MAT216: Linear Algebra and Fourier Transformation

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LECTURE-01

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simple system

System of equations is the most fascinating topic in Linear Algebra. Because we can see it through both vector space and Linear Transformation perspective respectively. For example, take a system of linear equations,

$$\begin{aligned} x - y &= 1 \\ 2x + y &= 6 \end{aligned} \tag{1}$$

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$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} x + \begin{pmatrix} -1 \\ 1 \end{pmatrix} y = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

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Interpretation of the solution: Here, we want to find such scalars x, y for which our vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ can reach the vector $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$ by **vector addition**.

Matrix form

Likewise, we can rewrite the system as,

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

Here, we are considering the linear transformation (don't worry we will discuss it in our upcoming lectures, or if you want to understand it right now have a look at the course Essence of linear algebra, YouTube playlist from 3b1b channel) and want to find such vectors which are mapped to the specific vector $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$ after the transformation applied.

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Interpretation of the solution: So, basically we want all those vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ which are mapped to $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$.

Bigger picture

System of linear Equations

- Algebraic form
- Vector form
- Transformation form
- $\cdots$

Bigger picture

- System of linear Equations
  - Algebraic form
  - Vector form
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  - $\cdots$

madness.jpeg

Welcome to Linear Algebra, where the notation is made up and you might gain your third eye.

> *"Life isn't linear, but written words are. Like our system of linear equations."*

L1 lines.png

System of equation: Three variables

L1 planes.png

continued...

Like for a three-variable system:

$$\begin{cases} a_1x + b_1y + c_1z & = d_1 \\ a_2x + b_2y + c_2z & = d_2 \\ a_3x + b_3y + c_3z & = d_3 \end{cases}$$

We can rewrite it in the vector form:

$$\underbrace{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}}_{\in\mathbb{R}_3} x + \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}_{\in\mathbb{R}_3} y + \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\in\mathbb{R}_3} z = \underbrace{\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}}_{\in\mathbb{R}_3}$$

continued...

So, our solution $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ actually tell us how much we should scale our column vectors $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ and $\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$ respectively in order to reach the right hand side vector $\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$.

Two questions that we will revisit many times throughout our course: (1) Does a given linear system have a solution? In other words, is it consistent? (2) If it is consistent, is the solution unique?

Course outline

outline.png

Questions

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continued...

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