Canonical Correlation Analysis I

Mike Strube

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1 Preliminaries

The RStudio workspace and console panes are cleared of old output, variables, and other miscellaneous debris. Then some packages are loaded and the required data files are input.

1.1 Clear the Console Panes and Load Packages

```
library(psych)
## Warning: package 'psych' was built under R version 3.5.1
library(ggplot2)
## Warning: package 'ggplot2' was built under R version 3.5.1
##
## Attaching package: 'ggplot2'
## The following objects are masked from 'package:psych':
##
## %+%, alpha
library(MASS)
## Warning: package 'MASS' was built under R version 3.5.1
library(sciplot)
library(dplyr)
```

```
## Warning: package 'dplyr' was built under R version 3.5.1
##
## Attaching package: 'dplyr'
## The following object is masked from 'package:MASS':
##
##
      select
## The following objects are masked from 'package:stats':
##
##
      filter, lag
## The following objects are masked from 'package:base':
##
      intersect, setdiff, setequal, union
library(aod)
library(MVN)
## sROC 0.1-2 loaded
library(boot)
##
## Attaching package: 'boot'
## The following object is masked from 'package:psych':
##
##
      logit
library(car)
## Warning: package 'car' was built under R version 3.5.1
## Loading required package: carData
## Warning: package 'carData' was built under R version 3.5.1
##
## Attaching package: 'car'
## The following object is masked from 'package:boot':
##
##
      logit
## The following object is masked from 'package:dplyr':
##
##
      recode
## The following object is masked from 'package:psych':
##
##
      logit
library(LogisticDx)
library(biotools)
## Loading required package: rpanel
## Loading required package: tcltk
## Package 'rpanel', version 1.1-4: type help(rpanel) for summary information
##
## Attaching package: 'rpanel'
## The following object is masked from 'package:boot':
##
      poisons
## Loading required package: tkrplot
## Warning: package 'tkrplot' was built under R version 3.5.1
## Loading required package: lattice
```

```
## Warning: package 'lattice' was built under R version 3.5.1
##
## Attaching package: 'lattice'
## The following object is masked from 'package:boot':
##
##
      melanoma
## Loading required package: SpatialEpi
## Loading required package: sp
## ---
## biotools version 3.1
##
library(multcomp)
## Loading required package: mutnorm
## Loading required package: survival
## Warning: package 'survival' was built under R version 3.5.1
## Attaching package: 'survival'
## The following object is masked from 'package:boot':
##
##
## The following object is masked from 'package:aod':
##
##
      rats
## Loading required package: TH.data
## Warning: package 'TH.data' was built under R version 3.5.1
##
## Attaching package: 'TH. data'
## The following object is masked from 'package:MASS':
##
##
      qeyser
library(candisc)
## Loading required package: heplots
##
## Attaching package: 'heplots'
## The following object is masked from 'package:biotools':
##
##
      boxM
##
## Attaching package: 'candisc'
## The following object is masked from 'package:stats':
##
##
      cancor
library(ez)
library(GGally)
## Attaching package: 'GGally'
## The following object is masked from 'package:dplyr':
##
## nasa
```

```
library(qqplotr)
## Warning: package 'qqplotr' was built under R version 3.5.1
##
## Attaching package: 'qqplotr'
## The following objects are masked from 'package:ggplot2':
##
##
      stat_qq_line, StatQqLine
library(gridExtra)
##
## Attaching package: 'gridExtra'
## The following object is masked from 'package:dplyr':
##
##
      combine
library(reshape)
## Warning: package 'reshape' was built under R version 3.5.1
##
## Attaching package: 'reshape'
## The following object is masked from 'package:dplyr':
##
##
      rename
library(emmeans)
## Warning: package 'emmeans' was built under R version 3.5.1
##
## Attaching package: 'emmeans'
## The following object is masked from 'package: GGally':
##
##
## The following object is masked from 'package:multcomp':
##
##
      cld
library(profileR)
## Warning: package 'profileR' was built under R version 3.5.1
## Loading required package: RColorBrewer
## Loading required package: lavaan
## Warning: package 'lavaan' was built under R version 3.5.1
## This is lavaan 0.6-3
## lavaan is BETA software! Please report any bugs.
## Attaching package: 'lavaan'
## The following object is masked from 'package:psych':
##
##
      cor2cov
library(Rmisc)
## Loading required package: plyr
## You have loaded plyr after dplyr - this is likely to cause problems.
## If you need functions from both plyr and dplyr, please load plyr first, then dplyr:
## library(plyr); library(dplyr)
```

```
##
##
##
## Attaching package: 'plyr'
## The following objects are masked from 'package:reshape':
##
## rename, round_any
## The following objects are masked from 'package:dplyr':
##
## arrange, count, desc, failwith, id, mutate, rename,
## summarise, summarize
```

