



MAT215: Complex Variables & Laplace Transformations

Quiz-03

January 7, 2026

Total - 20 Marks

(You need to answer all questions)

Name:

ID:

Section:

1. Solve the differential equation:

$$y'' + 9y = \sin(2t), \quad y(0) = 1, \quad y\left(\frac{\pi}{2}\right) = -1$$

(10 Marks)

2. Solve the differential equation:

$$y'' + 2y' + 5y = e^{-t} \cos(t), \quad y(0) = 0, \quad y'(0) = 1$$

OR,

Solve the system,

$$\begin{aligned}\frac{dx}{dt} &= 2x + y \\ \frac{dy}{dt} &= -x + 2y\end{aligned}$$

with the initial conditions, $x(0) = 1, y(0) = 0$

(10 Marks)

1. **Laplace Transformation of the function $f(t)$:**

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$$

2. **Laplace Transformation table:** Here s is a complex variable and $Re(s)$ indicates the real part of s .

| $f(t)$ | $\mathcal{L}\{f(t)\} = F(s)$ |
|---------------------------------|---|
| 1 | $\frac{1}{s} \quad Re(s) > 0$ |
| t | $\frac{1}{s^2} \quad Re(s) > 0$ |
| $t^n; \quad n = 0, 1, 2, \dots$ | $\frac{n!}{s^{n+1}} \quad Re(s) > 0$ |
| e^{at} | $\frac{1}{s - a} \quad Re(s) > a$ |
| $\sin(at)$ | $\frac{a}{s^2 + a^2} \quad Re(s) > a $ |
| $\cos(at)$ | $\frac{s}{s^2 + a^2} \quad Re(s) > a $ |
| $\sinh(at)$ | $\frac{a}{s^2 - a^2} \quad Re(s) > a $ |
| $\cosh(at)$ | $\frac{s}{s^2 - a^2} \quad Re(s) > a $ |

3. Laplace Transformations of derivatives:

$$\mathcal{L}\{y^n(t)\} = s^n \mathcal{L}\{y(t)\} - s^{n-1}y(0) - s^{n-2}y'(0) - \dots - sy^{n-2}(0) - y^{n-1}(0)$$

4. First Translation Theorem: If $F(s) = \mathcal{L}\{f(t)\}$, then

- $\mathcal{L}\{e^{at}f(t)\} = F(s - a)$
- $\mathcal{L}^{-1}\{F(s - a)\} = e^{at}f(t) \quad \text{or,} \quad \mathcal{L}^{-1}\{F(s)\} = e^{at}\mathcal{L}^{-1}\{F(s + a)\}$

5. Unit Step Function:

$$u(t - a) = \begin{cases} 0 & t \leq a \\ 1 & t > a \end{cases}$$

6. If $F(s) = \mathcal{L}\{f(t)\}$, then $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} \{F(s)\}$, for $n = 1, 2, 3, \dots$

In remembrance of Sharif Osman Bin Hadi, whose courage and sacrifice will shape our nation.