Multidimensional Scaling II

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1 Preliminaries

In this section, the RStudio workspace and console panes are cleared of old output, variables, and other miscellaneous debris. Packages are loaded and any required data files are retrieved.

```
options(replace.assign = TRUE, width = 65, digits = 4, scipen = 4, fig.width = 4,
    fig.height = 4)
# Clear the workspace and console.
rm(list = ls(all = TRUE))
cat("\f")
```

```
# Turn off showing of significance asterisks.
options(show.signif.stars = F)
# Set the contrast option; important for ANOVAs.
options(contrasts = c("contr.sum", "contr.poly"))
how_long <- Sys.time()
set.seed(123)
library(knitr)
```

```
library(psych)
library(ggplot2)

##
## Attaching package: 'ggplot2'
## The following objects are masked from 'package:psych':
##
##     %+%, alpha

library(MASS)
library(sciplot)
library(ggplot2)
library(vegan)

## Warning: package 'vegan' was built under R version 3.5.1
## Loading required package: permute
## Warning: package 'permute' was built under R version 3.5.1
## Loading required package: lattice
## This is vegan 2.5-2

library(smacof)
```

```
## Warning:  package 'smacof' was built under R version 3.5.1
## Loading required package:  plotrix
##
## Attaching package:  'plotrix'
## The following object is masked from 'package:psych':
##
##    rescale
##
## Attaching package:  'smacof'
## The following object is masked from 'package:base':
##
##    transform
```

```
library(ape)
library(ade4)
```

```
## Warning:  package 'ade4' was built under R version 3.5.1
```

```
library(ecodist)
```

```
## Warning:  package 'ecodist' was built under R version 3.5.1
##
## Attaching package:  'ecodist'
## The following object is masked from 'package:vegan':
##
##    mantel
```

```
library(scatterplot3d)
```

1.1 Data Files

The data come from a 2017 C-SPAN survey of 91 historians who were asked to rank the presidents from George Washington through Barak Obama on 10 characteristics:

(a) Public persuasion (PP)

(b) Crisis leadership (CL)

(c) Economic management (EM)

(d) Moral authority (MA)

(e) International relations (IR)

(f) Administrative skills (AS)

(g) Relations with Congress (RC)

(h) Vision/Setting an agenda (VSA)

(i) Pursued equal justice for all (PEJ)

(j) Performance within context of times (PCT)

The composite ranks (across the 91 historians) are used in the analyses that follow.

```
# Get the drug use data from the working directory.
setwd("C:\\Courses\\Psychology 516\\PowerPoint\\2018")
Presidents <- read.table("presidents.csv", sep = ",", header = TRUE)
Presidents <- as.data.frame(Presidents)
row.names(Presidents) <- Presidents$President

# The euclidean distances are created from the ranks.
Presidents_Dist <- dist(Presidents[, 2:ncol(Presidents)], method = "euclidean",
    diag = TRUE)
Presidents_Dist
```

```
##             Washington   Adams Jefferson Madison  Monroe
## Washington       0.000
## Adams           49.629   0.000
## Jefferson       21.703  34.641     0.000
## Madison         46.098  16.733    27.129   0.000
## Monroe          37.014  25.080    20.616  21.048   0.000
## JQ_Adams        60.083  17.578    45.420  28.125  37.068
## Jackson         54.936  34.684    38.510  30.806  29.017
## Van_Buren       91.717  47.927    72.270  48.959  56.569
## WH_Harrison    107.620  62.490    88.312  63.474  74.740
## Tyler          108.849  62.921    90.405  66.821  73.851
## Polk            45.078  35.986    31.321  31.097  19.950
## Taylor          87.698  42.332    69.498  45.519  54.945
## Fillmore       106.353  60.581    87.966  63.403  71.868
## Pierce         118.966  73.212    99.910  75.260  84.350
## Buchanan       125.674  79.894   106.644  81.994  91.082
## Lincoln         12.490  51.118    26.363  48.570  42.237
## A_Johnson      121.165  74.619   102.548  77.807  87.000
## Grant           65.376  24.062    48.343  27.221  38.601
## Hayes           86.081  42.107    67.683  43.555  52.991
## Garfield        79.310  35.875    59.908  34.828  49.699
## Arthur          93.381  48.466    74.679  49.890  59.766
## Cleveland       64.962  26.702    46.314  25.710  31.369
## B_Harrison      85.988  40.410    67.000  42.367  52.134
## McKinley        42.685  22.561    26.211  16.643  14.900
## T_Roosevelt      9.381  43.093    16.882  40.137  32.094
## Taft            66.558  25.826    49.346  27.946  34.525
## Wilson          34.132  32.955    22.361  29.155  20.567
## Harding        110.833  64.831    92.081  67.209  76.929
## Coolidge        71.372  30.512    52.488  27.731  39.268
## Hoover          99.695  56.639    80.511  57.096  65.947
## F_Roosevelt      7.937  48.000    20.881  44.811  36.401
## Truman          26.476  30.166    22.405  31.686  26.777
## Eisenhower      19.698  33.897    17.176  31.129  24.739
## Kennedy         29.155  29.850    20.518  28.018  26.981
## L_Johnson       52.288  44.193    38.301  34.132  42.000
## Nixon           81.756  40.062    66.265  48.693  50.577
## Ford            76.276  32.078    58.626  33.601  44.944
## Carter          85.610  40.596    67.320  43.405  58.915
## Reagan          40.212  34.234    28.671  31.591  27.550
## G_H_W_Bush      50.130  16.733    35.805  21.071  21.048
## Clinton         55.254  34.699    44.159  36.959  38.432
```

```
## G_W_Bush        90.144  48.010    71.141  47.106  59.414
## Obama           53.179  28.054    42.767  33.926  44.587
##             JQ_Adams Jackson Van_Buren WH_Harrison   Tyler
## Washington
## Adams
## Jefferson
## Madison
## Monroe
## JQ_Adams       0.000
## Jackson       45.913   0.000
## Van_Buren     47.011  51.049     0.000
## WH_Harrison   61.498  62.081    25.612       0.000
## Tyler         60.564  64.140    23.707      19.799   0.000
## Polk          46.883  23.409    57.585      72.979  72.746
## Taylor        43.301  44.011    20.174      22.293  24.352
## Fillmore      58.626  63.190    22.517      17.292  10.247
## Pierce        70.349  74.034    31.225      17.861  15.000
## Buchanan      76.681  80.486    37.630      23.195  20.833
## Lincoln       59.883  60.249    94.053     110.209 111.937
## A_Johnson     71.379  76.974    34.799      20.421  16.643
## Grant         30.594  38.497    39.497      51.807  54.754
## Hayes         42.802  45.387    18.547      25.338  27.459
## Garfield      36.878  42.048    26.000      31.812  40.447
## Arthur        46.000  54.827    15.492      24.413  24.536
## Cleveland     33.407  25.179    29.665      44.362  45.497
## B_Harrison    39.192  48.724    12.083      28.107  28.213
## McKinley      35.100  28.178    51.904      67.424  68.775
## T_Roosevelt   53.796  49.173    84.947     100.767 102.372
## Taft          26.192  41.473    31.016      49.477  48.000
## Wilson        44.238  28.125    66.461      80.156  80.889
## Harding       62.944  68.088    25.417      15.362  16.673
## Coolidge      35.721  37.041    26.758      38.781  42.732
## Hoover        52.259  63.977    18.894      32.634  31.000
## F_Roosevelt   58.455  54.305    89.739     106.099 107.596
## Truman        39.762  45.967    73.478      90.161  90.371
## Eisenhower    45.563  46.797    76.000      92.542  93.424
## Kennedy       41.328  39.243    70.385      84.664  87.338
## L_Johnson     47.937  51.865    66.015      79.649  85.440
## Nixon         38.158  49.254    30.887      46.562  40.497
## Ford          32.802  48.374    27.276      43.336  43.749
## Carter        32.140  58.643    35.875      43.255  47.760
## Reagan        48.177  28.513    66.866      79.190  81.173
## G_H_W_Bush    28.302  36.674    46.787      65.521  64.008
## Clinton       41.964  45.508    58.078      71.715  73.695
## G_W_Bush      47.791  50.239    24.779      28.810  37.175
## Obama         30.397  46.000    64.560      73.185  77.679
##                Polk  Taylor Fillmore  Pierce Buchanan Lincoln
## Washington
## Adams
## Jefferson
## Madison
## Monroe
## JQ_Adams
## Jackson
```

```
## Van_Buren
## WH_Harrison
## Tyler
## Polk          0.000
## Taylor       53.768   0.000
## Fillmore     70.221  21.863    0.000
## Pierce       82.310  33.196   14.071   0.000
## Buchanan     88.566  39.686   20.322   7.141    0.000
## Lincoln      51.381  90.648  109.366 121.840  128.445   0.000
## A_Johnson    85.094  34.699   16.125   7.348    7.937 123.818
## Grant        46.776  35.369   52.754  63.820   70.071  65.605
## Hayes        51.049  10.724   22.338  34.191   40.373  88.904
## Garfield     51.575  20.518   36.729  45.858   51.981  80.585
## Arthur       59.279  18.788   18.193  28.896   34.670  95.341
## Cleveland    30.067  25.120   42.907  54.836   61.417  68.527
## B_Harrison   54.424  16.941   24.799  35.735   42.095  87.864
## McKinley     18.385  48.052   65.276  77.827   84.368  47.371
## T_Roosevelt  40.546  81.000  100.045 112.477  119.139  12.728
## Taft         38.210  32.109   44.215  56.560   62.881  68.717
## Wilson       21.378  60.415   78.842  90.951   97.658  42.178
## Harding      75.710  26.363   12.369  14.248   18.493 113.102
## Coolidge     40.792  22.956   37.961  49.568   56.125  74.686
## Hoover       67.845  33.615   28.531  33.317   38.613 101.514
## F_Roosevelt  45.727  86.475  105.319 117.754  124.447   9.000
## Truman       39.230  69.986   88.057 100.827  107.373  24.597
## Eisenhower   37.014  72.767   90.438 103.271  109.973  22.935
## Kennedy      33.971  65.184   84.493  96.825  103.189  28.249
## L_Johnson    43.589  65.612   80.306  90.637   96.187  50.418
## Nixon        51.127  31.129   41.000  51.643   56.886  82.861
## Ford         51.342  29.816   39.459  51.000   57.131  77.162
## Carter       66.280  36.000   45.453  52.839   58.352  84.812
## Reagan       33.302  60.067   79.737  91.553   98.056  44.621
## G_H_W_Bush   33.272  46.043   61.498  74.431   81.062  51.990
## Clinton      38.588  54.000   70.314  82.024   87.401  54.323
## G_W_Bush     58.498  24.515   32.573  40.037   45.122  90.841
## Obama        50.279  55.830   75.399  86.284   92.326  50.892
##             A_Johnson   Grant   Hayes Garfield  Arthur Cleveland
## Washington
## Adams
## Jefferson
## Madison
## Monroe
## JQ_Adams
## Jackson
## Van_Buren
## WH_Harrison
## Tyler
## Polk
## Taylor
## Fillmore
## Pierce
## Buchanan
## Lincoln
## A_Johnson       0.000
```

```
## Grant          64.969   0.000
## Hayes          36.042  36.442   0.000
## Garfield       47.823  24.900  20.050    0.000
## Arthur         31.097  40.817  12.806   24.617   0.000
## Cleveland      57.437  27.459  23.281   23.875  31.843     0.000
## B_Harrison     37.776  31.113  14.071   18.762  12.166    25.573
## McKinley       80.169  35.665  45.056   42.214  52.745    24.739
## T_Roosevelt   114.564  58.344  79.498   72.180  86.856    58.429
## Taft           58.745  32.711  26.608   29.631  30.100    19.748
## Wilson         93.552  50.170  59.607   56.630  68.593    38.536
## Harding        13.892  53.310  26.533   36.905  21.260    47.497
## Coolidge       52.086  28.071  18.815   18.439  26.192    15.232
## Hoover         37.656  52.393  30.282   36.837  22.956    42.202
## F_Roosevelt   119.917  62.330  85.000   77.246  91.962    63.804
## Truman        102.518  45.133  68.447   61.490  74.344    49.608
## Eisenhower    105.418  50.774  70.626   64.607  77.201    51.029
## Kennedy        98.382  40.743  63.561   55.209  70.640    44.317
## L_Johnson      92.978  47.833  59.967   50.872  63.340    46.797
## Nixon          51.624  38.210  30.757   36.986  33.226    30.757
## Ford           52.943  23.917  26.420   23.875  25.259    27.785
## Carter         53.907  32.202  36.290   24.556  33.838    40.878
## Reagan         93.360  37.537  61.115   53.451  69.477    41.000
## G_H_W_Bush     76.498  27.875  44.328   41.316  49.020    27.911
## Clinton        82.377  37.323  50.050   46.314  56.223    37.961
## G_W_Bush       42.226  34.088  23.108   18.547  21.071    32.125
## Obama          86.977  39.217  56.321   45.848  63.103    44.654
##             B_Harrison McKinley T_Roosevelt    Taft  Wilson
## Washington
## Adams
## Jefferson
## Madison
## Monroe
## JQ_Adams
## Jackson
## Van_Buren
## WH_Harrison
## Tyler
## Polk
## Taylor
## Fillmore
## Pierce
## Buchanan
## Lincoln
## A_Johnson
## Grant
## Hayes
## Garfield
## Arthur
## Cleveland
## B_Harrison       0.000
## McKinley        46.303    0.000
## T_Roosevelt     79.284   37.202       0.000
## Taft            25.495   28.531      60.366   0.000
## Wilson          62.330   22.023      30.545  44.576   0.000
```

```
## Harding         26.683   69.771     104.096  49.417  84.894
## Coolidge        20.199   30.529      65.269  19.748  46.787
## Hoover          25.159   61.066      93.236  36.318  75.100
## F_Roosevelt     84.125   42.485       6.856  65.429  36.249
## Truman          66.910   32.419      20.567  48.466  33.793
## Eisenhower      69.943   29.257      16.310  49.940  28.548
## Kennedy         63.403   27.092      21.541  47.455  31.097
## L_Johnson       59.313   36.028      47.032  45.453  48.713
## Nixon           28.460   46.583      74.337  30.033  59.958
## Ford            18.762   40.398      70.071  22.091  57.905
## Carter          28.337   54.470      78.797  36.865  67.587
## Reagan          60.918   30.545      35.454  52.067  31.081
## G_H_W_Bush      41.122   22.023      44.238  25.040  36.222
## Clinton         50.843   31.953      47.739  38.743  46.947
## G_W_Bush        20.591   52.000      83.042  37.363  68.096
## Obama           57.184   40.694      45.629  45.188  43.440
##             Harding Coolidge  Hoover F_Roosevelt  Truman
## Washington
## Adams
## Jefferson
## Madison
## Monroe
## JQ_Adams
## Jackson
## Van_Buren
## WH_Harrison
## Tyler
## Polk
## Taylor
## Fillmore
## Pierce
## Buchanan
## Lincoln
## A_Johnson
## Grant
## Hayes
## Garfield
## Arthur
## Cleveland
## B_Harrison
## McKinley
## T_Roosevelt
## Taft
## Wilson
## Harding       0.000
## Coolidge     41.425    0.000
## Hoover       32.388   36.973   0.000
## F_Roosevelt 109.229   70.406  97.898       0.000
## Truman       92.071   55.937  81.609      23.452   0.000
## Eisenhower   94.984   55.767  83.193      19.261  15.067
## Kennedy      87.293   50.833  80.256      26.096  16.401
## L_Johnson    81.449   48.125  71.617      50.388  42.907
## Nixon        43.474   36.414  41.581      79.341  62.185
## Ford         41.809   21.817  35.029      74.209  55.381
```

```
## Carter       45.398   35.482  39.013      83.114  65.299
## Reagan       82.662   48.528  80.511      38.859  33.793
## G_H_W_Bush   66.023   32.357  55.245      48.042  30.430
## Clinton      70.930   43.046  68.819      52.421  38.936
## G_W_Bush     30.529   29.086  35.285      87.755  71.421
## Obama        78.115   48.703  71.645      50.804  36.318
##             Eisenhower Kennedy L_Johnson   Nixon    Ford  Carter
## Washington
## Adams
## Jefferson
## Madison
## Monroe
## JQ_Adams
## Jackson
## Van_Buren
## WH_Harrison
## Tyler
## Polk
## Taylor
## Fillmore
## Pierce
## Buchanan
## Lincoln
## A_Johnson
## Grant
## Hayes
## Garfield
## Arthur
## Cleveland
## B_Harrison
## McKinley
## T_Roosevelt
## Taft
## Wilson
## Harding
## Coolidge
## Hoover
## F_Roosevelt
## Truman
## Eisenhower       0.000
## Kennedy         21.401   0.000
## L_Johnson       43.635  34.699     0.000
## Nixon           67.690  59.464    63.530   0.000
## Ford            58.720  54.203    51.284  35.944   0.000
## Carter          70.491  63.095    57.836  43.578  28.054   0.000
## Reagan          35.398  26.134    49.810  60.158  54.927  64.885
## G_H_W_Bush      32.604  32.939    45.244  40.559  30.017  47.686
## Clinton         44.193  29.950    36.263  42.485  45.837  56.921
## G_W_Bush        75.432  64.946    56.780  38.105  28.036  32.481
## Obama           43.220  32.311    46.712  53.889  51.108  48.239
##              Reagan G_H_W_Bush Clinton G_W_Bush   Obama
## Washington
## Adams
## Jefferson
```

```
## Madison
## Monroe
## JQ_Adams
## Jackson
## Van_Buren
## WH_Harrison
## Tyler
## Polk
## Taylor
## Fillmore
## Pierce
## Buchanan
## Lincoln
## A_Johnson
## Grant
## Hayes
## Garfield
## Arthur
## Cleveland
## B_Harrison
## McKinley
## T_Roosevelt
## Taft
## Wilson
## Harding
## Coolidge
## Hoover
## F_Roosevelt
## Truman
## Eisenhower
## Kennedy
## L_Johnson
## Nixon
## Ford
## Carter
## Reagan        0.000
## G_H_W_Bush   36.797      0.000
## Clinton      42.190     35.917   0.000
## G_W_Bush     63.159     49.346  52.355    0.000
## Obama        45.177     41.316  39.737   58.310   0.000

# If the ratings are provided on quite different scales, then they
# should be standardized before distances are calculated. Here is
# way to do that and modify the names if files are later combined.
# Standardization is not needed here because the data are ranks
# and so all scales have identical standard deviations.
Presidents_Z <- scale(Presidents[, 2:ncol(Presidents)])
Presidents_Z <- as.data.frame(Presidents_Z)
names(Presidents_Z) <- paste(names(Presidents[-1]), "_Z", sep = "")
Presidents_Dist_Z <- dist(scale(Presidents[, 2:ncol(Presidents)]),
    method = "euclidean", diag = TRUE)
# Presidents_Dist_Z

# Raw ranks can be converted to normalized ranks as follows. This
```

```r
# can be useful if there are missing data and thus different
# numbers of objects ranked across scales. This step is also
# unnecessary for for the current data because all objects were
# ranked for all scales.
Presidents_r <- Presidents[, 2:ncol(Presidents)] - 1
Presidents_NR <- matrix(NA, ncol = (length(Presidents_r[1, ])), nrow = length(Presidents_r[,
    1]))
for (j in seq(1, ncol(Presidents_r))) {
    for (i in seq(1, nrow(Presidents_r))) {
        Presidents_NR[i, j] <- Presidents_r[i, j]/(42)
    }
}

Presidents_NR <- as.data.frame(Presidents_NR)
names(Presidents_NR) <- paste(names(Presidents[-1]), "_NR", sep = "")
Presidents_Dist_NR <- dist(Presidents_NR, method = "euclidean", diag = TRUE)

Presidents_All <- cbind(Presidents, Presidents_Z, Presidents_NR)
# cor(Presidents_All[-1],,use='pairwise.complete.obs')

cor(Presidents[, 2:11])
```

```
##         PP     CL     EM     MA     IR     AS     RC    VSA
## PP  1.0000 0.9186 0.8686 0.7448 0.7440 0.7159 0.8127 0.9281
## CL  0.9186 1.0000 0.9002 0.8043 0.8706 0.7945 0.8408 0.9106
## EM  0.8686 0.9002 1.0000 0.7545 0.7981 0.8163 0.7753 0.8807
## MA  0.7448 0.8043 0.7545 1.0000 0.7419 0.7346 0.7085 0.8217
## IR  0.7440 0.8706 0.7981 0.7419 1.0000 0.7596 0.7066 0.7735
## AS  0.7159 0.7945 0.8163 0.7346 0.7596 1.0000 0.8025 0.7915
## RC  0.8127 0.8408 0.7753 0.7085 0.7066 0.8025 1.0000 0.8043
## VSA 0.9281 0.9106 0.8807 0.8217 0.7735 0.7915 0.8043 1.0000
## PEJ 0.5512 0.5817 0.6616 0.6392 0.5159 0.5667 0.5435 0.6222
## PCT 0.9230 0.9641 0.9166 0.8664 0.8443 0.8303 0.8774 0.9449
##        PEJ    PCT
## PP  0.5512 0.9230
## CL  0.5817 0.9641
## EM  0.6616 0.9166
## MA  0.6392 0.8664
## IR  0.5159 0.8443
## AS  0.5667 0.8303
## RC  0.5435 0.8774
## VSA 0.6222 0.9449
## PEJ 1.0000 0.6237
## PCT 0.6237 1.0000
```

