Introduction

1. Logic : relation of consequence between the premises and the conclusion of sound argument
   1. Sound(an argument): if conclusion is a consequence of its premises ~ correct, valid
   2. Unsound : not Sound
   3. Argument : System of declarative sentences
      1. Conclusion
      2. Premises
      3. Sentences : linguistic expression stating a complete thought
         1. Declarative
         2. Interrogative
         3. Imperative
   4. Criterion for the soundness of an argument : An argument is sound iff it is not possible for its premises to be true and its conclusion false
      1. Necessary truth : a Sentence is necessary truth iff no conceivable circumstances of being false
      2. Conditional, consequent : If (corresponding conditional) then (consequent)
      3. A sentence is a necessary truth iff no unsound argument of which it is the conclusion
   5. Parenthetical remark
      1. If a sentence is known to be true, it is true,
         1. Even if a sentence is true, it could not be known to be true.
      2. Logically possible (a sentence) : the hypothesis it occurs is compatible with the laws of logic
      3. Proposition, statement, thought, judgement
   6. Logical form
      1. If a sentence is necessary truth then all sentences of the same logical form are also necessarily true.
      2. A truth of logic ( a sentence) iff it is a substitution-instance of a matrix all instances of which are necessary truths
         1. Matrix : formal expressions built with logical words together with sentential, class, or individual letters
            1. Logical words : and, or, if…then, not, all, is, etc
            2. Sentential letters : sentence
            3. Class letters : a set of elements
            4. Individual letters : elements
   7. Artificial language (Formalized language) : grammatically simpler, regular than natural language
2. Further preliminaries
   1. Use and mention
      1. Use : name (identifier)
      2. Mention : use the name for reference (object which identifier indicates)
   2. Sense and denotation
      1. Sense (an expression) : meaning
      2. Denotation (an expression) : objects which it refers
      3. Condition of sense and denotation
         1. The denotation (a complex expression) is a function of the denotations of its parts
         2. The sense (a complex expression) is a function of the senses of its parts
      4. Ordinary vs Oblique sense/denotation
         1. Oblique denotation = Ordinary sense
         2. Direct occurrence (an occurrence of a name or description in a expression) : in the context it has ordinary sense
   3. Variables : expression of generalization, must specify substituents
      1. Substituents : a set of values which could substitute a variable
      2. Values : names
   4. Sentence forms
      1. Replacement of a direct occurrence of a sentence with the expression with same ordinary denotation does not change its truth value
      2. Replacement of an indirect occurrence of a sentence with the expression with same ordinary denotation can change its truth value
      3. Sentence form : it is an expression that is a sentence or is obtainable from a sentence by replacing some or all direct occurrences of names by variables
      4. Quantifier : often asserted in sentence forms
         1. Universal quantifier : For every x, ..
         2. Existential quantifier : There is an x such that …
      5. Variables
         1. Free occurrences : needs additional quantifiers or replacements to obtain a sentence
         2. Bound : already has those
   5. Description forms
      1. Replacement of a direct occurrence of a description with the expression with same ordinary denotation does not change its truth value
   6. Set ~ Class
      1. Elements : objects constituting a set ~ members
         1. Sets having same members are identical
      2. Empty set : no members
      3. Universal set : set of all objects satisfying (x is identical with x)
      4. 원소나열법 : notation for any finite number of occurrences of variables, names, or descriptions
      5. A subset of B : every element of a set A is also an element of a set B ~ included in
         1. Φ⊂A
         2. A⊂A
         3. A⊂(Universal set)
         4. If A⊂B and B⊂A, then A = B
         5. If A⊂B and B⊂C, then A⊂C
      6. The Union of A and B : a set of all members belonging to A or to B
      7. The Intersection of A and B : a set of all members belonging to A and to B
      8. The Complement of A : a set of all members not belonging to A
      9. Russell’s Antinomy : K = {x | x is not an element of x} K ∈K or K not∈ K ?
         1. Give up Assumption that all sets can themselves be members of sets
      10. Relation : any set of ordered n-tuples of objects is an n-ary relation
          1. Binary Relation : x R y = <x,y> ∈ R
          2. Domain : a set of all objects x s.t. for some y , x R y
          3. Converse domain : a set of all objects y s.t. for some x, x R y
          4. Field (binary relation) : (Domain) U (Converse domain)
          5. Converse (binary relation R) : for all objects x, y , x R y iff y S x
          6. Function (a binary relation R) : for all objects x, y, z , if x R y and x R z , then y = z
          7. 1-1 relation (binary relation R) : R and its converse are both function
          8. N-ary operation (n+1-ary relation R) : with respect to set D, for each n-tuple <x1, x2, … , 투> of objects in D there uniquely exists an object y in D s.t. <x1,x2,..,투,y> ∈ R
   7. Object-language and Metalanguage
      1. Explain one language by using another, then former is Object-language and latter is Metalanguage (ex. artificial language explained by English)
      2. Metalinguistic variables : variables of the metalanguage, Greek letters
3. Formalized Language L
   1. Grammar of L
      1. Expression of L : finite length strings of symbols
         1. Variables : u~z with/w/o subscripts
            1. Subscript : lower right Arabic numerals
         2. Constants
            1. –, V, (, ), &, ->, <->, ∃
            2. Non-logical constants

