MAT092: Remedial Course in Mathematics

Emon Hossain[1]

[1]Lecturer
MNS department
Brac University

Lecture on Permutations

Rule of sum and product

Definition

If there are n choices for one action, and m choices for another action and the two actions can't be done at the same time, then there are $n + m$ ways to choose one of the actions.

Definition

If there are n choices for one action, and m choices for another action and the two actions can be done one after another, then there are $n \times m$ ways to choose both actions.

Examples

Example

How many positive divisors does 2000 have?

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$$2000 = 2^4 \times 5^3$$

Any positive divisor of 2000 is of the form $2^a \times 5^b$ where $0 \leq a \leq 4$ and $0 \leq b \leq 3$.

$$0 \leq a \leq 4, 0 \leq b \leq 3$$

So, there are 5 choices for a and 4 choices for b. By the rule of product, the total number of positive divisors of 2000 is $5 \times 4 = 20$.

Example

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$$\binom{5}{2} \times \binom{4}{2} = 10 \times 6 = 60$$

Permutations of a set of Distinct Objects

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1234, 1243, 1324, 1342, 1423, 1432,
2134, 2143, 2314, 2341, 2413, 2431,
3124, 3142, 3214, 3241, 3412, 3421,
4123, 4132, 4213, 4231, 4312, 4321.

Example

Out of a class of 30 students, how many ways are there to choose a class president, a secretary, and a treasurer? A student may hold at most one post.

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$$30 \times 29 \times 28 = 24360$$

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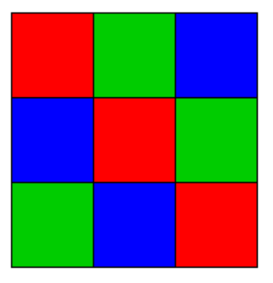

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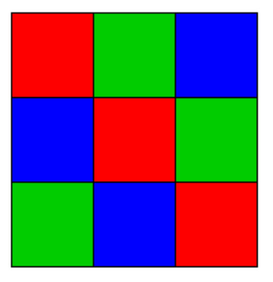

$$\binom{9}{3}\binom{6}{3}\binom{3}{3} = 1680$$

Restrictions on Topology

Example

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Solution 1: Since rotations are considered the same, we may fix the position of one of the friends, and then proceed to arrange the 5 remaining friends clockwise around him. Thus, there are $5! = 120$ ways to arrange the friends.

Solution 2: Consider arranging the 6 friends in a line. There are $6! = 720$ ways to do this. Now, for each arrangement in a line, there are 6 rotations that correspond to the same arrangement around the round table. Thus, the number of distinct arrangements around the round table is $\frac{6!}{6} = 120$.

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Suppose Ellie is choosing a secret passcode consisting of the digits $0, 1, 2, \ldots, k$ for some $k \leq 9$. She would like her passcode to use each digit at most once and because she is concerned about security, she would like to choose a value of k such that the number of possible permutations is at least 250,000 . What is the smallest value of k Ellie can use?

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Solution: Note that $8! = 40320$ and $9! = 362880$. Therefore, Ellie needs at least 9 digits in her passcode. Since the digits start from 0 , the smallest value of k Ellie can choose is $k = 8$.

Example

Example

How many 5-digit numbers without repetition of digits can be formed using the digits $0, 2, 4, 6, 8$?

Example

If Anna has 12 different ornaments and would like to place k of them on a necklace and if Lisa has 13 different ornaments and would like to place k of them on a necklace, for what values of k does Anna have more choices in the possible number of ways to place all of her ornaments?

Example

Example

There are 10 people at a party who shakes hands with each other. If each two of them shake hands with each other, how many handshakes happen at the party?

Example

How many diagonals are there of a 12-sided polygon?

Example

Example

How many triangles can be formed using 10 points in a plane out of which 4 are collinear?

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How many triangles can be formed using 10 points in a plane out of which 4 are collinear? **Answer:**

$$\binom{10}{3} - \binom{4}{3} = 120 - 4 = 116$$

Or,

$$\binom{6}{3} + \binom{4}{2} \times 6 = 20 + 72 = 92$$

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A team of four has to be selected from 6 boys and 4 girls. How many different ways a team can be selected if at least one boy must be there in the team?

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$$\{(BGGG), (BBGG), (BBBG), (BBBB)\}$$

Stratergy

Given a permutation problem, how do we determine which category the problem falls under and which technique should be applied to solve the problem? It may be useful to first ask yourself a few questions:

- Are the objects all distinct?
- How many objects are there in total?
- How many objects are we asked to place into an ordering?
- Are there any restrictions on the orderings?