Confirmatory Factor Analysis

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1 Preliminaries

In this section, the RStudio workspace and console panes are cleared of old output, variables, and other miscellaneous debris. Packages are loaded and any required data files are retrieved.

```
options(replace.assign = TRUE, width = 65, digits = 4, scipen = 4, fig.width = 4,
    fig.height = 4)
# Clear the workspace and console.
rm(list = ls(all = TRUE))
cat("\f")
```

```
# Turn off showing of significance asterisks.
options(show.signif.stars = F)
# Set the contrast option; important for ANOVAs.
options(contrasts = c("contr.sum", "contr.poly"))
how_long <- Sys.time()
set.seed(123)
library(knitr)
```

```
library(psych)
library(car)

## Loading required package:  carData
##
## Attaching package:  'car'
## The following object is masked from 'package:psych':
##
##    logit

library(multcomp)

## Loading required package:  mvtnorm
## Loading required package:  survival
## Loading required package:  TH.data
## Loading required package:  MASS
##
## Attaching package:  'TH.data'
## The following object is masked from 'package:MASS':
##
##    geyser
```

```
library(ggplot2)
```

```
##
## Attaching package:  'ggplot2'
## The following objects are masked from 'package:psych':
##
##    %+%, alpha
```

```
library(MASS)
library(parallel)
library(ellipse)
```

```
##
## Attaching package:  'ellipse'
## The following object is masked from 'package:car':
##
##    ellipse
## The following object is masked from 'package:graphics':
##
##    pairs
```

```
library(FactoMineR)
```

```
## Warning:  package 'FactoMineR' was built under R version 3.5.1
```

```
library(PerformanceAnalytics)
```

```
## Warning:  package 'PerformanceAnalytics' was built under R version 3.5.1
## Loading required package:  xts
## Loading required package:  zoo
##
## Attaching package:  'zoo'
## The following objects are masked from 'package:base':
##
##    as.Date, as.Date.numeric
##
## Attaching package:  'PerformanceAnalytics'
## The following object is masked from 'package:graphics':
##
##    legend
```

```
library(plotpc)
```

```
## Loading required package:  grid
```

```
library(sciplot)
library(GPArotation)
library(GGally)
library(MVN)
```

```
## sROC 0.1-2 loaded
```

```
library(qqplotr)
library(scatterplot3d)
library(rgl)
```

```
## Warning:  package 'rgl' was built under R version 3.5.1
```

```
library(cowplot)
```

```
##
## Attaching package:  'cowplot'
## The following object is masked from 'package:ggplot2':
##
##    ggsave
```

```
library(lavaan)
```

```
## This is lavaan 0.6-1
## lavaan is BETA software!  Please report any bugs.
##
## Attaching package:  'lavaan'
## The following object is masked from 'package:psych':
##
##    cor2cov
```

```
library(semPlot)
library(semTools)
```

```
##
## ################################################################################
## This is semTools 0.4-14
## All users of R (or SEM) are invited to submit functions or ideas for functions.
## ################################################################################
##
## Attaching package:  'semTools'
## The following object is masked from 'package:PerformanceAnalytics':
##
##    kurtosis
## The following object is masked from 'package:psych':
##
##    skew
```

```
library(MVN)
```

1.1 Data Files

We will use the full mental abilities data set for the example analyses.

```
# Get the drug use data from the working directory.
setwd("C:\\Courses\\Psychology 516\\PowerPoint\\2018")
Mental <- read.table("mental.csv", sep = ",", header = TRUE)
Mental <- as.data.frame(Mental)
Mental <- na.omit(Mental)
names(Mental) <- c("Grammar", "Paragraph_Comprehension", "Vocabulary",
    "Sentence_Completion", "Geometry", "Algebra", "Numerical_Puzzles",
    "Series_Completion", "Practical_Problem_Solving", "Symbol_Manipulation",
    "Analytical_Ability", "Formal_Logic")
describe(Mental)
```

```
##                           vars   n  mean   sd median trimmed
## Grammar                      1 500 -0.01 0.98  -0.02   -0.01
## Paragraph_Comprehension      2 500  0.00 1.06   0.04    0.01
## Vocabulary                   3 500 -0.01 1.00   0.02    0.01
## Sentence_Completion          4 500 -0.09 1.09  -0.09   -0.09
```

```
## Geometry                     5 500  0.01 0.98  -0.03    0.00
## Algebra                      6 500 -0.03 0.96   0.00   -0.02
## Numerical_Puzzles            7 500 -0.02 0.99  -0.06   -0.02
## Series_Completion            8 500 -0.01 0.96  -0.03    0.00
## Practical_Problem_Solving    9 500 -0.02 0.94  -0.04   -0.03
## Symbol_Manipulation         10 500  0.00 0.97  -0.06   -0.01
## Analytical_Ability          11 500  0.00 0.98   0.01    0.01
## Formal_Logic                12 500 -0.06 1.02  -0.09   -0.07
##                              mad   min  max range  skew kurtosis
## Grammar                     0.99 -3.00 2.24  5.24 -0.06    -0.21
## Paragraph_Comprehension     1.09 -3.02 2.95  5.98 -0.09    -0.19
## Vocabulary                  0.98 -3.49 2.76  6.25 -0.25    -0.05
## Sentence_Completion         1.12 -3.09 3.01  6.10  0.02    -0.24
## Geometry                    0.94 -2.52 2.78  5.30  0.11    -0.17
## Algebra                     0.87 -2.91 2.89  5.80 -0.07     0.17
## Numerical_Puzzles           0.93 -2.80 2.88  5.69  0.01     0.05
## Series_Completion           0.96 -3.04 2.29  5.33 -0.11    -0.21
## Practical_Problem_Solving   0.96 -2.64 3.03  5.67  0.13    -0.02
## Symbol_Manipulation         0.94 -3.07 2.74  5.81  0.09     0.12
## Analytical_Ability          1.02 -3.75 2.64  6.39 -0.15     0.00
## Formal_Logic                0.93 -3.20 3.73  6.93  0.06     0.44
##                               se
## Grammar                     0.04
## Paragraph_Comprehension     0.05
## Vocabulary                  0.04
## Sentence_Completion         0.05
## Geometry                    0.04
## Algebra                     0.04
## Numerical_Puzzles           0.04
## Series_Completion           0.04
## Practical_Problem_Solving   0.04
## Symbol_Manipulation         0.04
## Analytical_Ability          0.04
## Formal_Logic                0.05
```

