Confirmatory Factor Analysis III

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October 8, 2018

1 Preliminaries

In this section, the RStudio workspace and console panes are cleared of old output, variables, and other miscellaneous debris. Packages are loaded and any required data files are retrieved.

```
options(replace.assign = TRUE, width = 65, digits = 4, scipen = 4, fig.width = 4,
    fig.height = 4)
# Clear the workspace and console.
rm(list = ls(all = TRUE))
cat("\f")
```

```
# Turn off showing of significance asterisks.
options(show.signif.stars = F)
# Set the contrast option; important for ANOVAs.
options(contrasts = c("contr.sum", "contr.poly"))
how_long <- Sys.time()
set.seed(123)
library(knitr)
```

```
library(psych)
library(car)

## Loading required package:  carData
##
## Attaching package:  'car'
## The following object is masked from 'package:psych':
##
##    logit

library(multcomp)

## Loading required package:  mvtnorm
## Loading required package:  survival
## Loading required package:  TH.data
## Loading required package:  MASS
##
## Attaching package:  'TH.data'
## The following object is masked from 'package:MASS':
##
##    geyser
```

```
library(ggplot2)
```

```
##
## Attaching package:  'ggplot2'
## The following objects are masked from 'package:psych':
##
##    %+%, alpha
```

```
library(MASS)
library(parallel)
library(ellipse)
```

```
##
## Attaching package:  'ellipse'
## The following object is masked from 'package:car':
##
##    ellipse
## The following object is masked from 'package:graphics':
##
##    pairs
```

```
library(FactoMineR)
```

```
## Warning:  package 'FactoMineR' was built under R version 3.5.1
```

```
library(PerformanceAnalytics)
```

```
## Warning:  package 'PerformanceAnalytics' was built under R version 3.5.1
## Loading required package:  xts
## Loading required package:  zoo
##
## Attaching package:  'zoo'
## The following objects are masked from 'package:base':
##
##    as.Date, as.Date.numeric
##
## Attaching package:  'PerformanceAnalytics'
## The following object is masked from 'package:graphics':
##
##    legend
```

```
library(plotpc)
```

```
## Loading required package:  grid
```

```
library(sciplot)
library(GPArotation)
library(GGally)
library(MVN)
```

```
## sROC 0.1-2 loaded
```

```
library(qqplotr)
library(scatterplot3d)
library(rgl)
```

```
## Warning:  package 'rgl' was built under R version 3.5.1
```

```
library(cowplot)
```

```
##
## Attaching package:  'cowplot'
## The following object is masked from 'package:ggplot2':
##
##    ggsave
```

```
library(lavaan)
```

```
## This is lavaan 0.6-1
## lavaan is BETA software!  Please report any bugs.
##
## Attaching package:  'lavaan'
## The following object is masked from 'package:psych':
##
##    cor2cov
```

```
library(semPlot)
library(semTools)
```

```
##
## ################################################################################
## This is semTools 0.4-14
## All users of R (or SEM) are invited to submit functions or ideas for functions.
## ################################################################################
##
## Attaching package:  'semTools'
## The following object is masked from 'package:PerformanceAnalytics':
##
##    kurtosis
## The following object is masked from 'package:psych':
##
##    skew
```

```
library(MVN)
```

1.1 Data Files

We will use the mental abilities data sets (full and split), the need for cognition data set, the self-esteem data set, and the HolzingerSwineford data set for the example analyses.

```
# Get the drug use data from the working directory.
setwd("C:\\Courses\\Psychology 516\\PowerPoint\\2018")
Mental <- read.table("mental.csv", sep = ",", header = TRUE)
Mental <- as.data.frame(Mental)
Mental <- na.omit(Mental)
names(Mental) <- c("Grammar", "Paragraph_Comprehension", "Vocabulary",
    "Sentence_Completion", "Geometry", "Algebra", "Numerical_Puzzles",
    "Series_Completion", "Practical_Problem_Solving", "Symbol_Manipulation",
    "Analytical_Ability", "Formal_Logic")
Mental_1 <- read.table("mental_1.csv", sep = ",", header = TRUE)
Mental_1 <- as.data.frame(Mental_1)
Mental_1 <- na.omit(Mental_1)
names(Mental_1) <- c("Grammar", "Paragraph_Comprehension", "Vocabulary",
    "Sentence_Completion", "Geometry", "Algebra", "Numerical_Puzzles",
```

```
    "Series_Completion", "Practical_Problem_Solving", "Symbol_Manipulation",
    "Analytical_Ability", "Formal_Logic")
Mental_2 <- read.table("mental_2.csv", sep = ",", header = TRUE)
Mental_2 <- as.data.frame(Mental_2)
Mental_2 <- na.omit(Mental_2)
names(Mental_2) <- c("Grammar", "Paragraph_Comprehension", "Vocabulary",
    "Sentence_Completion", "Geometry", "Algebra", "Numerical_Puzzles",
    "Series_Completion", "Practical_Problem_Solving", "Symbol_Manipulation",
    "Analytical_Ability", "Formal_Logic")
Mental_G <- read.table("mental_groups.csv", sep = ",", header = TRUE)
Mental_G <- as.data.frame(Mental_G)
Mental_G <- na.omit(Mental_G)
names(Mental_G) <- c("Grammar", "Paragraph_Comprehension", "Vocabulary",
    "Sentence_Completion", "Geometry", "Algebra", "Numerical_Puzzles",
    "Series_Completion", "Practical_Problem_Solving", "Symbol_Manipulation",
    "Analytical_Ability", "Formal_Logic", "Group")
Mental_G$Group <- as.factor(Mental_G$Group)
NC <- read.table("need_for_cognition.csv", sep = ",", header = TRUE)
NC <- as.data.frame(NC[, 1:18])
NC <- na.omit(NC)
names(NC) <- c("nc1", "nc2", "nc3", "nc4", "nc5", "nc6", "nc7", "nc8",
    "nc9", "nc10", "nc11", "nc12", "nc13", "nc14", "nc15", "nc16",
    "nc17", "nc18")
SE <- read.table("Set_4.csv", sep = ",", header = TRUE)
SE <- as.data.frame(SE)
SE <- na.omit(SE[, 2:21])

HS <- as.data.frame(HolzingerSwineford1939)
HS <- HS[, c(5, 7:15)]
HS <- na.omit(HS)
```

2 Screen the Data

For each data set we do a quick screen to make sure there are no severe multivariate outliers or multivariate normality problems.

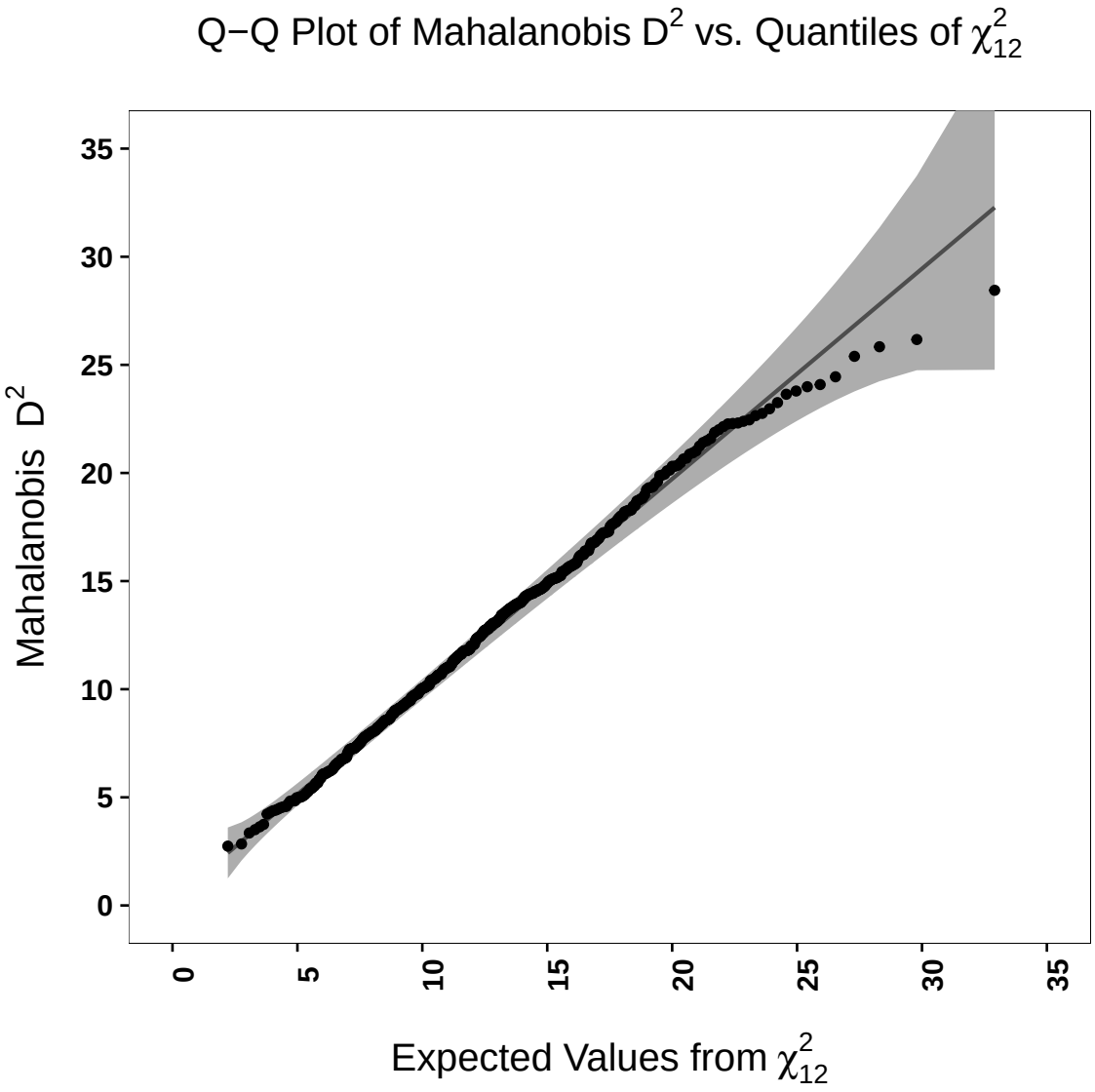
2.1 Mental Abilities

First we will do a quick screen of the data to make sure there are no severe multivariate outliers.

2.1.1 Mahalanobis Distance

```
CV <- cov(Mental)
D2 <- mahalanobis(Mental, center = colMeans(Mental), cov = CV)
D2 <- as.data.frame(D2)
describe(D2)

##    vars   n  mean   sd median trimmed  mad  min   max range skew
## X1    1 500 11.98 4.75   11.4   11.65 4.66 2.74 28.45  25.7 0.61
##    kurtosis   se
## X1     0.05 0.21
```

```
ggplot(D2, aes(sample = D2)) + stat_qq_band(distribution = "chisq",
    dparams = list(df = 12)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 12)) + stat_qq(distribution = "qchisq", dparams = list(df = 12)) +
    scale_y_continuous(breaks = seq(0, 35, 5)) + scale_x_continuous(breaks = seq(0,
    35, 5)) + coord_cartesian(xlim = c(0, 35), ylim = c(0, 35)) +
    xlab(expression("Expected Values from" * ~chi[12]^2)) + ylab(expression("Mahalanobis " *
    ~D^2)) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" *
    ~D^2 * " vs. Quantiles of" * ~chi[12]^2))
```



The squared Mahalanobis distances suggest the data are well behaved.

2.1.2 Multivariate Normality

We can also test for multivariate normality.

```
mvn(Mental, mvnTest = "mardia")

## $multivariateNormality
##              Test          Statistic           p value Result
## 1 Mardia Skewness   367.600071004817 0.437353917013108    YES
## 2 Mardia Kurtosis -0.866369530731211 0.386287547416061    YES
## 3             MVN               <NA>              <NA>    YES
##
## $univariateNormality
```

```
##            Test                   Variable Statistic   p value
## 1  Shapiro-Wilk          Grammar             0.9955    0.1597
## 2  Shapiro-Wilk  Paragraph_Comprehension     0.9980    0.8256
## 3  Shapiro-Wilk        Vocabulary            0.9951    0.1166
## 4  Shapiro-Wilk    Sentence_Completion       0.9972    0.5408
## 5  Shapiro-Wilk         Geometry             0.9961    0.2666
## 6  Shapiro-Wilk          Algebra             0.9964    0.3168
## 7  Shapiro-Wilk     Numerical_Puzzles        0.9966    0.3863
## 8  Shapiro-Wilk     Series_Completion        0.9959    0.2195
## 9  Shapiro-Wilk Practical_Problem_Solving    0.9974    0.6374
## 10 Shapiro-Wilk    Symbol_Manipulation       0.9962    0.2767
## 11 Shapiro-Wilk    Analytical_Ability        0.9971    0.5221
## 12 Shapiro-Wilk       Formal_Logic           0.9961    0.2500
##    Normality
## 1        YES
## 2        YES
## 3        YES
## 4        YES
## 5        YES
## 6        YES
## 7        YES
## 8        YES
## 9        YES
## 10       YES
## 11       YES
## 12       YES
##
## $Descriptives
##                               n      Mean Std.Dev    Median    Min
## Grammar                     500 -0.007632  0.9765 -0.019820 -3.003
## Paragraph_Comprehension     500 -0.003750  1.0633  0.035079 -3.024
## Vocabulary                  500 -0.014783  0.9989  0.023294 -3.492
## Sentence_Completion         500 -0.087844  1.0863 -0.087127 -3.092
## Geometry                    500  0.010634  0.9789 -0.028235 -2.522
## Algebra                     500 -0.028644  0.9615 -0.002082 -2.906
## Numerical_Puzzles           500 -0.015062  0.9890 -0.062496 -2.804
## Series_Completion           500 -0.006348  0.9638 -0.029178 -3.038
## Practical_Problem_Solving   500 -0.016828  0.9361 -0.040021 -2.636
## Symbol_Manipulation         500  0.000833  0.9718 -0.056943 -3.073
## Analytical_Ability          500 -0.004649  0.9834  0.009074 -3.751
## Formal_Logic                500 -0.062950  1.0165 -0.092532 -3.197
##                               Max    25th   75th     Skew
## Grammar                     2.235 -0.6782 0.6886 -0.05736
## Paragraph_Comprehension     2.952 -0.7485 0.7003 -0.09118
## Vocabulary                  2.763 -0.6567 0.6660 -0.24969
## Sentence_Completion         3.005 -0.8542 0.6525  0.01663
## Geometry                    2.779 -0.6281 0.6738  0.10710
## Algebra                     2.893 -0.6320 0.5580 -0.07334
## Numerical_Puzzles           2.881 -0.5965 0.6013  0.01468
## Series_Completion           2.288 -0.6444 0.6381 -0.11470
## Practical_Problem_Solving   3.033 -0.6044 0.6685  0.12662
## Symbol_Manipulation         2.740 -0.6110 0.6397  0.08702
## Analytical_Ability          2.636 -0.6867 0.6939 -0.15307
## Formal_Logic                3.729 -0.6654 0.5589  0.05588
```

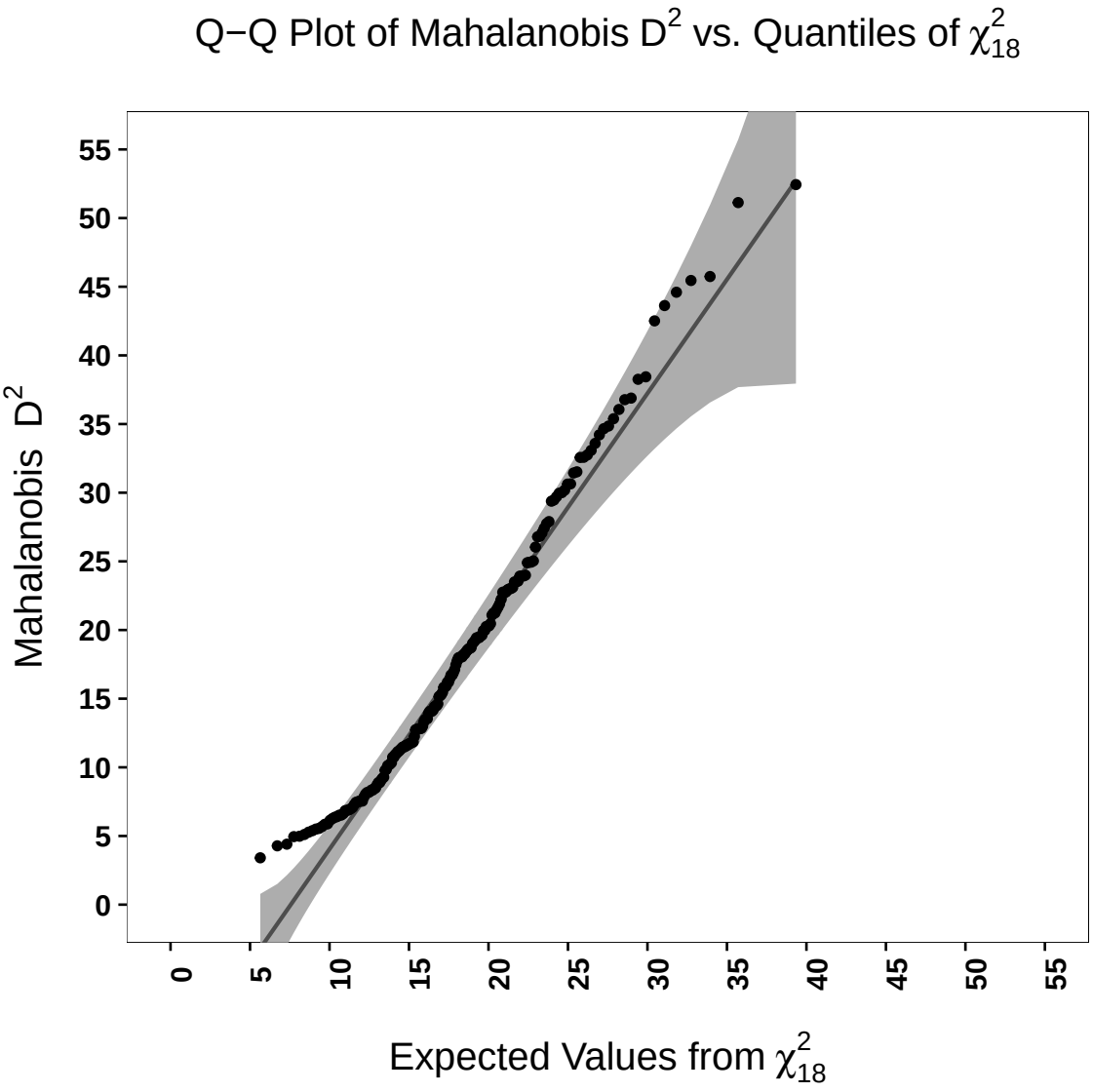
```
##                              Kurtosis
## Grammar                     -0.210884
## Paragraph_Comprehension     -0.185360
## Vocabulary                  -0.047192
## Sentence_Completion         -0.238030
## Geometry                    -0.174019
## Algebra                      0.171153
## Numerical_Puzzles            0.046084
## Series_Completion           -0.214526
## Practical_Problem_Solving   -0.019846
## Symbol_Manipulation          0.120057
## Analytical_Ability           0.001184
## Formal_Logic                 0.441237
```

The data are multivariate normal, which is of course not surprising given that they were generated from a multivariate normal distribution.

2.2 Need for Cognition

2.2.1 Mahalanobis Distance

```
CV <- cov(NC)
D2 <- mahalanobis(NC, center = colMeans(NC), cov = CV)
D2 <- as.data.frame(D2)
describe(D2)

##    vars   n  mean    sd median trimmed   mad  min   max range
## X1    1 195 17.91 10.18  15.92    16.8 10.15 3.41 52.44 49.03
##    skew kurtosis   se
## X1 0.97     0.61 0.73

ggplot(D2, aes(sample = D2)) + stat_qq_band(distribution = "chisq",
    dparams = list(df = 18)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 18)) + stat_qq(distribution = "qchisq", dparams = list(df = 18)) +
    scale_y_continuous(breaks = seq(0, 55, 5)) + scale_x_continuous(breaks = seq(0,
    55, 5)) + coord_cartesian(xlim = c(0, 55), ylim = c(0, 55)) +
    xlab(expression("Expected Values from" * ~chi[18]^2)) + ylab(expression("Mahalanobis " *
    ~D^2)) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" *
    ~D^2 * " vs. Quantiles of" * ~chi[18]^2))
```



The squared Mahalanobis distances suggest no multivariate outliers but the data appear to violate multivariate normality.

2.2.2 Multivariate Normality

We can also test for multivariate normality.

```
mvn(NC, mvnTest = "mardia")

## $multivariateNormality
##              Test        Statistic              p value Result
## 1 Mardia Skewness 2396.78535052338 1.67732326269208e-91     NO
## 2 Mardia Kurtosis 17.7171063217399                    0     NO
## 3             MVN             <NA>                 <NA>     NO
##
```

```
## $univariateNormality
##            Test  Variable Statistic   p value Normality
## 1  Shapiro-Wilk    nc1       0.8666  <0.001      NO
## 2  Shapiro-Wilk    nc2       0.8308  <0.001      NO
## 3  Shapiro-Wilk    nc3       0.7518  <0.001      NO
## 4  Shapiro-Wilk    nc4       0.7873  <0.001      NO
## 5  Shapiro-Wilk    nc5       0.7576  <0.001      NO
## 6  Shapiro-Wilk    nc6       0.8931  <0.001      NO
## 7  Shapiro-Wilk    nc7       0.8703  <0.001      NO
## 8  Shapiro-Wilk    nc8       0.8739  <0.001      NO
## 9  Shapiro-Wilk    nc9       0.8156  <0.001      NO
## 10 Shapiro-Wilk   nc10       0.7986  <0.001      NO
## 11 Shapiro-Wilk   nc11       0.7450  <0.001      NO
## 12 Shapiro-Wilk   nc12       0.7249  <0.001      NO
## 13 Shapiro-Wilk   nc13       0.9039  <0.001      NO
## 14 Shapiro-Wilk   nc14       0.8485  <0.001      NO
## 15 Shapiro-Wilk   nc15       0.8637  <0.001      NO
## 16 Shapiro-Wilk   nc16       0.8466  <0.001      NO
## 17 Shapiro-Wilk   nc17       0.7721  <0.001      NO
## 18 Shapiro-Wilk   nc18       0.8907  <0.001      NO
##
## $Descriptives
##        n  Mean Std.Dev Median Min Max 25th 75th    Skew Kurtosis
## nc1  195 3.241  1.2387      4   1   5  2.0    4 -0.5261  -0.8380
## nc2  195 3.913  1.0242      4   1   5  3.5    5 -0.9719   0.4887
## nc3  195 1.821  1.0618      1   1   5  1.0    2  1.3361   1.0606
## nc4  195 1.974  1.1462      2   1   5  1.0    2  1.1327   0.3874
## nc5  195 1.831  1.0439      2   1   5  1.0    2  1.2589   0.7722
## nc6  195 3.287  1.2012      4   1   5  2.0    4 -0.4373  -0.7488
## nc7  195 2.487  1.3174      2   1   5  1.0    4  0.4179  -1.0842
## nc8  195 2.518  1.2156      2   1   5  2.0    4  0.5151  -0.8168
## nc9  195 2.267  1.2644      2   1   5  1.0    3  0.8479  -0.4672
## nc10 195 4.056  0.9481      4   1   5  4.0    5 -1.1948   1.4206
## nc11 195 4.205  0.9627      4   1   5  4.0    5 -1.5193   2.2967
## nc12 195 1.754  1.0605      1   1   5  1.0    2  1.4276   1.2688
## nc13 195 3.287  1.1485      3   1   5  3.0    4 -0.3693  -0.6502
## nc14 195 3.749  1.0952      4   1   5  3.0    5 -0.8062  -0.1036
## nc15 195 3.677  1.0713      4   1   5  3.0    4 -0.7868   0.1836
## nc16 195 2.292  1.1892      2   1   5  1.0    3  0.7793  -0.3685
## nc17 195 1.903  1.1287      2   1   5  1.0    2  1.1535   0.3776
## nc18 195 3.328  1.2078      4   1   5  2.0    4 -0.3654  -0.9283
```

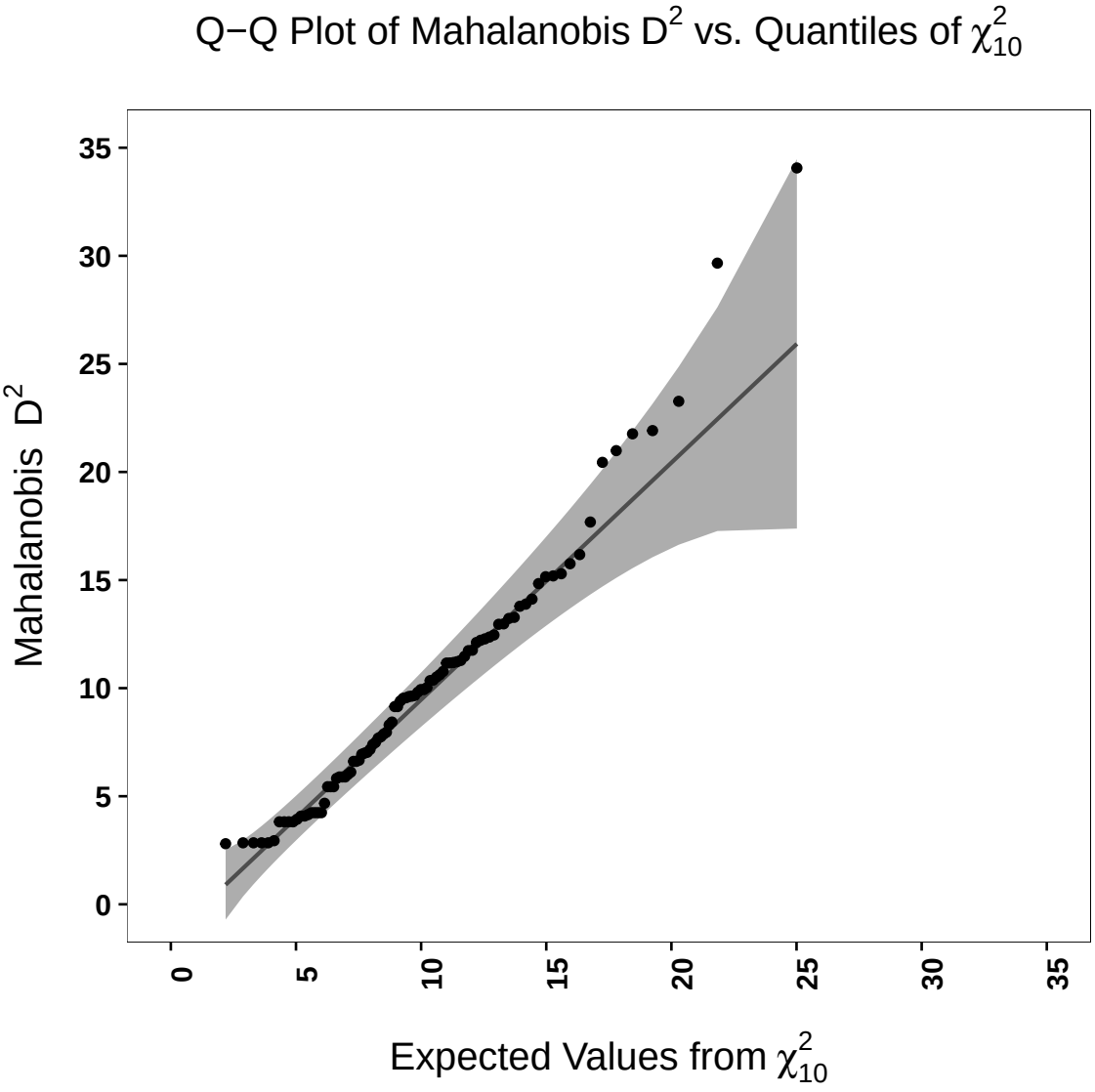
The data are not univariate or multivariate normal.

2.3 Self-Esteem Sample 1

2.3.1 Mahalanobis Distance

```
CV <- cov(SE[, 1:10])
D2 <- mahalanobis(SE[, 1:10], center = colMeans(SE[, 1:10]), cov = CV)
D2 <- as.data.frame(D2)
describe(D2)

##    vars  n mean   sd median trimmed  mad min   max range skew
```

```
## X1    1 94 9.89 5.72   9.54    9.15 5.08 2.8 34.06 31.26 1.51
##    kurtosis   se
## X1     3.33 0.59

ggplot(D2, aes(sample = D2)) + stat_qq_band(distribution = "chisq",
    dparams = list(df = 10)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 10)) + stat_qq(distribution = "qchisq", dparams = list(df = 10)) +
    scale_y_continuous(breaks = seq(0, 35, 5)) + scale_x_continuous(breaks = seq(0,
    35, 5)) + coord_cartesian(xlim = c(0, 35), ylim = c(0, 35)) +
    xlab(expression("Expected Values from" * ~chi[10]^2)) + ylab(expression("Mahalanobis " *
    ~D^2)) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" *
    ~D^2 * " vs. Quantiles of" * ~chi[10]^2))
```



The squared Mahalanobis distances suggest no multivariate outliers but the data appear to violate multivariate normality.

2.3.2 Multivariate Normality

We can also test for multivariate normality.

```
mvn(SE[, 1:10], mvnTest = "mardia")

## $multivariateNormality
##              Test        Statistic              p value Result
## 1 Mardia Skewness 500.450121412264 6.81205353279162e-24     NO
## 2 Mardia Kurtosis 4.10286690060031 0.000040806202145971     NO
## 3             MVN             <NA>                  <NA>     NO
##
```

```
## $univariateNormality
##            Test  Variable Statistic   p value Normality
## 1  Shapiro-Wilk   r_1_1      0.5919  <0.001      NO
## 2  Shapiro-Wilk   r_1_2      0.5753  <0.001      NO
## 3  Shapiro-Wilk   r_1_3      0.6562  <0.001      NO
## 4  Shapiro-Wilk   r_1_4      0.7373  <0.001      NO
## 5  Shapiro-Wilk   r_1_5      0.6739  <0.001      NO
## 6  Shapiro-Wilk   r_1_6      0.7866  <0.001      NO
## 7  Shapiro-Wilk   r_1_7      0.7878  <0.001      NO
## 8  Shapiro-Wilk   r_1_8      0.8678  <0.001      NO
## 9  Shapiro-Wilk   r_1_9      0.8704  <0.001      NO
## 10 Shapiro-Wilk  r_1_10      0.8231  <0.001      NO
##
## $Descriptives
##         n  Mean Std.Dev Median Min Max 25th 75th     Skew
## r_1_1  94 3.670  0.5937      4   1   4    3    4 -1.89147
## r_1_2  94 3.723  0.4730      4   2   4    3    4 -1.28100
## r_1_3  94 3.596  0.6103      4   2   4    3    4 -1.20663
## r_1_4  94 3.372  0.6219      3   1   4    3    4 -0.70899
## r_1_5  94 3.457  0.8252      4   1   4    3    4 -1.56957
## r_1_6  94 3.202  0.6968      3   1   4    3    4 -0.66526
## r_1_7  94 3.202  0.6653      3   1   4    3    4 -0.46013
## r_1_8  94 2.755  0.9912      3   1   4    2    4 -0.22450
## r_1_9  94 2.638  0.8902      3   1   4    2    3  0.03653
## r_1_10 94 3.064  0.9482      3   1   4    2    4 -0.57445
##        Kurtosis
## r_1_1   3.80261
## r_1_2   0.35707
## r_1_3   0.36166
## r_1_4   0.77997
## r_1_5   1.84237
## r_1_6   0.55887
## r_1_7   0.09848
## r_1_8  -1.06337
## r_1_9  -0.86502
## r_1_10 -0.81931
```

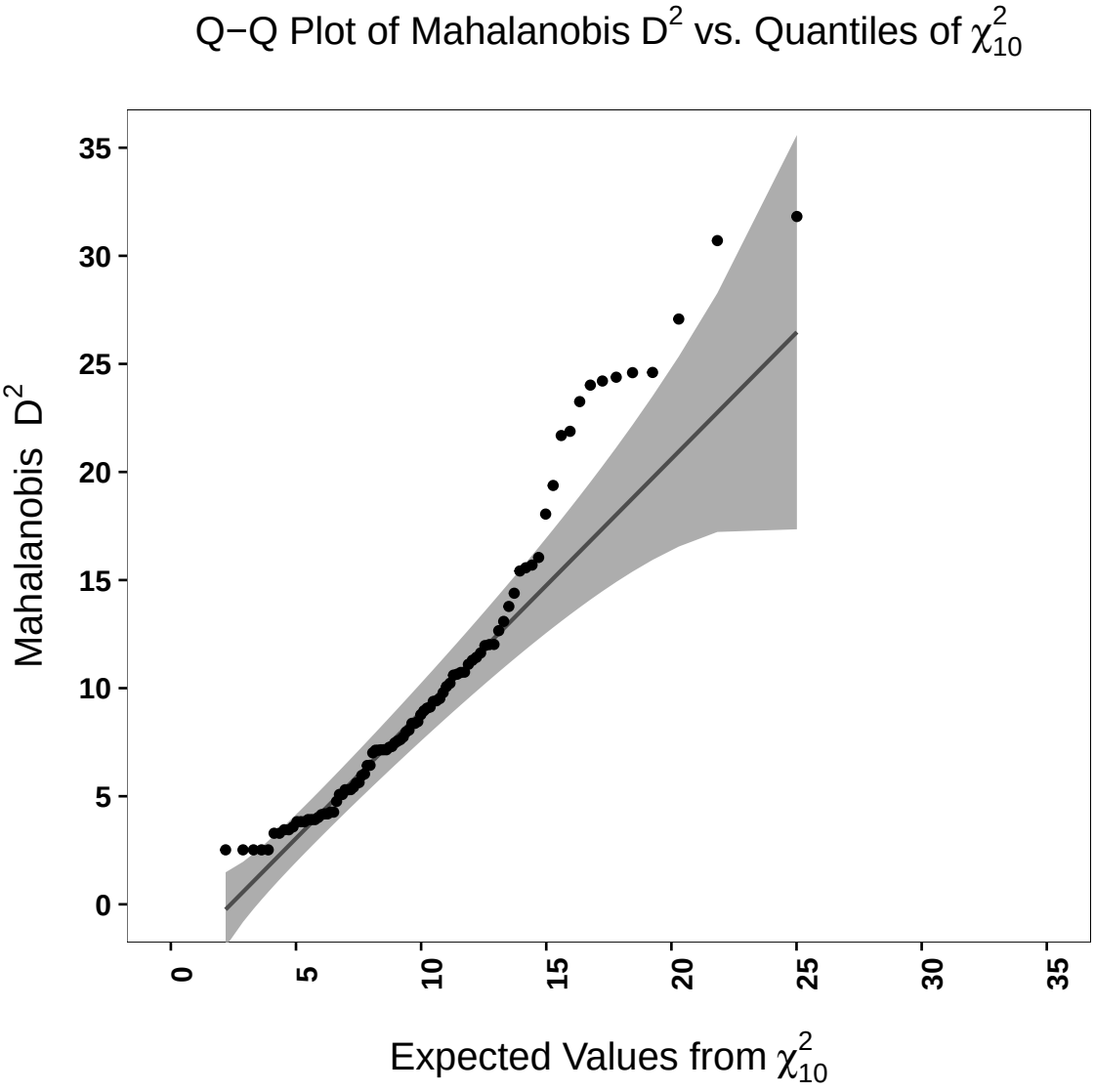
The data are not univariate or multivariate normal.