Predicates : capital Italic letter

N-ary predicate : superscript n

Superscript : upper right Arabic numerals

Sentential letter : no superscript

Individual constants : a ~ t

* + - 1. Individual symbol : variable or individual constant
      2. Atomic formula : a sentential letter or n-ary predicate with a string of length n of individual symbols
         1. General : a formula not an atomic formula, begins with a universal / existential quantifier
         2. Molecular : a formula not an atomic formula, otherwise
      3. Formula : atomic formula or built up from one or more atomic formulas by a finite number of applications as :
         1. If Φ is a formula, then -Φ is a formula

Negation

* + - * 1. If Φ and Ψ are formulas, then (ΦVΨ) , (Φ&Ψ), (Φ->Ψ), (Φ<->Ψ) are formulas

Disjunction(ΦVΨ), Conjunction(Φ&Ψ), conditional(Φ->Ψ) ( antecedent, consequent), biconditional(Φ<->Ψ)

* + - * 1. If Φ is a formula and (α)Φ and (∃α)Φ are formulas

Universal Quantifier : (α)

Existential Quantifier : (∃α)

Universal generalization (α)Φ, Existential generalization (∃α)Φ

* + - 1. Occurrence of a variable α in formula Φ
         1. Bound : within an occurrence in Φ of a formula of the form (α)Ψ or of the form (∃α)Ψ
         2. Free occurrence
      2. Occurrence of a formula Ψ in formula Φ
         1. Bound : within an occurrence in Φ of a formula of the form (α)θ or (∃α)θ
         2. Free occurrence
      3. Order of a formula
         1. Order 1 : Atomic formulas
         2. Order of Disjunction, Conjunction, conditional, bidirectional is 1 + (Maximum order of two formulas)
         3. Order n formula Φ -> -Φ, (α)Φ (∃α)Φ is order n+1 where α is variable
  1. Notational conventions
     1. Outermost parentheses are occasionally omitted

