MAT216: Linear Algebra and Fourier Transformation

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LECTURE-10

Linear Transformation

Definition

Let U and V be two vector spaces over the same field $\mathbb{F}$. A linear Transformation T of U into V, written as $T : U \to V$, is a function of U into V such that

$$T\left(\overrightarrow{u_1} + \overrightarrow{u_2}\right) = T\left(\overrightarrow{u_1}\right) + T\left(\overrightarrow{u_2}\right) \text{ for all } \overrightarrow{u_1}, \overrightarrow{u_2} \in U$$

$$T(\alpha\vec{u}) = \alpha T(\vec{u}) \text{ for all } \vec{u} \in U \text{ and for all } \alpha \in F$$

Example

Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the transformation defined by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x+y \\ x-y \end{pmatrix}$$

Example

Example

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation defined by

$$T(x,y,z) = (xy, x+y, x+z)$$

Show that T is not a linear transformation.

N.B.: Let V be a vector space over a field $\mathbb{F}$. If the transformation $T : V \to V$, is a linear transformation from V into itself, then T is called a linear operator.

Example

Example

Consider a linear Transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$. Given that,

$$T(1,0) = (-1,1), \; T(0,1) = (2,1)$$

Then find $T(x,y)$.

Matrix Representation

Example

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by

$$T(x, y) = (x - 4y, 3x + 2y)$$

Find the matrix representation with respect to the standard basis.

Example

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x - 3y + z \\ x + y - 4z \\ 2y + 5z \end{pmatrix}$$

Find the matrix representation with respect to the standard basis.

Kernel of a Linear Transformation

Kernel of T = Null space of T

Example

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$$T(x, y, z) = (2x - y + z, x + 2y - z, x + 7y - 4z)$$

Find the Basis and Dimension of Ker(T).

Image of a Linear Transformation

Image of T = Column space of T

Example

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$$T(x,y,z) = (2x - y + z, x + 2y - z, x + 7y - 4z)$$

Find the Basis and Dimension of $\mathrm{Im}(T)$.