

MAT215: Complex Variables And Laplace Transformations

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INTEGRATION LECTURE-01

Parametrization

The key to parametrization is to realize that the goal of this method is to describe the location of all points on a geometric object, such as a curve, a surface, or a region. This description must be one-to-one and onto: every point must be described once and only once.

$$\gamma := \{(x, y) : x^2 + y^2 = 1\}$$

If you want to geometrically analyze the curve γ (length/enclosed area, etc), we need a manageable way to produce the points (production scheme).

Motivation of Parametrization

Parametric curve: A curve in the 2-D plane can be described by,

$$\left. \begin{array}{l} x = f(t) \\ y = g(t) \end{array} \right\} \text{ where } a \leq t \leq b$$

These are called parametric equations for that curve, and t is the parameter.

What is the difference between general and parametric curves?

Example

Plot the parametric curves defined by,

$$x = \cos t, y = -\sin t; 0 \leq t \leq \pi$$

$$x = t, y = 1; 0 \leq t \leq 4$$

$$z = 1 + it; 0 \leq t \leq 1$$

Straight line from $(0, 3)$ to $(2, 3)$

Straight line from $(2, 3)$ to $(2, 4)$

Straight line from $(0, 3)$ to $(2, 4)$

Line segment: $\gamma(t) = a + t(b - a), 0 \leq t \leq 1$

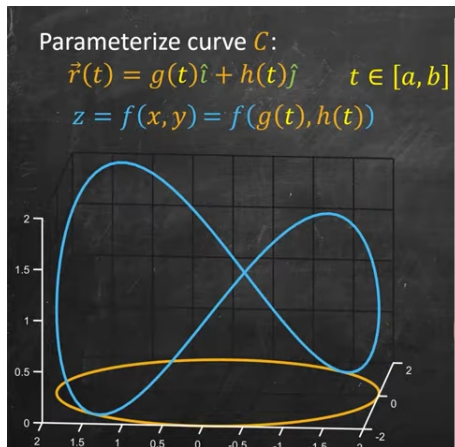
Circle: $\gamma(t) = (a + r \cos(t), b + r \sin(t)), 0 \leq t \leq 2\pi$

Explicit function: $\gamma(t) \stackrel{y=f(x)}{=} (t, f(t))$

Explicit function in polar: $\gamma(t) \stackrel{r=f(\theta)}{=} (f(t) \cos(t), f(t) \sin(t))$

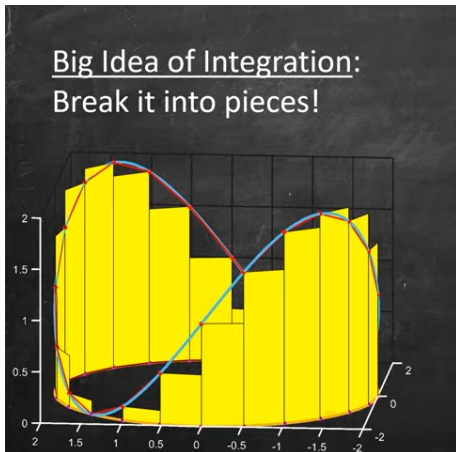
Derivative exists at all points and is continuous

Motivation of line integral



continued...

Big Idea of Integration:
Break it into pieces!



Path in complex integration

Is there any meaning of

$$\int_{z_1}^{z_2} f(z) dz = ?$$

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Consider $C : z(t) = h(t) + ig(t); a \leq t \leq b$ Then line integral (or complex line integration) of f along C is defined as,

$$\int_C f(z) dz = \int_a^b f(\gamma(t)) \gamma'(t) dt$$

Example

$$C_1 : z(t) := \gamma(t) = t, 0 \leq t \leq 1$$

$$C_2 : z(t) = 1 + it, 0 \leq t \leq 1$$

$$f(z) = \bar{z} \quad C_3 : z(t) = \gamma(t) = t(1 + i), 0 \leq t \leq 1$$