LNN-EL: A Neuro-Symbolic Approach to Short-text Entity Linking

Hang Jiang[1], Sairam Gurajada[2], Qiuhao Lu[3], Sumit Neelam[2], Lucian Popa[2], Prithviraj Sen[2], Alexander Gray[2], Yunyao Li[2]
[1]MIT [2]IBM Research University of Oregon
hjian42@mit.edu, {sairam.gurajada, alexander.gray}@ibm.com, luqh@cs.uoregon.edu,
sumit.neelam@in.ibm.com, {lpopa,senp,yunyaoli}@us.ibm.com

Abstract

Entity linking (EL), the task of disambiguating mentions in text by linking them to entities in a knowledge graph, is crucial for text understanding, question answering or conversational systems. Entity linking on short text (e.g., single sentence or question) poses particular challenges due to limited context. While prior approaches use either heuristics or black-box neural methods, here we propose LNN-EL, a neuro-symbolic approach that combines the advantages of using interpretable rules based on first-order logic with the performance of neural learning. Even though constrained to using rules, LNN-EL performs competitively against SotA black-box neural approaches, with the added benefits of extensibility and transferability. In particular, we show that we can easily blend existing rule templates given by a human expert, with multiple types of features (priors, BERT encodings, box embeddings, etc), and even scores resulting from previous EL methods, thus improving on such methods. For instance, on the LC-QuAD-1.0 dataset, we show more than 4% increase in F1 score over previous SotA. Finally, we show that the inductive bias offered by using logic results in learned rules that transfer well across datasets, even without fine tuning, while maintaining high accuracy.

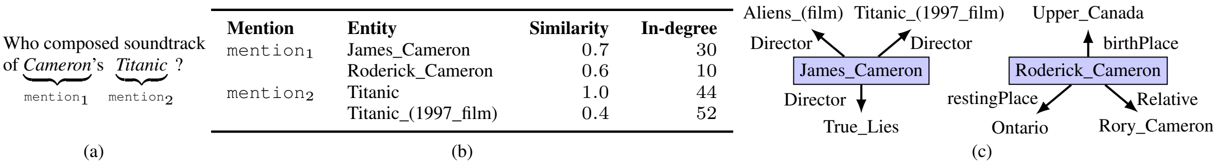
1 Introduction

Entity Linking (EL) is the task of disambiguating textual *mentions* by linking them to canonical entities provided by a knowledge graph (KG) such as DBpedia, YAGO (Suchanek et al., 2007) or Wikidata (Vrandečić and Krötzsch, 2014). A large body of existing work deals with EL in the context of longer text (i.e., comprising of multiple sentences) (Bunescu and Pasca, 2006). The general approach is: 1) extract *features* measuring some degree of similarity between the textual mention and any one of several candidate entities (Mihalcea and Csomai, 2007; Cucerzan, 2007; Ratinov et al., 2011), followed by 2) the disambiguation step, either heuristics-based (non-learning) (Hoffart et al., 2011; Sakor et al., 2019; Ferragina and Scaiella, 2012) or learning-based (Mihalcea and Csomai, 2007; Cucerzan, 2007; Ratinov et al., 2011; Hoffart et al., 2012; Ganea and Hofmann, 2017), to link the mention to an actual entity.

A particular type of entity linking, focused on short text (i.e., a single sentence or question), has attracted recent attention due to its relevance for downstream applications such as question answering and conversational systems. Short-text EL is particularly challenging because the limited context surrounding mentions results in greater ambiguity (Sakor et al., 2019). To address this challenge, one needs to exploit as many features from as many sources of evidence as possible.

Consider the question in Fig. 1(a), containing mention$_1$ (Cameron) and mention$_2$ (Titanic).[1] DBpedia contains several person entities whose last name matches Cameron. Two such entities are shown in Fig. 3(b), James_Cameron and Roderick_Cameron, along with their string similarity scores (in this case, character-level Jaccard similarity) to mention$_1$. In this case, the string similarities are quite close. In the absence of reliable discerning information, one can employ a prior such as using the more popular candidate entity, as measured by the in-degree of the entity in the KG (see Fig. 3(b)). Given the higher in-degree, we can (correctly) link mention$_1$ to James_Cameron. However, for mention$_2$, the correct entry is Titanic_(1997_film) as opposed to Titanic the ship, but it actually has a *lower* string similarity. To link to the correct entity, one needs to exploit the fact that James_Cameron has an edge

[1] Note that we assume that mention extraction has already been applied and we are given the textual mentions.