2 Screen the Data

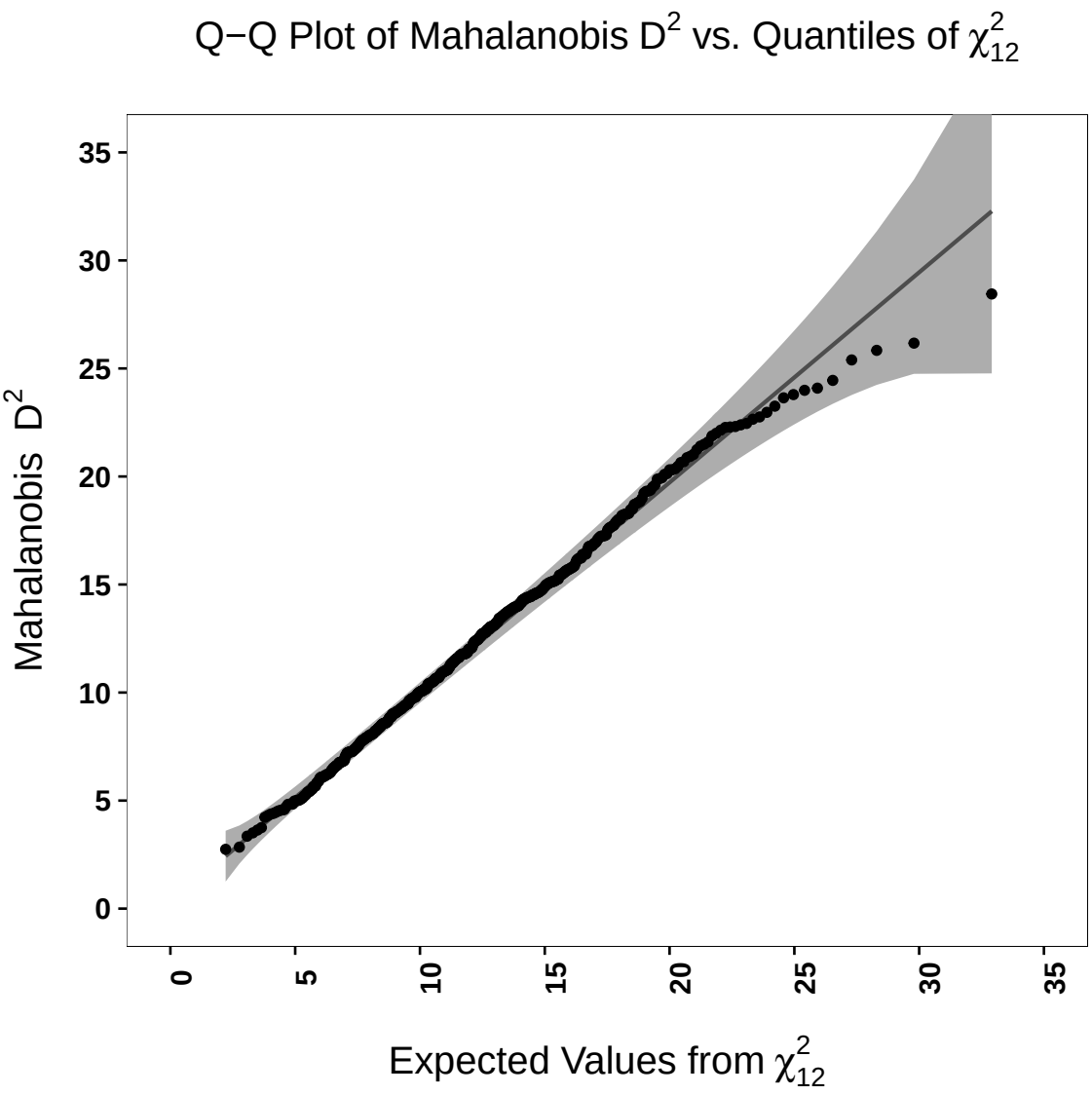
First we will do a quick screen of the data to make sure there are no severe multivariate outliers.

2.1 Mahalanobis Distance

```
CV <- cov(Mental)
D2 <- mahalanobis(Mental, center = colMeans(Mental), cov = CV)
D2 <- as.data.frame(D2)
describe(D2)

##    vars   n  mean   sd median trimmed  mad  min   max range skew
## X1    1 500 11.98 4.75   11.4   11.65 4.66 2.74 28.45  25.7 0.61
##    kurtosis   se
## X1     0.05 0.21
```

```
ggplot(D2, aes(sample = D2)) + stat_qq_band(distribution = "chisq",
    dparams = list(df = 12)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 12)) + stat_qq(distribution = "qchisq", dparams = list(df = 12)) +
    scale_y_continuous(breaks = seq(0, 35, 5)) + scale_x_continuous(breaks = seq(0,
    35, 5)) + coord_cartesian(xlim = c(0, 35), ylim = c(0, 35)) +
    xlab(expression("Expected Values from" * ~chi[12]^2)) + ylab(expression("Mahalanobis " *
    ~D^2)) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" *
    ~D^2 * " vs. Quantiles of" * ~chi[12]^2))
```



The squared Mahalanobis distances suggest the data are well behaved. We can also test for multivariate normality.

```
mvn(Mental, mvnTest = "mardia")

## $multivariateNormality
##              Test          Statistic           p value Result
## 1 Mardia Skewness   367.600071004817 0.437353917013108    YES
## 2 Mardia Kurtosis -0.866369530731211 0.386287547416061    YES
## 3             MVN               <NA>              <NA>    YES
##
## $univariateNormality
##            Test                  Variable Statistic   p value
## 1  Shapiro-Wilk          Grammar             0.9955    0.1597
## 2  Shapiro-Wilk  Paragraph_Comprehension     0.9980    0.8256
```

```
## 3  Shapiro-Wilk         Vocabulary             0.9951    0.1166
## 4  Shapiro-Wilk    Sentence_Completion         0.9972    0.5408
## 5  Shapiro-Wilk          Geometry              0.9961    0.2666
## 6  Shapiro-Wilk           Algebra              0.9964    0.3168
## 7  Shapiro-Wilk     Numerical_Puzzles          0.9966    0.3863
## 8  Shapiro-Wilk     Series_Completion          0.9959    0.2195
## 9  Shapiro-Wilk Practical_Problem_Solving      0.9974    0.6374
## 10 Shapiro-Wilk    Symbol_Manipulation         0.9962    0.2767
## 11 Shapiro-Wilk    Analytical_Ability          0.9971    0.5221
## 12 Shapiro-Wilk       Formal_Logic             0.9961    0.2500
##    Normality
## 1        YES
## 2        YES
## 3        YES
## 4        YES
## 5        YES
## 6        YES
## 7        YES
## 8        YES
## 9        YES
## 10       YES
## 11       YES
## 12       YES
##
## $Descriptives
##                              n      Mean Std.Dev    Median    Min
## Grammar                    500 -0.007632  0.9765 -0.019820 -3.003
## Paragraph_Comprehension    500 -0.003750  1.0633  0.035079 -3.024
## Vocabulary                 500 -0.014783  0.9989  0.023294 -3.492
## Sentence_Completion        500 -0.087844  1.0863 -0.087127 -3.092
## Geometry                   500  0.010634  0.9789 -0.028235 -2.522
## Algebra                    500 -0.028644  0.9615 -0.002082 -2.906
## Numerical_Puzzles          500 -0.015062  0.9890 -0.062496 -2.804
## Series_Completion          500 -0.006348  0.9638 -0.029178 -3.038
## Practical_Problem_Solving  500 -0.016828  0.9361 -0.040021 -2.636
## Symbol_Manipulation        500  0.000833  0.9718 -0.056943 -3.073
## Analytical_Ability         500 -0.004649  0.9834  0.009074 -3.751
## Formal_Logic               500 -0.062950  1.0165 -0.092532 -3.197
##                               Max    25th   75th     Skew
## Grammar                     2.235 -0.6782 0.6886 -0.05736
## Paragraph_Comprehension     2.952 -0.7485 0.7003 -0.09118
## Vocabulary                  2.763 -0.6567 0.6660 -0.24969
## Sentence_Completion         3.005 -0.8542 0.6525  0.01663
## Geometry                    2.779 -0.6281 0.6738  0.10710
## Algebra                     2.893 -0.6320 0.5580 -0.07334
## Numerical_Puzzles           2.881 -0.5965 0.6013  0.01468
## Series_Completion           2.288 -0.6444 0.6381 -0.11470
## Practical_Problem_Solving   3.033 -0.6044 0.6685  0.12662
## Symbol_Manipulation         2.740 -0.6110 0.6397  0.08702
## Analytical_Ability          2.636 -0.6867 0.6939 -0.15307
## Formal_Logic                3.729 -0.6654 0.5589  0.05588
##                             Kurtosis
## Grammar                     -0.210884
## Paragraph_Comprehension     -0.185360
```

