Multivariate Analysis of Variance

Today . . .

- Resampling when assumptions fail
- Incorporating covariates into a MANOVA
- Extending MANOVA to collections of different measures—profile analysis and doubly repeated designs.

If the assumptions underlying the MANOVA (homogeneous covariance matrices, multivariate normality) are not viable, alternative approaches can be taken that do not make these assumptions: bootstrapping and randomization tests.

When participants are randomly assigned to groups, we assume under the null hypothesis that these assignments are inconsequential or arbitrary. In a randomization test, we actually make the group assignments arbitrary.

On each of a large number of trials, the group assignments are shuffled randomly so that participants will get new group assignments that may not match their original assignments.

The statistical analyses are conducted on each of these reshuffled samples.

The location of parameter estimates (e.g., Wilks' Λ) from the original analysis in the resulting randomization distributions is examined. If the original estimates are rare in the null-consistent randomization distributions, the null is rejected.

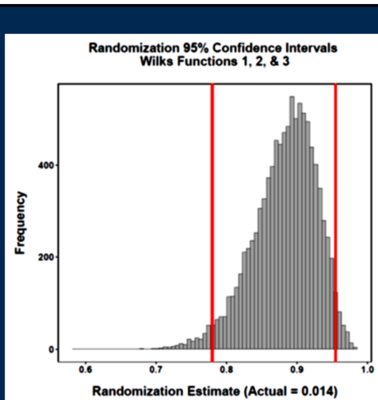
```
LM_1 <- lm(cbind(P_Verbal, P_Quant, C_Verbal, C_Quant) ~ as.factor(Group),
    data = Skills)
LDA_1 <- candisc(LM_1, data = Skills)
```

```
LR test stat approx F numDF denDF Pr(> F)
       0.014     82.3    12   246  <2e-16
       0.103     66.3     6   188  <2e-16
       0.638              2
```

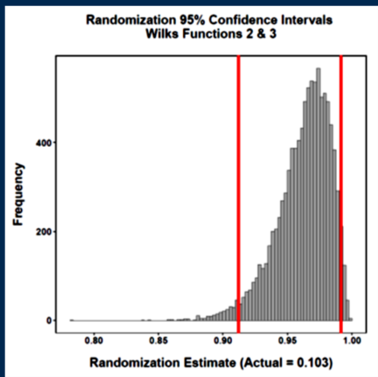
```
DA_Chi_Square(Skills, LDA_1)

##   Chi_Sq df          p
## 1 405.01 12 3.292e-79
## 2 215.99  6 7.437e-44
## 3  42.68  2 5.402e-10
```

All three functions were significant in the original analysis.

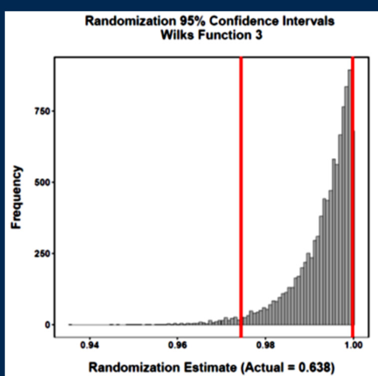
```
LDA_1$coeffs.std

##               Can1    Can2    Can3
## P_Verbal  0.8044 -0.7131 -1.3283
## P_Quant   0.5891  1.2191  1.4285
## C_Verbal -0.4769 -0.1206 -0.5982
## C_Quant  -0.1877 -1.0700  0.8519

LDA_1$structure

##             Can1    Can2     Can3
## P_Verbal 0.9196 -0.3851 -0.01672
## P_Quant  0.9350 -0.1684  0.26787
## C_Verbal 0.4047 -0.7919  0.22928
## C_Quant  0.3313 -0.8866  0.30943
```



The obtained empirical value falls outside the 95% CI from the null-consistent randomization distribution. We reject the null. The groups can be separated by at least one discriminant function.



The obtained empirical value falls outside the 95% CI from the null-consistent randomization distribution. The groups can be separated by at least two discriminant functions.



The obtained empirical value falls outside the 95% CI from the null-consistent randomization distribution. The groups can be separated by the third discriminant function.

Adding covariates to a multivariate analysis (MANCOVA) is a direct extension of univariate analysis of covariance.

