Exploratory Factor Analysis

Today . . .

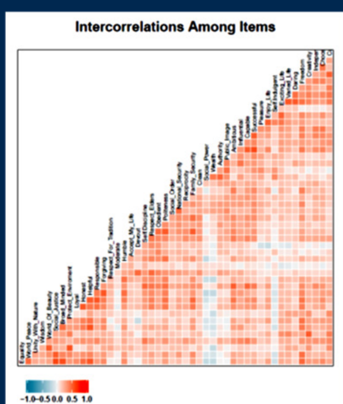
- Examples
- Method comparison

The example data set comes from a sample of 538 university students who completed the Schwartz Values Inventory (1992). Participants rated the importance of 46 values representing 10 basic groups of values:

- Universalism
- Benevolence
- Tradition
- Conformity
- Security
- Power
- Achievement
- Hedonism
- Stimulation
- Self-Direction

Please rate the importance of these values as guiding principles in your life using the 7-point scale below.

- 1 = not at all important
- 2 = slightly important
- 3 = somewhat important
- 4 = moderately important
- 5 = very important
- 6 = extremely important
- 7 = completely important



The correlation heat map is a bit of a Rorschach. One big factor? Four, perhaps correlated, factors?

Factor rotation might provide some help.

The analysis begins in the same way as principal components analysis. It would make little sense to search for common factors in an identity matrix:

```
Kaiser-Meyer-Olkin factor adequacy
Call: KMO(r = SVI[, 2:47])
Overall MSA =  0.92
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```
cortest.bartlett(SVI[, 2:47])

$chisq
[1] 10344

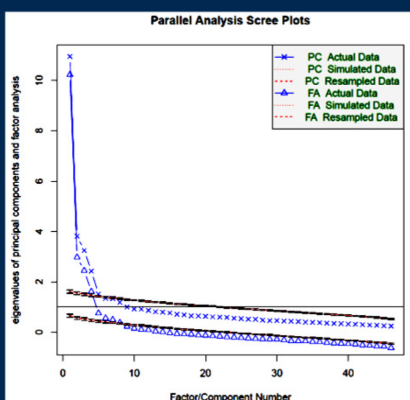
$p.value
[1] 0

$df
[1] 1035
```

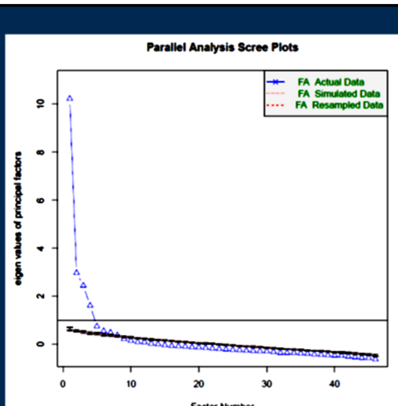
Rotation takes place in a subspace defined by the number of factors retained. Different “meanings” will result with different numbers of factors extracted.

Factors can be expected to replicate and hold their meaning to the extent they are dominating the correlation matrix, defined strongly by relatively large numbers of variables.

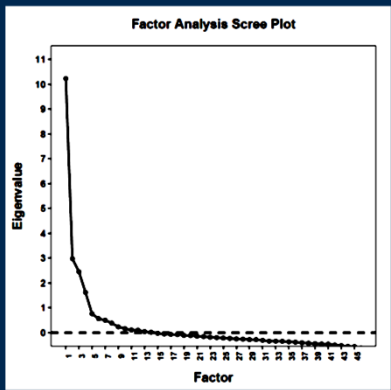
Generally it is a good idea to be conservative in factor extraction unless cross-validation will be conducted to verify the presence of weak factors.



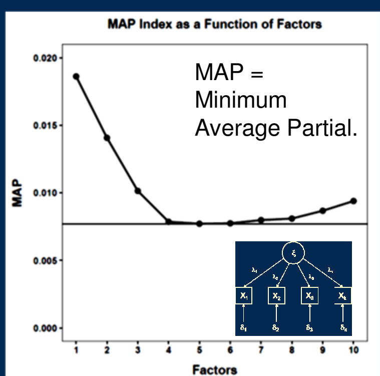
Factor analysis eigenvalues are attenuated relative to the principal components analysis eigenvalues.



