

MAT216: Linear Algebra and Fourier Transformation

Emon Hossain¹

¹Lecturer
MNS department
Brac University

LECTURE-15

Definition

The **Fourier Transform** of a function $f(x)$ is defined as,

$$F(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx$$

where ω is the frequency variable. And the **Inverse Fourier Transform** is given by,

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{i\omega x} d\omega$$

Example

Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} \pi, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

Hence, evaluate the integral,

$$\int_0^\infty \frac{\sin x}{x} dx$$

Hint:

$$F(\omega) = \frac{2\pi \sin \omega}{\omega}$$

Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Hence evaluate the integral,

$$\int_0^\infty \frac{\sin(ax)\cos(ax)}{x} dx$$

Hint:

$$F(\omega) = \frac{2\sin(a\omega)}{\omega}$$

Example

Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence, evaluate the integral,

$$\int_0^\infty \left(\frac{x \cos(x) - \sin(x)}{x^3} \right) \cos\left(\frac{x}{2}\right) dx$$

Example

Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence, evaluate the integral,

$$\int_0^\infty \frac{\sin^2 x}{x^2} dx$$

Hint:

$$F(\omega) = \frac{4}{\omega^2} \sin^2 \left(\frac{\omega}{2} \right)$$