Introduction

1. Logic : relation of consequence between the premises and the conclusion of sound argument
   1. Sound(an argument): if conclusion is a consequence of its premises ~ correct, valid
   2. Unsound : not Sound
   3. Argument : System of declarative sentences
      1. Conclusion
      2. Premises
      3. Sentences : linguistic expression stating a complete thought
         1. Declarative
         2. Interrogative
         3. Imperative
   4. Criterion for the soundness of an argument : An argument is sound iff it is not possible for its premises to be true and its conclusion false
      1. Necessary truth : a Sentence is necessary truth iff no conceivable circumstances of being false
      2. Conditional, consequent : If (corresponding conditional) then (consequent)
      3. A sentence is a necessary truth iff no unsound argument of which it is the conclusion
   5. Parenthetical remark
      1. If a sentence is known to be true, it is true,
         1. Even if a sentence is true, it could not be known to be true.
      2. Logically possible (a sentence) : the hypothesis it occurs is compatible with the laws of logic
      3. Proposition, statement, thought, judgement
   6. Logical form
      1. If a sentence is necessary truth then all sentences of the same logical form are also necessarily true.
      2. A truth of logic ( a sentence) iff it is a substitution-instance of a matrix all instances of which are necessary truths
         1. Matrix : formal expressions built with logical words together with sentential, class, or individual letters
            1. Logical words : and, or, if…then, not, all, is, etc
            2. Sentential letters : sentence
            3. Class letters : a set of elements
            4. Individual letters : elements
   7. Artificial language (Formalized language) : grammatically simpler, regular than natural language
2. Further preliminaries
   1. Use and mention
      1. Use : name (identifier)
      2. Mention : use the name for reference (object which identifier indicates)
   2. Sense and denotation
      1. Sense (an expression) : meaning
      2. Denotation (an expression) : objects which it refers
      3. Condition of sense and denotation
         1. The denotation (a complex expression) is a function of the denotations of its parts
         2. The sense (a complex expression) is a function of the senses of its parts
      4. Ordinary vs Oblique sense/denotation
         1. Oblique denotation = Ordinary sense
         2. Direct occurrence (an occurrence of a name or description in a expression) : in the context it has ordinary sense
   3. Variables : expression of generalization, must specify substituents
      1. Substituents : a set of values which could substitute a variable
      2. Values : names
   4. Sentence forms
      1. Replacement of a direct occurrence of a sentence with the expression with same ordinary denotation does not change its truth value
      2. Replacement of an indirect occurrence of a sentence with the expression with same ordinary denotation can change its truth value
      3. Sentence form : it is an expression that is a sentence or is obtainable from a sentence by replacing some or all direct occurrences of names by variables
      4. Quantifier : often asserted in sentence forms
         1. Universal quantifier : For every x, ..
         2. Existential quantifier : There is an x such that …
      5. Variables
         1. Free occurrences : needs additional quantifiers or replacements to obtain a sentence
         2. Bound : already has those
   5. Description forms
      1. Replacement of a direct occurrence of a description with the expression with same ordinary denotation does not change its truth value
   6. Set ~ Class
      1. Elements : objects constituting a set ~ members
         1. Sets having same members are identical
      2. Empty set : no members
      3. Universal set : set of all objects satisfying (x is identical with x)
      4. 원소나열법 : notation for any finite number of occurrences of variables, names, or descriptions
      5. A subset of B : every element of a set A is also an element of a set B ~ included in
         1. Φ⊂A
         2. A⊂A
         3. A⊂(Universal set)
         4. If A⊂B and B⊂A, then A = B
         5. If A⊂B and B⊂C, then A⊂C
      6. The Union of A and B : a set of all members belonging to A or to B
      7. The Intersection of A and B : a set of all members belonging to A and to B
      8. The Complement of A : a set of all members not belonging to A
      9. Russell’s Antinomy : K = {x | x is not an element of x} K ∈K or K not∈ K ?
         1. Give up Assumption that all sets can themselves be members of sets
      10. Relation : any set of ordered n-tuples of objects is an n-ary relation
          1. Binary Relation : x R y = <x,y> ∈ R
          2. Domain : a set of all objects x s.t. for some y , x R y
          3. Converse domain : a set of all objects y s.t. for some x, x R y
          4. Field (binary relation) : (Domain) U (Converse domain)
          5. Converse (binary relation R) : for all objects x, y , x R y iff y S x
          6. Function (a binary relation R) : for all objects x, y, z , if x R y and x R z , then y = z
          7. 1-1 relation (binary relation R) : R and its converse are both function
          8. N-ary operation (n+1-ary relation R) : with respect to set D, for each n-tuple <x1, x2, … , 투> of objects in D there uniquely exists an object y in D s.t. <x1,x2,..,투,y> ∈ R
   7. Object-language and Metalanguage
      1. Explain one language by using another, then former is Object-language and latter is Metalanguage (ex. artificial language explained by English)
      2. Metalinguistic variables : variables of the metalanguage, Greek letters
3. Formalized Language L
   1. Grammar of L
      1. Expression of L : finite length strings of symbols
         1. Variables : u~z with/w/o subscripts
            1. Subscript : lower right Arabic numerals
         2. Constants
            1. –, V, (, ), &, ->, <->, ∃
            2. Non-logical constants

Predicates : capital Italic letter

N-ary predicate : superscript n

Superscript : upper right Arabic numerals

Sentential letter : no superscript

Individual constants : a ~ t

* + - 1. Individual symbol : variable or individual constant
      2. Atomic formula : a sentential letter or n-ary predicate with a string of length n of individual symbols
         1. General : a formula not an atomic formula, begins with a universal / existential quantifier
         2. Molecular : a formula not an atomic formula, otherwise
      3. Formula : atomic formula or built up from one or more atomic formulas by a finite number of applications as :
         1. If Φ is a formula, then -Φ is a formula

Negation

* + - * 1. If Φ and Ψ are formulas, then (ΦVΨ) , (Φ&Ψ), (Φ->Ψ), (Φ<->Ψ) are formulas

Disjunction(ΦVΨ), Conjunction(Φ&Ψ), conditional(Φ->Ψ) ( antecedent, consequent), biconditional(Φ<->Ψ)

* + - * 1. If Φ is a formula and (α)Φ and (∃α)Φ are formulas

Universal Quantifier : (α)

Existential Quantifier : (∃α)