The president ranks are best thought of as ordinal level, so a non-metric MDS is most appropriate.

2 Stress Plot

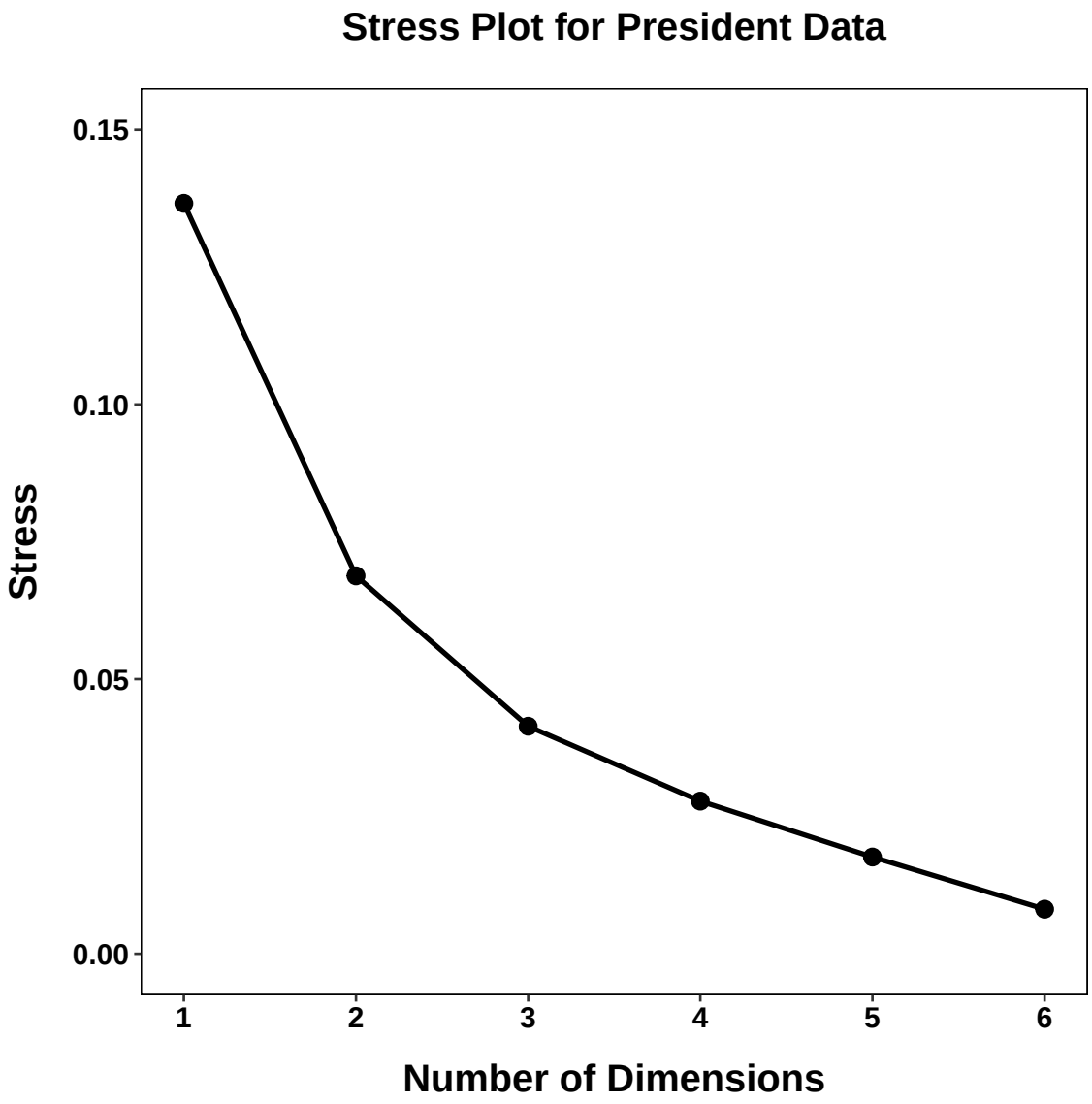
Stress is defined as:

$$Stress = \sqrt{\frac{\sum_{i=1}^{k}\sum_{j=1}^{k}(d_{ij} - \delta_{ij})^2}{\sum_{i=1}^{k}\sum_{j=1}^{k} d_{ij}^2}} \quad (i \neq j)$$

Here we run the smacofSym( ) function iteratively for 1 to 6 dimensions in order to get the stress values for plotting. The stress plot can help us determine the best number of dimensions to describing the data.

```
mds_stress <- matrix(NA, nrow = 6, ncol = 2)
for (i in 1:6) {
    mds_1 <- smacofSym(Presidents_Dist, ndim = i, verbose = FALSE,
        type = "ordinal", itmax = 1000)
    mds_stress[i, 1] <- i
    mds_stress[i, 2] <- mds_1$stress
}
```

```
plot_data <- as.data.frame(mds_stress)
names(plot_data) <- c("D", "Stress")

ggplot(plot_data, aes(x = D, y = Stress)) + geom_point(shape = 19,
    size = 3, color = "black", na.rm = TRUE) + geom_line(size = 1) +
    scale_y_continuous(breaks = c(seq(0, 0.15, 0.05))) + scale_x_continuous(breaks = c(seq(1,
    6, 1))) + coord_cartesian(xlim = c(1, 6), ylim = c(0, 0.15)) +
    xlab("Number of Dimensions") + ylab("Stress") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Stress Plot for President Data")
```



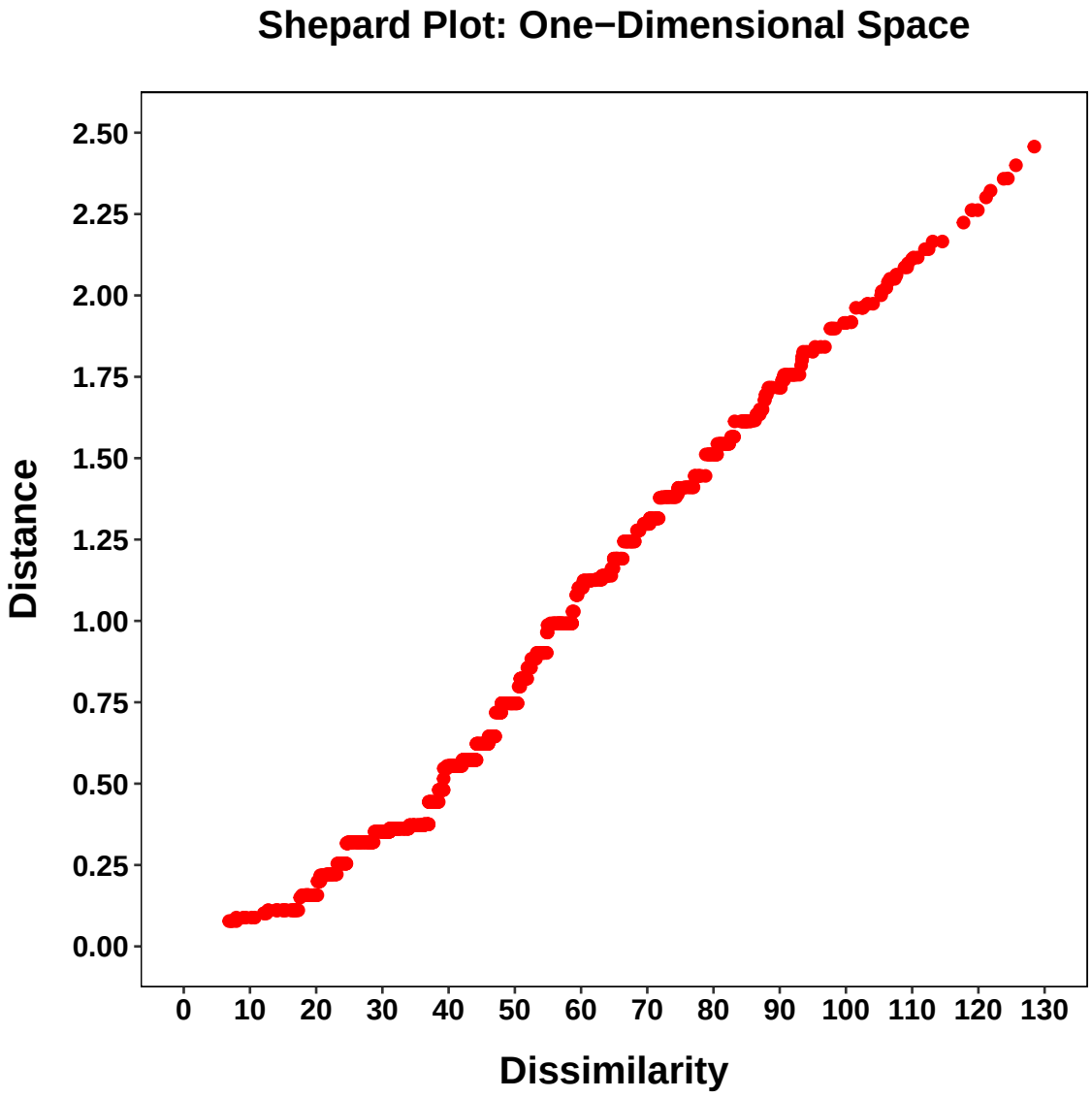
3 Shepard Plots

Shepard plots for up to 4 dimensions.

```
mds_2 <- smacofSym(Presidents_Dist, ndim = 1, verbose = FALSE, type = "ordinal",
    itmax = 1000)
Presidents_1_fits <- Shepard(Presidents_Dist, mds_2$conf)
mds_2 <- smacofSym(Presidents_Dist, ndim = 2, verbose = FALSE, type = "ordinal",
    itmax = 1000)
Presidents_2_fits <- Shepard(Presidents_Dist, mds_2$conf)
mds_2 <- smacofSym(Presidents_Dist, ndim = 3, verbose = FALSE, type = "ordinal",
    itmax = 1000)
Presidents_3_fits <- Shepard(Presidents_Dist, mds_2$conf)
```

