



MAT215: Complex Variables & Laplace Transformations

Quiz-04

January 7, 2026

Total - 20 Marks

(You need to answer **All questions**)

Name:

ID:

Section:

- Evaluate the following integral using cauchy integral theorem:

$$\int_{|z|=3} \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$$

(10 Marks)

- Compute

$$\int_C \frac{1}{z} dz$$

Where C is a square centred at the origin with the vertices $\pm 1 \pm i$.

- Compute

$$\int_{|z|=3} \frac{z}{z^2 + 4} dz$$

(10 Marks)

Bonus Question:

- Compute the integral

$$\int_{|z|=1} \frac{e^{z^2}}{z-2} dz$$

(1 Marks)

1 Formulas

Cauchy's Integral Formula:

Let $f(z)$ be analytic inside and on a simple closed curve C and let $z = z_0$ be any point inside C . Then,

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z - z_0)^{n+1}} dz$$

Residue Theorem:

Let $f(z)$ be single-valued and analytic inside and on a simple closed curve C except at the singularities $z_1, z_2, z_3, \dots, z_k$ inside C . Then the residue theorem states that:

$$\oint f(z) dz = 2\pi i \sum_{i=1}^k \operatorname{Re}(z = z_i)$$

where the Residue $\operatorname{Re}(z = z_i)$ can be calculated by,

$$\operatorname{Re}(z = z_i) = \lim_{z \rightarrow z_i} \frac{1}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} \{(z - z_i)^m f(z)\}$$

where $z = z_i$ is the Pole of order m .

Best of Luck!