MAT216: Linear Algebra and Fourier Transformation

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LECTURE-06

Example

Consider the set $V = \mathbb{R}^2$. A generic element of $\mathbb{R}^2$ is given by the pair (x, y) where $x, y \in \mathbb{R}$ and $k \in \mathbb{R}$. The operations addition and scalar multiplication is defined on $\mathbb{R}^2$ by

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$$
$$k(x_1, y_1) = (kx_1, ky_1)$$

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Check Axiom M3.

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$$\begin{aligned}(x_1, y_1) + (x_2, y_2) &= (x_1 + x_2, y_1 + y_2) \\ k(x_1, y_1) &= (k^2 x_1, k^2 y_1)\end{aligned}$$

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$$k(x_1, y_1) = (0, 0)$$

Check Axiom M5. `https://math.stackexchange.com/questions/22179/does-the-multiplicative-identity-have-to-be-1`

Example

Let $V = \{(0,0,0)\}$. Is V a vector space over $\mathbb{R}$ with respect to the usual operations? Justify your answer.

Example

Let $V = \{(1, 1, 1)\}$. Is V a vector space over $\mathbb{R}$ with respect to the usual operations? Justify your answer.

Small buddy! Big catch

Definition

If W be a non-empty subset of a vector space $V(\mathbb{F})$, then W is called a subspace of V if W satisfies all the axioms of vector space V with respect to vector addition and scalar multiplication. So, formally we can write the laws as:

$W \neq \emptyset$

W is closure under addition, $\forall u, v \in W \implies u + v \in W$

W is closed under scalar multiplication, $\alpha \in \mathbb{F}, u \in W \implies \alpha u \in W$

Example

Problem

Determine whether the following sets are subspace of $\mathbb{R}^2$ or not.

$$S = \left\{ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R} \right\},\ T = \left\{ \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R} \right\},$$

$$U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R}, x \geq 0 \right\},\ V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R}, x \geq y \right\}$$

Remark

Remark: If S and T are two subspaces of a vector space $V(\mathbb{F})$ then $S \cap T$ is a subspace of $V(\mathbb{F})$. But what about $S \cup T$? For example, take W_1 to be the x-axis and W_2 the y-axis, both subspaces of $\mathbb{R}^2$. Their union includes both $(3, 0)$ and $(0, 5)$, whose sum, $(3, 5)$, is not in the union. Hence, the union is not a vector space.