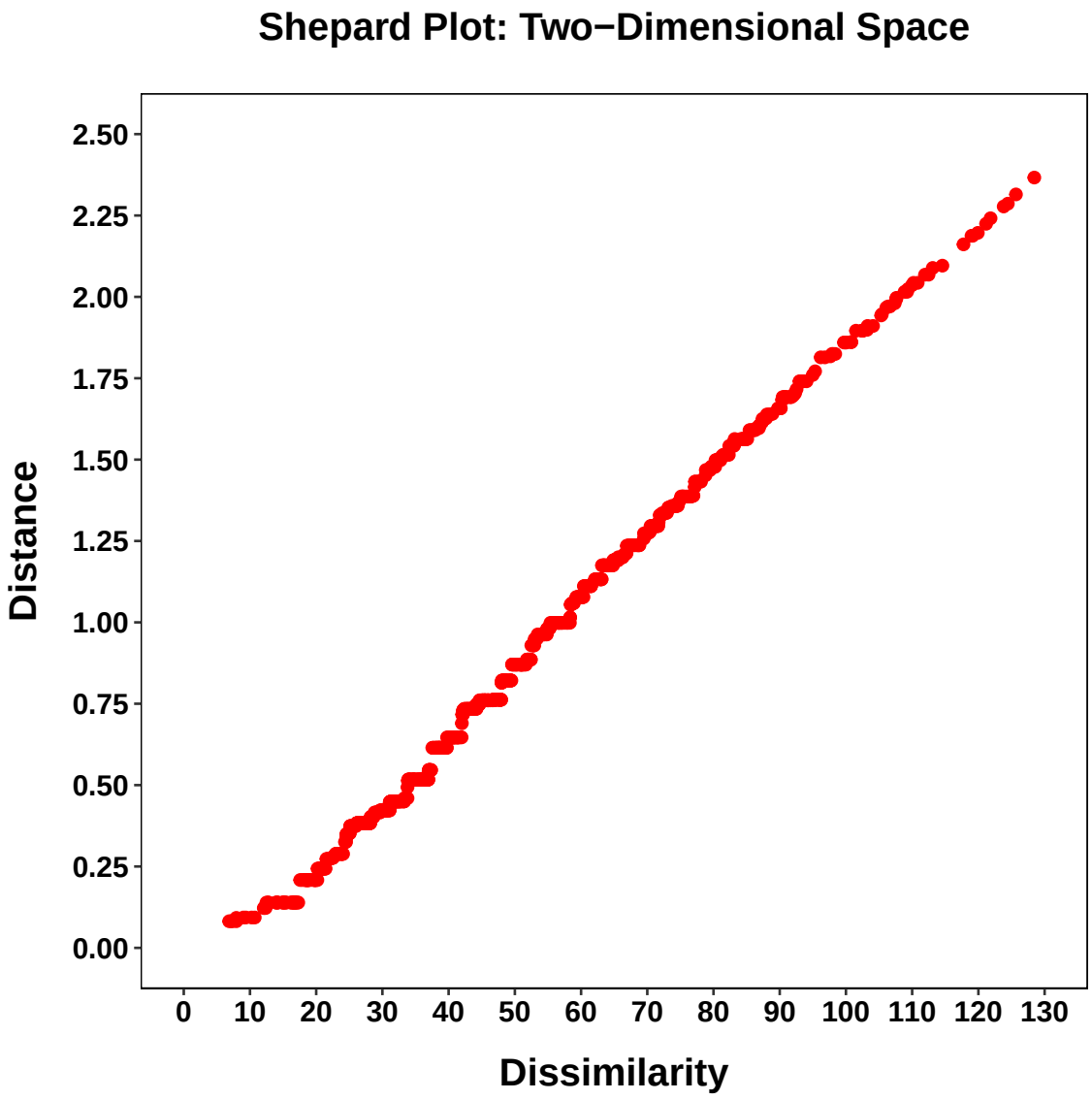
```
mds_2 <- smacofSym(Presidents_Dist, ndim = 4, verbose = FALSE, type = "ordinal",
    itmax = 1000)
Presidents_4_fits <- Shepard(Presidents_Dist, mds_2$conf)
```

```
plot_data <- as.data.frame(Presidents_1_fits)

ggplot(plot_data, aes(x = x, y = yf)) + geom_point(shape = 19, size = 2,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(0,
    2.5, 0.25))) + scale_x_continuous(breaks = c(seq(0, 130, 10))) +
    coord_cartesian(xlim = c(0, 130), ylim = c(0, 2.5)) + xlab("Dissimilarity") +
    ylab("Distance") + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Shepard Plot: One-Dimensional Space")
```



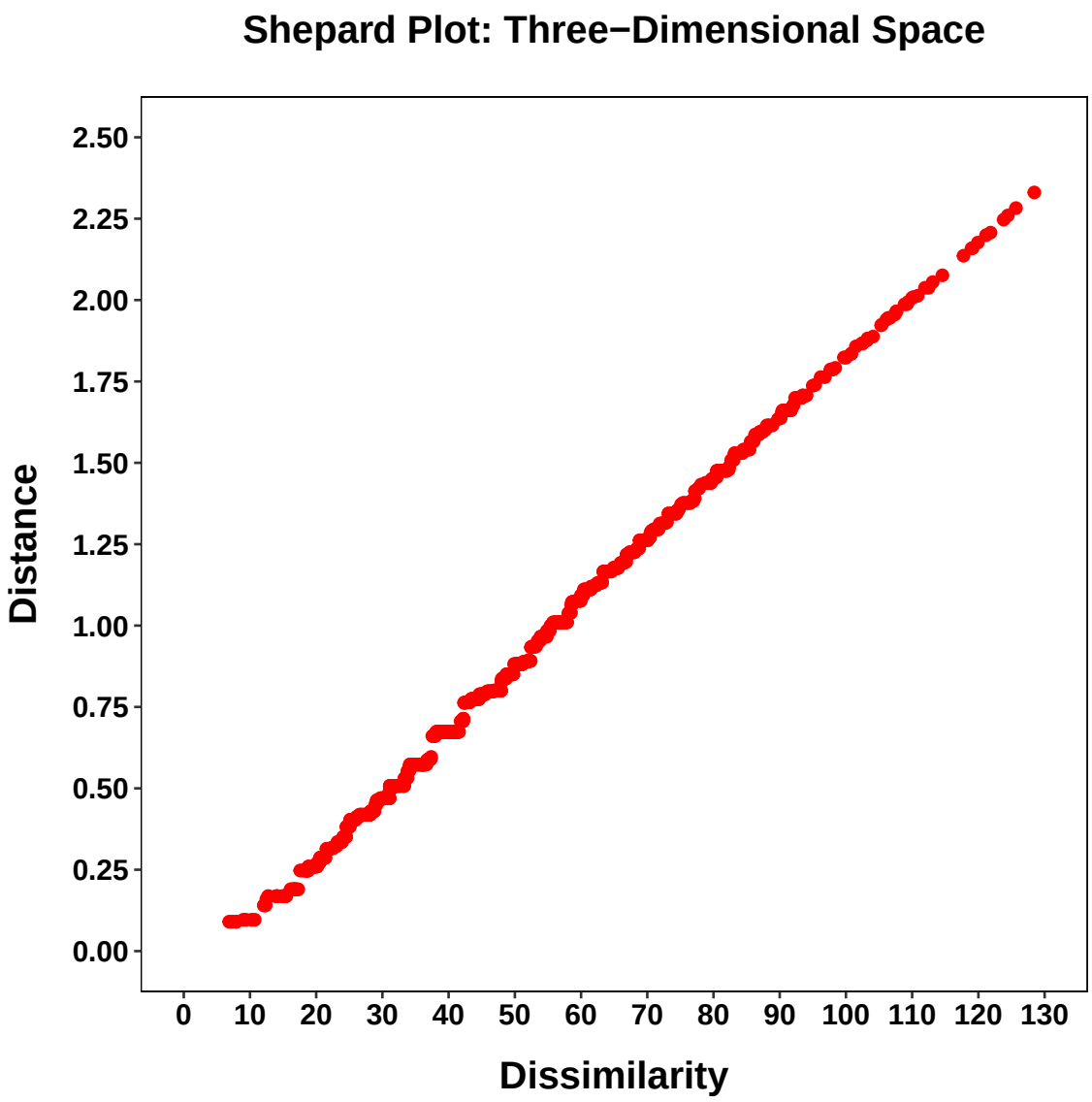

```
plot_data <- as.data.frame(Presidents_2_fits)

ggplot(plot_data, aes(x = x, y = yf)) + geom_point(shape = 19, size = 2,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(0,
    2.5, 0.25))) + scale_x_continuous(breaks = c(seq(0, 130, 10))) +
    coord_cartesian(xlim = c(0, 130), ylim = c(0, 2.5)) + xlab("Dissimilarity") +
    ylab("Distance") + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
```

```
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Shepard Plot: Two-Dimensional Space")
```



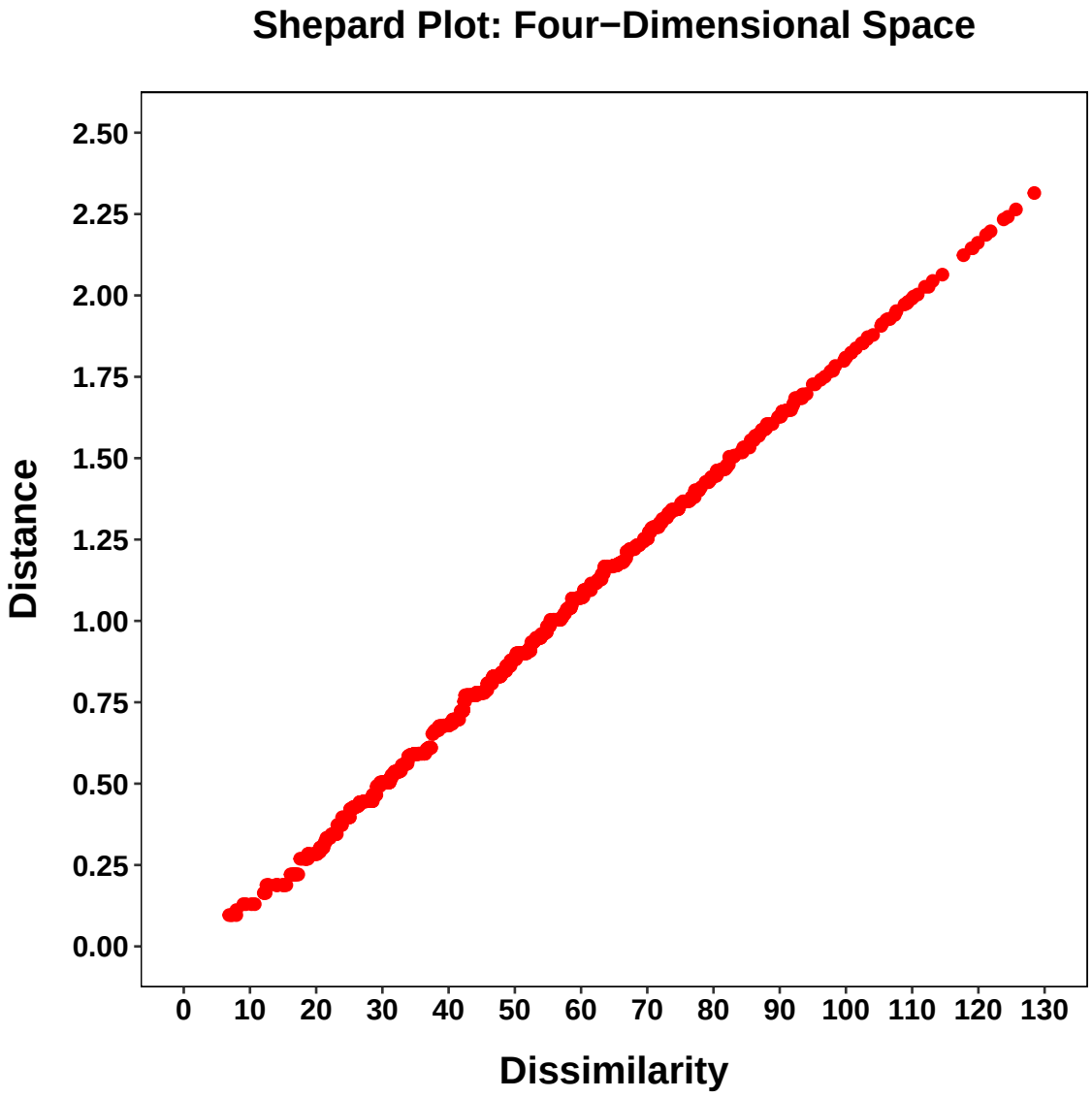

```
plot_data <- as.data.frame(Presidents_3_fits)

ggplot(plot_data, aes(x = x, y = yf)) + geom_point(shape = 19, size = 2,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(0,
    2.5, 0.25))) + scale_x_continuous(breaks = c(seq(0, 130, 10))) +
    coord_cartesian(xlim = c(0, 130), ylim = c(0, 2.5)) + xlab("Dissimilarity") +
    ylab("Distance") + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
```

```
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Shepard Plot: Three-Dimensional Space")
```



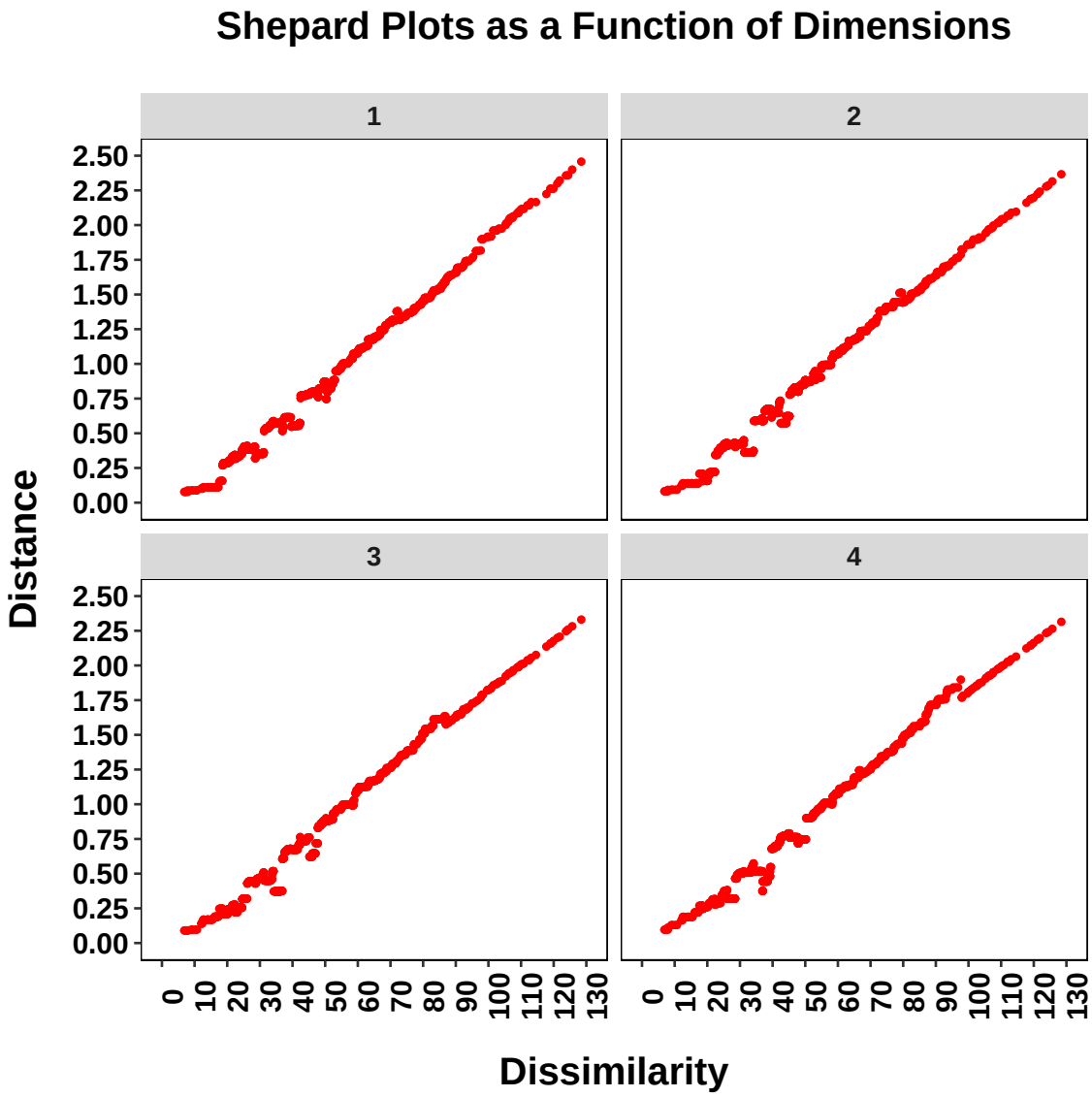

```
plot_data <- as.data.frame(Presidents_4_fits)
```

```
ggplot(plot_data, aes(x = x, y = yf)) + geom_point(shape = 19, size = 2,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(0,
    2.5, 0.25))) + scale_x_continuous(breaks = c(seq(0, 130, 10))) +
    coord_cartesian(xlim = c(0, 130), ylim = c(0, 2.5)) + xlab("Dissimilarity") +
    ylab("Distance") + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Shepard Plot: Four-Dimensional Space")
```




```
plot_data <- as.data.frame(Presidents_1_fits)
plot_data <- rbind(plot_data, Presidents_2_fits)
plot_data <- rbind(plot_data, Presidents_3_fits)
plot_data <- rbind(plot_data, Presidents_4_fits)
plot_data$D <- c(rep(1, 43), rep(2, 43), rep(3, 43), rep(4, 43))

p <- ggplot(plot_data, aes(x = x, y = yf)) + geom_point(shape = 19,
    size = 1, color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(0,
    2.5, 0.25))) + scale_x_continuous(breaks = c(seq(0, 130, 10))) +
    coord_cartesian(xlim = c(0, 130), ylim = c(0, 2.5)) + xlab("Dissimilarity") +
    ylab("Distance") + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
```