Universal generalization (α)Φ, Existential generalization (∃α)Φ

* + - 1. Occurrence of a variable α in formula Φ
         1. Bound : within an occurrence in Φ of a formula of the form (α)Ψ or of the form (∃α)Ψ
         2. Free occurrence
      2. Occurrence of a formula Ψ in formula Φ
         1. Bound : within an occurrence in Φ of a formula of the form (α)θ or (∃α)θ
         2. Free occurrence
      3. Order of a formula
         1. Order 1 : Atomic formulas
         2. Order of Disjunction, Conjunction, conditional, bidirectional is 1 + (Maximum order of two formulas)
         3. Order n formula Φ -> -Φ, (α)Φ (∃α)Φ is order n+1 where α is variable
  1. Notational conventions
     1. Outermost parentheses are occasionally omitted

1. Interpretations and Validity
   1. Interpretations of L : Given a sentence Φ in L assign a denotation to each non-logical constant occurring in Φ
      1. Specify Universe of discourse : non-empty domain D
      2. Assign to each individual constant an element of D
      3. Assign to each n-ary predicate an n-ary relation among element of D
      4. Assign to each sentential letter one of truth values T or F
   2. Truth : Φ is true under I
      1. Let I be any interpretation and Φ be any quantifier-free sentence of L
         1. If Φ is sentential letter, then Φ is true under I iff I assigns T to Φ
         2. If Φ is atomic and not a sentential , then Φ is true under I iff the objects I assigns to the individual constants of Φ are related by the relation that I assigns to the predicate of Φ
         3. If Φ = - Ψ, then Φ is true under I iff Ψ is not true under I
         4. If Φ = (Ψ V χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I or χ is true under I or both
         5. If Φ = (Ψ & χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I and χ is true under I
         6. If Φ = (Ψ -> χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is not true under I or χ is true under I or both
         7. If Φ = (Ψ <-> χ) for sentences Ψ, χ, then Φ is true under I iff either Ψ and χ are both true or both not true under I.
      2. Φ is false under I iff Φ is not true under I
      3. Every quantifier-free sentence is an atomic sentence or is a molecular compound of shorter sentences
      4. Let I and I’ be interpretations of L, and β be an individual constant; the I is β-variant of I’ iff I and I’ are the same or differ only in what they assign to β
      5. Let Φ be any sentence of L, α a variable, and β the first individual constant not occurring in Φ.
         1. If Φ is sentential letter, then Φ is true under I iff I assigns T to Φ
         2. If Φ is atomic and not a sentential , then Φ is true under I iff the objects I assigns to the individual constants of Φ are related by the relation that I assigns to the predicate of Φ
         3. If Φ = - Ψ, then Φ is true under I iff Ψ is not true under I
         4. If Φ = (Ψ V χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I or χ is true under I or both
         5. If Φ = (Ψ & χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I and χ is true under I
         6. If Φ = (Ψ -> χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is not true under I or χ is true under I or both
         7. If Φ = (Ψ <-> χ) for sentences Ψ, χ, then Φ is true under I iff either Ψ and χ are both true or both not true under I.
         8. If Φ = (α)Ψ, then Φ is true under I iff Ψ α/β is true under every β-variant of I
            1. α/β : replace all free occurrences of α by occurrences of some individual constant β
         9. If Φ = (∃α)Ψ, then Φ is true under I iff Ψ α/β is true under at least one β-variant of I
      6. Φ is true/false under I = I assigns the truth-value T/F to Φ
      7. Complete Interpretations : for each element of the domain of I, there is an individual constant to which I assigns that element as denotation
         1. Φ is true under all complete interpretations iff Φ is true under all interpretations
   3. Validity, Consequence, Consistency
      1. Valid (a sentence Φ) : Φ is true under every interpretation
         1. Not valid : a nonempty set D and an assignment of appropriate entities to the non-logical constants of L which makes Φ false exists
      2. Consequence of a set of sentences Γ ( a sentence Φ) : there is no interpretation under which all the sentences of Γ are true and Φ is false
      3. Consistent ( a set of sentences Γ ) : there is an interpretation under which all the sentences of Γ are true
      4. Deduction theorem : For any sentence Φ, Φ is a consequence of the sentences Γ together with a sentence Ψ iff (Ψ -> Φ) is a consequence of Γ alone.
2. Translating Natural Language into L
   1. Introduction
      1. For any Interpretation I,
         1. ‘Ds’ is true under I iff the object I assigns to ‘s’ belongs to the set that I assigns to ‘D’
         2. ‘-Ds’ is true under I iff the object I assigns to ‘s’ does not belong to the set that I assigns to ‘D’
         3. ‘(∃x)Dx’ is true under I iff some element of the domain of I belongs to the set that I assigns to ‘D’.
         4. ‘Ds -> (∃x)Dx’ is true under I iff either the object that I assigns to ‘s’ does not belong to the set that I assigns to ‘D’ or some element of the domain of I belongs to the set that I assigns to ‘D’, or both
   2. Interpretation and Translation
      1. Given the sense, the denotation(if any) is thereby fixed, but converse does not hold.
   3. Translating connectives and quantifiers
      1. Only relative to an interpretation do the expressions of L have denotations
         1. Futile to translate unless an interpretation is given
      2. If an Interpretation is given a standard translation con be obtained.
      3. No simple correspondence between the form of an ordinary sentence and its counterpart in L exists.
      4. Two formal sentences equivalent in L may not be equivalently represented in Natural Language.
3. Tautologous Sentences
   1. Definition of tautology
      1. No atomic sentence is tautologous
      2. Normal assignment
         1. An assignment A of the truth values T and F to all the sentences of L is called normal iff for each sentence Φ of L,
            1. A assigns exactly one of the truth values T/F to Φ
            2. If Φ = -Ψ, then A assigns T to Φ iff A do not assign T to Ψ
            3. If Φ = ( Ψ V χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns T to Ψ or T to χ or both
            4. If Φ = ( Ψ & χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns T to Ψ and T to χ
            5. If Φ = ( Ψ -> χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns F to Ψ or T to χ or both
            6. If Φ = ( Ψ <-> χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns T to both Ψ and χ or assigns F to both
      3. A sentence Φ is tautologous (is a tautology) iff it is assigned the truth value T by every normal assignment of truth values T and F to the sentences of L
         1. Every tautologous sentence is valid