Parallel analysis is particularly important in factor analysis where the eigenvalue > 1.0 rule has less meaning. Confidence intervals are less useful as sample size increases.



Eigenvalues can drop below 0 in factor analysis because not all of the variability in the variables is common factor variance.



MAP = Minimum Average Partial.

The MAP index is a function of the average partial correlation among variables controlling for factors. The minimum indicates the optimal number of factors.

	PC1	PC2	PC3	PC4
SS loadings	10.95	3.81	3.24	2.42
Proportion Var	0.24	0.08	0.07	0.05
Cumulative Var	0.24	0.32	0.39	0.44
Proportion Explained	0.54	0.19	0.16	0.12
Cumulative Proportion	0.54	0.72	0.88	1.00

	PA1	PA2	PA3	PA4
SS loadings	10.37	3.23	2.65	1.85
Proportion Var	0.23	0.07	0.06	0.04
Cumulative Var	0.23	0.30	0.35	0.39
Proportion Explained	0.57	0.18	0.15	0.10
Cumulative Proportion	0.57	0.75	0.90	1.00

Unlike principal components analysis, factor analysis will not attempt to explain all of the variance in each variable. Only common factor variance is of interest. For similar numbers of linear combinations, factor analysis will account for less variance.

To the extent there is random error in the measures (always) or there are systematic but specific sources of variance (usually), the eigenvalues for factor analysis will be smaller than the corresponding eigenvalues in principal components analysis.

They are guided by the same general goals but there is less to work with in factor analysis.

	PA1	PA2	PA3	PA4	h2
Equality	0.55	-0.48	0.07	-0.13	0.549
World_Peace	0.50	-0.33	0.00	0.05	0.361
Unity_With_Nature	0.36	-0.21	0.16	0.57	0.523
Wisdom	0.55	0.01	0.01	0.05	0.308
World_Of_Beauty	0.44	-0.21	0.30	0.37	0.458
Social_Justice	0.52	-0.42	0.00	-0.01	0.446
Broad_Minded	0.48	-0.39	0.18	-0.16	0.442
Protect_Enviroment	0.36	-0.27	0.24	0.38	0.405
Loyal	0.56	-0.08	-0.20	-0.21	0.409
Honest	0.63	-0.23	-0.20	-0.25	0.553
Helpful	0.59	-0.41	-0.16	-0.05	0.539
Responsible	0.63	0.03	-0.20	-0.17	0.472
Forgiving	0.51	-0.29	-0.17	0.07	0.385
Respect_For_Tradition	0.40	0.23	-0.33	0.37	0.458
Moderate	0.11	0.09	-0.13	0.16	0.061
Humble	0.50	-0.19	-0.29	0.06	0.373
Accept_My_Life	0.29	0.11	-0.22	0.30	0.232
Devout	0.21	0.11	-0.43	0.27	0.317
Self-Discipline	0.57	0.07	-0.23	0.12	0.391
Respect_Elders	0.59	0.08	-0.41	0.06	0.517
Obedient	0.48	0.14	-0.34	0.17	0.389
Politeness	0.61	0.05	-0.31	-0.06	0.472

The unrotated solution does not suggest a meaningful interpretation beyond an initial "values are important" dimension. A single dominant factor is a typical result.

