MAT092: Remedial Course in Mathematics

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Lecture on Permutations

Rule of sum and product

Definition

If there are n choices for one action, and m choices for another action and the two actions can't be done at the same time, then there are $n + m$ ways to choose one of the actions.

Definition

If there are n choices for one action, and m choices for another action and the two actions can be done one after another, then there are $n \times m$ ways to choose both actions.

Examples

Example

How many positive divisors does 2000 have?

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$$2000 = 2^4 \times 5^3$$

Any positive divisor of 2000 is of the form $2^a \times 5^b$ where $0 \leq a \leq 4$ and $0 \leq b \leq 3$.

$$0 \leq a \leq 4, 0 \leq b \leq 3$$

So, there are 5 choices for a and 4 choices for b. By the rule of product, the total number of positive divisors of 2000 is $5 \times 4 = 20$.

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$$\binom{5}{2} \times \binom{4}{2} = 10 \times 6 = 60$$

Permutations of a set of Distinct Objects

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$1234, 1243, 1324, 1342, 1423, 1432,$
$2134, 2143, 2314, 2341, 2413, 2431,$
$3124, 3142, 3214, 3241, 3412, 3421,$
$4123, 4132, 4213, 4231, 4312, 4321.$

Example

Out of a class of 30 students, how many ways are there to choose a class president, a secretary, and a treasurer? A student may hold at most one post.

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$$30 \times 29 \times 28 = 24360$$

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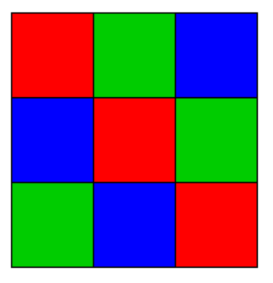
How many different ways are there to color a 3×3 grid with green, red, and blue paints, using each color 3 times?



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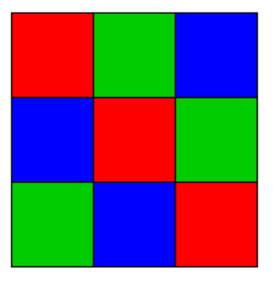

$$\binom{9}{3}\binom{6}{3}\binom{3}{3} = 1680$$

Restrictions on Topology

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Solution 2: Consider arranging the 6 friends in a line. There are $6! = 720$ ways to do this. Now, for each arrangement in a line, there are 6 rotations that correspond to the same arrangement around the round table. Thus, the number of distinct arrangements around the round table is $\frac{6!}{6} = 120$.

Example

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Suppose Ellie is choosing a secret passcode consisting of the digits $0, 1, 2, \ldots, k$ for some $k \leq 9$. She would like her passcode to use each digit at most once and because she is concerned about security, she would like to choose a value of k such that the number of possible permutations is at least 250,000 . What is the smallest value of k Ellie can use?

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Solution: Note that $8! = 40320$ and $9! = 362880$. Therefore, Ellie needs at least 9 digits in her passcode. Since the digits start from 0 , the smallest value of k Ellie can choose is $k = 8$.

Example

Example

How many 5-digit numbers without repetition of digits can be formed using the digits $0, 2, 4, 6, 8$?

Example

If Anna has 12 different ornaments and would like to place k of them on a necklace and if Lisa has 13 different ornaments and would like to place k of them on a necklace, for what values of k does Anna have more choices in the possible number of ways to place all of her ornaments?

Stratergy

Given a permutation problem, how do we determine which category the problem falls under and which technique should be applied to solve the problem? It may be useful to first ask yourself a few questions:

- Are the objects all distinct?
- How many objects are there in total?
- How many objects are we asked to place into an ordering?
- Are there any restrictions on the orderings?