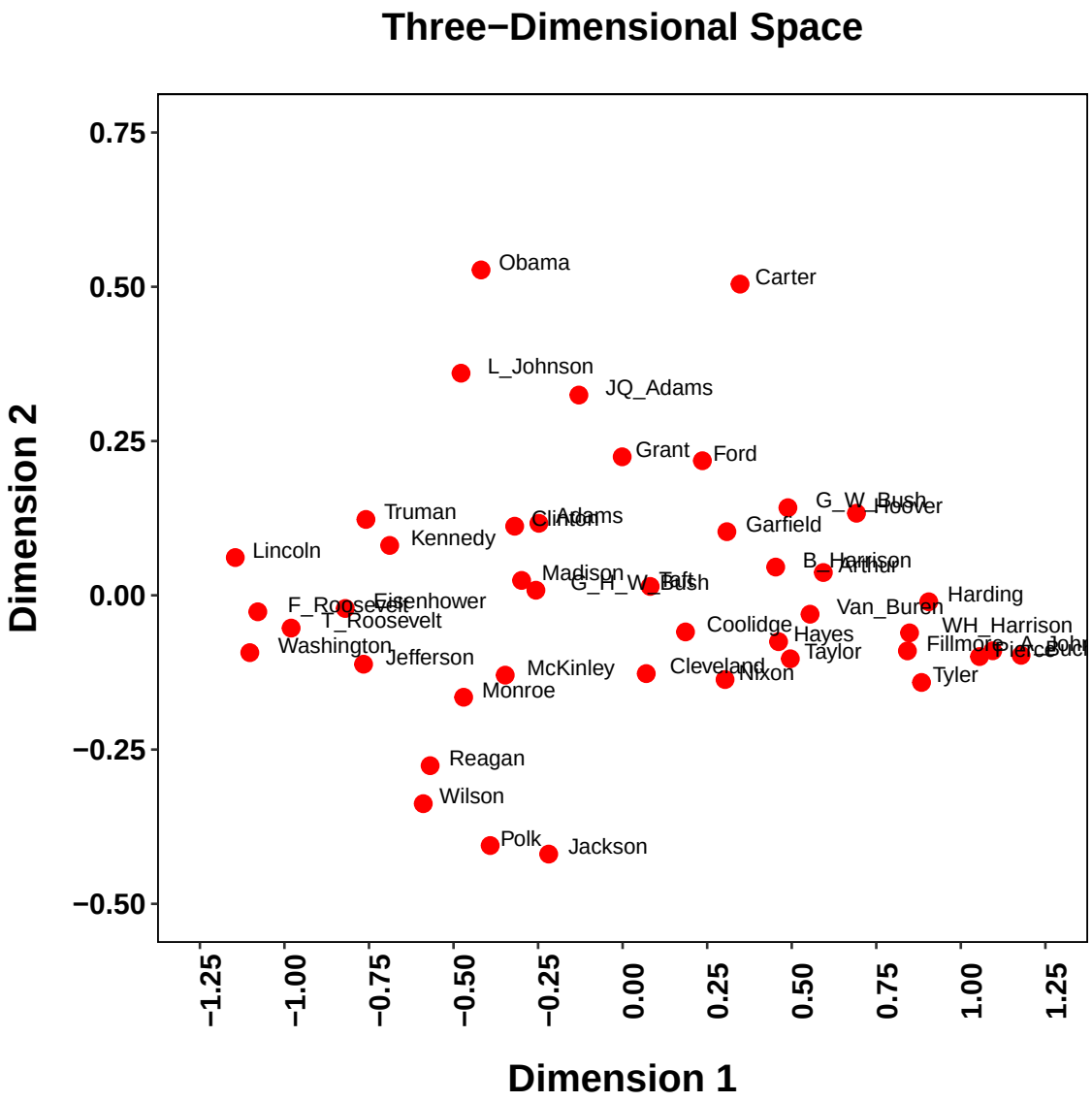
```
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Shepard Plots as a Function of Dimensions")
p + facet_wrap(~D, nrow = 2)
```



4 Scatterplots

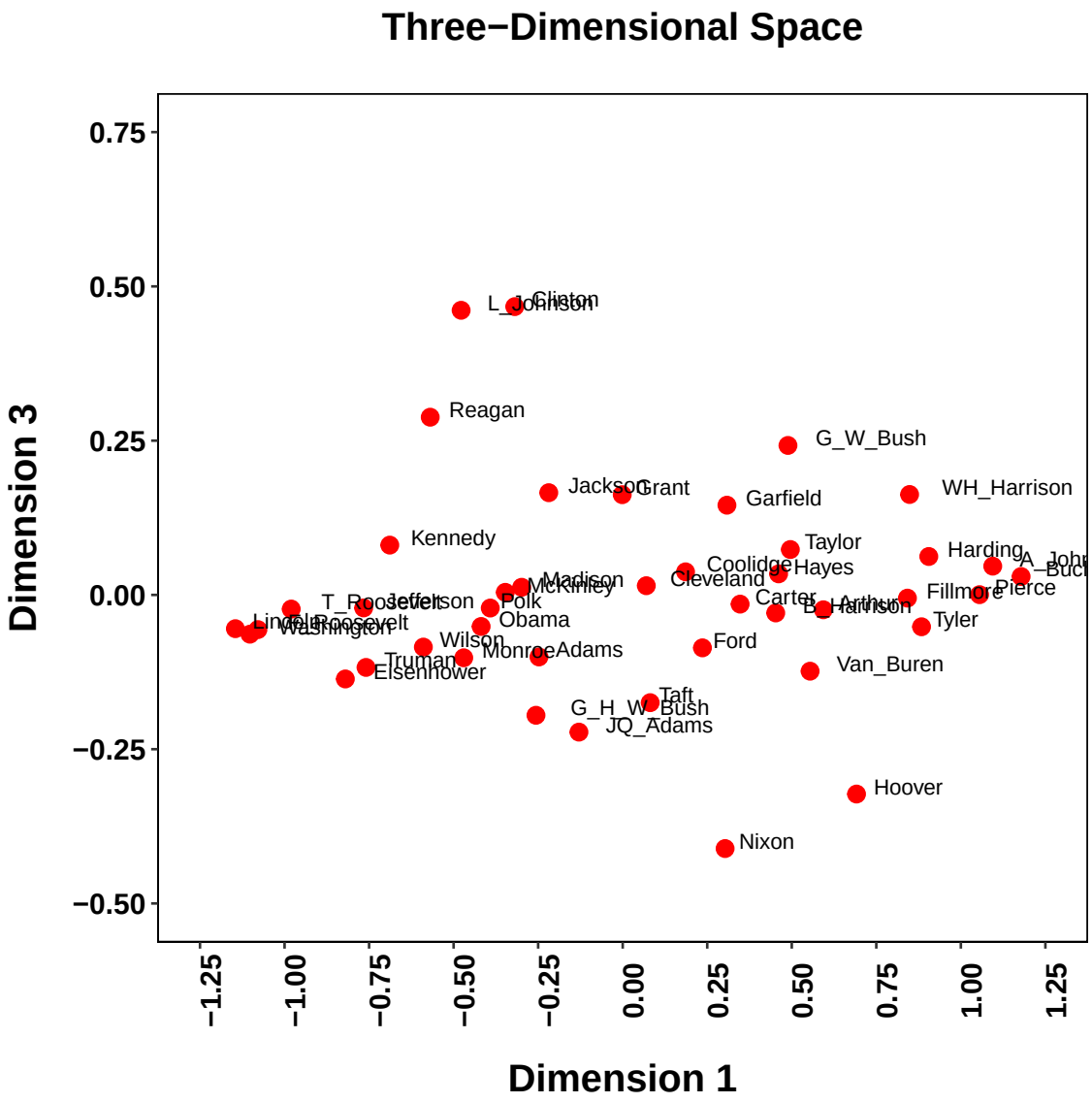
```
mds_3 <- smacofSym(Presidents_Dist, ndim = 3, verbose = FALSE, type = "ordinal",
    itmax = 1000)
plot_data <- as.data.frame(mds_3$conf)
names(plot_data) <- c("D1", "D2", "D3")
plot_data$Name <- row.names(plot_data)
```

```
ggplot(plot_data, aes(x = D1, y = D2)) + geom_point(shape = 19, size = 3,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(-0.5,
    0.75, 0.25))) + scale_x_continuous(breaks = c(seq(-1.25, 1.25,
    0.25))) + geom_text(aes(label = Name), hjust = -0.25, vjust = 0,
    size = 3) + coord_cartesian(xlim = c(-1.25, 1.25), ylim = c(-0.5,
    0.75)) + xlab("Dimension 1") + ylab("Dimension 2") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Three-Dimensional Space")
```



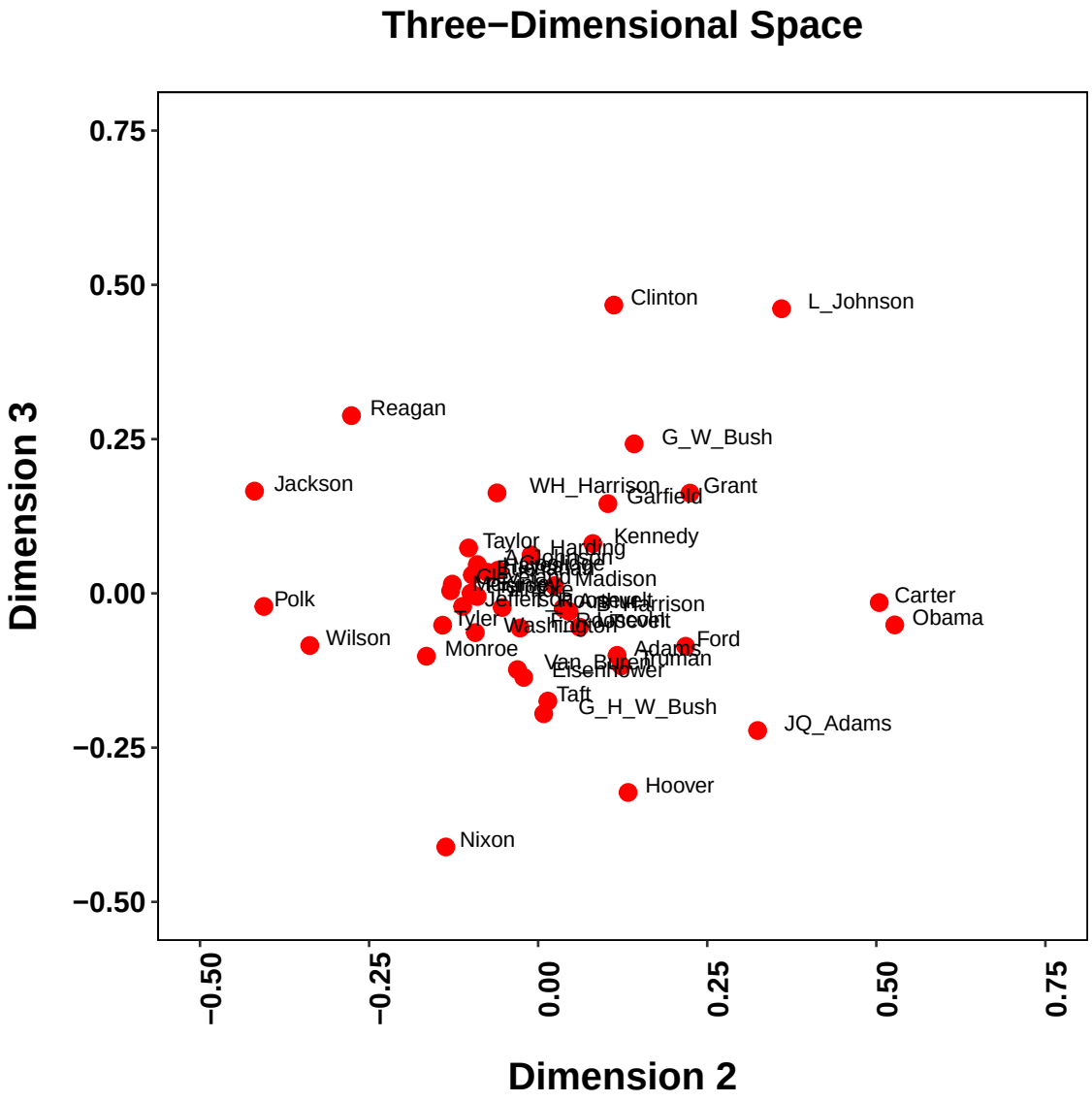

```
ggplot(plot_data, aes(x = D1, y = D3)) + geom_point(shape = 19, size = 3,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(-0.5,
    0.75, 0.25))) + scale_x_continuous(breaks = c(seq(-1.25, 1.25,
    0.25))) + geom_text(aes(label = Name), hjust = -0.25, vjust = 0,
    size = 3) + coord_cartesian(xlim = c(-1.25, 1.25), ylim = c(-0.5,
    0.75)) + xlab("Dimension 1") + ylab("Dimension 3") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
```

```
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Three-Dimensional Space")
```



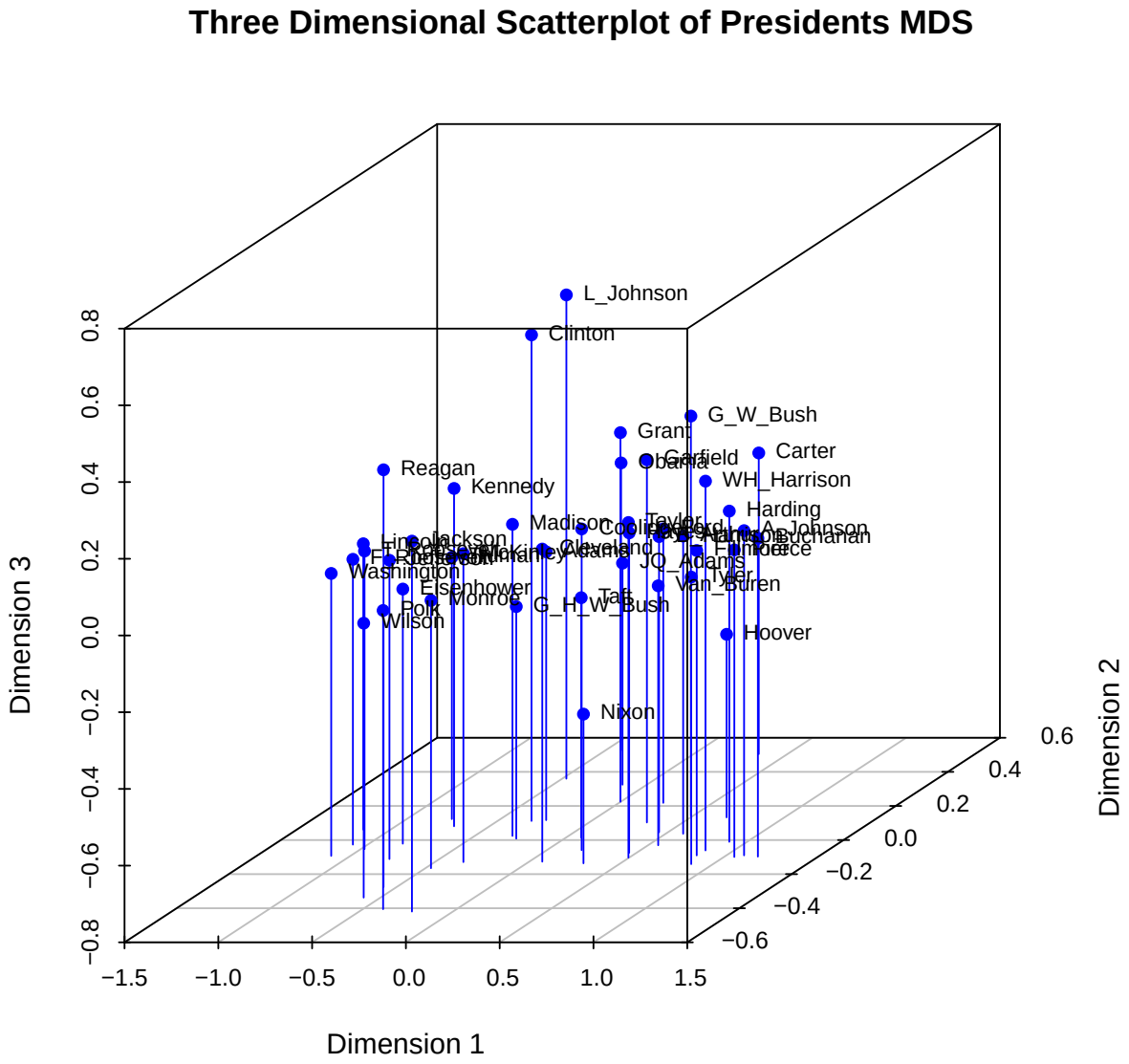

```
ggplot(plot_data, aes(x = D2, y = D3)) + geom_point(shape = 19, size = 3,
    color = "red", na.rm = TRUE) + scale_y_continuous(breaks = c(seq(-0.5,
    0.75, 0.25))) + scale_x_continuous(breaks = c(seq(-0.5, 0.75,
    0.25))) + geom_text(aes(label = Name), hjust = -0.25, vjust = 0,
    size = 3) + coord_cartesian(xlim = c(-0.5, 0.75), ylim = c(-0.5,
    0.75)) + xlab("Dimension 2") + ylab("Dimension 3") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
```

```
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle("Three-Dimensional Space")
```




```
mds_points <- as.data.frame(mds_3$conf)
with(mds_points, {
    s3d <- scatterplot3d(mds_points$D1, mds_points$D2, mds_points$D3,
        color = "blue", pch = 16, type = "h", main = "Three Dimensional Scatterplot of Presidents MDS",
```