      4. A sentence Φ is a tautological consequence of a set of sentences Γ iff Φ is assigned the truth value T by every normal assignment that assigns the truth value T to all sentences of Γ.
         1. A sentence Φ is a tautological consequence of a set of sentences Γ iff
            1. Γ is empty and Φ is tautologous or
            2. There are sentences Ψ\_1, Ψ\_2, … ,Ψ\_n which belongs to Γ and are such that ((…(Ψ\_1 & Ψ\_2)& … & Ψ\_n) -> Φ) is tautologous.
      5. A set of sentences Γ is truth-functionally consistent iff there is at least one normal assignment that assigns the truth value T to all members of Γ
         1. Truth-functionally inconsistent
   2. Tautologous SC sentences; truth tables
      1. For any sentence Φ, if Φ does not contain any quantifiers, then Φ is tautologous iff Φ is valid.
         1. For any SC sentence Φ, Φ is tautologous iff Φ is valid.
         2. For any sentence Φ, Φ is tautologous iff there is a tautologous SC sentence Ψ s.t. Φ is substitution instance of Ψ.
            1. Substitution instance of SC sentence Ψ ( sentence Φ) : result of replacing sentential letters of Ψ by sentences.
      2. Truth table method
         1. A Truth value normal assignment gives to an SC sentence is determined by truth values it gives to occurring sentential letters
         2. Suppose an SC sentence Φ containing n distinct sentential letters, there are 2^n different ways truth values T and F can be assigned to these n letters.
   3. Deciding whether sentences are tautologous
      1. Basic truth-functional component of a sentence Ψ( a sentence Φ) : Φ is atomic or general and occurs free in Ψ at least once.
      2. Associated with a sentence Ψ (a SC sentence Φ) : Φ is obtained from Ψ by putting occurrences of sentential letters for all free occurrences in Ψ of its basic truth-functional components
      3. For any sentences Φ, Ψ , if Ψ is an SC sentence associated with Φ, then Φ is tautologous iff Ψ is tautologous.
         1. Construct an SC sentence Ψ associated with Φ.
         2. By a truth-table, test Ψ for tautologousness.
         3. Decide whether Φ is tautologous.
   4. Rules of derivation for SC sentences
      1. SC derivation : finite sequence of consecutively numbered lines, each consisting of an SC sentence together with list of numbers. ~ proof
         1. P (Introduction of premises) Any SC sentence may be entered on a line with the line number taken as the only premise-number
         2. MP (Modus Ponens) Ψ may be entered on a line if Φ and (Φ -> Ψ) appear on earlier lines; as premise-numbers of the new line take all premise-numbers of those earlier lines.
         3. MT (Modus Tollens) Φ may be entered on a line if Ψ and (-Φ -> -Ψ) appear on earlier lines ; as premise-numbers of the new line take all premise-numbers of those earlier lines.
         4. C (Conditionalization) (Φ ->Ψ) may be entered on a line if Ψ appear on earlier lines ; as premise-numbers of the new line take all premise-numbers of those earlier lines. With the exception of any that is the line number of a line on which Φ appears.
         5. D (Definitional interchange) if Ψ is obtained from Φ by replacing an occurrence of a sentence χ in Φ by an occurrence of a sentence to which χ is definitionally equivalent, then Ψ may be entered on a line if Φ appears on an earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
            1. (Φ V Ψ) is definitionally equivalent to (-Φ->-Ψ)
            2. (Φ & Ψ) is definitionally equivalent to -(Φ->-Ψ)
            3. (Φ <-> Ψ) is definitionally equivalent to ((Φ->Ψ)&(Ψ->Φ))
      2. SC derivation of Φ from Γ : an SC sentence Φ appears on the last line and premises belongs to a set of SC sentences Γ
         1. SC derivable from a set of SC sentences of Γ (an SC sentence Φ) : exists SC derivation of Φ from Γ
         2. SC Theorem ( an SC sentence Φ) : Φ is SC derivable from (empty set of sentences)
      3. Any SC sentence that is a substitution instance of a previously proved SC theorem may enter on a line of proof with the empty set of premise-numbers.
         1. Ψ may enter on a line if Φ\_1, Φ\_2, ..,Φ\_n appear on earlier line and the conditional (Φ\_1 -> (Φ\_2 -> … -> (Φ\_n -> Ψ)..)) is a substitution instance of already proved SC theorem.
4. Rules of Inference for L
   1. Basic rules; derivability
      1. Impossibility of a decision procedure for validity
      2. Impossibility of step-by-step procedure for deciding whether given sentence is a consequence of other sentences
      3. Derivation : finite sequence of consecutively numbered lines with premise-numbers of the line
         1. P (Introduction of premises) : Any sentence may be entered on a line with the line number taken as the only premise-number
         2. T (Tautological inference) : Any sentence may enter on a line if it is a tautological consequence of a set of sentences that appear on earlier lines; as premise-numbers of the new line take all premise-numbers of those earlier lines
         3. C (Conditionalization) : (Φ ->Ψ) may be entered on a line if Ψ appear on earlier lines ; as premise-numbers of the new line take all premise-numbers of those earlier lines. With the exception of any that is the line number of a line on which Φ appears.
         4. US (Universal specification) The sentence Φ α/β may enter on a line if (α)Φ appears on an earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
         5. UG (Universal generalization) The sentence (α)Φ may enter on a line if Φ α/β appears on an earlier line and β occurs neither in Φ nor in any premise of that earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
         6. E (Existential quantification) The sentence (∃α)Φ may enter on a line if –(α)-Φ appears on an earlier line or vice versa; as premise-numbers of the new line take all premise-numbers of those earlier lines
      4. Derivation of Φ from Γ : a derivation in which a sentence Φ appears on the last line and all premises of that line belong to a set of sentences Γ
         1. derivable from the set of sentences Γ ( a sentence Φ ) : there is a derivation of Φ from Γ
   2. The short-cut rules EG, ES and Q
      1. EG (Existential generalization) : The sentence (∃α)Φ may enter on a line if Φ α/β appears on an earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
      2. ES (Existential specification) : Suppose (∃α)Ψ appears on line I of a derivation, that Φ α/β appears as a premise on a later line j, and that Φ appears on a still later line k; and suppose further that the constant β occurs neither in Φ,Ψ, nor in any premise of line k other than Φ α/β ; than may enter on a new line. As premise-numbers of the new line take all those of lines I and k, except the number j.
      3. Q (Quantifier exchange) The sentence –(∃α)Φ may enter on a line if (α)-Φ appears on earlier line, or vice versa; similarly for the pairs {(∃α)-Φ, -(α)Φ },{–(∃α)-Φ ,(α)Φ },{–(∃α)Φ , -(α)-Φ }; as premise-numbers of the new line take all premise-numbers of those earlier lines.
