MAT215: Complex Variables And Laplace Transformations

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Lecture-05

Derivative formula

Example

Show that,

$$\mathscr{L}\{y'(t)\} = sY(s) - y(0)$$

Example

Show that,

$$\mathscr{L}\{y''(t)\} = s^2 Y(s) - sy(0) - y'(0)$$

Example

Show that,

$$\mathscr{L}\{y'''(t)\} = s^3 Y(s) - s^2 y(0) - sy'(0) - y''(0)$$

Example

Example

Solve the given differential equation:

$$y'' - 4y' + 4y = t^3 e^{2t},\ y(0) = 0, y'(0) = 0$$

Example

Solve the given differential equation:

$$y'' - 3y' + 2y = 4, y(0) = 1, y'(0) = -1$$

Hint:

$$\frac{s-2}{s(s-1)} = \frac{2}{s} + \frac{-1}{s-1}$$

Example

Example

Solve the given differential equation:

$$y''' - 3y'' + 3y' - y = e^t t^2,\ y(0) = 0, y'(0) = 1, y''(0) = -2$$

Example

Solve the given differential equation:

$$\frac{dy}{dt} + y = 13 \sin 2t,\ y(0) = 6$$

Hint:

$$\frac{6s^2 + 50}{(s+3)(s^2+4)} = \frac{8}{(s+3)} + \frac{-2s+6}{(s^2+4)}$$

Example

Example

Solve the given differential equation:

$$2y''' + 3y'' - 3y' - 2y = e^{-t},\ y(0) = 0, y'(0) = 0, y''(0) = 1$$

Example

Solve the given differential equation:

$$y'' + 2y' + 5y = e^{-t} \sin t,\ y(0) = 0, y'(0) = 1$$

Example

Example

Solve the given differential equation:

$$y' + y = f(t),\ y(0) = 5$$

$$f(t) = \begin{cases} 0, & 0 \le t < \pi \\ 2\cos t, & t \ge \pi \end{cases}$$

Example

Solve the given differential equation:

$$y'' + 4y = \sin t u(t - 2\pi),\ y(0) = 1, y'(0) = 0$$

Important

Example

Solve the given differential equation:

$$y'' + 9y = \cos 2t,\ y(0) = 0, y\left(\frac{\pi}{2}\right) = -1$$

Example

Solve the given system of differential equations:

$$\frac{dx}{dt} = -x + y$$
$$\frac{dy}{dt} = 2x$$

with initial conditions $x(0) = 0, y(0) = 1$.

Answer:

$$x' = -x + y$$
$$sX(s) - x(0) = -X(s) + Y(s)$$
$$Y(s) = (s+1)X(s)$$

And,

$$
\begin{aligned}
y' &= 2x \\
sY(s) - y(0) &= 2X(s) \\
sY(s) - 1 &= 2X(s) \\
2X(s) - s(s+1)X(s) &= -1 \\
X(s) &= \frac{1}{s^2+s-2} = \frac{1/3}{s-1} + \frac{-1/3}{s+2} \\
x(t) &= \frac{1}{3}e^{t} - \frac{1}{3}e^{-2t}
\end{aligned}
$$

Now,

$$
\begin{aligned}
Y(s) &= (s+1)X(s) = \frac{s+1}{s^2+s-2} = \frac{2/3}{s-1} + \frac{1/3}{s+2} \\
y(t) &= \frac{2}{3}e^{t} + \frac{1}{3}e^{-2t}
\end{aligned}
$$