1. Interpretations and Validity
   1. Interpretations of L : Given a sentence Φ in L assign a denotation to each non-logical constant occurring in Φ
      1. Specify Universe of discourse : non-empty domain D
      2. Assign to each individual constant an element of D
      3. Assign to each n-ary predicate an n-ary relation among element of D
      4. Assign to each sentential letter one of truth values T or F
   2. Truth : Φ is true under I
      1. Let I be any interpretation and Φ be any quantifier-free sentence of L
         1. If Φ is sentential letter, then Φ is true under I iff I assigns T to Φ
         2. If Φ is atomic and not a sentential , then Φ is true under I iff the objects I assigns to the individual constants of Φ are related by the relation that I assigns to the predicate of Φ
         3. If Φ = - Ψ, then Φ is true under I iff Ψ is not true under I
         4. If Φ = (Ψ V χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I or χ is true under I or both
         5. If Φ = (Ψ & χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I and χ is true under I
         6. If Φ = (Ψ -> χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is not true under I or χ is true under I or both
         7. If Φ = (Ψ <-> χ) for sentences Ψ, χ, then Φ is true under I iff either Ψ and χ are both true or both not true under I.
      2. Φ is false under I iff Φ is not true under I
      3. Every quantifier-free sentence is an atomic sentence or is a molecular compound of shorter sentences
      4. Let I and I’ be interpretations of L, and β be an individual constant; the I is β-variant of I’ iff I and I’ are the same or differ only in what they assign to β
      5. Let Φ be any sentence of L, α a variable, and β the first individual constant not occurring in Φ.
         1. If Φ is sentential letter, then Φ is true under I iff I assigns T to Φ
         2. If Φ is atomic and not a sentential , then Φ is true under I iff the objects I assigns to the individual constants of Φ are related by the relation that I assigns to the predicate of Φ
         3. If Φ = - Ψ, then Φ is true under I iff Ψ is not true under I
         4. If Φ = (Ψ V χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I or χ is true under I or both
         5. If Φ = (Ψ & χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is true under I and χ is true under I
         6. If Φ = (Ψ -> χ) for sentences Ψ, χ, then Φ is true under I iff Ψ is not true under I or χ is true under I or both
         7. If Φ = (Ψ <-> χ) for sentences Ψ, χ, then Φ is true under I iff either Ψ and χ are both true or both not true under I.
         8. If Φ = (α)Ψ, then Φ is true under I iff Ψ α/β is true under every β-variant of I
            1. α/β : replace all free occurrences of α by occurrences of some individual constant β
         9. If Φ = (∃α)Ψ, then Φ is true under I iff Ψ α/β is true under at least one β-variant of I
      6. Φ is true/false under I = I assigns the truth-value T/F to Φ
      7. Complete Interpretations : for each element of the domain of I, there is an individual constant to which I assigns that element as denotation
         1. Φ is true under all complete interpretations iff Φ is true under all interpretations
   3. Validity, Consequence, Consistency
      1. Valid (a sentence Φ) : Φ is true under every interpretation
         1. Not valid : a nonempty set D and an assignment of appropriate entities to the non-logical constants of L which makes Φ false exists
      2. Consequence of a set of sentences Γ ( a sentence Φ) : there is no interpretation under which all the sentences of Γ are true and Φ is false
      3. Consistent ( a set of sentences Γ ) : there is an interpretation under which all the sentences of Γ are true
      4. Deduction theorem : For any sentence Φ, Φ is a consequence of the sentences Γ together with a sentence Ψ iff (Ψ -> Φ) is a consequence of Γ alone.
2. Translating Natural Language into L
   1. Introduction
      1. For any Interpretation I,
         1. ‘Ds’ is true under I iff the object I assigns to ‘s’ belongs to the set that I assigns to ‘D’
         2. ‘-Ds’ is true under I iff the object I assigns to ‘s’ does not belong to the set that I assigns to ‘D’
         3. ‘(∃x)Dx’ is true under I iff some element of the domain of I belongs to the set that I assigns to ‘D’.
         4. ‘Ds -> (∃x)Dx’ is true under I iff either the object that I assigns to ‘s’ does not belong to the set that I assigns to ‘D’ or some element of the domain of I belongs to the set that I assigns to ‘D’, or both
   2. Interpretation and Translation
