UNIVERSITY of HOUSTON | ECE

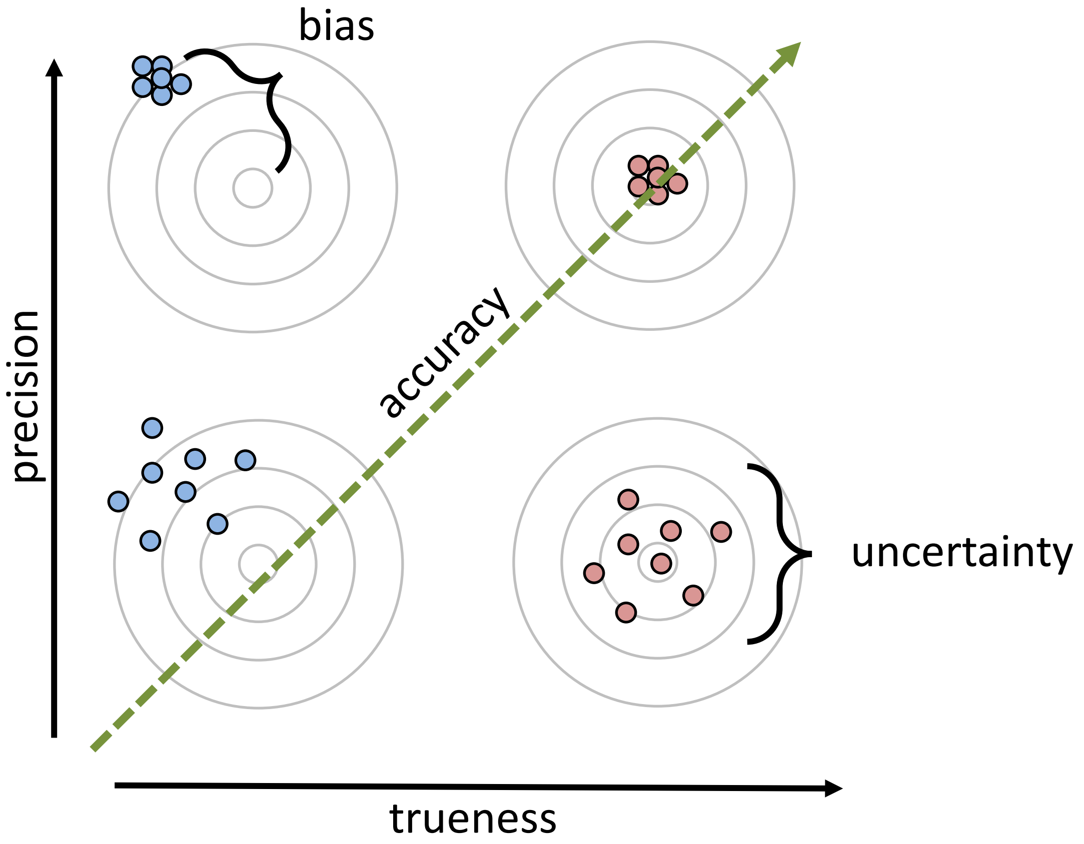
Quantifying Error

ECE 3340 – David Mayerich

ISO 5725 – Precision, Trueness, and Accuracy

- ***Trueness*** – how well results agree with *true* values $t(x)$
- ***Precision*** – how well results agree with *each other*
- **Accuracy** – how closely a result agrees with reality

- ***Bias*** – systematic error that deviates in a predictable way (affects trueness)
- ***Uncertainty*** – unpredictable deviations in the result (affects precision)