2.4 Self-Esteem Sample 2

2.4.1 Mahalanobis Distance

```
CV <- cov(SE[, 11:20])
D2 <- mahalanobis(SE[, 11:20], center = colMeans(SE[, 11:20]), cov = CV)
D2 <- as.data.frame(D2)
describe(D2)

##    vars  n mean   sd median trimmed  mad  min   max range skew
## X1    1 94 9.89 6.82   7.86    8.81 5.31 2.52 31.82  29.3 1.37
##    kurtosis  se
## X1     1.22 0.7

ggplot(D2, aes(sample = D2)) + stat_qq_band(distribution = "chisq",
```

```
    dparams = list(df = 10)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 10)) + stat_qq(distribution = "qchisq", dparams = list(df = 10)) +
    scale_y_continuous(breaks = seq(0, 35, 5)) + scale_x_continuous(breaks = seq(0,
    35, 5)) + coord_cartesian(xlim = c(0, 35), ylim = c(0, 35)) +
    xlab(expression("Expected Values from" * ~chi[10]^2)) + ylab(expression("Mahalanobis " *
    ~D^2)) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" *
    ~D^2 * " vs. Quantiles of" * ~chi[10]^2))
```



The squared Mahalanobis distances suggest no multivariate outliers but the data appear to violate multivariate normality.

2.4.2 Multivariate Normality

We can also test for multivariate normality.

```
mvn(SE[, 11:20], mvnTest = "mardia")

## $multivariateNormality
##              Test        Statistic              p value Result
## 1 Mardia Skewness 579.546878261195 3.64214052345009e-34     NO
## 2 Mardia Kurtosis 8.45722209724254                    0     NO
## 3             MVN             <NA>                 <NA>     NO
##
```

```
## $univariateNormality
##            Test  Variable Statistic   p value Normality
## 1  Shapiro-Wilk   r_2_1      0.5926  <0.001      NO
## 2  Shapiro-Wilk   r_2_2      0.5513  <0.001      NO
## 3  Shapiro-Wilk   r_2_3      0.6786  <0.001      NO
## 4  Shapiro-Wilk   r_2_4      0.7554  <0.001      NO
## 5  Shapiro-Wilk   r_2_5      0.6884  <0.001      NO
## 6  Shapiro-Wilk   r_2_6      0.7947  <0.001      NO
## 7  Shapiro-Wilk   r_2_7      0.7767  <0.001      NO
## 8  Shapiro-Wilk   r_2_8      0.8491  <0.001      NO
## 9  Shapiro-Wilk   r_2_9      0.8298  <0.001      NO
## 10 Shapiro-Wilk  r_2_10      0.7418  <0.001      NO
##
## $Descriptives
##         n  Mean Std.Dev Median Min Max 25th 75th    Skew
## r_2_1  94 3.702  0.5044      4   2   4    3    4 -1.3574
## r_2_2  94 3.734  0.4442      4   3   4    3    4 -1.0425
## r_2_3  94 3.564  0.6145      4   2   4    3    4 -1.0659
## r_2_4  94 3.351  0.6988      3   1   4    3    4 -0.9632
## r_2_5  94 3.479  0.7722      4   1   4    3    4 -1.4545
## r_2_6  94 3.181  0.6873      3   2   4    3    4 -0.2421
## r_2_7  94 3.191  0.6439      3   1   4    3    4 -0.4308
## r_2_8  94 2.904  0.9624      3   1   4    2    4 -0.3124
## r_2_9  94 2.979  0.9501      3   1   4    2    4 -0.3304
## r_2_10 94 3.266  0.9636      4   1   4    3    4 -0.9710
##        Kurtosis
## r_2_1   0.80618
## r_2_2  -0.92260
## r_2_3   0.05539
## r_2_4   0.98720
## r_2_5   1.55834
## r_2_6  -0.91969
## r_2_7   0.32236
## r_2_8  -1.06265
## r_2_9  -1.15290
## r_2_10 -0.34993
```

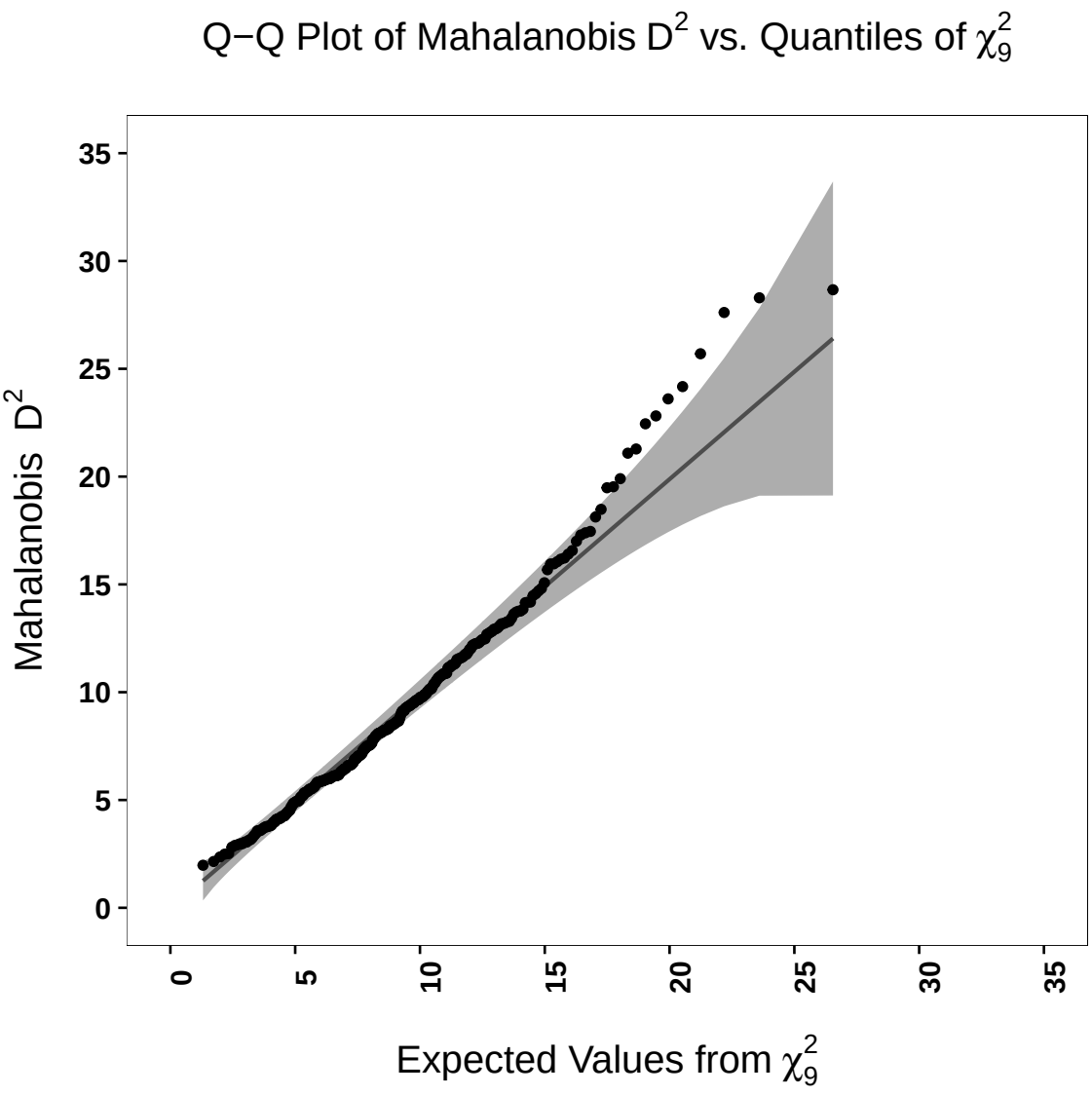
The data are not univariate or multivariate normal.

2.5 Holzinger and Swineford (1939)

2.5.1 Mahalanobis Distance

```
CV <- cov(HS[, 2:10])
D2 <- mahalanobis(HS[, 2:10], center = colMeans(HS[, 2:10]), cov = CV)
D2 <- as.data.frame(D2)
describe(D2)

##    vars   n mean   sd median trimmed  mad  min   max range skew
## X1    1 301 8.97 4.75   8.09    8.38 3.97 1.97 28.67 26.69 1.41
##    kurtosis   se
## X1     2.66 0.27

ggplot(D2, aes(sample = D2)) + stat_qq_band(distribution = "chisq",
```

```
    dparams = list(df = 9)) + stat_qq_line(distribution = "chisq",
    dparams = list(df = 9)) + stat_qq(distribution = "qchisq", dparams = list(df = 9)) +
    scale_y_continuous(breaks = seq(0, 35, 5)) + scale_x_continuous(breaks = seq(0,
    35, 5)) + coord_cartesian(xlim = c(0, 35), ylim = c(0, 35)) +
    xlab(expression("Expected Values from" * ~chi[9]^2)) + ylab(expression("Mahalanobis " *
    ~D^2)) + theme(text = element_text(size = 14, family = "sans",
    color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 90), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + ggtitle(expression("Q-Q Plot of Mahalanobis" *
    ~D^2 * " vs. Quantiles of" * ~chi[9]^2))
```



The squared Mahalanobis distances suggest no multivariate outliers but the data appear to violate multivariate normality.

2.5.2 Multivariate Normality

We can also test for multivariate normality.

```
mvn(HS[, 2:10], mvnTest = "mardia")

## $multivariateNormality
##              Test         Statistic              p value Result
## 1 Mardia Skewness 344.905276936947 8.86473544030093e-15     NO
## 2 Mardia Kurtosis 2.83021589156651  0.00465166039260279     NO
## 3             MVN              <NA>                  <NA>     NO
##
```

```
## $univariateNormality
##            Test  Variable Statistic   p value Normality
## 1 Shapiro-Wilk    x1        0.9928  0.1582      YES
## 2 Shapiro-Wilk    x2        0.9697  <0.001      NO
## 3 Shapiro-Wilk    x3        0.9523  <0.001      NO
## 4 Shapiro-Wilk    x4        0.9827  0.0011      NO
## 5 Shapiro-Wilk    x5        0.9769  0.0001      NO
## 6 Shapiro-Wilk    x6        0.9538  <0.001      NO
## 7 Shapiro-Wilk    x7        0.9908   0.056      YES
## 8 Shapiro-Wilk    x8        0.9807  0.0004      NO
## 9 Shapiro-Wilk    x9        0.9942   0.307      YES
##
## $Descriptives
##       n  Mean Std.Dev Median    Min    Max  25th  75th    Skew
## x1 301 4.936   1.167  5.000 0.6667  8.500 4.167 5.667 -0.2543
## x2 301 6.088   1.177  6.000 2.2500  9.250 5.250 6.750  0.4701
## x3 301 2.250   1.131  2.125 0.2500  4.500 1.375 3.125  0.3834
## x4 301 3.061   1.164  3.000 0.0000  6.333 2.333 3.667  0.2675
## x5 301 4.341   1.290  4.500 1.0000  7.000 3.500 5.250 -0.3498
## x6 301 2.186   1.096  2.000 0.1429  6.143 1.429 2.714  0.8579
## x7 301 4.186   1.090  4.087 1.3043  7.435 3.478 4.913  0.2491
## x8 301 5.527   1.013  5.500 3.0500 10.000 4.850 6.100  0.5253
## x9 301 5.374   1.009  5.417 2.7778  9.250 4.750 6.083  0.2039
##    Kurtosis
## x1  0.30753
## x2  0.33239
## x3 -0.90753
## x4  0.08013
## x5 -0.55254
## x6  0.81656
## x7 -0.30740
## x8  1.17156
## x9  0.28991
```

The data are not univariate or multivariate normal.

3 CFA With Two Approaches to Scaling

The scale for the latent variables can either be set by using one of the manifest variables or by specifying the variances of the latent variables directly (usually by standardizing them). The two approaches will produce identical fits to the data but will differ in the particular parameters that get estimated.

```
scaling.model.1 <- "
# Latent variable definitions.
# Scale of the latent variables is set by the first listed manifest variable.
Verbal =~ Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion
Math =~ Geometry+Algebra+Numerical_Puzzles+Series_Completion
Reasoning =~ Practical_Problem_Solving+Symbol_Manipulation+Analytical_Ability+Formal_Logic
# Latent variable covariances.
Verbal ~~ Math
Verbal ~~ Reasoning
Math ~~ Reasoning
```

```
"

scaling.model.2 <- "
# Latent variable definitions.
# Scale of the latent variables is set directly.
Verbal =~ NA*Grammar+Paragraph_Comprehension+Vocabulary+Sentence_Completion
Math =~ NA*Geometry+Algebra+Numerical_Puzzles+Series_Completion
Reasoning =~ NA*Practical_Problem_Solving+Symbol_Manipulation+Analytical_Ability+Formal_Logic
# Latent variable covariances.
Verbal ~~ Math
Verbal ~~ Reasoning
Math ~~ Reasoning
# Latent variable variances.
Verbal ~~ 1*Verbal
Math ~~ 1*Math
Reasoning ~~ 1*Reasoning
"
```

```
CFA_Fit_1 <- cfa(scaling.model.1, data = Mental, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_1, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  29 iterations
##
##   Number of observations                           500
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                      55.498      56.546
##   Degrees of freedom                                51          51
##   P-value (Chi-square)                           0.309       0.276
##   Scaling correction factor                                  0.981
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1962.116    1980.927
##   Degrees of freedom                                66          66
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.998       0.997
##   Tucker-Lewis Index (TLI)                       0.997       0.996
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7510.369   -7510.369
##   Loglikelihood unrestricted model (H1)      -7482.565   -7482.565
##
```

```
##   Number of free parameters                         39          39
##   Akaike (AIC)                                15098.739   15098.739
##   Bayesian (BIC)                              15263.108   15263.108
##   Sample-size adjusted Bayesian (BIC)         15139.320   15139.320
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                           0.013       0.015
##   90 Percent Confidence Interval          0.000   0.032       0.000   0.033
##   P-value RMSEA <= 0.05                           1.000       1.000
##
##   Robust RMSEA                                                   NA
##   90 Percent Confidence Interval                              0.000      NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                            0.026       0.026
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal =~
##     Grammar           1.000                               0.716
##     Prgrph_Cmprhns    1.065    0.080   13.256    0.000    0.762
##     Vocabulary        0.960    0.070   13.772    0.000    0.687
##     Sentenc_Cmpltn    1.108    0.079   13.940    0.000    0.793
##   Math =~
##     Geometry          1.000                               0.669
##     Algebra           1.002    0.082   12.225    0.000    0.670
##     Numericl_Pzzls    0.994    0.080   12.357    0.000    0.664
##     Series_Compltn    1.031    0.079   13.081    0.000    0.689
##   Reasoning =~
##     Prctcl_Prblm_S    1.000                               0.650
##     Symbol_Manpltn    1.030    0.081   12.652    0.000    0.670
##     Analytcl_Ablty    1.097    0.085   12.899    0.000    0.713
##     Formal_Logic      1.143    0.090   12.647    0.000    0.744
##   Std.all
##
##     0.734
##     0.718
##     0.689
##     0.731
##
##     0.684
##     0.698
##     0.672
##     0.716
##
##     0.695
```

```
##     0.690
##     0.726
##     0.732
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal ~~
##     Math              0.199    0.029    6.782    0.000    0.417
##     Reasoning         0.215    0.029    7.442    0.000    0.461
##   Math ~~
##     Reasoning         0.176    0.026    6.759    0.000    0.405
##   Std.all
##
##     0.417
##     0.461
##
##     0.405
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          -0.008    0.044   -0.175    0.861   -0.008
##    .Prgrph_Cmprhns   -0.004    0.048   -0.079    0.937   -0.004
##    .Vocabulary       -0.015    0.045   -0.331    0.741   -0.015
##    .Sentenc_Cmpltn   -0.088    0.049   -1.808    0.071   -0.088
##    .Geometry          0.011    0.044    0.243    0.808    0.011
##    .Algebra          -0.029    0.043   -0.666    0.505   -0.029
##    .Numericl_Pzzls   -0.015    0.044   -0.341    0.733   -0.015
##    .Series_Compltn   -0.006    0.043   -0.147    0.883   -0.006
##    .Prctcl_Prblm_S   -0.017    0.042   -0.402    0.688   -0.017
##    .Symbol_Manpltn    0.001    0.043    0.019    0.985    0.001
##    .Analytcl_Ablty   -0.005    0.044   -0.106    0.916   -0.005
##    .Formal_Logic     -0.063    0.045   -1.385    0.166   -0.063
##     Verbal            0.000                               0.000
##     Math              0.000                               0.000
##     Reasoning         0.000                               0.000
##   Std.all
##    -0.008
##    -0.004
##    -0.015
##    -0.081
##     0.011
##    -0.030
##    -0.015
##    -0.007
##    -0.018
##     0.001
##    -0.005
##    -0.062
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
```

```
##    .Grammar           0.439    0.036   12.355    0.000    0.439
##    .Prgrph_Cmprhns    0.547    0.044   12.390    0.000    0.547
##    .Vocabulary        0.524    0.041   12.747    0.000    0.524
##    .Sentenc_Cmpltn    0.548    0.044   12.378    0.000    0.548
##    .Geometry          0.509    0.046   11.188    0.000    0.509
##    .Algebra           0.474    0.041   11.462    0.000    0.474
##    .Numericl_Pzzls    0.535    0.045   11.762    0.000    0.535
##    .Series_Compltn    0.452    0.040   11.349    0.000    0.452
##    .Prctcl_Prblm_S    0.452    0.039   11.562    0.000    0.452
##    .Symbol_Manpltn    0.494    0.042   11.846    0.000    0.494
##    .Analytcl_Ablty    0.456    0.042   10.850    0.000    0.456
##    .Formal_Logic      0.478    0.041   11.787    0.000    0.478
##     Verbal            0.513    0.060    8.520    0.000    1.000
##     Math              0.447    0.056    7.997    0.000    1.000
##     Reasoning         0.423    0.049    8.639    0.000    1.000
##   Std.all
##     0.461
##     0.485
##     0.526
##     0.466
##     0.533
##     0.513
##     0.548
##     0.488
##     0.516
##     0.524
##     0.473
##     0.464
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     Grammar           0.539
##     Prgrph_Cmprhns    0.515
##     Vocabulary        0.474
##     Sentenc_Cmpltn    0.534
##     Geometry          0.467
##     Algebra           0.487
##     Numericl_Pzzls    0.452
##     Series_Compltn    0.512
##     Prctcl_Prblm_S    0.484
##     Symbol_Manpltn    0.476
##     Analytcl_Ablty    0.527
##     Formal_Logic      0.536

CFA_Fit_2 <- cfa(scaling.model.2, data = Mental, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_2, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  17 iterations
##
##   Number of observations                           500
```

```
##   Number of missing patterns                          1
##
##   Estimator                                          ML      Robust
##   Model Fit Test Statistic                       55.498      56.546
##   Degrees of freedom                                 51          51
##   P-value (Chi-square)                            0.309       0.276
##   Scaling correction factor                                   0.981
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1962.116    1980.927
##   Degrees of freedom                                 66          66
##   P-value                                         0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                     0.998       0.997
##   Tucker-Lewis Index (TLI)                        0.997       0.996
##
##   Robust Comparative Fit Index (CFI)                             NA
##   Robust Tucker-Lewis Index (TLI)                                NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -7510.369   -7510.369
##   Loglikelihood unrestricted model (H1)      -7482.565   -7482.565
##
##   Number of free parameters                          39          39
##   Akaike (AIC)                                15098.739   15098.739
##   Bayesian (BIC)                              15263.108   15263.108
##   Sample-size adjusted Bayesian (BIC)         15139.320   15139.320
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                           0.013       0.015
##   90 Percent Confidence Interval          0.000   0.032       0.000  0.033
##   P-value RMSEA <= 0.05                           1.000       1.000
##
##   Robust RMSEA                                                   NA
##   90 Percent Confidence Interval                              0.000     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                            0.026       0.026
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
```

```
##   Verbal =~
##     Grammar           0.716    0.042   17.040    0.000    0.716
##     Prgrph_Cmprhns    0.762    0.044   17.213    0.000    0.762
##     Vocabulary        0.687    0.041   16.598    0.000    0.687
##     Sentenc_Cmpltn    0.793    0.046   17.357    0.000    0.793
##   Math =~
##     Geometry          0.669    0.042   15.995    0.000    0.669
##     Algebra           0.670    0.045   14.906    0.000    0.670
##     Numericl_Pzzls    0.664    0.044   15.098    0.000    0.664
##     Series_Compltn    0.689    0.041   16.783    0.000    0.689
##   Reasoning =~
##     Prctcl_Prblm_S    0.650    0.038   17.277    0.000    0.650
##     Symbol_Manpltn    0.670    0.045   14.787    0.000    0.670
##     Analytcl_Ablty    0.713    0.044   16.360    0.000    0.713
##     Formal_Logic      0.744    0.046   16.102    0.000    0.744
##   Std.all
##
##     0.734
##     0.718
##     0.689
##     0.731
##
##     0.684
##     0.698
##     0.672
##     0.716
##
##     0.695
##     0.690
##     0.726
##     0.732
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Verbal ~~
##     Math              0.417    0.046    9.108    0.000    0.417
##     Reasoning         0.461    0.048    9.528    0.000    0.461
##   Math ~~
##     Reasoning         0.405    0.048    8.412    0.000    0.405
##   Std.all
##
##     0.417
##     0.461
##
##     0.405
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .Grammar          -0.008    0.044   -0.175    0.861   -0.008
##    .Prgrph_Cmprhns   -0.004    0.048   -0.079    0.937   -0.004
##    .Vocabulary       -0.015    0.045   -0.331    0.741   -0.015
##    .Sentenc_Cmpltn   -0.088    0.049   -1.808    0.071   -0.088
##    .Geometry          0.011    0.044    0.243    0.808    0.011
##    .Algebra          -0.029    0.043   -0.666    0.505   -0.029
```

```
##    .Numericl_Pzzls   -0.015    0.044   -0.341    0.733   -0.015
##    .Series_Compltn   -0.006    0.043   -0.147    0.883   -0.006
##    .Prctcl_Prblm_S   -0.017    0.042   -0.402    0.688   -0.017
##    .Symbol_Manpltn    0.001    0.043    0.019    0.985    0.001
##    .Analytcl_Ablty   -0.005    0.044   -0.106    0.916   -0.005
##    .Formal_Logic     -0.063    0.045   -1.385    0.166   -0.063
##     Verbal            0.000                              0.000
##     Math              0.000                              0.000
##     Reasoning         0.000                              0.000
##   Std.all
##    -0.008
##    -0.004
##    -0.015
##    -0.081
##     0.011
##    -0.030
##    -0.015
##    -0.007
##    -0.018
##     0.001
##    -0.005
##    -0.062
##     0.000
##     0.000
##     0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     Verbal           1.000                              1.000
##     Math             1.000                              1.000
##     Reasoning        1.000                              1.000
##    .Grammar          0.439    0.036   12.355    0.000    0.439
##    .Prgrph_Cmprhns   0.547    0.044   12.390    0.000    0.547
##    .Vocabulary       0.524    0.041   12.747    0.000    0.524
##    .Sentenc_Cmpltn   0.548    0.044   12.378    0.000    0.548
##    .Geometry         0.509    0.046   11.188    0.000    0.509
##    .Algebra          0.474    0.041   11.462    0.000    0.474
##    .Numericl_Pzzls   0.535    0.045   11.762    0.000    0.535
##    .Series_Compltn   0.452    0.040   11.349    0.000    0.452
##    .Prctcl_Prblm_S   0.452    0.039   11.562    0.000    0.452
##    .Symbol_Manpltn   0.494    0.042   11.846    0.000    0.494
##    .Analytcl_Ablty   0.456    0.042   10.850    0.000    0.456
##    .Formal_Logic     0.478    0.041   11.787    0.000    0.478
##   Std.all
##     1.000
##     1.000
##     1.000
##     0.461
##     0.485
##     0.526
##     0.466
##     0.533
##     0.513
##     0.548
```

```
##     0.488
##     0.516
##     0.524
##     0.473
##     0.464
##
## R-Square:
##                    Estimate
##     Grammar           0.539
##     Prgrph_Cmprhns    0.515
##     Vocabulary        0.474
##     Sentenc_Cmpltn    0.534
##     Geometry          0.467
##     Algebra           0.487
##     Numericl_Pzzls    0.452
##     Series_Compltn    0.512
##     Prctcl_Prblm_S    0.484
##     Symbol_Manpltn    0.476
##     Analytcl_Ablty    0.527
##     Formal_Logic      0.536
```

4 CFA With Covariance versus Correlation Matrices

Confirmatory factor analysis models are usually based on the decomposition of covariance matrices, not correlation matrices. The solutions hold, strictly speaking, for the analysis of covariance matrices. To the extent that the solution depends on the scale of the variables, analyses based on covariance matrices and correlation matrices can differ. The differences are a function of the differences in the variances across the variables.

```
NC.model.1 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
"
```

```
CFA_Fit_3 <- cfa(NC.model.1, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_3, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  29 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     267.315     228.948
##   Degrees of freedom                               135         135
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.168
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
```

```
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.854       0.870
##   Tucker-Lewis Index (TLI)                       0.835       0.853
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4984.773   -4984.773
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         54          54
##   Akaike (AIC)                               10077.545   10077.545
##   Bayesian (BIC)                             10254.287   10254.287
##   Sample-size adjusted Bayesian (BIC)        10083.223   10083.223
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.071       0.060
##   90 Percent Confidence Interval          0.059  0.084       0.047  0.072
##   P-value RMSEA <= 0.05                          0.004       0.095
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.060       0.060
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.691
##     nc2               1.072    0.155    6.922    0.000    0.741
##     nc3              -0.884    0.152   -5.818    0.000   -0.611
##     nc4              -1.038    0.189   -5.496    0.000   -0.717
##     nc5              -1.018    0.184   -5.543    0.000   -0.704
##     nc6               0.682    0.155    4.395    0.000    0.472
##     nc7              -0.957    0.164   -5.826    0.000   -0.662
##     nc8              -0.787    0.143   -5.519    0.000   -0.544
##     nc9              -0.951    0.182   -5.210    0.000   -0.657
```

```
##     nc10              0.801    0.134    5.980    0.000    0.554
##     nc11              0.906    0.149    6.095    0.000    0.626
##     nc12             -0.910    0.162   -5.602    0.000   -0.629
##     nc13              0.865    0.122    7.095    0.000    0.598
##     nc14              0.800    0.129    6.218    0.000    0.553
##     nc15              0.567    0.148    3.819    0.000    0.392
##     nc16             -0.604    0.153   -3.956    0.000   -0.418
##     nc17             -0.836    0.170   -4.912    0.000   -0.578
##     nc18              0.319    0.140    2.277    0.023    0.221
##   Std.all
##
##     0.560
##     0.725
##    -0.577
##    -0.627
##    -0.676
##     0.394
##    -0.504
##    -0.448
##    -0.521
##     0.586
##     0.652
##    -0.594
##     0.522
##     0.506
##     0.367
##    -0.352
##    -0.513
##     0.183
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                             0.000
##   Std.all
##     2.623
##     3.830
```

```
##     1.719
##     1.727
##     1.758
##     2.744
##     1.893
##     2.077
##     1.797
##     4.290
##     4.379
##     1.658
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.049    0.131    7.997    0.000    1.049
##    .nc2               0.495    0.077    6.450    0.000    0.495
##    .nc3               0.748    0.116    6.440    0.000    0.748
##    .nc4               0.793    0.129    6.153    0.000    0.793
##    .nc5               0.588    0.085    6.945    0.000    0.588
##    .nc6               1.213    0.127    9.541    0.000    1.213
##    .nc7               1.289    0.129    9.990    0.000    1.289
##    .nc8               1.175    0.111   10.615    0.000    1.175
##    .nc9               1.159    0.136    8.526    0.000    1.159
##    .nc10              0.588    0.093    6.305    0.000    0.588
##    .nc11              0.530    0.084    6.311    0.000    0.530
##    .nc12              0.724    0.125    5.766    0.000    0.724
##    .nc13              0.955    0.092   10.336    0.000    0.955
##    .nc14              0.887    0.103    8.645    0.000    0.887
##    .nc15              0.988    0.132    7.515    0.000    0.988
##    .nc16              1.232    0.145    8.477    0.000    1.232
##    .nc17              0.934    0.113    8.255    0.000    0.934
##    .nc18              1.402    0.112   12.569    0.000    1.402
##     NC                0.478    0.125    3.823    0.000    1.000
##   Std.all
##     0.687
##     0.474
##     0.667
##     0.606
##     0.543
##     0.845
##     0.746
##     0.799
##     0.728
##     0.657
##     0.574
##     0.647
##     0.728
##     0.744
```

```
##     0.866
##     0.876
##     0.737
##     0.966
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.313
##     nc2               0.526
##     nc3               0.333
##     nc4               0.394
##     nc5               0.457
##     nc6               0.155
##     nc7               0.254
##     nc8               0.201
##     nc9               0.272
##     nc10              0.343
##     nc11              0.426
##     nc12              0.353
##     nc13              0.272
##     nc14              0.256
##     nc15              0.134
##     nc16              0.124
##     nc17              0.263
##     nc18              0.034

CFA_Fit_4 <- cfa(NC.model.1, data = as.data.frame(scale(NC)), missing = "ML",
    estimator = "MLR", likelihood = "wishart", representation = "LISREL")
summary(CFA_Fit_4, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  26 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     267.315     228.948
##   Degrees of freedom                               135         135
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.168
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic            1061.451     874.878
##   Degrees of freedom                              153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.854       0.870
##   Tucker-Lewis Index (TLI)                       0.835       0.853
##
```

```
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4572.337   -4572.337
##   Loglikelihood unrestricted model (H1)      -4437.990   -4437.990
##
##   Number of free parameters                         54          54
##   Akaike (AIC)                                9252.673    9252.673
##   Bayesian (BIC)                              9429.415    9429.415
##   Sample-size adjusted Bayesian (BIC)         9258.351    9258.351
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.071       0.060
##   90 Percent Confidence Interval          0.059  0.084       0.047  0.072
##   P-value RMSEA <= 0.05                          0.004       0.095
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.060       0.060
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.558
##     nc2               1.296    0.187    6.922    0.000    0.724
##     nc3              -1.031    0.177   -5.818    0.000   -0.575
##     nc4              -1.121    0.204   -5.496    0.000   -0.626
##     nc5              -1.208    0.218   -5.543    0.000   -0.674
##     nc6               0.704    0.160    4.395    0.000    0.393
##     nc7              -0.900    0.155   -5.826    0.000   -0.502
##     nc8              -0.801    0.145   -5.519    0.000   -0.447
##     nc9              -0.931    0.179   -5.210    0.000   -0.520
##     nc10              1.046    0.175    5.980    0.000    0.584
##     nc11              1.166    0.191    6.095    0.000    0.651
##     nc12             -1.062    0.190   -5.602    0.000   -0.593
##     nc13              0.933    0.131    7.095    0.000    0.521
##     nc14              0.905    0.146    6.218    0.000    0.505
##     nc15              0.655    0.172    3.819    0.000    0.366
##     nc16             -0.630    0.159   -3.956    0.000   -0.351
##     nc17             -0.917    0.187   -4.912    0.000   -0.512
##     nc18              0.328    0.144    2.277    0.023    0.183
```

```
##   Std.all
##
##     0.560
##     0.725
##    -0.577
##    -0.627
##    -0.676
##     0.394
##    -0.504
##    -0.448
##    -0.521
##     0.586
##     0.652
##    -0.594
##     0.522
##     0.506
##     0.367
##    -0.352
##    -0.513
##     0.183
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1              -0.000    0.072   -0.000    1.000   -0.000
##    .nc2               0.000    0.072    0.000    1.000    0.000
##    .nc3               0.000    0.072    0.000    1.000    0.000
##    .nc4              -0.000    0.072   -0.000    1.000   -0.000
##    .nc5               0.000    0.072    0.000    1.000    0.000
##    .nc6               0.000    0.072    0.000    1.000    0.000
##    .nc7              -0.000    0.072   -0.000    1.000   -0.000
##    .nc8               0.000    0.072    0.000    1.000    0.000
##    .nc9               0.000    0.072    0.000    1.000    0.000
##    .nc10              0.000    0.072    0.000    1.000    0.000
##    .nc11             -0.000    0.072   -0.000    1.000   -0.000
##    .nc12              0.000    0.072    0.000    1.000    0.000
##    .nc13              0.000    0.072    0.000    1.000    0.000
##    .nc14              0.000    0.072    0.000    1.000    0.000
##    .nc15              0.000    0.072    0.000    1.000    0.000
##    .nc16              0.000    0.072    0.000    1.000    0.000
##    .nc17              0.000    0.072    0.000    1.000    0.000
##    .nc18             -0.000    0.072   -0.000    1.000   -0.000
##     NC                0.000                              0.000
##   Std.all
##    -0.000
##     0.000
##     0.000
##    -0.000
##     0.000
##     0.000
##    -0.000
##     0.000
##     0.000
##     0.000
##    -0.000
```

```
##     0.000
##     0.000
##     0.000
##     0.000
##     0.000
##     0.000
##    -0.000
##     0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1              0.683    0.085    7.997    0.000    0.683
##    .nc2              0.471    0.073    6.450    0.000    0.471
##    .nc3              0.664    0.103    6.440    0.000    0.664
##    .nc4              0.603    0.098    6.153    0.000    0.603
##    .nc5              0.540    0.078    6.945    0.000    0.540
##    .nc6              0.841    0.088    9.541    0.000    0.841
##    .nc7              0.742    0.074    9.990    0.000    0.742
##    .nc8              0.795    0.075   10.615    0.000    0.795
##    .nc9              0.725    0.085    8.526    0.000    0.725
##    .nc10             0.654    0.104    6.305    0.000    0.654
##    .nc11             0.571    0.091    6.311    0.000    0.571
##    .nc12             0.643    0.112    5.766    0.000    0.643
##    .nc13             0.724    0.070   10.336    0.000    0.724
##    .nc14             0.740    0.086    8.645    0.000    0.740
##    .nc15             0.861    0.115    7.515    0.000    0.861
##    .nc16             0.871    0.103    8.477    0.000    0.871
##    .nc17             0.733    0.089    8.255    0.000    0.733
##    .nc18             0.961    0.076   12.569    0.000    0.961
##     NC               0.311    0.081    3.823    0.000    1.000
##   Std.all
##     0.687
##     0.474
##     0.667
##     0.606
##     0.543
##     0.845
##     0.746
##     0.799
##     0.728
##     0.657
##     0.574
##     0.647
##     0.728
##     0.744
##     0.866
##     0.876
##     0.737
##     0.966
##     1.000
##
## R-Square:
##                   Estimate
##     nc1              0.313
```

```
##     nc2               0.526
##     nc3               0.333
##     nc4               0.394
##     nc5               0.457
##     nc6               0.155
##     nc7               0.254
##     nc8               0.201
##     nc9               0.272
##     nc10              0.343
##     nc11              0.426
##     nc12              0.353
##     nc13              0.272
##     nc14              0.256
##     nc15              0.134
##     nc16              0.124
##     nc17              0.263
##     nc18              0.034
```

```
cov(NC)
```

```
##          nc1     nc2     nc3     nc4     nc5     nc6      nc7
## nc1   1.5344  0.5778 -0.3122 -0.3855 -0.3920  0.3634 -0.46854
## nc2   0.5778  1.0491 -0.5054 -0.4146 -0.5303  0.3138 -0.46246
## nc3  -0.3122 -0.5054  1.1274  0.4541  0.4952 -0.2420  0.44872
## nc4  -0.3855 -0.4146  0.4541  1.3138  0.6812 -0.2297  0.52802
## nc5  -0.3920 -0.5303  0.4952  0.6812  1.0898 -0.2810  0.53132
## nc6   0.3634  0.3138 -0.2420 -0.2297 -0.2810  1.4429 -0.38805
## nc7  -0.4685 -0.4625  0.4487  0.5280  0.5313 -0.3881  1.73566
## nc8  -0.3574 -0.4443  0.2893  0.4257  0.3922 -0.2217  0.54018
## nc9  -0.5027 -0.4148  0.5172  0.5945  0.4165 -0.2522  0.36426
## nc10  0.3368  0.3709 -0.4073 -0.3748 -0.3512  0.3239 -0.33175
## nc11  0.4297  0.5077 -0.3290 -0.4432 -0.3878  0.3377 -0.33241
## nc12 -0.4764 -0.4649  0.3113  0.4627  0.5715 -0.3207  0.48652
## nc13  0.5593  0.4582 -0.2936 -0.3844 -0.3790  0.2882 -0.27465
## nc14  0.4475  0.4367 -0.3134 -0.3209 -0.2902  0.3663 -0.37695
## nc15  0.3669  0.3634 -0.2799 -0.1939 -0.1684  0.2170 -0.09437
## nc16 -0.4162 -0.2270  0.2022  0.4354  0.3178 -0.2132  0.32593
## nc17 -0.3785 -0.4828  0.3381  0.5233  0.3855 -0.2966  0.37761
## nc18  0.1731  0.2349 -0.1831 -0.1204 -0.1194  0.2455 -0.23804
##           nc8      nc9     nc10    nc11    nc12     nc13    nc14
## nc1  -0.35744 -0.50275  0.33685  0.4297 -0.4764  0.55929  0.4475
## nc2  -0.44430 -0.41478  0.37092  0.5077 -0.4649  0.45815  0.4367
## nc3   0.28932  0.51718 -0.40735 -0.3290  0.3113 -0.29355 -0.3134
## nc4   0.42572  0.59450 -0.37483 -0.4432  0.4627 -0.38435 -0.3209
## nc5   0.39223  0.41649 -0.35123 -0.3878  0.5715 -0.37898 -0.2902
## nc6  -0.22168 -0.25223  0.32392  0.3377 -0.3207  0.28824  0.3663
## nc7   0.54018  0.36426 -0.33175 -0.3324  0.4865 -0.27465 -0.3769
## nc8   1.47777  0.47457 -0.33865 -0.1635  0.1333 -0.32992 -0.4156
## nc9   0.47457  1.59863 -0.44811 -0.4055  0.3340 -0.35533 -0.2058
## nc10 -0.33865 -0.44811  0.89886  0.4729 -0.2799  0.34454  0.3648
## nc11 -0.16349 -0.40550  0.47291  0.9268 -0.4029  0.45625  0.3302
## nc12  0.13331  0.33402 -0.27986 -0.4029  1.1247 -0.41348 -0.4230
## nc13 -0.32992 -0.35533  0.34454  0.4563 -0.4135  1.31916  0.5674
## nc14 -0.41557 -0.20584  0.36476  0.3302 -0.4230  0.56738  1.1994
```