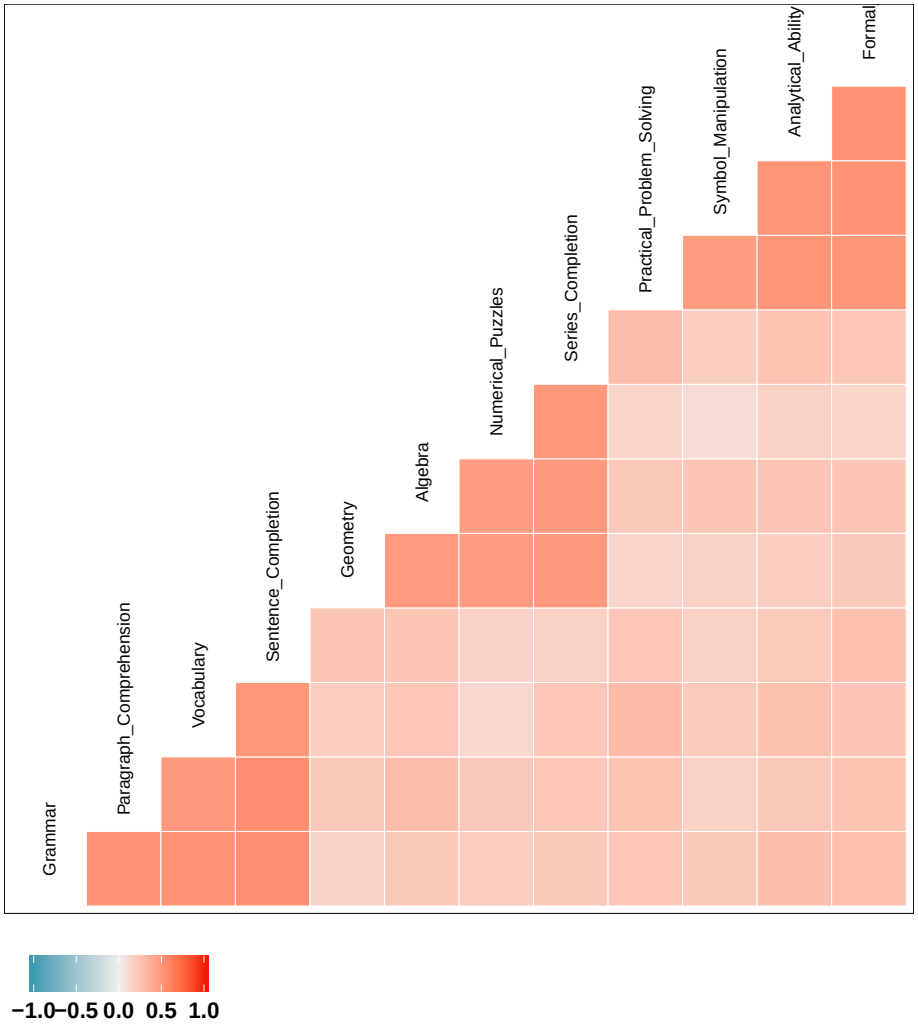
```
## Vocabulary                -0.047192
## Sentence_Completion       -0.238030
## Geometry                  -0.174019
## Algebra                    0.171153
## Numerical_Puzzles          0.046084
## Series_Completion         -0.214526
## Practical_Problem_Solving -0.019846
## Symbol_Manipulation        0.120057
## Analytical_Ability         0.001184
## Formal_Logic               0.441237
```

The data are multivariate normal, which is of course not surprising given that they were generated from a multivariate normal distribution.

3 Correlations

A heat map for the correlation matrix easily identifies the pattern of correlations in the simulated data.

```
ggcorr(Mental, label = FALSE, angle = 90, hjust = 0.1, size = 2.5,
    digits = 2) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Intercorrelations Among Items")
```



4 Confirmatory Factor Analysis

In the following sections, we will examine several confirmatory factor analysis models that range in their complexity, some of which are nested and so can be compared statistically for goodness of fit.

4.1 Model 1

```
mental.model.1 <- "
# Latent variable definitions.
# Scale of the latent variables is set by the first listed manifest variable.
Verbal =~ Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion
```

