1. Given below are several pairs of functions f(n) and g(n). For each pair, $\lim_{n \to \infty} \frac{f(n)}{g(n)}$.
   Show work. For some of these, you may need to use L'Hospital's rule. (20 points)

   (a) $f(n) = 15n^2$ and $g(n) = 5$.
   (b) $f(n) = 2^n$ and $g(n) = n^2$.
   (c) $f(n) = n \ln n$ and $g(n) = 17n^2 + 4$. (Note that "ln" denotes the natural logarithm.)
   (d) $f(n) = 3^n$ and $g(n) = 2^n$.

2. (a) Use induction on n to prove that for all integers n >= 0, the value $4^n + 1$ is not divisible by 3. (20 points)

   (b) Consider the infinite sequence of integers $f_0, f_1, f_2, \ldots$, defined by $f_0 = 1$, $f_1 = 2$ and $f_n = f_{n-1} + f_{n-2}$ for all n >= 2. Use *induction on n* to prove that $f_n > \left(\frac{3}{2}\right)^n$ for all n >= 1. (20 points)

3. Let A = {x; y; z; w} and let B = {1; 2; 3}. Determine the number of functions from A to B that are neither one-to-one nor map y to 3. Show work. (20 points)
   Hint: Use the principle of inclusion-exclusion.

4. Let $A = \{x_1; x_2; \ldots; x_{10}\}$ be a set of 10 positive integers, not necessarily distinct, such that $x_i <= 10$, $1 <= i <= 10$. Prove that there are at least two different 5-element subsets S_1 and S_2 of A such that the sum of the elements in S_1 is equal to the sum of the elements in S_2. (20 points)
   Hint: Use the pigeonhole principle.