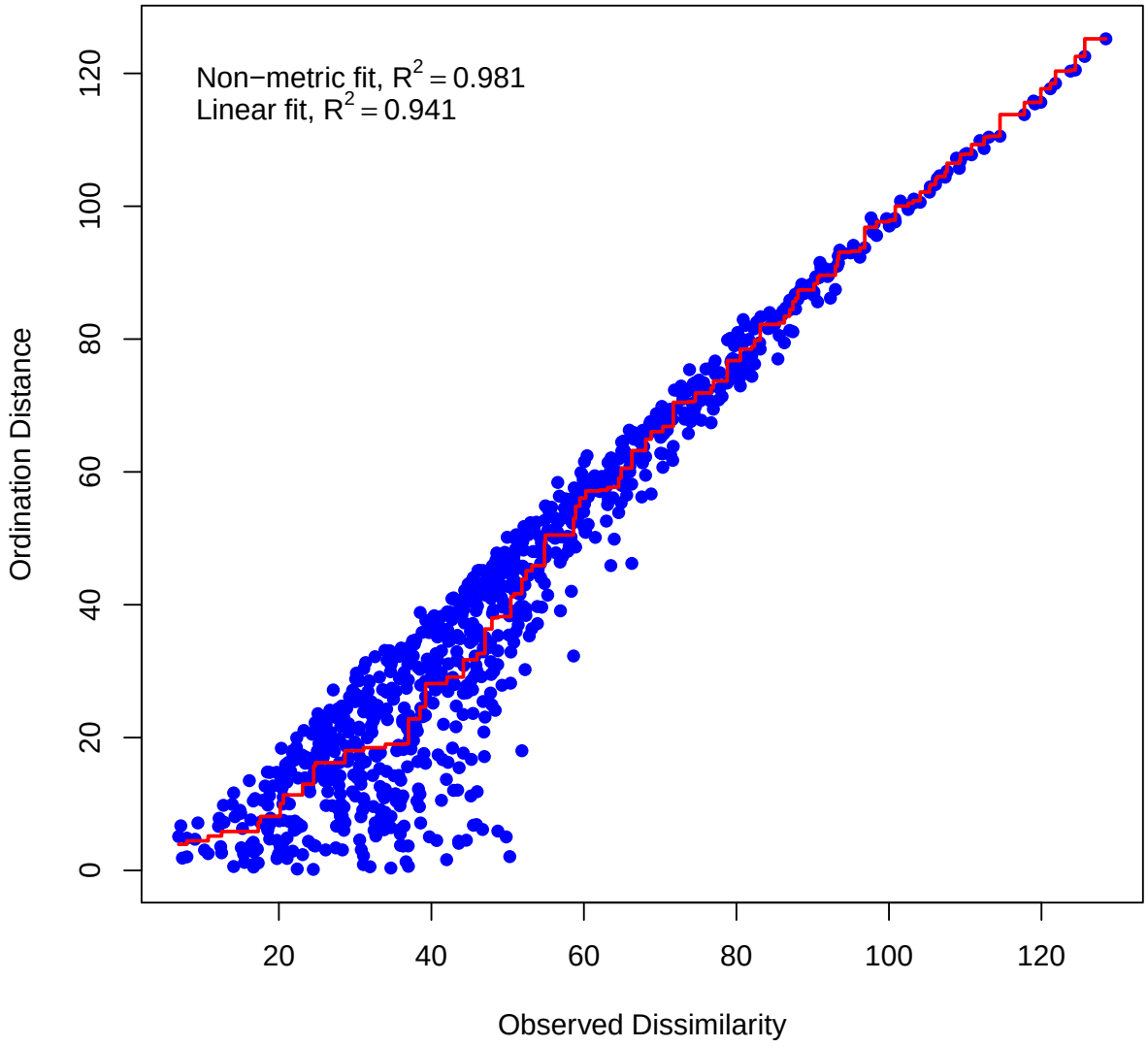
```
        xlab = "Dimension 1", ylab = "Dimension 2", zlab = "Dimension 3")
    s3d.coords <- s3d$xyz.convert(mds_points$D1, mds_points$D2, mds_points$D3)
    text(s3d.coords$x, s3d.coords$y, labels = row.names(mds_points),
        cex = 0.75, pos = 4)
})
```



5 Additional Functions and Output

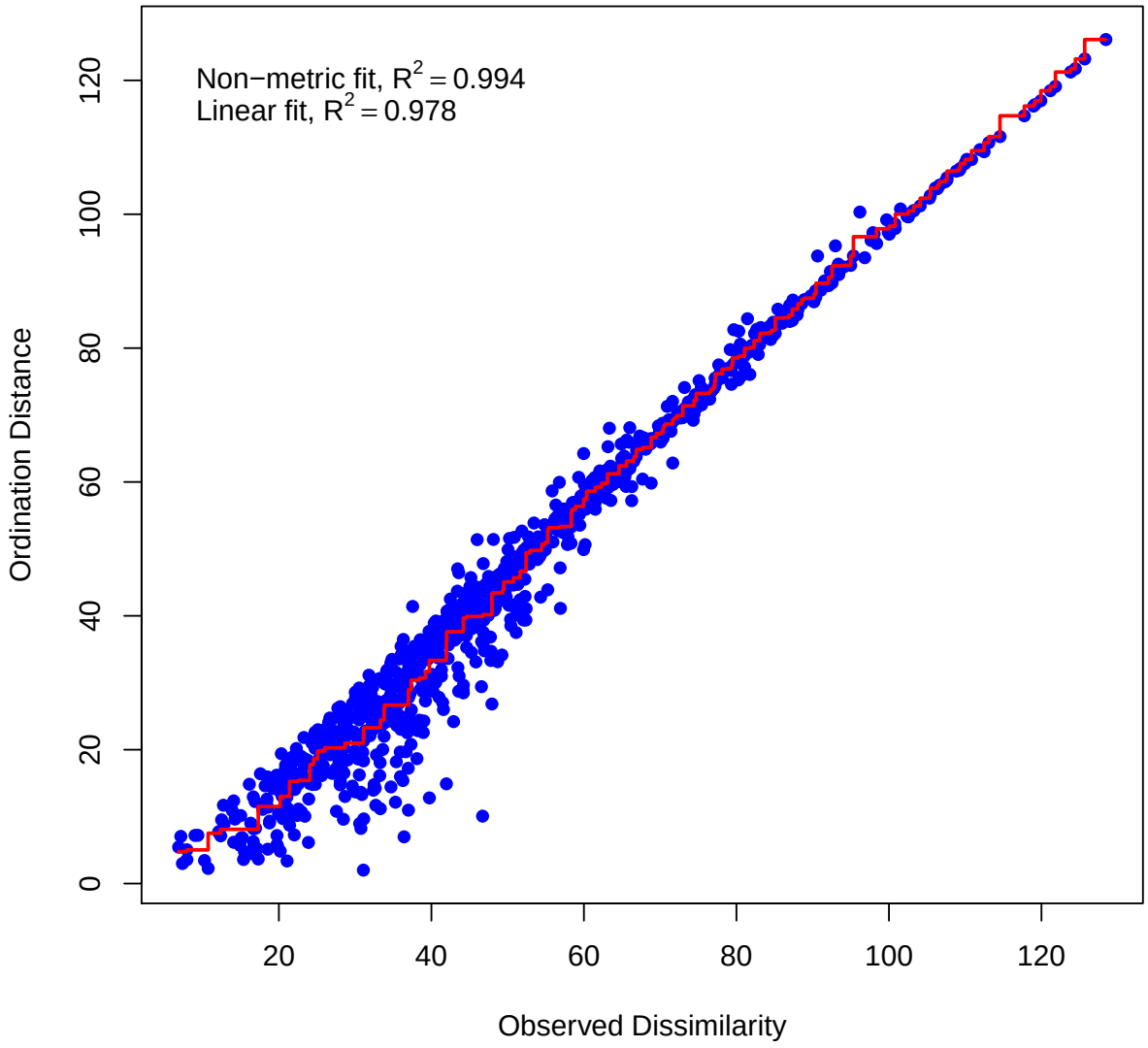
The isoMDS( ) function in the MASS package along with the stressplot( ) function in the vegan package can produce nice looking Shepard plots. These include linear and nonmetric fit indices. The former is the usual linear squared multiple correlation. The latter is 1 minus the stress squared.

```r
mds_4 <- isoMDS(Presidents_Dist, k = 1)

## initial  value 14.492070
## final  value 13.698046
## converged

stressplot(mds_4, Presidents_Dist, pch = 16)
```

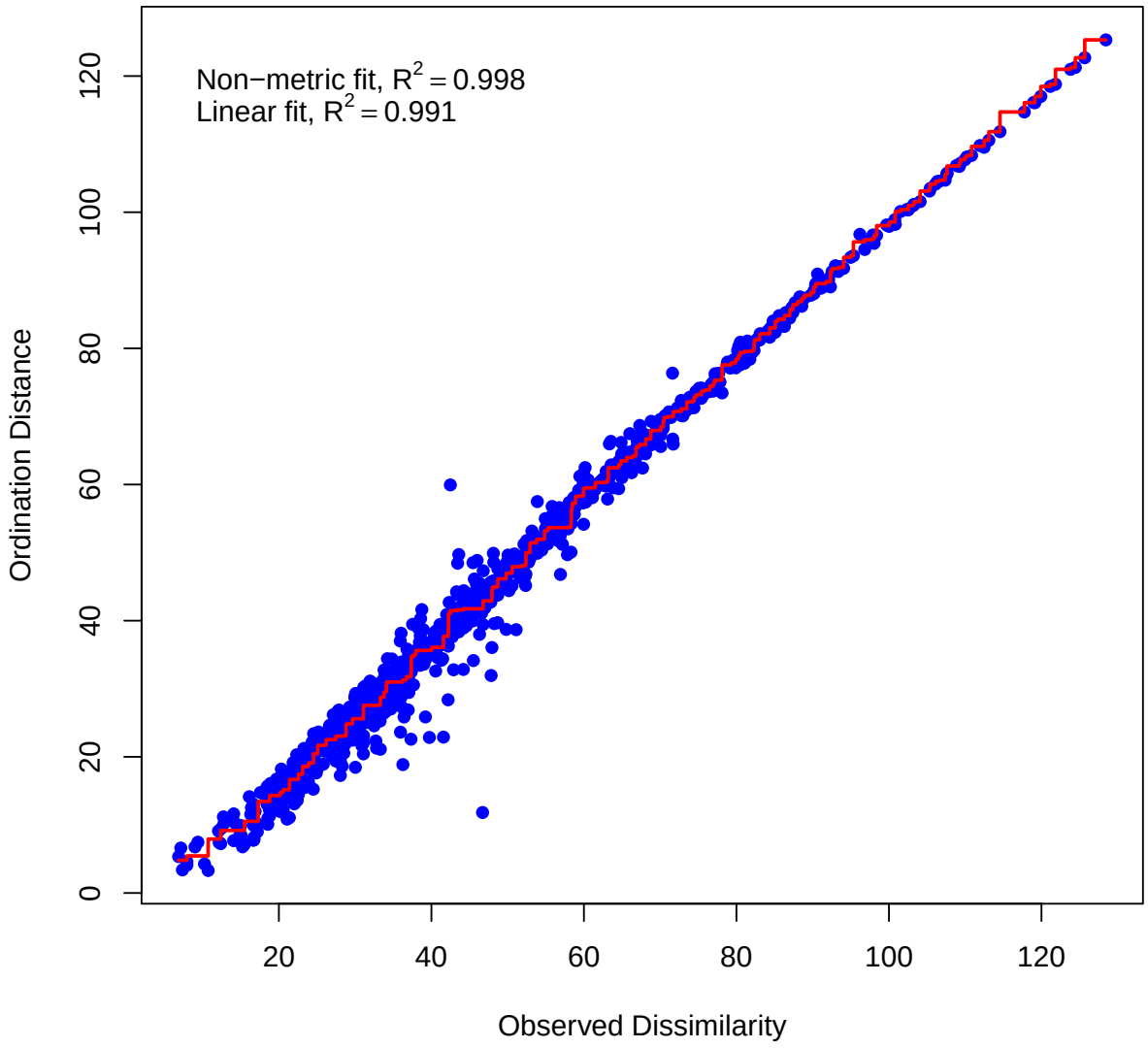



```r
mds_4 <- isoMDS(Presidents_Dist, k = 2)

## initial  value 9.572724
## iter   5 value 8.145550
## iter  10 value 7.656842
## final  value 7.563124
```

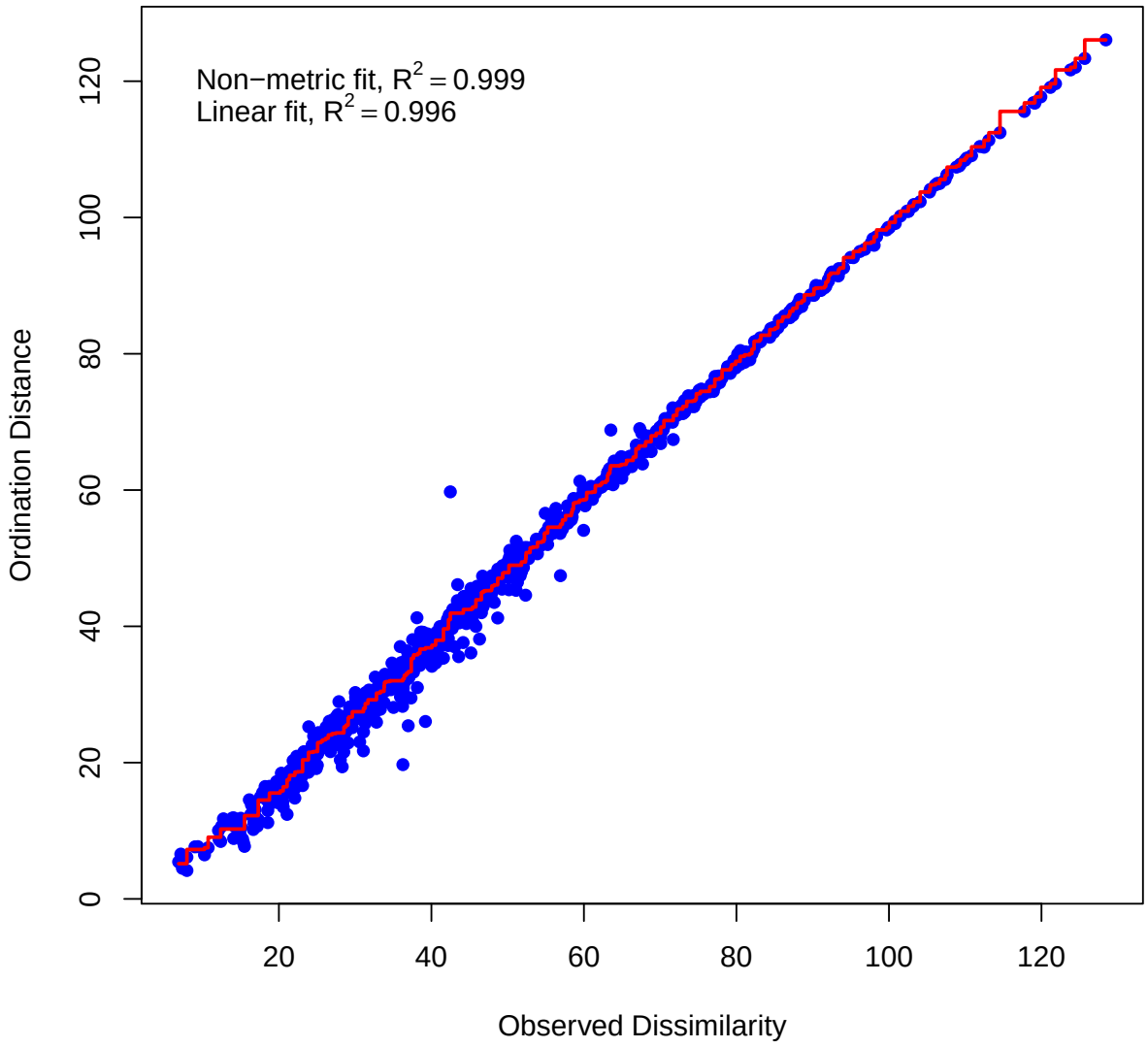
```
## converged

stressplot(mds_4, Presidents_Dist, pch = 16)
```




```
mds_4 <- isoMDS(Presidents_Dist, k = 3)

## initial  value 7.417783
## iter   5 value 5.895790
## iter  10 value 4.968203
## iter  15 value 4.623696
## final  value 4.603599
## converged

stressplot(mds_4, Presidents_Dist, pch = 16)
```



