COMPSCI 240: Reasoning Under Uncertainty

Profs. Ivan Lee and Jie Xiong

University of Massachusetts at Amherst

Fall 2020

Lecture 1

General Information

- Instructors: Sunghoon (Ivan) Lee (silee@cs.umass.edu) and Jie Xiong (jxiong@cs.umass.edu)
- Lectures: MoWe 2:30 PM – 3:45 PM Class Zoom with passcode: cs240rocks
- Course Webpage: Moodle
- Other Important Class Resources: Piazza and Gradescope
- Teaching Assistants:
  - Zack While (zwhile@cs.umass.edu)
  - Mengxue Zhang (mengxuezhang@cs.umass.edu)
  - Yuda Feng (yudafeng@umass.edu)
  - Brandon Oubre (boubre@cs.umass.edu)
  - Binbin Xie (binbinxie@cs.umass.edu)
  - Minhao Cui (minhaocui@umass.edu)

General Information

- Discussions: (start from this Friday (8/28/2020))
  - Section AA (TA: Zack While): Friday 1:25pm-2:15pm
  - Section AB (TA: Minhao Cui): Fr Friday 11:15am-12:05pm
  - Section AC (TA: Yuda Feng): Friday 12:20pm-1:10pm
  - Section AD (TA: Brandon Oubre): Friday 1:25pm-2:15pm
  - Section AE (TA: Mengxue Zhang): Friday 11:15am-12:05pm
  - Section AF (TA: Binbin Xie): Friday 12:20pm-1:10pm
- Zoom links to all discussion sessions are available on Moodle.

More Information

- Office Hours: (start from next week (8/31/2020))
  - Prof. Lee (First Half Semester): Mon 10:00 AM - 11:00 AM
  - TA Mengxue Zhang: Mon 9:00 AM - 10:00 AM
  - TA Yuda Feng: Tue 12:00 PM - 1:00 PM
  - TA Brandon Oubre: Wed 9:30 AM - 10:30 AM
  - TA Binbin Xie: Thur 1:00 PM - 2:00 PM
  - TA Minhao Cui: Fri 10:00 AM - 11:00 AM
  - UCA Eli Rotondo: Mon 1:00 PM - 2:00 PM
  - UCA Apurv Shah: Tue 8:00 AM - 9:00 AM
  - UCA Long Le: Wed 6:00 PM - 7:00 PM (only Long's office hour will start from 9/9/2020)
  - UCA Raghav Gupta: Fri 3:30 PM - 4:30 PM
- Zoom links to all office hours are available on Moodle.

More Information

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<tr><td>9:30-10</td><td></td><td rowspan="2">Brandon</td><td></td><td></td></tr>
<tr><td>10-10:30</td><td></td><td></td><td rowspan="2">Ivan</td><td rowspan="2">Minghao</td></tr>
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<tr><td>11-11:30</td><td></td><td></td><td></td><td></td><td rowspan="7">Discussions</td></tr>
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<tr><td>12-12:30</td><td></td><td rowspan="2">Yuda</td><td></td><td></td></tr>
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<tr><td>6-6:30</td><td></td><td></td><td rowspan="2">Long Le</td><td></td><td></td></tr>
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More Information

- Lecture Slides: Slides and lecture recordings will be available on Moodle.
- Textbook: Introduction to Probability, 2nd Edition by Dimitri P. Bertsekas and John N. Tsitsiklis
- Outgoing emails: Course emails will be sent via Moodle.
- Incoming emails: No math questions to either instructors or TAs.

Class Schedule

- Basic counting problems
- Probability
- Discrete random variables
- Exam #1 (9/28)
- Continuous random variables
- Central Limit Theorem
- Probabilistic reasoning
- Exam #2 (10/28)
- Game Theory
- Markov Chains
- Bayesian Network.
- Final Exam

Grade Breakdown

There will be 10 weekly online quizzes, 4 homework assignments, 3 in-class exams.

- Weekly Quiz in Moodle (10 of them) 10%
- Homeworks (all 4 HWs count) 20%
- Exam 1 20%
- Exam 2 20%
- Final Exam 20%

Important Dates

- Homework 1 Assignment Sep 2, Due Sep 9
- Homework 2 Assignment Sep 16, Due Sep 23
- Online Exam #1: Sep 28
- Homework 3 Assignment Oct 14, Due Oct 21
- Online Exam #2: Oct 28
- Homework 4 Assignment Nov 9, Due Dec 16
- Online Final Exam:

Homework Policy

- Homework must be submitted to Gradescope by **2 pm** on the deadline date. Late submissions will not be accepted under any circumstances (Gradescope will simply disable the submission). NO exceptions.
  - If Gradscope server magically fails on you right before the deadline, you should email the instructors your homework BEFORE the deadline to be considered for grading.
- Graded homework will be made available via Gradescope on the Friday following the deadline. Once returned, students have until the next Friday discussion session to let the instructor/TAs know about any grading dispute/concern. No change in grades will be recorded after one week of returning the homeworks. NO exceptions.

