Remedial Course in Mathematics
Final Examination
Total - 40 Marks
(You need to answer **ANY FOUR** questions)

1. (a) Sketch step-by-step the graph of

$$y = 1 + 5\cos\left(2x - \frac{\pi}{2}\right).$$

   Mark at least one full period.

   (b) On the same set of axes, sketch the graphs of

$$y = |\sin x| \quad \text{and} \quad y = \cos x$$

   for $0 \leq x < 2\pi$. Clearly mark all points of intersection. Hence, solve the equation

$$|\sin x| = \cos x$$

   for $0 \leq x < 2\pi$.

   (5+5 Marks)

2. Given the system of equations:

$$x = 1$$
$$2y + 3z = 2$$
$$y + z = 3$$

   (a) Write the system in matrix form, $A\vec{x} = \vec{b}$.

   (b) Determine whether the system is solvable or not.

   (c) If the system is solvable find the solution.

   (2+4+4 Marks)

3. (a) Find the fourth root of $z = -1 - i$. And sketch the root in the complex plane.

   (b) Write the complex number

$$z = -1 + i\sqrt{3}$$

in the polar form
$$z = r(\cos\theta + i\sin\theta)$$
Hence, compute z^{100}.

<div align="right">(6+4 Marks)</div>

4. (a) Eight friends go out for dinner. How many ways are there to sit them around a round table? Rotations of a sitting arrangement are considered the same, but a reflection is considered different.

   (b) If the reflections are also considered the same, how many ways are there to sit them around a round table?

   (c) A team of four has to be selected from 6 boys and 4 girls. How many different ways can a team be selected if at least one boy must be in the team?

   (d) How many different diagonals does a 12-sided polygon have?

   (e) How many triangles can be formed using 10 points in a plane, out of which 4 are collinear?

<div align="right">(2+2+2+2+2 Marks)</div>

5. A **Riemann sum** approximates this area using rectangles:
$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x \quad \text{where} \quad \Delta x = \frac{b-a}{n}.$$

   As $n \to \infty$, rectangles become very thin and the approximation becomes exact.

   (a) Find the value of the definite integral using formula
$$\int_0^1 x^2\,dx.$$

   (b) We want to approximate the same integral. Take $n = 4$ equal subintervals and use **left endpoints** to compute the Riemann sum.

   (c) Consider the differential equation
$$\frac{dy}{dx} = x.$$
   Find the slope field and the solution family.

<div align="right">(2+5+3 Marks)</div>

6. (a)
$$f(x) = \begin{cases} x^2 + k, & x \le 2 \\ 4x - 3, & x > 2 \end{cases}$$

   Find k such that $\lim_{x \to 2} f(x)$ exists.

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(b) Evaluate:
$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}.$$

(6+4 Marks)

7. (a) Identify and sketch:
$$|z - 1| + |z - 1| = 4$$
where $z = x + iy$.

(b) Prove using $\epsilon - \delta$ definition that $\lim_{x \to 2}(3x - 1) = 5$.

(6+4 Marks)

In remembrance of Sharif Osman Bin Hadi, whose courage and sacrifice will shape our nation.