The goals in ANCOVA are:

(a) to reduce the error for testing a target effect

(b) to adjust for differences that might otherwise make inferences ambiguous

The interest is in asking a question that begins, “controlling for . . .”

This simple MANOVA determines the linear combination of the two computer GRE measures that best separates participants who received computer training from those who did not. Only one linear combination is possible here.

```
LM_2 <- lm(cbind(C_Verbal, C_Quant) ~ Tx_C, data = Skills_Trimmed)
LDA_2 <- candisc(LM_2, data = Skills_Trimmed)
LDA_2
```

```
  CanRsq Eigenvalue Difference Percent Cumulative
1  0.543       1.19                100        100
```

```
DA_Chi_Square(Skills_Trimmed, LDA_2)

##   Chi_Sq df         p
## 1  75.09  2 4.939e-17
```

```
LDA_2$coeffs.std

##              Can1
## C_Verbal -1.204
## C_Quant   2.007

LDA_2$structure

##              Can1
## C_Verbal 0.7887
## C_Quant  0.9457
```

Both measures are positively related to the linear combination that best separates the groups, but the standardized coefficients have opposite signs because of the high correlation between the two measures.

```
MANOVA_2 <- manova(cbind(C_Verbal, C_Quant) ~ Tx_C, data = Skills_Trimmed)
summary(MANOVA_2, test = "Wilks")

##           Df Wilks approx F num Df den Df Pr(>F)
## Tx_C       1 0.457     56.9      2     96 <2e-16
## Residuals 97
```

```
cor(MANOVA_2$residuals)

##          C_Verbal C_Quant
## C_Verbal   1.0000  0.9265
## C_Quant    0.9265  1.0000
```

```
LM_3 <- lm(cbind(C_Verbal, C_Quant) ~ P_Verbal + P_Quant + Tx_C, data = Skills_Trimmed)
LDA_3 <- candisc(LM_3, data = Skills_Trimmed)
```

This MANOVA asks the same question, but qualified by "controlling for performance on standard paper GRE measures . . ."

This analysis might be motivated by a concern for whether computer training has a unique effect on computer measures or whether it simply boosts performance on any measure of the talent that was the focus of training.

```
  CanRsq Eigenvalue Difference Percent Cumulative
1  0.108      0.121                 100        100
```

```
DA_Chi_Square(Skills_Trimmed, LDA_3)

##   Chi_Sq df        p
## 1  10.97  2 0.004154
```

```
LDA_3$coeffs.std
##                Can1
## C_Verbal -0.01967
## C_Quant   1.01462

LDA_3$structure

##            Can1
## C_Verbal 0.944
## C_Quant  1.000
```

The multivariate tests indicate that it is still possible to separate the groups significantly with a linear combination of the computer measures, controlling for the differences between the groups on the standard measures. But, controlling for the standard measures largely eliminates the independent contribution that the computer verbal measure made to group separation.

```
MANOVA_3 <- manova(cbind(C_Verbal, C_Quant) ~ P_Verbal + P_Quant +
    Tx_C, data = Skills_Trimmed)
summary(MANOVA_3, test = "Wilks")

##           Df Wilks approx F num Df den Df Pr(>F)
## P_Verbal   1 0.243    146.7      2     94 <2e-16
## P_Quant    1 0.863      7.4      2     94  0.001
## Tx_C       1 0.246    144.0      2     94 <2e-16
## Residuals 95
```

```
cor(MANOVA_3$residuals)

##          C_Verbal C_Quant
## C_Verbal   1.0000  0.7476
## C_Quant    0.7476  1.0000
```

```
cor(MANOVA_2$residuals)

##          C_Verbal C_Quant
## C_Verbal   1.0000  0.9265
## C_Quant    0.9265  1.0000
```

Controlling for the standard paper measures reduces the within-group correlation between the computer measures.

A MANCOVA carries the same assumption of homogeneity of regression that accompanies a univariate analysis of covariance. It is tested in the same way, by examining interactions involving the covariates.

```
MANOVA_4 <- manova(cbind(C_Verbal, C_Quant) ~ P_Verbal + P_Quant +
    Tx_C + Tx_C:P_Verbal + Tx_C:P_Quant, data = Skills_Trimmed)
summary(MANOVA_4, test = "Wilks")

##                Df Wilks approx F num Df den Df      Pr(>F)
## P_Verbal        1 0.219    164.0      2     92   < 2e-16
## P_Quant         1 0.863      7.3      2     92    0.0011
## Tx_C            1 0.237    148.0      2     92   < 2e-16
## P_Verbal:Tx_C   1 0.995      0.2      2     92    0.7898
## P_Quant:Tx_C    1 0.737     16.4      2     92 0.00000082
## Residuals      93
```

Used carefully, a MANCOVA allows separating the influence of highly correlated variables and isolating the unique influence of treatment effects.