      1. Given the sense, the denotation(if any) is thereby fixed, but converse does not hold.
   3. Translating connectives and quantifiers
      1. Only relative to an interpretation do the expressions of L have denotations
         1. Futile to translate unless an interpretation is given
      2. If an Interpretation is given a standard translation con be obtained.
      3. No simple correspondence between the form of an ordinary sentence and its counterpart in L exists.
      4. Two formal sentences equivalent in L may not be equivalently represented in Natural Language.
3. Tautologous Sentences
   1. Definition of tautology
      1. No atomic sentence is tautologous
      2. Normal assignment
         1. An assignment A of the truth values T and F to all the sentences of L is called normal iff for each sentence Φ of L,
            1. A assigns exactly one of the truth values T/F to Φ
            2. If Φ = -Ψ, then A assigns T to Φ iff A do not assign T to Ψ
            3. If Φ = ( Ψ V χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns T to Ψ or T to χ or both
            4. If Φ = ( Ψ & χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns T to Ψ and T to χ
            5. If Φ = ( Ψ -> χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns F to Ψ or T to χ or both
            6. If Φ = ( Ψ <-> χ ) for sentences Ψ,χ then A assigns T to Φ iff A assigns T to both Ψ and χ or assigns F to both
      3. A sentence Φ is tautologous (is a tautology) iff it is assigned the truth value T by every normal assignment of truth values T and F to the sentences of L
         1. Every tautologous sentence is valid
      4. A sentence Φ is a tautological consequence of a set of sentences Γ iff Φ is assigned the truth value T by every normal assignment that assigns the truth value T to all sentences of Γ.
         1. A sentence Φ is a tautological consequence of a set of sentences Γ iff
            1. Γ is empty and Φ is tautologous or
            2. There are sentences Ψ\_1, Ψ\_2, … ,Ψ\_n which belongs to Γ and are such that ((…(Ψ\_1 & Ψ\_2)& … & Ψ\_n) -> Φ) is tautologous.
      5. A set of sentences Γ is truth-functionally consistent iff there is at least one normal assignment that assigns the truth value T to all members of Γ
         1. Truth-functionally inconsistent
   2. Tautologous SC sentences; truth tables
      1. For any sentence Φ, if Φ does not contain any quantifiers, then Φ is tautologous iff Φ is valid.
         1. For any SC sentence Φ, Φ is tautologous iff Φ is valid.
         2. For any sentence Φ, Φ is tautologous iff there is a tautologous SC sentence Ψ s.t. Φ is substitution instance of Ψ.
            1. Substitution instance of SC sentence Ψ ( sentence Φ) : result of replacing sentential letters of Ψ by sentences.
      2. Truth table method
         1. A Truth value normal assignment gives to an SC sentence is determined by truth values it gives to occurring sentential letters
         2. Suppose an SC sentence Φ containing n distinct sentential letters, there are 2^n different ways truth values T and F can be assigned to these n letters.
   3. Deciding whether sentences are tautologous
      1. Basic truth-functional component of a sentence Ψ( a sentence Φ) : Φ is atomic or general and occurs free in Ψ at least once.
      2. Associated with a sentence Ψ (a SC sentence Φ) : Φ is obtained from Ψ by putting occurrences of sentential letters for all free occurrences in Ψ of its basic truth-functional components
      3. For any sentences Φ, Ψ , if Ψ is an SC sentence associated with Φ, then Φ is tautologous iff Ψ is tautologous.
         1. Construct an SC sentence Ψ associated with Φ.
         2. By a truth-table, test Ψ for tautologousness.
         3. Decide whether Φ is tautologous.
   4. Rules of derivation for SC sentences
      1. SC derivation : finite sequence of consecutively numbered lines, each consisting of an SC sentence together with list of numbers. ~ proof
         1. P (Introduction of premises) Any SC sentence may be entered on a line with the line number taken as the only premise-number
         2. MP (Modus Ponens) Ψ may be entered on a line if Φ and (Φ -> Ψ) appear on earlier lines; as premise-numbers of the new line take all premise-numbers of those earlier lines.
         3. MT (Modus Tollens) Φ may be entered on a line if Ψ and (-Φ -> -Ψ) appear on earlier lines ; as premise-numbers of the new line take all premise-numbers of those earlier lines.