   3. Theorems of logic
      1. A Theorem of logic (a sentence Φ) : Φ is derivable from the empty set of sentences ~ theorem
         1. A sentence Φ is derivable from a set of sentences Γ iff Φ is consequence of Γ
         2. Φ is a theorem of logic iff Φ is valid.
5. Some Metatheorems
   1. Replacement, negation and duality, prenex normal form
      1. Φ^{α\_1, α\_2, …,α\_n}\_{β\_1,β\_2, …,β\_n} : For any formula Φ, result of replacing all free occurrences of distinct variables (α\_1, α\_2, …,α\_n) by occurrences of individual symbols (β\_1,β\_2, …,β\_n) respectively.
      2. String of quantifiers : a quantifier | result of prefixing a quantifier to a shorter string of quantifiers
         1. A string of universal quantifiers : a universal quantifier | result of prefixing a universal quantifier to a shorter string of quantifiers
         2. A string of existential quantifiers : an existential quantifier | result of prefixing an existential quantifier to a shorter string of quantifiers
      3. Closure of formula Ψ (a formula Φ) : Φ is sentence and Φ = Ψ or Φ is the result of prefixing a string of universal quantifiers to Ψ
      4. ㅐㅡΦ : every closure of Φ is a theorem of logic
      5. ( I )If Φ^{α\_1, α\_2, …,α\_n}\_{β\_1,β\_2, …,β\_n} is a theorem of logic (where β\_1,β\_2, …,β\_n are distinct individual constants not occurring in Φ) , thenㅐㅡΦ
         1. Pf) UG
      6. ( II )If one closure of Φ is a theorem of logic, then ㅐㅡΦ
         1. Pf) US
      7. ( III )If ㅐㅡΦ and ㅐㅡΦ -> Ψ, then ㅐㅡΨ
         1. Pf) US
      8. ( IV )Generalization
         1. ㅐㅡ(α)(α’)Φ <-> (α)(α’)Φ
         2. ㅐㅡ(∃α)(∃α’)Φ <-> (∃α)(∃α’)Φ
         3. ㅐㅡ(α)(Φ & Ψ) <-> (Φ & (α)Ψ) if α does not occur free in Φ
         4. ㅐㅡ(α)(∃α’)(Φ & Ψ) <-> ((α)Φ & (∃α’)Ψ) if α does not occur free in Φ
      9. ( V ) If ㅐㅡΨ<->Ψ’, then
         1. ㅐㅡ-Ψ<->-Ψ
         2. ㅐㅡ(Ψ & χ) <-> (Ψ’ & χ)
         3. ㅐㅡ( χ & Ψ ) <-> ( χ & Ψ’ )
         4. om ( Ψ V χ ) <-> ( Ψ’ V χ )
         5. om ( χ V Ψ ) <-> ( χ V Ψ’ )
         6. om (Ψ -> χ) <-> (Ψ’->χ)
         7. om (χ -> Ψ) <-> (χ ->Ψ’)
         8. om (Ψ <-> χ) <-> (Ψ’ <-> χ)
         9. om (χ <-> Ψ) <-> ( χ <-> Ψ’)
         10. om (α)Ψ <-> (α)Ψ’
         11. om (∃α)Ψ <-> (∃α)Ψ
             1. pf) I, III, IV
      10. (Replacement)
          1. Suppose that Φ’ is like Φ except for containing an occurrence of Ψ’ where Φ contains an occurrence of Ψ, and suppose that omΨ<->Ψ’. Then omΦ<->Φ’, and omΦ iff omΦ’
             1. Pf) V, I, III
      11. (Definition of equivalence) : for any formulas Φ and Ψ, Φ is equivalent to Ψ iff omΦ<->Ψ
      12. (Rewriting of bound variables) if formulas (α)Φ and (α’)Φ’ are alike except that former has occurrences of α where and only where the latter has occurrences of α’, then they are equivalent. Similarly for (∃α)Φ and (∃α’)Φ’.
          1. Pf) I
      13. (Negation theorem) Suppose that a formula Φ contains no occurrences of -> and <-> and that Φ’ is obtained from Φ by exchanging & and v, exchanging universal and the corresponding existential quantifiers, and by repacing atomic formulas by their negations. Then Φ’ is equivalent to -Φ.
      14. (Definition of Prenex normal form) ( a formula Φ ) : Φ is either quantifier-free or consists of a string of quantifiers followed by a quantifier-free formula.
      15. (Prenex normal form) : For any formula Φ there is an equivalent formula Ψ that is in prenex normal form.
          1. Pf) reduction to prenex normal form, (Replacement), (Negation theorem), IV
          2. If a sentence Φ has a prenex normal form in the prefix of which no existential quantifier precedes any universal quantifier, then the validity of Φ is decidable.
      16. (X) if omΦ and if Ψ is the result of replacing all atomic formulas in Φ by their negations, then omΨ
      17. (Duality) if omΦ<->Ψ and neither -> nor <-> occurs in Φ and Ψ, and if Φ\* and Ψ\* are obtained from Φ and Ψ, respectively, by exchanging & and v, and universal and the corresponding existential quantifiers, then omΦ\*<->Ψ\* ; similarly if omΦ<->Ψ, then omΨ\* -> Φ\*
          1. Pf) (Replacement), (Negation theorem), (X)
   2. Soundness and consistency
      1. Sound ( a system of inference rules) : any conclusion derived by their use will be a consequence of the premises from which it is obtained
         1. Assertion
            1. Any sentence appearing on the first line of a derivation is a consequence of the premises of that line
            2. Any sentence appearing on the latter line is a consequence of its premises if all sentences appearing on earlier lines are consequence of theirs
         2. Any sentence appearing on a line of a derivation is a consequence of the premises of that line
         3. If a sentence Φ is derivable from a set of sentences Γ, then Φ is a consequence of Γ.
      2. Consistent ( a system of inference rules ) : there is no sentence Φ such that both Φ and -Φ are derivable from (Universal set)
         1. Transform T(Φ) (a sentence Φ) : SC sentence which is the result of deleting all individual symbols, quantifiers, and predicate superscripts from Φ
   3. Completeness
      1. Complete ( a system of inference rules ) : One can derive, from any given set of sentences, any consequence of that set
         1. Consistent with respect to derivability ( a set of sentences Γ ) : the sentence ‘P&-P’ is not derivable from Γ ~ d-consistent
            1. For any sentence Φ and set of sentences Γ, Φ is derivable form Γ iff ΓU{-Φ} is not d-consistent
            2. For any set of sentences Γ, Γ is d-consistent iff at least one sentence Φ from L is not derivable from Γ
         2. Maximal d-consistent (a set of sentences Γ) iff Γ is d-consistent and not properly included in any d-consistent set Δ.
            1. Φ€Δ iff -Φ !∈ Δ
            2. Φ∈Δ iff Φ is derivable from Δ
            3. (Φ v Ψ) ∈ Δ iff Φ ∈ Δ or Ψ ∈ Δ
            4. (Φ & Ψ) ∈ Δ iff Φ ∈ Δ and Ψ ∈ Δ
            5. (Φ -> Ψ) ∈ Δ iff Φ !∈ Δ or Ψ ∈ Δ, or both
            6. (Φ <-> Ψ) ∈ Δ iff Φ, Ψ ∈ Δ or Φ, Ψ !∈ Δ
         3. ω-complete ( a set of sentences Γ ) : for every formula Φ and variable α, if (∃α)Φ belongs to Γ, then there is an individual constant β s.t. Φ α/β also belongs to Γ.
         4. Suppose Δ is maximal d-consistent and ω-complete, then
            1. (α)Φ ∈ Δ iff for every individual constant β, Φ α/β ∈ Δ
            2. (∃α)Φ ∈ Δ iff for some individual constant β, Φ α/β ∈ Δ
      2. Principle lemma for completeness
         1. ( I ) for any set of sentences Γ, if Γ is d-consistent, then it is consistent.
            1. ( I’ ) for any set of sentences Γ, if Γ is d-consistent and all indices of individual constants occurring in the sentences of Γ are even, then it is consistent.
            2. ( II ) For any set of sentences Γ, if Γ satisfies the hypothesis of I’, then there is a set of sentences Δ that includes Γ and is maximal d-consistent and ω-complete.