Here it shows that the computer training has a large impact on GRE performance, but seems to have an especially important or unique effect on the computer quantitative measure.

The Covariate x Group interactions would, however, require additional exploration.

Extending MANOVA to repeated measures has some advantages. One of the simplest is a profile analysis.

In a profile analysis, several different measures that use the same scale are compared—their profile is assessed, often by comparing groups.

A key assumption is that the measures can be directly compared—<u>their metrics are the same</u>.

This is no problem when the “different measures” are simple replications over time. The assumption is very important to consider when the measures are truly different conceptually.

When the same measures are collected over time, a profile analysis is the multivariate approach to repeated measures that might be used if the assumptions for a univariate repeated measures ANOVA (e.g., sphericity) are not met.

When used with different measures, collected on one occasion with a common measurement scale, a profile analysis addresses questions such as “is Measure X elevated relative to Measures Y and Z?” If groups are included, the question becomes “is the difference between X and Y greater in Group A than in Group B?”

A profile analysis addresses three major questions:

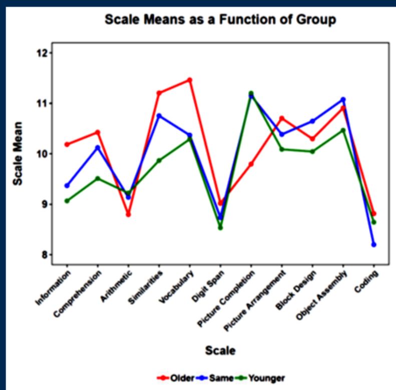
- Are the profiles parallel? Is the pattern of differences across measures similar for the several groups compared (the interaction)?
- If the profiles are parallel, are they coincident? Coincident and parallel profiles will have no group differences in the between-subjects part of the design (there will be no group main effect).
- If the profiles are parallel, are they also level? The flatness of the profiles is addressed by collapsing across groups and testing for whether the measures are similar in their means (the main effect from the within-subjects part of the design).

In this data set, 11 subtest scores from the WISC, measured on the same scale, were compared for three groups of learning disabled children.

- Information
- Comprehension
- Arithmetic
- Similarities
- Vocabulary
- Digit span
- Picture completion
- Picture arrangement
- Block design
- Object assembly
- Coding

The three groups of children were formed based on their age preferences for playmates:

- 1 = younger
- 2 = older
- 3 = same




```r
Measure <- factor(c("info", "comp", "arith", "simil", "vocab", "digit",
    "pictcomp", "parang", "block", "object", "coding"), levels = c("info",
    "comp", "arith", "simil", "vocab", "digit", "pictcomp", "parang",
    "block", "object", "coding"))
idata <- data.frame(Measure)

LM_1 <- lm(cbind(info, comp, arith, simil, vocab, digit, pictcomp,
    parang, block, object, coding) ~ Profiles$AgeMate, data = Profiles)
```

```r
ANOVA_1 <- Anova(LM_1, idata = idata, idesign = ~Measure, type = 2)
summary(ANOVA_1, multivariate = FALSE)
```

In this analysis, we define the profile as if it had "levels" on a factor called "Measure". Reminder: the separate measures must be on the same scale or this definition will not make any sense.

```
Univariate Type II Repeated-Measures ANOVA Assuming Sphericity

                             SS num Df Error SS den Df       F
(Intercept)              179322      1     4916    161 5872.55
Profiles$AgeMate             50      2     4916    161    0.81
Measure                    1190     10     9408   1610   20.37
Profiles$AgeMate:Measure    218     20     9408   1610    1.87
                         Pr(>F)
(Intercept)              <2e-16
Profiles$AgeMate          0.446
Measure                  <2e-16
Profiles$AgeMate:Measure  0.011


Mauchly Tests for Sphericity

                         Test statistic  p-value
Measure                           0.281 1.32e-18
Profiles$AgeMate:Measure          0.281 1.32e-18
```

The groups are not different in their overall means. The measures are different collapsed across groups.