	PA1	PA2	PA3	PA4	h2
Social_Order	0.40	0.29	-0.05	0.12	0.262
National_Security	0.50	0.22	-0.10	0.04	0.311
Reciprocity	0.43	0.22	-0.13	0.04	0.258
Family_Security	0.55	-0.06	-0.21	-0.18	0.380
Clean	0.42	0.21	-0.12	0.14	0.254
Social_Power	0.13	0.60	0.26	0.12	0.463
Wealth	0.14	0.63	0.09	-0.18	0.464
Authority	0.39	0.45	-0.01	0.11	0.369
Public_Image	0.36	0.49	-0.06	0.01	0.372
Ambitious	0.63	0.08	-0.04	-0.26	0.468
Influential	0.51	0.19	0.06	0.04	0.305
Capable	0.58	0.18	0.04	-0.17	0.396
Successful	0.60	0.30	0.07	-0.29	0.535
Pleasure	0.37	0.30	0.33	-0.14	0.351
Enjoy_Life	0.44	0.07	0.32	-0.27	0.377
Self-Indulgent	0.10	0.43	0.19	0.07	0.240
Exciting_Life	0.51	0.09	0.48	-0.04	0.500
Varied_Life	0.45	-0.01	0.47	0.08	0.435
Daring	0.36	0.08	0.37	0.22	0.323
Freedom	0.65	-0.11	0.18	-0.21	0.504
Creativity	0.48	-0.12	0.37	0.20	0.419
Independent	0.48	0.04	0.15	-0.15	0.277
Choose_Own_Goals	0.52	-0.03	0.17	-0.19	0.343
Curious	0.46	-0.10	0.39	0.23	0.436

The unrotated solution does not suggest a meaningful interpretation beyond an initial "values are important" dimension. A single dominant factor is a typical result.

	PA1	PA2	PA3	PA4
Honest	0.72	0.00	0.06	0.18
Helpful	0.64	-0.19	0.24	0.20
Equality	0.64	-0.15	0.34	-0.03
Freedom	0.60	0.25	0.29	0.00
Loyal	0.59	0.10	0.01	0.21
Responsible	0.59	0.20	0.03	0.30
Ambitious	0.58	0.32	0.06	0.15
Family_Security	0.56	0.09	0.01	0.24
Broad_Minded	0.55	-0.05	0.35	-0.12
Social_Justice	0.55	-0.17	0.33	0.09
Politeness	0.52	0.14	0.00	0.43
Forgiving	0.48	-0.15	0.24	0.28
Choose_Own_Goals	0.47	0.26	0.22	-0.02
World_Peace	0.46	-0.11	0.34	0.14
Humble	0.45	-0.11	0.11	0.37

What could we name this factor?

Varimax rotation maintains orthogonal factors but shifts the variance in an attempt to achieve simpler structure. The total amount of variance accounted for is maintained.

	PA1	PA2	PA3	PA4
Wealth	-0.05	0.65	-0.18	0.08
Social_Power	-0.23	0.62	0.10	0.13
Successful	0.48	0.54	0.03	0.12
Pleasure	0.18	0.53	0.18	-0.06
Public_Image	0.09	0.50	-0.04	0.33
Authority	0.06	0.48	0.08	0.36
Self-Indulgent	-0.16	0.45	0.07	0.08

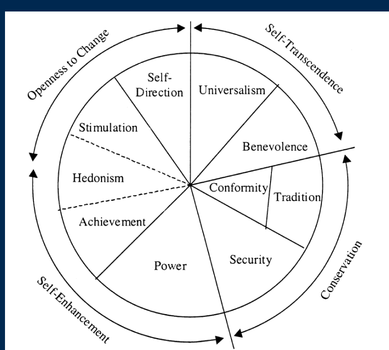
What could we name this factor?

	PA1	PA2	PA3	PA4
World_Of_Beauty	0.15	0.01	0.65	0.14
Unity_With_Nature	0.01	-0.11	0.64	0.31
Curious	0.19	0.18	0.61	0.04
Protect_Enviroment	0.12	-0.10	0.60	0.13
Creativity	0.23	0.16	0.58	0.03
Varied_Life	0.22	0.31	0.53	-0.08
Daring	0.05	0.29	0.49	0.06
Exciting_Life	0.29	0.44	0.46	-0.10

What could we name this factor?

	PA1	PA2	PA3	PA4
Respect_For_Tradition	0.07	0.12	0.10	0.66
Respect_Elders	0.45	0.09	-0.01	0.56
Devout	0.05	-0.06	-0.06	0.55
Obedient	0.27	0.11	0.03	0.55
Self-Discipline	0.37	0.14	0.14	0.47
Accept_My_Life	0.05	0.04	0.12	0.46

What could we name this factor?



