

calculus for deep learning

1. Power rule:

$$\frac{\partial}{\partial x} (x^n) = nx^{n-1}, \quad \frac{\partial}{\partial y} (y^n) = ny^{n-1}.$$

Example:-

$$f(x, y) = x^3$$

$$\frac{\partial f}{\partial x} = 3x^2$$

$$g(x, y) = y^4$$

$$\frac{\partial g}{\partial y} = 4y^3$$

2. constant multiple rule:

$$f(x, y) = 5x^2$$

$$\frac{\partial f}{\partial x} = 5 \cdot 2x^{2-1} = 10x$$

$$g(x, y) = 7y^3$$

$$\frac{\partial g}{\partial y} = 7 \cdot 3 \cdot y^{3-1} = 21y^2$$

3. Sum Rule:

$$f(x, y) = x^2 + y^2$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (x^2) + \frac{\partial}{\partial x} (y^2) \\ &= 2x + 0 = 2x \end{aligned}$$

$$g(x, y) = 3x^3 + 4y$$

$$\begin{aligned} \frac{\partial g}{\partial y} &= \frac{\partial}{\partial y} (3x^3) + \frac{\partial}{\partial y} (4y) \quad [y^0 = 1] \\ &= 0 + 4 \cdot y^{1-1} = 4 \cdot y^0 = 4 \cdot 1 = 4 \end{aligned}$$

4. Product Rule;

$$\frac{\partial f}{\partial x} = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial u}{\partial x}$$

$$f(x, y) = x^2 \cdot y^3$$

$$\begin{aligned}\frac{\partial f}{\partial x} &= x^2 \cdot \frac{\partial}{\partial x} (y^3) + y^3 \cdot \frac{\partial}{\partial x} (x^2) \\ &= x^2 \cdot 0 + y^3 \cdot 2x \\ &= 2xy^3\end{aligned}$$

$$g(x, y) = (3x) \cdot (4y^2)$$

$$\begin{aligned}\frac{\partial g}{\partial y} &= 3x \cdot \frac{\partial}{\partial y} (4y^2) + 4y^2 \cdot \frac{\partial}{\partial y} (3x) \\ &= 3x \cdot 8y + 4y^2 \cdot 0 \\ &= 24xy\end{aligned}$$

④ Partial Derivative with respect to x .

$$u(x, y) = x^2 + y^2, \quad v(x, y) = x^2y + y^2x$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} (x^2) + \frac{\partial}{\partial x} (y^2) = 2x$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x} (x^2y) + \frac{\partial}{\partial x} (y^2x) = 2xy + y^2$$

$$\begin{aligned}\frac{\partial f}{\partial x} &= u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial u}{\partial x} \\ &= (x^2 + y^2) \cdot (2xy + y^2) + (x^2y + y^2x) \cdot 2x \\ &= 4x^3y + x^2y^2 + 4xy^3 + y^4\end{aligned}$$

5. Quotient Rule:

$$f(x, y) = \frac{u(x, y)}{v(x, y)}$$

$$\frac{\partial f}{\partial x} = \frac{v \cdot \frac{\partial u}{\partial x} - u \cdot \frac{\partial v}{\partial x}}{v^2}$$

$$f(x, y) = \frac{x^2}{y}$$

$$\frac{\partial f}{\partial x} = \frac{y \cdot \frac{\partial}{\partial x}(x^2) - x^2 \cdot \frac{\partial}{\partial x}(y)}{y^2}$$

$$= \frac{2xy - x^2 \cdot 0}{y^2}$$

$$= \frac{2xy}{y^2}$$

6. Chain Rule:

$$z = f(g(x, y), h(x, y))$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial g} \cdot \frac{\partial g}{\partial x} + \frac{\partial z}{\partial h}$$

Example.

$$f(x, y) = e^{x^2 + y^2}$$

$$\text{Let, } u = x^2 + y^2, f = e^u$$

$$\frac{\partial u}{\partial x} = 2x, \frac{\partial f}{\partial u} = e^{x^2 + y^2}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}$$

$$= e^{x^2 + y^2} \cdot 2x$$

$$= 2x e^{x^2 + y^2}$$