         4. C (Conditionalization) (Φ ->Ψ) may be entered on a line if Ψ appear on earlier lines ; as premise-numbers of the new line take all premise-numbers of those earlier lines. With the exception of any that is the line number of a line on which Φ appears.
         5. D (Definitional interchange) if Ψ is obtained from Φ by replacing an occurrence of a sentence χ in Φ by an occurrence of a sentence to which χ is definitionally equivalent, then Ψ may be entered on a line if Φ appears on an earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
            1. (Φ V Ψ) is definitionally equivalent to (-Φ->-Ψ)
            2. (Φ & Ψ) is definitionally equivalent to -(Φ->-Ψ)
            3. (Φ <-> Ψ) is definitionally equivalent to ((Φ->Ψ)&(Ψ->Φ))
      2. SC derivation of Φ from Γ : an SC sentence Φ appears on the last line and premises belongs to a set of SC sentences Γ
         1. SC derivable from a set of SC sentences of Γ (an SC sentence Φ) : exists SC derivation of Φ from Γ
         2. SC Theorem ( an SC sentence Φ) : Φ is SC derivable from (empty set of sentences)
      3. Any SC sentence that is a substitution instance of a previously proved SC theorem may enter on a line of proof with the empty set of premise-numbers.
         1. Ψ may enter on a line if Φ\_1, Φ\_2, ..,Φ\_n appear on earlier line and the conditional (Φ\_1 -> (Φ\_2 -> … -> (Φ\_n -> Ψ)..)) is a substitution instance of already proved SC theorem.
4. Rules of Inference for L
   1. Basic rules; derivability
      1. Impossibility of a decision procedure for validity
      2. Impossibility of step-by-step procedure for deciding whether given sentence is a consequence of other sentences
      3. Derivation : finite sequence of consecutively numbered lines with premise-numbers of the line
         1. P (Introduction of premises) : Any sentence may be entered on a line with the line number taken as the only premise-number
         2. T (Tautological inference) : Any sentence may enter on a line if it is a tautological consequence of a set of sentences that appear on earlier lines; as premise-numbers of the new line take all premise-numbers of those earlier lines
         3. C (Conditionalization) : (Φ ->Ψ) may be entered on a line if Ψ appear on earlier lines ; as premise-numbers of the new line take all premise-numbers of those earlier lines. With the exception of any that is the line number of a line on which Φ appears.
         4. US (Universal specification) The sentence Φ α/β may enter on a line if (α)Φ appears on an earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
         5. UG (Universal generalization) The sentence (α)Φ may enter on a line if Φ α/β appears on an earlier line and β occurs neither in Φ nor in any premise of that earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
         6. E (Existential quantification) The sentence (∃α)Φ may enter on a line if –(α)-Φ appears on an earlier line or vice versa; as premise-numbers of the new line take all premise-numbers of those earlier lines
      4. Derivation of Φ from Γ : a derivation in which a sentence Φ appears on the last line and all premises of that line belong to a set of sentences Γ
         1. derivable from the set of sentences Γ ( a sentence Φ ) : there is a derivation of Φ from Γ
   2. The short-cut rules EG, ES and Q
      1. EG (Existential generalization) : The sentence (∃α)Φ may enter on a line if Φ α/β appears on an earlier line; as premise-numbers of the new line take all premise-numbers of those earlier lines
      2. ES (Existential specification) : Suppose (∃α)Ψ appears on line I of a derivation, that Φ α/β appears as a premise on a later line j, and that Φ appears on a still later line k; and suppose further that the constant β occurs neither in Φ,Ψ, nor in any premise of line k other than Φ α/β ; than may enter on a new line. As premise-numbers of the new line take all those of lines I and k, except the number j.
      3. Q (Quantifier exchange) The sentence –(∃α)Φ may enter on a line if (α)-Φ appears on earlier line, or vice versa; similarly for the pairs {(∃α)-Φ, -(α)Φ },{–(∃α)-Φ ,(α)Φ },{–(∃α)Φ , -(α)-Φ }; as premise-numbers of the new line take all premise-numbers of those earlier lines.
   3. Theorems of logic
      1. A Theorem of logic (a sentence Φ) : Φ is derivable from the empty set of sentences ~ theorem
         1. A sentence Φ is derivable from a set of sentences Γ iff Φ is consequence of Γ
         2. Φ is a theorem of logic iff Φ is valid.
5. Some Metatheorems
   1. Replacement, negation and duality, prenex normal form
      1. Φ^{α\_1, α\_2, …,α\_n}\_{β\_1,β\_2, …,β\_n} : For any formula Φ, result of replacing all free occurrences of distinct variables (α\_1, α\_2, …,α\_n) by occurrences of individual symbols (β\_1,β\_2, …,β\_n) respectively.