Homework Submission Instruction

- Please send an email to the instructor by the end of this week (Friday) if you are in any ways opposed to use Gradescope to collect/grade homework and exams.
- Code to enroll to CS240 on Gradescope will be shared via Email sometime this week.
- For every online homework and exam, please upload a PDF (Latex or hand-written and scanned) of your submission. For every question, please carefully assign the corresponding pages in your PDF to that question (you can select one or more pages per question). If there is an overlap, you can assign the same page to multiple questions. If you do not assign the correct pages to the corresponding questions, it is likely that you will not receive any points for that question.

Weekly Quiz

- Every Wednesday, a short online quiz will be posted on Moodle. The quiz needs to be taken by Friday 11:59 PM ET. The quiz should take about 30 minutes but you are allowed 1 hour.

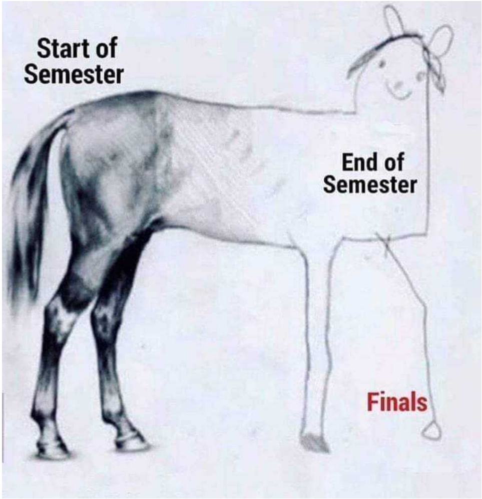
Exam Policy

- All exams will be taken online. It will be basically a longer homework with shorter time to complete.
- Once graded exams are returned, the students have exactly 1 week to let the TAs/instructors know about any grading dispute/concern. No change in grades will be recorded after one week of returning the exams. NO exceptions.
- Concerns/dispute regarding midterms/homework must be reported during the instructors/TAs office hours.
- Missing exams: With doctor's note and/or legal documents that comply the University's Class Absence Policy, you will be able to take a make-up exam. See HERE.
- Formal UMass academic honesty policy applies to all homework, quizzes, and exams. See HERE.

Homework / Exam Submission Instruction

- The submission deadline is strict and Gradescope will stop accepting submissions beyond that time. Please give yourselves sufficient time buffer (e.g., 30 mins) before the deadline to properly upload your PDF and assign pages to questions.
- Failure to meet the deadline for any reasons (e.g., poor Internet connection) will not be accepted. Please find a reliable Internet connection or have a secondary Internet option and provide yourself with a enough time buffer to upload the PDFs.
- In case Gradescope servers fail during the submission process, you may email the course instructor with the PDF BEFORE the deadline.
- You will be able to see feedback on your assignment as well as the grading rubric once we have finished grading. You will be able to submit regrade requests for your assignment at that point (until the following Friday tutorial session).

How to Ace CS 240? (or Any Other Courses)



How to Ace CS 240? (or Any Other Courses)

How to Ace CS 240? (or Any Other Courses)

- Please attend every class and every discussion if possible.
  - Instructors sneak-peak exam questions during lectures.
- Try to be on schedule to follow the course materials if cannot attend class.
- Do all homework and quiz yourself.
- Read the materials before and re-visit the materials after each class.
- Get help during instructor and TAs' office hours.

Why do we need reasoning under uncertainty?

- There are things that are completely certain.
  - $1 + 1 = 2$
- But most things in the world are uncertain.
  - It will be raining tomorrow
  - Whether mom just called me while listening to a loud heavy metal music.
  - To (or not to) gamble
- We often make decisions that maximize the benefit given uncertainty
  - Should I bring my umbrella?
  - Should I take off my headphone and yell back to mom: "WHY?!" (please be nice to your moms).
  - I call that $100K bet! Bring it on!

Why do we need reasoning under uncertainty?