Assume that all the sentences of L can be arranged in an infinite list Φ\_1, Φ\_2, … Φ\_n with following properties.

Each sentence of L occurs at least one in the list

For each sentence of the form (∃α)Φ, there is at least one I such that Φ\_i = (∃α)Φ and Φ\_(i+1) = Φ α/β, where β is a ‘new’ individual constant

* + - * 1. ( III ) every maximal d-consistent, ω-complete set of sentences is consistent.

Corollary : every maximal d-consistent ,ω-complete set of sentences is satisfiable by an interpretation having a domain equinumerous with the positive integers. ( denumerably infinite domain)

* + - * 1. Corollary of ( I ) : For any set of sentences Γ, if Γ is d-consistent, then it is satisfiable by an interpretation having a denumerably infinite domain.
        2. (Löwenheim-Skolem theorem) : if Γ is a consistent set of sentences, then Γ is satisfiable by an interpretation having a denumerably infinite domain.
  1. A proof procedure for valid sentences
     1. If a given sentence is valid, then there exists procedures which generates a proof of that sentence.
        1. Given an arbitrary valid sentence Φ, we can reduce Φ to a sentence Ψ in prenex normal form.
        2. Procedure ( a valid sentence Φ in prenex normal form ) :
           1. Enter -Φ on the first line as a premise.
           2. Derive a prenex normal form Ψ from -Φ.
           3. Construct a sequence of lines satisfying the following two conditions, until a truth-functional inconsistency appears:

Whenever any universal generalization (α)θ appears on a line, particular instances θ α/β (for all individual constants β occurring in the sequence, and in any case for at least one individual constant β) shall appear on later lines, inferred by US.

Whenever any existential generalization (∃α)θ appears on a line, θ α/β (for some new individual constant β ) shall appear as a premise on a later line.

* + - * 1. When a truth-functional inconsistency appears, derive ‘P&-P’ by rule T, apply ES to transfer dependence to -Φ, conditionate to obtain the theorem -Φ -> (P&-P), and apply T to obtain the theorem Φ.

1. Identity and Terms
   1. Identity; the language L\_{1}
      1. Leibniz’s Law : if two things are identical, then whatever is true of the one is true of the other.
      2. I^{2}\_{1} : a logical constant that stands for the relation of identity in the domain of the relevant interpretation
      3. L\_{1} : first order predicate calculus with identity
         1. Interpretation for the language L\_{1} : nonempty domain D together with an assignment that associates with each individual constant of L\_{1} an element of D, with each n-ary predicate other than I^{2}\_{1} an n-ary relation among elements of D, with the predicate I^{2}\_{1} the identity relation among elements of D, and with each sentential letter of L\_{1} one of truth values T or F.
            1. Every Interpretation of L\_{1} is an interpretation of L, but not for converse.
         2. Derivation in L\_{1} : finite sequence of consecutively numbered lines each consisting of a sentence of L\_{1} together with a set of premise-numbers.
            1. Rules : P,T,C,US,UG,E and..
            2. The sentence β = β may be entered on a line, with the empty set of premise-numbers
            3. If a sentence Φ is like a sentence Ψ except that β and γ have been exchanged at one or more places, then Φ may be entered on a line if Ψ and β = γ appear on earlier lines; as premise-numbers of the new line take those of the earlier lines.
         3. Derivation of Φ from Γ : a sentence Φ appears on the last line and all premises of that line belong to a set of sentences Γ
         4. Derivable from a set of sentences Γ (a sentence Φ) : there is a derivation of Φ from Γ.
         5. Φ is derivable from Γ iff Φ is a consequence of Γ.
         6. Class : Division of all the individual constants of L\_{1} s.t. constants α, β are assigned to the same class iff the sentence α = β is in Δ.
            1. [(metalinguistic variable)] = the class to which (metalinguistic variable) belongs
         7. 8.( III )
         8. Completeness of the predicate calculus with identity
         9. (Löwenheim-Skolem theorem of L\_{1}) : if Γ is a consistent set of sets of L\_{1}, then Γ is satisfiable by an interpretation having a finite or denumerably infinite domain.
         10. A theorem of Logic (a sentence Φ) : Φ is derivable from the empty set of sentences.
   2. Parenthetical remark
      1. Frege’s solution about identity : The truth of an identity sentence requires only that the two terms have same denotation
      2. Objectionable feature
         1. The identity relation among the element of one domain will be different from that among the elements of another
         2. Identity relation cannot be a member of themselves.
   3. Terms ; the Language L’
      1. Operation symbol : new kind of symbol ~ functor, function sign
      2. L’ : first order predicate calculus with identity and operation symbols
         1. Result of adding operation symbols to L\_{1}.
         2. Symbol classification
            1. Variables
            2. Constants

Logical constants U { I^{2}\_{1} }

Non-logical constants

Predicates – { I^{2}\_{1} }

Operation symbols : lower-case italic letters ‘a’ through ‘t’ with or without numerical subscripts and superscripts

Individual constant : operation symbol without superscript

N-ary operation symbol : operation symbol having as superscript a numeral for the positive integer n.