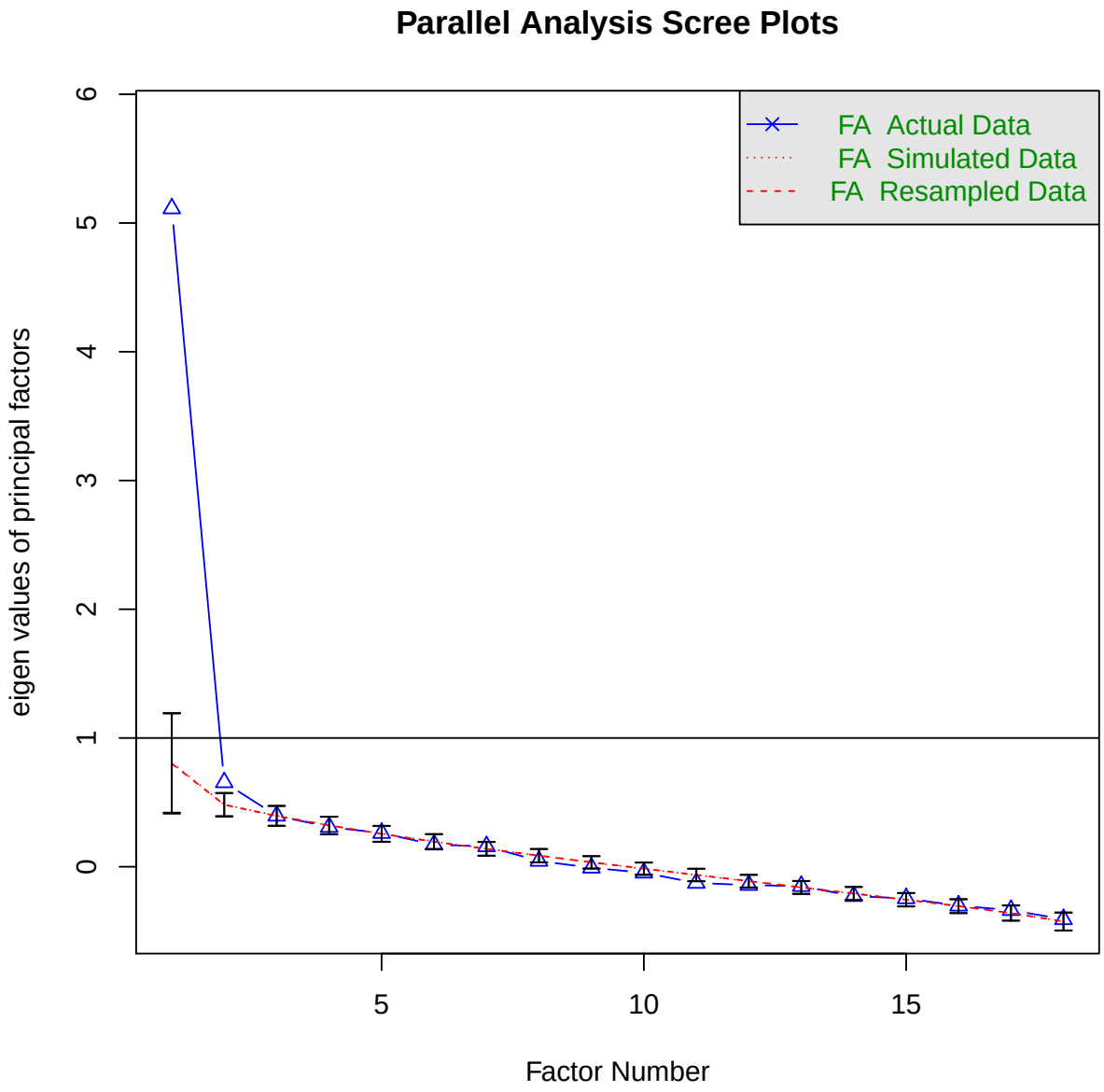
```
## nc15 -0.23386 -0.23814  0.24512  0.2470 -0.2294  0.36130  0.2689
## nc16  0.20349  0.30825 -0.15575 -0.2716  0.2579 -0.06891 -0.1736
## nc17  0.49403  0.52612 -0.24190 -0.3871  0.3316 -0.13682 -0.1019
## nc18 -0.03685 -0.01581 -0.02376  0.1231 -0.1404  0.02897  0.2427
##          nc15     nc16    nc17     nc18
## nc1   0.36693 -0.41618 -0.3785  0.17306
## nc2   0.36344 -0.22696 -0.4828  0.23495
## nc3  -0.27994  0.20222  0.3381 -0.18306
## nc4  -0.19389  0.43537  0.5233 -0.12041
## nc5  -0.16836  0.31776  0.3855 -0.11943
## nc6   0.21697 -0.21324 -0.2966  0.24547
## nc7  -0.09437  0.32593  0.3776 -0.23804
## nc8  -0.23386  0.20349  0.4940 -0.03685
## nc9  -0.23814  0.30825  0.5261 -0.01581
## nc10  0.24512 -0.15575 -0.2419 -0.02376
## nc11  0.24703 -0.27161 -0.3871  0.12305
## nc12 -0.22942  0.25789  0.3316 -0.14044
## nc13  0.36130 -0.06891 -0.1368  0.02897
## nc14  0.26891 -0.17359 -0.1019  0.24269
## nc15  1.14766 -0.08549 -0.2017  0.12720
## nc16 -0.08549  1.41412  0.4410 -0.28200
## nc17 -0.20174  0.44100  1.2740 -0.12765
## nc18  0.12720 -0.28200 -0.1277  1.45874

cov(scale(NC))

##          nc1     nc2     nc3      nc4      nc5     nc6      nc7
## nc1   1.0000  0.4554 -0.2374 -0.27155 -0.30314  0.2442 -0.28711
## nc2   0.4554  1.0000 -0.4647 -0.35317 -0.49596  0.2551 -0.34272
## nc3  -0.2374 -0.4647  1.0000  0.37315  0.44680 -0.1897  0.32077
## nc4  -0.2715 -0.3532  0.3732  1.00000  0.56931 -0.1668  0.34967
## nc5  -0.3031 -0.4960  0.4468  0.56931  1.00000 -0.2241  0.38633
## nc6   0.2442  0.2551 -0.1897 -0.16684 -0.22413  1.0000 -0.24521
## nc7  -0.2871 -0.3427  0.3208  0.34967  0.38633 -0.2452  1.00000
## nc8  -0.2374 -0.3568  0.2241  0.30554  0.30908 -0.1518  0.33729
## nc9  -0.3210 -0.3203  0.3852  0.41022  0.31555 -0.1661  0.21868
## nc10  0.2868  0.3820 -0.4046 -0.34493 -0.35488  0.2844 -0.26560
## nc11  0.3603  0.5149 -0.3218 -0.40162 -0.38587  0.2920 -0.26209
## nc12 -0.3627 -0.4280  0.2764  0.38068  0.51625 -0.2518  0.34822
## nc13  0.3931  0.3895 -0.2407 -0.29196 -0.31609  0.2089 -0.18151
## nc14  0.3298  0.3894 -0.2695 -0.25564 -0.25380  0.2785 -0.26126
## nc15  0.2765  0.3312 -0.2461 -0.15791 -0.15054  0.1686 -0.06686
## nc16 -0.2825 -0.1863  0.1602  0.31941  0.25597 -0.1493  0.20804
## nc17 -0.2707 -0.4176  0.2821  0.40446  0.32716 -0.2188  0.25394
## nc18  0.1157  0.1899 -0.1427 -0.08698 -0.09472  0.1692 -0.14960
##          nc8      nc9     nc10    nc11    nc12     nc13     nc14
## nc1  -0.2374 -0.32100  0.28683  0.3603 -0.3627  0.39312  0.32985
## nc2  -0.3568 -0.32029  0.38198  0.5149 -0.4280  0.38946  0.38935
## nc3   0.2241  0.38524 -0.40465 -0.3218  0.2764 -0.24071 -0.26949
## nc4   0.3055  0.41022 -0.34493 -0.4016  0.3807 -0.29196 -0.25564
## nc5   0.3091  0.31555 -0.35488 -0.3859  0.5163 -0.31609 -0.25380
## nc6  -0.1518 -0.16608  0.28443  0.2920 -0.2518  0.20892  0.27848
## nc7   0.3373  0.21868 -0.26560 -0.2621  0.3482 -0.18151 -0.26126
## nc8   1.0000  0.30876 -0.29383 -0.1397  0.1034 -0.23630 -0.31214
```

```
## nc9   0.3088  1.00000 -0.37382 -0.3331  0.2491 -0.24468 -0.14865
## nc10 -0.2938 -0.37382  1.00000  0.5181 -0.2783  0.31641  0.35130
## nc11 -0.1397 -0.33314  0.51813  1.0000 -0.3946  0.41264  0.31315
## nc12  0.1034  0.24911 -0.27834 -0.3946  1.0000 -0.33947 -0.36420
## nc13 -0.2363 -0.24468  0.31641  0.4126 -0.3395  1.00000  0.45107
## nc14 -0.3121 -0.14865  0.35130  0.3132 -0.3642  0.45107  1.00000
## nc15 -0.1796 -0.17582  0.24134  0.2395 -0.2019  0.29364  0.22920
## nc16  0.1408  0.20501 -0.13815 -0.2373  0.2045 -0.05046 -0.13329
## nc17  0.3601  0.36866 -0.22605 -0.3563  0.2770 -0.10554 -0.08246
## nc18 -0.0251 -0.01035 -0.02075  0.1058 -0.1096  0.02089  0.18348
##          nc15     nc16     nc17     nc18
## nc1   0.27651 -0.28253 -0.27069  0.11568
## nc2   0.33123 -0.18634 -0.41760  0.18992
## nc3  -0.24610  0.16016  0.28211 -0.14274
## nc4  -0.15791  0.31941  0.40446 -0.08698
## nc5  -0.15054  0.25597  0.32716 -0.09472
## nc6   0.16861 -0.14929 -0.21878  0.16920
## nc7  -0.06686  0.20804  0.25394 -0.14960
## nc8  -0.17958  0.14077  0.36005 -0.02510
## nc9  -0.17582  0.20501  0.36866 -0.01035
## nc10  0.24134 -0.13815 -0.22605 -0.02075
## nc11  0.23952 -0.23725 -0.35628  0.10583
## nc12 -0.20194  0.20449  0.27700 -0.10965
## nc13  0.29364 -0.05046 -0.10554  0.02089
## nc14  0.22920 -0.13329 -0.08246  0.18348
## nc15  1.00000 -0.06710 -0.16685  0.09831
## nc16 -0.06710  1.00000  0.32856 -0.19634
## nc17 -0.16685  0.32856  1.00000 -0.09364
## nc18  0.09831 -0.19634 -0.09364  1.00000

(as.matrix(diag(cov(NC))))

##        [,1]
## nc1  1.5344
## nc2  1.0491
## nc3  1.1274
## nc4  1.3138
## nc5  1.0898
## nc6  1.4429
## nc7  1.7357
## nc8  1.4778
## nc9  1.5986
## nc10 0.8989
## nc11 0.9268
## nc12 1.1247
## nc13 1.3192
## nc14 1.1994
## nc15 1.1477
## nc16 1.4141
## nc17 1.2740
## nc18 1.4587
```

5 Model Modification

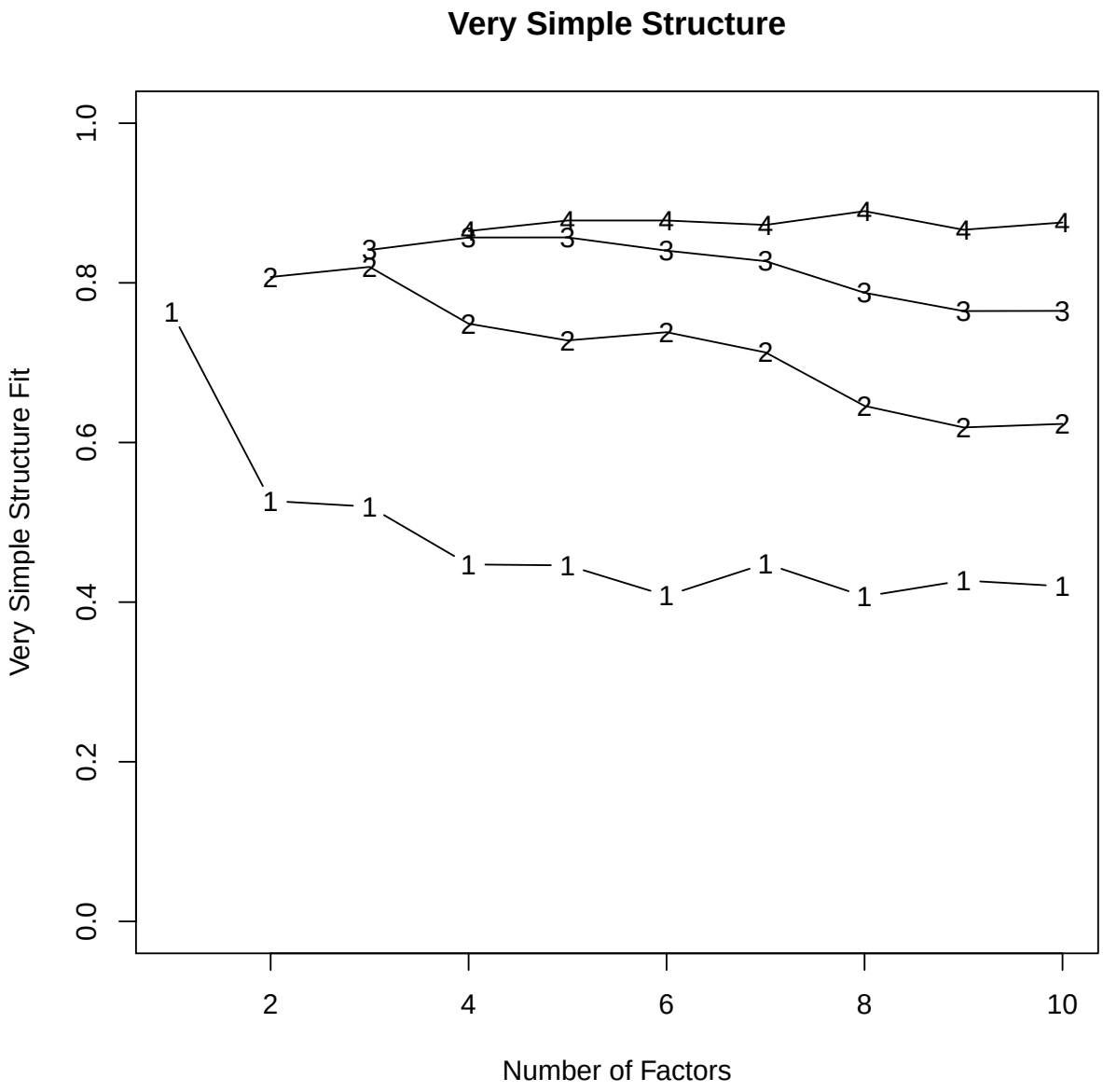
The Need for Cognition Scale is supposedly a measure of a single underlying latent variable. Is that true? We'll first see if one factor seems sensible using methods from exploratory factor analysis. Then we'll apply CFA and explore model modification.

```
fa.parallel(NC, fa = "fa", fm = "ml", error.bars = TRUE, n.iter = 1000)
```



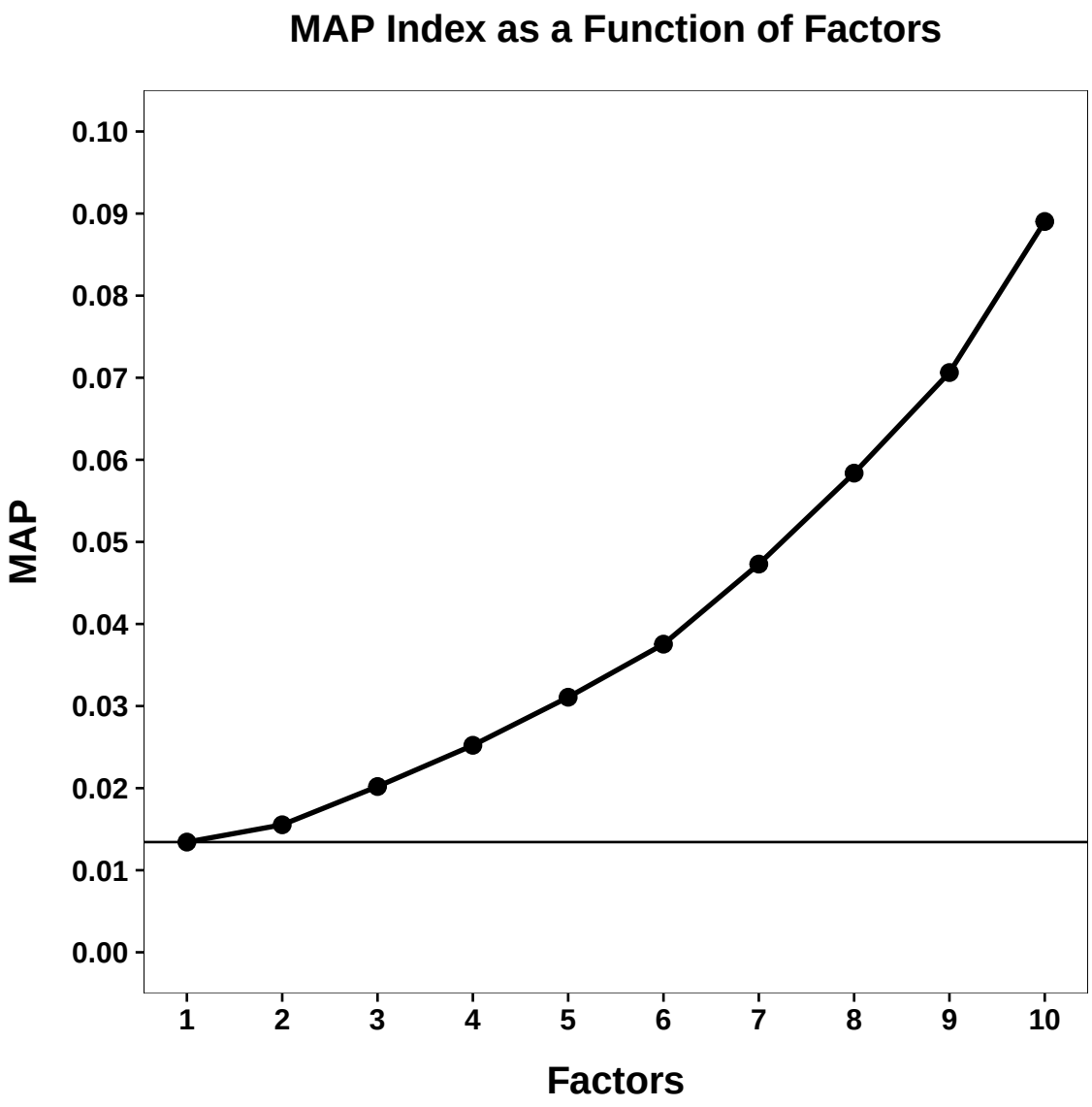

```
## Parallel analysis suggests that the number of factors =  2  and the number of components =  NA
```

```
M <- vss(NC, n = 10, fm = "pc")
```




```
plot_data <- M$map
plot_data <- as.data.frame(plot_data)
names(plot_data) <- c("map")
plot_data$component <- seq(1, 10, 1)

ggplot(plot_data, aes(x = component, y = map)) + geom_line(size = 1) +
    geom_point(size = 3) + coord_cartesian(xlim = c(1, 10), ylim = c(0,
    0.1)) + scale_x_continuous(breaks = c(seq(1, 10, 1))) + scale_y_continuous(breaks = seq(0,
    0.1, 0.01)) + xlab("Factors") + ylab("MAP") + theme(text = element_text(size = 14,
    family = "sans", color = "black", face = "bold"), axis.text.y = element_text(colour = "black",
    size = 12, face = "bold"), axis.text.x = element_text(colour = "black",
    size = 12, face = "bold", angle = 0), axis.title.x = element_text(margin = margin(15,
    0, 0, 0), size = 16), axis.title.y = element_text(margin = margin(0,
    15, 0, 0), size = 16), axis.line.x = element_blank(), axis.line.y = element_blank(),
```

```
    plot.title = element_text(size = 16, face = "bold", margin = margin(0,
        0, 20, 0), hjust = 0.5), panel.background = element_rect(fill = "white",
        linetype = 1, color = "black"), panel.grid.major = element_blank(),
    panel.grid.minor = element_blank(), plot.background = element_rect(fill = "white"),
    plot.margin = unit(c(1, 1, 1, 1), "cm"), legend.position = "bottom",
    legend.title = element_blank()) + geom_hline(yintercept = min(plot_data$map,
    size = 1, linetype = 2)) + ggtitle("MAP Index as a Function of Factors")
```




```
fit_NC <- fa(NC, nfactors = 1, rotate = "none", fm = "ml", n.obs = 195,
    n.iter = 1000)
fit_NC

## Factor Analysis with confidence intervals using method = fa(r = NC, nfactors = 1, n.obs = 195, n.iter
##     fm = "ml")
```

```
## Factor Analysis using method =  ml
## Call: fa(r = NC, nfactors = 1, n.obs = 195, n.iter = 1000, rotate = "none",
##     fm = "ml")
## Standardized loadings (pattern matrix) based upon correlation matrix
##        ML1    h2   u2 com
## nc1  -0.56 0.313 0.69   1
## nc2  -0.73 0.526 0.47   1
## nc3   0.58 0.333 0.67   1
## nc4   0.63 0.394 0.61   1
## nc5   0.68 0.457 0.54   1
## nc6  -0.39 0.155 0.84   1
## nc7   0.50 0.254 0.75   1
## nc8   0.45 0.201 0.80   1
## nc9   0.52 0.272 0.73   1
## nc10 -0.59 0.343 0.66   1
## nc11 -0.65 0.426 0.57   1
## nc12  0.59 0.353 0.65   1
## nc13 -0.52 0.272 0.73   1
## nc14 -0.51 0.256 0.74   1
## nc15 -0.37 0.134 0.87   1
## nc16  0.35 0.124 0.88   1
## nc17  0.51 0.263 0.74   1
## nc18 -0.18 0.034 0.97   1
##
##                 ML1
## SS loadings    5.11
## Proportion Var 0.28
##
## Mean item complexity =  1
## Test of the hypothesis that 1 factor is sufficient.
##
## The degrees of freedom for the null model are  153  and the objective function was  5.47 with Chi Squ
## The degrees of freedom for the model are 135  and the objective function was  1.38
##
## The root mean square of the residuals (RMSR) is  0.07
## The df corrected root mean square of the residuals is  0.07
##
## The harmonic number of observations is  195 with the empirical chi square  264  with prob <  2.2e-10
## The total number of observations was  195  with Likelihood Chi Square =  257  with prob <  1.3e-09
##
## Tucker Lewis Index of factoring reliability =  0.841
## RMSEA index =  0.071  and the 90 % confidence intervals are  0.055 0.081
## BIC =  -454.9
## Fit based upon off diagonal values = 0.95
## Measures of factor score adequacy
##                                                 ML1
## Correlation of (regression) scores with factors   0.94
## Multiple R square of scores with factors          0.89
## Minimum correlation of possible factor scores     0.78
##
##  Coefficients and bootstrapped confidence intervals
##        low   ML1 upper
## nc1  -1.22 -0.56  0.58
## nc2  -1.58 -0.73  0.72
```

```
## nc3  -0.53  0.58  1.25
## nc4  -0.56  0.63  1.36
## nc5  -0.62  0.68  1.47
## nc6  -0.88 -0.39  0.42
## nc7  -0.47  0.50  1.09
## nc8  -0.43  0.45  0.97
## nc9  -0.48  0.52  1.13
## nc10 -1.28 -0.59  0.58
## nc11 -1.42 -0.65  0.65
## nc12 -0.58  0.59  1.31
## nc13 -1.14 -0.52  0.55
## nc14 -1.12 -0.51  0.55
## nc15 -0.82 -0.37  0.41
## nc16 -0.34  0.35  0.78
## nc17 -0.45  0.51  1.11
## nc18 -0.44 -0.18  0.24
```

```
NC.model.1 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
"
```

```
NC_Fit_1 <- cfa(NC.model.1, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_1, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  29 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     267.315     228.948
##   Degrees of freedom                               135         135
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.168
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.854       0.870
##   Tucker-Lewis Index (TLI)                       0.835       0.853
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
```

```
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4984.773   -4984.773
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         54          54
##   Akaike (AIC)                               10077.545   10077.545
##   Bayesian (BIC)                             10254.287   10254.287
##   Sample-size adjusted Bayesian (BIC)        10083.223   10083.223
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.071       0.060
##   90 Percent Confidence Interval          0.059  0.084       0.047  0.072
##   P-value RMSEA <= 0.05                          0.004       0.095
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.060       0.060
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.691
##     nc2               1.072    0.155    6.922    0.000    0.741
##     nc3              -0.884    0.152   -5.818    0.000   -0.611
##     nc4              -1.038    0.189   -5.496    0.000   -0.717
##     nc5              -1.018    0.184   -5.543    0.000   -0.704
##     nc6               0.682    0.155    4.395    0.000    0.472
##     nc7              -0.957    0.164   -5.826    0.000   -0.662
##     nc8              -0.787    0.143   -5.519    0.000   -0.544
##     nc9              -0.951    0.182   -5.210    0.000   -0.657
##     nc10              0.801    0.134    5.980    0.000    0.554
##     nc11              0.906    0.149    6.095    0.000    0.626
##     nc12             -0.910    0.162   -5.602    0.000   -0.629
##     nc13              0.865    0.122    7.095    0.000    0.598
##     nc14              0.800    0.129    6.218    0.000    0.553
##     nc15              0.567    0.148    3.819    0.000    0.392
##     nc16             -0.604    0.153   -3.956    0.000   -0.418
##     nc17             -0.836    0.170   -4.912    0.000   -0.578
##     nc18              0.319    0.140    2.277    0.023    0.221
##   Std.all
##
##     0.560
##     0.725
```

```
##    -0.577
##    -0.627
##    -0.676
##     0.394
##    -0.504
##    -0.448
##    -0.521
##     0.586
##     0.652
##    -0.594
##     0.522
##     0.506
##     0.367
##    -0.352
##    -0.513
##     0.183
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                               0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.727
##     1.758
##     2.744
##     1.893
##     2.077
##     1.797
##     4.290
##     4.379
##     1.658
##     2.869
##     3.432
##     3.441
```

```
##     1.933
##     1.690
##     2.763
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.049    0.131    7.997    0.000    1.049
##    .nc2               0.495    0.077    6.450    0.000    0.495
##    .nc3               0.748    0.116    6.440    0.000    0.748
##    .nc4               0.793    0.129    6.153    0.000    0.793
##    .nc5               0.588    0.085    6.945    0.000    0.588
##    .nc6               1.213    0.127    9.541    0.000    1.213
##    .nc7               1.289    0.129    9.990    0.000    1.289
##    .nc8               1.175    0.111   10.615    0.000    1.175
##    .nc9               1.159    0.136    8.526    0.000    1.159
##    .nc10              0.588    0.093    6.305    0.000    0.588
##    .nc11              0.530    0.084    6.311    0.000    0.530
##    .nc12              0.724    0.125    5.766    0.000    0.724
##    .nc13              0.955    0.092   10.336    0.000    0.955
##    .nc14              0.887    0.103    8.645    0.000    0.887
##    .nc15              0.988    0.132    7.515    0.000    0.988
##    .nc16              1.232    0.145    8.477    0.000    1.232
##    .nc17              0.934    0.113    8.255    0.000    0.934
##    .nc18              1.402    0.112   12.569    0.000    1.402
##     NC                0.478    0.125    3.823    0.000    1.000
##   Std.all
##     0.687
##     0.474
##     0.667
##     0.606
##     0.543
##     0.845
##     0.746
##     0.799
##     0.728
##     0.657
##     0.574
##     0.647
##     0.728
##     0.744
##     0.866
##     0.876
##     0.737
##     0.966
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.313
##     nc2               0.526
##     nc3               0.333
##     nc4               0.394
##     nc5               0.457
```

```
##     nc6               0.155
##     nc7               0.254
##     nc8               0.201
##     nc9               0.272
##     nc10              0.343
##     nc11              0.426
##     nc12              0.353
##     nc13              0.272
##     nc14              0.256
##     nc15              0.134
##     nc16              0.124
##     nc17              0.263
##     nc18              0.034

MI <- modificationIndices(NC_Fit_1)
subset(MI, mi > 10)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 105  nc4 ~~  nc5 15.72  0.218   0.218    0.319    0.319
## 157  nc8 ~~ nc11 11.43  0.205   0.205    0.260    0.260
## 158  nc8 ~~ nc12 11.24 -0.234  -0.234   -0.254   -0.254
## 174 nc10 ~~ nc11 11.57  0.149   0.149    0.268    0.268
## 195 nc13 ~~ nc14 13.90  0.258   0.258    0.281    0.281
## 198 nc13 ~~ nc17 10.62  0.232   0.232    0.246    0.246
## 202 nc14 ~~ nc17 12.34  0.240   0.240    0.264    0.264
```

