



Linear Algebra & Fourier Analysis

Quiz-01 (Solution)

1. (a) Solve the linear system (using Gaussian Elimination or Gauss-Jordan method):

$$\begin{cases} x + 3y + 2z = 2 \\ 2x + 7y + 7z = -1 \\ 2x + 5y + 2z = 7 \end{cases}$$

Hint: You can reach row echelon form within 3 steps or reduced row echelon form within 6 steps.

Solution: Subtract twice the first equation from the second one and replace the second equation with the result to get

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 2 & 5 & 2 & 7 \end{array} \right)$$

Subtract twice the first equation from the third one and replace the third equation with the result to get

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & -1 & -2 & 3 \end{array} \right)$$

Adding the last two equations

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & -2 \end{array} \right)$$

So, the system becomes,

$$\begin{cases} x + 3y + 2z = 2 \\ y + 3z = -5 \\ z = -2 \end{cases}$$

Now, using the back substitution we get:

$$x = 3, y = 1, z = -2.$$

- (b) Suppose you have the following augmented matrix and the reduced row echelon form of that matrix respectively. Can you determine under what condition for a, b and c the system is consistent and inconsistent?

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -3 & a \\ 1 & 1 & 3 & 1 & b \\ 1 & 0 & 2 & -1 & c \end{array} \right) \sim \dots \sim \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & c \\ 0 & 1 & 1 & 2 & -a+c \\ 0 & 0 & 0 & 0 & a+b-2c \end{array} \right)$$

Solution: Consistent if $a + b - 2c = 0$ and inconsistent if $a + b - 2c \neq 0$.

- (c) Now, if we consider the columns as the vectors $v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ and $v_4 = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$ in \mathbb{R}^3 . Can $w = \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix}$ be written as a linear combination of v_1, v_2, v_3 and v_4 ?

Hint: Linear combination mean $w = \alpha v_1 + \beta v_2 + \gamma v_3 + \mu v_4$. Insert those vectors in this equation. Did you see something similar to (b)? What can you say if we consider $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix}$.

Solution: The above equation becomes:

$$\begin{aligned} \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix} &= \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix} \\ \begin{pmatrix} \alpha \\ \alpha \\ \alpha \end{pmatrix} + \begin{pmatrix} -\beta \\ \beta \\ 0 \end{pmatrix} + \begin{pmatrix} \gamma \\ 3\gamma \\ 2\gamma \end{pmatrix} + \begin{pmatrix} -3\mu \\ \mu \\ -\mu \end{pmatrix} &= \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix} \\ \begin{pmatrix} \alpha - \beta + \gamma - 3\mu \\ \alpha + \beta + 3\gamma + \mu \\ \alpha + 2\gamma - \mu \end{pmatrix} &= \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix} \\ \underbrace{\begin{pmatrix} 1 & -1 & 1 & -3 \\ 1 & 1 & 3 & 1 \\ 1 & 0 & 2 & -1 \end{pmatrix}}_{Q1(b)} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \mu \end{pmatrix} &= \begin{pmatrix} 13 \\ 3 \\ 8 \end{pmatrix} \end{aligned}$$

We already know when this is consistent by inspecting the right-hand side. Just check our w satisfy the condition, $a + b - 2c = 13 + 3 - 16 = 0$. Hence, we can write w as a linear combination.

(10+2+3 Marks)

2. Using Row Operations to find A^{-1} :

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$

(15 Marks)

Solution:

$$A^{-1} = \begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$$

3. Write **True** if the statement is correct otherwise **False**. A short explanation is needed to justify your answer.

- (a) If a linear system has more unknowns than equations, then it has infinitely many solutions.

True. Because in this case, we always have free variables.

- (b) If the number of equations in a linear system exceeds the number of unknowns, then the system must be inconsistent.

False. Counterexample:

$$\begin{cases} x + y &= 2 \\ x - y &= 1 \\ 2x - 2y &= 2 \end{cases}$$

- (c) The linear system,

$$\begin{cases} x - y &= 3 \\ 2x - 2y &= k \end{cases}$$

can't have a unique solution, regardless of the value of k .

True, because either the lines will be the same or parallel as their slopes are the same. In both cases, we have either infinitely many solutions or no solution.

- (d) The linear system with corresponding augmented matrix

$$\left(\begin{array}{cc|c} 2 & -1 & 4 \\ 0 & 0 & -1 \end{array} \right)$$

is consistent.

False. The last equation give $0x + 0y = -1$ which is invalid.

- (e) If A is an $n \times n$ matrix that is not invertible, then the linear system $A\mathbf{x} = 0$ has infinitely many solution.

True. Not invertible of A implies we have at least one row with zeros in our row echelon form. Which give us free variables.

(10 Marks)

Best of Luck!