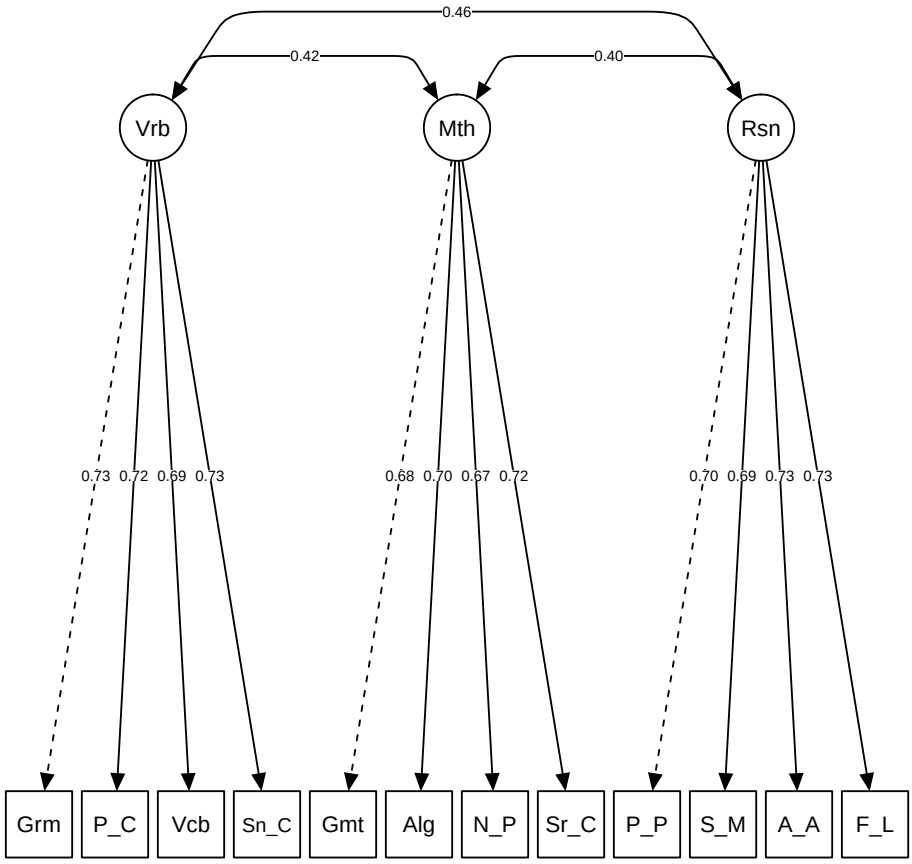
```
Math =~ Geometry+Algebra+Numerical_Puzzles+Series_Completion
Reasoning =~ Practical_Problem_Solving+Symbol_Manipulation+Analytical_Ability+Formal_Logic
# Latent variable covariances.
Verbal ~~ Math
Verbal ~~ Reasoning
Math ~~ Reasoning
"
CFA_Fit_1 <- cfa(mental.model.1, data = Mental, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_1, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
##   90 Percent Confidence Interval                              0.000     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                            0.026       0.026
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal =~
##     Grammar           1.000                               0.716
##     Prgrph_Cmprhns    1.065    0.080   13.256    0.000    0.762
##     Vocabulary        0.960    0.070   13.772    0.000    0.687
##     Sentenc_Cmpltn    1.108    0.079   13.940    0.000    0.793
##   Math =~
##     Geometry          1.000                               0.669
##     Algebra           1.002    0.082   12.225    0.000    0.670
##     Numericl_Pzzls    0.994    0.080   12.357    0.000    0.664
##     Series_Compltn    1.031    0.079   13.081    0.000    0.689
##   Reasoning =~
##     Prctcl_Prblm_S    1.000                               0.650
##     Symbol_Manpltn    1.030    0.081   12.652    0.000    0.670
##     Analytcl_Ablty    1.097    0.085   12.899    0.000    0.713
##     Formal_Logic      1.143    0.090   12.647    0.000    0.744
##   Std.all
##
##     0.734
##     0.718
##     0.689
##     0.731
##
##     0.684
##     0.698
##     0.672
##     0.716
##
##     0.695
##     0.690
##     0.726
##     0.732
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal ~~
##     Math              0.199    0.029    6.782    0.000    0.417
##     Reasoning         0.215    0.029    7.442    0.000    0.461
##   Math ~~
##     Reasoning         0.176    0.026    6.759    0.000    0.405
##   Std.all
```

```
##
##     0.417
##     0.461
##
##     0.405
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          -0.008    0.044   -0.175    0.861   -0.008
##    .Prgrph_Cmprhns   -0.004    0.048   -0.079    0.937   -0.004
##    .Vocabulary       -0.015    0.045   -0.331    0.741   -0.015
##    .Sentenc_Cmpltn   -0.088    0.049   -1.808    0.071   -0.088
##    .Geometry          0.011    0.044    0.243    0.808    0.011
##    .Algebra          -0.029    0.043   -0.666    0.505   -0.029
##    .Numericl_Pzzls   -0.015    0.044   -0.341    0.733   -0.015
##    .Series_Compltn   -0.006    0.043   -0.147    0.883   -0.006
##    .Prctcl_Prblm_S   -0.017    0.042   -0.402    0.688   -0.017
##    .Symbol_Manpltn    0.001    0.043    0.019    0.985    0.001
##    .Analytcl_Ablty   -0.005    0.044   -0.106    0.916   -0.005
##    .Formal_Logic     -0.063    0.045   -1.385    0.166   -0.063
##     Verbal            0.000                              0.000
##     Math              0.000                              0.000
##     Reasoning         0.000                              0.000
##   Std.all
##    -0.008
##    -0.004
##    -0.015
##    -0.081
##     0.011
##    -0.030
##    -0.015
##    -0.007
##    -0.018
##     0.001
##    -0.005
##    -0.062
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          0.439    0.036   12.355    0.000    0.439
##    .Prgrph_Cmprhns   0.547    0.044   12.390    0.000    0.547
##    .Vocabulary       0.524    0.041   12.747    0.000    0.524
##    .Sentenc_Cmpltn   0.548    0.044   12.378    0.000    0.548
##    .Geometry         0.509    0.046   11.188    0.000    0.509
##    .Algebra          0.474    0.041   11.462    0.000    0.474
##    .Numericl_Pzzls   0.535    0.045   11.762    0.000    0.535
##    .Series_Compltn   0.452    0.040   11.349    0.000    0.452
##    .Prctcl_Prblm_S   0.452    0.039   11.562    0.000    0.452
##    .Symbol_Manpltn   0.494    0.042   11.846    0.000    0.494
##    .Analytcl_Ablty   0.456    0.042   10.850    0.000    0.456
##    .Formal_Logic     0.478    0.041   11.787    0.000    0.478
```

```
##     Verbal            0.513    0.060    8.520    0.000    1.000
##     Math              0.447    0.056    7.997    0.000    1.000
##     Reasoning         0.423    0.049    8.639    0.000    1.000
##   Std.all
##     0.461
##     0.485
##     0.526
##     0.466
##     0.533
##     0.513
##     0.548
##     0.488
##     0.516
##     0.524
##     0.473
##     0.464
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     Grammar           0.539
##     Prgrph_Cmprhns    0.515
##     Vocabulary        0.474
##     Sentenc_Cmpltn    0.534
##     Geometry          0.467
##     Algebra           0.487
##     Numericl_Pzzls    0.452
##     Series_Compltn    0.512
##     Prctcl_Prblm_S    0.484
##     Symbol_Manpltn    0.476
##     Analytcl_Ablty    0.527
##     Formal_Logic      0.536
```

```
semPaths(CFA_Fit_1, title = FALSE, "std", edge.label.cex = 0.5, curvePivot = TRUE,
    layout = "tree2", style = "lisrel", residuals = FALSE, sizeLat = 5,
    sizeMan = 5, intercepts = FALSE, what = "path", edge.color = "black")
```



4.2 Model 2

```
mental.model.2 <- "
# Latent variable definitions.
# Scale of the latent variables is set by the first listed manifest variable.
Verbal =~ Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion
Math =~ Geometry+Algebra+Numerical_Puzzles+Series_Completion
Reasoning =~ Practical_Problem_Solving+Symbol_Manipulation+Analytical_Ability+Formal_Logic
# Latent variable covariances are set to 0.
Verbal ~~ 0*Math
Verbal ~~ 0*Reasoning
Math ~~ 0*Reasoning
"
```

```r
CFA_Fit_2 <- cfa(mental.model.2, data = Mental, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_2, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

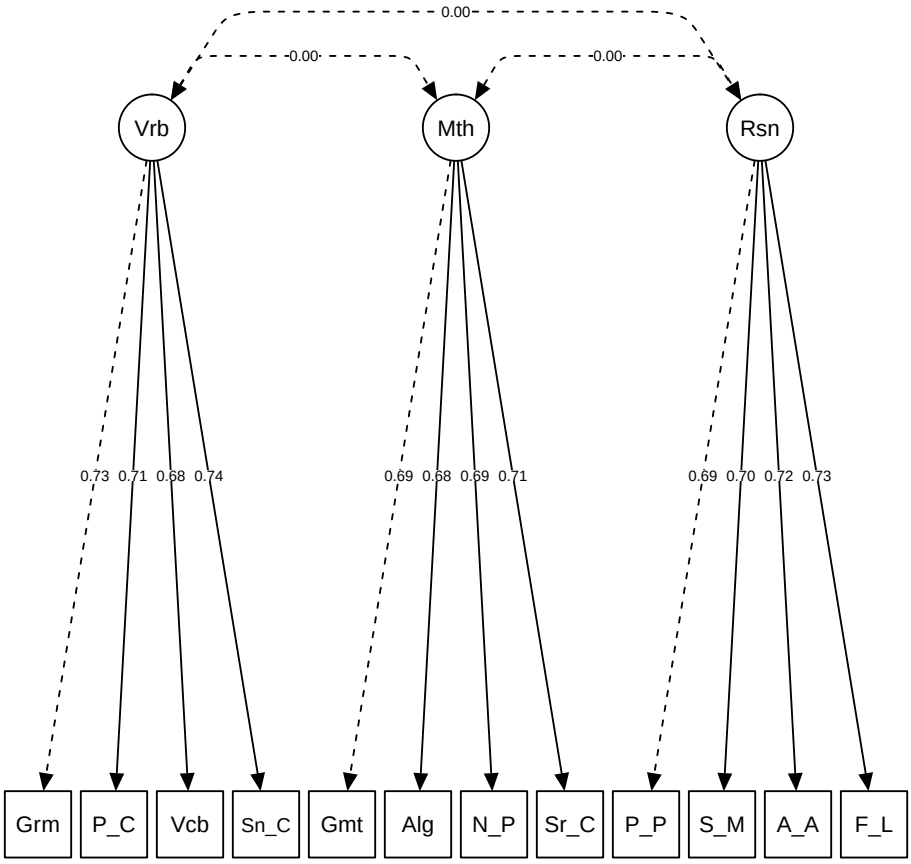
```
## lavaan (0.6-1) converged normally after  22 iterations
##
##   Number of observations                           500
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     214.022     216.879
##   Degrees of freedom                                54          54
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  0.987
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1962.116    1980.927
##   Degrees of freedom                                66          66
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.916       0.915
##   Tucker-Lewis Index (TLI)                       0.897       0.896
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7589.790   -7589.790
##   Loglikelihood unrestricted model (H1)      -7482.565   -7482.565
##
##   Number of free parameters                         36          36
##   Akaike (AIC)                                15251.580   15251.580
##   Bayesian (BIC)                              15403.306   15403.306
##   Sample-size adjusted Bayesian (BIC)         15289.040   15289.040
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.077       0.078
##   90 Percent Confidence Interval          0.066  0.088       0.067  0.089
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.158       0.158
##
## Parameter Estimates:
```