We will free a parameter at a time, depending on the highest modification index at each step.

```
NC.model.2 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
"
```

```
NC_Fit_2 <- cfa(NC.model.2, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_2, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  29 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     251.756     216.307
##   Degrees of freedom                               134         134
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.164
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
```

```
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.870       0.886
##   Tucker-Lewis Index (TLI)                       0.852       0.870
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4976.953   -4976.953
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         55          55
##   Akaike (AIC)                               10063.906   10063.906
##   Bayesian (BIC)                             10243.921   10243.921
##   Sample-size adjusted Bayesian (BIC)        10069.689   10069.689
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.067       0.056
##   90 Percent Confidence Interval          0.054  0.080       0.043  0.069
##   P-value RMSEA <= 0.05                          0.015       0.205
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.059       0.059
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.704
##     nc2               1.066    0.152    7.022    0.000    0.750
##     nc3              -0.864    0.149   -5.800    0.000   -0.608
##     nc4              -0.964    0.174   -5.548    0.000   -0.679
##     nc5              -0.958    0.170   -5.632    0.000   -0.674
##     nc6               0.682    0.153    4.463    0.000    0.480
##     nc7              -0.932    0.158   -5.882    0.000   -0.656
##     nc8              -0.771    0.139   -5.532    0.000   -0.543
##     nc9              -0.928    0.174   -5.330    0.000   -0.653
##     nc10              0.793    0.132    6.009    0.000    0.558
```

```
##     nc11               0.898    0.145    6.209    0.000    0.632
##     nc12              -0.886    0.158   -5.602    0.000   -0.623
##     nc13               0.861    0.119    7.223    0.000    0.606
##     nc14               0.801    0.127    6.318    0.000    0.564
##     nc15               0.573    0.146    3.920    0.000    0.403
##     nc16              -0.583    0.148   -3.924    0.000   -0.410
##     nc17              -0.815    0.165   -4.930    0.000   -0.573
##     nc18               0.320    0.138    2.313    0.021    0.225
##   Std.all
##
##     0.570
##     0.735
##    -0.574
##    -0.594
##    -0.648
##     0.401
##    -0.499
##    -0.448
##    -0.518
##     0.590
##     0.658
##    -0.589
##     0.529
##     0.516
##     0.377
##    -0.346
##    -0.509
##     0.187
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.220    0.070    3.134    0.002    0.220
##   Std.all
##
##     0.301
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
```

```
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                               0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.727
##     1.758
##     2.744
##     1.893
##     2.077
##     1.797
##     4.290
##     4.379
##     1.658
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
##     0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1              1.031    0.130    7.932    0.000    1.031
##    .nc2              0.481    0.074    6.480    0.000    0.481
##    .nc3              0.752    0.116    6.460    0.000    0.752
##    .nc4              0.846    0.132    6.412    0.000    0.846
##    .nc5              0.629    0.083    7.576    0.000    0.629
##    .nc6              1.205    0.128    9.427    0.000    1.205
##    .nc7              1.296    0.129   10.044    0.000    1.296
##    .nc8              1.176    0.111   10.625    0.000    1.176
##    .nc9              1.164    0.137    8.520    0.000    1.164
##    .nc10             0.583    0.093    6.251    0.000    0.583
##    .nc11             0.523    0.084    6.246    0.000    0.523
##    .nc12             0.730    0.126    5.791    0.000    0.730
##    .nc13             0.945    0.092   10.267    0.000    0.945
##    .nc14             0.875    0.102    8.547    0.000    0.875
##    .nc15             0.979    0.131    7.474    0.000    0.979
##    .nc16             1.239    0.146    8.511    0.000    1.239
##    .nc17             0.939    0.113    8.334    0.000    0.939
##    .nc18             1.401    0.112   12.559    0.000    1.401
##     NC               0.496    0.124    3.983    0.000    1.000
##   Std.all
##     0.675
##     0.460
##     0.670
##     0.648
##     0.580
##     0.839
##     0.751
```

```
##     0.800
##     0.732
##     0.651
##     0.567
##     0.653
##     0.720
##     0.733
##     0.858
##     0.880
##     0.741
##     0.965
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.325
##     nc2               0.540
##     nc3               0.330
##     nc4               0.352
##     nc5               0.420
##     nc6               0.161
##     nc7               0.249
##     nc8               0.200
##     nc9               0.268
##     nc10              0.349
##     nc11              0.433
##     nc12              0.347
##     nc13              0.280
##     nc14              0.267
##     nc15              0.142
##     nc16              0.120
##     nc17              0.259
##     nc18              0.035

MI <- modificationIndices(NC_Fit_2)
subset(MI, mi > 10)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 125  nc5 ~~ nc12 10.44  0.162   0.162    0.238    0.238
## 157  nc8 ~~ nc11 12.03  0.210   0.210    0.268    0.268
## 158  nc8 ~~ nc12 10.79 -0.231  -0.231   -0.249   -0.249
## 174 nc10 ~~ nc11 10.91  0.145   0.145    0.263    0.263
## 195 nc13 ~~ nc14 13.07  0.249   0.249    0.274    0.274
## 198 nc13 ~~ nc17 10.95  0.236   0.236    0.250    0.250
## 202 nc14 ~~ nc17 13.00  0.247   0.247    0.272    0.272

anova(NC_Fit_1, NC_Fit_2)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_2 134 10064 10244   252
## NC_Fit_1 135 10078 10254   267       15.6       1    0.00008
```

```
NC.model.3 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
"
```

```
NC_Fit_3 <- cfa(NC.model.3, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_3, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  30 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     238.584     204.920
##   Degrees of freedom                               133         133
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.164
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.884       0.900
##   Tucker-Lewis Index (TLI)                       0.866       0.885
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4970.333   -4970.333
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         56          56
##   Akaike (AIC)                               10052.666   10052.666
##   Bayesian (BIC)                             10235.954   10235.954
##   Sample-size adjusted Bayesian (BIC)        10058.554   10058.554
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.064       0.053
##   90 Percent Confidence Interval          0.051  0.077       0.039  0.066
##   P-value RMSEA <= 0.05                          0.042       0.352
```

```
##
##   Robust RMSEA                                                 NA
##   90 Percent Confidence Interval                               NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                          0.057       0.057
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.699
##     nc2               1.073    0.156    6.895    0.000    0.751
##     nc3              -0.875    0.150   -5.825    0.000   -0.612
##     nc4              -0.978    0.177   -5.511    0.000   -0.684
##     nc5              -0.972    0.174   -5.576    0.000   -0.680
##     nc6               0.683    0.154    4.429    0.000    0.478
##     nc7              -0.944    0.160   -5.900    0.000   -0.660
##     nc8              -0.775    0.141   -5.488    0.000   -0.542
##     nc9              -0.944    0.178   -5.290    0.000   -0.660
##     nc10              0.797    0.134    5.957    0.000    0.557
##     nc11              0.903    0.149    6.072    0.000    0.631
##     nc12             -0.889    0.161   -5.528    0.000   -0.621
##     nc13              0.832    0.118    7.050    0.000    0.582
##     nc14              0.772    0.126    6.108    0.000    0.540
##     nc15              0.570    0.148    3.841    0.000    0.399
##     nc16             -0.596    0.149   -4.001    0.000   -0.417
##     nc17             -0.838    0.168   -4.985    0.000   -0.586
##     nc18              0.321    0.140    2.301    0.021    0.225
##   Std.all
##
##     0.566
##     0.735
##    -0.578
##    -0.598
##    -0.653
##     0.399
##    -0.502
##    -0.447
##    -0.524
##     0.589
##     0.657
##    -0.588
##     0.508
##     0.494
##     0.373
##    -0.352
##    -0.520
```

```
##     0.187
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.213    0.070    3.030    0.002    0.213
##  .nc13 ~~
##    .nc14              0.250    0.085    2.952    0.003    0.250
##   Std.all
##
##     0.295
##
##     0.267
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                             0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.727
##     1.758
##     2.744
##     1.893
##     2.077
##     1.797
##     4.290
##     4.379
##     1.658
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
```

```
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.038    0.130    7.964    0.000    1.038
##    .nc2               0.480    0.075    6.396    0.000    0.480
##    .nc3               0.747    0.116    6.456    0.000    0.747
##    .nc4               0.839    0.131    6.385    0.000    0.839
##    .nc5               0.622    0.082    7.552    0.000    0.622
##    .nc6               1.207    0.127    9.476    0.000    1.207
##    .nc7               1.291    0.128   10.051    0.000    1.291
##    .nc8               1.177    0.112   10.522    0.000    1.177
##    .nc9               1.154    0.136    8.459    0.000    1.154
##    .nc10              0.584    0.094    6.244    0.000    0.584
##    .nc11              0.524    0.084    6.252    0.000    0.524
##    .nc12              0.733    0.127    5.790    0.000    0.733
##    .nc13              0.974    0.092   10.613    0.000    0.974
##    .nc14              0.902    0.104    8.696    0.000    0.902
##    .nc15              0.983    0.131    7.497    0.000    0.983
##    .nc16              1.233    0.145    8.474    0.000    1.233
##    .nc17              0.924    0.112    8.241    0.000    0.924
##    .nc18              1.401    0.112   12.558    0.000    1.401
##     NC                0.489    0.125    3.919    0.000    1.000
##   Std.all
##     0.680
##     0.460
##     0.666
##     0.642
##     0.574
##     0.841
##     0.748
##     0.800
##     0.726
##     0.653
##     0.568
##     0.655
##     0.742
##     0.756
##     0.861
##     0.876
##     0.729
##     0.965
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.320
##     nc2               0.540
##     nc3               0.334
##     nc4               0.358
##     nc5               0.426
##     nc6               0.159
##     nc7               0.252
##     nc8               0.200
```

```
##     nc9               0.274
##     nc10              0.347
##     nc11              0.432
##     nc12              0.345
##     nc13              0.258
##     nc14              0.244
##     nc15              0.139
##     nc16              0.124
##     nc17              0.271
##     nc18              0.035

MI <- modificationIndices(NC_Fit_3)
subset(MI, mi > 10)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 126  nc5 ~~ nc12 10.37  0.161   0.161    0.239    0.239
## 158  nc8 ~~ nc11 11.89  0.210   0.210    0.267    0.267
## 159  nc8 ~~ nc12 10.58 -0.229  -0.229   -0.247   -0.247
## 175 nc10 ~~ nc11 11.14  0.147   0.147    0.266    0.266

anova(NC_Fit_2, NC_Fit_3)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_3 133 10053 10236   239
## NC_Fit_2 134 10064 10244   252       13.2       1    0.00028
```

```
NC.model.4 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
nc8 ~~ nc11
"
```

```
NC_Fit_4 <- cfa(NC.model.4, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_4, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  31 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     225.806     193.454
##   Degrees of freedom                               132         132
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.167
##     for the Yuan-Bentler correction (Mplus variant)
##
```

```
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.897       0.915
##   Tucker-Lewis Index (TLI)                       0.880       0.901
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4963.911   -4963.911
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         57          57
##   Akaike (AIC)                               10041.823   10041.823
##   Bayesian (BIC)                             10228.384   10228.384
##   Sample-size adjusted Bayesian (BIC)        10047.816   10047.816
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.061       0.049
##   90 Percent Confidence Interval          0.047  0.074       0.034  0.062
##   P-value RMSEA <= 0.05                          0.099       0.535
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.056       0.056
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.694
##     nc2               1.081    0.156    6.918    0.000    0.750
##     nc3              -0.870    0.151   -5.773    0.000   -0.603
##     nc4              -0.984    0.178   -5.524    0.000   -0.683
##     nc5              -0.969    0.173   -5.599    0.000   -0.672
##     nc6               0.686    0.155    4.424    0.000    0.476
##     nc7              -0.950    0.161   -5.892    0.000   -0.659
##     nc8              -0.836    0.148   -5.661    0.000   -0.580
```

```
##     nc9               -0.955    0.179   -5.340    0.000   -0.663
##     nc10               0.813    0.136    5.960    0.000    0.564
##     nc11               0.933    0.153    6.096    0.000    0.647
##     nc12              -0.875    0.160   -5.475    0.000   -0.607
##     nc13               0.842    0.119    7.053    0.000    0.584
##     nc14               0.781    0.127    6.152    0.000    0.542
##     nc15               0.573    0.150    3.817    0.000    0.398
##     nc16              -0.597    0.151   -3.957    0.000   -0.414
##     nc17              -0.854    0.170   -5.012    0.000   -0.592
##     nc18               0.314    0.141    2.230    0.026    0.218
##   Std.all
##
##     0.561
##     0.734
##    -0.570
##    -0.597
##    -0.646
##     0.397
##    -0.502
##    -0.478
##    -0.525
##     0.596
##     0.674
##    -0.574
##     0.510
##     0.496
##     0.372
##    -0.349
##    -0.526
##     0.181
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.218    0.070    3.135    0.002    0.218
##  .nc13 ~~
##    .nc14              0.248    0.085    2.917    0.004    0.248
##  .nc8 ~~
##    .nc11              0.213    0.059    3.620    0.000    0.213
##   Std.all
##
##     0.300
##
##     0.265
##
##     0.281
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
```

```
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                      0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.727
##     1.758
##     2.744
##     1.893
##     2.077
##     1.797
##     4.290
##     4.379
##     1.658
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.045    0.130    8.050    0.000    1.045
##    .nc2               0.482    0.074    6.490    0.000    0.482
##    .nc3               0.758    0.117    6.466    0.000    0.758
##    .nc4               0.841    0.129    6.502    0.000    0.841
##    .nc5               0.632    0.083    7.575    0.000    0.632
##    .nc6               1.209    0.127    9.488    0.000    1.209
##    .nc7               1.292    0.127   10.184    0.000    1.292
##    .nc8               1.134    0.110   10.272    0.000    1.134
##    .nc9               1.152    0.135    8.527    0.000    1.152
##    .nc10              0.576    0.091    6.347    0.000    0.576
##    .nc11              0.503    0.080    6.253    0.000    0.503
##    .nc12              0.750    0.128    5.846    0.000    0.750
##    .nc13              0.971    0.092   10.608    0.000    0.971
##    .nc14              0.899    0.103    8.720    0.000    0.899
##    .nc15              0.984    0.130    7.543    0.000    0.984
##    .nc16              1.235    0.146    8.462    0.000    1.235
##    .nc17              0.917    0.111    8.244    0.000    0.917
```

```
##    .nc18              1.404    0.112   12.587    0.000    1.404
##     NC                0.481    0.124    3.895    0.000    1.000
##   Std.all
##     0.685
##     0.461
##     0.676
##     0.643
##     0.583
##     0.842
##     0.748
##     0.771
##     0.724
##     0.644
##     0.546
##     0.671
##     0.740
##     0.754
##     0.862
##     0.878
##     0.723
##     0.967
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.315
##     nc2               0.539
##     nc3               0.324
##     nc4               0.357
##     nc5               0.417
##     nc6               0.158
##     nc7               0.252
##     nc8               0.229
##     nc9               0.276
##     nc10              0.356
##     nc11              0.454
##     nc12              0.329
##     nc13              0.260
##     nc14              0.246
##     nc15              0.138
##     nc16              0.122
##     nc17              0.277
##     nc18              0.033

MI <- modificationIndices(NC_Fit_4)
subset(MI, mi > 10)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 127  nc5 ~~ nc12 11.45  0.171   0.171    0.248    0.248
## 159  nc8 ~~ nc12 14.05 -0.261  -0.261   -0.283   -0.283
## 175 nc10 ~~ nc11 11.31  0.144   0.144    0.268    0.268

anova(NC_Fit_3, NC_Fit_4)

## Chi Square Difference Test
```

```
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_4 132 10042 10228   226
## NC_Fit_3 133 10053 10236   239       12.8       1    0.00035
```

```
NC.model.5 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
nc8 ~~ nc11
nc8 ~~ nc12
"
```

```
NC_Fit_5 <- cfa(NC.model.5, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_5, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  30 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     210.413     179.711
##   Degrees of freedom                               131         131
##   P-value (Chi-square)                           0.000       0.003
##   Scaling correction factor                                  1.171
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.913       0.933
##   Tucker-Lewis Index (TLI)                       0.898       0.921
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4956.175   -4956.175
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         58          58
##   Akaike (AIC)                               10028.350   10028.350
```

```
##   Bayesian (BIC)                              10218.184   10218.184
##   Sample-size adjusted Bayesian (BIC)         10034.448   10034.448
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.056       0.044
##   90 Percent Confidence Interval          0.041  0.070       0.028  0.058
##   P-value RMSEA <= 0.05                          0.236       0.755
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.054       0.054
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.688
##     nc2               1.081    0.154    7.011    0.000    0.744
##     nc3              -0.854    0.148   -5.762    0.000   -0.587
##     nc4              -0.993    0.176   -5.639    0.000   -0.683
##     nc5              -0.984    0.175   -5.628    0.000   -0.677
##     nc6               0.686    0.156    4.395    0.000    0.472
##     nc7              -0.969    0.165   -5.865    0.000   -0.667
##     nc8              -0.895    0.150   -5.955    0.000   -0.616
##     nc9              -0.953    0.177   -5.392    0.000   -0.656
##     nc10              0.812    0.133    6.103    0.000    0.559
##     nc11              0.940    0.152    6.192    0.000    0.647
##     nc12             -0.927    0.166   -5.588    0.000   -0.638
##     nc13              0.851    0.121    7.051    0.000    0.586
##     nc14              0.798    0.128    6.229    0.000    0.549
##     nc15              0.570    0.148    3.860    0.000    0.392
##     nc16             -0.595    0.150   -3.965    0.000   -0.410
##     nc17             -0.863    0.167   -5.173    0.000   -0.594
##     nc18              0.306    0.142    2.151    0.032    0.211
##   Std.all
##
##     0.557
##     0.728
##    -0.555
##    -0.598
##    -0.650
##     0.394
##    -0.507
##    -0.507
```

```
##    -0.520
##     0.591
##     0.674
##    -0.603
##     0.511
##     0.503
##     0.367
##    -0.345
##    -0.528
##     0.175
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.215    0.069    3.135    0.002    0.215
##  .nc13 ~~
##    .nc14              0.243    0.083    2.906    0.004    0.243
##  .nc8 ~~
##    .nc11              0.240    0.058    4.126    0.000    0.240
##    .nc12             -0.266    0.064   -4.165    0.000   -0.266
##   Std.all
##
##     0.297
##
##     0.261
##
##     0.324
##    -0.301
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                      0.000
##   Std.all
##     2.623
##     3.830
##     1.719
```

```
##     1.727
##     1.758
##     2.744
##     1.893
##     2.074
##     1.797
##     4.290
##     4.379
##     1.658
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.053    0.129    8.139    0.000    1.053
##    .nc2               0.490    0.074    6.636    0.000    0.490
##    .nc3               0.777    0.120    6.494    0.000    0.777
##    .nc4               0.840    0.128    6.590    0.000    0.840
##    .nc5               0.626    0.083    7.580    0.000    0.626
##    .nc6               1.212    0.128    9.503    0.000    1.212
##    .nc7               1.282    0.126   10.176    0.000    1.282
##    .nc8               1.095    0.108   10.175    0.000    1.095
##    .nc9               1.160    0.135    8.583    0.000    1.160
##    .nc10              0.582    0.089    6.504    0.000    0.582
##    .nc11              0.504    0.080    6.265    0.000    0.504
##    .nc12              0.712    0.126    5.642    0.000    0.712
##    .nc13              0.969    0.090   10.755    0.000    0.969
##    .nc14              0.891    0.101    8.787    0.000    0.891
##    .nc15              0.988    0.129    7.666    0.000    0.988
##    .nc16              1.239    0.147    8.451    0.000    1.239
##    .nc17              0.914    0.110    8.325    0.000    0.914
##    .nc18              1.407    0.112   12.608    0.000    1.407
##     NC                0.474    0.122    3.889    0.000    1.000
##   Std.all
##     0.690
##     0.469
##     0.692
##     0.643
##     0.577
##     0.845
##     0.743
##     0.742
##     0.729
##     0.651
##     0.546
##     0.636
##     0.739
##     0.747
##     0.865
```

```
##     0.881
##     0.721
##     0.969
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.310
##     nc2               0.531
##     nc3               0.308
##     nc4               0.357
##     nc5               0.423
##     nc6               0.155
##     nc7               0.257
##     nc8               0.258
##     nc9               0.271
##     nc10              0.349
##     nc11              0.454
##     nc12              0.364
##     nc13              0.261
##     nc14              0.253
##     nc15              0.135
##     nc16              0.119
##     nc17              0.279
##     nc18              0.031

MI <- modificationIndices(NC_Fit_5)
subset(MI, mi > 10)

##      lhs op  rhs    mi  epc sepc.lv sepc.all sepc.nox
## 175 nc10 ~~ nc11 10.54 0.14    0.14    0.258    0.258

anova(NC_Fit_4, NC_Fit_5)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_5 131 10028 10218   210
## NC_Fit_4 132 10042 10228   226       15.4       1   0.000087
```

```
NC.model.6 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
nc8 ~~ nc11
nc8 ~~ nc12
nc10 ~~ nc11
"
```

```
NC_Fit_6 <- cfa(NC.model.6, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_6, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  30 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     200.120     171.533
##   Degrees of freedom                               130         130
##   P-value (Chi-square)                           0.000       0.009
##   Scaling correction factor                                  1.167
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.923       0.942
##   Tucker-Lewis Index (TLI)                       0.909       0.932
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4951.002   -4951.002
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         59          59
##   Akaike (AIC)                                10020.004   10020.004
##   Bayesian (BIC)                              10213.111   10213.111
##   Sample-size adjusted Bayesian (BIC)         10026.208   10026.208
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.053       0.041
##   90 Percent Confidence Interval          0.038  0.067       0.023  0.055
##   P-value RMSEA <= 0.05                          0.365       0.849
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.053       0.053
##
## Parameter Estimates:
```

```
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.695
##     nc2               1.078    0.153    7.031    0.000    0.749
##     nc3              -0.848    0.147   -5.777    0.000   -0.589
##     nc4              -0.987    0.177   -5.584    0.000   -0.686
##     nc5              -0.985    0.177   -5.574    0.000   -0.684
##     nc6               0.676    0.157    4.317    0.000    0.470
##     nc7              -0.970    0.165   -5.881    0.000   -0.674
##     nc8              -0.882    0.148   -5.943    0.000   -0.613
##     nc9              -0.940    0.178   -5.278    0.000   -0.653
##     nc10              0.756    0.128    5.922    0.000    0.526
##     nc11              0.899    0.146    6.143    0.000    0.625
##     nc12             -0.928    0.166   -5.595    0.000   -0.645
##     nc13              0.838    0.121    6.944    0.000    0.582
##     nc14              0.787    0.127    6.182    0.000    0.546
##     nc15              0.564    0.146    3.876    0.000    0.392
##     nc16             -0.599    0.148   -4.057    0.000   -0.416
##     nc17             -0.863    0.166   -5.199    0.000   -0.600
##     nc18              0.320    0.142    2.247    0.025    0.222
##   Std.all
##
##     0.562
##     0.733
##    -0.556
##    -0.600
##    -0.657
##     0.392
##    -0.513
##    -0.506
##    -0.518
##     0.556
##     0.651
##    -0.610
##     0.508
##     0.500
##     0.367
##    -0.351
##    -0.533
##     0.184
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.208    0.069    3.002    0.003    0.208
##  .nc13 ~~
##    .nc14              0.246    0.084    2.924    0.003    0.246
##  .nc8 ~~
```

```
##    .nc11              0.219    0.061    3.570    0.000    0.219
##    .nc12             -0.257    0.063   -4.076    0.000   -0.257
##  .nc10 ~~
##    .nc11              0.140    0.074    1.896    0.058    0.140
##   Std.all
##
##     0.290
##
##     0.264
##
##     0.288
##    -0.294
##
##     0.245
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                             0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.727
##     1.758
##     2.744
##     1.893
##     2.079
##     1.797
##     4.290
##     4.381
##     1.658
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
```

```
##     2.763
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.044    0.130    8.012    0.000    1.044
##    .nc2               0.482    0.073    6.646    0.000    0.482
##    .nc3               0.775    0.119    6.493    0.000    0.775
##    .nc4               0.837    0.128    6.511    0.000    0.837
##    .nc5               0.616    0.083    7.444    0.000    0.616
##    .nc6               1.215    0.128    9.503    0.000    1.215
##    .nc7               1.272    0.126   10.118    0.000    1.272
##    .nc8               1.091    0.107   10.166    0.000    1.091
##    .nc9               1.164    0.136    8.538    0.000    1.164
##    .nc10              0.618    0.100    6.203    0.000    0.618
##    .nc11              0.531    0.087    6.088    0.000    0.531
##    .nc12              0.703    0.125    5.631    0.000    0.703
##    .nc13              0.974    0.090   10.788    0.000    0.974
##    .nc14              0.895    0.102    8.770    0.000    0.895
##    .nc15              0.988    0.129    7.637    0.000    0.988
##    .nc16              1.234    0.146    8.437    0.000    1.234
##    .nc17              0.908    0.110    8.269    0.000    0.908
##    .nc18              1.402    0.112   12.519    0.000    1.402
##     NC                0.483    0.124    3.878    0.000    1.000
##   Std.all
##     0.684
##     0.462
##     0.691
##     0.640
##     0.568
##     0.846
##     0.737
##     0.744
##     0.732
##     0.691
##     0.576
##     0.628
##     0.742
##     0.750
##     0.865
##     0.877
##     0.716
##     0.966
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.316
##     nc2               0.538
##     nc3               0.309
##     nc4               0.360
##     nc5               0.432
##     nc6               0.154
##     nc7               0.263
```

```
##     nc8               0.256
##     nc9               0.268
##     nc10              0.309
##     nc11              0.424
##     nc12              0.372
##     nc13              0.258
##     nc14              0.250
##     nc15              0.135
##     nc16              0.123
##     nc17              0.284
##     nc18              0.034

MI <- modificationIndices(NC_Fit_6)
subset(MI, mi > 5)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 80   nc2 ~~  nc4 7.524  0.134   0.134    0.211    0.211
## 99   nc3 ~~  nc8 5.883 -0.167  -0.167   -0.181   -0.181
## 101  nc3 ~~ nc10 6.198 -0.125  -0.125   -0.180   -0.180
## 102  nc3 ~~ nc11 5.041  0.107   0.107    0.167    0.167
## 113  nc4 ~~  nc9 5.728  0.169   0.169    0.172    0.172
## 129  nc5 ~~ nc12 8.664  0.145   0.145    0.220    0.220
## 198 nc13 ~~ nc17 5.849  0.164   0.164    0.174    0.174
## 202 nc14 ~~ nc17 8.331  0.187   0.187    0.208    0.208
## 207 nc16 ~~ nc17 6.678  0.203   0.203    0.192    0.192

anova(NC_Fit_5, NC_Fit_6)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_6 130 10020 10213   200
## NC_Fit_5 131 10028 10218   210       10.3       1     0.0013
```

```
NC.model.7 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
nc8 ~~ nc11
nc8 ~~ nc12
nc10 ~~ nc11
nc5 ~~ nc12
"
```

```
NC_Fit_7 <- cfa(NC.model.7, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_7, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  31 iterations
##
##   Number of observations                           195
```

```
##   Number of missing patterns                          1
##
##   Estimator                                          ML      Robust
##   Model Fit Test Statistic                      191.642     164.452
##   Degrees of freedom                                129         129
##   P-value (Chi-square)                            0.000       0.019
##   Scaling correction factor                                   1.165
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              1061.451     874.878
##   Degrees of freedom                                153         153
##   P-value                                         0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                     0.931       0.951
##   Tucker-Lewis Index (TLI)                        0.918       0.942
##
##   Robust Comparative Fit Index (CFI)                             NA
##   Robust Tucker-Lewis Index (TLI)                                NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)               -4946.741   -4946.741
##   Loglikelihood unrestricted model (H1)       -4850.426   -4850.426
##
##   Number of free parameters                          60          60
##   Akaike (AIC)                                10013.482   10013.482
##   Bayesian (BIC)                              10209.862   10209.862
##   Sample-size adjusted Bayesian (BIC)         10019.791   10019.791
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                           0.050       0.038
##   90 Percent Confidence Interval          0.034  0.064       0.018  0.053
##   P-value RMSEA <= 0.05                           0.484       0.908
##
##   Robust RMSEA                                                   NA
##   90 Percent Confidence Interval                                 NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                            0.052       0.052
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
```

```
##   NC =~
##     nc1               1.000                               0.700
##     nc2               1.071    0.151    7.090    0.000    0.750
##     nc3              -0.843    0.148   -5.716    0.000   -0.590
##     nc4              -0.980    0.175   -5.586    0.000   -0.686
##     nc5              -0.940    0.165   -5.693    0.000   -0.658
##     nc6               0.674    0.155    4.359    0.000    0.472
##     nc7              -0.956    0.161   -5.953    0.000   -0.669
##     nc8              -0.874    0.147   -5.953    0.000   -0.612
##     nc9              -0.944    0.177   -5.347    0.000   -0.661
##     nc10              0.758    0.129    5.872    0.000    0.531
##     nc11              0.899    0.145    6.181    0.000    0.629
##     nc12             -0.889    0.157   -5.662    0.000   -0.622
##     nc13              0.833    0.119    6.980    0.000    0.583
##     nc14              0.783    0.127    6.166    0.000    0.548
##     nc15              0.569    0.146    3.887    0.000    0.399
##     nc16             -0.598    0.148   -4.041    0.000   -0.418
##     nc17             -0.863    0.166   -5.197    0.000   -0.604
##     nc18              0.321    0.142    2.261    0.024    0.225
##   Std.all
##
##     0.567
##     0.734
##    -0.557
##    -0.600
##    -0.633
##     0.394
##    -0.509
##    -0.506
##    -0.524
##     0.561
##     0.655
##    -0.589
##     0.509
##     0.502
##     0.373
##    -0.353
##    -0.537
##     0.186
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.219    0.065    3.365    0.001    0.219
##  .nc13 ~~
##    .nc14              0.245    0.085    2.867    0.004    0.245
##  .nc8 ~~
##    .nc11              0.216    0.061    3.532    0.000    0.216
##    .nc12             -0.233    0.062   -3.731    0.000   -0.233
##  .nc10 ~~
##    .nc11              0.136    0.074    1.846    0.065    0.136
##  .nc5 ~~
##    .nc12              0.144    0.065    2.233    0.026    0.144
##   Std.all
```

```
##
##     0.298
##
##     0.263
##
##     0.285
##    -0.261
##
##     0.239
##
##     0.210
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                               0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.727
##     1.761
##     2.744
##     1.893
##     2.082
##     1.797
##     4.290
##     4.380
##     1.660
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
##     0.000
##
```

```
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     .nc1              1.037    0.129    8.006    0.000    1.037
##     .nc2              0.481    0.072    6.644    0.000    0.481
##     .nc3              0.773    0.119    6.474    0.000    0.773
##     .nc4              0.837    0.129    6.501    0.000    0.837
##     .nc5              0.647    0.089    7.300    0.000    0.647
##     .nc6              1.213    0.128    9.465    0.000    1.213
##     .nc7              1.279    0.125   10.192    0.000    1.279
##     .nc8              1.088    0.106   10.216    0.000    1.088
##     .nc9              1.154    0.135    8.567    0.000    1.154
##     .nc10             0.613    0.100    6.127    0.000    0.613
##     .nc11             0.526    0.088    6.007    0.000    0.526
##     .nc12             0.728    0.125    5.842    0.000    0.728
##     .nc13             0.972    0.091   10.644    0.000    0.972
##     .nc14             0.893    0.103    8.685    0.000    0.893
##     .nc15             0.983    0.130    7.558    0.000    0.983
##     .nc16             1.232    0.146    8.437    0.000    1.232
##     .nc17             0.903    0.109    8.278    0.000    0.903
##     .nc18             1.401    0.112   12.519    0.000    1.401
##      NC               0.490    0.124    3.940    0.000    1.000
##    Std.all
##      0.679
##      0.461
##      0.689
##      0.640
##      0.599
##      0.845
##      0.741
##      0.744
##      0.726
##      0.685
##      0.570
##      0.653
##      0.741
##      0.748
##      0.861
##      0.876
##      0.712
##      0.965
##      1.000
##
## R-Square:
##                    Estimate
##      nc1              0.321
##      nc2              0.539
##      nc3              0.311
##      nc4              0.360
##      nc5              0.401
##      nc6              0.155
##      nc7              0.259
##      nc8              0.256
##      nc9              0.274
##      nc10             0.315
```

```
##     nc11              0.430
##     nc12              0.347
##     nc13              0.259
##     nc14              0.252
##     nc15              0.139
##     nc16              0.124
##     nc17              0.288
##     nc18              0.035

MI <- modificationIndices(NC_Fit_7)
subset(MI, mi > 5)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 81   nc2 ~~  nc4 9.007  0.148   0.148    0.233    0.233
## 97   nc3 ~~  nc5 6.003  0.122   0.122    0.172    0.172
## 100  nc3 ~~  nc8 5.889 -0.167  -0.167   -0.182   -0.182
## 102  nc3 ~~ nc10 5.941 -0.122  -0.122   -0.178   -0.178
## 103  nc3 ~~ nc11 5.313  0.110   0.110    0.173    0.173
## 104  nc3 ~~ nc12 5.233 -0.127  -0.127   -0.170   -0.170
## 198 nc13 ~~ nc17 6.127  0.168   0.168    0.179    0.179
## 202 nc14 ~~ nc17 8.709  0.191   0.191    0.213    0.213
## 207 nc16 ~~ nc17 6.552  0.201   0.201    0.190    0.190

anova(NC_Fit_6, NC_Fit_7)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_7 129 10013 10210   192
## NC_Fit_6 130 10020 10213   200       8.48       1     0.0036
```

```
NC.model.8 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
nc8 ~~ nc11
nc8 ~~ nc12
nc10 ~~ nc11
nc5 ~~ nc12
nc2 ~~ nc4
"
```

```
NC_Fit_8 <- cfa(NC.model.8, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_8, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  31 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
```

```
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     182.118     156.424
##   Degrees of freedom                               128         128
##   P-value (Chi-square)                           0.001       0.044
##   Scaling correction factor                                  1.164
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.940       0.961
##   Tucker-Lewis Index (TLI)                       0.929       0.953
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4941.954   -4941.954
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         61          61
##   Akaike (AIC)                                10005.909   10005.909
##   Bayesian (BIC)                              10205.562   10205.562
##   Sample-size adjusted Bayesian (BIC)         10012.323   10012.323
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.047       0.034
##   90 Percent Confidence Interval          0.030  0.062       0.010  0.050
##   P-value RMSEA <= 0.05                          0.626       0.954
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.052       0.052
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.694
```

```
##     nc2               1.104    0.158    6.998    0.000    0.766
##     nc3              -0.853    0.149   -5.738    0.000   -0.592
##     nc4              -1.036    0.182   -5.690    0.000   -0.719
##     nc5              -0.945    0.168   -5.634    0.000   -0.656
##     nc6               0.664    0.155    4.271    0.000    0.461
##     nc7              -0.958    0.160   -5.977    0.000   -0.665
##     nc8              -0.873    0.145   -6.034    0.000   -0.606
##     nc9              -0.952    0.178   -5.351    0.000   -0.661
##     nc10              0.756    0.130    5.826    0.000    0.525
##     nc11              0.904    0.147    6.159    0.000    0.627
##     nc12             -0.891    0.155   -5.730    0.000   -0.618
##     nc13              0.834    0.120    6.956    0.000    0.579
##     nc14              0.782    0.128    6.122    0.000    0.543
##     nc15              0.572    0.148    3.867    0.000    0.397
##     nc16             -0.604    0.148   -4.085    0.000   -0.419
##     nc17             -0.879    0.167   -5.256    0.000   -0.610
##     nc18              0.326    0.142    2.295    0.022    0.226
##   Std.all
##
##     0.562
##     0.750
##    -0.559
##    -0.627
##    -0.630
##     0.385
##    -0.506
##    -0.501
##    -0.524
##     0.555
##     0.653
##    -0.585
##     0.505
##     0.497
##     0.371
##    -0.354
##    -0.542
##     0.188
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.212    0.064    3.339    0.001    0.212
##  .nc13 ~~
##    .nc14              0.250    0.086    2.903    0.004    0.250
##  .nc8 ~~
##    .nc11              0.212    0.061    3.497    0.000    0.212
##    .nc12             -0.228    0.062   -3.682    0.000   -0.228
##  .nc10 ~~
##    .nc11              0.142    0.074    1.929    0.054    0.142
##  .nc5 ~~
##    .nc12              0.156    0.064    2.430    0.015    0.156
##  .nc2 ~~
##    .nc4               0.148    0.058    2.534    0.011    0.148
##   Std.all
```

```
##
##     0.294
##
##     0.267
##
##     0.278
##    -0.255
##
##     0.248
##
##     0.226
##
##     0.245
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                               0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.723
##     1.759
##     2.744
##     1.893
##     2.083
##     1.797
##     4.290
##     4.379
##     1.659
##     2.869
##     3.432
##     3.441
##     1.933
##     1.690
##     2.763
```

```
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.045    0.129    8.120    0.000    1.045
##    .nc2               0.457    0.070    6.489    0.000    0.457
##    .nc3               0.771    0.120    6.427    0.000    0.771
##    .nc4               0.796    0.127    6.274    0.000    0.796
##    .nc5               0.653    0.090    7.282    0.000    0.653
##    .nc6               1.223    0.128    9.570    0.000    1.223
##    .nc7               1.285    0.124   10.332    0.000    1.285
##    .nc8               1.095    0.106   10.296    0.000    1.095
##    .nc9               1.153    0.135    8.560    0.000    1.153
##    .nc10              0.619    0.100    6.179    0.000    0.619
##    .nc11              0.529    0.088    6.028    0.000    0.529
##    .nc12              0.735    0.124    5.917    0.000    0.735
##    .nc13              0.978    0.091   10.710    0.000    0.978
##    .nc14              0.899    0.103    8.686    0.000    0.899
##    .nc15              0.984    0.130    7.552    0.000    0.984
##    .nc16              1.231    0.145    8.509    0.000    1.231
##    .nc17              0.895    0.110    8.139    0.000    0.895
##    .nc18              1.400    0.112   12.525    0.000    1.400
##     NC                0.482    0.123    3.914    0.000    1.000
##   Std.all
##     0.684
##     0.438
##     0.688
##     0.606
##     0.603
##     0.852
##     0.744
##     0.749
##     0.725
##     0.692
##     0.573
##     0.658
##     0.745
##     0.753
##     0.862
##     0.875
##     0.706
##     0.965
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.316
##     nc2               0.562
##     nc3               0.312
##     nc4               0.394
##     nc5               0.397
##     nc6               0.148
##     nc7               0.256
##     nc8               0.251
```

```
##     nc9               0.275
##     nc10              0.308
##     nc11              0.427
##     nc12              0.342
##     nc13              0.255
##     nc14              0.247
##     nc15              0.138
##     nc16              0.125
##     nc17              0.294
##     nc18              0.035

MI <- modificationIndices(NC_Fit_8)
subset(MI, mi > 5)

##      lhs op  rhs    mi    epc sepc.lv sepc.all sepc.nox
## 97   nc3 ~~  nc5 6.074  0.121   0.121    0.171    0.171
## 100  nc3 ~~  nc8 5.406 -0.158  -0.158   -0.172   -0.172
## 102  nc3 ~~ nc10 6.193 -0.124  -0.124   -0.180   -0.180
## 103  nc3 ~~ nc11 5.173  0.108   0.108    0.168    0.168
## 104  nc3 ~~ nc12 5.042 -0.124  -0.124   -0.165   -0.165
## 198 nc13 ~~ nc17 6.102  0.166   0.166    0.178    0.178
## 202 nc14 ~~ nc17 8.607  0.189   0.189    0.211    0.211
## 207 nc16 ~~ nc17 6.355  0.196   0.196    0.187    0.187

anova(NC_Fit_7, NC_Fit_8)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_8 128 10006 10206   182
## NC_Fit_7 129 10013 10210   192       9.52       1      0.002
```

```
NC.model.9 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
nc4 ~~ nc5
nc13 ~~ nc14
nc8 ~~ nc11
nc8 ~~ nc12
nc10 ~~ nc11
nc5 ~~ nc12
nc2 ~~ nc4
nc14 ~~ nc17
"
```

```
NC_Fit_9 <- cfa(NC.model.9, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_9, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  33 iterations
##
##   Number of observations                           195
```

```
##   Number of missing patterns                          1
##
##   Estimator                                          ML      Robust
##   Model Fit Test Statistic                      172.279     147.771
##   Degrees of freedom                                127         127
##   P-value (Chi-square)                            0.005       0.100
##   Scaling correction factor                                   1.166
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              1061.451     874.878
##   Degrees of freedom                                153         153
##   P-value                                         0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                     0.950       0.971
##   Tucker-Lewis Index (TLI)                        0.940       0.965
##
##   Robust Comparative Fit Index (CFI)                             NA
##   Robust Tucker-Lewis Index (TLI)                                NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)               -4937.009   -4937.009
##   Loglikelihood unrestricted model (H1)       -4850.426   -4850.426
##
##   Number of free parameters                          62          62
##   Akaike (AIC)                                 9998.019    9998.019
##   Bayesian (BIC)                              10200.945   10200.945
##   Sample-size adjusted Bayesian (BIC)         10004.538   10004.538
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                           0.043       0.029
##   90 Percent Confidence Interval          0.025   0.058       0.000  0.046
##   P-value RMSEA <= 0.05                           0.762       0.982
##
##   Robust RMSEA                                                   NA
##   90 Percent Confidence Interval                              0.000     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                            0.050       0.050
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
```

```
##   NC =~
##     nc1               1.000                               0.694
##     nc2               1.101    0.154    7.131    0.000    0.764
##     nc3              -0.847    0.148   -5.740    0.000   -0.588
##     nc4              -1.030    0.179   -5.762    0.000   -0.715
##     nc5              -0.936    0.165   -5.690    0.000   -0.650
##     nc6               0.670    0.155    4.318    0.000    0.465
##     nc7              -0.956    0.158   -6.036    0.000   -0.664
##     nc8              -0.883    0.144   -6.123    0.000   -0.613
##     nc9              -0.946    0.175   -5.409    0.000   -0.656
##     nc10              0.755    0.129    5.862    0.000    0.524
##     nc11              0.901    0.144    6.267    0.000    0.625
##     nc12             -0.893    0.155   -5.759    0.000   -0.619
##     nc13              0.834    0.120    6.973    0.000    0.578
##     nc14              0.817    0.129    6.334    0.000    0.567
##     nc15              0.572    0.147    3.888    0.000    0.397
##     nc16             -0.607    0.147   -4.142    0.000   -0.421
##     nc17             -0.897    0.164   -5.458    0.000   -0.622
##     nc18              0.332    0.142    2.339    0.019    0.230
##   Std.all
##
##     0.562
##     0.748
##    -0.555
##    -0.623
##    -0.624
##     0.388
##    -0.505
##    -0.507
##    -0.520
##     0.554
##     0.651
##    -0.586
##     0.505
##     0.523
##     0.371
##    -0.355
##    -0.552
##     0.191
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##  .nc4 ~~
##    .nc5               0.221    0.065    3.422    0.001    0.221
##  .nc13 ~~
##    .nc14              0.184    0.084    2.177    0.029    0.184
##  .nc8 ~~
##    .nc11              0.214    0.060    3.556    0.000    0.214
##    .nc12             -0.234    0.061   -3.831    0.000   -0.234
##  .nc10 ~~
##    .nc11              0.143    0.073    1.952    0.051    0.143
##  .nc5 ~~
##    .nc12              0.159    0.065    2.447    0.014    0.159
##  .nc2 ~~
```

```
##    .nc4               0.146    0.058    2.502    0.012    0.146
##  .nc14 ~~
##    .nc17              0.209    0.066    3.173    0.002    0.209
##   Std.all
##
##     0.303
##
##     0.201
##
##     0.282
##    -0.262
##
##     0.249
##
##     0.229
##
##     0.240
##
##     0.241
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              3.328    0.086   38.480    0.000    3.328
##     NC                0.000                              0.000
##   Std.all
##     2.623
##     3.830
##     1.719
##     1.722
##     1.759
##     2.744
##     1.893
##     2.083
##     1.797
##     4.290
##     4.380
##     1.659
```

```
##     2.869
##     3.461
##     3.441
##     1.933
##     1.690
##     2.763
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               1.045    0.128    8.165    0.000    1.045
##    .nc2               0.460    0.071    6.492    0.000    0.460
##    .nc3               0.776    0.121    6.432    0.000    0.776
##    .nc4               0.804    0.127    6.315    0.000    0.804
##    .nc5               0.661    0.091    7.270    0.000    0.661
##    .nc6               1.219    0.128    9.525    0.000    1.219
##    .nc7               1.286    0.124   10.358    0.000    1.286
##    .nc8               1.086    0.105   10.390    0.000    1.086
##    .nc9               1.160    0.135    8.579    0.000    1.160
##    .nc10              0.620    0.100    6.216    0.000    0.620
##    .nc11              0.531    0.088    6.022    0.000    0.531
##    .nc12              0.734    0.125    5.879    0.000    0.734
##    .nc13              0.978    0.091   10.724    0.000    0.978
##    .nc14              0.852    0.095    8.931    0.000    0.852
##    .nc15              0.984    0.130    7.566    0.000    0.984
##    .nc16              1.229    0.144    8.519    0.000    1.229
##    .nc17              0.881    0.108    8.154    0.000    0.881
##    .nc18              1.398    0.112   12.502    0.000    1.398
##     NC                0.481    0.122    3.955    0.000    1.000
##   Std.all
##     0.685
##     0.441
##     0.692
##     0.611
##     0.610
##     0.849
##     0.745
##     0.743
##     0.729
##     0.693
##     0.576
##     0.657
##     0.745
##     0.726
##     0.862
##     0.874
##     0.695
##     0.963
##     1.000
##
## R-Square:
##                    Estimate
##     nc1               0.315
##     nc2               0.559
```

```
##     nc3               0.308
##     nc4               0.389
##     nc5               0.390
##     nc6               0.151
##     nc7               0.255
##     nc8               0.257
##     nc9               0.271
##     nc10              0.307
##     nc11              0.424
##     nc12              0.343
##     nc13              0.255
##     nc14              0.274
##     nc15              0.138
##     nc16              0.126
##     nc17              0.305
##     nc18              0.037

MI <- modificationIndices(NC_Fit_9)
subset(MI, mi > 5)

##      lhs op  rhs     mi    epc sepc.lv sepc.all sepc.nox
## 98   nc3 ~~  nc5  6.463  0.125   0.125    0.175    0.175
## 101  nc3 ~~  nc8  5.543 -0.159  -0.159   -0.174   -0.174
## 103  nc3 ~~ nc10  6.312 -0.126  -0.126   -0.181   -0.181
## 105  nc3 ~~ nc12  5.097 -0.124  -0.124   -0.165   -0.165
## 198 nc13 ~~ nc16  5.082  0.177   0.177    0.161    0.161
## 199 nc13 ~~ nc17 12.611  0.249   0.249    0.268    0.268
## 207 nc16 ~~ nc17  6.012  0.186   0.186    0.179    0.179

anova(NC_Fit_8, NC_Fit_9)

## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_9 127  9998 10201   172
## NC_Fit_8 128 10006 10206   182       9.84       1     0.0017
```

