

MAT216: Linear Algebra and Fourier Transformation

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LECTURE-08

Span

Definition

Let S be a nonempty subset of a vector space V . The span of S , denoted by $\text{span}(S)$, is the set containing all linear combinations of vectors in S . For convenience, we define $\text{span}(\emptyset) = \{0\}$.

Example

Does $\left\{ \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ span \mathbb{R}^2 ?

A set of vectors $S = \{v_1, \dots, v_n\} \subset V$ spans the vector space V if every vector in V can be generated using a linear combination of S .

Mathematically, for any arbitrary $\vec{v}_{\text{random}} \in V$ the system,

$$\alpha_1 v_1 + \cdots + \alpha_n v_n = v_{\text{random}}$$

has a solution (unique or many).

Example

Question-1: Determine whether or not the vectors $u = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ span \mathbb{R}^3 .

Question-2: Determine whether the set of vectors S span \mathbb{R}^3 :

$$S = \{(3, 0, 4), (6, 2, -1), (12, 4, -2), (3, 2, -5)\}$$

Basis

Definition

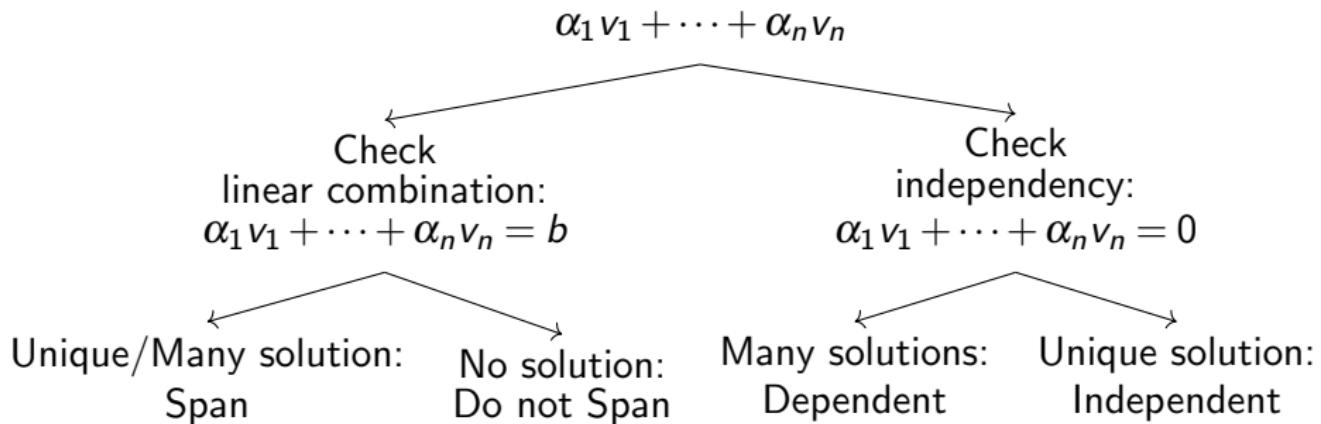
A basis is a set of vectors, S , that generates all elements of the vector space V , and the vectors in the set are linearly independent.

That means to be a basis, we need to satisfy two conditions:

S is linearly independent

S spans V

Summary



Example

Example

Is $S = \{(1, 2), (2, 3)\}$ a basis of \mathbb{R}^2 ?

Hint: Check linear independence and spanning condition.

Example

What about $S = \{(1, 2), (2, 4)\}$?

Example

Determine whether the set of vectors S form a basis of \mathbb{R}^3 :

$$S = \{(2, -3, 1), (4, 1, 1)\}$$

More example

Example

Find the basis of the space $M_{2 \times 2}(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b + c + d = 0 \right\}$

Hint: How many way you can pair 1 and -1 inside the matrix entries?

Example

Find the basis from

$$\{t^3 - 2t^2 + 4t + 1, 2t^3 - 3t^2 + 9t - 1, t^3 + 6t - 5, 2t^3 - 5t^2 + 7t + 5\}.$$