A Pseudo Real-World Problem:

You have studied all night and woke up 15 minutes before your exam. You could catch a 10 minute-long bus ride. Each minute, a bus to the school passes your house with probability $1/5$. Your other choice is to bike to the University from home, but it will take you 16 minutes. Should you wait for the bus or hop on your bike?

Why do we need reasoning under uncertainty?

- Prediction and Estimation (Weather! and Stock Market!)
- Detection (Communication!)
- Almost all branches of Computer Science

Why do we need reasoning under uncertainty?

What do you see?

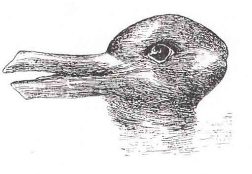


Figure: A duck or a rabbit?

Figure: A young or old woman?

Why do we need reasoning under uncertainty? What if I don't do research

You need this for

- Market analysis (will this product sale?)
- Stock Market (Wall Street, Hedge Funds)
- Medical tests (Pharmaceuticals)
- etc. etc.

But also to enable logical thought processes. To think rationally! To understand the meaning of life, universe and everything!

This is a core course!

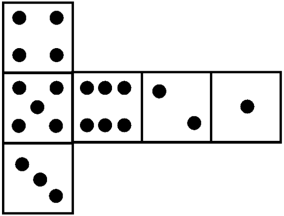
How do we reason under uncertainty?

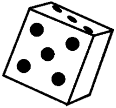
- Using Probability Theory
- Main idea: Assign each event a measure between 0 to 1: to signify its likelihood
- Then proceed very carefully - or our intuitions and observations will not match

Back to basics: Set theory

- A set is a collection of objects. The objects are called elements of the set.
- If S is a set and x is an element of S, we write $x \in S$.
- If x is not an element of S, we write $x \notin S$. A set can have no elements, in which case it is called an empty set, denoted by $\emptyset$.
- Apple $\in$ { Orange, Apple, Pear }
  Strawberry $\notin$ { Orange, Apple, Pear }

Back to basics: Set theory





- Various ways of writing a set containing the possible outcomes of a dice throw:

$$S = \{1, 2, 3, 4, 5, 6\}$$

or

$$S = \{x | x \text{ is a possible outcome of a throw of a dice}\}$$

A slightly more challenging question

- I throw a dice twice. What is the possible sum of the two outcomes?

  $$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

  For element 2 and 4 in set S, which one has a larger probability?

Set theory

- Size of a set S is denoted by $|S|$
- $|\{$ Orange, Apple, Pear $\}| = 3$
- S is a subset of T, $S \subset T$, means every element of S is also an element of T
  - $\{$Apple, Pear $\} \subset \{$ Orange, Apple, Pear $\}$
  - $\{$ Orange, Apple, Pear $\} \subset \{$ Orange, Apple, Pear $\}$
  - $\{$Apple, Banana $\} \not\subset \{$ Orange, Apple, Pear $\}$
- If $S \subset T$ and $T \subset S$ then,

$$S = T$$

Universal Set

- Ω: contains all objects that could conceivably be of interest in a particular context.
- In the context of coin tossing, $\Omega = \{H, T\}$.
- In the context of dice, $\Omega = \{1, 2, 3, 4, 5, 6\}$.

Set operations

- Complement: $S^c = \{x \in \Omega | x \notin S\}$
  Example: $\Omega = \{1, 2, 3, 4, 5\}; \quad S = \{2, 5\}$
  $S^c = \{1, 3, 4\}$
  Note that, $\Omega^c = \emptyset$
- The union of two sets S and T is the set of all elements that belong to S or T (or both), and is denoted by $S \cup T$.

$$S \cup T = \{x | x \in S \text{ or } x \in T\}$$

- The intersection of two sets S and T is the set of all elements that belong to both S and T, and is denoted by $S \cap T$.

$$S \cap T = \{x | x \in S \text{ and } x \in T\}$$

Set operations

$$\Omega = \{1,2,3,4,5,6\}; \quad S = \{1,2,5\} \quad T = \{2,3,4,5\}$$

$$S \cup T = \{1,2,3,4,5\}$$

$$S \cap T = \{2,5\}$$

Power Set

By default: $\emptyset \subset S \subset \Omega$.
Power Set: Set of all subsets

$$S = \{1, 2, 3\}$$

$$2^S = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

A slightly more challenging question

for set

$$S = \{1, 2, 3, 4, 5, 6\}$$

How many subsets will the power set of S have?