An alternative would be to specify a model alternative that might be conceptually viable.

```
NC.model.10 <- "
# Latent variable definitions.
NC_N =~ NA*nc1 + nc2 + nc6 + nc10 + nc11 + nc13 + nc14 + nc15 + nc18
NC_P =~ NA*nc3 + nc4 + nc5 + nc7 + nc8 + nc9 + nc12 + nc16 + nc17
NC_N ~~ 1*NC_N
NC_P ~~ 1*NC_P

NC_N ~~ NC_P
"
```

```
NC_Fit_10 <- cfa(NC.model.10, data = NC, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_10, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  22 iterations
```

```
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     230.668     198.045
##   Degrees of freedom                               134         134
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.165
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.894       0.911
##   Tucker-Lewis Index (TLI)                       0.879       0.899
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -4966.355   -4966.355
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         55          55
##   Akaike (AIC)                                10042.710   10042.710
##   Bayesian (BIC)                              10222.725   10222.725
##   Sample-size adjusted Bayesian (BIC)         10048.493   10048.493
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.061       0.050
##   90 Percent Confidence Interval          0.047  0.074       0.035  0.063
##   P-value RMSEA <= 0.05                          0.088       0.504
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.055       0.055
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
```

```
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC_N =~
##     nc1               0.722    0.088    8.193    0.000    0.722
##     nc2               0.757    0.076   10.002    0.000    0.757
##     nc6               0.493    0.096    5.148    0.000    0.493
##     nc10              0.570    0.087    6.567    0.000    0.570
##     nc11              0.663    0.087    7.643    0.000    0.663
##     nc13              0.658    0.083    7.882    0.000    0.658
##     nc14              0.599    0.100    6.022    0.000    0.599
##     nc15              0.433    0.089    4.880    0.000    0.433
##     nc18              0.224    0.099    2.258    0.024    0.224
##   NC_P =~
##     nc3               0.617    0.075    8.199    0.000    0.617
##     nc4               0.793    0.089    8.948    0.000    0.793
##     nc5               0.760    0.068   11.201    0.000    0.760
##     nc7               0.694    0.082    8.483    0.000    0.694
##     nc8               0.554    0.091    6.051    0.000    0.554
##     nc9               0.679    0.101    6.748    0.000    0.679
##     nc12              0.626    0.094    6.649    0.000    0.626
##     nc16              0.452    0.103    4.400    0.000    0.452
##     nc17              0.609    0.104    5.845    0.000    0.609
##   Std.all
##
##     0.585
##     0.741
##     0.411
##     0.603
##     0.691
##     0.574
##     0.549
##     0.405
##     0.186
##
##     0.583
##     0.694
##     0.730
##     0.528
##     0.457
##     0.539
##     0.592
##     0.381
##     0.541
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC_N ~~
##     NC_P             -0.820    0.051  -15.934    0.000   -0.820
##   Std.all
##
##    -0.820
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
```

```
##    .nc1               3.241    0.089   36.537    0.000    3.241
##    .nc2               3.913    0.073   53.347    0.000    3.913
##    .nc6               3.287    0.086   38.214    0.000    3.287
##    .nc10              4.056    0.068   59.746    0.000    4.056
##    .nc11              4.205    0.069   60.997    0.000    4.205
##    .nc13              3.287    0.082   39.966    0.000    3.287
##    .nc14              3.749    0.078   47.799    0.000    3.749
##    .nc15              3.677    0.077   47.929    0.000    3.677
##    .nc18              3.328    0.086   38.480    0.000    3.328
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##     NC_N              0.000                             0.000
##     NC_P              0.000                             0.000
##   Std.all
##     2.623
##     3.830
##     2.744
##     4.290
##     4.379
##     2.869
##     3.432
##     3.441
##     2.763
##     1.719
##     1.727
##     1.758
##     1.893
##     2.077
##     1.797
##     1.658
##     1.933
##     1.690
##     0.000
##     0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     NC_N             1.000                             1.000
##     NC_P             1.000                             1.000
##    .nc1              1.005    0.133    7.569    0.000    1.005
##    .nc2              0.471    0.076    6.211    0.000    0.471
##    .nc6              1.193    0.130    9.141    0.000    1.193
##    .nc10             0.569    0.093    6.104    0.000    0.569
##    .nc11             0.482    0.081    5.969    0.000    0.482
##    .nc13             0.879    0.090    9.768    0.000    0.879
##    .nc14             0.834    0.103    8.114    0.000    0.834
##    .nc15             0.954    0.132    7.209    0.000    0.954
```

```
##    .nc18              1.401    0.112   12.546    0.000    1.401
##    .nc3               0.741    0.118    6.256    0.000    0.741
##    .nc4               0.678    0.123    5.506    0.000    0.678
##    .nc5               0.507    0.086    5.873    0.000    0.507
##    .nc7               1.245    0.130    9.566    0.000    1.245
##    .nc8               1.164    0.113   10.296    0.000    1.164
##    .nc9               1.129    0.139    8.150    0.000    1.129
##    .nc12              0.727    0.125    5.839    0.000    0.727
##    .nc16              1.203    0.146    8.233    0.000    1.203
##    .nc17              0.896    0.114    7.843    0.000    0.896
##   Std.all
##     1.000
##     1.000
##     0.658
##     0.451
##     0.831
##     0.637
##     0.523
##     0.670
##     0.699
##     0.836
##     0.965
##     0.661
##     0.518
##     0.467
##     0.721
##     0.791
##     0.710
##     0.650
##     0.855
##     0.707
##
## R-Square:
##                    Estimate
##     nc1               0.342
##     nc2               0.549
##     nc6               0.169
##     nc10              0.363
##     nc11              0.477
##     nc13              0.330
##     nc14              0.301
##     nc15              0.164
##     nc18              0.035
##     nc3               0.339
##     nc4               0.482
##     nc5               0.533
##     nc7               0.279
##     nc8               0.209
##     nc9               0.290
##     nc12              0.350
##     nc16              0.145
##     nc17              0.293

MI <- modificationIndices(NC_Fit_10)
```

```
subset(MI, mi > 5)
```

```
##      lhs op  rhs     mi    epc sepc.lv sepc.all sepc.nox
## 61  NC_N =~  nc4  5.233  0.403   0.403    0.353    0.353
## 66  NC_N =~ nc12  6.590 -0.435  -0.435   -0.411   -0.411
## 126 nc10 ~~ nc11  8.545  0.130   0.130    0.249    0.249
## 130 nc10 ~~ nc18  6.271 -0.169  -0.169   -0.190   -0.190
## 148 nc11 ~~  nc8 10.047  0.187   0.187    0.250    0.250
## 153 nc13 ~~ nc14  9.301  0.206   0.206    0.240    0.240
## 164 nc13 ~~ nc17  7.484  0.187   0.187    0.210    0.210
## 175 nc14 ~~ nc17  8.947  0.198   0.198    0.229    0.229
## 203  nc4 ~~  nc5  5.381  0.127   0.127    0.216    0.216
## 213  nc5 ~~ nc12  6.709  0.136   0.136    0.225    0.225
## 222  nc8 ~~ nc12 12.317 -0.250  -0.250   -0.272   -0.272
## 224  nc8 ~~ nc17  5.053  0.175   0.175    0.172    0.172
## 230 nc16 ~~ nc17  5.384  0.182   0.182    0.175    0.175
```

```
anova(NC_Fit_9, NC_Fit_10)
```

```
## Chi Square Difference Test
##
##            Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_9  127  9998 10201   172
## NC_Fit_10 134 10043 10223   231       58.4       7    3.2e-10
```

```
anova(NC_Fit_1, NC_Fit_9)
```

```
## Chi Square Difference Test
##
##           Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_9 127  9998 10201   172
## NC_Fit_1 135 10078 10254   267         95       8     <2e-16
```

```
anova(NC_Fit_1, NC_Fit_10)
```

```
## Chi Square Difference Test
##
##            Df   AIC   BIC Chisq Chisq diff Df diff Pr(>Chisq)
## NC_Fit_10 134 10043 10223   231
## NC_Fit_1  135 10078 10254   267       36.6       1    1.4e-09
```

6 Reliability

The reliability of a scale can be obtained from CFA output. It is first important to score the items in a consistent direction.

```
NC_rescaled <- as.data.frame(NC)
NC_rescaled$nc1 <- 6 - NC_rescaled$nc1
NC_rescaled$nc2 <- 6 - NC_rescaled$nc2
NC_rescaled$nc6 <- 6 - NC_rescaled$nc6
NC_rescaled$nc10 <- 6 - NC_rescaled$nc10
NC_rescaled$nc11 <- 6 - NC_rescaled$nc11
NC_rescaled$nc13 <- 6 - NC_rescaled$nc13
```

```r
NC_rescaled$nc14 <- 6 - NC_rescaled$nc14
NC_rescaled$nc15 <- 6 - NC_rescaled$nc15
NC_rescaled$nc18 <- 6 - NC_rescaled$nc18
cor(NC_rescaled)
```

```
##         nc1    nc2    nc3     nc4     nc5    nc6     nc7    nc8
## nc1  1.0000 0.4554 0.2374 0.27155 0.30314 0.2442 0.28711 0.2374
## nc2  0.4554 1.0000 0.4647 0.35317 0.49596 0.2551 0.34272 0.3568
## nc3  0.2374 0.4647 1.0000 0.37315 0.44680 0.1897 0.32077 0.2241
## nc4  0.2715 0.3532 0.3732 1.00000 0.56931 0.1668 0.34967 0.3055
## nc5  0.3031 0.4960 0.4468 0.56931 1.00000 0.2241 0.38633 0.3091
## nc6  0.2442 0.2551 0.1897 0.16684 0.22413 1.0000 0.24521 0.1518
## nc7  0.2871 0.3427 0.3208 0.34967 0.38633 0.2452 1.00000 0.3373
## nc8  0.2374 0.3568 0.2241 0.30554 0.30908 0.1518 0.33729 1.0000
## nc9  0.3210 0.3203 0.3852 0.41022 0.31555 0.1661 0.21868 0.3088
## nc10 0.2868 0.3820 0.4046 0.34493 0.35488 0.2844 0.26560 0.2938
## nc11 0.3603 0.5149 0.3218 0.40162 0.38587 0.2920 0.26209 0.1397
## nc12 0.3627 0.4280 0.2764 0.38068 0.51625 0.2518 0.34822 0.1034
## nc13 0.3931 0.3895 0.2407 0.29196 0.31609 0.2089 0.18151 0.2363
## nc14 0.3298 0.3894 0.2695 0.25564 0.25380 0.2785 0.26126 0.3121
## nc15 0.2765 0.3312 0.2461 0.15791 0.15054 0.1686 0.06686 0.1796
## nc16 0.2825 0.1863 0.1602 0.31941 0.25597 0.1493 0.20804 0.1408
## nc17 0.2707 0.4176 0.2821 0.40446 0.32716 0.2188 0.25394 0.3601
## nc18 0.1157 0.1899 0.1427 0.08698 0.09472 0.1692 0.14960 0.0251
##          nc9     nc10   nc11   nc12    nc13    nc14    nc15
## nc1  0.32100  0.28683 0.3603 0.3627 0.39312 0.32985 0.27651
## nc2  0.32029  0.38198 0.5149 0.4280 0.38946 0.38935 0.33123
## nc3  0.38524  0.40465 0.3218 0.2764 0.24071 0.26949 0.24610
## nc4  0.41022  0.34493 0.4016 0.3807 0.29196 0.25564 0.15791
## nc5  0.31555  0.35488 0.3859 0.5163 0.31609 0.25380 0.15054
## nc6  0.16608  0.28443 0.2920 0.2518 0.20892 0.27848 0.16861
## nc7  0.21868  0.26560 0.2621 0.3482 0.18151 0.26126 0.06686
## nc8  0.30876  0.29383 0.1397 0.1034 0.23630 0.31214 0.17958
## nc9  1.00000  0.37382 0.3331 0.2491 0.24468 0.14865 0.17582
## nc10 0.37382  1.00000 0.5181 0.2783 0.31641 0.35130 0.24134
## nc11 0.33314  0.51813 1.0000 0.3946 0.41264 0.31315 0.23952
## nc12 0.24911  0.27834 0.3946 1.0000 0.33947 0.36420 0.20194
## nc13 0.24468  0.31641 0.4126 0.3395 1.00000 0.45107 0.29364
## nc14 0.14865  0.35130 0.3132 0.3642 0.45107 1.00000 0.22920
## nc15 0.17582  0.24134 0.2395 0.2019 0.29364 0.22920 1.00000
## nc16 0.20501  0.13815 0.2373 0.2045 0.05046 0.13329 0.06710
## nc17 0.36866  0.22605 0.3563 0.2770 0.10554 0.08246 0.16685
## nc18 0.01035 -0.02075 0.1058 0.1096 0.02089 0.18348 0.09831
##         nc16    nc17     nc18
## nc1  0.28253 0.27069  0.11568
## nc2  0.18634 0.41760  0.18992
## nc3  0.16016 0.28211  0.14274
## nc4  0.31941 0.40446  0.08698
## nc5  0.25597 0.32716  0.09472
## nc6  0.14929 0.21878  0.16920
## nc7  0.20804 0.25394  0.14960
## nc8  0.14077 0.36005  0.02510
## nc9  0.20501 0.36866  0.01035
## nc10 0.13815 0.22605 -0.02075
```

```
## nc11 0.23725 0.35628  0.10583
## nc12 0.20449 0.27700  0.10965
## nc13 0.05046 0.10554  0.02089
## nc14 0.13329 0.08246  0.18348
## nc15 0.06710 0.16685  0.09831
## nc16 1.00000 0.32856  0.19634
## nc17 0.32856 1.00000  0.09364
## nc18 0.19634 0.09364  1.00000
```

6.1 Single Factor Model

A single factor model is estimated in order to get the raw information for calculating reliability.

```
NC.model.11 <- "
# Latent variable definitions.
NC =~ nc1 + nc2 + nc3 + nc4 + nc5 + nc6 + nc7 + nc8 + nc9 + nc10 +
nc11 + nc12 + nc13 + nc14 + nc15 + nc16 + nc17 + nc18
"
```

```
NC_Fit_11 <- cfa(NC.model.11, data = NC_rescaled, missing = "ML",
    estimator = "MLR", likelihood = "wishart", representation = "LISREL")
summary(NC_Fit_11, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  29 iterations
##
##   Number of observations                           195
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     267.315     228.948
##   Degrees of freedom                               135         135
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.168
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1061.451     874.878
##   Degrees of freedom                               153         153
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.854       0.870
##   Tucker-Lewis Index (TLI)                       0.835       0.853
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
```

```
##   Loglikelihood user model (H0)              -4984.773   -4984.773
##   Loglikelihood unrestricted model (H1)      -4850.426   -4850.426
##
##   Number of free parameters                         54          54
##   Akaike (AIC)                                10077.545   10077.545
##   Bayesian (BIC)                              10254.287   10254.287
##   Sample-size adjusted Bayesian (BIC)         10083.223   10083.223
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.071       0.060
##   90 Percent Confidence Interval          0.059  0.084       0.047  0.072
##   P-value RMSEA <= 0.05                          0.004       0.095
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.060       0.060
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   NC =~
##     nc1               1.000                               0.691
##     nc2               1.072    0.155    6.922    0.000    0.741
##     nc3               0.884    0.152    5.818    0.000    0.611
##     nc4               1.038    0.189    5.496    0.000    0.717
##     nc5               1.018    0.184    5.543    0.000    0.704
##     nc6               0.682    0.155    4.395    0.000    0.472
##     nc7               0.957    0.164    5.826    0.000    0.662
##     nc8               0.787    0.143    5.519    0.000    0.544
##     nc9               0.951    0.182    5.210    0.000    0.657
##     nc10              0.801    0.134    5.980    0.000    0.554
##     nc11              0.906    0.149    6.095    0.000    0.626
##     nc12              0.910    0.162    5.602    0.000    0.629
##     nc13              0.865    0.122    7.095    0.000    0.598
##     nc14              0.800    0.129    6.218    0.000    0.553
##     nc15              0.567    0.148    3.819    0.000    0.392
##     nc16              0.604    0.153    3.956    0.000    0.418
##     nc17              0.836    0.170    4.912    0.000    0.578
##     nc18              0.319    0.140    2.277    0.023    0.221
##   Std.all
##
##     0.560
##     0.725
##     0.577
```

```
##     0.627
##     0.676
##     0.394
##     0.504
##     0.448
##     0.521
##     0.586
##     0.652
##     0.594
##     0.522
##     0.506
##     0.367
##     0.352
##     0.513
##     0.183
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1               2.759    0.089   31.103    0.000    2.759
##    .nc2               2.087    0.073   28.456    0.000    2.087
##    .nc3               1.821    0.076   23.943    0.000    1.821
##    .nc4               1.974    0.082   24.054    0.000    1.974
##    .nc5               1.831    0.075   24.490    0.000    1.831
##    .nc6               2.713    0.086   31.537    0.000    2.713
##    .nc7               2.487    0.094   26.363    0.000    2.487
##    .nc8               2.518    0.087   28.924    0.000    2.518
##    .nc9               2.267    0.091   25.034    0.000    2.267
##    .nc10              1.944    0.068   28.627    0.000    1.944
##    .nc11              1.795    0.069   26.035    0.000    1.795
##    .nc12              1.754    0.076   23.094    0.000    1.754
##    .nc13              2.713    0.082   32.983    0.000    2.713
##    .nc14              2.251    0.078   28.705    0.000    2.251
##    .nc15              2.323    0.077   30.281    0.000    2.323
##    .nc16              2.292    0.085   26.918    0.000    2.292
##    .nc17              1.903    0.081   23.538    0.000    1.903
##    .nc18              2.672    0.086   30.891    0.000    2.672
##     NC                0.000                               0.000
##   Std.all
##     2.233
##     2.043
##     1.719
##     1.727
##     1.758
##     2.264
##     1.893
##     2.077
##     1.797
##     2.055
##     1.869
##     1.658
##     2.368
##     2.061
##     2.174
##     1.933
```

```
##     1.690
##     2.218
##     0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .nc1              1.049    0.131    7.997    0.000    1.049
##    .nc2              0.495    0.077    6.450    0.000    0.495
##    .nc3              0.748    0.116    6.440    0.000    0.748
##    .nc4              0.793    0.129    6.153    0.000    0.793
##    .nc5              0.588    0.085    6.945    0.000    0.588
##    .nc6              1.213    0.127    9.541    0.000    1.213
##    .nc7              1.289    0.129    9.990    0.000    1.289
##    .nc8              1.175    0.111   10.615    0.000    1.175
##    .nc9              1.159    0.136    8.526    0.000    1.159
##    .nc10             0.588    0.093    6.305    0.000    0.588
##    .nc11             0.530    0.084    6.311    0.000    0.530
##    .nc12             0.724    0.125    5.766    0.000    0.724
##    .nc13             0.955    0.092   10.336    0.000    0.955
##    .nc14             0.887    0.103    8.645    0.000    0.887
##    .nc15             0.988    0.132    7.515    0.000    0.988
##    .nc16             1.232    0.145    8.477    0.000    1.232
##    .nc17             0.934    0.113    8.255    0.000    0.934
##    .nc18             1.402    0.112   12.569    0.000    1.402
##     NC               0.478    0.125    3.823    0.000    1.000
##   Std.all
##     0.687
##     0.474
##     0.667
##     0.606
##     0.543
##     0.845
##     0.746
##     0.799
##     0.728
##     0.657
##     0.574
##     0.647
##     0.728
##     0.744
##     0.866
##     0.876
##     0.737
##     0.966
##     1.000
##
## R-Square:
##                   Estimate
##     nc1               0.313
##     nc2               0.526
##     nc3               0.333
##     nc4               0.394
##     nc5               0.457
##     nc6               0.155
```

```
##     nc7               0.254
##     nc8               0.201
##     nc9               0.272
##     nc10              0.343
##     nc11              0.426
##     nc12              0.353
##     nc13              0.272
##     nc14              0.256
##     nc15              0.134
##     nc16              0.124
##     nc17              0.263
##     nc18              0.034
```

6.2 Coefficient Alpha: Traditional

```
psych::alpha(NC_rescaled)

##
## Reliability analysis
## Call: psych::alpha(x = NC_rescaled)
##
##   raw_alpha std.alpha G6(smc) average_r S/N   ase mean   sd
##       0.86      0.87    0.88      0.27 6.6 0.014  2.2 0.62
##  median_r
##      0.27
##
##  lower alpha upper     95% confidence boundaries
## 0.84 0.86 0.89
##
##  Reliability if an item is dropped:
##      raw_alpha std.alpha G6(smc) average_r S/N alpha se var.r
## nc1       0.85      0.86    0.88      0.26 6.1    0.015 0.014
## nc2       0.85      0.85    0.87      0.25 5.8    0.016 0.012
## nc3       0.85      0.86    0.88      0.26 6.1    0.015 0.013
## nc4       0.85      0.86    0.87      0.26 6.0    0.015 0.012
## nc5       0.85      0.86    0.87      0.26 5.9    0.015 0.012
## nc6       0.86      0.87    0.88      0.27 6.4    0.014 0.014
## nc7       0.86      0.86    0.88      0.27 6.2    0.015 0.014
## nc8       0.86      0.86    0.88      0.27 6.3    0.015 0.013
## nc9       0.86      0.86    0.88      0.27 6.2    0.015 0.013
## nc10      0.86      0.86    0.87      0.26 6.1    0.015 0.013
## nc11      0.85      0.86    0.87      0.26 6.0    0.015 0.012
## nc12      0.85      0.86    0.87      0.26 6.1    0.015 0.013
## nc13      0.86      0.86    0.88      0.27 6.2    0.015 0.013
## nc14      0.86      0.86    0.88      0.27 6.2    0.015 0.013
## nc15      0.86      0.87    0.88      0.28 6.5    0.014 0.013
## nc16      0.86      0.87    0.88      0.28 6.5    0.014 0.013
## nc17      0.86      0.86    0.88      0.27 6.2    0.015 0.013
## nc18      0.87      0.87    0.89      0.29 6.9    0.014 0.010
##      med.r
## nc1   0.26
## nc2   0.26
```

```
## nc3   0.27
## nc4   0.26
## nc5   0.26
## nc6   0.28
## nc7   0.28
## nc8   0.27
## nc9   0.27
## nc10  0.26
## nc11  0.26
## nc12  0.27
## nc13  0.27
## nc14  0.27
## nc15  0.28
## nc16  0.28
## nc17  0.27
## nc18  0.29
##
##  Item statistics
##        n raw.r std.r r.cor r.drop mean   sd
## nc1  195  0.61  0.60  0.57   0.53  2.8 1.24
## nc2  195  0.72  0.73  0.72   0.67  2.1 1.02
## nc3  195  0.59  0.60  0.57   0.53  1.8 1.06
## nc4  195  0.65  0.64  0.63   0.58  2.0 1.15
## nc5  195  0.67  0.67  0.66   0.61  1.8 1.04
## nc6  195  0.47  0.47  0.41   0.38  2.7 1.20
## nc7  195  0.56  0.55  0.51   0.47  2.5 1.32
## nc8  195  0.51  0.50  0.47   0.43  2.5 1.22
## nc9  195  0.56  0.56  0.52   0.48  2.3 1.26
## nc10 195  0.59  0.60  0.58   0.53  1.9 0.95
## nc11 195  0.64  0.66  0.65   0.59  1.8 0.96
## nc12 195  0.60  0.61  0.59   0.54  1.8 1.06
## nc13 195  0.55  0.55  0.52   0.47  2.7 1.15
## nc14 195  0.56  0.56  0.53   0.48  2.3 1.10
## nc15 195  0.42  0.43  0.37   0.34  2.3 1.07
## nc16 195  0.44  0.43  0.37   0.34  2.3 1.19
## nc17 195  0.56  0.55  0.52   0.48  1.9 1.13
## nc18 195  0.29  0.28  0.20   0.18  2.7 1.21
##
## Non missing response frequency for each item
##         1    2    3    4    5 miss
## nc1  0.12 0.42 0.18 0.14 0.14    0
## nc2  0.31 0.44 0.14 0.08 0.03    0
## nc3  0.50 0.31 0.09 0.07 0.03    0
## nc4  0.44 0.32 0.10 0.09 0.05    0
## nc5  0.49 0.33 0.07 0.09 0.02    0
## nc6  0.14 0.36 0.24 0.15 0.11    0
## nc7  0.30 0.26 0.16 0.19 0.08    0
## nc8  0.21 0.39 0.14 0.18 0.07    0
## nc9  0.32 0.40 0.06 0.15 0.08    0
## nc10 0.35 0.46 0.11 0.05 0.03    0
## nc11 0.45 0.41 0.07 0.04 0.03    0
## nc12 0.56 0.25 0.10 0.06 0.03    0
## nc13 0.14 0.34 0.28 0.15 0.09    0
## nc14 0.26 0.44 0.14 0.12 0.04    0
```

```
## nc15 0.22 0.42 0.23 0.07 0.06    0
## nc16 0.28 0.40 0.12 0.13 0.06    0
## nc17 0.49 0.27 0.11 0.09 0.04    0
## nc18 0.16 0.37 0.18 0.21 0.08    0
```

6.3 Coefficient Alpha: CFA

```
# Extract standardized loadings and error variances.
NC_Loadings <- inspect(NC_Fit_11, what = "std")$lambda
NC_error_variances <- diag(inspect(NC_Fit_11, what = "std")$theta)

# Standardized coefficient alpha, version 1.
Mean_Item_Reliability <- mean(NC_Loadings^2)
(18 * Mean_Item_Reliability)/(1 + 17 * Mean_Item_Reliability)

## [1] 0.8771

# Standardized coefficient alpha, version 2.
sum(NC_Loadings)^2/(sum(NC_Loadings)^2 + sum(NC_error_variances))

## [1] 0.8705
```

7 Cross-Validation

7.1 Across Samples

The most convincing cross-validation occurs across independent samples. For this demonstration, the state self-esteem data will be used. Cross-validation in this case is usually described in the context of measurement invariance. Several increasingly stringent forms of invariance are possible, estimated from a set of models that place increasing equality constraints across groups.