Mention	Entity	Similarity	In-degree
mention$_1$	James_Cameron	0.7	30
	Roderick_Cameron	0.6	10
mention$_2$	Titanic	1.0	44
	Titanic_(1997_film)	0.4	52

Figure 1: (a) Question with 2 mentions that need to be disambiguated against DBpedia. (b) For each mention-candidate entity pair, the character-level Jaccard similarity is shown along with the in-degree of the entity in the knowledge graph. (c) (Partial) Ego networks for entities `James_Cameron` and `Roderick_Cameron`.

connecting it to `Titanic_(1997_film)` in the KG (see ego network on the left in Fig. 1(c)). Linking co-occurring mentions from text to connected entities in the KG is an instance of *collective entity linking*. This example provides some intuition as to how priors, local features (string similarity) and collective entity linking can be exploited to overcome the limited context in short-text EL.

While the use of priors, local features and non-local features (for collective linking) has been proposed before (Ratinov et al., 2011), our goal in this paper is to provide an *extensible* framework that can combine any number of such features and more, including contextual embeddings such as BERT encodings (Devlin et al., 2019) and Query2box embeddings (Ren et al., 2020), and even the results of previously developed neural EL models (e.g., BLINK (Wu et al., 2020)). Additionally, such a framework must not only allow for easy inclusion of new sources of evidence but also for *interpretability* of the resulting model (Guidotti et al., 2018). An approach that combines disparate features should, at the very least, be able to state, post-training, which features are detrimental and which features aid EL performance and under what conditions, in order to enable actionable insights in the next iteration of model improvement.

Our Approach. We propose to use rules in first-order logic (FOL), an interpretable fragment of logic, as a glue to combine EL features into a coherent model. Each rule in itself is a disambiguation model capturing specific characteristics of the overall linking. While inductive logic programming (Muggleton, 1996) and statistical relational learning (Getoor and Taskar, 2007) have for long focused on learning FOL rules from labeled data, more recent approaches based on neuro-symbolic AI have led to impressive advances. In this work, we start with an input set of rule templates (given by an expert or available as a library), and learn the parameters of these rules (namely, the thresholds of the various similarity predicates as well as the weights of the predicates that appear in the rules), based on a labeled dataset. We use logical neural networks (LNN) (Riegel et al., 2020), a powerful neuro-symbolic AI approach based on real-valued logic that employs neural networks to learn the parameters of the rules. Learning of the rule templates themselves will be the focus of future work.

Summary of contributions

- We propose, to the best of our knowledge, the first neuro-symbolic method for entity linking (coined "*LNN-EL*") that provides a principled approach to learning EL rules.
- Our approach is extensible and can combine disparate types of local and global features as well as results of prior black-box neural methods, thus building on top of such approaches.
- Our approach produces interpretable rules that humans can inspect toward actionable insights.
- We evaluate our approach on three benchmark datasets and show competitive (or better) performance with SotA black-box neural approaches (e.g., BLINK (Wu et al., 2020)) even though we are constrained on using rules.
- By leveraging rules, the learned model shows a desirable *transferability* property: it performs well not only on the dataset on which it was trained, but also on other datasets from the same domain without further training.

2 Related Work

Entity Linking Models. Entity Linking is a well-studied problem in NLP, especially for long text. Approaches such as (Bunescu and Pasca, 2006; Ratinov et al., 2011; Sil et al., 2012; Hoffart et al., 2011; Shen et al., 2015) use a myriad of classical ML and deep learning models to combine priors, local and global features. These techniques, in general, can be applied to short text, but the lack of sufficient context may render them ineffective. The recently proposed BLINK (Logeswaran et al., 2019; Wu et al., 2020) uses powerful transformer-based encoder architectures trained on massive amounts of data (such as Wikipedia, Wikia) to achieve SotA performance on entity disambiguation tasks, and is shown to be especially effective in zero-shot settings. BLINK is quite effective on short text (as observed in our findings); in our approach, we use BLINK both as a baseline and as a component that is combined in larger rules.

