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CS 431
Test 1

1)

a) $\sigma^2(x) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu_x)^2$ $\mu_x = \frac{\sum x}{n}$

$\frac{15}{5} = 3$

$= \frac{1}{5}\sum_{i=1}^{5}(x_i-\mu_x)^2$

$= \frac{1}{5}((1-3)^2+(-1-3)^2+(6-3)^2+(3-3)^2+(6-3)^2)$

$= \frac{1}{5}(4+16+9+0+9)$

$= \frac{1}{5}(38) = \boxed{7.6}$

$\sigma^2(y) = \frac{1}{n}\sum_{i=1}^{n}(y_i-\mu_y)$ $\mu_y = \frac{10}{5} = 2$

$= \frac{1}{5}((5-2)^2+(2-2)^2+(-10-2)^2+(12-2)^2+(1-2)^2)$

$= \frac{1}{5}(9+0+144+100+1)$

$= \boxed{50.8}$

$\sigma^2(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu_x)(y_i-\mu_y)$

$= \frac{1}{10}((1-3)(5-2)+(-1-3)(2-2)+(6-3)(-10-2)$
$+(3-3)(12-2)+(6-3)(1-2))$

$= \frac{1}{10}((-2)(3)+(-4)(0)+(3)(-12)+$
$+(0)(10)+(3)(-1))$

$= \frac{1}{10}(-6+0-36+0-3)$

$= \frac{1}{10}(-45) = \boxed{-4.5}$

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2) $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ $v = \begin{bmatrix} \sqrt{3}/\sqrt{3} \\ -\sqrt{3}/2\sqrt{3} \\ -\sqrt{3}/2\sqrt{3} \end{bmatrix}$ goal: $\det(A - \lambda I_n) = 0$

$$\det\left(\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = \det\left(\begin{bmatrix} 3-\lambda & 1 & 1 \\ 1 & 2-\lambda & 2 \\ 1 & 2 & 2-\lambda \end{bmatrix}\right)$$

$$= (3-\lambda)\det\begin{bmatrix} 2-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix} + 1\det\begin{bmatrix} 1 & 2 \\ 1 & 2-\lambda \end{bmatrix}$$
$$+ \det\begin{bmatrix} 1 & 2-\lambda \\ 1 & 2 \end{bmatrix}$$

$$= (3-\lambda)((2-\lambda)^2 - (2)(2)) + ((2-\lambda) - (1)(2))$$
$$+ ((1)(2) - (2-\lambda)(1))$$

$$= (3-\lambda)((2-\lambda)(2-\lambda) - 4) + (2 - \lambda - 2)$$
$$+ (2 - 2 + \lambda)$$
$$= (3-\lambda)(4 - 4\lambda + \lambda^2 - 4) + (-\lambda) + (\lambda)$$
$$= (-12\lambda + 7\lambda^2 - \lambda^3)$$

$$(-1)\lambda^3 + (7)\lambda^2 + (-12)\lambda = 0$$

$$\lambda = \frac{-7 \pm \sqrt{7^2 - 4(-1)(-12)}}{2(-1)} = \frac{-7 \pm \sqrt{49 - 48}}{-2}$$

$$= \frac{-7 \pm 1}{-2} \qquad \lambda = 0, 3, 4$$

cont. ⟶

$\lambda_1 = 0$

$\lambda_2 = 3$

$\lambda_3 = 4$

$\cancel{\mathrm{Nul}(A - \lambda_1 I_n) = \mathrm{Nul}\left(\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)}$

$Au \stackrel{?}{=} \lambda u$?

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} \sqrt{2}/\sqrt{3} \\ -\sqrt{2}/2\sqrt{3} \\ -\sqrt{2}/2\sqrt{3} \end{bmatrix} = \begin{bmatrix} \sqrt{2} - \sqrt{\frac{2}{3}} \\ \frac{\sqrt{6} - 6\sqrt{2}}{3\sqrt{3}} \\ \frac{\sqrt{6} - 6\sqrt{2}}{3\sqrt{3}} \end{bmatrix}$$

$$\lambda_2 u = \begin{bmatrix} \sqrt{2} \\ -\sqrt{\frac{3}{2}} \\ -\sqrt{3/2} \end{bmatrix} \qquad \lambda_1 u = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_3 u = \begin{bmatrix} \frac{4\sqrt{2}}{3} \\ -\frac{2\sqrt{2}}{\sqrt{3}} \\ -\frac{2\sqrt{2}}{\sqrt{3}} \end{bmatrix}$$

No it is not a eigenvector of A

4)

b) Def Centroid(D):
For i in Drow : $0 \le i \le n$
for j in Dcol, $0 \le j \le d$
Val = mean($D_1, \ldots, D_i$) - mean($D_i, \ldots, D$
return smallest Val
This will run in $O(n^2)$ time assuming taking the mean of each row is $O(n)$

c) KCentroid(D, $\phi(x)$)
For