MAT216: Linear Algebra and Fourier Transformation

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Lecture-09

Null Space

Definition

The **Null space** of a $m \times n$ matrix A, written as $\text{Null}(A)$ is the set of all solutions to the homogeneous equation $Ax = 0$.

$$\text{Null}(A) = \{x : x \text{ is in } \mathbb{R}^n, Ax = 0\}$$

$\text{Null}(A)$ is a subspace of $\mathbb{R}^n$.

Example

Example

Find the null space of the following matrix:

$$T = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 7 & -4 \end{pmatrix}$$

Example

Example

Determine whether the following set is a vector space:

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x - y = 0, y + z = 0 \right\}$$

Hint:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Row Space

Definition

The **Row space** of $m \times n$ matrix A is the set of all linear combinations of the rows of A.

$$\text{Row}(A) = \text{Span}\{R_1, \cdots, R_n\}$$

The row space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.

Example

Example

Find the row space of the following matrix:

$$T = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 7 & -4 \end{pmatrix}$$

Definition

The **Column space** of $m \times n$ matrix A is the set of all linear combinations of the columns of A.

$$\mathrm{Col}(A) = \mathrm{Span}\{C_1, \cdots, C_n\}$$

or,

$$\mathrm{Col}(A) = \{b : b = Ax \text{ for some } x \in \mathbb{R}^n\}$$

The column space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^m$.

Example

Example

Find the column space of the following matrix:

$$T = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 7 & -4 \end{pmatrix}$$

Example

Find a matrix A such that $W = \text{Col}(A)$ where

$$W = \left\{ \begin{bmatrix} x - 2y \\ 3y \\ x + y \end{bmatrix} : x, y \in \mathbb{R} \right\}$$

Determine whether W is a vector space or not.

Hint:

$$\begin{bmatrix} x - 2y \\ 3y \\ x + y \end{bmatrix} = x \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$$

Rank-Nullity Theorem

$$\text{row-rank}(T) + \text{nullity}(T) = \# \text{ of columns}$$