```
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal =~
##     Grammar          1.000                               0.716
##     Prgrph_Cmprhns   1.060    0.080   13.247    0.000    0.759
##     Vocabulary       0.953    0.070   13.573    0.000    0.682
##     Sentenc_Cmpltn   1.118    0.082   13.680    0.000    0.801
##   Math =~
##     Geometry         1.000                               0.674
##     Algebra          0.976    0.080   12.177    0.000    0.657
##     Numericl_Pzzls   1.008    0.082   12.316    0.000    0.679
##     Series_Compltn   1.015    0.077   13.223    0.000    0.684
##   Reasoning =~
##     Prctcl_Prblm_S   1.000                               0.645
##     Symbol_Manpltn   1.055    0.084   12.629    0.000    0.681
##     Analytcl_Ablty   1.102    0.089   12.436    0.000    0.711
##     Formal_Logic     1.147    0.093   12.319    0.000    0.740
##   Std.all
##
##     0.734
##     0.715
##     0.684
##     0.738
##
##     0.689
##     0.684
##     0.687
##     0.710
##
##     0.690
##     0.701
##     0.724
##     0.729
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal ~~
##     Math             0.000                               0.000
##     Reasoning        0.000                               0.000
##   Math ~~
##     Reasoning        0.000                               0.000
##   Std.all
##
##     0.000
##     0.000
##
##     0.000
##
## Intercepts:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     .Grammar         -0.008    0.044   -0.175    0.861   -0.008
##     .Prgrph_Cmprhns  -0.004    0.048   -0.079    0.937   -0.004
##     .Vocabulary      -0.015    0.045   -0.331    0.741   -0.015
##     .Sentenc_Cmpltn  -0.088    0.049   -1.808    0.071   -0.088
##     .Geometry         0.011    0.044    0.243    0.808    0.011
##     .Algebra         -0.029    0.043   -0.666    0.505   -0.029
##     .Numericl_Pzzls  -0.015    0.044   -0.341    0.733   -0.015
##     .Series_Compltn  -0.006    0.043   -0.147    0.883   -0.006
##     .Prctcl_Prblm_S  -0.017    0.042   -0.402    0.688   -0.017
##     .Symbol_Manpltn   0.001    0.043    0.019    0.985    0.001
##     .Analytcl_Ablty  -0.005    0.044   -0.106    0.916   -0.005
##     .Formal_Logic    -0.063    0.045   -1.385    0.166   -0.063
##      Verbal           0.000                              0.000
##      Math             0.000                              0.000
##      Reasoning        0.000                              0.000
##    Std.all
##     -0.008
##     -0.004
##     -0.015
##     -0.081
##      0.011
##     -0.030
##     -0.015
##     -0.007
##     -0.018
##      0.001
##     -0.005
##     -0.062
##      0.000
##      0.000
##      0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     .Grammar         0.439    0.036   12.051    0.000    0.439
##     .Prgrph_Cmprhns  0.552    0.046   11.987    0.000    0.552
##     .Vocabulary      0.530    0.042   12.621    0.000    0.530
##     .Sentenc_Cmpltn  0.536    0.046   11.740    0.000    0.536
##     .Geometry        0.503    0.045   11.059    0.000    0.503
##     .Algebra         0.491    0.042   11.698    0.000    0.491
##     .Numericl_Pzzls  0.515    0.046   11.318    0.000    0.515
##     .Series_Compltn  0.459    0.040   11.403    0.000    0.459
##     .Prctcl_Prblm_S  0.458    0.039   11.639    0.000    0.458
##     .Symbol_Manpltn  0.479    0.041   11.659    0.000    0.479
##     .Analytcl_Ablty  0.459    0.044   10.539    0.000    0.459
##     .Formal_Logic    0.484    0.043   11.355    0.000    0.484
##      Verbal          0.513    0.061    8.444    0.000    1.000
##      Math            0.454    0.056    8.122    0.000    1.000
##      Reasoning       0.417    0.050    8.357    0.000    1.000
##    Std.all
##     0.461
##     0.489
```

```
##     0.533
##     0.455
##     0.525
##     0.532
##     0.528
##     0.496
##     0.524
##     0.508
##     0.476
##     0.469
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     Grammar           0.539
##     Prgrph_Cmprhns    0.511
##     Vocabulary        0.467
##     Sentenc_Cmpltn    0.545
##     Geometry          0.475
##     Algebra           0.468
##     Numericl_Pzzls    0.472
##     Series_Compltn    0.504
##     Prctcl_Prblm_S    0.476
##     Symbol_Manpltn    0.492
##     Analytcl_Ablty    0.524
##     Formal_Logic      0.531

semPaths(CFA_Fit_2, title = FALSE, "std", edge.label.cex = 0.5, curvePivot = TRUE,
    layout = "tree2", style = "lisrel", residuals = FALSE, sizeLat = 5,
    sizeMan = 5, intercepts = FALSE, what = "path", edge.color = "black")
```



4.3 Model 3

```
mental.model.3 <- "
# Latent variable definition
# Scale of the latent variable is set by the first listed manifest variable.
Ability =~ Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion+
 Geometry+Algebra+Numerical_Puzzles+Series_Completion+
 Practical_Problem_Solving+Symbol_Manipulation+Analytical_Ability+Formal_Logic
"
CFA_Fit_3 <- cfa(mental.model.3, data = Mental, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_3, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