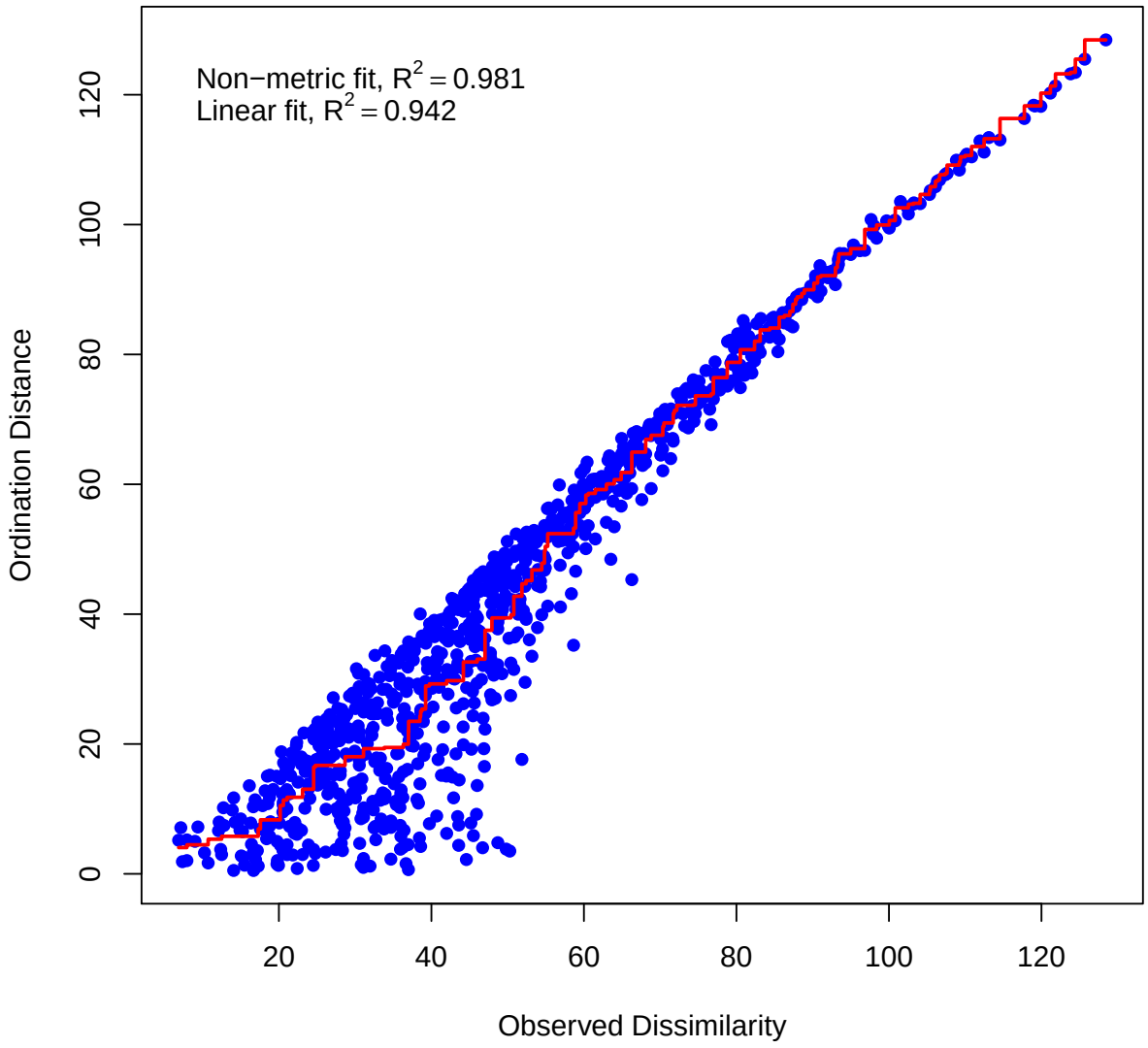

```
mds_4 <- isoMDS(Presidents_Dist, k = 4)

## initial  value 4.995663
## iter   5 value 3.619898
## iter  10 value 3.002937
## iter  15 value 2.889910
## final  value 2.865268
## converged

stressplot(mds_4, Presidents_Dist, pch = 16)
```



The metaMDS( ) function in the vegan package can provide some other useful features. In particular it runs the analysis from multiple start points to find the best solution and insure that a local minimum has not been found.

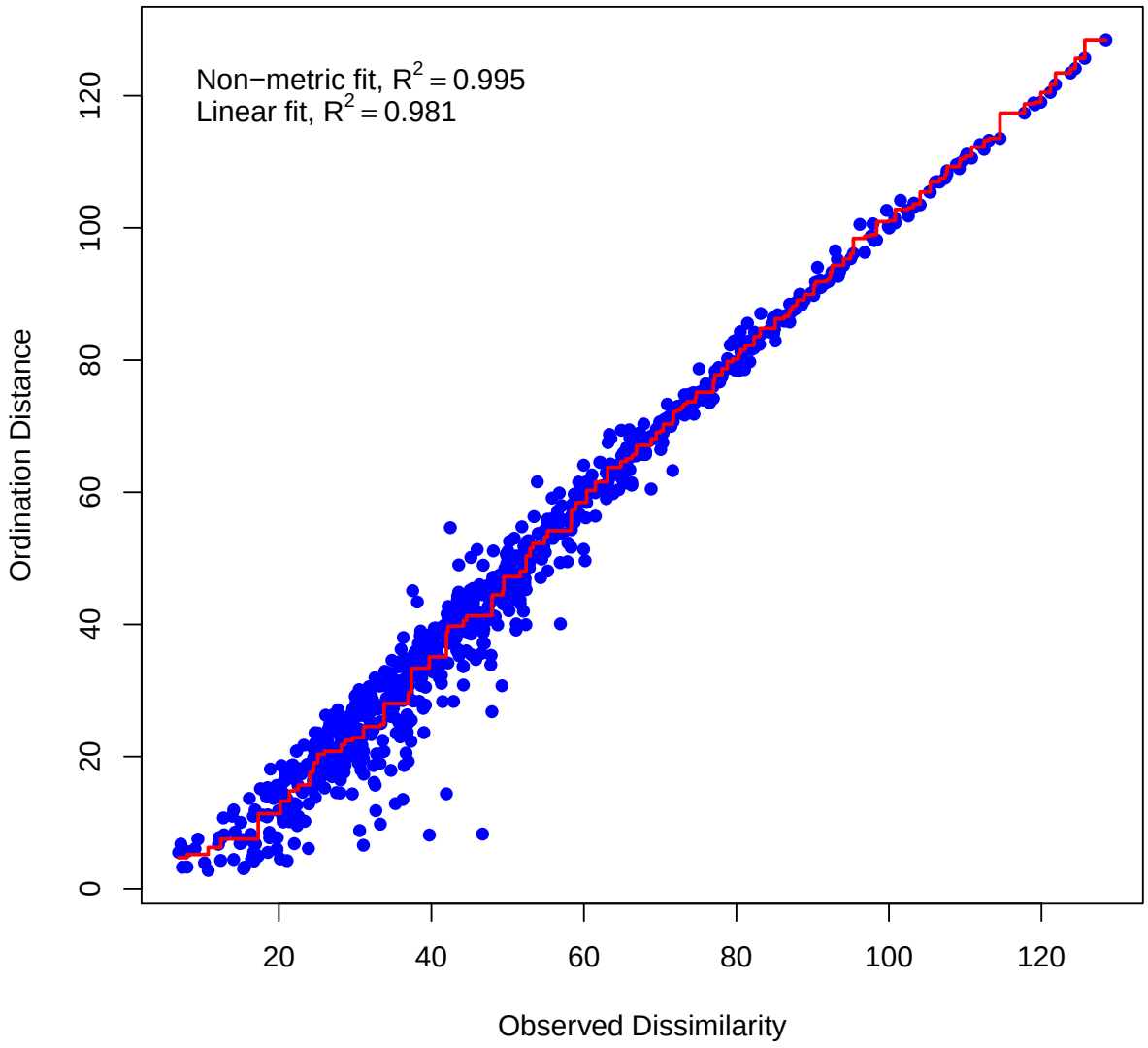
```
mds_4 <- metaMDS(Presidents_Dist, k = 1, distance = "euclidean", autotransform = FALSE,
    trymax = 100)

## Run 0 stress 0.1372
## Run 1 stress 0.1368
## ... New best solution
## ... Procrustes: rmse 0.002882  max resid 0.008896
## ... Similar to previous best
## Run 2 stress 0.545
## Run 3 stress 0.5595
## Run 4 stress 0.5405
```

```
## Run 5 stress 0.5618
## Run 6 stress 0.5557
## Run 7 stress 0.5611
## Run 8 stress 0.5488
## Run 9 stress 0.1366
## ... New best solution
## ... Procrustes: rmse 0.003321  max resid 0.01244
## Run 10 stress 0.5569
## Run 11 stress 0.547
## Run 12 stress 0.5612
## Run 13 stress 0.5637
## Run 14 stress 0.5607
## Run 15 stress 0.1364
## ... New best solution
## ... Procrustes: rmse 0.002447  max resid 0.007383
## ... Similar to previous best
## Run 16 stress 0.562
## Run 17 stress 0.1371
## Run 18 stress 0.1371
## Run 19 stress 0.5225
## Run 20 stress 0.137
## *** Solution reached

stressplot(mds_4, pch = 16)
```



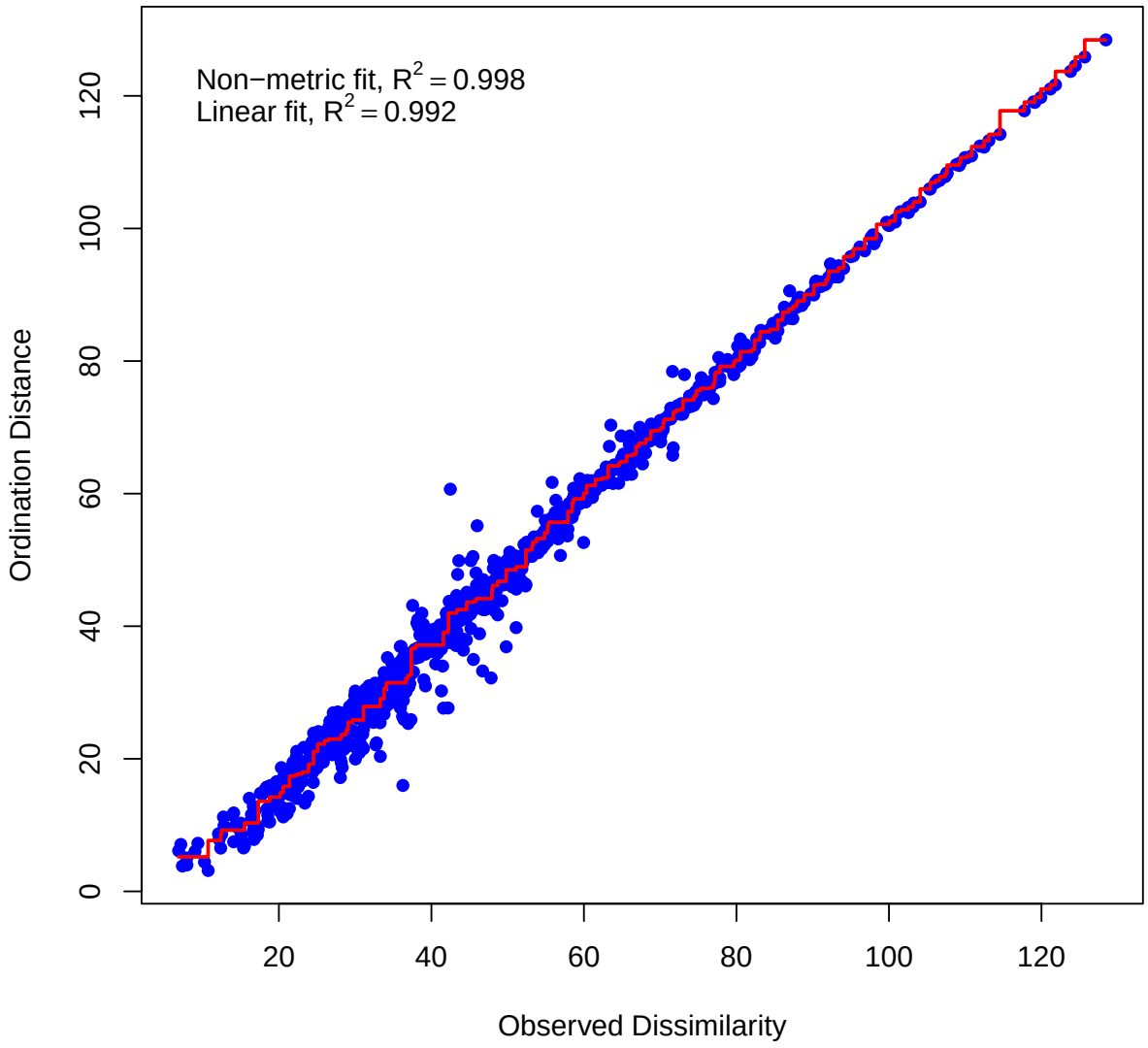

```
mds_4 <- metaMDS(Presidents_Dist, k = 2, distance = "euclidean", autotransform = FALSE,
    trymax = 100)

## Run 0 stress 0.06875
## Run 1 stress 0.08134
## Run 2 stress 0.08679
## Run 3 stress 0.07337
## Run 4 stress 0.07778
## Run 5 stress 0.07682
## Run 6 stress 0.0868
## Run 7 stress 0.07679
## Run 8 stress 0.08376
## Run 9 stress 0.06897
## ... Procrustes: rmse 0.01915  max resid 0.1163
```

```
## Run 10 stress 0.08545
## Run 11 stress 0.06897
## ... Procrustes: rmse 0.01916  max resid 0.1163
## Run 12 stress 0.08812
## Run 13 stress 0.08812
## Run 14 stress 0.07778
## Run 15 stress 0.07991
## Run 16 stress 0.07775
## Run 17 stress 0.08176
## Run 18 stress 0.07386
## Run 19 stress 0.08212
## Run 20 stress 0.07359
## Run 21 stress 0.06875
## ... New best solution
## ... Procrustes: rmse 0.00006904  max resid 0.0002139
## ... Similar to previous best
## *** Solution reached

stressplot(mds_4, pch = 16)
```