For short-text EL, some prior work (Sakor et al., 2019; Ferragina and Scaiella, 2012; Mendes et al., 2011) addresses the joint problem of mention detection and linking, with primary focus on identifying mention spans, while linking is done via heuristic methods without learning. (Sakor et al., 2019) also jointly extracts relation spans which aide in overall linking performance. The recent ELQ (Li et al., 2020) extends BLINK to jointly learn mention detection and linking. In contrast, we focus solely on linking and take a different strategy based on combining logic rules with learning. This facilitates a principled way combining multiple types of EL features with interpretability and learning using promising gradient-based techniques.

Rule-based Learning. FOL rules and learning have been successfully applied in some NLP tasks and also other domains. Of these, the task that is closest to ours is entity resolution (ER), which is the task of linking two entities across two structured datasets. In this context, works like (Chaudhuri et al., 2007; Arasu et al., 2010; Wang et al., 2012; Hernández et al., 2013) use FOL rules for ER. Approaches such as (Singla and Domingos, 2006; Pujara and Getoor, 2016) induce probabilistic rules using MLNs (Richardson and Domingos, 2006) and PSL (Bach et al., 2017), respectively. None of these approaches use any recent advances in neural-based learning; moreover, they are focused on entity resolution, which is a related task but distinct from short-text EL.

3 Preliminaries

3.1 Entity Linking.

Given text T, a set $M = \{m_1, m_2, ...\}$ of mentions, where each m_i is contained in T, and a knowledge graph (KG) comprising of a set $\mathcal{E}$ of entities, entity linking is a many-to-one function that links each mention $m_i \in M$ to an entity $e_{ij} \in C_i$, where $C_i \subseteq \mathcal{E}$ is a subset of relevant candidates for mention m_i. More generally, we formulate the problem as a ranking of the candidates in C_i so that the "correct" entity for m_i is ranked highest. Following existing approaches(e.g. (Sakor et al., 2019; Wu et al., 2020), we use off-the-shelf lookup tools such as DBpedia lookup[2] to retrieve top-100 candidates for each mention. While this service is specific to DBpedia, we assume that similar services exist or can be implemented on top of other KGs.

3.2 Logical Neural Networks

Fueled by the rise in complexity of deep learning, recently there has been a push towards learning interpretable models (Guidotti et al., 2018; Danilevsky et al., 2020). While linear classifiers, decision lists/trees may also be considered interpretable, rules expressed in first-order logic (FOL) form a much more powerful, closed language that offer semantics clear enough for human interpretation and a larger range of operators facilitating the expression of richer models. To learn these rules, neuro-symbolic AI typically substitutes conjunctions (disjunctions) with differentiable t-norms (t-conorms) (Esteva and Godo, 2001). However, since these norms do not have any learnable parameters (more details in App. A.1), their behavior cannot be adjusted, thus limiting their ability to model well the data.

In contrast, logical neural networks (LNN) (Riegel et al., 2020) offer operators that include parameters, thus allowing to better learn from the data. To maintain the crisp semantics of FOL, LNNs enforce constraints when learning operators such as conjunction. Concretely, LNN-$\wedge$ is expressed as:

$$\max(0, \min(1, \beta - w_1(1-x) - w_2(1-y)))$$

$$\text{subject to: } \beta - (1-\alpha)(w_1 + w_2) \geq \alpha \quad (1)$$

$$\beta - \alpha w_1 \leq 1 - \alpha \quad (2)$$

$$\beta - \alpha w_2 \leq 1 - \alpha \quad (3)$$

$$w_1, w_2 \geq 0$$

where β, w_1, w_2 are *learnable* parameters, $x, y \in [0,1]$ are inputs and $\alpha \in [\frac{1}{2}, 1]$ is a hyperparameter. Note that $\max(0, \min(1, \cdot))$ clamps the output of LNN-$\wedge$ between 0 and 1 regardless of β, w_1, w_2, x, and y. The more interesting aspects are in the constraints. While Boolean conjunction only returns 1 or `true` when both inputs are 1, LNNs relax this condition by using α as a proxy for 1 (and conversely, $1-\alpha$ as a proxy for 0). In particular, Constraint (1) forces the output of LNN-$\wedge$ to be greater than α when both inputs are greater than α. Similarly, Constraints (2) and (3) constrain the behavior of LNN-$\wedge$ when one input is low and the other is high. For instance, Constraint (2) forces the output of LNN-$\wedge$ to be less than $1-\alpha$ for $y = 1$