* + - 1. N-ary predicate, sentential letter, individual symbol, formula ,sentence, bound/free occurrences are equally defined
      2. Term : individual symbol or built up from individual and operation symbol by a finite number of following rules
         1. If τ\_{1}, τ\_{2}, …, τ\_{n} are terms and θ is an operation symbol of degree n, then θτ\_{1}τ\_{2}…τ\_{n} is a term.
      3. Atomic formula : expression either a sentential letter or the form πτ\_{1}τ\_{2}…τ\_{n}, where π is n-ary predicate and τ\_{1},τ\_{2},…,τ\_{n} are terms.
    1. Interpretation of L’ : non-empty domain D together with an assignment that associates
       1. Each individual constant of L’ an element of D
       2. Each n-ary operation symbol an n-ary operation with respect to D
       3. Each sentential letter of L’ one of truth values T or F
       4. Each n-ary predicate an n-ary relation among elements of D
       5. The binary predicate I^{2}\_{1} the identity relation among elements of D
    2. the value under an interpretation (a constant term τ)
       1. if τ is an individual constant, the value of τ under I is the element of D which I assign to τ
       2. if τ = θτ\_{1}τ\_{2}…τ\_{n}, where τ\_{1}, τ\_{2}, …, τ\_{n} are terms and θ is an operation symbol of degree n, then the value of the function I (θ)
    3. To define ‘True under I’ we need only one change the definition given for L and L\_{1}
       1. If Φ is atomic and not a sentential letter, the Φ is true under I iff the values under I of the (constant) terms of Φ are related by the relation that I assigns to the predicate of Φ.
    4. Notion valid, consequence, consistet, normal assignment, tautologous, tautological consequence, truth-functionally consistent, Derivation, derivation of Φ from Γ, derivable are all same.
    5. Amended US, I from inference rules. It is complete.

1. Axioms for L\_{1}
   1. Introduction
      1. Axioms : easily recognizable valid sentences
      2. (Modus Ponens) : A sentence Φ follows from sentences Ψ and χ by modus ponens iff χ = (Ψ -> Φ)
      3. Definitionally equivalent to a formula Ψ ( a formula Φ ) : formulas χ, θ, Φ\_{1}, Φ\_{2} s.t. Φ and Ψ are alike except that one contains an occurrence of χ at some place where the other contains an occurrence of θ, and either
         1. Χ = (Φ\_{1} v Φ\_{2}) and θ = (-Φ\_{1} -> Φ\_{2})
         2. Χ = (Φ\_{1} & Φ\_{2}) and θ = -(Φ\_{1} -> -Φ\_{2})
         3. Χ = (Φ\_{1} <-> Φ\_{2}) and θ = ((Φ\_{1}-> Φ\_{2}) & (Φ\_{2}-> Φ\_{1}))
         4. Χ = (∃α)Φ\_{1} and θ = -(α)-Φ\_{1}
      4. Proof : a finite sequence of sentences, each of which is an axiom or is definitionally equivalent to an earlier sentence of the sequence or follows from earlier sentences by modus ponens.
      5. Theorem (a sentence Φ) : Φ is the last line of a proof.
         1. ㅏ Φ : all closures of Φ are theorems.
      6. Notation : Φ, Ψ, χ are formulas. α is variables. β γ are individual symbols. P, Q, R, T are expressions.
      7. If Φ and Φ ->Ψ are theorems, then Ψ is a theorem.
      8. If Φ is a sentence, Φ->Φ is a theorem.
      9. If Φ, Ψ, χ are sentences, (Ψ -> χ)-> ((Φ->Ψ) ->(Φ->χ)) is a theorem.
      10. If Φ->Ψ and Ψ->χ are theorems, then Φ->χ is a theorem.
      11. (Mathematical induction) : All the positive integers have some given property P ?
          1. (weak induction)
             1. 1 has the property P
             2. For every positive integer k, if k has the property P, then k+1 has the property P
          2. (Strong induction)
             1. For each positive integer k, if all positive integers less than k have the property P, then k has the property P.
      12. Q(Φ->Ψ) ->(QΦ->QΨ) is a theorem, if Q is a string of universal quantifiers containing every variable that occurs free in Φ or Ψ.
      13. If QΦ and Q(Φ->Ψ) are theorems, then QΨ is a theorem, where Q is a string of universal quantifiers.
      14. ㅏ(Ψ->χ)->((Φ->Ψ)->(Φ->χ))
      15. If Q(Φ->Ψ) and Q(Ψ->χ) are theorems, then Q(Φ->χ) is a theorem.
      16. ㅏQ(Φ->Ψ) ->(QΦ->QΨ)
      17. ㅏQΦ->Φ
      18. ㅏΦ->QΦ, if no variable in Q occurs free in Φ.
      19. ㅏQΦ->PΦ, if every variable in P is either in Q or does not occur free in Φ.
      20. If Ψ and χ are closures of Φ, then Ψ->χ is a theorem, and χ is a theorem if Ψ is a theorem.
      21. If ㅏ Φ and ㅏΦ->Ψ, then ㅏΨ.
      22. If ㅏΦ and Ψ is definitionally equivalent to Φ, then ㅏΨ.
      23. If ㅏΦ->Ψ and ㅏΨ->χ, then ㅏΦ->χ
      24. ㅏΦ iff ㅏQΦ->QΨ
      25. If ㅏΦ->Ψ, then ㅏQΦ->QΨ
   2. Sentential calculus (look book ,full of one-liner theorems)
      1. (Replacement) : If ㅏΦ<->Ψ and χ is like θ except for having an occurrence of Φ at some place where θ has an occurrence of Ψ, then ㅏχ<->θ and ㅏχ iff ㅏθ.