The profiles are not parallel. The sphericity assumption is violated. A MANOVA does not have that assumption and can be used to verify the inferences.

```
MANOVA_1 <- Manova(LM_1, idata = idata, idesign = ~Measure, type = 2)
summary(MANOVA_1)
```

```
Multivariate Tests: Profiles$AgeMate
                 Df test stat approx F num Df den Df Pr(>F)
Pillai            2    0.0100   0.8124      2    161  0.446
Wilks             2    0.9900   0.8124      2    161  0.446
Hotelling-Lawley  2    0.0101   0.8124      2    161  0.446
Roy               2    0.0101   0.8124      2    161  0.446
```

```
Multivariate Tests: Measure
                 Df test stat approx F num Df den Df Pr(>F)
Pillai            1    0.4752    13.76     10    152 <2e-16
Wilks             1    0.5248    13.76     10    152 <2e-16
Hotelling-Lawley  1    0.9053    13.76     10    152 <2e-16
Roy               1    0.9053    13.76     10    152 <2e-16
```

```
Multivariate Tests: Profiles$AgeMate:Measure
                 Df test stat approx F num Df den Df  Pr(>F)
Pillai            2    0.2224    1.915     20    306 0.01131
Wilks             2    0.7840    1.967     20    304 0.00868
Hotelling-Lawley  2    0.2674    2.019     20    302 0.00666
Roy               2    0.2321    3.551     10    153 0.00030
```

The significant interaction means that the profiles of subscore means are different for the three groups.

There are quite a number of ways that this interaction could be explored to determine the nature of the differences.

A priori hypotheses should be tested with focused contrasts, without any need for Type I error protection.

Exploratory tests should adjust for the number of tests conducted.

When variables have different metrics (are not commensurate), but are measured repeatedly, a <u>doubly multivariate analysis</u> is used.

This approach can be thought of as a multivariate analysis of transformed scores (sums and differences). It can help identify patterns of outcomes that distinguish groups.

In this study, 38 healthy young men and 37 age-matched psychiatric male in-patients were asked to engage in brief 10-minute conversations with two other people (the targets, actually research assistants blind to the study purpose or participant status).

Participants were given some brief background information about the targets before meeting them. Both targets were described as holding steady jobs, having hobbies, and going to school part time.

One target (A) was described as having had a lifetime problem with seasonal allergies. The other target (B) was described has having been hospitalized in the past for a psychiatric problem.

During each interview, the distance the participant sat from the other person (in cm) and the amount of eye contact (in seconds) were assessed. At the end of the 10-minute conversation, participants were asked to rate their liking for the target (on a 7-point scale).

The researchers hypothesized that all participants would distance themselves more from targets believed to have had a psychiatric problem and that they would like this target less than the target with no apparent history of psychiatric problems.

Eye contact, however, was expected to show a different pattern. Participants were expected to engage in more eye contact with targets who were different from them. Healthy participants were expected to have more eye contact with targets thought to have psychiatric problems than with targets believed to be healthy. Psychiatric patients were expected to show the opposite pattern.

```
# Create a matrix that represents the sums and differences of the
# measures. The first three columns in the following matrix create
# sums of the distance, liking, and eye contact variables,
# collapsing over the two targets. The last three columns create
# difference scores comparing the responses to each target,
# separately for each measure (distance, liking, eye contact).
imatrix <- matrix(c(1, 0, 0, 1, 0, 0, 1, 0, 0, -1, 0, 0, 0, 1, 0,
    0, 1, 0, 0, 1, 0, 0, -1, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, -1),
    nrow = 6, ncol = 6, byrow = TRUE)
colnames(imatrix) <- c("Distance_Sum", "Liking_Sum", "Eye_Contant_Sum",
    "Distance_Diff", "Liking_Diff", "Eye_Contanct_Diff")
rownames(imatrix) <- colnames(Double)[-c(7:9)]
(imatrix <- list(measure = imatrix[, 1:3], target = imatrix[, 4:6]))

## $measure
##                          Distance_Sum Liking_Sum Eye_Contant_Sum
## Interpersonal_Distance_A            1          0               0
## Interpersonal_Distance_B            1          0               0
## Liking_A                            0          1               0
## Liking_B                            0          1               0
## Eye_Contact_A                       0          0               1
## Eye_Contact_B                       0          0               1
```

