MAT216: Linear Algebra and Fourier Transformation

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LECTURE-05

Physicist: Arrow

CSE: Array

Mathematician: 10 Axioms

What we need before we talk about vector space: A set, V, where we define an operator $\oplus$ which operates on set elements and another operator $\odot$ which operates on the field and set elements

Example:

$$\begin{pmatrix}1\\2\end{pmatrix} \oplus \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}3\\5\end{pmatrix}$$

$$2 \odot \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$$

Axioms

Operations on vector addition:

Closure law (A1): $\forall u, v \in V \implies u \oplus v \in V$.

Commutative law (A2): $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$

Associative law (A3): $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$.

Existence of Additive identity (A4): There exists an element $\mathbf{0}_V \in V$, called the zero vector, such that $\mathbf{v} \oplus \mathbf{0}_V = \mathbf{v}$ for all $\mathbf{v} \in V$.

Existence of Additive inverse (A5): For every $\mathbf{v} \in V$, there exists an element $-\mathbf{v} \in V$, called the additive inverse of $\mathbf{v}$, such that $\mathbf{v} \oplus (-\mathbf{v}) = \mathbf{0}$.

Axioms

Operations on scalar multiplication:

Closure law (M1): $\forall \alpha \in \mathbb{F}$ and $\mathbf{v} \in V \implies \alpha \odot \mathbf{v} \in V$.

Distributive law respect to vector addition (M2):
$\alpha \odot (\mathbf{u} \oplus \mathbf{v}) = \alpha \odot \mathbf{u} \oplus \alpha \odot \mathbf{v}$

Distributive law respect to field addition (M3):
$(\alpha \tilde{\oplus} \beta) \odot \mathbf{v} = \alpha \odot \mathbf{v} \oplus \beta \odot \mathbf{v}$

Compatibility of scalar multiplication with field multiplication (M4): $\alpha \odot (\beta \odot \mathbf{v}) = (\alpha \star \beta) \odot \mathbf{v}$.

Existence of multiplicative identity (M5): $1_{\mathbb{F}}\mathbf{v} = \mathbf{v}$, where $1_{\mathbb{F}}$ denotes the multiplicative identity in $\mathbb{F}$.

Example

Consider the set $V = \mathbb{R}^2$. A generic element of $\mathbb{R}^2$ is given by the pair (x, y) where $x, y \in \mathbb{R}$ and $k \in \mathbb{R}$. The operations of addition and scalar multiplication are defined on $\mathbb{R}^2$ by

$$\begin{aligned} (x_1, y_1) + (x_2, y_2) &= (x_1 + x_2, y_1 y_2) \\ k(x_1, y_1) &= (kx_1, ky_1) \end{aligned}$$

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$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$k(x_1, y_1) = (0, 0)$$

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Example

Let $V = \{(0,0,0)\}$. Is V a vector space over $\mathbb{R}$ with respect to the usual operations? Justify your answer.

Example

Let $V = \{(1,1,1)\}$. Is V a vector space over $\mathbb{R}$ with respect to the usual operations? Justify your answer.

Some example of vector space:

$\mathbb{R}^n$ under operations over $\mathbb{R}$

Set of all $m \times n$ matrices