

# MAT215: Complex Variables And Laplace Transformations

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LECTURE-04

## Example

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^4} \right\}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{12}{4 - 3s} \right\}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{23s - 15}{s^2 + 5} \right\}$$

# First Translation

First Translation Theorem states that:

$$\mathcal{L}^{-1}\{F(s)\} = e^{at}\mathcal{L}^{-1}F(s+a)$$

## Example

- Evaluate

$$\mathcal{L}^{-1}\left\{\frac{6}{(s-2)^6}\right\}$$

- Evaluate

$$\mathcal{L}^{-1}\left\{\frac{5}{(s-3)^2+15}\right\}$$

- Evaluate

$$\mathcal{L}^{-1}\left\{\frac{4s}{(s-3)^2+25}\right\}$$

# First Translation

## Example

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{5s}{s^2 + 2s + 5} \right\}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{6s - 4}{s^2 - 8s - 9} \right\}$$

## Second Translation

Second Translation Theorem states that:

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$$

$$\mathcal{L}^{-1}\{F(s)e^{-as}\} = f(t-a)u(t-a)$$

### Example

- Evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{s^4}e^{-2s}\right\}$$

- Evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-3)e^{-2s}}\right\}$$

- Evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}e^{-\pi s}\right\}$$

## Second Translation

### Example

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{6s - 3}{s^2 + 4} e^{-\pi s} \right\}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{6s - 4}{s^2 - 8s - 9} e^{-\pi s} \right\}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{1}{s(s+1)}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{1}{s(s+1)} e^{-s} \right\}$$

**Hint:**

$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{2s+3}{(s+1)(2s^3+3s^2-3s-2)}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{2s+3}{(s+1)(2s^3+3s^2-3s-2)} \right\}$$

**Hint:**

$$\frac{2s+3}{(s+1)(2s^3+3s^2-3s-2)} = \frac{1/9}{s+1} + \frac{5/18}{s+2} + \frac{1/2}{s-1} + \frac{-16/9}{2s+1}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{4s^2 - 5s}{(s+1)(s-2)^2}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{4s^2 - 5s}{(s+1)(s-2)^2} \right\}$$

**Hint:**

$$\frac{4s^2 - 5s}{(s+1)(s-2)^2} = \frac{1}{s+1} + \frac{3}{s-2} + \frac{2}{(s-2)^2}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{-s}{(s^2+1)(s+1)}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{-s}{(s^2+1)(s+1)} e^{-\pi s} \right\}$$

**Hint:**

$$\frac{-s}{(s^2+1)(s+1)} = \frac{-1/2s - 1/2}{s^2+1} + \frac{1/2}{s+1}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{1}{(s^2 + 1)(s^2 + 4)}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + 1)(s^2 + 4)} \right\}$$

**Hint:**

$$\frac{1}{(s^2 + 1)(s^2 + 4)} = \frac{1/3}{u+1} + \frac{-1/3}{u+4}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{1}{(s^2 + 1)(s^2 + 4)}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + 1)(s^2 + 4)} e^{-2\pi s} \right\}$$

**Hint:**

$$\frac{1}{(s^2 + 1)(s^2 + 4)} = \frac{1/3}{s^2 + 1} + \frac{-1/3}{s^2 + 4}$$

# Using Partial Fraction Decomposition

## Example

- Find the partial fraction decomposition of

$$\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)}$$

- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)} \right\}$$

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- Evaluate

$$\mathcal{L}^{-1} \left\{ \frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)} e^{-3\pi s} \right\}$$

**Hint:**

$$\frac{s^2 + 2s + 3}{(s^2 + 2s + 2)(s^2 + 2s + 5)} = \frac{1/3}{u+2} + \frac{2/3}{u+5}$$