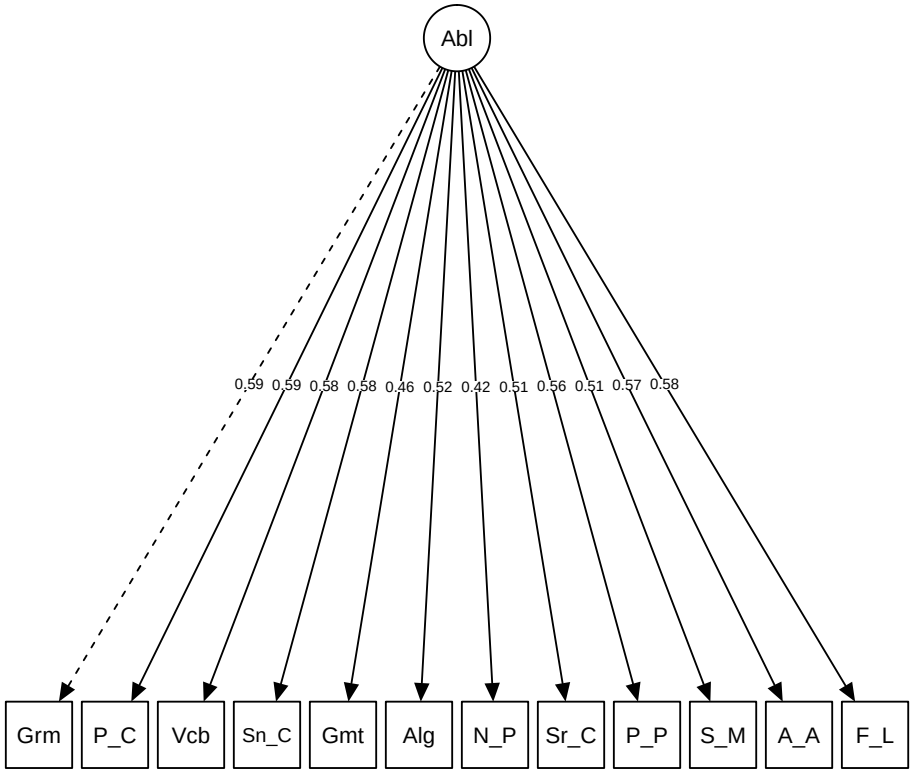
```
## lavaan (0.6-1) converged normally after  26 iterations
##
##   Number of observations                           500
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     770.567     848.990
##   Degrees of freedom                                54          54
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  0.908
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1962.116    1980.927
##   Degrees of freedom                                66          66
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.622       0.585
##   Tucker-Lewis Index (TLI)                       0.538       0.493
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7868.620   -7868.620
##   Loglikelihood unrestricted model (H1)      -7482.565   -7482.565
##
##   Number of free parameters                         36          36
##   Akaike (AIC)                                15809.241   15809.241
##   Bayesian (BIC)                              15960.967   15960.967
##   Sample-size adjusted Bayesian (BIC)         15846.700   15846.700
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.163       0.172
##   90 Percent Confidence Interval          0.153  0.173       0.161  0.183
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.112       0.112
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
```

```
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Ability =~
##     Grammar           1.000                               0.578
##     Prgrph_Cmprhns    1.084    0.092   11.844    0.000    0.627
##     Vocabulary        0.999    0.078   12.853    0.000    0.578
##     Sentenc_Cmpltn    1.082    0.089   12.141    0.000    0.626
##     Geometry          0.773    0.137    5.630    0.000    0.447
##     Algebra           0.861    0.150    5.743    0.000    0.498
##     Numericl_Pzzls    0.720    0.131    5.501    0.000    0.417
##     Series_Compltn    0.845    0.139    6.067    0.000    0.489
##     Prctcl_Prblm_S    0.907    0.156    5.803    0.000    0.524
##     Symbol_Manpltn    0.857    0.167    5.121    0.000    0.495
##     Analytcl_Ablty    0.967    0.172    5.607    0.000    0.559
##     Formal_Logic      1.012    0.178    5.694    0.000    0.585
##   Std.all
##
##     0.593
##     0.590
##     0.579
##     0.577
##     0.457
##     0.518
##     0.422
##     0.508
##     0.561
##     0.510
##     0.569
##     0.577
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          -0.008    0.044   -0.175    0.861   -0.008
##    .Prgrph_Cmprhns   -0.004    0.048   -0.079    0.937   -0.004
##    .Vocabulary       -0.015    0.045   -0.331    0.741   -0.015
##    .Sentenc_Cmpltn   -0.088    0.049   -1.808    0.071   -0.088
##    .Geometry          0.011    0.044    0.243    0.808    0.011
##    .Algebra          -0.029    0.043   -0.666    0.505   -0.029
##    .Numericl_Pzzls   -0.015    0.044   -0.341    0.733   -0.015
##    .Series_Compltn   -0.006    0.043   -0.147    0.883   -0.006
##    .Prctcl_Prblm_S   -0.017    0.042   -0.402    0.688   -0.017
##    .Symbol_Manpltn    0.001    0.043    0.019    0.985    0.001
##    .Analytcl_Ablty   -0.005    0.044   -0.106    0.916   -0.005
##    .Formal_Logic     -0.063    0.045   -1.385    0.166   -0.063
##     Ability           0.000                               0.000
##   Std.all
##    -0.008
##    -0.004
##    -0.015
##    -0.081
##     0.011
##    -0.030
```

```
##    -0.015
##    -0.007
##    -0.018
##     0.001
##    -0.005
##    -0.062
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar           0.617    0.054   11.492    0.000    0.617
##    .Prgrph_Cmprhns    0.735    0.065   11.303    0.000    0.735
##    .Vocabulary        0.662    0.056   11.857    0.000    0.662
##    .Sentenc_Cmpltn    0.786    0.070   11.263    0.000    0.786
##    .Geometry          0.757    0.060   12.660    0.000    0.757
##    .Algebra           0.675    0.053   12.733    0.000    0.675
##    .Numericl_Pzzls    0.803    0.058   13.861    0.000    0.803
##    .Series_Compltn    0.688    0.049   13.934    0.000    0.688
##    .Prctcl_Prblm_S    0.600    0.058   10.269    0.000    0.600
##    .Symbol_Manpltn    0.697    0.062   11.331    0.000    0.697
##    .Analytcl_Ablty    0.653    0.061   10.625    0.000    0.653
##    .Formal_Logic      0.688    0.067   10.351    0.000    0.688
##     Ability           0.334    0.069    4.864    0.000    1.000
##   Std.all
##     0.649
##     0.651
##     0.664
##     0.667
##     0.791
##     0.731
##     0.822
##     0.742
##     0.686
##     0.740
##     0.676
##     0.668
##     1.000
##
## R-Square:
##                    Estimate
##     Grammar           0.351
##     Prgrph_Cmprhns    0.349
##     Vocabulary        0.336
##     Sentenc_Cmpltn    0.333
##     Geometry          0.209
##     Algebra           0.269
##     Numericl_Pzzls    0.178
##     Series_Compltn    0.258
##     Prctcl_Prblm_S    0.314
##     Symbol_Manpltn    0.260
##     Analytcl_Ablty    0.324
##     Formal_Logic      0.332

semPaths(CFA_Fit_3, title = FALSE, "std", edge.label.cex = 0.5, curvePivot = TRUE,
```

```
    layout = "tree2", style = "lisrel", residuals = FALSE, sizeLat = 5,
    sizeMan = 5, intercepts = FALSE, what = "path", edge.color = "black")
```



4.4 Model 3 Alternative

```
mental.model.3b <- "
# Latent variable definitions.
# Scale of the latent variables is not set by the first listed manifest variable.
Verbal =~ NA*Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion
Math =~ NA*Geometry+Algebra+Numerical_Puzzles+Series_Completion
Reasoning =~ NA*Practical_Problem_Solving+Symbol_Manipulation+Analytical_Ability+Formal_Logic
# Latent variable covariances are set equal and to 1. Along with the standardization,
```