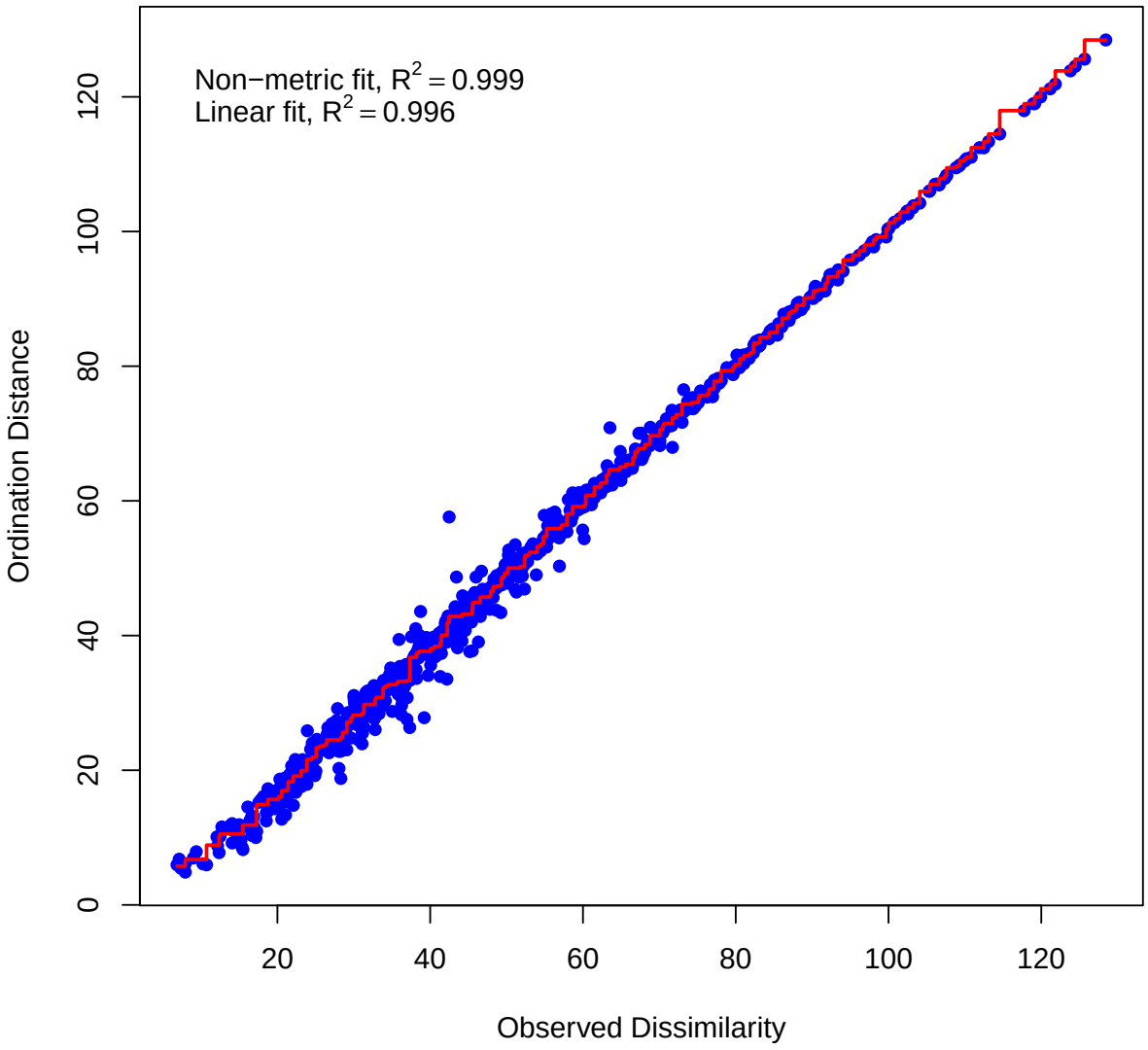

```
mds_4 <- metaMDS(Presidents_Dist, k = 3, distance = "euclidean", autotransform = FALSE,
    trymax = 100)

## Run 0 stress 0.04125
## Run 1 stress 0.04592
## Run 2 stress 0.0413
## ... Procrustes: rmse 0.01065  max resid 0.05273
## Run 3 stress 0.04245
## Run 4 stress 0.04484
## Run 5 stress 0.04782
## Run 6 stress 0.04245
## Run 7 stress 0.04645
## Run 8 stress 0.04615
## Run 9 stress 0.04436
```

```
## Run 10 stress 0.04125
## ... Procrustes: rmse 0.0005428  max resid 0.002195
## ... Similar to previous best
## Run 11 stress 0.04125
## ... Procrustes: rmse 0.0006414  max resid 0.001778
## ... Similar to previous best
## Run 12 stress 0.04244
## Run 13 stress 0.04484
## Run 14 stress 0.0413
## ... Procrustes: rmse 0.01066  max resid 0.05289
## Run 15 stress 0.04615
## Run 16 stress 0.04942
## Run 17 stress 0.04503
## Run 18 stress 0.04125
## ... New best solution
## ... Procrustes: rmse 0.0003057  max resid 0.0007012
## ... Similar to previous best
## Run 19 stress 0.04521
## Run 20 stress 0.04259
## *** Solution reached

stressplot(mds_4, pch = 16)
```




```
mds_4 <- metaMDS(Presidents_Dist, k = 4, distance = "euclidean", autotransform = FALSE,
    trymax = 100)

## Run 0 stress 0.02735
## Run 1 stress 0.02826
## Run 2 stress 0.02903
## Run 3 stress 0.02912
## Run 4 stress 0.02896
## Run 5 stress 0.02826
## Run 6 stress 0.0274
## ... Procrustes: rmse 0.004032  max resid 0.01093
## Run 7 stress 0.02859
## Run 8 stress 0.02866
## Run 9 stress 0.02802
```

```
## Run 10 stress 0.02826
## Run 11 stress 0.02858
## Run 12 stress 0.02826
## Run 13 stress 0.02908
## Run 14 stress 0.02856
## Run 15 stress 0.02879
## Run 16 stress 0.02797
## Run 17 stress 0.02735
## ... New best solution
## ... Procrustes: rmse 0.001718  max resid 0.005703
## ... Similar to previous best
## Run 18 stress 0.02929
## Run 19 stress 0.02735
## ... Procrustes: rmse 0.0003186  max resid 0.0009356
## ... Similar to previous best
## Run 20 stress 0.02798
## *** Solution reached

stressplot(mds_4, pch = 16)
```



The smacof package provides a permutation test that can be used to determine if the obtained stress value is different from what would be expected based on random data. It also provides a jackknife function that gives an indication of stability for the solution.

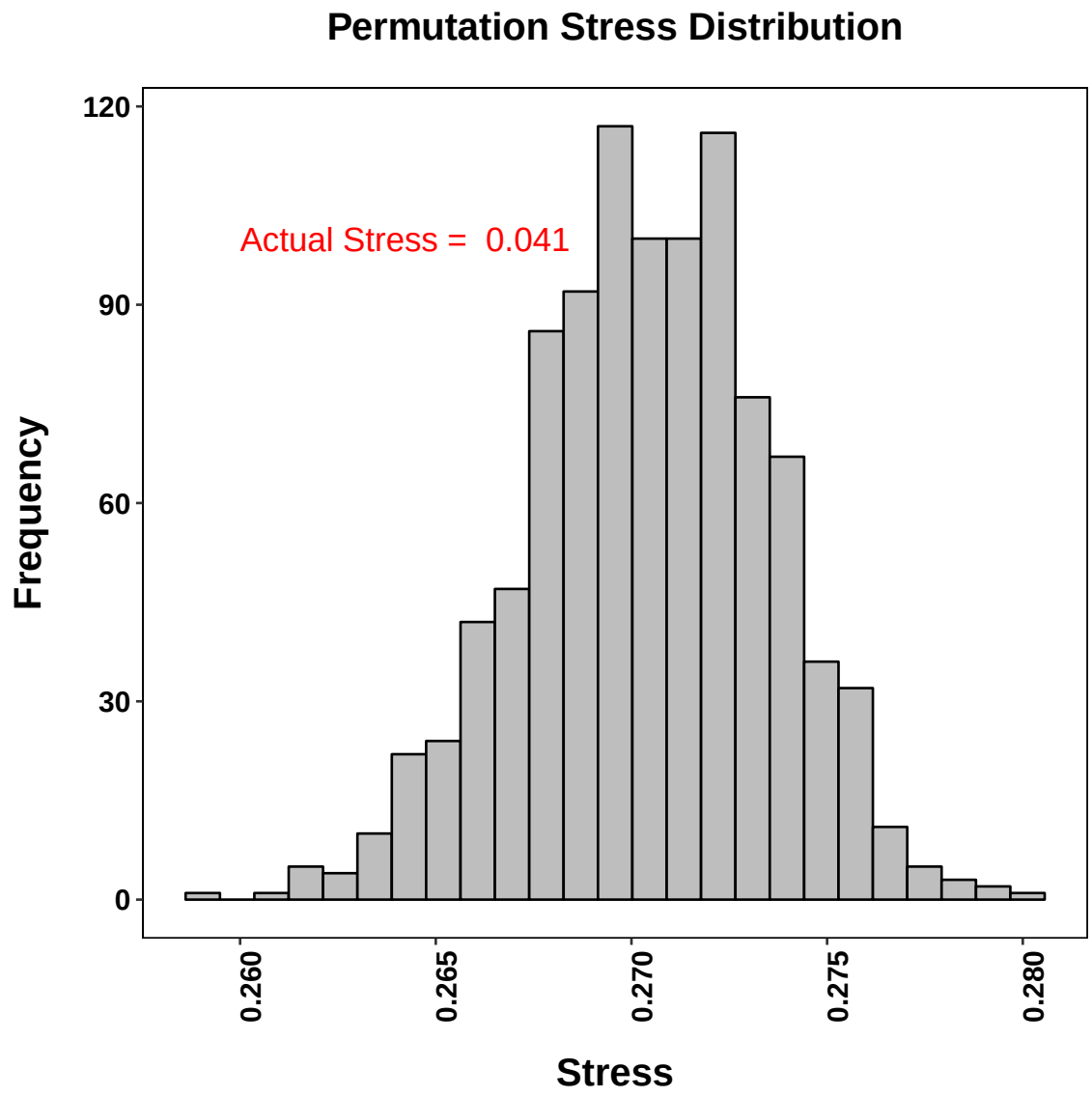
```
mds_5 <- smacofSym(Presidents_Dist, ndim = 3, verbose = FALSE, type = "ordinal",
    itmax = 1000)
mds_5

##
## Call:
## smacofSym(delta = Presidents_Dist, ndim = 3, type = "ordinal",
##     verbose = FALSE, itmax = 1000)
##
## Model: Symmetric SMACOF
## Number of objects: 43
```

```
## Stress-1 value: 0.041
## Number of iterations: 46

perm_mds_5 <- permtest(mds_5, nrep = 1000, verbose = FALSE)

## Warning in smacofSym(delta = structure(c(0, 69.476614770727, 34.132096331752, :  Iteration
limit reached!  You may want to increase the itmax argument!
## Warning in smacofSym(delta = structure(c(0, 42.0951303596984, 35.8050275799363, :  Iteration
limit reached!  You may want to increase the itmax argument!
## Warning in smacofSym(delta = structure(c(0, 38.4317577011513, 23.7065391822594, :  Iteration
limit reached!  You may want to increase the itmax argument!
## Warning in smacofSym(delta = structure(c(0, 46.7974358271904, 58.9151932866217, :  Iteration
limit reached!  You may want to increase the itmax argument!
## Warning in smacofSym(delta = structure(c(0, 39.2428337406972, 23.9165214862028, :  Iteration
limit reached!  You may want to increase the itmax argument!
## Warning in smacofSym(delta = structure(c(0, 91.0823802938856, 21.3775583264319, :  Iteration
limit reached!  You may want to increase the itmax argument!

perm_mds_5

##
## Call: permtest.smacof(object = mds_5, nrep = 1000, verbose = FALSE)
##
## SMACOF Permutation Test
## Number of objects: 43
## Number of replications (permutations): 1000
##
## Observed stress value: 0.041
## p-value: <0.001

perm_stress <- as.data.frame(perm_mds_5$stressvec)

jackfit <- jackknife(mds_5, itmax = 1000)
jackfit

##
## Call: jackknife.smacofB(object = mds_5, itmax = 1000)
##
## SMACOF Jackknife
## Number of objects: 43
## Value loss function: 14.1
## Number of iterations: 3
##
## Stability measure: 0.9921
## Cross validity: 0.9997
## Dispersion: 0.0082

jackknife_D1 <- matrix(jackfit$jackknife.conf[, 1, 1], nrow = 43,
    ncol = 1)

for (i in seq(2, 43)) {
    jackknife_D1 <- cbind(jackknife_D1, jackfit$jackknife.conf[, 1,
        i])
}
```

```
jackknife_D2 <- matrix(jackfit$jackknife.conf[, 2, 1], nrow = 43,
    ncol = 1)

for (i in seq(2, 43)) {
    jackknife_D2 <- cbind(jackknife_D1, jackfit$jackknife.conf[, 2,
        i])
}

jackknife_D3 <- matrix(jackfit$jackknife.conf[, 3, 1], nrow = 43,
    ncol = 1)

for (i in seq(2, 43)) {
    jackknife_D3 <- cbind(jackknife_D1, jackfit$jackknife.conf[, 3,
        i])
}

jackknife_var <- matrix(apply(jackknife_D1, 1, var), nrow = 43)
jackknife_var <- cbind(jackknife_var, apply(jackknife_D2, 1, var))
jackknife_var <- cbind(jackknife_var, apply(jackknife_D3, 1, var))
jackknife_var <- as.data.frame(jackknife_var)
jackknife_var$President <- row.names(Presidents)
describe(jackknife_var[, 1:3])

##    vars  n mean   sd median trimmed  mad min  max range skew
## V1    1 43 0.00 0.00   0.00    0.00 0.00   0 0.00  0.00 0.50
## V2    2 43 0.01 0.01   0.01    0.01 0.01   0 0.03  0.03 0.44
## V3    3 43 0.01 0.01   0.01    0.01 0.01   0 0.03  0.03 0.60
##    kurtosis se
## V1    -1.11  0
## V2    -1.03  0
## V3    -1.02  0
```

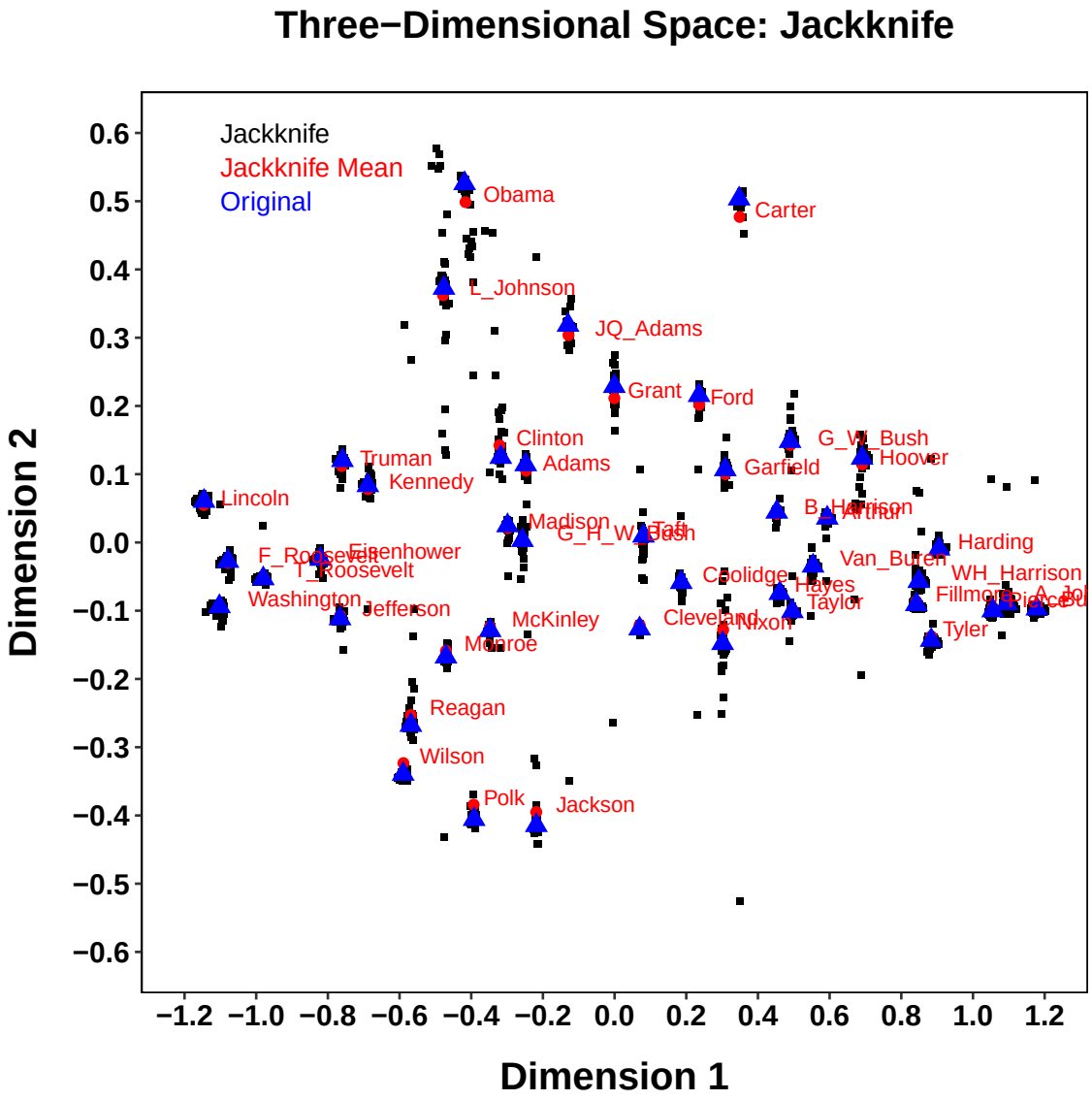
```
plot_data <- as.data.frame(perm_mds_5$stressvec)
names(plot_data) <- c("Stress")
plot_data$Stress_obs <- rep(perm_mds_5$stress.obs, 1000)

Actual_Stress <- perm_mds_5$stress.obs

ggplot(plot_data, aes(x = Stress)) + geom_histogram(bins = round((max(plot_data[,
    "Stress"]) - min(plot_data[, "Stress"]))/(2 * IQR(plot_data[,
    "Stress"]) * length(plot_data[, "Stress"])^(-1/3))), color = "black",
    fill = "grey", size = 0.5, na.rm = TRUE) + # scale_x_continuous(breaks=c(seq(0,.30,.05))) +
# coord_cartesian(xlim = c(0,.3), ylim = c(0,120)) +
xlab("Stress") + ylab("Frequency") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
```

```
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + annotate("text", x = 0.26, y = 100,
    label = paste("Actual Stress = ", toString(round(Actual_Stress,
        3))), hjust = 0, size = 5, color = "red") + ggtitle("Permutation Stress Distribution")
```