   3. Quantification theory (look book, full of one-liner theorems)
   4. Identity, further quantification theory, and substitution
      1. (Substitution) If Φ is a theorem and Ψ is a substitution-instance of Φ, then Ψ is a theorem.
         1. Stencil : an expression either a sentence of L\_{1} or is obtainable from such a sentence by putting the counters ①, ②, ③, .. where that sentence contains occurrences of individual constants.
         2. Substitute σ for θ in Φ ( σ : stencil of degree n, θ : n-ary predicate other than I^{2}\_{1}, Φ : a sentence in L\_{1} ) : for each occurrence of θδ\_{1}δ\_{2}..δ\_{n} in Φ where δ\_{1},δ\_{2},..,δ\_{n} are individual symbols, put σ(δ\_{1},δ\_{2},..,δ\_{n})
         3. Legitimate ( a substitution) : If in this substitution we have at all places obeyed the rule that no variable shall occur bound at any place in any substituted σ(δ\_{1},δ\_{2},..,δ\_{n}) unless it already occurs bound at that place in σ
         4. A substitution-instance of a sentence Φ ( a sentence Ψ) : Ψ can be obtained from Φ by a legitimate substitution.
   5. Proofs from assumptions
      1. Proof of the sentence Φ from the assumptions Γ (a set of sentences Γ) : Φ is the last term of the sequence and each term of the sequence is either an axiom or one of the assumptions Γ or follows from earlier terms by modus ponens or definitional interchange.
      2. ΓokΦ : there is a proof of Φ from the assumptions Γ
      3. (empty set) ok Φ iff Φ is a theorem
      4. If Φ ∈ Γ, then Γ ok Φ
      5. If Γ ok Φ and Γ ⊂ Δ, then Δ ok Φ
      6. If Γ ok Φ and Δ ok Φ -> Ψ, then Γ U Δ ok Ψ
      7. (Deduction Theorem) Γ U { Φ } ok Ψ iff Γ ok Φ -> Ψ
      8. Γ ok Φ iff Γ U {-Φ} ok ‘P&-P’
      9. Γ ok Φ iff for every sentence Ψ, Γ U { - Φ } ok Ψ
      10. Γ ok Φ iff Φ is a consequence of Γ.
2. Formalized Theory
   1. Elementary theory : a theory formalized by means of elementary logic
   2. Introduction
      1. Deductive theory T formalized in the first order predicate calculus : a pair of sets T = <Δ,Γ> where Δ is a set of non-logical constants of L containing at least one predicate of degree ≥ 1 and Γ is a set of sentences of L satisfying the conditions
         1. All non-logical constants occurring in members of Γ are members of Δ
         2. Deductively closed : Every sentence of T that is a consequence of Γ is a member of Γ
      2. Non-logical Vocabulary of T = <Δ,Γ> : Δ
      3. Assertions, Theses of T = <Δ,Γ> : Γ
         1. Deductive theory is uniquely determined by its non-logical vocabulary and its assertions.
      4. Deductive theory T formalized in the first order predicate calculus with identity ( or with identity and operation symbols) : Similar, but now referring to L\_{1} (or L’)
      5. Consistent ( a theory T) : no sentence and its negation are both asserted by T
      6. Complete ( a theory T) : For any sentence of T, either that sentence or its negation is asserted by T.
         1. Every inconsistent theory is automatically complete
      7. Independent (a set of sentences Γ) : no element Φ of Γ ~ {Φ} (special notation ~)
      8. Decidable ( a set of sentences Γ) : there is a step-by-step procedure which, applied to an arbitrary expression Φ, will decide in a finite number of steps whether or not Φ belongs to Γ.
      9. Axiomatizable ( a theory T) : among the assertions of a given theory T there is a decidable subset ① of which all the others are consequences
         1. Set of axioms of T : ①
         2. Axiomatic theory : T
      10. Heteronomous system : an axiom system proposed next and judged first on the basis of whether or not the resulting theorems coincide with the assertions
      11. Autonomous : axiom system proposed first and the theses of the theory are defined as those sentences of the theory following from the axioms
      12. Evaluation of axiom system
          1. Consistency
             1. Semantically : give an interpretation under which all the axioms are true
             2. Syntactically : without reference to interpretations, there is no sentence Φ s.t. both Φ and -Φ are derivable from the axioms
          2. Complete : theory from an axiom system is complete
          3. Independence : give each axiom an interpretation under which it is false but the remaining axioms are true
          4. Categorical : all its models are isomorphic
             1. Any consistent axiom system has infinitely many models.
             2. (Isomorphic model) Given a set of sentences Γ of L’, interpretation I\_{1} and I\_{2} are isomorphic models of Γ iff