1.2 Data

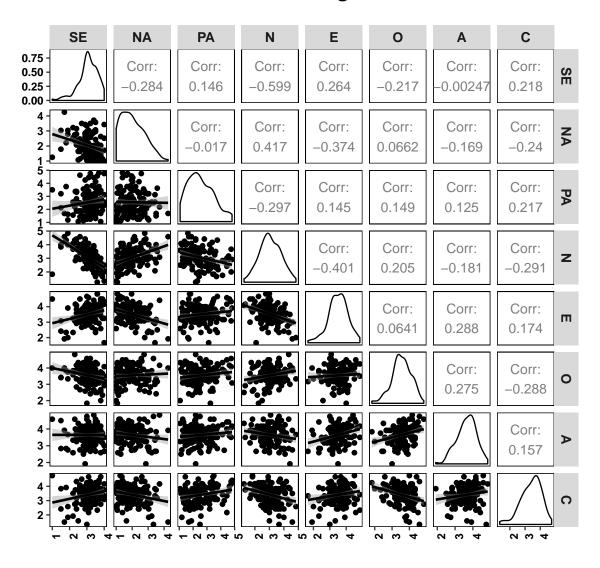
```
setwd("C:\\Courses\\Psychology 516\\PowerPoint\\2018")

# Get the data for the main MANOVA examples.
CCA <- read.table("cancorr_neo.csv", sep = ",", header = TRUE)
CCA <- as.data.frame(CCA)</pre>
```

1.3 Basic Visualization

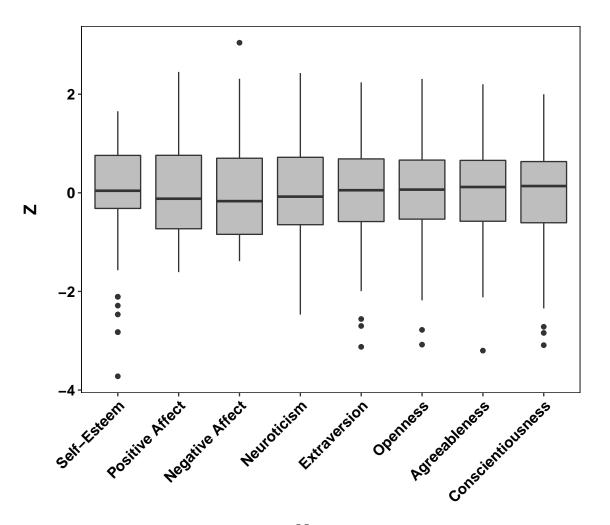
The basic nature of the data is easily viewed with some simple graphics.

