

# MAT216: Linear Algebra and Fourier Transformation

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LECTURE-15

The **Fourier Transform** of a function  $f(x)$  is defined as,

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$$

where  $\omega$  is the frequency variable. And the **Inverse Fourier Transform** is given by,

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega x} d\omega$$

# Example

## Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} \pi, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

Hence, evaluate the integral,

$$\int_0^{\infty} \frac{\sin x}{x} dx$$

**Hint:**

$$F(\omega) = \frac{2\pi \sin \omega}{\omega}$$

## Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Hence evaluate the integral,

$$\int_0^{\infty} \frac{\sin(ax) \cos(ax)}{x} dx$$

**Hint:**

$$F(\omega) = \frac{2 \sin(a\omega)}{\omega}$$

# Example

## Example

Find the Fourier transformation of the function,

$$f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence, evaluate the integral,

$$\int_0^{\infty} \left( \frac{x \cos(x) - \sin(x)}{x^3} \right) \cos\left(\frac{x}{2}\right) dx$$

# Example

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Find the Fourier transformation of the function,

$$f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Hence, evaluate the integral,

$$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$$

**Hint:**

$$F(\omega) = \frac{4}{\omega^2} \sin^2 \left( \frac{\omega}{2} \right)$$