MAT215: Complex Variables And Laplace Transformations

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Lecture-01

Complex plane

As a set, the complex number field $\mathbb C$ be the set $\mathbb R^2=\mathbb R\times\mathbb R$. The set is a plane, so we call it the complex plane. To make it a field, we define addition and product:

$$(a,b)+(c,d) \stackrel{\text{def}}{=} (a+c,b+d)$$
$$(a,b)(c,d) \stackrel{\text{def}}{=} (ac-bd,bc+da)$$

Why do we define the operations in such a way? One answer is to mimic the complex operation. Another is to consider the multiplication as the map $z_1 \mapsto z_1 z_2$ (which is a real-linear operator).

$$a+ib=a\begin{pmatrix}1&0\\0&1\end{pmatrix}+b\begin{pmatrix}0&-1\\1&0\end{pmatrix}=\begin{pmatrix}a&-b\\b&a\end{pmatrix}$$

Problem

Show that the additive identity is $\mathbf{0} = (0,0)$, the multiplicative identity is $\mathbf{1} = (1,0)$ and the multiplicative inverse $(a,b)^{-1} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

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Complex numbers

Problem

Find the real numbers x and y such that 3x + 2iy - ix + 5y = 7 + 5i

Problem

Prove that every real-linear operator on \mathbb{C} , that is every 2×2 real matrix M, can be represented by two complex numbers ξ and ζ and the formula $z \mapsto \xi z + \zeta \bar{z}$.

Hint: Use T(z) = T(x + iy)

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Complex number

Given a complex number $z \stackrel{\text{def}}{=} x + iy$, its "evil twin" is the complex conjugate of z:

$$\bar{z} \stackrel{\text{def}}{=} x - iy$$

Find the real and imaginary parts.

We can show that,

$$x^{3} + y^{3} + 3ixy = \left(\frac{z + \overline{z}}{2}\right)^{3} + \left(\frac{z - \overline{z}}{2i}\right)^{3} + 3i\left(\frac{z + \overline{z}}{2}\right)\left(\frac{z - \overline{z}}{2i}\right)$$

Problem

Guess all the properties of Conjugate.

z and \bar{z} are not independent variables, why?

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Problem

Problem

Find the square root of -15-8i.

Problem

Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$.

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Argument

The function $\mathbf{Arg}(z): \mathbb{C}\backslash\{0\} \to (-\pi,\pi]$ is defined as follows:

$$\operatorname{Arg}(z) = \left\{ \begin{array}{ll} \operatorname{arctan} \frac{y}{x} & \text{if } x > 0, y \in \mathbb{R} \\ \operatorname{arctan} \frac{y}{x} + \pi & \text{if } x < 0, y \geq 0 \\ \operatorname{arctan} \frac{y}{x} - \pi & \text{if } x < 0, y < 0 \\ \frac{\pi}{2} & \text{if } x = 0, y > 0 \\ -\frac{\pi}{2} & \text{if } x = 0, y < 0 \\ \text{undefined} & \text{if } x = 0, y = 0 \end{array} \right.$$

Thus, if $z=r(\cos\Theta+i\sin\Theta)$, with r>0 and $-\pi<\Theta\leq\pi$, then $\arg(z)=\{\operatorname{Arg}(z)+2n\pi\mid n\in\mathbb{Z}\}.$

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Polar form

Properties of Modulus:

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

$$|z^m| = |z|^m$$

Properties of Arguments:

$$Arg(z_1 \cdot z_2) = Arg(z_1) + Arg(z_2)$$

$$Arg\left(\frac{z_1}{z_2}\right) = Arg(z_1) - Arg(z_2)$$

$$Arg(z^m) = mArg(z)$$

https://scipp.ucsc.edu/~haber/ph116A/arg_11.pdf Express each of the following complex numbers in polar form:

(a)
$$2 + 2\sqrt{3}i$$

(b)
$$-5 + 5i$$

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problems

Problem

Solve the equation: $e^{4z} = i$

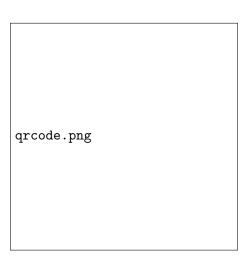
Problem

Solve for x and y,

$$\left(\frac{3}{2} + \frac{\sqrt{3}}{2}i\right)^{2024} = 3^{1012}(x + iy)$$

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Good-bye



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