# MAT216: Linear Algebra and Fourier Transformation

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LECTURE-06

# Example

Consider the set $V = \mathbb{R}^2$. A generic element of $\mathbb{R}^2$ is given by the pair $(x, y)$ where $x, y \in \mathbb{R}$ and $k \in \mathbb{R}$. The operations addition and scalar multiplication is defined on $\mathbb{R}^2$ by

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1)$$
$$k(x_1, y_1) = (kx_1, ky_1)$$

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**Check Axiom M3**.

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**Check Axiom M5**. https://math.stackexchange.com/questions/22179/does-the-multiplicative-identity-have-to-be-1

Let $V = \{(0,0,0)\}$. Is $V$ a vector space over $\mathbb{R}$ with respect to the usual operations? Justify your answer.

Let $V = \{(1,1,1)\}$. Is $V$ a vector space over $\mathbb{R}$ with respect to the usual operations? Justify your answer.

# Small buddy! Big catch

## Definition

If $W$ be a non-empty subset of a vector space $V(\mathbb{F})$, then $W$ is called a subspace of $V$ if $W$ satisfies all the axioms of vector space $V$ with respect to vector addition and scalar multiplication. So, formally we can write the laws as:

$W \neq \emptyset$

$W$ is closure under addition, $\forall u, v \in W \implies u + v \in W$

$W$ is closed under scalar multiplication, $\alpha \in \mathbb{F}, u \in W \implies \alpha u \in W$



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Determine whether the following sets are subspace of $\mathbb{R}^3$. i.
$W = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$ ii. $W = \{(x, y, 1) \mid x, y \in \mathbb{R}\}$ iii.
$W = \{(x, y, z) \in \mathbb{R}^3 \mid x \geq 0\}$ iv. $W = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$ v.
$W = \{(x, y, z) \in \mathbb{R}^3 \mid z = x + y + 3\}$

$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\} = M_{2 \times 2}(\mathbb{R})$. Determine whether the

following sets are subspace of V . i. $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a + b + c + d = 0 \right\}$

ii. $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, \text{Det} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 0 \right\}$ iii.

$W = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| \, a, b \in \mathbb{R} \right\}$

# Example

## Problem

*Determine whether the following sets are subspace of $\mathbb{R}^2$ or not.*

$$S = \left\{ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R} \right\}, \ T = \left\{ \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R} \right\},$$

$$U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R}, x \geq 0 \right\}, \ V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x, y \in \mathbb{R}, x \geq y \right\}$$

# Remark

**Remark:** If $S$ and $T$ are two subspaces of a vector space $V(\mathbb{F})$ then $S \cap T$ is a subspace of $V(\mathbb{F})$. But what about $S \cup T$? For example, take $W_1$ to be the $x$-axis and $W_2$ the $y$-axis, both subspaces of $\mathbb{R}^2$. Their union includes both $(3, 0)$ and $(0, 5)$, whose sum, $(3, 5)$, is not in the union. Hence, the union is not a vector space.