

# MAT215: Complex Variables And Laplace Transformations

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LECTURE-03

# Polar Form

## Problem

Find the Polar form of:

$$-1 + \sqrt{3}i$$

$$12i$$

Find the expression:

$$z_1 z_2 = ?$$

$$\frac{z_1}{z_2} = ?$$

$$z^{-1} ?$$

**Hint:** You need to pray to get the formula :3

# Exponential Form

Find the expression:

$$z_1 z_2 = ?$$

$$\frac{z_1}{z_2} = ?$$

$$z^{-1} ?$$

**Hint:** Use the polar form of each complex number and manipulate them.

## Issues

Let for multiplication,  $z_1 = i, z_2 = -1$ . But

$$\operatorname{Arg}(z_1) + \operatorname{Arg}(z_2) = \frac{\pi}{2} + \pi = \frac{3\pi}{2} \neq -\frac{\pi}{2} = \operatorname{Arg}(z_1 z_2)$$

To bring it back into the principal range, subtract.

Let for division,  $z_1 = -i, z_2 = -1$ . Then:  $\operatorname{Arg}(z_1) = -\frac{\pi}{2}, \operatorname{Arg}(z_2) = \pi$ .

Then:

$$-\frac{\pi}{2} - \pi = -\frac{3\pi}{2}$$

Now  $-\frac{3\pi}{2} \notin (-\pi, \pi]$ . But  $\frac{3\pi}{2} + 2\pi = \frac{\pi}{2}$

Let for power,  $z = -1$ , so  $\operatorname{Arg}(z) = \pi$ . Then  $z^2 = (-1)^2 = 1$ , and  $\operatorname{Arg}(z^2) = 0$ .

But  $n\operatorname{Arg}(z) = 2\pi \notin (-\pi, \pi]$ . We must subtract  $2\pi$  to bring it back:  $2\pi - 2\pi = 0$ .

# Problems

To find a large power or nth root, use exponential form.

## Problem

Solve the equation:  $e^{4z} = i$

**Hint:** Start with known complex number

## Problem

Solve for  $x$  and  $y$ ,

$$\left(\frac{3}{2} + \frac{\sqrt{3}}{2}i\right)^{2024} = 3^{1012}(x+iy)$$

**Hint:** Start with known complex number

## Examples

Find  $(1 + i\sqrt{3})^{10}$ ,  $(-16)^{\frac{1}{4}}$ , all the cubic roots of unity.

### Problem

*Find all the values of  $z$  for which  $z^5 = 32\sqrt{-1}$ , and locate them in the complex plane.*

# Problems

## Problem

*Graph the lines,*

$$|z| = 2$$

$$|z - 2i| = 3$$

$$\left| \frac{z-3}{z+3} \right| = 2$$

$$\operatorname{Im}(z^2) = 4$$

$$\operatorname{Re}(z^2) = 4$$

$$\operatorname{Re}\left(\frac{1}{z^2}\right) = 1$$

$$\arg(z) = \frac{\pi}{3}$$

# Problems

## Problem

*Graph the regions,*

$$|z| > 2$$

$$|z - 2i| \leq 3$$

$$\left| \frac{z-3}{z+3} \right| < 2$$

$$\operatorname{Im}(z^2) > 4$$

$$\operatorname{Re}(z^2) < 4$$

$$\frac{\pi}{4} \leq \arg(z) \leq \frac{2\pi}{3}$$

# Issues

$$|z_1 z_2| = |z_1| |z_2| \text{ but } |z_1 + z_2| \leq |z_1| + |z_2|$$

$$\sqrt{z_1 z_2} \neq \sqrt{z_1} \sqrt{z_2}$$

$$(z^a)^b \neq z^{ab}$$

$$\log_a b^r \neq r \log_a b$$

Consider the following examples:  $z_1 = 1 + i, z_2 = -1 + i,$   
 $\sqrt{(-1)(-1)} \neq \sqrt{(-1)}\sqrt{(-1)}.$

# Complex Power