[2] https://lookup.dbpedia.org/

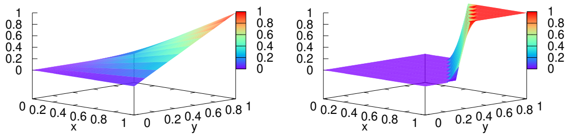


Figure 2: (left) Product t-norm. (right) LNN-$\wedge$ ($\alpha = 0.7$).

and $x \leq 1 - \alpha$. This formulation allows for unconstrained learning when $x, y \in [1 - \alpha, \alpha]$. By changing α a user can control how much learning to enable (increase to make region of unconstrained learning wider or decrease for the opposite). Fig. 2 depicts product t-norm and LNN-$\wedge$ ($\alpha = 0.7$). While the former increases slowly with increasing x, y, LNN-$\wedge$ produces a high output when both inputs are $\geq \alpha$ and stays high thereafter, thus closely modeling Boolean conjunction semantics.

In case the application requires even more degrees of freedom, the hard constraints (1), (2) and (3) can be relaxed via the inclusion of slacks:

$$\begin{aligned}
&\text{LNN-}\wedge(x, y) = \\
&\max(0, \min(1, \beta - w_1(1 - x) - w_2(1 - y))) \\
&\text{subject to: } \beta - (1 - \alpha)(w_1 + w_2) + \Delta \geq \alpha \\
&\beta - \alpha w_1 \leq 1 - \alpha + \delta_1 \\
&\beta - \alpha w_2 \leq 1 - \alpha + \delta_2 \\
&w_1, w_2, \delta_1, \delta_2, \Delta \geq 0
\end{aligned}$$

where δ_1, δ_2, and Δ denote slack variables. If any of Constraints (1), (2) and (3) in LNN-$\wedge$ are unsatisfied then slacks help correct the direction of the inequality without putting pressure on parameters w_1, w_2, and β during training. For the rest of the paper, by LNN-$\wedge$ we refer to the above formulation. LNN negation is a pass-through operator: LNN-$\neg(x) = 1 - x$, and LNN disjunction is defined in terms of LNN-$\wedge$:

$$\text{LNN-}\vee(x, y) = 1 - \text{LNN-}\wedge(1 - x, 1 - y)$$

While vanilla backpropagation cannot handle linear inequality constraints such as Constraint (1), specialized learning algorithms are available within the LNN framework. For more details, please check Riegel et al. (2020)

4 LNN-EL

An overview of our neuro-symbolic approach for entity linking is depicted in Figure 3. We next discuss the details about feature generation component that generates features using a catalogue of feature functions (Sec. 4.1) followed by proposed model that does neuro-symbolic learning over user provided EL algorithm in Sec. 4.2.

Features	Description
Name	$sim(m_i, e_{ij})$, where sim is a general purpose string similarity function such as Jaccard ($jacc$), JaroWinkler (jw), Levenshtein (lev), Partial Ratio (pr), etc.
Context	$Ctx(m_i, e_{ij})$ $= \sum_{m_k \in M \setminus \{m_i\}} pr(m_k, e_{ij}.\mathsf{desc})$ where m_k is a mention in the context of m_i
Type	$Type(m_i, e_{ij})$ $= \begin{cases} 1 & \text{if } m_i.\mathsf{type} \in e_{ij}.\mathsf{dom} \\ 0, & \text{otherwise} \end{cases}$ where $m_i.\mathsf{type}$ is the type of the mention and $e_{ij}.\mathsf{dom}$ is the set of domains
Entity Prominence	$Prom(e_{ij}) = indegree(e_{ij})$, i.e., number of links pointing to entity e_{ij}

Table 1: Non-embedding based feature functions.