Correlations Among Measures



```
size = 12, face = "bold", angle = 45, hjust = 1), axis.title.x = element_text(margin = margin(15, 0, 0, 0), size = 14), axis.title.y = element_text(margin = margin(0, 15, 0, 0), size = 14), axis.line.x = element_blank(), axis.line.y = element_blank(), plot.title = element_text(size = 16, face = "bold", margin = margin(0, 0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white", linetype = 1, color = "black"), panel.grid.major = element_blank(), panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"), plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom", legend.title = element_blank()) + ggtitle("Standardized Outcomes")
```

Standardized Outcomes



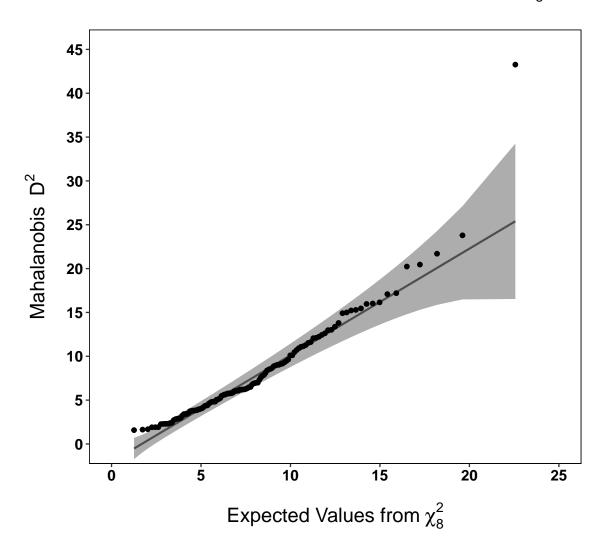
Measure

2 Multivariate Normality Assumption

2.1 Full Sample

```
mvn(CCA[, 2:9], mvnTest = "mardia")
## $multivariateNormality
                 Test
                               Statistic
                                                        p value Result
## 1 Mardia Skewness 299.119034297691 2.71647280885726e-17
## 2 Mardia Kurtosis 7.1931311285724 6.33271213246189e-13
                                                                     NO
                  MVN
                                    < N A >
                                                                     NO
## $univariateNormality
              Test Variable Statistic p value Normality
## 1 Shapiro-Wilk rosen_se 0.9457 0.0001
## 2 Shapiro-Wilk panas_n
                                0.9529
                                          0.0002
## 3 Shapiro-Wilk panas_p 0.9617 0.0013
## 4 Shapiro-Wilk neo_n 0.9934 0.8261
## 5 Shapiro-Wilk neo_e 0.9848 0.1705
## 6 Shapiro-Wilk neo_o 0.9853 0.1906
## 7 Shapiro-Wilk neo_a 0.9859 0.2162
## 8 Shapiro-Wilk neo_c 0.9712 0.0085
                                                       YES
                                                     YES
                                                     YES
                                                       YES
##
## $Descriptives
              n Mean Std.Dev Median Min
                                                 Max 25th 75th
## rosen_se 126 3.076 0.5580 3.100 1.000 4.000 2.900 3.500
## panas n 126 2.017 0.7336 1.893 1.000 4.250 1.399 2.531
## panas_p 126 2.548 0.9232 2.438 1.062 4.812 1.875 3.250
## neo_n 126 3.056 0.7314 3.000 1.250 4.833 2.583 3.583
## neo_e 126 3.510 0.5902 3.542 1.667 4.833 3.167 3.917
## neo_o 126 3.547 0.5565 3.583 1.833 4.833 3.250 3.917
## neo_a 126 3.645 0.5399 3.708 1.917 4.833 3.333 4.000
## neo_c 126 3.408 0.6713 3.500 1.333 4.750 3.000 3.833
                Skew Kurtosis
## rosen_se -0.89779 1.25663
## panas_n 0.55904 -0.42080
## panas_p 0.51108 -0.46035
## neo_n
             0.01739 -0.38834
## neo_e -0.35119 0.39208
## neo_o -0.33584 0.14004
## neo_a -0.34782 -0.08027
## neo_c -0.61659 0.31163
```

