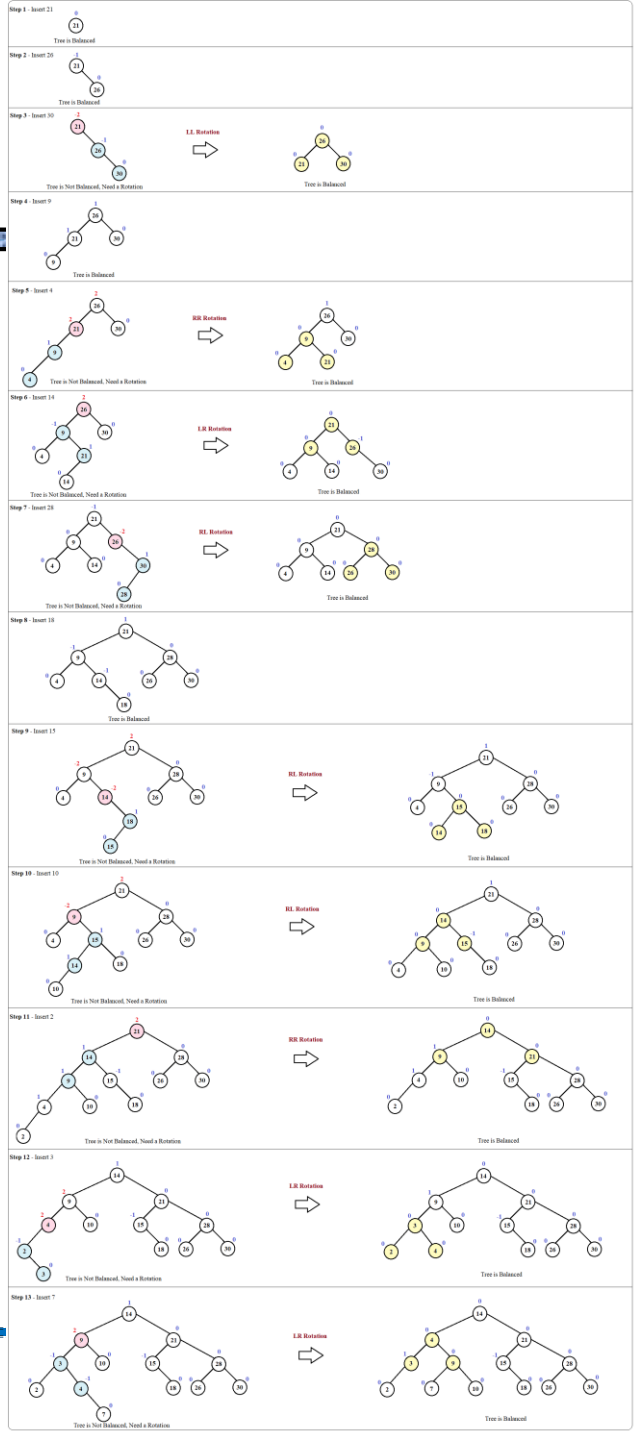
AVL Tree

Step-by-Step Construction of the AVL Tree for the given Sequence → 21, 26, 30, 9, 4, 14, 28, 18,15,10, 2, 3, 7



Tree is Balanced

AVL Tree


Step 1 - Insert 21
Tree is Balanced
Step 2 - Insert 26
Tree is Balanced
Step 3 - Insert 30
Tree is Not Balanced, Need a Rotation
LL Rotation
Tree is Balanced
Step 4 - Insert 9
Tree is Balanced
Step 5 - Insert 4
Tree is Not Balanced, Need a Rotation
RR Rotation
Tree is Balanced
Step 6 - Insert 14
Tree is Not Balanced, Need a Rotation
LR Rotation
Tree is Balanced
Step 7 - Insert 28
Tree is Not Balanced, Need a Rotation
RL Rotation
Tree is Balanced
Step 8 - Insert 18
Tree is Balanced
Step 9 - Insert 15
Tree is Not Balanced, Need a Rotation
RL Rotation
Tree is Balanced
Step 10 - Insert 10
Tree is Not Balanced, Need a Rotation
RL Rotation
Tree is Balanced
Step 11 - Insert 2
Tree is Not Balanced, Need a Rotation
RR Rotation
Tree is Balanced
Step 12 - Insert 3
Tree is Not Balanced, Need a Rotation
LR Rotation
Tree is Balanced
Step 13 - Insert 7
Tree is Not Balanced, Need a Rotation
LR Rotation
Tree is Balanced

