

# MAT216: Linear Algebra and Fourier Transformation

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LECTURE-01

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## simple system

System of equations is the most fascinating topic in Linear Algebra. Because we can see it through both vector space and Linear Transformation perspective respectively. For example, take a system of linear equations,

$$\begin{aligned}x - y &= 1 \\ 2x + y &= 6\end{aligned}\tag{1}$$

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$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} x + \begin{pmatrix} -1 \\ 1 \end{pmatrix} y = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

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$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}x + \begin{pmatrix} -1 \\ 1 \end{pmatrix}y = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

**Interpretation of the solution:** Here, we want to find such scalars  $x, y$  for which our vectors  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$  can reach the vector  $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$  by **vector addition**.

## Matrix form

Likewise, we can rewrite the system as,

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

Here, we are considering the linear transformation (don't worry we will discuss it in our upcoming lectures, or if you want to understand it right now have a look at the course Essence of linear algebra, YouTube playlist from 3b1b channel) and want to find such vectors which are mapped to the specific vector  $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$  after the transformation applied.

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**Interpretation of the solution:** So, basically we want all those vectors  $\begin{pmatrix} x \\ y \end{pmatrix}$  which are mapped to  $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$ .

# Bigger picture

## System of linear Equations

Algebraic form      Vector form      Transformation form      ...

# Bigger picture

## System of linear Equations

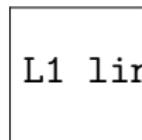
Algebraic form      Vector form      Transformation form      ...

madness.jpeg

Welcome to Linear Algebra, where the notation is made up and you might gain your third eye.

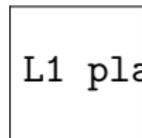
*“Life isn’t linear, but written words are. Like our system of linear equations.”*

# System of equation: Two variables



L1\_lines.png

# System of equation: Three variables



L1\_planes.png

continued...

Like for a three-variable system:

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

We can rewrite it in the vector form:

$$\underbrace{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}}_{\in \mathbb{R}_3} x + \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}_{\in \mathbb{R}_3} y + \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\in \mathbb{R}_3} z = \underbrace{\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}}_{\in \mathbb{R}_3}$$

continued...

So, our solution  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$  actually tell us how much we should scale our column vectors  $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ ,  $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$  and  $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$  respectively in order to reach the right hand side vector  $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ .

Two questions that we will revisit many times throughout our course: (1) Does a given linear system have a solution? In other words, is it consistent? (2) If it is consistent, is the solution unique?

# Course outline

outline.png

# Questions

continued...

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# Bibliography I