MAT215: Complex Variables And Laplace Transformations

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LECTURE-01

Motivation

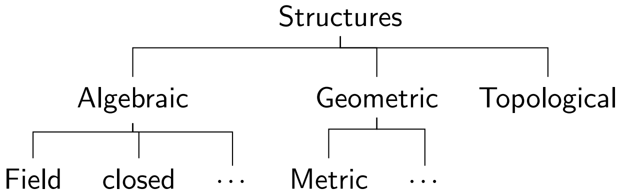
You need motivation right?

Motivation

You need motivation right?

flop_motivation.png

Structures



Structures
Algebraic
Geometric
Topological
Field
closed
$\cdots$
Metric
$\cdots$

Nothing to lose, only gain!

`only_gain.jpg`

$$(i) + (-i) > 0 \qquad (i) + (-i) < 0$$

Imposter!

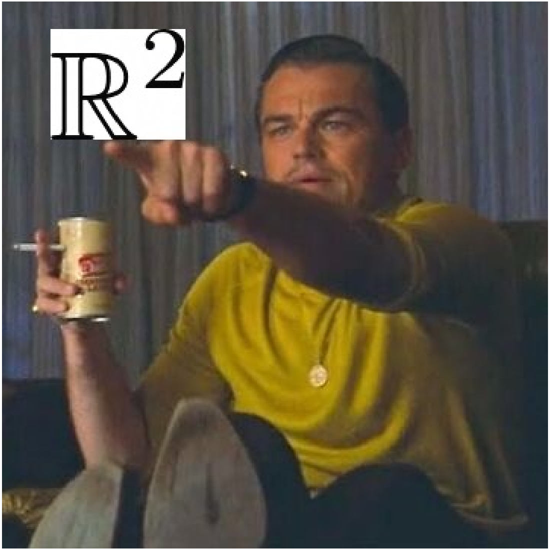


Figure: Imposter

Mate who cancel tours!

Figure: *i*

https://math.stackexchange.com/q/1760416/803654

Serious, study!

Take $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} e^{-\frac{1}{x}}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

This function is smooth, but this is not analytic (Taylor expandable). Because $f^n = 0$ for every n. So, the Taylor series about 0 gives us

$$f(0) + \sum_{n=1}^{\infty} \frac{f^n(0)}{n!} x^n = 0 \neq f(x)$$

But this is not the case for functions with complex variables. Every smooth function on $\mathbb{C}$ is also analytic.

Continued...

Consider the function,

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

This function is once differentiable on $\mathbb{R}$, but the second derivative does not exist. However, for functions with complex variables, if a function is once differentiable, then it is infinitely differentiable.

continued...

The Analytic Miracle: Cauchy's Integral Formula:

$$f(z_0) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{z - z_0}, dz$$

> *This means: to know a function inside a region, it is enough to know it only along the boundary. No other branch of analysis offers this generosity.*

And:

> *Differentiation and integration are no longer enemies — they become two faces of the same formula.*

It's almost as if the function remembers its boundary perfectly.

Fundamental Theorem of Algebra: Every non-constant polynomial has a root in ($\mathbb{C}$).

Good-bye

qrcode.png