Error

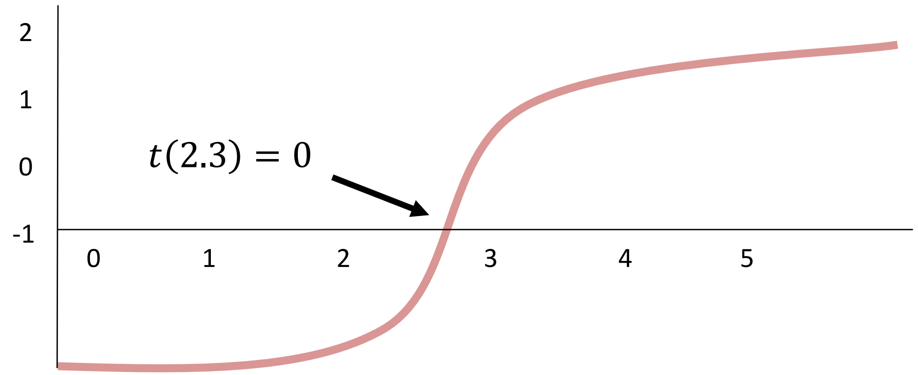
- An error is a deviation of a result from reality
  - A ***systematic error*** is introduced by *bias* – this can often be corrected
  - A ***random error*** is a product of *uncertainty* and impossible to correct
  - Random errors can often be **controlled** or **bounded**
- **Calculating Error:** assume $t(x)$ is the true result, and $c(x)$ is the result of a computation:
  - Absolute error: $E_a = |t(x) - c(x)|$
  - Relative error: $E_r = \left|\frac{E_a}{t(x)}\right|$
- What are the relative and absolute errors for the evaluations below?
  - $t(x) = 1.5$ $\quad c(x) = 1$
  - $t(x) = 1{,}000{,}000.5$ $\quad c(x) = 1{,}000{,}000$

Absolute vs. Relative Error

- Scientific and engineering applications are less sensitive to small errors in large values
- Relative error can compare values at widely varying scales – often represented as a ***percentage error*** $E_R \times 100$
- Most computing systems are designed to minimize relative error

Relative Error – true values near zero

- Undefined when the true value is zero
  - there isn't a defined way to deal with a true value of $t(x) = 0$



- Options include:
  - regularization – specify a small value ϵ:

$$E_r = \frac{E_a}{|t(x)| + \epsilon}$$

  - specify a value for $f(x) = 0$:

$$E_r = \begin{cases} \dfrac{|t(x) - p(x)|}{t(x)} & \text{if } t(x) \neq 0 \\ 1 & \text{otherwise} \end{cases}$$

Relative Error – Interval Measurements

- Invalid for **interval measurements**:

	Celsius	Kelvin	
true ⟶	10° C	283 K	true
measured ⟶	11° C	284 K	calculated

$$E_r = \left|\frac{10 - 11}{10}\right| \qquad E_r = \left|\frac{283 - 284}{283}\right|$$

$$E_r = 0.1 = 10\% \qquad E_r = 0.0035 = .35\%$$

- relative error requires a ***ratio scale*** with a **natural zero**
- ratio measurements allow quantitative comparisons
  - zero in a ratio measurement indicates that there is zero of something
  - "it's twice as cold today as it was yesterday" only makes sense in Kelvin

Uncertainty Due To Imprecision

- First major form of error we will encounter is ***imprecision***:
  - Form of *uncertainty* due to limits in numerical representation
  - limited number of digits available to represent any value
- How do we represent $x_1 = 31.26$, $x_2 = 31.24$ with only 3 digits of precision? How about $x_3 = 31.25$?
- We obey standard rounding rules:
  - chop off excess digits
  - look at the most significant digit chopped: if it is ≥ 5, increase the least significant digit kept
- This affects *precision*, but does it affect *trueness*?
  - Note that 5 is the **midpoint** between representable numbers, but we consistently round **up**. This introduces a slight upward bias to all of our calculations.

Summary

- Absolute error: $E_a = |true - calculated|$
- Relative error: $E_r = \left|\frac{E_a}{true}\right|$
- If the true value is zero:
  - $E_r = 0$ if E_a is zero (there is no error)
  - $E_r = 1$ otherwise (the error is 100%)
- Relative error only makes sense for ratio measurements
  - **ratio measurements have a natural zero point**
- Relative error is more important in practice
  - ± 1 ton is a lot for a truck, but not a lot for a cargo ship