This model suggests that the factors could be correlated.

Schwartz, S. H. (1992). Universals in the content and structure of values: Theoretical advances and empirical tests in 20 countries. In M. P. Zanna (Ed.), *Advances in experimental social psychology* (pp. 1-65). San Diego, CA: Academic Press.

$$\Lambda^* = \Lambda T$$

T is the transformation matrix of cosines that indicate the location of the new axes compared to the original axes.

	PA1*	PA2*	PA3*	PA4*
PA1	0.7403	0.3418	0.4106	0.4081
PA2	-0.3650	0.8289	-0.3146	0.2843
PA3	-0.1708	0.3873	0.6313	-0.6498
PA4	-0.5381	-0.2148	0.5778	0.5748

In terms of angles:

	PA1*	PA2*	PA3*	PA4*
PA1	42.24	70.01	65.76	65.92
PA2	111.41	34.02	108.33	73.49
PA3	99.83	67.22	50.85	130.53
PA4	122.56	102.40	54.70	54.91

Scores on the underlying common factors can be obtained in much the same way as in principal components analysis. The key difference is that variables are assumed to be measured with error in factor analysis.

$$\Xi = XB$$

$$\frac{1}{N-1}X'\Xi = \frac{1}{N-1}X'XB$$

$$\Lambda = RB$$

$$B = R^{-1}\Lambda$$

$$\Xi = XR^{-1}\Lambda$$

These are often referred to as regression-based factor scores.

Because they are estimated, not exact calculations, factor scores can have small correlations despite the orthogonal nature of the factors.

	PA1	PA2	PA3	PA4
PA1	1.00000	0.01892	0.0664423	0.0629665
PA2	0.01892	1.00000	0.0176323	0.0350169
PA3	0.06644	0.01763	1.0000000	-0.0008299
PA4	0.06297	0.03502	-0.0008299	1.0000000

It is rare for the different rotational criteria to produce quite different results:

	Varimax				Quartamax				Equamax			
	PA1	PA2	PA3	PA4	PA1	PA2	PA4	PA3	PA1	PA2	PA4	PA3
Equality	0.64	-0.15	0.34	-0.03	0.62	-0.28	0.24	-0.18	0.62	-0.28	0.24	-0.18
World_Peace	0.46	-0.11	0.34	0.14	0.50	-0.19	0.26	0.02	0.50	-0.19	0.26	0.02
Unity_With_Nature	0.01	-0.11	0.64	0.31	0.17	-0.10	0.64	0.27	0.17	-0.10	0.64	0.27
Wisdom	0.38	0.19	0.26	0.25	0.48	0.14	0.21	0.11	0.48	0.14	0.21	0.11
World_Of_Beauty	0.15	0.01	0.65	0.14	0.27	-0.02	0.62	0.05	0.27	-0.02	0.62	0.05
Social_Justice	0.55	-0.17	0.33	0.09	0.56	-0.26	0.25	-0.04	0.56	-0.26	0.25	-0.04
Broad_Minded	0.55	-0.05	0.35	-0.12	0.53	-0.18	0.25	-0.26	0.53	-0.18	0.25	-0.26
Protect_Enviroment	0.12	-0.10	0.60	0.13	0.21	-0.12	0.58	0.07	0.21	-0.12	0.58	0.07
Loyal	0.59	0.10	0.01	0.21	0.63	0.01	-0.08	0.05	0.63	0.01	-0.08	0.05
Honest	0.72	0.00	0.06	0.18	0.73	-0.11	-0.05	0.00	0.73	-0.11	-0.05	0.00
Helpful	0.64	-0.19	0.24	0.20	0.66	-0.28	0.15	0.05	0.66	-0.28	0.15	0.05
Responsible	0.59	0.20	0.03	0.30	0.66	0.12	-0.05	0.12	0.66	0.12	-0.05	0.12
Forgiving	0.48	-0.15	0.24	0.28	0.53	-0.21	0.17	0.16	0.53	-0.21	0.17	0.16
Respect_For_Tradition	0.07	0.12	0.10	0.66	0.27	0.17	0.11	0.59	0.27	0.17	0.11	0.59
Moderate	-0.02	0.03	0.03	0.24	0.05	0.06	0.04	0.23	0.05	0.06	0.04	0.23
Humble	0.45	-0.11	0.11	0.37	0.53	-0.15	0.05	0.26	0.53	-0.15	0.05	0.26