Q–Q Plot of Mahalanobis D^2 vs. Quantiles of χ_8^2

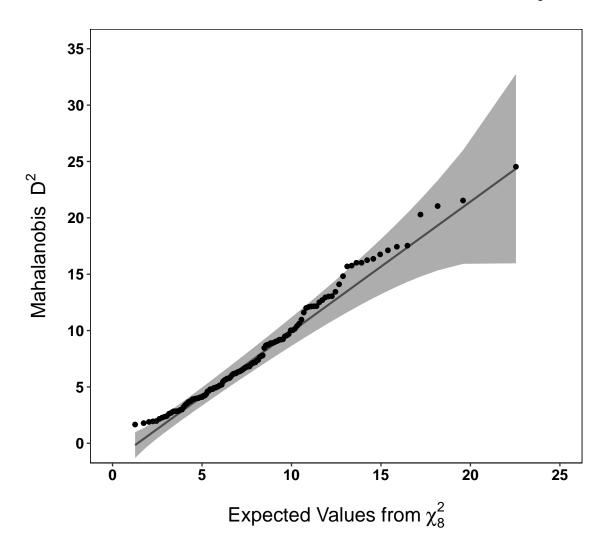


2.2 Outlier Excluded

```
CCA$D2 1 <- D2 1
CCA_Trimmed <- CCA[which(CCA$D2_1 != max(CCA$D2_1)), ]</pre>
mvn(CCA_Trimmed[, 2:9], mvnTest = "mardia")
## $multivariateNormality
                                                      p value Result
               Test
                             Statistic
## 1 Mardia Skewness 205.806610974633 0.0000018153144861216
## 2 Mardia Kurtosis 3.33221975072556 0.000861561958803936
                                                                    NO
## 3
          MVN
                                  < N A >
                                                          < NA >
                                                                    NΠ
##
## $univariateNormality
             Test Variable Statistic p value Normality
## 1 Shapiro-Wilk rosen_se 0.9589 0.0008
## 2 Shapiro-Wilk panas_n 0.9543 0.0003
## 3 Shapiro-Wilk panas_n 0.9606 0.0011 NO ## 4 Shapiro-Wilk neo_n 0.9935 0.8380 YES ## 5 Shapiro-Wilk neo_e 0.9844 0.1612 YES ## 6 Shapiro-Wilk neo_o 0.9853 0.1948 YES ## 7 Shapiro-Wilk neo_a 0.9855 0.2042 YES ## 8 Shapiro-Wilk neo c 0.9669 0.0037 NO
## 8 Shapiro-Wilk neo_c
                               0.9669 0.0037
                                                     NO
## $Descriptives
             n Mean Std.Dev Median Min Max 25th 75th
## rosen se 125 3.093 0.5280 3.100 1.500 4.000 2.900 3.500
## panas_n 125 2.023 0.7333 1.893 1.000 4.250 1.417 2.536
## panas_p 125 2.540 0.9229 2.438 1.062 4.812 1.875 3.250
## neo_n 125 3.066 0.7261 3.000 1.250 4.833 2.583 3.583
## neo_e 125 3.503 0.5858 3.500 1.667 4.833 3.167 3.917
## neo_o 125 3.543 0.5573 3.583 1.833 4.833 3.250 3.917
## neo_a 125 3.637 0.5355 3.667 1.917 4.833 3.333 4.000
## neo_c 125 3.397 0.6631 3.500 1.333 4.667 3.000 3.833
## Skew Kurtosis
## rosen_se -0.67507 0.51642
## panas_n 0.54845 -0.42426
## panas_p 0.53140 -0.42896
## neo n
            0.01604 -0.35486
## neo_e -0.36034 0.43692
## neo_o -0.32212 0.13615
## neo_a -0.36084 -0.05427
## neo_c -0.66322 0.32854
CV <- cov(CCA_Trimmed[, 2:9])</pre>
D2_1 <- mahalanobis(CCA_Trimmed[, 2:9], center = colMeans(CCA_Trimmed[,
    2:9]), cov = CV)
D2_1 <- as.data.frame(D2_1)</pre>
ggplot(D2_1, aes(sample = D2_1)) + stat_qq_band(distribution = "chisq",
    dparams = list(df = 8)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 8)) + stat_qq(distribution = "qchisq", dparams = list(df = 8)) +
    scale_y_continuous(breaks = seq(0, 35, 5)) + scale_x_continuous(breaks = seq(0,
  (25, 5)) + coord_cartesian(xlim = c(0, 25), ylim = c(0, 35)) +
```