1. The first model simply requires estimation of the same basic factor model in each group. Configural invariance is said to exist if this first model has a good fit and the same loadings are significant in all groups.
2. The next constraint is to require the factor loadings to be equal across groups. This constraint is called metric or weak invariance and tests whether respondents across groups attribute the same meaning to the latent constructs.
3. Next the intercepts are constrained to be equal. This is called scalar or strong invariance and implies that the meaning of the construct (the factor loadings), and the levels of the underlying items (intercepts) are equal in both groups. If this constraint holds, the groups can be compared on their latent variable scores.
4. Finally, the residual variances are fixed to be equal across groups. This is called strict invariance and means that the explained variance for every item is the same across groups. In other words, the latent construct is measured identically across groups. If error variances are not equal, groups can still be compared on the latent variable, but this is measured with different amounts of error between groups.

It is also possible to extend the sequence of tests by requiring the latent means, variances, and covariances to be equal.

The sequence of tests compares each newly constrained model to the previous one. The level of invariance is determined by the furthest step at which model equivalence holds.

It is possible that partial invariance holds, occurring when a level of invariance is found for some, but not all items.

Comparison of models is usuall conducted using both chi-square difference tests as well as comparison of CFI. A nonsignificant chi-square difference and a CFI difference less than .01 are conventionally taken as evidence for invariance at a particular step.

```
HS.model <- "
Visual =~ x1+x2+x3
Textual =~ x4+x5+x6
Speed =~ x7+x8+x9
"
no_groups <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(no_groups, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  36 iterations
##
##   Number of observations                           301
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                      85.022      86.842
##   Degrees of freedom                                24          24
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  0.979
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              915.799     877.158
##   Degrees of freedom                                36          36
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.931       0.925
##   Tucker-Lewis Index (TLI)                       0.896       0.888
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
```

```
##   Loglikelihood user model (H0)              -3737.745   -3737.745
##   Loglikelihood unrestricted model (H1)      -3695.092   -3695.092
##
##   Number of free parameters                         30          30
##   Akaike (AIC)                                7535.490    7535.490
##   Bayesian (BIC)                              7646.703    7646.703
##   Sample-size adjusted Bayesian (BIC)         7551.560    7551.560
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.092       0.093
##   90 Percent Confidence Interval          0.071  0.114       0.073  0.115
##   P-value RMSEA <= 0.05                          0.001       0.001
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.060       0.060
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.900
##     x2                0.554    0.132    4.184    0.000    0.498
##     x3                0.729    0.141    5.161    0.000    0.656
##   Textual =~
##     x4                1.000                               0.990
##     x5                1.113    0.066   16.917    0.000    1.102
##     x6                0.926    0.061   15.064    0.000    0.917
##   Speed =~
##     x7                1.000                               0.619
##     x8                1.180    0.131    9.031    0.000    0.731
##     x9                1.082    0.267    4.053    0.000    0.670
##   Std.all
##
##     0.772
##     0.424
##     0.581
##
##     0.852
##     0.855
##     0.838
##
##     0.570
##     0.723
```

```
##     0.665
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual          0.408    0.099    4.104    0.000    0.459
##     Speed            0.262    0.060    4.359    0.000    0.471
##   Textual ~~
##     Speed            0.173    0.056    3.076    0.002    0.283
##   Std.all
##
##     0.459
##     0.471
##
##     0.283
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1               4.936    0.067   73.351    0.000    4.936
##    .x2               6.088    0.068   89.705    0.000    6.088
##    .x3               2.250    0.065   34.522    0.000    2.250
##    .x4               3.061    0.067   45.618    0.000    3.061
##    .x5               4.341    0.074   58.355    0.000    4.341
##    .x6               2.186    0.063   34.609    0.000    2.186
##    .x7               4.186    0.063   66.655    0.000    4.186
##    .x8               5.527    0.058   94.697    0.000    5.527
##    .x9               5.374    0.058   92.392    0.000    5.374
##     Visual           0.000                               0.000
##     Textual          0.000                               0.000
##     Speed            0.000                               0.000
##   Std.all
##     4.235
##     5.179
##     1.993
##     2.634
##     3.369
##     1.998
##     3.848
##     5.467
##     5.334
##     0.000
##     0.000
##     0.000
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1               0.549    0.157    3.503    0.000    0.549
##    .x2               1.134    0.112   10.118    0.000    1.134
##    .x3               0.844    0.100    8.405    0.000    0.844
##    .x4               0.371    0.050    7.370    0.000    0.371
##    .x5               0.446    0.057    7.857    0.000    0.446
##    .x6               0.356    0.047    7.645    0.000    0.356
##    .x7               0.799    0.097    8.209    0.000    0.799
##    .x8               0.488    0.120    4.073    0.000    0.488
```

```
##    .x9                0.566    0.119    4.760    0.000    0.566
##     Visual            0.809    0.181    4.479    0.000    1.000
##     Textual           0.979    0.121    8.062    0.000    1.000
##     Speed             0.384    0.107    3.590    0.000    1.000
##   Std.all
##     0.404
##     0.821
##     0.662
##     0.275
##     0.269
##     0.298
##     0.676
##     0.477
##     0.558
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442

config <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL", group = "school")
summary(config, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  66 iterations
##
##   Number of observations per group
##   Pasteur                                        156
##   Grant-White                                    145
##   Number of missing patterns per group
##   Pasteur                                          1
##   Grant-White                                      1
##
##   Estimator                                       ML      Robust
##   Model Fit Test Statistic                   115.084     120.934
##   Degrees of freedom                              48          48
##   P-value (Chi-square)                         0.000       0.000
##   Scaling correction factor                                0.952
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                    63.897      67.145
##   Grant-White                                51.187      53.789
```

```
##
## Model test baseline model:
##
##   Minimum Function Test Statistic              951.384     928.080
##   Degrees of freedom                            72          72
##   P-value                                      0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                  0.924       0.915
##   Tucker-Lewis Index (TLI)                     0.886       0.872
##
##   Robust Comparative Fit Index (CFI)                          NA
##   Robust Tucker-Lewis Index (TLI)                             NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3682.198   -3682.198
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                     60          60
##   Akaike (AIC)                               7484.395    7484.395
##   Bayesian (BIC)                             7706.822    7706.822
##   Sample-size adjusted Bayesian (BIC)        7516.536    7516.536
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                        0.097       0.101
##   90 Percent Confidence Interval          0.074  0.119       0.078  0.124
##   P-value RMSEA <= 0.05                        0.001       0.000
##
##   Robust RMSEA                                                NA
##   90 Percent Confidence Interval                              NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                         0.068       0.068
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               1.047
##     x2                0.394    0.198    1.990    0.047    0.412
##     x3                0.570    0.202    2.828    0.005    0.597
##   Textual =~
```

```
##     x4                1.000                               0.946
##     x5                1.183    0.106   11.176    0.000    1.119
##     x6                0.875    0.094    9.357    0.000    0.827
##   Speed =~
##     x7                1.000                               0.591
##     x8                1.125    0.263    4.283    0.000    0.665
##     x9                0.922    0.298    3.089    0.002    0.545
##   Std.all
##
##     0.887
##     0.336
##     0.515
##
##     0.823
##     0.856
##     0.838
##
##     0.547
##     0.682
##     0.551
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.479    0.144    3.323    0.001    0.484
##     Speed             0.185    0.076    2.441    0.015    0.299
##   Textual ~~
##     Speed             0.182    0.076    2.394    0.017    0.325
##   Std.all
##
##     0.484
##     0.299
##
##     0.325
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                4.941    0.095   52.075    0.000    4.941
##    .x2                5.984    0.099   60.746    0.000    5.984
##    .x3                2.487    0.093   26.689    0.000    2.487
##    .x4                2.823    0.092   30.587    0.000    2.823
##    .x5                3.995    0.105   38.056    0.000    3.995
##    .x6                1.922    0.079   24.240    0.000    1.922
##    .x7                4.432    0.087   51.011    0.000    4.432
##    .x8                5.563    0.078   70.977    0.000    5.563
##    .x9                5.418    0.079   68.212    0.000    5.418
##     Visual            0.000                               0.000
##     Textual           0.000                               0.000
##     Speed             0.000                               0.000
##   Std.all
##     4.183
##     4.880
##     2.144
##     2.457
```

```
##     3.057
##     1.947
##     4.098
##     5.702
##     5.480
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                0.298    0.339    0.879    0.379    0.298
##    .x2                1.334    0.180    7.416    0.000    1.334
##    .x3                0.989    0.154    6.422    0.000    0.989
##    .x4                0.425    0.073    5.848    0.000    0.425
##    .x5                0.456    0.086    5.286    0.000    0.456
##    .x6                0.290    0.055    5.250    0.000    0.290
##    .x7                0.820    0.133    6.190    0.000    0.820
##    .x8                0.510    0.101    5.031    0.000    0.510
##    .x9                0.680    0.126    5.388    0.000    0.680
##     Visual            1.097    0.380    2.889    0.004    1.000
##     Textual           0.894    0.161    5.558    0.000    1.000
##     Speed             0.350    0.140    2.494    0.013    1.000
##   Std.all
##     0.214
##     0.887
##     0.735
##     0.322
##     0.267
##     0.297
##     0.701
##     0.535
##     0.696
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.786
##     x2                0.113
##     x3                0.265
##     x4                0.678
##     x5                0.733
##     x6                0.703
##     x7                0.299
##     x8                0.465
##     x9                0.304
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
```

```
##   Visual =~
##     x1                1.000                               0.777
##     x2                0.736    0.191    3.857    0.000    0.572
##     x3                0.925    0.217    4.267    0.000    0.719
##   Textual =~
##     x4                1.000                               0.971
##     x5                0.990    0.089   11.142    0.000    0.961
##     x6                0.963    0.091   10.552    0.000    0.935
##   Speed =~
##     x7                1.000                               0.679
##     x8                1.226    0.163    7.535    0.000    0.833
##     x9                1.058    0.251    4.213    0.000    0.719
##   Std.all
##
##     0.677
##     0.517
##     0.694
##
##     0.866
##     0.829
##     0.826
##
##     0.659
##     0.796
##     0.701
##
## Covariances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.408    0.116    3.523    0.000    0.541
##     Speed             0.276    0.101    2.722    0.006    0.523
##   Textual ~~
##     Speed             0.222    0.105    2.108    0.035    0.336
##   Std.all
##
##     0.541
##     0.523
##
##     0.336
##
## Intercepts:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                4.930    0.096   51.524    0.000    4.930
##    .x2                6.200    0.092   67.192    0.000    6.200
##    .x3                1.996    0.086   23.118    0.000    1.996
##    .x4                3.317    0.093   35.507    0.000    3.317
##    .x5                4.712    0.097   48.823    0.000    4.712
##    .x6                2.469    0.094   26.189    0.000    2.469
##    .x7                3.921    0.086   45.666    0.000    3.921
##    .x8                5.488    0.087   62.963    0.000    5.488
##    .x9                5.327    0.085   62.363    0.000    5.327
##     Visual            0.000                               0.000
##     Textual           0.000                               0.000
##     Speed             0.000                               0.000
```

```
##   Std.all
##     4.293
##     5.599
##     1.926
##     2.959
##     4.068
##     2.182
##     3.805
##     5.246
##     5.196
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                0.715    0.182    3.925    0.000    0.715
##    .x2                0.899    0.142    6.317    0.000    0.899
##    .x3                0.557    0.122    4.552    0.000    0.557
##    .x4                0.315    0.066    4.748    0.000    0.315
##    .x5                0.419    0.072    5.857    0.000    0.419
##    .x6                0.406    0.076    5.345    0.000    0.406
##    .x7                0.601    0.100    5.976    0.000    0.601
##    .x8                0.401    0.160    2.508    0.012    0.401
##    .x9                0.535    0.141    3.801    0.000    0.535
##     Visual            0.604    0.195    3.094    0.002    1.000
##     Textual           0.942    0.163    5.787    0.000    1.000
##     Speed             0.461    0.121    3.826    0.000    1.000
##   Std.all
##     0.542
##     0.733
##     0.519
##     0.251
##     0.312
##     0.317
##     0.566
##     0.367
##     0.509
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.458
##     x2                0.267
##     x3                0.481
##     x4                0.749
##     x5                0.688
##     x6                0.683
##     x7                0.434
##     x8                0.633
##     x9                0.491
```

```
weak <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL", group = "school",
    group.equal = "loadings")
summary(weak, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  52 iterations
##
##   Number of observations per group
##   Pasteur                                        156
##   Grant-White                                    145
##   Number of missing patterns per group
##   Pasteur                                          1
##   Grant-White                                      1
##
##   Estimator                                       ML      Robust
##   Model Fit Test Statistic                   123.222     125.163
##   Degrees of freedom                              54          54
##   P-value (Chi-square)                         0.000       0.000
##   Scaling correction factor                                0.984
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                     68.384      69.461
##   Grant-White                                 54.838      55.702
##
## Model test baseline model:
##
##   Minimum Function Test Statistic            951.384     928.080
##   Degrees of freedom                              72          72
##   P-value                                      0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                  0.921       0.917
##   Tucker-Lewis Index (TLI)                     0.895       0.889
##
##   Robust Comparative Fit Index (CFI)                          NA
##   Robust Tucker-Lewis Index (TLI)                             NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3686.294   -3686.294
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                       54          54
##   Akaike (AIC)                                7480.587    7480.587
##   Bayesian (BIC)                              7680.771    7680.771
##   Sample-size adjusted Bayesian (BIC)         7509.514    7509.514
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.093       0.094
##   90 Percent Confidence Interval          0.071  0.114       0.072  0.116
```

```
##   P-value RMSEA <= 0.05                          0.001       0.001
##
##   Robust RMSEA                                                NA
##   90 Percent Confidence Interval                              NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.072       0.072
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.897
##     x2      (.p2.)    0.599    0.140    4.271    0.000    0.537
##     x3      (.p3.)    0.784    0.153    5.129    0.000    0.704
##   Textual =~
##     x4                1.000                               0.956
##     x5      (.p5.)    1.083    0.070   15.557    0.000    1.035
##     x6      (.p6.)    0.912    0.069   13.239    0.000    0.871
##   Speed =~
##     x7                1.000                               0.552
##     x8      (.p8.)    1.201    0.136    8.855    0.000    0.663
##     x9      (.p9.)    1.038    0.207    5.006    0.000    0.573
##   Std.all
##
##     0.771
##     0.432
##     0.600
##
##     0.823
##     0.824
##     0.860
##
##     0.514
##     0.679
##     0.577
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.416    0.135    3.070    0.002    0.485
##     Speed             0.169    0.066    2.556    0.011    0.340
##   Textual ~~
##     Speed             0.176    0.061    2.895    0.004    0.333
##   Std.all
```

```
##
##     0.485
##     0.340
##
##     0.333
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                4.941    0.095   52.075    0.000    4.941
##    .x2                5.984    0.099   60.746    0.000    5.984
##    .x3                2.487    0.093   26.689    0.000    2.487
##    .x4                2.823    0.092   30.587    0.000    2.823
##    .x5                3.995    0.105   38.056    0.000    3.995
##    .x6                1.922    0.079   24.240    0.000    1.922
##    .x7                4.432    0.087   51.011    0.000    4.432
##    .x8                5.563    0.078   70.977    0.000    5.563
##    .x9                5.418    0.079   68.212    0.000    5.418
##     Visual            0.000                               0.000
##     Textual           0.000                               0.000
##     Speed             0.000                               0.000
##   Std.all
##     4.243
##     4.812
##     2.119
##     2.432
##     3.180
##     1.898
##     4.126
##     5.692
##     5.457
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                0.551    0.176    3.133    0.002    0.551
##    .x2                1.258    0.166    7.587    0.000    1.258
##    .x3                0.882    0.129    6.824    0.000    0.882
##    .x4                0.434    0.072    6.013    0.000    0.434
##    .x5                0.508    0.080    6.366    0.000    0.508
##    .x6                0.266    0.057    4.660    0.000    0.266
##    .x7                0.849    0.113    7.495    0.000    0.849
##    .x8                0.515    0.095    5.430    0.000    0.515
##    .x9                0.658    0.118    5.555    0.000    0.658
##     Visual            0.805    0.227    3.547    0.000    1.000
##     Textual           0.913    0.139    6.559    0.000    1.000
##     Speed             0.305    0.083    3.689    0.000    1.000
##   Std.all
##     0.406
##     0.813
##     0.640
##     0.322
##     0.322
```

```
##     0.260
##     0.736
##     0.539
##     0.667
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.594
##     x2                0.187
##     x3                0.360
##     x4                0.678
##     x5                0.678
##     x6                0.740
##     x7                0.264
##     x8                0.461
##     x9                0.333
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.850
##     x2      (.p2.)    0.599    0.140    4.271    0.000    0.509
##     x3      (.p3.)    0.784    0.153    5.129    0.000    0.667
##   Textual =~
##     x4                1.000                               0.952
##     x5      (.p5.)    1.083    0.070   15.557    0.000    1.031
##     x6      (.p6.)    0.912    0.069   13.239    0.000    0.868
##   Speed =~
##     x7                1.000                               0.689
##     x8      (.p8.)    1.201    0.136    8.855    0.000    0.828
##     x9      (.p9.)    1.038    0.207    5.006    0.000    0.715
##   Std.all
##
##     0.727
##     0.466
##     0.651
##
##     0.857
##     0.857
##     0.795
##
##     0.665
##     0.793
##     0.700
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
```

```
##     Textual            0.437    0.108    4.054    0.000    0.540
##     Speed              0.314    0.097    3.242    0.001    0.536
##   Textual ~~
##     Speed              0.226    0.099    2.280    0.023    0.345
##   Std.all
##
##     0.540
##     0.536
##
##     0.345
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                4.930    0.096   51.524    0.000    4.930
##    .x2                6.200    0.092   67.192    0.000    6.200
##    .x3                1.996    0.086   23.118    0.000    1.996
##    .x4                3.317    0.093   35.507    0.000    3.317
##    .x5                4.712    0.097   48.823    0.000    4.712
##    .x6                2.469    0.094   26.189    0.000    2.469
##    .x7                3.921    0.086   45.666    0.000    3.921
##    .x8                5.488    0.087   62.963    0.000    5.488
##    .x9                5.327    0.085   62.363    0.000    5.327
##     Visual            0.000                              0.000
##     Textual           0.000                              0.000
##     Speed             0.000                              0.000
##   Std.all
##     4.216
##     5.679
##     1.948
##     2.985
##     3.918
##     2.263
##     3.783
##     5.253
##     5.214
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                0.645    0.171    3.780    0.000    0.645
##    .x2                0.933    0.146    6.392    0.000    0.933
##    .x3                0.605    0.113    5.352    0.000    0.605
##    .x4                0.329    0.066    4.999    0.000    0.329
##    .x5                0.384    0.072    5.324    0.000    0.384
##    .x6                0.437    0.075    5.838    0.000    0.437
##    .x7                0.599    0.095    6.297    0.000    0.599
##    .x8                0.406    0.145    2.805    0.005    0.406
##    .x9                0.532    0.126    4.216    0.000    0.532
##     Visual            0.722    0.176    4.094    0.000    1.000
##     Textual           0.906    0.147    6.172    0.000    1.000
##     Speed             0.475    0.113    4.193    0.000    1.000
##   Std.all
```

```
##     0.472
##     0.783
##     0.577
##     0.266
##     0.265
##     0.367
##     0.558
##     0.372
##     0.510
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.528
##     x2                0.217
##     x3                0.423
##     x4                0.734
##     x5                0.735
##     x6                0.633
##     x7                0.442
##     x8                0.628
##     x9                0.490
```

```
strong <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL", group = "school",
    group.equal = c("loadings", "intercepts"))
summary(strong, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
## lavaan (0.6-1) converged normally after  61 iterations
##
##   Number of observations per group
##   Pasteur                                         156
##   Grant-White                                     145
##   Number of missing patterns per group
##   Pasteur                                           1
##   Grant-White                                       1
##
##   Estimator                                        ML      Robust
##   Model Fit Test Statistic                    163.015     165.643
##   Degrees of freedom                               60          60
##   P-value (Chi-square)                          0.000       0.000
##   Scaling correction factor                                 0.984
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                      89.632      91.077
##   Grant-White                                  73.383      74.566
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             951.384     928.080
```

```
##   Degrees of freedom                                72          72
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.883       0.877
##   Tucker-Lewis Index (TLI)                       0.859       0.852
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3706.323   -3706.323
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                         48          48
##   Akaike (AIC)                                7508.647    7508.647
##   Bayesian (BIC)                              7686.588    7686.588
##   Sample-size adjusted Bayesian (BIC)         7534.359    7534.359
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.107       0.109
##   90 Percent Confidence Interval          0.088  0.127       0.089  0.129
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.082       0.082
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.892
##     x2      (.p2.)    0.576    0.131    4.380    0.000    0.514
##     x3      (.p3.)    0.798    0.173    4.612    0.000    0.712
##   Textual =~
##     x4                1.000                               0.938
##     x5      (.p5.)    1.120    0.068   16.568    0.000    1.050
##     x6      (.p6.)    0.932    0.064   14.603    0.000    0.874
##   Speed =~
```

```
##     x7                1.000                                    0.568
##     x8      (.p8.)    1.130    0.134    8.459    0.000    0.641
##     x9      (.p9.)    1.009    0.208    4.848    0.000    0.573
##   Std.all
##
##     0.768
##     0.411
##     0.591
##
##     0.815
##     0.829
##     0.862
##
##     0.516
##     0.657
##     0.578
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.410    0.135    3.033    0.002    0.490
##     Speed             0.178    0.067    2.652    0.008    0.351
##   Textual ~~
##     Speed             0.180    0.063    2.880    0.004    0.338
##   Std.all
##
##     0.490
##     0.351
##
##     0.338
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    5.001    0.095   52.874    0.000    5.001
##    .x2      (.26.)    6.151    0.087   70.631    0.000    6.151
##    .x3      (.27.)    2.271    0.095   23.937    0.000    2.271
##    .x4      (.28.)    2.778    0.087   32.065    0.000    2.778
##    .x5      (.29.)    4.035    0.104   38.965    0.000    4.035
##    .x6      (.30.)    1.926    0.075   25.665    0.000    1.926
##    .x7      (.31.)    4.242    0.079   53.523    0.000    4.242
##    .x8      (.32.)    5.630    0.075   74.753    0.000    5.630
##    .x9      (.33.)    5.465    0.072   75.861    0.000    5.465
##     Visual            0.000                                0.000
##     Textual           0.000                                0.000
##     Speed             0.000                                0.000
##   Std.all
##     4.302
##     4.925
##     1.885
##     2.413
##     3.184
##     1.900
##     3.855
##     5.771
```

```
##     5.516
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                0.555    0.185    3.007    0.003    0.555
##    .x2                1.296    0.162    8.021    0.000    1.296
##    .x3                0.944    0.147    6.424    0.000    0.944
##    .x4                0.445    0.073    6.093    0.000    0.445
##    .x5                0.502    0.081    6.177    0.000    0.502
##    .x6                0.263    0.058    4.510    0.000    0.263
##    .x7                0.888    0.128    6.929    0.000    0.888
##    .x8                0.541    0.089    6.052    0.000    0.541
##    .x9                0.654    0.115    5.666    0.000    0.654
##     Visual            0.796    0.239    3.325    0.001    1.000
##     Textual           0.879    0.135    6.491    0.000    1.000
##     Speed             0.322    0.092    3.488    0.000    1.000
##   Std.all
##     0.411
##     0.831
##     0.650
##     0.336
##     0.313
##     0.256
##     0.734
##     0.568
##     0.666
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.589
##     x2                0.169
##     x3                0.350
##     x4                0.664
##     x5                0.687
##     x6                0.744
##     x7                0.266
##     x8                0.432
##     x9                0.334
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.841
##     x2      (.p2.)    0.576    0.131    4.380    0.000    0.484
##     x3      (.p3.)    0.798    0.173    4.612    0.000    0.672
```

```
##   Textual =~
##     x4                1.000                               0.933
##     x5      (.p5.)    1.120    0.068   16.568    0.000    1.045
##     x6      (.p6.)    0.932    0.064   14.603    0.000    0.869
##   Speed =~
##     x7                1.000                               0.711
##     x8      (.p8.)    1.130    0.134    8.459    0.000    0.803
##     x9      (.p9.)    1.009    0.208    4.848    0.000    0.717
##   Std.all
##
##     0.721
##     0.442
##     0.643
##
##     0.847
##     0.862
##     0.796
##
##     0.668
##     0.773
##     0.704
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.427    0.107    3.990    0.000    0.544
##     Speed             0.329    0.101    3.257    0.001    0.550
##   Textual ~~
##     Speed             0.236    0.098    2.415    0.016    0.356
##   Std.all
##
##     0.544
##     0.550
##
##     0.356
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    5.001    0.095   52.874    0.000    5.001
##    .x2      (.26.)    6.151    0.087   70.631    0.000    6.151
##    .x3      (.27.)    2.271    0.095   23.937    0.000    2.271
##    .x4      (.28.)    2.778    0.087   32.065    0.000    2.778
##    .x5      (.29.)    4.035    0.104   38.965    0.000    4.035
##    .x6      (.30.)    1.926    0.075   25.665    0.000    1.926
##    .x7      (.31.)    4.242    0.079   53.523    0.000    4.242
##    .x8      (.32.)    5.630    0.075   74.753    0.000    5.630
##    .x9      (.33.)    5.465    0.072   75.861    0.000    5.465
##     Visual           -0.148    0.141   -1.050    0.294   -0.176
##     Textual           0.576    0.119    4.824    0.000    0.618
##     Speed            -0.177    0.104   -1.706    0.088   -0.250
##   Std.all
##     4.286
##     5.618
##     2.174
```

```
##     2.522
##     3.330
##     1.763
##     3.991
##     5.422
##     5.369
##    -0.176
##     0.618
##    -0.250
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1                0.654    0.178    3.679    0.000    0.654
##    .x2                0.964    0.152    6.330    0.000    0.964
##    .x3                0.641    0.130    4.933    0.000    0.641
##    .x4                0.343    0.065    5.258    0.000    0.343
##    .x5                0.376    0.073    5.146    0.000    0.376
##    .x6                0.437    0.074    5.902    0.000    0.437
##    .x7                0.625    0.106    5.882    0.000    0.625
##    .x8                0.434    0.148    2.938    0.003    0.434
##    .x9                0.522    0.126    4.149    0.000    0.522
##     Visual            0.708    0.187    3.791    0.000    1.000
##     Textual           0.870    0.144    6.048    0.000    1.000
##     Speed             0.505    0.125    4.040    0.000    1.000
##   Std.all
##     0.480
##     0.804
##     0.587
##     0.283
##     0.256
##     0.366
##     0.553
##     0.403
##     0.504
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.520
##     x2                0.196
##     x3                0.413
##     x4                0.717
##     x5                0.744
##     x6                0.634
##     x7                0.447
##     x8                0.597
##     x9                0.496

strict <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL", group = "school",
    group.equal = c("loadings", "intercepts", "residuals"))
summary(strict, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
## lavaan (0.6-1) converged normally after  60 iterations
##
##   Number of observations per group
##   Pasteur                                        156
##   Grant-White                                    145
##   Number of missing patterns per group
##   Pasteur                                          1
##   Grant-White                                      1
##
##   Estimator                                       ML      Robust
##   Model Fit Test Statistic                   180.305     180.223
##   Degrees of freedom                              69          69
##   P-value (Chi-square)                         0.000       0.000
##   Scaling correction factor                                1.000
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                    92.496      92.454
##   Grant-White                                87.809      87.769
##
## Model test baseline model:
##
##   Minimum Function Test Statistic           951.384     928.080
##   Degrees of freedom                             72          72
##   P-value                                      0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                  0.873       0.870
##   Tucker-Lewis Index (TLI)                     0.868       0.864
##
##   Robust Comparative Fit Index (CFI)                          NA
##   Robust Tucker-Lewis Index (TLI)                             NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3715.028   -3715.028
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                         39          39
##   Akaike (AIC)                                7508.055    7508.055
##   Bayesian (BIC)                              7652.632    7652.632
##   Sample-size adjusted Bayesian (BIC)         7528.947    7528.947
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.104       0.104
##   90 Percent Confidence Interval          0.085  0.123       0.085  0.123
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                NA
##   90 Percent Confidence Interval                              NA     NA
```

```
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.088       0.088
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.876
##     x2      (.p2.)    0.591    0.142    4.159    0.000    0.518
##     x3      (.p3.)    0.837    0.213    3.936    0.000    0.733
##   Textual =~
##     x4                1.000                               0.945
##     x5      (.p5.)    1.125    0.066   16.932    0.000    1.064
##     x6      (.p6.)    0.933    0.061   15.245    0.000    0.882
##   Speed =~
##     x7                1.000                               0.583
##     x8      (.p8.)    1.121    0.137    8.180    0.000    0.654
##     x9      (.p9.)    1.028    0.220    4.664    0.000    0.600
##   Std.all
##
##     0.739
##     0.438
##     0.641
##
##     0.837
##     0.850
##     0.829
##
##     0.554
##     0.678
##     0.620
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.367    0.150    2.455    0.014    0.444
##     Speed             0.174    0.070    2.480    0.013    0.341
##   Textual ~~
##     Speed             0.176    0.063    2.809    0.005    0.319
##   Std.all
##
##     0.444
##     0.341
##
```

```
##     0.319
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    5.012    0.095   52.568    0.000    5.012
##    .x2      (.26.)    6.133    0.085   71.826    0.000    6.133
##    .x3      (.27.)    2.314    0.097   23.915    0.000    2.314
##    .x4      (.28.)    2.784    0.087   32.046    0.000    2.784
##    .x5      (.29.)    4.029    0.102   39.323    0.000    4.029
##    .x6      (.30.)    1.927    0.075   25.696    0.000    1.927
##    .x7      (.31.)    4.271    0.078   54.872    0.000    4.271
##    .x8      (.32.)    5.622    0.074   75.588    0.000    5.622
##    .x9      (.33.)    5.461    0.072   76.041    0.000    5.461
##     Visual             0.000                             0.000
##     Textual            0.000                             0.000
##     Speed              0.000                             0.000
##   Std.all
##     4.227
##     5.186
##     2.023
##     2.464
##     3.219
##     1.811
##     4.056
##     5.834
##     5.644
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.638    0.176    3.616    0.000    0.638
##    .x2      (.11.)    1.130    0.111   10.192    0.000    1.130
##    .x3      (.12.)    0.771    0.134    5.751    0.000    0.771
##    .x4      (.13.)    0.383    0.050    7.642    0.000    0.383
##    .x5      (.14.)    0.435    0.057    7.568    0.000    0.435
##    .x6      (.15.)    0.354    0.047    7.484    0.000    0.354
##    .x7      (.16.)    0.769    0.094    8.185    0.000    0.769
##    .x8      (.17.)    0.501    0.092    5.445    0.000    0.501
##    .x9      (.18.)    0.576    0.099    5.841    0.000    0.576
##     Visual             0.767    0.225    3.416    0.001    1.000
##     Textual            0.894    0.135    6.601    0.000    1.000
##     Speed              0.340    0.103    3.312    0.001    1.000
##   Std.all
##     0.454
##     0.808
##     0.589
##     0.300
##     0.278
##     0.312
##     0.693
##     0.540
##     0.616
```

```
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.546
##     x2                0.192
##     x3                0.411
##     x4                0.700
##     x5                0.722
##     x6                0.688
##     x7                0.307
##     x8                0.460
##     x9                0.384
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.810
##     x2      (.p2.)    0.591    0.142    4.159    0.000    0.479
##     x3      (.p3.)    0.837    0.213    3.936    0.000    0.678
##   Textual =~
##     x4                1.000                               0.936
##     x5      (.p5.)    1.125    0.066   16.932    0.000    1.053
##     x6      (.p6.)    0.933    0.061   15.245    0.000    0.874
##   Speed =~
##     x7                1.000                               0.692
##     x8      (.p8.)    1.121    0.137    8.180    0.000    0.775
##     x9      (.p9.)    1.028    0.220    4.664    0.000    0.711
##   Std.all
##
##     0.712
##     0.411
##     0.611
##
##     0.834
##     0.847
##     0.827
##
##     0.619
##     0.738
##     0.684
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.422    0.109    3.888    0.000    0.556
##     Speed             0.331    0.100    3.309    0.001    0.590
##   Textual ~~
##     Speed             0.236    0.098    2.404    0.016    0.364
```

```
##   Std.all
##
##     0.556
##     0.590
##
##     0.364
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    5.012    0.095   52.568    0.000    5.012
##    .x2      (.26.)    6.133    0.085   71.826    0.000    6.133
##    .x3      (.27.)    2.314    0.097   23.915    0.000    2.314
##    .x4      (.28.)    2.784    0.087   32.046    0.000    2.784
##    .x5      (.29.)    4.029    0.102   39.323    0.000    4.029
##    .x6      (.30.)    1.927    0.075   25.696    0.000    1.927
##    .x7      (.31.)    4.271    0.078   54.872    0.000    4.271
##    .x8      (.32.)    5.622    0.074   75.588    0.000    5.622
##    .x9      (.33.)    5.461    0.072   76.041    0.000    5.461
##     Visual           -0.157    0.144   -1.096    0.273   -0.194
##     Textual           0.575    0.118    4.856    0.000    0.614
##     Speed            -0.176    0.105   -1.675    0.094   -0.255
##   Std.all
##     4.404
##     5.260
##     2.086
##     2.481
##     3.243
##     1.824
##     3.825
##     5.356
##     5.249
##    -0.194
##     0.614
##    -0.255
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.638    0.176    3.616    0.000    0.638
##    .x2      (.11.)    1.130    0.111   10.192    0.000    1.130
##    .x3      (.12.)    0.771    0.134    5.751    0.000    0.771
##    .x4      (.13.)    0.383    0.050    7.642    0.000    0.383
##    .x5      (.14.)    0.435    0.057    7.568    0.000    0.435
##    .x6      (.15.)    0.354    0.047    7.484    0.000    0.354
##    .x7      (.16.)    0.769    0.094    8.185    0.000    0.769
##    .x8      (.17.)    0.501    0.092    5.445    0.000    0.501
##    .x9      (.18.)    0.576    0.099    5.841    0.000    0.576
##     Visual            0.657    0.227    2.892    0.004    1.000
##     Textual           0.876    0.143    6.140    0.000    1.000
##     Speed             0.478    0.126    3.801    0.000    1.000
##   Std.all
##     0.493
##     0.831
##     0.626
##     0.304
```

```
##     0.282
##     0.317
##     0.616
##     0.455
##     0.533
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.507
##     x2                0.169
##     x3                0.374
##     x4                0.696
##     x5                0.718
##     x6                0.683
##     x7                0.384
##     x8                0.545
##     x9                0.467
```

```
strict_2 <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL", group = "school",
    group.equal = c("loadings", "intercepts", "residuals", "means"))
summary(strict_2, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  57 iterations
##
##   Number of observations per group
##   Pasteur                                        156
##   Grant-White                                    145
##   Number of missing patterns per group
##   Pasteur                                          1
##   Grant-White                                      1
##
##   Estimator                                       ML      Robust
##   Model Fit Test Statistic                   219.863     219.162
##   Degrees of freedom                              72          72
##   P-value (Chi-square)                         0.000       0.000
##   Scaling correction factor                                1.003
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                    110.637     110.284
##   Grant-White                                109.226     108.878
##
## Model test baseline model:
##
##   Minimum Function Test Statistic            951.384     928.080
##   Degrees of freedom                              72          72
##   P-value                                      0.000       0.000
##
```

```
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.832       0.828
##   Tucker-Lewis Index (TLI)                       0.832       0.828
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3734.939   -3734.939
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                         36          36
##   Akaike (AIC)                                7541.879    7541.879
##   Bayesian (BIC)                              7675.335    7675.335
##   Sample-size adjusted Bayesian (BIC)         7561.163    7561.163
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.117       0.117
##   90 Percent Confidence Interval          0.100  0.135       0.099  0.135
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.114       0.114
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                    Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.914
##     x2      (.p2.)    0.571    0.144    3.967    0.000    0.521
##     x3      (.p3.)    0.753    0.166    4.535    0.000    0.688
##   Textual =~
##     x4                1.000                               0.993
##     x5      (.p5.)    1.113    0.066   16.761    0.000    1.106
##     x6      (.p6.)    0.926    0.062   14.952    0.000    0.920
##   Speed =~
##     x7                1.000                               0.563
##     x8      (.p8.)    1.187    0.130    9.111    0.000    0.668
##     x9      (.p9.)    1.084    0.265    4.092    0.000    0.610
```

```
##   Std.all
##
##     0.770
##     0.441
##     0.602
##
##     0.852
##     0.856
##     0.839
##
##     0.533
##     0.693
##     0.630
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.385    0.154    2.494    0.013    0.424
##     Speed             0.181    0.069    2.621    0.009    0.351
##   Textual ~~
##     Speed             0.150    0.062    2.444    0.015    0.269
##   Std.all
##
##     0.424
##     0.351
##
##     0.269
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    4.939    0.077   64.457    0.000    4.939
##    .x2      (.26.)    6.090    0.069   88.522    0.000    6.090
##    .x3      (.27.)    2.253    0.071   31.566    0.000    2.253
##    .x4      (.28.)    3.070    0.073   42.211    0.000    3.070
##    .x5      (.29.)    4.350    0.083   52.704    0.000    4.350
##    .x6      (.30.)    2.194    0.070   31.346    0.000    2.194
##    .x7      (.31.)    4.201    0.067   62.871    0.000    4.201
##    .x8      (.32.)    5.545    0.066   84.185    0.000    5.545
##    .x9      (.33.)    5.391    0.061   88.256    0.000    5.391
##     Visual            0.000                               0.000
##     Textual           0.000                               0.000
##     Speed             0.000                               0.000
##   Std.all
##     4.162
##     5.150
##     1.973
##     2.635
##     3.368
##     2.001
##     3.972
##     5.746
##     5.562
##     0.000
##     0.000
```

```
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.574    0.172    3.333    0.001    0.574
##    .x2      (.11.)    1.126    0.114    9.842    0.000    1.126
##    .x3      (.12.)    0.830    0.110    7.539    0.000    0.830
##    .x4      (.13.)    0.371    0.051    7.348    0.000    0.371
##    .x5      (.14.)    0.446    0.058    7.628    0.000    0.446
##    .x6      (.15.)    0.357    0.048    7.418    0.000    0.357
##    .x7      (.16.)    0.802    0.096    8.374    0.000    0.802
##    .x8      (.17.)    0.485    0.115    4.202    0.000    0.485
##    .x9      (.18.)    0.567    0.119    4.779    0.000    0.567
##     Visual             0.835    0.219    3.816    0.000    1.000
##     Textual            0.986    0.151    6.517    0.000    1.000
##     Speed              0.317    0.100    3.180    0.001    1.000
##   Std.all
##     0.407
##     0.805
##     0.637
##     0.273
##     0.267
##     0.297
##     0.716
##     0.520
##     0.603
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.593
##     x2                0.195
##     x3                0.363
##     x4                0.727
##     x5                0.733
##     x6                0.703
##     x7                0.284
##     x8                0.480
##     x9                0.397
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.855
##     x2      (.p2.)    0.571    0.144    3.967    0.000    0.488
##     x3      (.p3.)    0.753    0.166    4.535    0.000    0.643
##   Textual =~
##     x4                1.000                               0.986
##     x5      (.p5.)    1.113    0.066   16.761    0.000    1.098
```

```
##     x6      (.p6.)    0.926    0.062   14.952    0.000    0.913
##   Speed =~
##     x7                1.000                             0.672
##     x8      (.p8.)    1.187    0.130    9.111    0.000    0.797
##     x9      (.p9.)    1.084    0.265    4.092    0.000    0.728
##   Std.all
##
##     0.748
##     0.418
##     0.577
##
##     0.851
##     0.854
##     0.837
##
##     0.600
##     0.753
##     0.695
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.417    0.114    3.670    0.000    0.494
##     Speed             0.342    0.093    3.664    0.000    0.596
##   Textual ~~
##     Speed             0.197    0.099    1.996    0.046    0.297
##   Std.all
##
##     0.494
##     0.596
##
##     0.297
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    4.939    0.077   64.457    0.000    4.939
##    .x2      (.26.)    6.090    0.069   88.522    0.000    6.090
##    .x3      (.27.)    2.253    0.071   31.566    0.000    2.253
##    .x4      (.28.)    3.070    0.073   42.211    0.000    3.070
##    .x5      (.29.)    4.350    0.083   52.704    0.000    4.350
##    .x6      (.30.)    2.194    0.070   31.346    0.000    2.194
##    .x7      (.31.)    4.201    0.067   62.871    0.000    4.201
##    .x8      (.32.)    5.545    0.066   84.185    0.000    5.545
##    .x9      (.33.)    5.391    0.061   88.256    0.000    5.391
##     Visual            0.000                             0.000
##     Textual           0.000                             0.000
##     Speed             0.000                             0.000
##   Std.all
##     4.324
##     5.214
##     2.020
##     2.649
##     3.386
```

```
##     2.011
##     3.754
##     5.240
##     5.148
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.574    0.172    3.333    0.001    0.574
##    .x2      (.11.)    1.126    0.114    9.842    0.000    1.126
##    .x3      (.12.)    0.830    0.110    7.539    0.000    0.830
##    .x4      (.13.)    0.371    0.051    7.348    0.000    0.371
##    .x5      (.14.)    0.446    0.058    7.628    0.000    0.446
##    .x6      (.15.)    0.357    0.048    7.418    0.000    0.357
##    .x7      (.16.)    0.802    0.096    8.374    0.000    0.802
##    .x8      (.17.)    0.485    0.115    4.202    0.000    0.485
##    .x9      (.18.)    0.567    0.119    4.779    0.000    0.567
##     Visual             0.731    0.222    3.296    0.001    1.000
##     Textual            0.972    0.162    5.990    0.000    1.000
##     Speed              0.451    0.130    3.479    0.001    1.000
##   Std.all
##     0.440
##     0.826
##     0.667
##     0.276
##     0.270
##     0.300
##     0.640
##     0.433
##     0.517
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.560
##     x2                0.174
##     x3                0.333
##     x4                0.724
##     x5                0.730
##     x6                0.700
##     x7                0.360
##     x8                0.567
##     x9                0.483
```

```
strict_3 <- cfa(HS.model, data = HS, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL", group = "school",
    group.equal = c("loadings", "intercepts", "residuals", "means",
        "lv.variances", "lv.covariances"))
summary(strict_3, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
## lavaan (0.6-1) converged normally after  55 iterations
##
##   Number of observations per group
##   Pasteur                                        156
##   Grant-White                                    145
##   Number of missing patterns per group
##   Pasteur                                          1
##   Grant-White                                      1
##
##   Estimator                                       ML      Robust
##   Model Fit Test Statistic                   225.436     223.115
##   Degrees of freedom                              78          78
##   P-value (Chi-square)                         0.000       0.000
##   Scaling correction factor                                1.010
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                    113.022     111.858
##   Grant-White                                112.414     111.257
##
## Model test baseline model:
##
##   Minimum Function Test Statistic            951.384     928.080
##   Degrees of freedom                              72          72
##   P-value                                      0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                  0.832       0.830
##   Tucker-Lewis Index (TLI)                     0.845       0.844
##
##   Robust Comparative Fit Index (CFI)                          NA
##   Robust Tucker-Lewis Index (TLI)                             NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3737.745   -3737.745
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                       30          30
##   Akaike (AIC)                                7535.490    7535.490
##   Bayesian (BIC)                              7646.703    7646.703
##   Sample-size adjusted Bayesian (BIC)         7551.560    7551.560
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.112       0.112
##   90 Percent Confidence Interval          0.095  0.130       0.095  0.129
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                NA
##   90 Percent Confidence Interval                              NA     NA
```

```
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.119       0.119
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.900
##     x2      (.p2.)    0.553    0.133    4.177    0.000    0.498
##     x3      (.p3.)    0.729    0.142    5.152    0.000    0.656
##   Textual =~
##     x4                1.000                               0.990
##     x5      (.p5.)    1.113    0.066   16.889    0.000    1.102
##     x6      (.p6.)    0.926    0.062   15.039    0.000    0.917
##   Speed =~
##     x7                1.000                               0.619
##     x8      (.p8.)    1.180    0.131    9.015    0.000    0.731
##     x9      (.p9.)    1.082    0.267    4.047    0.000    0.670
##   Std.all
##
##     0.772
##     0.424
##     0.581
##
##     0.852
##     0.855
##     0.838
##
##     0.570
##     0.723
##     0.665
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual (.22.)    0.408    0.100    4.097    0.000    0.459
##     Speed   (.23.)    0.262    0.060    4.352    0.000    0.471
##   Textual ~~
##     Speed   (.24.)    0.173    0.056    3.071    0.002    0.283
##   Std.all
##
##     0.459
##     0.471
##
```

```
##     0.283
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    4.936    0.067   73.229    0.000    4.936
##    .x2      (.26.)    6.088    0.068   89.556    0.000    6.088
##    .x3      (.27.)    2.250    0.065   34.464    0.000    2.250
##    .x4      (.28.)    3.061    0.067   45.542    0.000    3.061
##    .x5      (.29.)    4.341    0.075   58.258    0.000    4.341
##    .x6      (.30.)    2.186    0.063   34.552    0.000    2.186
##    .x7      (.31.)    4.186    0.063   66.544    0.000    4.186
##    .x8      (.32.)    5.527    0.058   94.539    0.000    5.527
##    .x9      (.33.)    5.374    0.058   92.238    0.000    5.374
##     Visual             0.000                             0.000
##     Textual            0.000                             0.000
##     Speed              0.000                             0.000
##   Std.all
##     4.235
##     5.179
##     1.993
##     2.634
##     3.369
##     1.998
##     3.848
##     5.467
##     5.334
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.549    0.157    3.497    0.000    0.549
##    .x2      (.11.)    1.134    0.112   10.101    0.000    1.134
##    .x3      (.12.)    0.844    0.101    8.391    0.000    0.844
##    .x4      (.13.)    0.371    0.050    7.357    0.000    0.371
##    .x5      (.14.)    0.446    0.057    7.844    0.000    0.446
##    .x6      (.15.)    0.356    0.047    7.632    0.000    0.356
##    .x7      (.16.)    0.799    0.098    8.195    0.000    0.799
##    .x8      (.17.)    0.488    0.120    4.066    0.000    0.488
##    .x9      (.18.)    0.566    0.119    4.752    0.000    0.566
##     Visual  (.19.)    0.809    0.181    4.471    0.000    1.000
##     Textual (.20.)    0.979    0.122    8.048    0.000    1.000
##     Speed   (.21.)    0.384    0.107    3.584    0.000    1.000
##   Std.all
##     0.404
##     0.821
##     0.662
##     0.275
##     0.269
##     0.298
##     0.676
##     0.477
##     0.558
```

```
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.900
##     x2      (.p2.)    0.553    0.133    4.177    0.000    0.498
##     x3      (.p3.)    0.729    0.142    5.152    0.000    0.656
##   Textual =~
##     x4                1.000                               0.990
##     x5      (.p5.)    1.113    0.066   16.889    0.000    1.102
##     x6      (.p6.)    0.926    0.062   15.039    0.000    0.917
##   Speed =~
##     x7                1.000                               0.619
##     x8      (.p8.)    1.180    0.131    9.015    0.000    0.731
##     x9      (.p9.)    1.082    0.267    4.047    0.000    0.670
##   Std.all
##
##     0.772
##     0.424
##     0.581
##
##     0.852
##     0.855
##     0.838
##
##     0.570
##     0.723
##     0.665
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual (.22.)    0.408    0.100    4.097    0.000    0.459
##     Speed   (.23.)    0.262    0.060    4.352    0.000    0.471
##   Textual ~~
##     Speed   (.24.)    0.173    0.056    3.071    0.002    0.283
```

```
##   Std.all
##
##     0.459
##     0.471
##
##     0.283
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    4.936    0.067   73.229    0.000    4.936
##    .x2      (.26.)    6.088    0.068   89.556    0.000    6.088
##    .x3      (.27.)    2.250    0.065   34.464    0.000    2.250
##    .x4      (.28.)    3.061    0.067   45.542    0.000    3.061
##    .x5      (.29.)    4.341    0.075   58.258    0.000    4.341
##    .x6      (.30.)    2.186    0.063   34.552    0.000    2.186
##    .x7      (.31.)    4.186    0.063   66.544    0.000    4.186
##    .x8      (.32.)    5.527    0.058   94.539    0.000    5.527
##    .x9      (.33.)    5.374    0.058   92.238    0.000    5.374
##     Visual            0.000                             0.000
##     Textual           0.000                             0.000
##     Speed             0.000                             0.000
##   Std.all
##     4.235
##     5.179
##     1.993
##     2.634
##     3.369
##     1.998
##     3.848
##     5.467
##     5.334
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.549    0.157    3.497    0.000    0.549
##    .x2      (.11.)    1.134    0.112   10.101    0.000    1.134
##    .x3      (.12.)    0.844    0.101    8.391    0.000    0.844
##    .x4      (.13.)    0.371    0.050    7.357    0.000    0.371
##    .x5      (.14.)    0.446    0.057    7.844    0.000    0.446
##    .x6      (.15.)    0.356    0.047    7.632    0.000    0.356
##    .x7      (.16.)    0.799    0.098    8.195    0.000    0.799
##    .x8      (.17.)    0.488    0.120    4.066    0.000    0.488
##    .x9      (.18.)    0.566    0.119    4.752    0.000    0.566
##     Visual  (.19.)    0.809    0.181    4.471    0.000    1.000
##     Textual (.20.)    0.979    0.122    8.048    0.000    1.000
##     Speed   (.21.)    0.384    0.107    3.584    0.000    1.000
##   Std.all
##     0.404
##     0.821
##     0.662
##     0.275
```

```
##     0.269
##     0.298
##     0.676
##     0.477
##     0.558
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.596
##     x2                0.179
##     x3                0.338
##     x4                0.725
##     x5                0.731
##     x6                0.702
##     x7                0.324
##     x8                0.523
##     x9                0.442
```

```
anova(config, weak, strong, strict, strict_2, strict_3)

## Chi Square Difference Test
##
##          Df  AIC  BIC Chisq Chisq diff Df diff  Pr(>Chisq)
## config   48 7484 7707   115
## weak     54 7481 7681   123        8.1       6       0.228
## strong   60 7509 7687   163       39.8       6 0.000000500
## strict   69 7508 7653   180       17.3       9       0.044
## strict_2 72 7542 7675   220       39.6       3 0.000000013
## strict_3 78 7535 7647   225        5.6       6       0.473
```

```
measurementInvariance(HS.model, data = HS, group = "school")

##
## Measurement invariance models:
##
## Model 1 : fit.configural
## Model 2 : fit.loadings
## Model 3 : fit.intercepts
## Model 4 : fit.means
##
## Chi Square Difference Test
##
##                Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## fit.configural 48 7484 7707   116
## fit.loadings   54 7481 7681   124        8.2       6       0.22
## fit.intercepts 60 7509 7687   164       40.1       6    4.4e-07
## fit.means      63 7543 7710   205       40.5       3    8.3e-09
##
##
## Fit measures:
```

```
##
##                   cfi rmsea cfi.delta rmsea.delta
## fit.configural 0.923 0.097        NA          NA
## fit.loadings   0.921 0.093     0.002       0.004
## fit.intercepts 0.882 0.107     0.038       0.015
## fit.means      0.840 0.122     0.042       0.015
```

