

Homework 1

Applied Multivariate Analysis

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1 Workspace

1.1 Packages

```
library(car)
library(knitr)
library(kableExtra)
library(plyr)
library(tidyverse)

## -- Attaching packages -----
tidyverse 1.2.1 --
## ggplot2 2.2.1      purrr  0.2.4
## tibble  1.4.2      dplyr  0.7.5
## tidyr   0.8.0      stringr 1.3.0
## readr   1.1.1      forcats 0.3.0
## -- Conflicts -----
tidyverse_conflicts() --
## x dplyr::arrange() masks plyr::arrange()
## x purrr::compact() masks plyr::compact()
## x dplyr::count()   masks plyr::count()
## x dplyr::failwith() masks plyr::failwith()
## x dplyr::filter()  masks stats::filter()
## x dplyr::id()       masks plyr::id()
## x dplyr::lag()      masks stats::lag()
## x dplyr::mutate()   masks plyr::mutate()
## x dplyr::recode()   masks car::recode()
## x dplyr::rename()   masks plyr::rename()
## x purrr::some()     masks car::some()
## x dplyr::summarise() masks plyr::summarise()
## x dplyr::summarize() masks plyr::summarize()
```

1.2 data

```
dat <- tribble(
  ~Group, ~V1, ~V2, ~V3, ~V4,
  1, 4, 9, 3, 8,
  2, 6, 7, 2, 1,
  3, 1, 6, 6, 2,
```

```
4, 3, 8, 7, 4
)
X <- dat %>% select(-Group) %>% as.matrix
```

```
dat %>%
  kable(., "latex", escape = F) %>%
  column_spec(1, bold = T)
```

Group	V1	V2	V3	V4
1	4	9	3	8
2	6	7	2	1
3	1	6	6	2
4	3	8	7	4

The general matrix equation, LXM, describes how to create linear combinations of the groups (the L matrix) and variables (the M matrix) to test different hypotheses.

2 Part 1

In words, describe what each of the following L vectors is trying to accomplish:

```
L1 <- c(1, 1, 1, 1)
L2 <- c(1, 0, 0, -1)
L3 <- c(1, 0, 0, 0)
L4 <- c(1, 1, -2, 0)
L5 <- c(1, 1, -1, -1)
```

2.1 Question 1

$[1 \ 1 \ 1 \ 1] = L1$

The grand mean across groups.

2.2 Question 2

$[1 \ 0 \ 0 \ -1] = L2$

The difference in means between the first and fourth groups.

2.3 Question 3

$[1 \ 0 \ 0 \ 0] = L3$

The mean of the first group.

2.4 Question 4

$[1 \ 1 \ -2 \ 0] = L4$

The difference in means between groups 1+2 and 3.

2.5 Question 5

5. $[1 \ 1 \ -1 \ -1] = L5$

The difference in means between groups 1+2 and 3+4.

3 Part 2

Assume that you want to answer the following "variable" questions. Give the appropriate M vector or matrix.

3.1 Question 6

6. The linear combination given by L should be performed separately on each of the variables (= M1).

```
(M1 <- diag(1, 4, 4))  
  
##      [,1] [,2] [,3] [,4]  
## [1,]    1    0    0    0  
## [2,]    0    1    0    0  
## [3,]    0    0    1    0  
## [4,]    0    0    0    1
```

3.2 Question 7

7. Variable 3 is the only variable of interest (= M2).

```
(M2 <- c(0,0,1,0))  
  
## [1] 0 0 1 0
```

3.3 Question 8

8. The difference between Variables 1 and 4 is of interest (=M3).

```
(M3 <- c(1, 0, -1, 0))  
  
## [1] 1 0 -1 0
```

3.4 Question 9

9. The sum of all variables is of interest (= M4).

```
(M4 <- rep(1,4))  
  
## [1] 1 1 1 1
```

3.5 Question 10

10. The difference between the first two variables and the difference between the second two variables is to be compared (= M5).

```
(M5 <- c(1, -1, -1, 1))  
  
## [1] 1 -1 -1 1
```

4 Part 3

Carry out the following matrix multiplications, in R.

4.1 Question 11

$$L1XM2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 4 & 9 & 3 & 8 \\ 6 & 7 & 2 & 1 \\ 1 & 6 & 6 & 2 \\ 3 & 8 & 7 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 10 \end{bmatrix}$$

```
L1 %*% X %*% M2
```

```
##      [,1]  
## [1,]   18
```

4.2 Question 12

$$L2XM5 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 4 & 9 & 3 & 8 \\ 6 & 7 & 2 & 1 \\ 1 & 6 & 6 & 2 \\ 3 & 8 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & -11 \end{bmatrix}$$

```
L2 %*% X %*% M5
```

```
##      [,1]  
## [1,]    8
```

4.3 Question 13

$$L3XM3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 4 & 9 & 3 & 8 \\ 6 & 7 & 2 & 1 \\ 1 & 6 & 6 & 2 \\ 3 & 8 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -10 \end{bmatrix}$$

```
L3 %*% X %*% M3
```

```
##      [,1]  
## [1,]    1
```

4.4 Question 14

$$L4XM4 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix} \begin{bmatrix} 4 & 9 & 3 & 8 \\ 6 & 7 & 2 & 1 \\ 1 & 6 & 6 & 2 \\ 3 & 8 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

```
L4 %*% X %*% M4
```

```
##      [,1]  
## [1,]   10
```

4.5 Question 15

$$L5XM1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \begin{bmatrix} 4 & 9 & 3 & 8 \\ 6 & 7 & 2 & 1 \\ 1 & 6 & 6 & 2 \\ 3 & 8 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
L5 %*% X %*% M1
```

```
##      [,1] [,2] [,3] [,4]  
## [1,]    6    2  -8    3
```