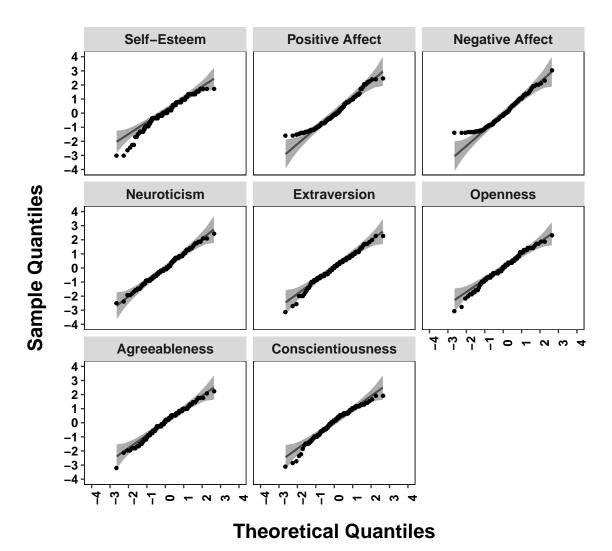
```
xlab(expression("Expected Values from" * ~chi[8]^2)) + ylab(expression("Mahalanobis " * ~D^2)) + theme(text = element_text(size = 14, family = "sans",
color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15, 0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0, 15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
plot.title = element_text(size = 16, face = "bold", margin = margin(0, 0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white", linetype = 1, color = "black"), panel.grid.major = element_blank(),
panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" * ~D^2 * " vs. Quantiles of" * ~chi[8]^2))
```

Q–Q Plot of Mahalanobis D^2 vs. Quantiles of χ_8^2



```
CCA_Trimmed_QQ <- scale(CCA_Trimmed[, 2:9])</pre>
plot_data <- melt(CCA_Trimmed_QQ)</pre>
names(plot_data) <- c("Case", "Measure", "Z")</pre>
plot_data$Measure_F <- factor(plot_data$Measure, levels = c("rosen_se",</pre>
    "panas_p", "panas_n", "neo_n", "neo_e", "neo_o", "neo_a", "neo_c"),
    labels = c("Self-Esteem", "Positive Affect", "Negative Affect",
        "Neuroticism", "Extraversion", "Openness", "Agreeableness",
        "Conscientiousness"))
p <- ggplot(plot_data, aes(sample = Z)) + stat_qq_band() + stat_qq_line() +</pre>
    stat_qq(distribution = qnorm, size = 1) + scale_y_continuous(breaks = seq(-4,
    4, 1)) + scale_x_continuous(breaks = seq(-4, 4, 1)) + coord_cartesian(xlim = c(-4,
    4), ylim = c(-4, 4)) + xlab("Theoretical Quantiles") + ylab("Sample Quantiles") +
    theme(text = element_text(size = 14, family = "sans", color = "black",
        face = "bold"), axis.text.y = element_text(colour = "black",
        size = 10, face = "bold"), axis.text.x = element_text(colour = "black",
        size = 10, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
        0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
        15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
        plot.title = element_text(size = 16, face = "bold", margin = margin(0,
            0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
            linetype = 1, color = "black"), panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
        plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
        legend.title = element_blank()) + ggtitle("Q-Q Plots for Measures")
p + facet_wrap(~Measure_F)
```

Q-Q Plots for Measures



The QQ-plot looks good but we still violate multivariate normality. It would probably be prudent to verify any claims about significance with a bootstrap or permutation test.