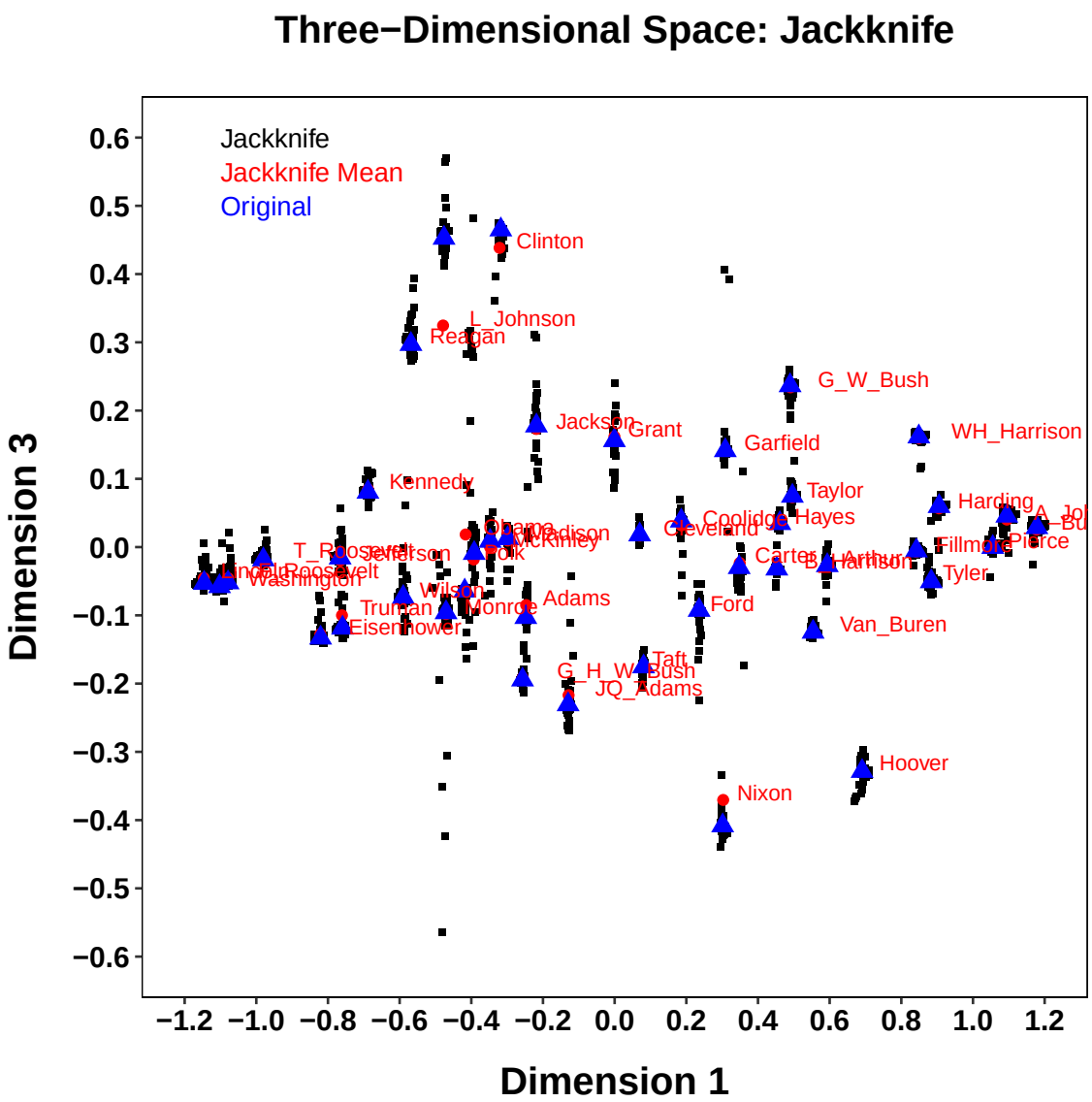

```
jackknife_long <- matrix(NA, nrow = 43 * 43, ncol = 6)
colnames(jackknife_long) <- c("President", "D1", "D2", "D3", "Iteration",
    "Group")
jackknife_long <- as.data.frame(jackknife_long)

for (i in seq(1, 43)) {
    jackknife_long[((i - 1) * 43 + 1):(i * 43), 1] <- 0
    jackknife_long[((i - 1) * 43 + 1):(i * 43), 2:4] <- jackfit$jackknife.conf[,
```

```
        , i]
    jackknife_long[((i - 1) * 43 + 1):(i * 43), 5] <- i
    jackknife_long[((i - 1) * 43 + 1):(i * 43), 6] <- 1
}

for (i in seq(1, 43)) {
    jackknife_long[((i - 1) * 43 + 1):(i * 43), 1] <- row.names((Presidents))
}

plot_data <- as.data.frame(jackknife_long)

plot_data_2 <- as.data.frame(cbind(rep(0, 43), jackfit$comparison.conf,
    rep(44, 43), rep(2, 43)))
names(plot_data_2) <- c("President", "D1", "D2", "D3", "Iteration",
    "Group")
plot_data_2$President <- row.names(Presidents)

plot_data_3 <- as.data.frame(cbind(rep(0, 43), jackfit$smacof.conf,
    rep(45, 43), rep(3, 43)))
names(plot_data_3) <- c("President", "D1", "D2", "D3", "Iteration",
    "Group")
plot_data_3$President <- row.names(Presidents)

plot_data <- rbind(plot_data, plot_data_2, plot_data_3)
plot_data$Group <- as.factor(plot_data$Group)

ggplot(plot_data, aes(x = D1, y = D2, color = Group)) + geom_point(aes(shape = Group,
    color = Group, size = Group)) + scale_color_manual(values = c("black",
    "red", "blue")) + scale_shape_manual(values = c(15, 16, 17)) +
    scale_size_manual(values = c(1, 2, 3)) + scale_y_continuous(breaks = c(round(seq(-0.6,
    0.6, 0.1), 2))) + scale_x_continuous(breaks = c(round(seq(-1.2,
    1.2, 0.2), 2))) + geom_text(data = subset(plot_data, Group ==
    2), aes(D1, D2, label = President), hjust = -0.25, vjust = 0,
    size = 3) + coord_cartesian(xlim = c(-1.2, 1.2), ylim = c(-0.6,
    0.6)) + xlab("Dimension 1") + ylab("Dimension 2") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "none",
    legend.title = element_blank()) + annotate("text", x = -1.1, y = 0.6,
    label = "Jackknife", color = "black", hjust = 0) + annotate("text",
    x = -1.1, y = 0.55, label = "Jackknife Mean", color = "red", hjust = 0) +
    annotate("text", x = -1.1, y = 0.5, label = "Original", color = "blue",
        hjust = 0) + ggtitle("Three-Dimensional Space: Jackknife")
```



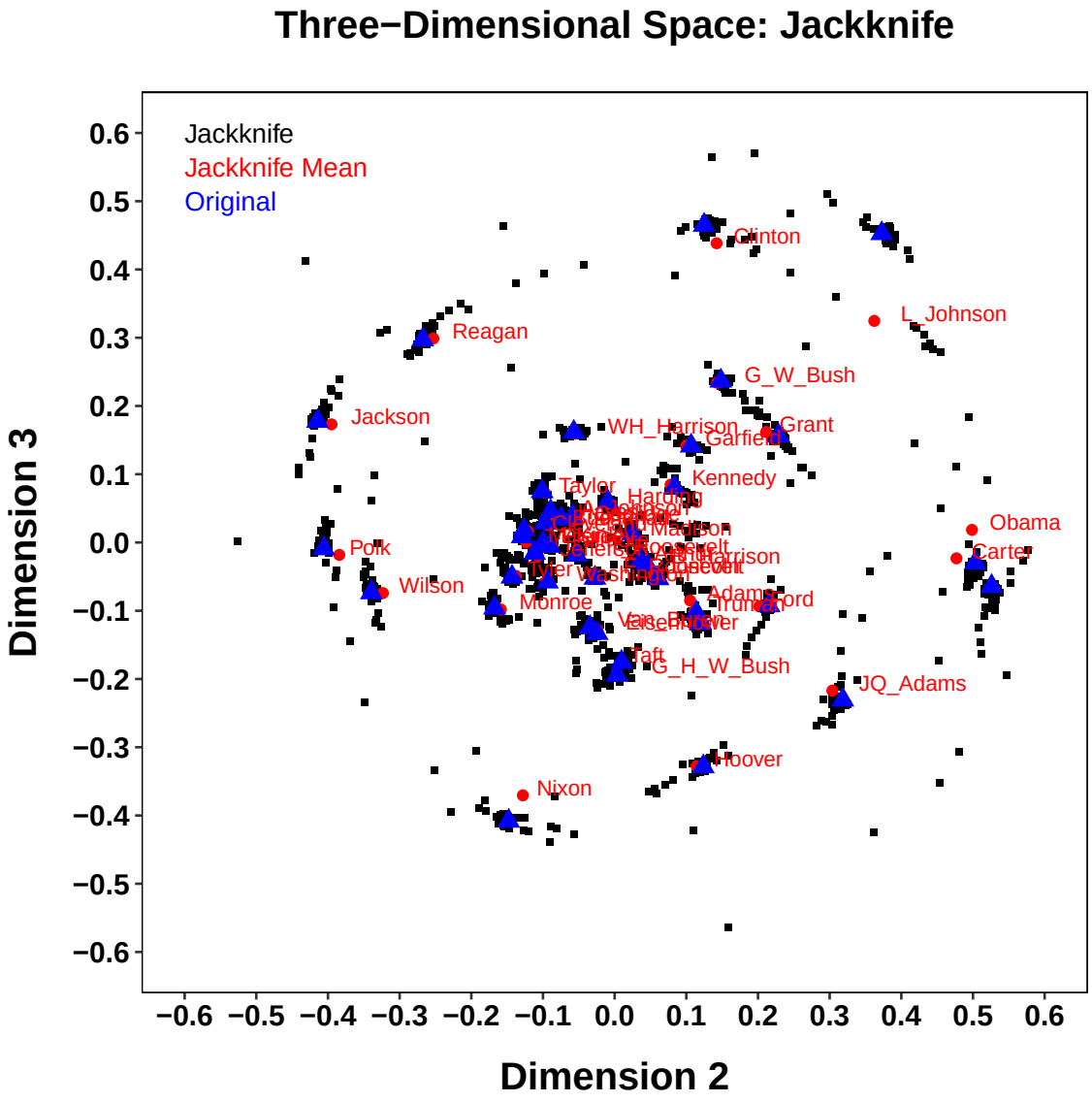

```
ggplot(plot_data, aes(x = D1, y = D3, color = Group)) + geom_point(aes(shape = Group,
    color = Group, size = Group)) + scale_color_manual(values = c("black",
    "red", "blue")) + scale_shape_manual(values = c(15, 16, 17)) +
    scale_size_manual(values = c(1, 2, 3)) + scale_y_continuous(breaks = c(round(seq(-0.6,
    0.6, 0.1), 2))) + scale_x_continuous(breaks = c(round(seq(-1.2,
    1.2, 0.2), 2))) + geom_text(data = subset(plot_data, Group ==
    2), aes(D1, D3, label = President), hjust = -0.25, vjust = 0,
    size = 3) + coord_cartesian(xlim = c(-1.2, 1.2), ylim = c(-0.6,
    0.6)) + xlab("Dimension 1") + ylab("Dimension 3") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
```

```
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "none",
    legend.title = element_blank()) + annotate("text", x = -1.1, y = 0.6,
    label = "Jackknife", color = "black", hjust = 0) + annotate("text",
    x = -1.1, y = 0.55, label = "Jackknife Mean", color = "red", hjust = 0) +
    annotate("text", x = -1.1, y = 0.5, label = "Original", color = "blue",
        hjust = 0) + ggtitle("Three-Dimensional Space: Jackknife")
```




```
ggplot(plot_data, aes(x = D2, y = D3, color = Group)) + geom_point(aes(shape = Group,
    color = Group, size = Group)) + scale_color_manual(values = c("black",
```

```
    "red", "blue")) + scale_shape_manual(values = c(15, 16, 17)) +
    scale_size_manual(values = c(1, 2, 3)) + scale_y_continuous(breaks = c(round(seq(-0.6,
    0.6, 0.1), 2))) + scale_x_continuous(breaks = c(round(seq(-0.6,
    0.6, 0.1), 2))) + geom_text(data = subset(plot_data, Group ==
    2), aes(D2, D3, label = President), hjust = -0.25, vjust = 0,
    size = 3) + coord_cartesian(xlim = c(-0.6, 0.6), ylim = c(-0.6,
    0.6)) + xlab("Dimension 2") + ylab("Dimension 3") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "none",
    legend.title = element_blank()) + annotate("text", x = -0.6, y = 0.6,
    label = "Jackknife", color = "black", hjust = 0) + annotate("text",
    x = -0.6, y = 0.55, label = "Jackknife Mean", color = "red", hjust = 0) +
    annotate("text", x = -0.6, y = 0.5, label = "Original", color = "blue",
        hjust = 0) + ggtitle("Three-Dimensional Space: Jackknife")
```



6 Follow-Up Analyses

One way to gain insight into the meaning of the dimensions is to correlate the original mean ratings with the coordinates of the MDS solution.

```
Presidents_2 <- as.data.frame(cbind(Presidents, mds_3$conf))
cor(Presidents_2[, 2:length(Presidents_2)], use = "pairwise.complete.obs")

##          PP       CL       EM      MA     IR      AS      RC
## PP   1.0000  0.91860  0.86862  0.7448 0.7440 0.71590  0.8127
## CL   0.9186  1.00000  0.90018  0.8043 0.8706 0.79450  0.8408
## EM   0.8686  0.90018  1.00000  0.7545 0.7981 0.81627  0.7753
## MA   0.7448  0.80429  0.75445  1.0000 0.7419 0.73464  0.7085
```

```
## IR   0.7440  0.87058  0.79810  0.7419 1.0000 0.75958  0.7066
## AS   0.7159  0.79450  0.81627  0.7346 0.7596 1.00000  0.8025
## RC   0.8127  0.84083  0.77529  0.7085 0.7066 0.80251  1.0000
## VSA  0.9281  0.91060  0.88070  0.8217 0.7735 0.79148  0.8043
## PEJ  0.5512  0.58170  0.66158  0.6392 0.5159 0.56670  0.5435
## PCT  0.9230  0.96406  0.91664  0.8664 0.8443 0.83032  0.8774
## D1   0.9224  0.96331  0.93889  0.8715 0.8657 0.87090  0.8803
## D2   0.1411  0.13653 -0.01907 -0.1504 0.1805 0.00866  0.1276
## D3  -0.2264 -0.01066 -0.05321  0.1486 0.3399 0.25138 -0.1578
##          VSA      PEJ      PCT        D1        D2         D3
## PP   0.92812  0.55119  0.92298 0.9223566  0.141069 -0.2263773
## CL   0.91060  0.58170  0.96406 0.9633115  0.136533 -0.0106613
## EM   0.88070  0.66158  0.91664 0.9388875 -0.019069 -0.0532063
## MA   0.82166  0.63923  0.86635 0.8715320 -0.150442  0.1486345
## IR   0.77348  0.51586  0.84431 0.8657271  0.180475  0.3398971
## AS   0.79148  0.56670  0.83032 0.8708968  0.008660  0.2513751
## RC   0.80429  0.54349  0.87738 0.8803035  0.127574 -0.1577770
## VSA  1.00000  0.62217  0.94488 0.9508638  0.035587 -0.1019317
## PEJ  0.62217  1.00000  0.62368 0.6898128 -0.677203 -0.0473194
## PCT  0.94488  0.62368  1.00000 0.9860184  0.067788 -0.0545041
## D1   0.95086  0.68981  0.98602 1.0000000  0.001394  0.0009921
## D2   0.03559 -0.67720  0.06779 0.0013940  1.000000  0.0430871
## D3  -0.10193 -0.04732 -0.05450 0.0009921  0.043087  1.0000000
```