Elijah M.
CS 401
HW3

3.1

1) We cannot use the Intermediate Value theorem because the function is discontinuous at 0

2)
a) We cannot use bisection search because the function is always positive

b) Yes we can use Newton's Method because if we pick a X_0 not equal to 0 it will converge after a single iteration

3.3

1) If the relative condition number is large then the function is ill conditioned

2) This would be the fault of the algorithm

3)
a) $K(X) = \left| \frac{X F'(X)}{F(X)} \right|$

$F(X) = (1 + \frac{x}{n})^n = I_n(x)$

$F'(X) = (1 + \frac{x}{n})^{n-1}$

$$\boxed{= \left| \frac{X(1 + \frac{x}{n})^{n-1}}{(1 + \frac{x}{n})^n} \right|}$$

For X = .05 this would be well conditioned.

(3.3)

c) $K(x) = \left| \frac{x z^{n-1}}{z^n} \right|$, $\left| \frac{(.05) z^{10^{15}-1}}{z^{10^{15}}} \right|$

It computed 1 when $n = 10^{15}$
b/c it converges toward it

d)

3.2

a) $\frac{(x^2+4)}{5} = 0$

$\Rightarrow x^2+4 = 0 \equiv x^2 = -4$

$x = \sqrt{-4}$???

NO fixed points

b) Lets say $x_0 = 1$:

$x_{k+1} = \frac{(x_0^2+4)}{5} = \frac{1+4}{5} = 1$ $\quad 0<1<2$

$x_0 = 0$:

$x_1 = \frac{(0+4)}{5} = \frac{4}{5}$ $\quad 0<\frac{4}{5}<2$

$x_0 = 2$:

$x_1 = \frac{(4+4)}{5} = \frac{8}{5}$ $\quad 0<\frac{8}{5}<2$

Yes it converges to a fixed point in the interval [0,2] when x_0 exists in [0,2]

(3.2) (Application Question)

1) $F(x) = x + rx\left(1 - \frac{x}{K}\right)$, $r > 0$, $K > 0$

$|F(z) - F(z')| \leq L \cdot |z - z'|$

$F'(z) \geq L$ $\quad\quad F'(x) = r - \frac{2rx}{K} + 1$

$\Rightarrow F'(x) < 1 = r - \frac{2rx}{K} + 1 < 1$

$= \frac{-2rx}{K} < -r = 2rx > rK$

$= x > \frac{rK}{2r} \Rightarrow \boxed{x > \frac{K}{2}}$

2) $x \leq K$, show $F(x) \geq x$

$x + rx\left(1 - \frac{x}{K}\right) \geq x$

$x \leq K \Rightarrow K + rK\left(1 - \frac{K}{K}\right) \geq K$

$\Rightarrow K + rK(0) \geq K \Rightarrow K \geq x,\ F(x) \geq x$

3) $x > K$, show $F(x) < x$

$x + rx\left(1 - \frac{x}{K}\right) < x$, $x > K$, so lets say $x = 2K$

$\Rightarrow 2K + 2rK\left(1 - \frac{2K}{K}\right) < K$

$\Rightarrow 2K + 2rK(1 - 2) < K$

$\Rightarrow 2K - 2Kr < K$

since $r > 0$, $2K - 2Kr < 0$ and since $K > 0$
$2K - 2Kr$ is some negative number

thus, $x > K$, $f(x) < x$