      2. String of quantifiers : a quantifier | result of prefixing a quantifier to a shorter string of quantifiers
         1. A string of universal quantifiers : a universal quantifier | result of prefixing a universal quantifier to a shorter string of quantifiers
         2. A string of existential quantifiers : an existential quantifier | result of prefixing an existential quantifier to a shorter string of quantifiers
      3. Closure of formula Ψ (a formula Φ) : Φ is sentence and Φ = Ψ or Φ is the result of prefixing a string of universal quantifiers to Ψ
      4. ㅐㅡΦ : every closure of Φ is a theorem of logic
      5. ( I )If Φ^{α\_1, α\_2, …,α\_n}\_{β\_1,β\_2, …,β\_n} is a theorem of logic (where β\_1,β\_2, …,β\_n are distinct individual constants not occurring in Φ) , thenㅐㅡΦ
         1. Pf) UG
      6. ( II )If one closure of Φ is a theorem of logic, then ㅐㅡΦ
         1. Pf) US
      7. ( III )If ㅐㅡΦ and ㅐㅡΦ -> Ψ, then ㅐㅡΨ
         1. Pf) US
      8. ( IV )Generalization
         1. ㅐㅡ(α)(α’)Φ <-> (α)(α’)Φ
         2. ㅐㅡ(∃α)(∃α’)Φ <-> (∃α)(∃α’)Φ
         3. ㅐㅡ(α)(Φ & Ψ) <-> (Φ & (α)Ψ) if α does not occur free in Φ
         4. ㅐㅡ(α)(∃α’)(Φ & Ψ) <-> ((α)Φ & (∃α’)Ψ) if α does not occur free in Φ
      9. ( V ) If ㅐㅡΨ<->Ψ’, then
         1. ㅐㅡ-Ψ<->-Ψ
         2. ㅐㅡ(Ψ & χ) <-> (Ψ’ & χ)
         3. ㅐㅡ( χ & Ψ ) <-> ( χ & Ψ’ )
         4. om ( Ψ V χ ) <-> ( Ψ’ V χ )
         5. om ( χ V Ψ ) <-> ( χ V Ψ’ )
         6. om (Ψ -> χ) <-> (Ψ’->χ)
         7. om (χ -> Ψ) <-> (χ ->Ψ’)
         8. om (Ψ <-> χ) <-> (Ψ’ <-> χ)
         9. om (χ <-> Ψ) <-> ( χ <-> Ψ’)
         10. om (α)Ψ <-> (α)Ψ’
         11. om (∃α)Ψ <-> (∃α)Ψ
             1. pf) I, III, IV
      10. (Replacement)
          1. Suppose that Φ’ is like Φ except for containing an occurrence of Ψ’ where Φ contains an occurrence of Ψ, and suppose that omΨ<->Ψ’. Then omΦ<->Φ’, and omΦ iff omΦ’
             1. Pf) V, I, III
      11. (Definition of equivalence) : for any formulas Φ and Ψ, Φ is equivalent to Ψ iff omΦ<->Ψ
      12. (Rewriting of bound variables) if formulas (α)Φ and (α’)Φ’ are alike except that former has occurrences of α where and only where the latter has occurrences of α’, then they are equivalent. Similarly for (∃α)Φ and (∃α’)Φ’.
          1. Pf) I
      13. (Negation theorem) Suppose that a formula Φ contains no occurrences of -> and <-> and that Φ’ is obtained from Φ by exchanging & and v, exchanging universal and the corresponding existential quantifiers, and by repacing atomic formulas by their negations. Then Φ’ is equivalent to -Φ.
      14. (Definition of Prenex normal form) ( a formula Φ ) : Φ is either quantifier-free or consists of a string of quantifiers followed by a quantifier-free formula.
      15. (Prenex normal form) : For any formula Φ there is an equivalent formula Ψ that is in prenex normal form.
          1. Pf) reduction to prenex normal form, (Replacement), (Negation theorem), IV
          2. If a sentence Φ has a prenex normal form in the prefix of which no existential quantifier precedes any universal quantifier, then the validity of Φ is decidable.
      16. (X) if omΦ and if Ψ is the result of replacing all atomic formulas in Φ by their negations, then omΨ
      17. (Duality) if omΦ<->Ψ and neither -> nor <-> occurs in Φ and Ψ, and if Φ\* and Ψ\* are obtained from Φ and Ψ, respectively, by exchanging & and v, and universal and the corresponding existential quantifiers, then omΦ\*<->Ψ\* ; similarly if omΦ<->Ψ, then omΨ\* -> Φ\*
          1. Pf) (Replacement), (Negation theorem), (X)
   2. Soundness and consistency
      1. Sound ( a system of inference rules) : any conclusion derived by their use will be a consequence of the premises from which it is obtained
         1. Assertion
            1. Any sentence appearing on the first line of a derivation is a consequence of the premises of that line
            2. Any sentence appearing on the latter line is a consequence of its premises if all sentences appearing on earlier lines are consequence of theirs
         2. Any sentence appearing on a line of a derivation is a consequence of the premises of that line
         3. If a sentence Φ is derivable from a set of sentences Γ, then Φ is a consequence of Γ.
      2. Consistent ( a system of inference rules ) : there is no sentence Φ such that both Φ and -Φ are derivable from (Universal set)
         1. Transform T(Φ) (a sentence Φ) : SC sentence which is the result of deleting all individual symbols, quantifiers, and predicate superscripts from Φ