I\_{1} and I\_{2} are models of Γ

There is a 1-1 relation between the domains of I\_{1} and I\_{2} associating with each element e of the domain of I\_{1} exactly one element e\* of the domain of I\_{2}

For each sentential letter Φ occurring in Γ, I\_{1}(Φ) = I\_{2}(Φ)

For each individual constant β occurring in Γ, I\_{2}(β) = I\_{1}(β)\*

For each n-ary predicate θ occurring in Γ, the n-tuple <t\_{1}, t\_{2}, …, t\_{n}> ∈ I\_{1}(θ) iff <t\_{1}\*, t\_{2}\*, .., t\_{n}\*> ∈ I\_{2}(θ), for all elements t\_{1}, t\_{2}, .. , t\_{n} of the domain of I\_{1}

For each n-ary operation θ occurring in Γ, the n-tuple <t\_{1}, t\_{2}, …, t\_{n}, η> ∈ I\_{1}(θ) iff <t\_{1}\*, t\_{2}\*, .., t\_{n}\*, η\*> ∈ I\_{2}(θ), for all elements t\_{1}, t\_{2}, .. , t\_{n}, η of the domain of I\_{1}

* + - * 1. If there are models with different cardinality, the theory is not categorical.
        2. Representation theorem : While not all models of the theory of groups are isomorphic to one another, every such model is isomorphic to a group of transformations.
  1. Aristotelian syllogistic
     1. T1
        1. Non-logical vocabulary of T1 : binary predicates A^{2}, E^{2}, I^{2}, O^{2}
        2. Axioms of assertions
           1. (x)(y)(z)((A\_{yz}&A\_{xy})->A\_{xz}) (Barbara)
           2. (x)(y)(z)((E\_{yz}&A\_{xy})->E\_{xz}) (Celarent)
           3. (x)(y)(I\_{xy} -> I\_{yx}) (Conversion of ‘I’)
           4. (x)(y)(E\_{xy} -> E\_{yx}) (Conversion of ‘E’)
           5. (x)(y)(A\_{xy} -> I\_{yx}) (Conversion of ‘A’)
           6. (x)(y)(E\_{xy} -> -I\_{yx}) (Definition of ‘E’)
           7. (x)(y)(O\_{xy} -> -A\_{yx}) (Definition of ‘O’)
        3. Evaluation of T1
           1. T1 is consistent
           2. T1 is not complete
           3. T1 is not independent
           4. T1 is not categorical
        4. TH : Brevity convention
           1. Any instance of an axiom or previously proved theorem may be entered on a line, with the empty set of premise numbers
           2. Ψ may enter the line if Φ\_{1}, …, Φ\_{n} appear on earlier lines and Ψ is a tautological consequence of { Φ\_{1}, …, Φ\_{n} } and an instance of an axiom or previously proved theorem; as premise numbers of the new line take all premise-numbers of those earlier lines.
  2. The theory of betweenness
     1. T2 : axiomatized theory formulated in the first order predicate calculus with identity
        1. Non-logical vocabulary : B^{3} (ternary)
        2. Assertions : sentences that are true under every interpretation I satisfying :
           1. The domain of I is the set of all points on some (Euclidean) straight line
           2. I assigns to B^{3} the relation of betweenness “② is between ① and ③” Among points on this straight line.
        3. Evaluation of T2
           1. T2 is consistent
           2. T2 is complete
        4. Axioms of assertions
           1. (x)(y)(Bxyx -> x = y)
           2. (x)(y)(z)(u)((Bxyu & Byzu) -> Bxyz)
           3. (x)(y)(z)(u)(((Bxyz & Byzu) & y != z) -> Bxyu)
           4. (x)(y)(z)((Bxyz v Bxzy ) b (Bzxy)
           5. (∃x)(∃y)x != y
           6. (x)(y) (x != y -> (∃z)(Bxyz & y != z))
           7. (x)(y)(x != y -> (∃z)(Bxzy & (z != x & z != y)))
  3. Groups; Boolean algebras
     1. T3 : Theory of groups formalized in the language L’
        1. Non-logical vocabulary of T3 :
           1. (Group addition) binary operation symbol f^{2}
           2. (Group complementation) : singulary operation symbol f^{1}
           3. (identity element) : individual constant e
        2. Assertions
           1. Conventions : Suppose τ, τ’ are terms or the result of previous application of these conventions

(τ + τ’) <- f^{2}ττ’

Τ\* for f^{1}τ

* + - * 1. Axioms of assertions

(x)(y)(z) x + (y + z) = (x + y) + z

(x) x + e = x

(x) x + x\* = e

* + - 1. Evaluation of T3
         1. T3 is consistent
         2. T3 is not complete
         3. T3 is independent
         4. T3 is not categorical
    1. T4 : Boolean algebras : first order predicate calculus with identity and operation symbols
       1. Non-logical vocabulary
          1. F^{1}, f^{2}, g^{2}, n, e
          2. Convention

(τ U τ’) <- f^{2}ττ’

(τ ∩ τ’) <- g^{2}ττ’

Τ\* <- f^{1}τ

0 <- n

1 <- e

* + - 1. Axioms for assertions
         1. (x)(y) x ∩ y = y ∩ x
         2. (x)(y) x U y = y U x
         3. (x)(y)(z) x ∩ ( y ∩ z ) = (x ∩ y) ∩ z
         4. (x)(y)(z) x U ( y U z ) = (x U y) U z
         5. (x)(y) x ∩ (x U y ) = x
         6. (x)(y) x U (x ∩ y ) = x
         7. (x)(y)(z) x ∩ ( y U z ) = (x ∩ y) U (x ∩ z)
         8. (x)(y)(z) x U ( y ∩ z ) = (x U y) ∩ (x U z)
         9. (x) x ∩ x\* = 0
         10. (x) x U x\* = 1
         11. 0 != 1
      2. Evaluation of T4
         1. T4 is consistent
         2. T4 is not complete
      3. Dual of a theorem is a theorem
         1. Dual : interchange of ∩ and U in a sentence of T4
  1. Definition
     1. To introduce notation that does not belong to the vocabulary of the theory but may improve readability of the formulas and render their content more clear
        1. Metalinguistic sentence asserting that a given symbol of the object language means the same as some other object language expression
        2. As an object language sentence of a certain form (usually an identity or biconditional or a generalized identity or biconditional, with the new symbol appearing on the left side alone or in a simple context)
     2. Definition can lead to contradiction not just by its forms but also by the content of the theory
     3. Criteria of Definition
        1. Creative ( a definition ) : Generates new theorems in which the defined symbol does not occur
        2. Definition should make the defined symbol eliminable
           1. Resistance to circular definitions
        3. Let T = (Δ,Γ) be theory. Let θ be a non-logical constant that does not belong to Δ. Let Φ be a sentence of L’ containing θ and otherwise formulated solely in terms of the vocabulary Δ. If we add Φ to the theses of T we get new theory T’ = (Δ U {θ} , {all sentences of T’ that are consequences of Γ U {Φ}}).
           1. As a definition of θ related to T, Satisfies the criterion of eliminability ( a sentence Φ) : for every formula Ψ of T’, there is a formula χ of T’ s.t. all closures of Ψ<->χ are theses of T’ and χ does not contain θ.
           2. As a definition of θ related to T, Satisfies the criterion of non-creativity ( a sentence Φ) : every thesis of T’ that does not contain θ is a thesis of T
     4. Construction of formally correct definitions
        1. Let T be a theory formulated in L’, and suppose θ is a non-logical constant which does not occur in T.
        2. If the constant θ is an n-ary predicate, Let definition be a sentence of form (α\_{1})..(α\_{n})(θα\_{1}…α\_{n} <-> ω), where ω is a formula of T and α\_{1}…α\_{n} are distinct and are all the variables occurring free in ω.
        3. If the constant θ is a sentential letter, Let definition be a sentence θ <-> ω where ω is a sentence of T.
        4. If the constant θ is an individual constant, Let definition be a sentence (α)(α = θ <-> ω) where ω is a formula of T and having α as its only free variable, and the corresponding sentence (∃β)(α)(α = β <-> ω) with β a variable distinct from α, is a thesis of T.
        5. If the constant θ is an n-ary operation symbol,, Let definition be a sentence of form (γ\_{1})..(γ\_{n})(α)(α = θγ\_{1}…γ\_{n} <-> ω), where α, γ\_{1}…α\_{n} are distinct and are all the variables occurring free in ω. Θ does not occur in ω, and the corresponding uniqueness condition (γ\_{1})..(γ\_{n})(∃β)(α)(α = β<-> ω) with β distinct from α, γ\_{1}, …, γ\_{n} is a thesis of T.
     5. Definability
        1. Definable in the theory T ( a non-logical constant θ) : there is among the theses of T a formally correct definition of θ relative to the theory T – θ
           1. T-θ : from the non-logical vocabulary of a theory T remove a constant θ, and from the theses of T we remove all those containing θ
        2. Padoa’s principle : Non-logical constant θ is not definable in a theory T If we can give two models of T that differ in what they assign to θ but are otherwise the same.