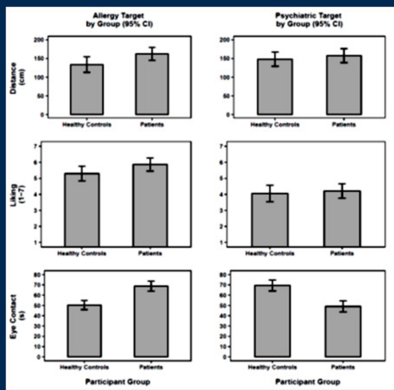
```
## $target
##                          Distance_Diff Liking_Diff
## Interpersonal_Distance_A             1           0
## Interpersonal_Distance_B            -1           0
## Liking_A                             0           1
## Liking_B                             0          -1
## Eye_Contact_A                        0           0
## Eye_Contact_B                        0           0
##                          Eye_Contanct_Diff
## Interpersonal_Distance_A                 0
## Interpersonal_Distance_B                 0
## Liking_A                                 0
## Liking_B                                 0
## Eye_Contact_A                            1
## Eye_Contact_B                           -1
```

```
# Contrast the two groups.
contrasts(Double$Group) <- matrix(c(1, -1), ncol = 1)
# Fit each measure in a linear model.
Double_Fit <- lm(cbind(Interpersonal_Distance_A, Interpersonal_Distance_B,
    Liking_A, Liking_B, Eye_Contact_A, Eye_Contact_B) ~ Group, data = Double)
# Get the doubly multivariate results.
Anova(Double_Fit, imatrix = imatrix, test = "Wilks")
```

```
Type II Repeated Measures MANOVA Tests: Wilks test statistic
              Df test stat approx F num Df den Df  Pr(>F)
measure        1     0.012     1939      3     71 < 2e-16
Group:measure  1     0.899        3      3     71   0.056
target         1     0.362       42      3     71 1.2e-15
Group:target   1     0.262       67      3     71 < 2e-16
```

There are group differences in both the sums (marginal) and differences.

Follow-up analyses can help identify the nature of the effects.




```
Univariate Type II Repeated-Measures ANOVA Assuming Sphericity

                   SS num Df Error SS den Df       F Pr(>F)
(Intercept)   3399344      1   356798     73  695.50 <2e-16
Group2          13481      1   356798     73    2.76   0.10
Measure           821      1   125958     73    0.48   0.49
Group2:Measure   3492      1   125958     73    2.02   0.16
```

Interpersonal Distance

```
Univariate Type II Repeated-Measures ANOVA Assuming Sphericity

                 SS num Df Error SS den Df       F Pr(>F)
(Intercept)    3543      1    234.6     73 1102.46 <2e-16
Group2            5      1    234.6     73    1.54   0.22
Measure          79      1     51.7     73  111.88 <2e-16
Group2:Measure    2      1     51.7     73    2.28   0.14
```

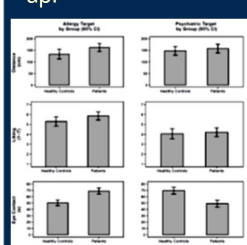
Liking

```
Univariate Type II Repeated-Measures ANOVA Assuming Sphericity

                   SS num Df Error SS den Df       F Pr(>F)
(Intercept)    529967      1    27811     73 1391.11 <2e-16
Group2             35      1    27811     73    0.09   0.76
Measure            14      1     6217     73    0.17   0.69
Group2:Measure  14124      1     6217     73  165.83 <2e-16
```

Eye Contact

```
summary.aov(Double_Fit)

##  Response Interpersonal_Distance_A :
##             Df Sum Sq Mean Sq F value Pr(>F)
## Group        1  15348   15348    4.56  0.036
## Residuals   73 245790    3367
##
##  Response Interpersonal_Distance_B :
##             Df Sum Sq Mean Sq F value Pr(>F)
## Group        1   1625    1625     0.5   0.48
## Residuals   73 236967    3246
##
##  Response Liking_A :
##             Df Sum Sq Mean Sq F value Pr(>F)
## Group        1    6.1    6.11    3.49  0.066
## Residuals   73  128.1    1.75
##
##  Response Liking_B :
##             Df Sum Sq Mean Sq F value Pr(>F)
## Group        1    0.5   0.459    0.21   0.65
## Residuals   73  158.2   2.167
##
##  Response Eye_Contact_A :
##             Df Sum Sq Mean Sq F value     Pr(>F)
## Group        1   6372    6372    32.4 0.00000025
## Residuals   73  14374     197
##
##  Response Eye_Contact_B :
##             Df Sum Sq Mean Sq F value     Pr(>F)
## Group        1   7787    7787    28.9 0.00000087
## Residuals   73  19654     269
```