Varimax Quartamax Equamax

Because factor analysis requires iterative methods, there are quite a number of ways to derive the factors. Among the most common are:

- Principal axes factoring
- Minimum residual
- Weighted least squares
- Maximum likelihood

Principal axes factoring uses the method we described originally. It replaces the main diagonal of the correlation matrix with an initial estimate of variable communalities, extracts principal components from this modified matrix, produces new communality estimates which are substituted into the main diagonal of the correlation matrix, and the process is repeated until estimates no longer change (below some change criterion).

Minimum residual factoring (sometime also called ordinary least squares factoring) derives factors that minimize the residual correlation matrix, the difference between the original correlation matrix and the correlation matrix that is reproduced by a given number of factors.

Weighted least squares factoring derives factors that also minimize the residual correlation matrix, but weights the variables by their uniqueness. Variables with higher uniqueness are weighted less than variables with lower uniqueness.

Maximum likelihood factoring uses the maximum likelihood method to find factor loadings (and later factor correlations) that maximize the likelihood of the data. It provides standard errors (and confidence intervals) for estimates, tests of significance, and goodness of fit tests. It also rests on an assumption of multivariate normality.

	PA1	PA2	PA3	PA4
SS loadings	6.73	3.91	3.74	3.72
Proportion Var	0.15	0.09	0.08	0.08
Cumulative Var	0.15	0.23	0.31	0.39
Proportion Explained	0.37	0.22	0.21	0.21
Cumulative Proportion	0.37	0.59	0.79	1.00

Principal Axes

	MR1	MR2	MR3	MR4
SS loadings	6.73	3.91	3.74	3.72
Proportion Var	0.15	0.09	0.08	0.08
Cumulative Var	0.15	0.23	0.31	0.39
Proportion Explained	0.37	0.22	0.21	0.21
Cumulative Proportion	0.37	0.59	0.79	1.00

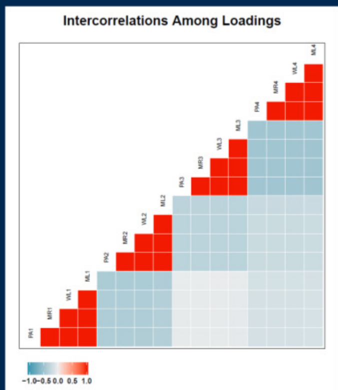
Minimum Residual

	WLS1	WLS2	WLS3	WLS4
SS loadings	6.73	3.92	3.74	3.72
Proportion Var	0.15	0.09	0.08	0.08
Cumulative Var	0.15	0.23	0.31	0.39
Proportion Explained	0.37	0.22	0.21	0.21
Cumulative Proportion	0.37	0.59	0.79	1.00

Weighted Least Squares

	ML1	ML2	ML4	ML3
SS loadings	6.66	3.99	3.76	3.70
Proportion Var	0.14	0.09	0.08	0.08
Cumulative Var	0.14	0.23	0.31	0.39
Proportion Explained	0.37	0.22	0.21	0.20
Cumulative Proportion	0.37	0.59	0.80	1.00

Maximum Likelihood



Does it matter? For well-defined latent variables, these methods will converge on very similar solutions.

Advocates of factor analysis often claim that it is inappropriate to apply principal components procedures in the search for meaning or latent constructs. But, does it really matter all that much?

To the extent that the communalities for all variables are high, the two procedures should give very similar results. When the commonalities are very low, then factor analysis results may depart from principal components.

Next time . . .

- Oblique rotation
- Cross-validation