3 Correlations

The correlations among the variables can be examined to detect simple patterns and to get a sense of the relations across sets.

```
## neo_a -0.1611 0.27112 0.26800 1.0000 0.1331
## neo_c -0.2719  0.15075 -0.30690  0.1331  1.0000
cor(CCA_Trimmed[, 2:4])
          rosen_se panas_n panas_p
## rosen_se 1.0000 -0.336316 0.188864
## panas_n -0.3363 1.000000 -0.008335
## panas_p
          0.1889 -0.008335 1.000000
cor(CCA_Trimmed[, 2:4], CCA_Trimmed[, 5:9])
            neo n neo e
                           neo o
                                   neo a
                                            neo c
## rosen_se -0.6966  0.3371 -0.20468  0.05339  0.2999
## panas_n 0.4093 -0.3656 0.07355 -0.15647 -0.2283
## panas_p -0.2875 0.1332 0.14284 0.11280 0.2043
```

4 Canonical Correlation Analysis

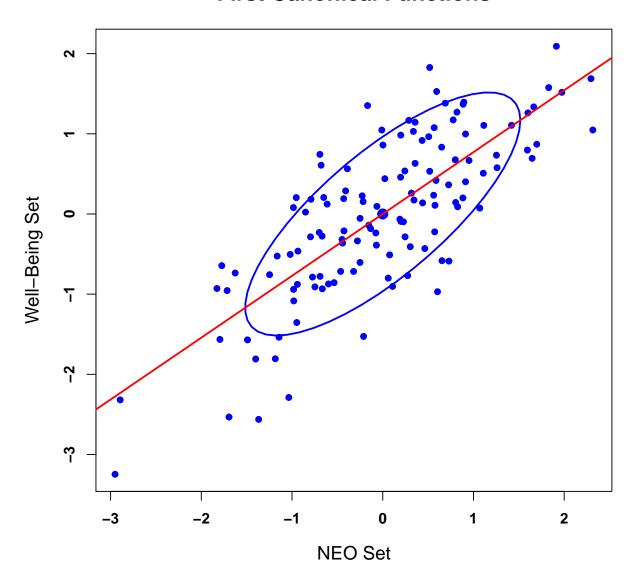
A simple canonical correlation analysis can be conducted using the cancor() function from the candisc package.

```
CCA_1 <- cancor(cbind(rosen_se, panas_n, panas_p) ~ neo_n + neo_e +</pre>
   neo_o + neo_a + neo_c, data = CCA_Trimmed, prefix = c("NEO", "Well_Being"),
    standardize = TRUE)
CCA_2 <- cancor(cbind(neo_n, neo_e, neo_o, neo_a, neo_c) ~ rosen_se +
    panas_n + panas_p, data = CCA_Trimmed, prefix = c("Well-Being",
    "NEO"), standardize = TRUE)
CCA_1
##
## Canonical correlation analysis of:
## 5 X variables: neo_n, neo_e, neo_o, neo_a, neo_c
   with 3 Y variables: rosen_se, panas_n, panas_p
##
##
      CanR CanRSQ Eigen percent
## 1 0.7715 0.59514 1.46997 91.875 91.87
## 2 0.2851 0.08127 0.08846 5.529 97.40
## 3 0.1997 0.03989 0.04155
                            2.597 100.00
                        scree
## 1 **************
## 2 **
## 3 *
## Test of HO: The canonical correlations in the
## current row and all that follow are zero
   CanR LR test stat approx F numDF denDF Pr(> F)
## 1 0.771
               0.357 9.75 15 323 <2e-16
                          1.91 8 236 0.059
## 2 0.285
                 0.882
## 3 0.200
                0.960
                          1.65
                                 3 119 0.182
CCA_1$coef
```