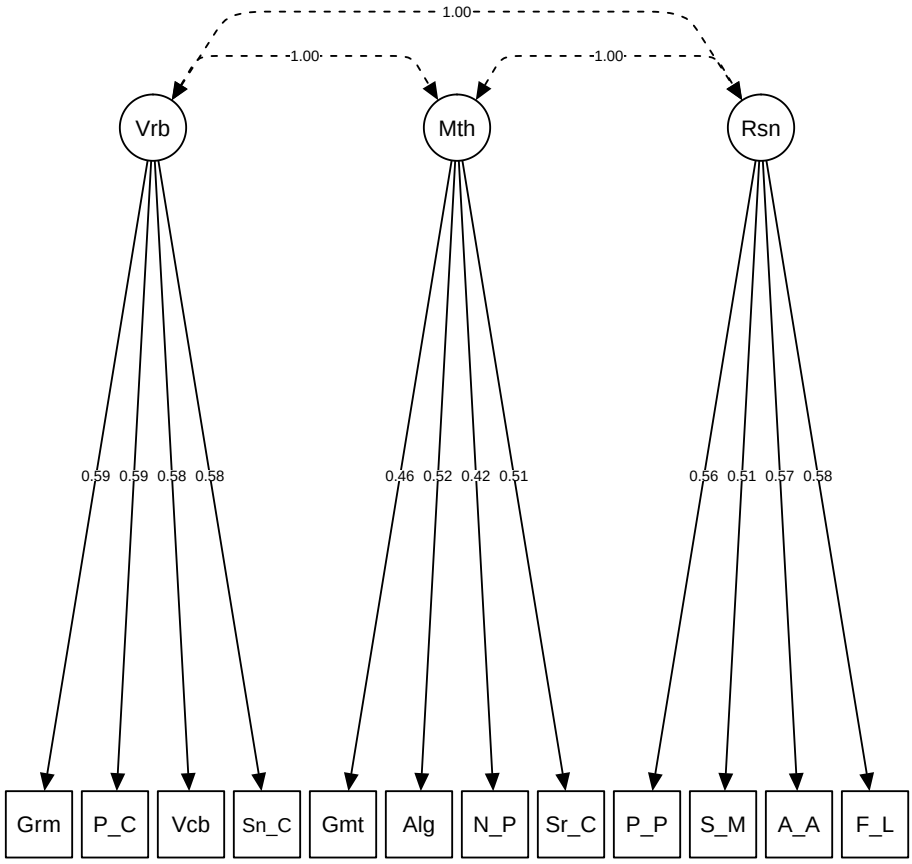
```
# this forces all three latent variables to be perfectly correlated, equivalent to
# a single latent variable, as in the previous model.
Verbal ~~ 1*Math
Verbal ~~ 1*Reasoning
Math ~~ 1*Reasoning
# Scale of the latent variables is set to 1 (standardized).
Verbal ~~ 1*Verbal
Math ~~ 1*Math
Reasoning ~~ 1*Reasoning
"
CFA_Fit_3b <- cfa(mental.model.3b, data = Mental, missing = "ML",
    estimator = "MLR", likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_3b, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                NA
##   90 Percent Confidence Interval                              NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.112       0.112
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal =~
##     Grammar           0.578    0.059    9.727    0.000    0.578
##     Prgrph_Cmprhns    0.627    0.063    9.913    0.000    0.627
##     Vocabulary        0.578    0.057   10.168    0.000    0.578
##     Sentenc_Cmpltn    0.626    0.067    9.345    0.000    0.626
##   Math =~
##     Geometry          0.447    0.058    7.701    0.000    0.447
##     Algebra           0.498    0.058    8.600    0.000    0.498
##     Numericl_Pzzls    0.417    0.056    7.468    0.000    0.417
##     Series_Compltn    0.489    0.052    9.397    0.000    0.489
##   Reasoning =~
##     Prctcl_Prblm_S    0.524    0.054    9.688    0.000    0.524
##     Symbol_Manpltn    0.495    0.066    7.488    0.000    0.495
##     Analytcl_Ablty    0.559    0.062    9.058    0.000    0.559
##     Formal_Logic      0.585    0.065    8.957    0.000    0.585
##   Std.all
##
##     0.593
##     0.590
##     0.579
##     0.577
##
##     0.457
##     0.518
##     0.422
##     0.508
##
##     0.561
##     0.510
##     0.569
##     0.577
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal ~~
##     Math              1.000                               1.000
##     Reasoning         1.000                               1.000
```

```
##   Math ~~
##     Reasoning         1.000                                1.000
##   Std.all
##
##     1.000
##     1.000
##
##     1.000
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          -0.008    0.044   -0.175    0.861   -0.008
##    .Prgrph_Cmprhns   -0.004    0.048   -0.079    0.937   -0.004
##    .Vocabulary       -0.015    0.045   -0.331    0.741   -0.015
##    .Sentenc_Cmpltn   -0.088    0.049   -1.808    0.071   -0.088
##    .Geometry          0.011    0.044    0.243    0.808    0.011
##    .Algebra          -0.029    0.043   -0.666    0.505   -0.029
##    .Numericl_Pzzls   -0.015    0.044   -0.341    0.733   -0.015
##    .Series_Compltn   -0.006    0.043   -0.147    0.883   -0.006
##    .Prctcl_Prblm_S   -0.017    0.042   -0.402    0.688   -0.017
##    .Symbol_Manpltn    0.001    0.043    0.019    0.985    0.001
##    .Analytcl_Ablty   -0.005    0.044   -0.106    0.916   -0.005
##    .Formal_Logic     -0.063    0.045   -1.385    0.166   -0.063
##     Verbal            0.000                                0.000
##     Math              0.000                                0.000
##     Reasoning         0.000                                0.000
##   Std.all
##    -0.008
##    -0.004
##    -0.015
##    -0.081
##     0.011
##    -0.030
##    -0.015
##    -0.007
##    -0.018
##     0.001
##    -0.005
##    -0.062
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     Verbal           1.000                                1.000
##     Math             1.000                                1.000
##     Reasoning        1.000                                1.000
##    .Grammar          0.617    0.054   11.492    0.000    0.617
##    .Prgrph_Cmprhns   0.735    0.065   11.303    0.000    0.735
##    .Vocabulary       0.662    0.056   11.857    0.000    0.662
##    .Sentenc_Cmpltn   0.786    0.070   11.263    0.000    0.786
##    .Geometry         0.757    0.060   12.660    0.000    0.757
##    .Algebra          0.675    0.053   12.733    0.000    0.675
```

```
##    .Numericl_Pzzls    0.803    0.058   13.861    0.000    0.803
##    .Series_Compltn    0.688    0.049   13.934    0.000    0.688
##    .Prctcl_Prblm_S    0.600    0.058   10.269    0.000    0.600
##    .Symbol_Manpltn    0.697    0.062   11.331    0.000    0.697
##    .Analytcl_Ablty    0.653    0.061   10.625    0.000    0.653
##    .Formal_Logic      0.688    0.067   10.351    0.000    0.688
##   Std.all
##     1.000
##     1.000
##     1.000
##     0.649
##     0.651
##     0.664
##     0.667
##     0.791
##     0.731
##     0.822
##     0.742
##     0.686
##     0.740
##     0.676
##     0.668
##
## R-Square:
##                    Estimate
##     Grammar           0.351
##     Prgrph_Cmprhns    0.349
##     Vocabulary        0.336
##     Sentenc_Cmpltn    0.333
##     Geometry          0.209
##     Algebra           0.269
##     Numericl_Pzzls    0.178
##     Series_Compltn    0.258
##     Prctcl_Prblm_S    0.314
##     Symbol_Manpltn    0.260
##     Analytcl_Ablty    0.324
##     Formal_Logic      0.332
```

```
semPaths(CFA_Fit_3b, title = FALSE, "std", edge.label.cex = 0.5, curvePivot = TRUE,
    layout = "tree2", style = "lisrel", residuals = FALSE, sizeLat = 5,
    sizeMan = 5, intercepts = FALSE, what = "path", edge.color = "black")
```



4.5 Model 4

```
mental.model.4 <- "
# Latent variable definitions.
# Scale of the latent variables is not set by the first listed manifest variable.
# This allows each loading to be estimated and all set equal.
Verbal =~ NA*Grammar+Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion
Math =~ NA*Geometry+Geometry+Algebra+Numerical_Puzzles+Series_Completion
Reasoning =~ NA*Practical_Problem_Solving+Practical_Problem_Solving+Symbol_Manipulation+Analytical_Abili
# Latent variable covariances are set equal.
Verbal ~~ a*Math
Verbal ~~ a*Reasoning
Math ~~ a*Reasoning
```

