MAT216: Linear Algebra and Fourier Transformation

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LECTURE-04

Elementary row operation

Swapping any two rows. $\left(R_i' \leftrightarrow R_j'\right)$

Multiplying a row by a non-zero constant. ($R_i' = k \cdot R_i'$)

Adding some multiple of a row to another row. $\left(R_i' = R_i' \pm k \cdot R_j'\right)$

An elementary matrix is a square matrix that has been obtained by performing an elementary row or column operation on an identity matrix.

Example

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Inverse matrix

a) Calculate the inverse of the matrix using Gauss-Jordan elimination method

$$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -3 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

b) Hence, solve the system

$$\begin{aligned} 2x + 2y + 3z &= 1 \\ x - 3y + z &= 2 \\ x + 2y &= 5 \end{aligned}$$

Binary operation

Suppose $*$ is a binary operation. A set G is called closed under $*$ if

$$a * b \in G \forall a, b \in G$$

Examples:

Addition on the set $\mathbb{R}$, operation $\boldsymbol{a} * \boldsymbol{b} = \boldsymbol{a} + \boldsymbol{b}$

Addition on the set $\mathbb{N}$, operation $a * b = a + b$

Addition on the set $\mathbb{Z}$, operation $a * b = a + b$

Subtraction on the set $\mathbb{R}$, operation $a * b = a - b$

Matrix addition on the set of all 3×3 matrices

Matrix multiplication on the set of all 2×2 matrices

Non-Example: Substraction on the set $\mathbb{N}$ oeprator: $a * b = a - b$

Commutativity

Suppose $*$ is a binary operation on the set G. The operation $*$ is called commutative if

$$a * b = b * a \forall a, b \in G$$

Examples:

Addition: $a * b = a + b$

Multiplication: $a * b = ab$

Matrix Addition

Non-Example: Substraction, Division, Matrix Multiplication

Associativity

Suppose $*$ is a binary operation on the set G. The operation $*$ is called associative if

$$(a * b) * c = a * (b * c) \forall a, b, c \in G$$

Examples:

Multiplication: $a * b = ab$

Matrix Addition: $(A + B) + C = A + (B + C)$

Matrix Multiplication

Non-Example: Substraction, Division

Identity

Suppose $*$ is a binary operation on the set G. The operation $*$ has an identity if there is a fixed unique element e such that

$$e * a = a * e = a \forall a \in G$$

Here e is called the identity of the operation.
Find the identity element of the operation $*$ on $\mathbb{R}$ where

$$a * b = a + b - 1$$

Find the identity of the operation $\oplus$ on $\mathbb{R}^2$ where

$$(a, b) \oplus (c, d) = (a + c + 2, b + d - 1)$$

Inverse

Suppose $*$ is a binary operation on the set G with the identity element e. Existence of Inverse means for each element $a \in G$ there will have a unique element a' which can be dependent of a, such that

$$a' * a = a * a' = e \text{ for all } a \in G$$

Here a' is called the Inverse of a.
Find the inverse

$$a * b = a + b - 1$$

Vector Space

To have a vector space, the ten following axioms must be satisfied for every u, v and w in V, and a and b in $\mathbb{F}$.

(Basically, we need a set, V where we define an operator $\oplus$ which operates on set elements and another operator $\odot$ which operates on the field and set elements)

Operations on vector addition:

Closure law (A1): $\forall u, v \in V \implies u \oplus v \in V$.

Commutative law (A2): $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$

Associative law (A3): $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$.

Existence of Additive identity (A4): There exists an element $\mathbf{0}_V \in V$, called the zero vector, such that $\mathbf{v} \oplus \mathbf{0}_V = \mathbf{v}$ for all $\mathbf{v} \in V$.

Existence of Additive inverse (A5): For every $\mathbf{v} \in V$, there exists an element $-\mathbf{v} \in V$, called the additive inverse of $\mathbf{v}$, such that $\mathbf{v} \oplus (-\mathbf{v}) = \mathbf{0}$.

Axioms

Operations on scalar multiplication:

Closure law (M1): $\forall \alpha \in \mathbb{F}$ and $\mathbf{v} \in V \implies \alpha \odot \mathbf{v} \in V$.

Distributive law respect to vector addition (M2):
$\alpha \odot (\mathbf{u} \oplus \mathbf{v}) = \alpha \odot \mathbf{u} \oplus \alpha \odot \mathbf{v}$

Distributive law respect to field addition (M3):
$(\alpha \tilde{\oplus} \beta) \odot \mathbf{v} = \alpha \odot \mathbf{v} \oplus \beta \odot \mathbf{v}$

Compatibility of scalar multiplication with field multiplication (M4): $\alpha \odot (\beta \odot \mathbf{v}) = (\alpha \star \beta) \odot \mathbf{v}$.

Existence of multiplicative identity (M5): $1_{\mathbb{F}}\mathbf{v} = \mathbf{v}$, where $1_{\mathbb{F}}$ denotes the multiplicative identity in $\mathbb{F}$.