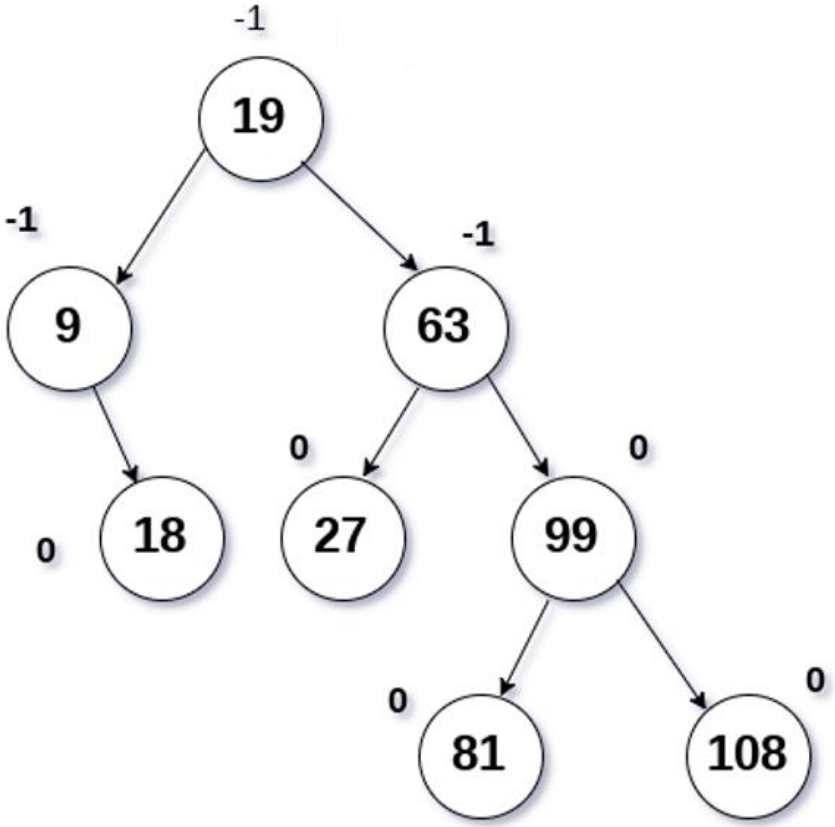
AVL Tree (Practice Problem)

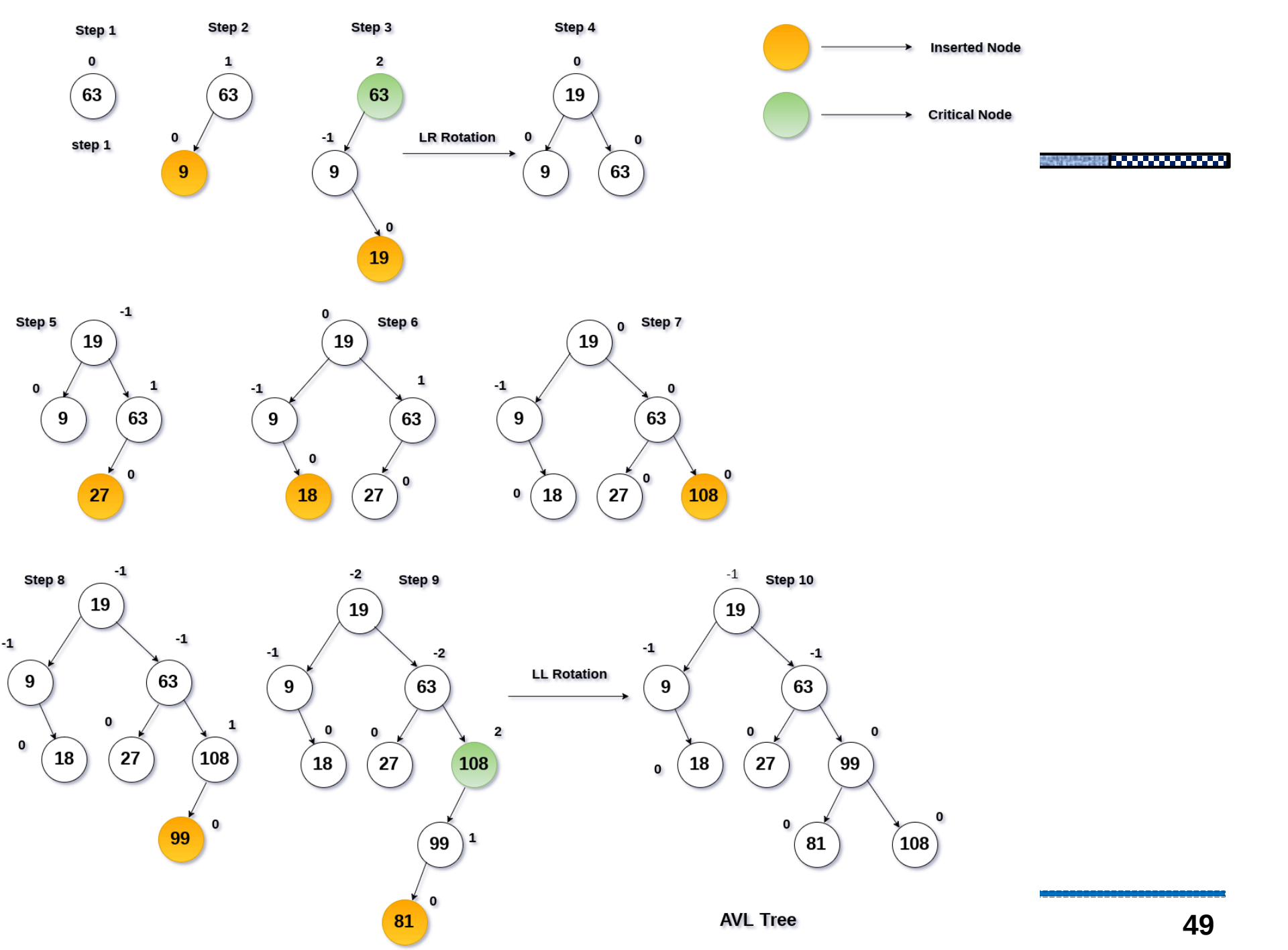
Construct an AVL tree by inserting the following elements in the given order →