Analyses of variance of each measure provide a low level of follow-up.



```
# Create the sums and differences for entry in discriminant
# analysis to produce additional results.
Double$Distance_Sum <- 0.7071 * Double$Interpersonal_Distance_A +
    0.7071 * Double$Interpersonal_Distance_B
Double$Liking_Sum <- 0.7071 * Double$Liking_A + 0.7071 * Double$Liking_B
Double$Eye_Contant_Sum <- 0.7071 * Double$Eye_Contact_A + 0.7071 *
    Double$Eye_Contact_B
Double$Distance_Diff <- 0.7071 * Double$Interpersonal_Distance_A -
    0.7071 * Double$Interpersonal_Distance_B
Double$Liking_Diff <- 0.7071 * Double$Liking_A - 0.7071 * Double$Liking_B
Double$Eye_Contact_Diff <- 0.7071 * Double$Eye_Contact_A - 0.7071 *
    Double$Eye_Contact_B

LM_Sum <- lm(cbind(Distance_Sum, Liking_Sum, Eye_Contant_Sum) ~ Group2,
    data = Double)
LDA_Sum <- candisc(LM_Sum, data = Double)
```

A discriminant analysis can provide additional insight by controlling the correlations among the multiple measures.

```
## Canonical Discriminant Analysis for Group2:
##
##   CanRsq Eigenvalue Difference Percent Cumulative
## 1  0.101      0.112                100        100
##
## Test of H0: The canonical correlations in the
## current row and all that follow are zero
##
##   LR test stat approx F num Df den Df Pr(> F)
## 1        0.899     8.16      1     73  0.0056
```

```
LDA_Sum$coeffs.std

##                    Can1
## Distance_Sum    1.03266
## Liking_Sum      0.92210
## Eye_Contant_Sum 0.01531

LDA_Sum$structure

##                    Can1
## Distance_Sum     0.6018
## Liking_Sum       0.4540
## Eye_Contant_Sum -0.1125
```

A linear combination of the sum variables (almost entirely due to distance and liking) can separate the healthy controls and patients.

```
LM_Diff <- lm(cbind(Distance_Diff, Liking_Diff, Eye_Contact_Diff) ~
    Group2, data = Double)
LDA_Diff <- candisc(LM_Diff, data = Double)
```

```
## Canonical Discriminant Analysis for Group2:
##
##   CanRsq Eigenvalue Difference Percent Cumulative
## 1  0.738       2.82                100        100
##
## Test of H0: The canonical correlations in the
## current row and all that follow are zero
##
##   LR test stat approx F num Df den Df Pr(> F)
## 1        0.262      206      1     73  <2e-16
```

```
LDA_Diff$coeffs.std
##                     Can1
## Distance_Diff     0.3898
## Liking_Diff      -0.1990
## Eye_Contact_Diff  1.0946
LDA_Diff$structure
##                    Can1
## Distance_Diff    0.1912
## Liking_Diff      0.2024
## Eye_Contact_Diff 0.9698
```

A linear combination of the difference variables (largely due to eye contact) can separate the healthy controls and patients.

	Distance_Sum	Liking_Sum	Eye_Contact_Sum
Distance_Sum	1.0000	-0.4795	-0.5912
Liking_Sum	-0.4795	1.0000	0.5297
Eye_Contact_Sum	-0.5912	0.5297	1.0000

	Distance_Diff	Liking_Diff	Eye_Contact_Diff
Distance_Diff	1.0000	-0.2710	-0.3148
Liking_Diff	-0.2710	1.0000	0.3744
Eye_Contact_Diff	-0.3148	0.3744	1.0000

Independent of participant group, distance is negatively related to liking and eye contact. Eye contact and liking are positively related. This is true for both sums and differences.

As the difference in distance from the two targets increases, the difference in liking (and in eye contact) increases in the opposite direction.

Next time . . .

Canonical correlation analysis—finding linear combinations that maximize correlations across sets of variables