7.2 Over Time, Within Samples

Invariance can also be tested within a sample, over time. The self-esteem data provide a good example. First, to establish configural invariance, a single-factor model is estimated at each time.

```
SE_Model_1 <- "
SE_1 =~ r_1_1 + r_1_2 + r_1_3 + r_1_4 + r_1_5 + r_1_6 + r_1_7 + r_1_8 + r_1_9 + r_1_10
SE_2 =~ r_2_1 + r_2_2 + r_2_3 + r_2_4 + r_2_5 + r_2_6 + r_2_7 + r_2_8 + r_2_9 + r_2_10
SE_1 ~ 0*1
SE_2 ~ 0*1
"
config <- cfa(SE_Model_1, data = SE, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(config, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  51 iterations
##
##   Number of observations                            94
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     545.371     512.917
##   Degrees of freedom                               169         169
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.063
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1469.287    1325.006
##   Degrees of freedom                               190         190
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.706       0.697
##   Tucker-Lewis Index (TLI)                       0.669       0.659
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1540.180   -1540.180
##   Loglikelihood unrestricted model (H1)      -1264.562   -1264.562
```

```
##
##   Number of free parameters                         61          61
##   Akaike (AIC)                                3202.360    3202.360
##   Bayesian (BIC)                              3357.501    3357.501
##   Sample-size adjusted Bayesian (BIC)         3164.924    3164.924
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.155       0.148
##   90 Percent Confidence Interval          0.140  0.169       0.134  0.162
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.084       0.084
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 =~
##     r_1_1             1.000                               0.462
##     r_1_2             0.715    0.117    6.134    0.000    0.330
##     r_1_3             1.032    0.144    7.190    0.000    0.477
##     r_1_4             0.913    0.111    8.215    0.000    0.422
##     r_1_5             1.001    0.196    5.105    0.000    0.462
##     r_1_6             1.189    0.163    7.306    0.000    0.549
##     r_1_7             1.069    0.149    7.193    0.000    0.494
##     r_1_8             1.294    0.282    4.593    0.000    0.598
##     r_1_9             1.226    0.249    4.924    0.000    0.566
##     r_1_10            1.374    0.275    4.993    0.000    0.635
##   SE_2 =~
##     r_2_1             1.000                               0.386
##     r_2_2             0.786    0.091    8.607    0.000    0.303
##     r_2_3             1.093    0.169    6.465    0.000    0.421
##     r_2_4             0.881    0.132    6.668    0.000    0.340
##     r_2_5             1.258    0.265    4.747    0.000    0.485
##     r_2_6             1.329    0.221    6.022    0.000    0.512
##     r_2_7             1.229    0.248    4.962    0.000    0.474
##     r_2_8             1.712    0.300    5.710    0.000    0.660
##     r_2_9             1.752    0.331    5.297    0.000    0.676
##     r_2_10            1.641    0.363    4.516    0.000    0.633
##   Std.all
##
##     0.782
##     0.702
##     0.786
```

```
##     0.682
##     0.563
##     0.793
##     0.746
##     0.606
##     0.639
##     0.673
##
##     0.769
##     0.686
##     0.689
##     0.489
##     0.632
##     0.750
##     0.740
##     0.690
##     0.715
##     0.660
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 ~~
##     SE_2              0.165    0.046    3.614    0.000    0.926
##   Std.all
##
##     0.926
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     SE_1              0.000                               0.000
##     SE_2              0.000                               0.000
##    .r_1_1             3.670    0.061   59.940    0.000    3.670
##    .r_1_2             3.723    0.049   76.318    0.000    3.723
##    .r_1_3             3.596    0.063   57.124    0.000    3.596
##    .r_1_4             3.372    0.064   52.575    0.000    3.372
##    .r_1_5             3.457    0.085   40.622    0.000    3.457
##    .r_1_6             3.202    0.072   44.552    0.000    3.202
##    .r_1_7             3.202    0.069   46.667    0.000    3.202
##    .r_1_8             2.755    0.102   26.951    0.000    2.755
##    .r_1_9             2.638    0.092   28.734    0.000    2.638
##    .r_1_10            3.064    0.098   31.327    0.000    3.064
##    .r_2_1             3.702    0.052   71.163    0.000    3.702
##    .r_2_2             3.734    0.046   81.499    0.000    3.734
##    .r_2_3             3.564    0.063   56.229    0.000    3.564
##    .r_2_4             3.351    0.072   46.493    0.000    3.351
##    .r_2_5             3.479    0.080   43.676    0.000    3.479
##    .r_2_6             3.181    0.071   44.874    0.000    3.181
##    .r_2_7             3.191    0.066   48.058    0.000    3.191
##    .r_2_8             2.904    0.099   29.258    0.000    2.904
##    .r_2_9             2.979    0.098   30.395    0.000    2.979
##    .r_2_10            3.266    0.099   32.861    0.000    3.266
##   Std.all
##     0.000
##     0.000
```

```
##     6.215
##     7.914
##     5.923
##     5.452
##     4.212
##     4.620
##     4.839
##     2.795
##     2.980
##     3.248
##     7.379
##     8.451
##     5.831
##     4.821
##     4.529
##     4.653
##     4.983
##     3.034
##     3.152
##     3.408
##
## Variances:
##                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .r_1_1            0.135    0.031    4.429    0.000    0.135
##    .r_1_2            0.112    0.022    5.147    0.000    0.112
##    .r_1_3            0.141    0.024    5.997    0.000    0.141
##    .r_1_4            0.205    0.030    6.935    0.000    0.205
##    .r_1_5            0.460    0.145    3.173    0.002    0.460
##    .r_1_6            0.179    0.027    6.664    0.000    0.179
##    .r_1_7            0.194    0.027    7.218    0.000    0.194
##    .r_1_8            0.615    0.089    6.888    0.000    0.615
##    .r_1_9            0.463    0.079    5.843    0.000    0.463
##    .r_1_10           0.486    0.104    4.680    0.000    0.486
##    .r_2_1            0.103    0.026    3.896    0.000    0.103
##    .r_2_2            0.103    0.027    3.840    0.000    0.103
##    .r_2_3            0.196    0.034    5.826    0.000    0.196
##    .r_2_4            0.368    0.103    3.559    0.000    0.368
##    .r_2_5            0.355    0.127    2.801    0.005    0.355
##    .r_2_6            0.205    0.031    6.600    0.000    0.205
##    .r_2_7            0.186    0.028    6.691    0.000    0.186
##    .r_2_8            0.480    0.068    7.036    0.000    0.480
##    .r_2_9            0.437    0.069    6.347    0.000    0.437
##    .r_2_10           0.518    0.121    4.293    0.000    0.518
##     SE_1             0.213    0.081    2.632    0.008    1.000
##     SE_2             0.149    0.043    3.418    0.001    1.000
##   Std.all
##     0.388
##     0.507
##     0.383
##     0.535
##     0.683
##     0.372
##     0.443
##     0.633
```

```
##     0.591
##     0.547
##     0.409
##     0.530
##     0.525
##     0.761
##     0.601
##     0.438
##     0.452
##     0.524
##     0.489
##     0.564
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     r_1_1             0.612
##     r_1_2             0.493
##     r_1_3             0.617
##     r_1_4             0.465
##     r_1_5             0.317
##     r_1_6             0.628
##     r_1_7             0.557
##     r_1_8             0.367
##     r_1_9             0.409
##     r_1_10            0.453
##     r_2_1             0.591
##     r_2_2             0.470
##     r_2_3             0.475
##     r_2_4             0.239
##     r_2_5             0.399
##     r_2_6             0.562
##     r_2_7             0.548
##     r_2_8             0.476
##     r_2_9             0.511
##     r_2_10            0.436
```

Across time, the corresponding items are constrained to have the same loadings.

```
SE_Model_2 <- "
SE_1 =~ a*r_1_1 + b*r_1_2 + c*r_1_3 + d*r_1_4 + e*r_1_5 + f*r_1_6 + g*r_1_7 + h*r_1_8 + i*r_1_9 + j*r_1_
SE_2 =~ a*r_2_1 + b*r_2_2 + c*r_2_3 + d*r_2_4 + e*r_2_5 + f*r_2_6 + g*r_2_7 + h*r_2_8 + i*r_2_9 + j*r_2_
SE_1 ~ 0*1
SE_2 ~ 0*1
"
weak <- cfa(SE_Model_2, data = SE, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(weak, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  40 iterations
##
##   Number of observations                            94
##   Number of missing patterns                         1
```

```
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     550.029     525.677
##   Degrees of freedom                               178         178
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.046
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1469.287    1325.006
##   Degrees of freedom                               190         190
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.709       0.694
##   Tucker-Lewis Index (TLI)                       0.690       0.673
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1542.534   -1542.534
##   Loglikelihood unrestricted model (H1)      -1264.562   -1264.562
##
##   Number of free parameters                        52          52
##   Akaike (AIC)                                3189.068    3189.068
##   Bayesian (BIC)                              3321.319    3321.319
##   Sample-size adjusted Bayesian (BIC)         3157.155    3157.155
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.150       0.145
##   90 Percent Confidence Interval          0.136  0.164       0.131  0.159
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.089       0.089
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 =~
```

```
##     r_1_1      (a)    1.000                                    0.430
##     r_1_2      (b)    0.753    0.067   11.190    0.000    0.324
##     r_1_3      (c)    1.076    0.124    8.708    0.000    0.463
##     r_1_4      (d)    0.926    0.101    9.171    0.000    0.399
##     r_1_5      (e)    1.134    0.162    7.010    0.000    0.488
##     r_1_6      (f)    1.264    0.143    8.855    0.000    0.544
##     r_1_7      (g)    1.150    0.158    7.284    0.000    0.495
##     r_1_8      (h)    1.513    0.252    5.999    0.000    0.651
##     r_1_9      (i)    1.480    0.233    6.358    0.000    0.637
##     r_1_10     (j)    1.514    0.271    5.578    0.000    0.651
##   SE_2 =~
##     r_2_1      (a)    1.000                                    0.409
##     r_2_2      (b)    0.753    0.067   11.190    0.000    0.308
##     r_2_3      (c)    1.076    0.124    8.708    0.000    0.440
##     r_2_4      (d)    0.926    0.101    9.171    0.000    0.379
##     r_2_5      (e)    1.134    0.162    7.010    0.000    0.464
##     r_2_6      (f)    1.264    0.143    8.855    0.000    0.517
##     r_2_7      (g)    1.150    0.158    7.284    0.000    0.470
##     r_2_8      (h)    1.513    0.252    5.999    0.000    0.618
##     r_2_9      (i)    1.480    0.233    6.358    0.000    0.605
##     r_2_10     (j)    1.514    0.271    5.578    0.000    0.619
##   Std.all
##
##     0.754
##     0.692
##     0.775
##     0.657
##     0.585
##     0.788
##     0.745
##     0.641
##     0.685
##     0.686
##
##     0.789
##     0.694
##     0.704
##     0.531
##     0.613
##     0.752
##     0.737
##     0.664
##     0.670
##     0.650
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 ~~
##     SE_2              0.164    0.046    3.585    0.000    0.931
##   Std.all
##
##     0.931
##
```

```
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##     SE_1              0.000                               0.000
##     SE_2              0.000                               0.000
##    .r_1_1             3.670    0.061   59.940    0.000    3.670
##    .r_1_2             3.723    0.049   76.318    0.000    3.723
##    .r_1_3             3.596    0.063   57.124    0.000    3.596
##    .r_1_4             3.372    0.064   52.575    0.000    3.372
##    .r_1_5             3.457    0.085   40.622    0.000    3.457
##    .r_1_6             3.202    0.072   44.552    0.000    3.202
##    .r_1_7             3.202    0.069   46.667    0.000    3.202
##    .r_1_8             2.755    0.102   26.951    0.000    2.755
##    .r_1_9             2.638    0.092   28.734    0.000    2.638
##    .r_1_10            3.064    0.098   31.327    0.000    3.064
##    .r_2_1             3.702    0.052   71.163    0.000    3.702
##    .r_2_2             3.734    0.046   81.499    0.000    3.734
##    .r_2_3             3.564    0.063   56.229    0.000    3.564
##    .r_2_4             3.351    0.072   46.493    0.000    3.351
##    .r_2_5             3.479    0.080   43.676    0.000    3.479
##    .r_2_6             3.181    0.071   44.874    0.000    3.181
##    .r_2_7             3.191    0.066   48.058    0.000    3.191
##    .r_2_8             2.904    0.099   29.258    0.000    2.904
##    .r_2_9             2.979    0.098   30.395    0.000    2.979
##    .r_2_10            3.266    0.099   32.861    0.000    3.266
##   Std.all
##     0.000
##     0.000
##     6.432
##     7.947
##     6.021
##     5.561
##     4.145
##     4.641
##     4.823
##     2.713
##     2.840
##     3.227
##     7.147
##     8.416
##     5.707
##     4.697
##     4.597
##     4.629
##     5.001
##     3.116
##     3.298
##     3.432
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .r_1_1             0.141    0.032    4.341    0.000    0.141
##    .r_1_2             0.115    0.022    5.289    0.000    0.115
##    .r_1_3             0.142    0.024    6.030    0.000    0.142
##    .r_1_4             0.209    0.030    6.890    0.000    0.209
```

```
##    .r_1_5             0.458    0.143    3.194    0.001    0.458
##    .r_1_6             0.180    0.026    6.875    0.000    0.180
##    .r_1_7             0.196    0.027    7.268    0.000    0.196
##    .r_1_8             0.608    0.089    6.852    0.000    0.608
##    .r_1_9             0.458    0.079    5.801    0.000    0.458
##    .r_1_10            0.477    0.101    4.721    0.000    0.477
##    .r_2_1             0.101    0.026    3.937    0.000    0.101
##    .r_2_2             0.102    0.026    3.917    0.000    0.102
##    .r_2_3             0.196    0.034    5.805    0.000    0.196
##    .r_2_4             0.366    0.104    3.526    0.000    0.366
##    .r_2_5             0.358    0.124    2.876    0.004    0.358
##    .r_2_6             0.205    0.031    6.625    0.000    0.205
##    .r_2_7             0.186    0.027    6.788    0.000    0.186
##    .r_2_8             0.486    0.069    7.021    0.000    0.486
##    .r_2_9             0.449    0.069    6.529    0.000    0.449
##    .r_2_10            0.523    0.119    4.391    0.000    0.523
##     SE_1              0.185    0.058    3.202    0.001    1.000
##     SE_2              0.167    0.043    3.925    0.000    1.000
##   Std.all
##     0.432
##     0.522
##     0.399
##     0.568
##     0.658
##     0.379
##     0.445
##     0.589
##     0.530
##     0.529
##     0.377
##     0.518
##     0.504
##     0.718
##     0.624
##     0.434
##     0.457
##     0.560
##     0.551
##     0.577
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     r_1_1             0.568
##     r_1_2             0.478
##     r_1_3             0.601
##     r_1_4             0.432
##     r_1_5             0.342
##     r_1_6             0.621
##     r_1_7             0.555
##     r_1_8             0.411
##     r_1_9             0.470
##     r_1_10            0.471
```

```
##     r_2_1             0.623
##     r_2_2             0.482
##     r_2_3             0.496
##     r_2_4             0.282
##     r_2_5             0.376
##     r_2_6             0.566
##     r_2_7             0.543
##     r_2_8             0.440
##     r_2_9             0.449
##     r_2_10            0.423
```