Given the input text T, together with labeled data in the form (m_i, C_i, L_i), where $m_i \in M$ is a mention in T, C_i is a list of candidate entities e_{ij} (drawn from lookup services[3]) for the mention m_i, and where each $l_{ij} \in L_i$ denotes a link/not-link label for the pair (m_i, e_{ij}). The first step is to generate a set $F_{ij} = \{f_k(m_i, e_{ij})\}$ of features for each pair (m_i, e_{ij}), where f_k is a feature function drawn from a catalog $\mathcal{F}$ of user provided functions.

4.1 Feature Functions

Our collection of feature functions include both non-embedding and embedding based functions.

Non-embedding based. We include here a multitude of functions (see Tab. 1) that measure the similarity between the mention m_i and the candidate entity e_{ij} based on multiple types of scores.

Name: a set of general purpose string similarity functions[4] such as *Jaccard*, *Jaro Winkler*, *Levenshtein*, *Partial Ratio*, etc. are used to compute the similarity between m_i and e_{ij}'s name.

Context: aggregated similarity of m_i's context to the description of e_{ij}. Here, we consider the list of all other mentions $m_k \in M$ ($k \neq i$) as m_i's context, together with e_{ij}'s textual description obtained using KG resources[5]. The exact formula we use is shown in Tab. 1, where *Partial Ratio*(pr) measures the similarity between each context mention and the description. (*Partial Ratio* computes the maximum similarity between a short input string and substrings of a second, longer string.) For normalizing the final score, we apply a min-max rescaling

[3] https://lookup.dbpedia.org

[4] pypi.org/project/py-stringmatching

[5] dbpedia.org/sparql

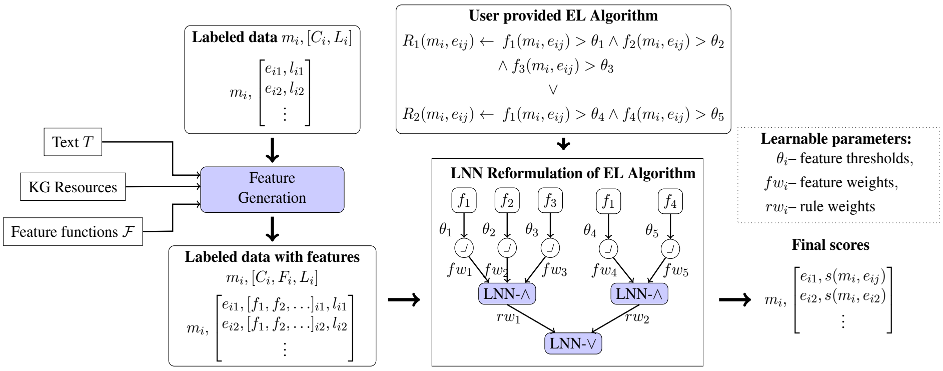

Labeled data $m_i, [C_i, L_i]$
$m_i, \begin{bmatrix} e_{i1}, l_{i1} \\ e_{i2}, l_{i2} \\ \vdots \end{bmatrix}$
Text T
KG Resources
Feature functions $\mathcal{F}$
Feature Generation
Labeled data with features
$m_i, [C_i, F_i, L_i]$
$m_i, \begin{bmatrix} e_{i1}, [f_1, f_2, \ldots]_{i1}, l_{i1} \\ e_{i2}, [f_1, f_2, \ldots]_{i2}, l_{i2} \\ \vdots \end{bmatrix}$
User provided EL Algorithm
$R_1(m_i, e_{ij}) \leftarrow f_1(m_i, e_{ij}) > \theta_1 \wedge f_2(m_i, e_{ij}) > \theta_2 \wedge f_3(m_i, e_{ij}) > \theta_3$
$\vee$
$R_2(m_i, e_{ij}) \leftarrow f_1(m_i, e_{ij}) > \theta_4 \wedge f_4(m_i, e_{ij}) > \theta_5$
LNN Reformulation of EL Algorithm
f_1 f_2 f_3 f_1 f_4
θ_1 θ_2 θ_3 θ_4 θ_5
fw_1 fw_2 fw_3 fw_4 fw_5
LNN-$\wedge$
LNN-$\wedge$
rw_1 rw_2
LNN-$\vee$
Learnable parameters:
θ_i– feature thresholds,
fw_i– feature weights,
rw_i– rule weights
Final scores
$m_i, \begin{bmatrix} e_{i1}, s(m_i, e_{ij}) \\ e_{i2}, s(m_i, e_{i2}) \\ \vdots \end{bmatrix}$

Figure 3: Overview of our approach

over all entities $e_{ij} \in C_i$.