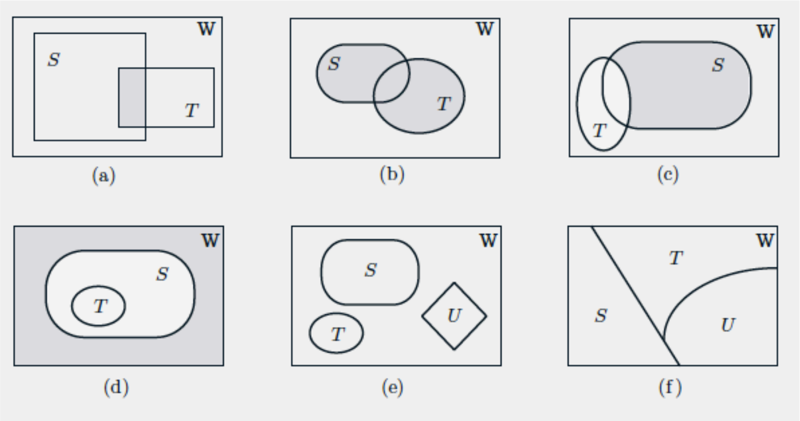
$$2^6 = 64$$

Disjoint Set

S and T are **disjoint** if $S \cap T = \emptyset$

$S_1, S_2, \ldots, S_n$ form a **partition** of S if S_i and S_j are disjoint for any $i \neq j$ and $S_1 \cup S_2 \cup \cdots \cup S_n = S$.

Venn Diagram



Courtesy: Textbook

Set Algebra

Using the above definitions, we can show that:

- Intersection Commutativity $S \cap T = T \cap S$
- Union Commutativity $S \cup T = T \cup S$
- Intersection Associativity $S \cap (T \cap U) = (S \cap T) \cap U$
- Union Associativity $S \cup (T \cup U) = (S \cup T) \cup U$
- Intersection Distributivity $S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$
- Union Distributivity $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$

Model of Probability

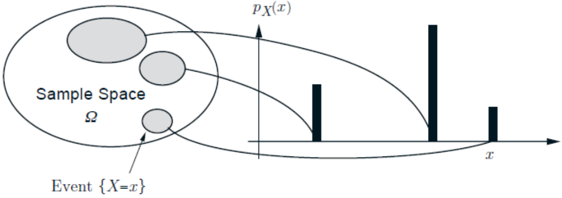
A probabilistic model is a mathematical description of an uncertain situation. Two fundamental elements of a probabilistic model is

- **Sample Space** Ω: all possible outcomes of an experiment
- **Probability Law**:

$$A \subset \Omega; \quad P(A),$$

  where A is an **event** (a set of possible outcomes) and $P(A)$ is a non-negative number presenting the **likelihood** of observing the event A.

Probabilistic model involves an **experiment**, which produces an **event** from the **sample space**.



Probability Laws

- Probability represents likelihood of any outcomes or of any set of possible outcomes.
- The probability law assigns to every event A, a number $P(A)$, call the **probability** of A.

Axioms of Probability

- Nonnegativity:

$$P(A) \geq 0$$

- Additivity: For any two disjoint sets A, B,

$$P(A \cup B) = P(A) + P(B)$$

  Holds for infinitely many disjoints events $A_1, A_2, A_3, \ldots$

$$P(\cup_i A_i) = \sum_i P(A_i).$$

- Normalization:

$$P(\Omega) = 1$$

Sample Problems

Question: Show that $P(A^c) = 1 - P(A)$?

$$\begin{aligned} 1 &= P(\Omega) \\ &= P(A \cup A^c) \text{ [particitions]} \\ &= P(A) + P(A^c) \text{ [additivity for two disjoint sets]} \end{aligned}$$

Hence,

$$P(A^c) = 1 - P(A).$$

Sample Problems

Question: What is $P(\emptyset)$?

Since $\emptyset = \Omega^c$, $P(\emptyset) = 1 - P(\Omega) = 0$.

Sample Problems

Question:

- Show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$
\begin{aligned}
P(A \cup B) &= P(A \cup B - A) \\
&= P(A) + P(B - A) \ [A \text{ and } B - A \text{ are disjoint}] \\
&= P(A) + P(B) - P(A \cap B)
\end{aligned}
$$

Sample Problems

Question:

- Show that $P(A \cup B) \leq P(A) + P(B)$

  Previously we showed that

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

  Because $P(A \cap B) \geq 0$,

  $$P(A \cup B) \leq P(A) + P(B)$$

What type of questions can be tested in exam?

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
How about $P(A \cup B \cup C)$?