Next the corresponding items are constrained to have the same intercepts.

```
SE_Model_3 <- "
SE_1 =~ a*r_1_1 + b*r_1_2 + c*r_1_3 + d*r_1_4 + e*r_1_5 + f*r_1_6 + g*r_1_7 + h*r_1_8 + i*r_1_9 + j*r_1_
SE_2 =~ a*r_2_1 + b*r_2_2 + c*r_2_3 + d*r_2_4 + e*r_2_5 + f*r_2_6 + g*r_2_7 + h*r_2_8 + i*r_2_9 + j*r_2_
r_1_1 ~ aa*1
r_2_1 ~ aa*1
r_1_2 ~ bb*1
r_2_2 ~ bb*1
r_1_3 ~ cc*1
r_2_3 ~ cc*1
r_1_4 ~ dd*1
r_2_4 ~ dd*1
r_1_5 ~ ee*1
r_2_5 ~ ee*1
r_1_6 ~ ff*1
r_2_6 ~ ff*1
r_1_7 ~ gg*1
r_2_7 ~ gg*1
r_1_8 ~ hh*1
r_2_8 ~ hh*1
r_1_9 ~ ii*1
r_2_9 ~ ii*1
r_1_10 ~ jj*1
r_2_10 ~ jj*1
SE_1 ~ 0*1
SE_2 ~ 0*1
"
strong <- cfa(SE_Model_3, data = SE, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(strong, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  65 iterations
##
##   Number of observations                            94
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     566.714     550.388
##   Degrees of freedom                               188         188
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.030
##     for the Yuan-Bentler correction (Mplus variant)
```

```
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1469.287    1325.006
##   Degrees of freedom                               190         190
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.704       0.681
##   Tucker-Lewis Index (TLI)                       0.701       0.677
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1550.966   -1550.966
##   Loglikelihood unrestricted model (H1)      -1264.562   -1264.562
##
##   Number of free parameters                         42          42
##   Akaike (AIC)                                3185.933    3185.933
##   Bayesian (BIC)                              3292.751    3292.751
##   Sample-size adjusted Bayesian (BIC)         3160.157    3160.157
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.147       0.144
##   90 Percent Confidence Interval          0.133  0.161       0.130  0.158
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.092       0.092
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 =~
##     r_1_1      (a)    1.000                               0.433
##     r_1_2      (b)    0.753    0.068   11.161    0.000    0.326
##     r_1_3      (c)    1.070    0.123    8.690    0.000    0.463
##     r_1_4      (d)    0.926    0.101    9.212    0.000    0.401
##     r_1_5      (e)    1.128    0.162    6.975    0.000    0.488
##     r_1_6      (f)    1.259    0.143    8.808    0.000    0.545
##     r_1_7      (g)    1.146    0.158    7.259    0.000    0.496
```

```
##     r_1_8      (h)    1.505    0.251    6.006    0.000    0.652
##     r_1_9      (i)    1.469    0.229    6.422    0.000    0.636
##     r_1_10     (j)    1.502    0.268    5.605    0.000    0.650
##   SE_2 =~
##     r_2_1      (a)    1.000                               0.410
##     r_2_2      (b)    0.753    0.068   11.161    0.000    0.309
##     r_2_3      (c)    1.070    0.123    8.690    0.000    0.439
##     r_2_4      (d)    0.926    0.101    9.212    0.000    0.380
##     r_2_5      (e)    1.128    0.162    6.975    0.000    0.463
##     r_2_6      (f)    1.259    0.143    8.808    0.000    0.516
##     r_2_7      (g)    1.146    0.158    7.259    0.000    0.470
##     r_2_8      (h)    1.505    0.251    6.006    0.000    0.617
##     r_2_9      (i)    1.469    0.229    6.422    0.000    0.602
##     r_2_10     (j)    1.502    0.268    5.605    0.000    0.616
##   Std.all
##
##     0.758
##     0.696
##     0.775
##     0.661
##     0.585
##     0.790
##     0.747
##     0.639
##     0.672
##     0.681
##
##     0.792
##     0.697
##     0.702
##     0.532
##     0.612
##     0.751
##     0.737
##     0.661
##     0.656
##     0.644
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 ~~
##     SE_2              0.164    0.046    3.597    0.000    0.926
##   Std.all
##
##     0.926
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .r_1_1     (aa)    3.693    0.050   74.423    0.000    3.693
##    .r_2_1     (aa)    3.693    0.050   74.423    0.000    3.693
##    .r_1_2     (bb)    3.733    0.041   91.257    0.000    3.733
##    .r_2_2     (bb)    3.733    0.041   91.257    0.000    3.733
##    .r_1_3     (cc)    3.589    0.056   63.898    0.000    3.589
```

```
##    .r_2_3     (cc)    3.589    0.056   63.898    0.000    3.589
##    .r_1_4     (dd)    3.372    0.056   59.732    0.000    3.372
##    .r_2_4     (dd)    3.372    0.056   59.732    0.000    3.372
##    .r_1_5     (ee)    3.475    0.066   52.456    0.000    3.475
##    .r_2_5     (ee)    3.475    0.066   52.456    0.000    3.475
##    .r_1_6     (ff)    3.200    0.063   50.756    0.000    3.200
##    .r_2_6     (ff)    3.200    0.063   50.756    0.000    3.200
##    .r_1_7     (gg)    3.203    0.058   55.628    0.000    3.203
##    .r_2_7     (gg)    3.203    0.058   55.628    0.000    3.203
##    .r_1_8     (hh)    2.845    0.096   29.731    0.000    2.845
##    .r_2_8     (hh)    2.845    0.096   29.731    0.000    2.845
##    .r_1_9     (ii)    2.818    0.093   30.353    0.000    2.818
##    .r_2_9     (ii)    2.818    0.093   30.353    0.000    2.818
##    .r_1_10    (jj)    3.169    0.093   34.226    0.000    3.169
##    .r_2_10    (jj)    3.169    0.093   34.226    0.000    3.169
##     SE_1              0.000                             0.000
##     SE_2              0.000                             0.000
##   Std.all
##     6.464
##     7.131
##     7.960
##     8.418
##     6.006
##     5.749
##     5.557
##     4.725
##     4.162
##     4.594
##     4.636
##     4.655
##     4.822
##     5.019
##     2.789
##     3.048
##     2.980
##     3.071
##     3.317
##     3.314
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .r_1_1             0.139    0.032    4.304    0.000    0.139
##    .r_1_2             0.114    0.022    5.274    0.000    0.114
##    .r_1_3             0.143    0.023    6.116    0.000    0.143
##    .r_1_4             0.207    0.030    6.932    0.000    0.207
##    .r_1_5             0.458    0.144    3.182    0.001    0.458
##    .r_1_6             0.179    0.026    6.768    0.000    0.179
##    .r_1_7             0.195    0.027    7.263    0.000    0.195
##    .r_1_8             0.616    0.091    6.784    0.000    0.616
##    .r_1_9             0.490    0.086    5.728    0.000    0.490
##    .r_1_10            0.490    0.105    4.675    0.000    0.490
```

```
##    .r_2_1             0.100    0.026    3.865    0.000    0.100
##    .r_2_2             0.101    0.026    3.862    0.000    0.101
##    .r_2_3             0.198    0.035    5.639    0.000    0.198
##    .r_2_4             0.365    0.105    3.482    0.000    0.365
##    .r_2_5             0.358    0.125    2.874    0.004    0.358
##    .r_2_6             0.206    0.032    6.490    0.000    0.206
##    .r_2_7             0.186    0.028    6.688    0.000    0.186
##    .r_2_8             0.491    0.070    7.020    0.000    0.491
##    .r_2_9             0.480    0.072    6.703    0.000    0.480
##    .r_2_10            0.535    0.118    4.522    0.000    0.535
##     SE_1              0.187    0.059    3.201    0.001    1.000
##     SE_2              0.168    0.043    3.950    0.000    1.000
##   Std.all
##     0.426
##     0.516
##     0.400
##     0.563
##     0.658
##     0.376
##     0.442
##     0.592
##     0.548
##     0.537
##     0.373
##     0.515
##     0.507
##     0.717
##     0.626
##     0.436
##     0.457
##     0.563
##     0.569
##     0.585
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     r_1_1             0.574
##     r_1_2             0.484
##     r_1_3             0.600
##     r_1_4             0.437
##     r_1_5             0.342
##     r_1_6             0.624
##     r_1_7             0.558
##     r_1_8             0.408
##     r_1_9             0.452
##     r_1_10            0.463
##     r_2_1             0.627
##     r_2_2             0.485
##     r_2_3             0.493
##     r_2_4             0.283
##     r_2_5             0.374
##     r_2_6             0.564
```

```
##     r_2_7             0.543
##     r_2_8             0.437
##     r_2_9             0.431
##     r_2_10            0.415
```

Finally, the corresponding items are constrained to have the same loadings, the same intercepts, and their error variances are constrained to be equal as well.

```
SE_Model_4 <- "
SE_1 =~ a*r_1_1 + b*r_1_2 + c*r_1_3 + d*r_1_4 + e*r_1_5 + f*r_1_6 + g*r_1_7 + h*r_1_8 + i*r_1_9 + j*r_1
SE_2 =~ a*r_2_1 + b*r_2_2 + c*r_2_3 + d*r_2_4 + e*r_2_5 + f*r_2_6 + g*r_2_7 + h*r_2_8 + i*r_2_9 + j*r_2
r_1_1 ~ aa*1
r_2_1 ~ aa*1
r_1_2 ~ bb*1
r_2_2 ~ bb*1
r_1_3 ~ cc*1
r_2_3 ~ cc*1
r_1_4 ~ dd*1
r_2_4 ~ dd*1
r_1_5 ~ ee*1
r_2_5 ~ ee*1
r_1_6 ~ ff*1
r_2_6 ~ ff*1
r_1_7 ~ gg*1
r_2_7 ~ gg*1
r_1_8 ~ hh*1
r_2_8 ~ hh*1
r_1_9 ~ ii*1
r_2_9 ~ ii*1
r_1_10 ~ jj*1
r_2_10 ~ jj*1
r_1_1 ~~ aaa*r_1_1
r_2_1 ~~ aaa*r_2_1
r_1_2 ~~ bbb*r_1_2
r_2_2 ~~ bbb*r_2_2
r_1_3 ~~ ccc*r_1_3
r_2_3 ~~ ccc*r_2_3
r_1_4 ~~ ddd*r_1_4
r_2_4 ~~ ddd*r_2_4
r_1_5 ~~ eee*r_1_5
r_2_5 ~~ eee*r_2_5
r_1_6 ~~ fff*r_1_6
r_2_6 ~~ fff*r_2_6
r_1_7 ~~ ggg*r_1_7
r_2_7 ~~ ggg*r_2_7
r_1_8 ~~ hhh*r_1_8
r_2_8 ~~ hhh*r_2_8
r_1_9 ~~ iii*r_1_9
r_2_9 ~~ iii*r_2_9
r_1_10 ~~ jjj*r_1_10
r_2_10 ~~ jjj*r_2_10
SE_1 ~ 0*1
SE_2 ~ 1
"
```

```
strict_1 <- cfa(SE_Model_4, data = SE, missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
summary(strict_1, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)

## lavaan (0.6-1) converged normally after  75 iterations
##
##   Number of observations                            94
##   Number of missing patterns                         1
##
##   Estimator                                         ML      Robust
##   Model Fit Test Statistic                     578.691     556.710
##   Degrees of freedom                               197         197
##   P-value (Chi-square)                           0.000       0.000
##   Scaling correction factor                                  1.039
##     for the Yuan-Bentler correction (Mplus variant)
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             1469.287    1325.006
##   Degrees of freedom                               190         190
##   P-value                                        0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                    0.702       0.683
##   Tucker-Lewis Index (TLI)                       0.712       0.694
##
##   Robust Comparative Fit Index (CFI)                            NA
##   Robust Tucker-Lewis Index (TLI)                               NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -1557.019   -1557.019
##   Loglikelihood unrestricted model (H1)      -1264.562   -1264.562
##
##   Number of free parameters                         33          33
##   Akaike (AIC)                                3180.039    3180.039
##   Bayesian (BIC)                              3263.967    3263.967
##   Sample-size adjusted Bayesian (BIC)         3159.786    3159.786
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.144       0.140
##   90 Percent Confidence Interval          0.131  0.158       0.127  0.154
##   P-value RMSEA <= 0.05                          0.000       0.000
##
##   Robust RMSEA                                                  NA
##   90 Percent Confidence Interval                                NA     NA
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.092       0.092
##
## Parameter Estimates:
```

```
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                   Robust.huber.white
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 =~
##     r_1_1      (a)    1.000                               0.433
##     r_1_2      (b)    0.743    0.074   10.105    0.000    0.322
##     r_1_3      (c)    1.061    0.125    8.469    0.000    0.459
##     r_1_4      (d)    0.895    0.103    8.683    0.000    0.388
##     r_1_5      (e)    1.124    0.157    7.167    0.000    0.487
##     r_1_6      (f)    1.251    0.143    8.759    0.000    0.542
##     r_1_7      (g)    1.140    0.150    7.582    0.000    0.494
##     r_1_8      (h)    1.493    0.253    5.901    0.000    0.646
##     r_1_9      (i)    1.485    0.231    6.420    0.000    0.643
##     r_1_10     (j)    1.515    0.268    5.662    0.000    0.656
##   SE_2 =~
##     r_2_1      (a)    1.000                               0.411
##     r_2_2      (b)    0.743    0.074   10.105    0.000    0.306
##     r_2_3      (c)    1.061    0.125    8.469    0.000    0.436
##     r_2_4      (d)    0.895    0.103    8.683    0.000    0.368
##     r_2_5      (e)    1.124    0.157    7.167    0.000    0.462
##     r_2_6      (f)    1.251    0.143    8.759    0.000    0.515
##     r_2_7      (g)    1.140    0.150    7.582    0.000    0.469
##     r_2_8      (h)    1.493    0.253    5.901    0.000    0.614
##     r_2_9      (i)    1.485    0.231    6.420    0.000    0.611
##     r_2_10     (j)    1.515    0.268    5.662    0.000    0.623
##   Std.all
##
##     0.779
##     0.697
##     0.744
##     0.584
##     0.607
##     0.776
##     0.748
##     0.657
##     0.683
##     0.679
##
##     0.763
##     0.678
##     0.727
##     0.565
##     0.587
##     0.760
##     0.731
##     0.637
##     0.664
##     0.660
##
## Covariances:
```

```
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   SE_1 ~~
##     SE_2              0.166    0.048    3.472    0.001    0.934
##   Std.all
##
##     0.934
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .r_1_1     (aa)    3.668    0.055   66.963    0.000    3.668
##    .r_2_1     (aa)    3.668    0.055   66.963    0.000    3.668
##    .r_1_2     (bb)    3.715    0.043   86.143    0.000    3.715
##    .r_2_2     (bb)    3.715    0.043   86.143    0.000    3.715
##    .r_1_3     (cc)    3.560    0.060   59.096    0.000    3.560
##    .r_2_3     (cc)    3.560    0.060   59.096    0.000    3.560
##    .r_1_4     (dd)    3.345    0.060   55.309    0.000    3.345
##    .r_2_4     (dd)    3.345    0.060   55.309    0.000    3.345
##    .r_1_5     (ee)    3.447    0.070   49.204    0.000    3.447
##    .r_2_5     (ee)    3.447    0.070   49.204    0.000    3.447
##    .r_1_6     (ff)    3.168    0.068   46.456    0.000    3.168
##    .r_2_6     (ff)    3.168    0.068   46.456    0.000    3.168
##    .r_1_7     (gg)    3.176    0.064   49.556    0.000    3.176
##    .r_2_7     (gg)    3.176    0.064   49.556    0.000    3.176
##    .r_1_8     (hh)    2.802    0.098   28.696    0.000    2.802
##    .r_2_8     (hh)    2.802    0.098   28.696    0.000    2.802
##    .r_1_9     (ii)    2.781    0.088   31.438    0.000    2.781
##    .r_2_9     (ii)    2.781    0.088   31.438    0.000    2.781
##    .r_1_10    (jj)    3.137    0.092   34.025    0.000    3.137
##    .r_2_10    (jj)    3.137    0.092   34.025    0.000    3.137
##     SE_1              0.000                               0.000
##     SE_2              0.037    0.026    1.412    0.158    0.091
##   Std.all
##     6.596
##     6.799
##     8.046
##     8.243
##     5.767
##     5.928
##     5.043
##     5.128
##     4.299
##     4.378
##     4.537
##     4.675
##     4.813
##     4.949
##     2.846
##     2.907
##     2.953
##     3.022
##     3.245
##     3.320
##     0.000
```

```
##     0.091
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .r_1_1    (aaa)    0.122    0.025    4.894    0.000    0.122
##    .r_2_1    (aaa)    0.122    0.025    4.894    0.000    0.122
##    .r_1_2    (bbb)    0.110    0.020    5.540    0.000    0.110
##    .r_2_2    (bbb)    0.110    0.020    5.540    0.000    0.110
##    .r_1_3    (ccc)    0.170    0.023    7.479    0.000    0.170
##    .r_2_3    (ccc)    0.170    0.023    7.479    0.000    0.170
##    .r_1_4    (ddd)    0.290    0.059    4.922    0.000    0.290
##    .r_2_4    (ddd)    0.290    0.059    4.922    0.000    0.290
##    .r_1_5    (eee)    0.406    0.094    4.338    0.000    0.406
##    .r_2_5    (eee)    0.406    0.094    4.338    0.000    0.406
##    .r_1_6    (fff)    0.194    0.021    9.143    0.000    0.194
##    .r_2_6    (fff)    0.194    0.021    9.143    0.000    0.194
##    .r_1_7    (ggg)    0.192    0.021    9.153    0.000    0.192
##    .r_2_7    (ggg)    0.192    0.021    9.153    0.000    0.192
##    .r_1_8    (hhh)    0.551    0.069    8.048    0.000    0.551
##    .r_2_8    (hhh)    0.551    0.069    8.048    0.000    0.551
##    .r_1_9    (iii)    0.474    0.069    6.835    0.000    0.474
##    .r_2_9    (iii)    0.474    0.069    6.835    0.000    0.474
##    .r_1_10   (jjj)    0.504    0.093    5.422    0.000    0.504
##    .r_2_10   (jjj)    0.504    0.093    5.422    0.000    0.504
##     SE_1              0.187    0.061    3.082    0.002    1.000
##     SE_2              0.169    0.044    3.852    0.000    1.000
##   Std.all
##     0.394
##     0.418
##     0.514
##     0.540
##     0.447
##     0.472
##     0.659
##     0.681
##     0.632
##     0.655
##     0.398
##     0.423
##     0.440
##     0.465
##     0.569
##     0.594
##     0.534
##     0.559
##     0.539
##     0.565
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     r_1_1             0.606
##     r_2_1             0.582
```

```
##     r_1_2             0.486
##     r_2_2             0.460
##     r_1_3             0.553
##     r_2_3             0.528
##     r_1_4             0.341
##     r_2_4             0.319
##     r_1_5             0.368
##     r_2_5             0.345
##     r_1_6             0.602
##     r_2_6             0.577
##     r_1_7             0.560
##     r_2_7             0.535
##     r_1_8             0.431
##     r_2_8             0.406
##     r_1_9             0.466
##     r_2_9             0.441
##     r_1_10            0.461
##     r_2_10            0.435

anova(config, weak, strong, strict_1)

## Chi Square Difference Test
##
##           Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config   169 3202 3358   545
## weak     178 3189 3321   550       4.66       9      0.863
## strong   188 3186 3293   567      16.69      10      0.082
## strict_1 197 3180 3264   579      11.98       9      0.215

anova(config, weak)

## Chi Square Difference Test
##
##         Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config 169 3202 3358   545
## weak   178 3189 3321   550       4.66       9       0.86

anova(config, strong)

## Chi Square Difference Test
##
##         Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config 169 3202 3358   545
## strong 188 3186 3293   567       21.3      19       0.32

anova(config, strict_1)

## Chi Square Difference Test
##
##           Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config   169 3202 3358   545
## strict_1 197 3180 3264   579       33.3      28       0.22
```

Across time, the corresponding items are constrained to have the same loadings, the same intercepts, and their error variances are constrained to be equal as well. In addi-

tion, the latent means are constrained to be equal (and 0).

```
SE_Model_5 <- "
SE_1 =~ a*r_1_1 + b*r_1_2 + c*r_1_3 + d*r_1_4 + e*r_1_5 + f*r_1_6 + g*r_1_7 + h*r_1_8 + i*r_1_9 + j*r_1_
SE_2 =~ a*r_2_1 + b*r_2_2 + c*r_2_3 + d*r_2_4 + e*r_2_5 + f*r_2_6 + g*r_2_7 + h*r_2_8 + i*r_2_9 + j*r_2_
r_1_1 ~ aa*1
r_2_1 ~ aa*1
r_1_2 ~ bb*1
r_2_2 ~ bb*1
r_1_3 ~ cc*1
r_2_3 ~ cc*1
r_1_4 ~ dd*1
r_2_4 ~ dd*1
r_1_5 ~ ee*1
r_2_5 ~ ee*1
r_1_6 ~ ff*1
r_2_6 ~ ff*1
r_1_7 ~ gg*1
r_2_7 ~ gg*1
r_1_8 ~ hh*1
r_2_8 ~ hh*1
r_1_9 ~ ii*1
r_2_9 ~ ii*1
r_1_10 ~ jj*1
r_2_10 ~ jj*1
r_1_1 ~~ aaa*r_1_1
r_2_1 ~~ aaa*r_2_1
r_1_2 ~~ bbb*r_1_2
r_2_2 ~~ bbb*r_2_2
r_1_3 ~~ ccc*r_1_3
r_2_3 ~~ ccc*r_2_3
r_1_4 ~~ ddd*r_1_4
r_2_4 ~~ ddd*r_2_4
r_1_5 ~~ eee*r_1_5
r_2_5 ~~ eee*r_2_5
r_1_6 ~~ fff*r_1_6
r_2_6 ~~ fff*r_2_6
r_1_7 ~~ ggg*r_1_7
r_2_7 ~~ ggg*r_2_7
r_1_8 ~~ hhh*r_1_8
r_2_8 ~~ hhh*r_2_8
r_1_9 ~~ iii*r_1_9
r_2_9 ~~ iii*r_2_9
r_1_10 ~~ jjj*r_1_10
r_2_10 ~~ jjj*r_2_10
SE_1 ~ 0*1
SE_2 ~ 0*1
"
strict_2 <- cfa(SE_Model_5, data = SE[, 2:21], missing = "ML", estimator = "MLR",
    likelihood = "wishart", representation = "LISREL")
```

```
## Error in `[.data.frame`(SE, , 2:21):  undefined columns selected
```

```
summary(strict_2, standardized = TRUE, rsq = TRUE, fit.measures = TRUE)
```

```
## lavaan (0.6-1) converged normally after  57 iterations
```

```
##
##   Number of observations per group
##   Pasteur                                         156
##   Grant-White                                     145
##   Number of missing patterns per group
##   Pasteur                                           1
##   Grant-White                                       1
##
##   Estimator                                        ML      Robust
##   Model Fit Test Statistic                    219.863     219.162
##   Degrees of freedom                               72          72
##   P-value (Chi-square)                          0.000       0.000
##   Scaling correction factor                                 1.003
##     for the Yuan-Bentler correction (Mplus variant)
##
## Chi-square for each group:
##
##   Pasteur                                     110.637     110.284
##   Grant-White                                 109.226     108.878
##
## Model test baseline model:
##
##   Minimum Function Test Statistic             951.384     928.080
##   Degrees of freedom                               72          72
##   P-value                                       0.000       0.000
##
## User model versus baseline model:
##
##   Comparative Fit Index (CFI)                   0.832       0.828
##   Tucker-Lewis Index (TLI)                      0.832       0.828
##
##   Robust Comparative Fit Index (CFI)                           NA
##   Robust Tucker-Lewis Index (TLI)                              NA
##
## Loglikelihood and Information Criteria:
##
##   Loglikelihood user model (H0)              -3734.939   -3734.939
##   Loglikelihood unrestricted model (H1)      -3624.272   -3624.272
##
##   Number of free parameters                         36          36
##   Akaike (AIC)                                7541.879    7541.879
##   Bayesian (BIC)                              7675.335    7675.335
##   Sample-size adjusted Bayesian (BIC)         7561.163    7561.163
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                         0.117       0.117
##   90 Percent Confidence Interval          0.100 0.135       0.099 0.135
##   P-value RMSEA <= 0.05                         0.000       0.000
##
##   Robust RMSEA                                                 NA
##   90 Percent Confidence Interval                               NA     NA
##
## Standardized Root Mean Square Residual:
```

```
##
##   SRMR                                           0.114       0.114
##
## Parameter Estimates:
##
##   Information                                 Observed
##   Observed information based on                Hessian
##   Standard Errors                    Robust.huber.white
##
##
## Group 1 [Pasteur]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.914
##     x2      (.p2.)    0.571    0.144    3.967    0.000    0.521
##     x3      (.p3.)    0.753    0.166    4.535    0.000    0.688
##   Textual =~
##     x4                1.000                               0.993
##     x5      (.p5.)    1.113    0.066   16.761    0.000    1.106
##     x6      (.p6.)    0.926    0.062   14.952    0.000    0.920
##   Speed =~
##     x7                1.000                               0.563
##     x8      (.p8.)    1.187    0.130    9.111    0.000    0.668
##     x9      (.p9.)    1.084    0.265    4.092    0.000    0.610
##   Std.all
##
##     0.770
##     0.441
##     0.602
##
##     0.852
##     0.856
##     0.839
##
##     0.533
##     0.693
##     0.630
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.385    0.154    2.494    0.013    0.424
##     Speed             0.181    0.069    2.621    0.009    0.351
##   Textual ~~
##     Speed             0.150    0.062    2.444    0.015    0.269
##   Std.all
##
##     0.424
##     0.351
##
##     0.269
##
```

```
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    4.939    0.077   64.457    0.000    4.939
##    .x2      (.26.)    6.090    0.069   88.522    0.000    6.090
##    .x3      (.27.)    2.253    0.071   31.566    0.000    2.253
##    .x4      (.28.)    3.070    0.073   42.211    0.000    3.070
##    .x5      (.29.)    4.350    0.083   52.704    0.000    4.350
##    .x6      (.30.)    2.194    0.070   31.346    0.000    2.194
##    .x7      (.31.)    4.201    0.067   62.871    0.000    4.201
##    .x8      (.32.)    5.545    0.066   84.185    0.000    5.545
##    .x9      (.33.)    5.391    0.061   88.256    0.000    5.391
##     Visual            0.000                             0.000
##     Textual           0.000                             0.000
##     Speed             0.000                             0.000
##   Std.all
##     4.162
##     5.150
##     1.973
##     2.635
##     3.368
##     2.001
##     3.972
##     5.746
##     5.562
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.574    0.172    3.333    0.001    0.574
##    .x2      (.11.)    1.126    0.114    9.842    0.000    1.126
##    .x3      (.12.)    0.830    0.110    7.539    0.000    0.830
##    .x4      (.13.)    0.371    0.051    7.348    0.000    0.371
##    .x5      (.14.)    0.446    0.058    7.628    0.000    0.446
##    .x6      (.15.)    0.357    0.048    7.418    0.000    0.357
##    .x7      (.16.)    0.802    0.096    8.374    0.000    0.802
##    .x8      (.17.)    0.485    0.115    4.202    0.000    0.485
##    .x9      (.18.)    0.567    0.119    4.779    0.000    0.567
##     Visual            0.835    0.219    3.816    0.000    1.000
##     Textual           0.986    0.151    6.517    0.000    1.000
##     Speed             0.317    0.100    3.180    0.001    1.000
##   Std.all
##     0.407
##     0.805
##     0.637
##     0.273
##     0.267
##     0.297
##     0.716
##     0.520
##     0.603
##     1.000
##     1.000
```

```
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.593
##     x2                0.195
##     x3                0.363
##     x4                0.727
##     x5                0.733
##     x6                0.703
##     x7                0.284
##     x8                0.480
##     x9                0.397
##
##
## Group 2 [Grant-White]:
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual =~
##     x1                1.000                               0.855
##     x2      (.p2.)    0.571    0.144    3.967    0.000    0.488
##     x3      (.p3.)    0.753    0.166    4.535    0.000    0.643
##   Textual =~
##     x4                1.000                               0.986
##     x5      (.p5.)    1.113    0.066   16.761    0.000    1.098
##     x6      (.p6.)    0.926    0.062   14.952    0.000    0.913
##   Speed =~
##     x7                1.000                               0.672
##     x8      (.p8.)    1.187    0.130    9.111    0.000    0.797
##     x9      (.p9.)    1.084    0.265    4.092    0.000    0.728
##   Std.all
##
##     0.748
##     0.418
##     0.577
##
##     0.851
##     0.854
##     0.837
##
##     0.600
##     0.753
##     0.695
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##   Visual ~~
##     Textual           0.417    0.114    3.670    0.000    0.494
##     Speed             0.342    0.093    3.664    0.000    0.596
##   Textual ~~
##     Speed             0.197    0.099    1.996    0.046    0.297
##   Std.all
##
```

```
##     0.494
##     0.596
##
##     0.297
##
## Intercepts:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.25.)    4.939    0.077   64.457    0.000    4.939
##    .x2      (.26.)    6.090    0.069   88.522    0.000    6.090
##    .x3      (.27.)    2.253    0.071   31.566    0.000    2.253
##    .x4      (.28.)    3.070    0.073   42.211    0.000    3.070
##    .x5      (.29.)    4.350    0.083   52.704    0.000    4.350
##    .x6      (.30.)    2.194    0.070   31.346    0.000    2.194
##    .x7      (.31.)    4.201    0.067   62.871    0.000    4.201
##    .x8      (.32.)    5.545    0.066   84.185    0.000    5.545
##    .x9      (.33.)    5.391    0.061   88.256    0.000    5.391
##     Visual             0.000                              0.000
##     Textual            0.000                              0.000
##     Speed              0.000                              0.000
##   Std.all
##     4.324
##     5.214
##     2.020
##     2.649
##     3.386
##     2.011
##     3.754
##     5.240
##     5.148
##     0.000
##     0.000
##     0.000
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv
##    .x1      (.10.)    0.574    0.172    3.333    0.001    0.574
##    .x2      (.11.)    1.126    0.114    9.842    0.000    1.126
##    .x3      (.12.)    0.830    0.110    7.539    0.000    0.830
##    .x4      (.13.)    0.371    0.051    7.348    0.000    0.371
##    .x5      (.14.)    0.446    0.058    7.628    0.000    0.446
##    .x6      (.15.)    0.357    0.048    7.418    0.000    0.357
##    .x7      (.16.)    0.802    0.096    8.374    0.000    0.802
##    .x8      (.17.)    0.485    0.115    4.202    0.000    0.485
##    .x9      (.18.)    0.567    0.119    4.779    0.000    0.567
##     Visual             0.731    0.222    3.296    0.001    1.000
##     Textual            0.972    0.162    5.990    0.000    1.000
##     Speed              0.451    0.130    3.479    0.001    1.000
##   Std.all
##     0.440
##     0.826
##     0.667
##     0.276
##     0.270
##     0.300
```

```
##     0.640
##     0.433
##     0.517
##     1.000
##     1.000
##     1.000
##
## R-Square:
##                    Estimate
##     x1                0.560
##     x2                0.174
##     x3                0.333
##     x4                0.724
##     x5                0.730
##     x6                0.700
##     x7                0.360
##     x8                0.567
##     x9                0.483

anova(config, weak, strong, strict_1, strict_2)

## Warning in lavTestLRT(object = new("lavaan", version = "0.6.1", call = lavaan::lavaan(model
= SE_Model_1, :  lavaan WARNING: some models are based on a different set of observed variables

## Chi Square Difference Test
##
##           Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## strict_2  72 7542 7675   220
## config   169 3202 3358   545        326      97     <2e-16
## weak     178 3189 3321   550          5       9      0.863
## strong   188 3186 3293   567         17      10      0.082
## strict_1 197 3180 3264   579         12       9      0.215

anova(config, weak)

## Chi Square Difference Test
##
##         Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config 169 3202 3358   545
## weak   178 3189 3321   550       4.66       9       0.86

anova(config, strong)

## Chi Square Difference Test
##
##         Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config 169 3202 3358   545
## strong 188 3186 3293   567       21.3      19       0.32

anova(config, strict_1)

## Chi Square Difference Test
##
##           Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## config   169 3202 3358   545
## strict_1 197 3180 3264   579       33.3      28       0.22
```

```r
anova(config, strict_2)
```

```
## Warning in lavTestLRT(object = new("lavaan", version = "0.6.1", call = lavaan::lavaan(model
= SE_Model_1, :  lavaan WARNING: some models are based on a different set of observed variables
```

```
## Chi Square Difference Test
##
##           Df  AIC  BIC Chisq Chisq diff Df diff Pr(>Chisq)
## strict_2  72 7542 7675   220
## config   169 3202 3358   545        326      97     <2e-16
```