MAT216: Linear Algebra and Fourier Transformation

Emon Hossain[1]

[1]Lecturer
MNS department
Brac University

LECTURE-08

Span

Definition

Let S be a nonempty subset of a vector space V. The span of S, denoted by span(S), is the set containing all linear combinations of vectors in S. For convenience, we define $\text{span}(\emptyset) = \{0\}$.

Example

Does $\left\{ \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ span $\mathbb{R}^2$?

A set of vectors $S = \{v_1, \cdots, v_n\} \subset V$ spans the vector space V if every vector in V can be generated using a linear combination of S. Mathematically, for any arbitrary $\vec{v}_{\text{random}} \in V$ the system,

$$\alpha_1 v_1 + \cdots + \alpha_n v_n = v_{\text{random}}$$

has a solution(unique or many).

Example

Question-1: Determine whether or not the vectors $u = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ span $\mathbb{R}^3$.

Question-2: Determine whether the set of vectors S span $\mathbb{R}^3$:

$$S = \{(3,0,4),(6,2,-1),(12,4,-2),(3,2,-5)\}$$

Basis

Definition

A basis is a set of vectors, S, that generates all elements of the vector space V, and the vectors in the set are linearly independent.

That means to be a basis, we need to satisfy two conditions:

- S is linearly independent
- S spans V

Summary

$$\alpha_1 v_1 + \cdots + \alpha_n v_n$$

- Check linear combination: $\alpha_1 v_1 + \cdots + \alpha_n v_n = b$
  - Unique/Many solution: Span
  - No solution: Do not Span
- Check independency: $\alpha_1 v_1 + \cdots + \alpha_n v_n = 0$
  - Many solutions: Dependent
  - Unique solution: Independent

Example

Example

Is $S = \{(1,2),(2,3)\}$ a basis of $\mathbb{R}^2$?

Hint: Check linear independence and spanning condition.

Example

What about $S = \{(1,2),(2,4)\}$?

Example

Determine whether the set of vectors S form a basis of $\mathbb{R}^3$:

$$S = \{(2,-3,1),(4,1,1)\}$$

More example

Example

Find the basis of the space $M_{2\times 2}(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a+b+c+d=0 \right\}$

Hint: How many way you can pair 1 and -1 inside the matrix entries?

Example

Find the basis from
$\{t^3-2t^2+4t+1, 2t^3-3t^2+9t-1, t^3+6t-5, 2t^3-5t^2+7t+5\}$.