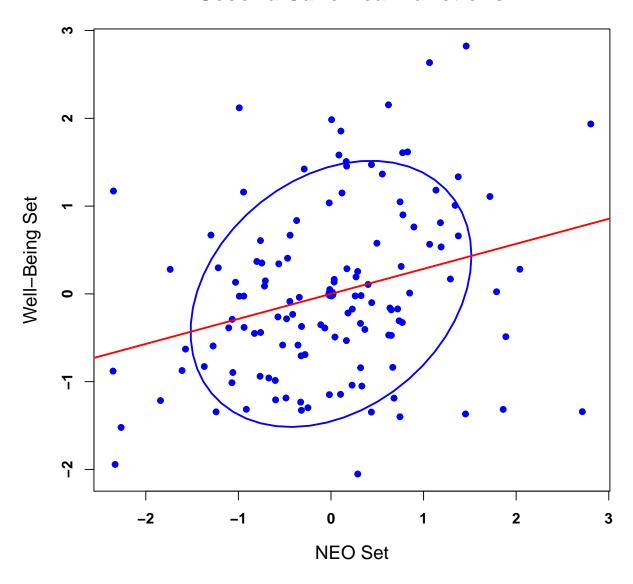
```
## NEO1 NEO2 NEO3
## neo_n -1.1781 0.09616 0.85933
## neo_e 0.3142 -0.11932 1.56070
## neo_o 0.1703 1.74253 -0.66598
## neo_a -0.1608  0.18465  0.80840
## neo_c 0.3678 0.85769 -0.08445
##
## $Y
       Well_Being1 Well_Being2 Well_Being3
## rosen_se 1.4263 -1.1365 -0.9394
## panas_n
             -0.4516 -0.3591
                                  -1.3310
             0.2981 1.0199 -0.3044
## panas_p
CCA_1$structure
## $X.xscores
## NE01 NE02 NE03
## neo_n -0.9582 0.13900 0.13441
## neo_e 0.5335 0.06792 0.76190
## neo_o -0.1804 0.83456 -0.05204
## neo_a 0.1595 0.40462 0.47332
## neo_c 0.4636 0.25432 0.08372
## $Y.xscores
## NEO1 NEO2 NEO3
## rosen_se 0.7070 -0.09514 -0.04411
## panas_n -0.4526 -0.01978 -0.16115
## panas_p 0.3241 0.23666 -0.07320
## $X.yscores
## Well_Being1 Well_Being2 Well_Being3
## neo_n -0.7392 0.03963 0.02685
                   0.01936
## neo_e
          0.4116
                               0.15217
## neo_o -0.1392 0.23791 -0.01039
## neo_a 0.1230 0.11535 0.09454
## neo_c 0.3576 0.07250 0.01672
## $Y.yscores
## Well_Being1 Well_Being2 Well_Being3
## rosen_se 0.9164 -0.33373 -0.2209
## panas_n
             -0.5867 -0.06938
                                  -0.8068
             0.4201
                       0.83018
                                  -0.3665
## panas_p
coef(CCA_1, type = "both", standardize = TRUE)
## [[1]]
          NEO1 NEO2 NEO3
## neo_n -0.85535 0.06982 0.6239
## neo_e 0.18407 -0.06990 0.9143
## neo_o 0.09491 0.97106 -0.3711
## neo a -0.08610 0.09887 0.4329
## neo_c 0.24388 0.56869 -0.0560
##
## [[2]]
```

```
Well_Being1 Well_Being2 Well_Being3
                 0.7531
                             -0.6001
## rosen_se
## panas_n
                -0.3311
                             -0.2633
                                          -0.976
                 0.2751
                              0.9413
                                          -0.281
## panas_p
CCA_1_Scores <- cbind(CCA_1$scores$X, CCA_1$scores$Y)</pre>
plot(CCA_1, which = 1, main = list("First Canonical Functions", cex = 1.5),
    xlab = list("NEO Set", cex = 1.25), ylab = list("Well-Being Set",
        cex = 1.25), font.axis = 2, pch = 16, col = "blue")
```

