MAT216: Linear Algebra and Fourier Transformation

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LECTURE-16

Definition

As we saw in our previous lecture, that we can split our Fourier series formula for even and odd functions. Now, we can do the same for Fourier Transform. The **Fourier Cosine Transform** of a function $f(x)$ is defined as,

$$F_c(\omega) = \int_0^\infty f(x)\cos(\omega x)\,dx$$

where ω is the frequency variable. And the **Inverse Fourier Transform** is given by,

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(\omega)\cos(\omega x)\,d\omega$$

Similarly, the **Fourier Sine Transform** of a function $f(x)$ is defined as,

$$F_s(\omega) = \int_0^{\infty} f(x) \sin(\omega x)\, dx$$

And the **Inverse Fourier Transform** is given by,

$$f(x) = \frac{2}{\pi} \int_0^{\infty} F_s(\omega) \sin(\omega x)\, d\omega$$

Example

Find the Fourier Sine Transform of

$$\begin{cases} 1, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$$

Hence, evaluate the integral

$$\int_0^\infty \frac{\sin^3 x}{x}\, dx$$

Hint:

$$F_s(\omega) = \frac{1 - \cos\omega}{\omega}$$

Example

Find the Fourier Cosine Transform of e^{-x}, $x \geq 0$. Hence, show that,

$$\int_0^\infty \frac{\cos(mx)}{x^2+1}dx = \frac{\pi}{2}e^{-m},\ m > 0$$

Hint:

$$F_c(\omega) = \frac{1}{1+\omega^2}$$

Example

Find the Fourier Sine Transform of e^{-x}, $x \geq 0$. Hence, show that,

$$\int_0^\infty \frac{x \sin(mx)}{x^2+1} dx = \frac{\pi}{2} e^{-m},\ m > 0$$

Hint:

$$F_s(\omega) = \frac{\omega}{1+\omega^2}$$

Example

Find the Fourier Cosine Transform of $e^{-mx}, x \geq 0$. Hence, show that,

$$\int_0^\infty \frac{\beta \cos(\rho v)}{v^2+\beta^2} dx = \frac{\pi}{2} e^{-\rho\beta}, \rho > 0, \beta > 0$$

Hint:

$$F_c(\omega) = \frac{m}{\omega^2+m^2}$$

Example

Find the Fourier Sine Transform of $e^{-mx}, x \geq 0$. Hence, show that,

$$\int_0^\infty \frac{x\sin(\rho x)}{x^2+m^2}dx = \frac{\pi}{2}e^{-\rho m}, \rho > 0, m > 0$$