63, 9, 19, 27, 18, 108, 99, 81

AVL Tree

Construct an AVL tree by inserting the following elements in the given order → 63, 9, 19, 27, 18, 108, 99, 81




Step 1
Step 2
Step 3
Step 4
step 1
LR Rotation
Inserted Node
Critical Node
Step 5
Step 6
Step 7
Step 8
Step 9
Step 10
LL Rotation
AVL Tree

AVL Tree (Practice Problem)

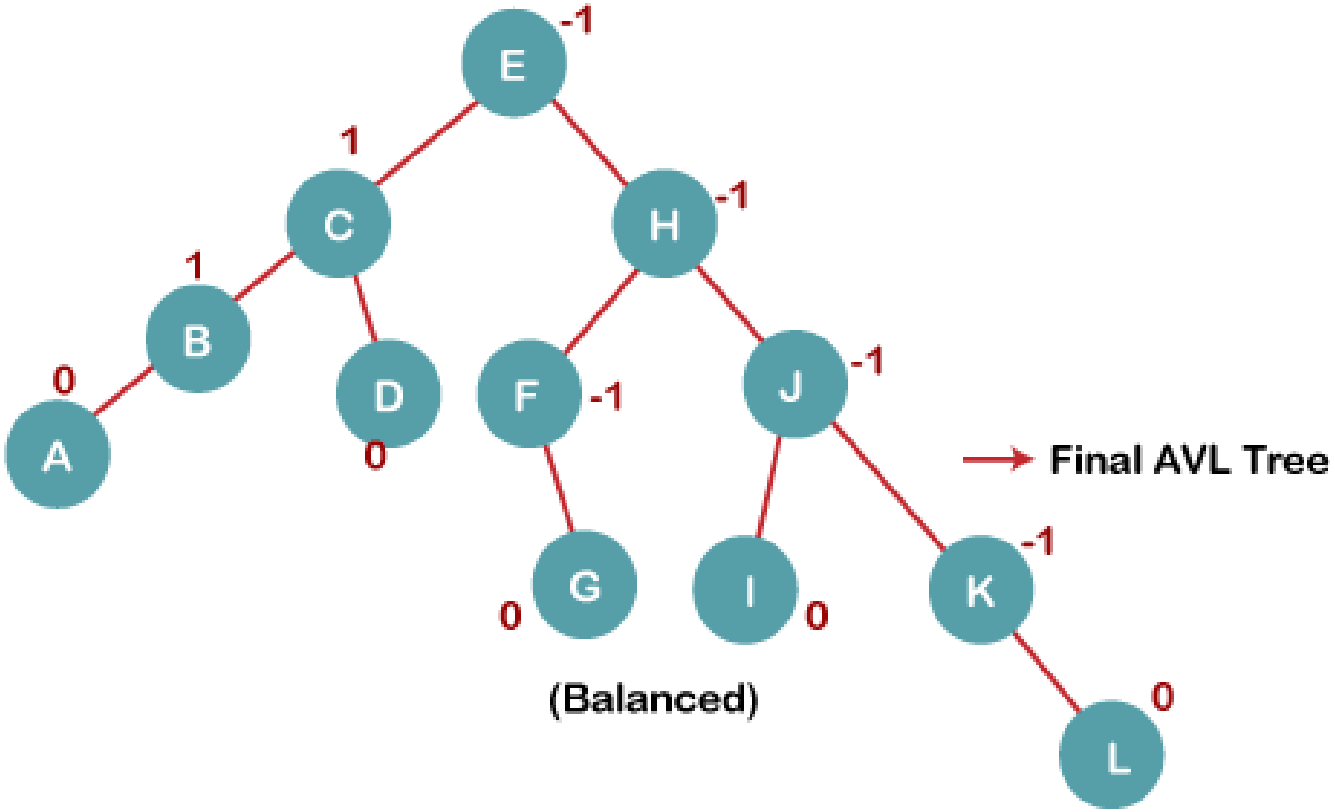
Construct an AVL tree having the following elements →

H, I, J, B, A, E, C, F, D, G, K, L

AVL Tree

Construct an AVL tree having the following elements

H, I, J, B, A, E, C, F, D, G, K, L




Thank You!