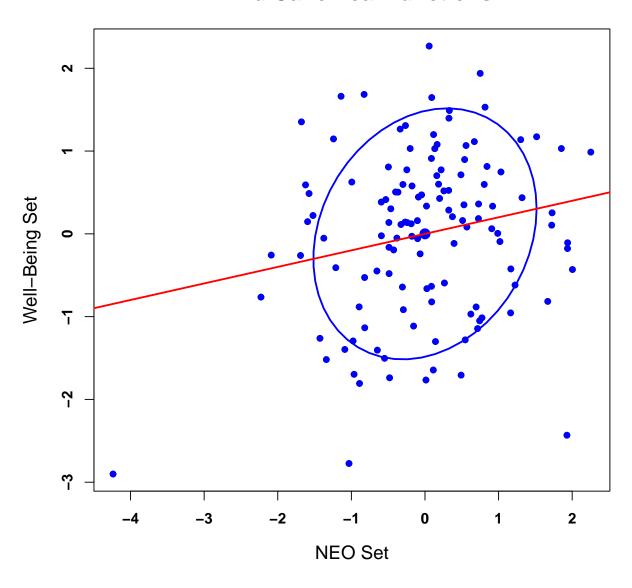
First Canonical Functions



Second Canonical Functions



Third Canonical Functions



```
Well_Being_Loadings <- CCA_1$structure$Y.yscores
NEO_Loadings <- CCA_1$structure$X.xscores

(Well_Being_Communalities <- rowSums((Well_Being_Loadings)^2))

## rosen_se panas_n panas_p
## 1 1 1

(NEO_Communalities <- rowSums((NEO_Loadings)^2))

## neo_n neo_e neo_o neo_a neo_c
## 0.9555 0.8697 0.7318 0.4132 0.2866

(Well_Being_Adequacy <- colSums((Well_Being_Loadings)^2)/length(Well_Being_Loadings[, 1]))</pre>
```

```
## Well_Being1 Well_Being2 Well_Being3
##
     0.4535
                 0.2685
                            0.2780
(NEO_Adequacy <- colSums((NEO_Loadings)^2)/length(NEO_Loadings[, 1]))
## NEO1 NEO2 NEO3
## 0.2951 0.1898 0.1665
(Well_Being_Redundancy <- Well_Being_Adequacy * (CCA_1$cancor)^2)</pre>
## Well_Being1 Well_Being2 Well_Being3
##
     0.26990
                0.02182
                           0.01109
(NEO_Redundancy <- NEO_Adequacy * (CCA_1$cancor)^2)
   NEO1 NEO2 NEO3
## 0.17565 0.01542 0.00664
redundancy(CCA_1)
##
## Redundancies for the X variables & total X canonical redundancy
##
      NEO1
               NEO2
                        NEO3 total X Y
## 0.17565 0.01542 0.00664 0.19771
##
## Redundancies for the Y variables & total Y canonical redundancy
## Well_Being1 Well_Being2 Well_Being3 total Y | X
     0.2699
                 0.0218
                            0.0111
                                       0.3028
# Including the outlier.
CCA_3 <- cancor(cbind(rosen_se, panas_n, panas_p) ~ neo_n + neo_e +
   neo_o + neo_a + neo_c, data = CCA, prefix = c("NEO", "Well-Being"),
   standardize = TRUE)
CCA_3
##
## Canonical correlation analysis of:
## 5 X variables: neo_n, neo_e, neo_o, neo_a, neo_c
   with 3 Y variables: rosen_se, panas_n, panas_p
##
      CanR CanRSQ Eigen percent
## 1 0.7130 0.50835 1.03395 86.536 86.54
## 2 0.3223 0.10391 0.11596
                           9.705 96.24
## 3 0.2073 0.04298 0.04491
                          3.759 100.00
                       scree
## 1 *************
## 2 ***
## 3 *
## Test of HO: The canonical correlations in the
## current row and all that follow are zero
## CanR LR test stat approx F numDF denDF Pr(> F)
          0.422 7.99 15 326 2.3e-15
## 1 0.713
                               8 238 0.018
## 2 0.322
               0.858
                         2.38
## 3 0.207     0.957     1.80     3     120     0.152
```

Correlations Among Measures