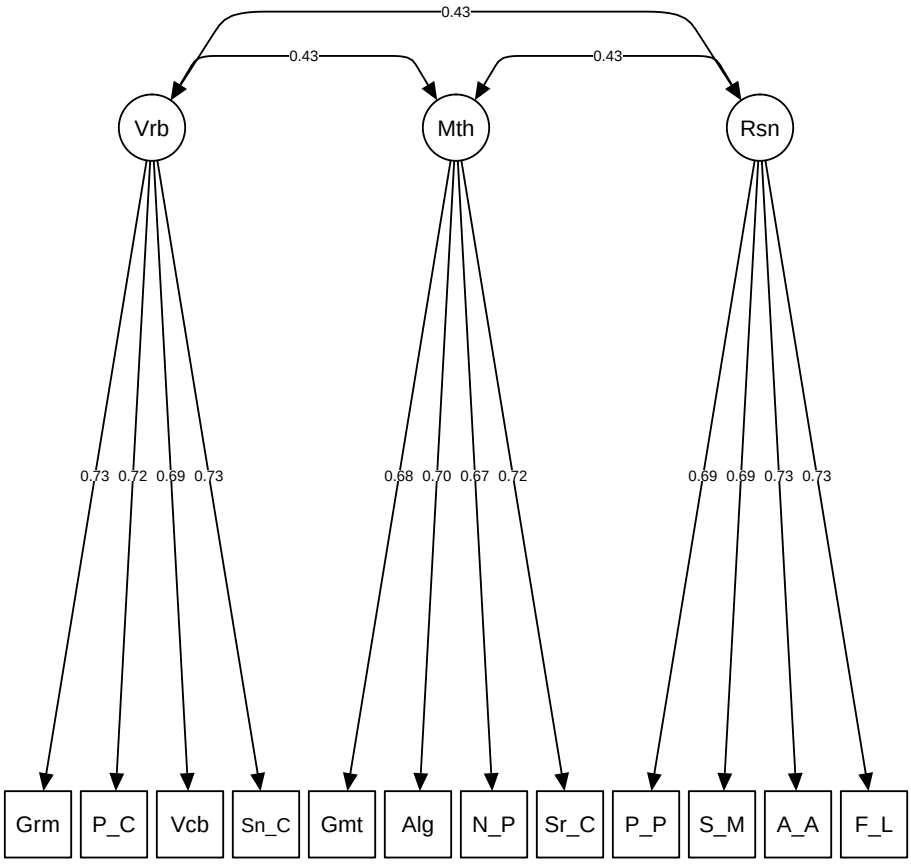
```
# Scale of the latent variables is set to 1 (standardized).
Verbal ~~ 1*Verbal
Math ~~ 1*Math
Reasoning ~~ 1*Reasoning
"
CFA_Fit_4 <- cfa(mental.model.4, data = Mental, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_4, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.027       0.027
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal =~
##     Grammar           0.715    0.042   17.024    0.000    0.715
##     Prgrph_Cmprhns    0.762    0.044   17.345    0.000    0.762
##     Vocabulary        0.685    0.041   16.598    0.000    0.685
##     Sentenc_Cmpltn    0.793    0.046   17.400    0.000    0.793
##   Math =~
##     Geometry          0.669    0.042   15.938    0.000    0.669
##     Algebra           0.672    0.045   14.980    0.000    0.672
##     Numericl_Pzzls    0.664    0.044   15.055    0.000    0.664
##     Series_Compltn    0.691    0.041   16.748    0.000    0.691
##   Reasoning =~
##     Prctcl_Prblm_S    0.650    0.038   17.251    0.000    0.650
##     Symbol_Manpltn    0.670    0.045   14.817    0.000    0.670
##     Analytcl_Ablty    0.713    0.044   16.335    0.000    0.713
##     Formal_Logic      0.743    0.046   16.102    0.000    0.743
##   Std.all
##
##     0.733
##     0.718
##     0.688
##     0.731
##
##     0.684
##     0.699
##     0.672
##     0.717
##
##     0.695
##     0.690
##     0.726
##     0.732
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal ~~
##     Math       (a)    0.428    0.034   12.716    0.000    0.428
##     Reasoning  (a)    0.428    0.034   12.716    0.000    0.428
##   Math ~~
##     Reasoning  (a)    0.428    0.034   12.716    0.000    0.428
##   Std.all
##
##     0.428
```

```
##     0.428
##
##     0.428
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          -0.008    0.044   -0.175    0.861   -0.008
##    .Prgrph_Cmprhns   -0.004    0.048   -0.079    0.937   -0.004
##    .Vocabulary       -0.015    0.045   -0.331    0.741   -0.015
##    .Sentenc_Cmpltn   -0.088    0.049   -1.808    0.071   -0.088
##    .Geometry          0.011    0.044    0.243    0.808    0.011
##    .Algebra          -0.029    0.043   -0.666    0.505   -0.029
##    .Numericl_Pzzls   -0.015    0.044   -0.341    0.733   -0.015
##    .Series_Compltn   -0.006    0.043   -0.147    0.883   -0.006
##    .Prctcl_Prblm_S   -0.017    0.042   -0.402    0.688   -0.017
##    .Symbol_Manpltn    0.001    0.043    0.019    0.985    0.001
##    .Analytcl_Ablty   -0.005    0.044   -0.106    0.916   -0.005
##    .Formal_Logic     -0.063    0.045   -1.385    0.166   -0.063
##     Verbal            0.000                              0.000
##     Math              0.000                              0.000
##     Reasoning         0.000                              0.000
##   Std.all
##    -0.008
##    -0.004
##    -0.015
##    -0.081
##     0.011
##    -0.030
##    -0.015
##    -0.007
##    -0.018
##     0.001
##    -0.005
##    -0.062
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     Verbal           1.000                              1.000
##     Math             1.000                              1.000
##     Reasoning        1.000                              1.000
##    .Grammar          0.439    0.036   12.336    0.000    0.439
##    .Prgrph_Cmprhns   0.546    0.044   12.363    0.000    0.546
##    .Vocabulary       0.524    0.041   12.719    0.000    0.524
##    .Sentenc_Cmpltn   0.547    0.044   12.368    0.000    0.547
##    .Geometry         0.511    0.046   11.183    0.000    0.511
##    .Algebra          0.473    0.041   11.478    0.000    0.473
##    .Numericl_Pzzls   0.537    0.045   11.864    0.000    0.537
##    .Series_Compltn   0.452    0.040   11.390    0.000    0.452
##    .Prctcl_Prblm_S   0.452    0.039   11.570    0.000    0.452
##    .Symbol_Manpltn   0.493    0.042   11.822    0.000    0.493
##    .Analytcl_Ablty   0.456    0.042   10.825    0.000    0.456
```

```
##    .Formal_Logic      0.478    0.041   11.752    0.000    0.478
##   Std.all
##     1.000
##     1.000
##     1.000
##     0.462
##     0.485
##     0.527
##     0.465
##     0.533
##     0.511
##     0.549
##     0.486
##     0.517
##     0.523
##     0.473
##     0.464
##
## R-Square:
##                    Estimate
##     Grammar           0.538
##     Prgrph_Cmprhns    0.515
##     Vocabulary        0.473
##     Sentenc_Cmpltn    0.535
##     Geometry          0.467
##     Algebra           0.489
##     Numericl_Pzzls    0.451
##     Series_Compltn    0.514
##     Prctcl_Prblm_S    0.483
##     Symbol_Manpltn    0.477
##     Analytcl_Ablty    0.527
##     Formal_Logic      0.536

semPaths(CFA_Fit_4, title = FALSE, "std", edge.label.cex = 0.5, curvePivot = TRUE,
    layout = "tree2", style = "lisrel", residuals = FALSE, sizeLat = 5,
    sizeMan = 5, intercepts = FALSE, what = "path", edge.color = "black")
```



5 Model Comparisons

Nested models can be compared. The difference between -2*log likelihood for each model is chi-square distributed, with degrees of freedom equal to the difference between the degrees of freedom for each model.

```
anova(CFA_Fit_1, CFA_Fit_2)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## CFA_Fit_1 51 15099 15263  55.5
## CFA_Fit_2 54 15252 15403 214.0        158       3     <2e-16
```

```r
anova(CFA_Fit_1, CFA_Fit_3)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## CFA_Fit_1 51 15099 15263  55.5
## CFA_Fit_3 54 15809 15961 770.6        715       3     <2e-16

anova(CFA_Fit_1, CFA_Fit_4)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## CFA_Fit_1 51 15099 15263  55.5
## CFA_Fit_4 53 15096 15252  56.5       1.02       2        0.6
```