Type: the overlap similarity of mention m_i's type to e_{ij}'s domain (class) set, similar to the domain-entity coherence score proposed in (Nguyen et al., 2014). Unlike in (Nguyen et al., 2014), instead of using a single type for all mentions in M, we obtain type information for each mention m_i using a trained BERT-based entity type detection model. We use KG resources [5] to obtain e_{ij}'s domain set, similar to Context similarity.

Entity Prominence: measure the prominence of entity e_{ij} as the number of entities that link to e_{ij} in target KG, i.e., $indegree(e_{ij})$. Similar to Context score normalization, we apply min-max rescaling over all entities $e_{ij} \in C_i$.

Embedding based. We also employ a suite of pre-trained or custom trained neural language models to compute the similarity of m_i and e_{ij}.

Pre-trained Embedding Models. These include SpaCy's semantic similarity[6] function that uses Glove (Pennington et al., 2014) trained on Common Crawl. In addition to SpaCy, we also use scores from an entity linking system such as BLINK (Wu et al., 2020) (a state-of-the-art entity linking model) as a feature function in our system.

BERT Embeddings. To further explore the semantics of the context in T and the inherent structure of the target KG, we incorporate an embedding-based similarity by training a mini entity linking model without any aforementioned prior information. We first tag the input text T with a special token $[\text{MENT}]$ to indicate the position of mention m_i, and then encode T with BERT, i.e., $\mathbf{m_i} = \text{BERT}(m_i, T)$. Each candidate e_{ij} is encoded with a pre-trained graph embedding Wiki2Vec (Yamada et al., 2020), i.e., $\mathbf{e_{ij}} = \text{Wiki2Vec}(e_{ij})$. The candidates are ranked in order of the cosine similarity to $\mathbf{m_i}$, i.e., $Sim_{cos}(\mathbf{m_i}, \mathbf{e_{ij}})$. The mini EL model is optimized with margin ranking loss so that the correct candidate is ranked higher.

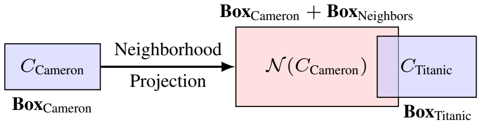

$\textbf{Box}_{\text{Cameron}} + \textbf{Box}_{\text{Neighbors}}$
C_{Cameron}
Neighborhood
Projection
$\mathcal{N}(C_{\text{Cameron}})$
C_{Titanic}
$\textbf{Box}_{\text{Cameron}}$
$\textbf{Box}_{\text{Titanic}}$

Figure 4: Candidates for linking the 'Titanic' mention appear in the intersection of the two boxes.

BERT with Box Embeddings. While features such as *Context* (see Tab. 1) can exploit other mentions appearing within the same piece of text, they only do so via textual similarity. A more powerful method is to jointly disambiguate the mentions in text to the actual entities in the KG, thus exploiting the structural context in the KG. Intuitively, the simultaneous linking of co-occurring mentions in text to related entities in the KG is a way to reinforce the links for each individual mention. To this end, we adapt the recent Query2Box (Ren et al., 2020), whose goal is to answer FOL queries over a KG. The main idea there is to represent sets of entities (i.e., queries) as contiguous regions in embedded space (e.g., axis-parallel hyper-rectangles or boxes), thus reducing logical operations to geometric operations (e.g., intersection).

Since Query2Box assumes a well-formed query as input, one complication in directly applying it to our setting is that we lack the information necessary to form such an FOL query. For instance, in the example from Sec. 1, while we may assume that the correct entities for our `Cameron` and `Titanic` mentions are connected in the KG, we do not know *how* these are connected, i.e., via which relation. To circumvent this challenge, we introduce a special *neighborhood* relation $\mathcal{N}$, such that $v \in \mathcal{N}(u)$ whenever there is some KG relation from entity u

[6] spacy.io/usage/vectors-similarity

to entity v. We next define two box operations:

$$\begin{aligned}\textbf{Box}(C_i) = \{\mathbf{v}|\, &\min(\{\mathbf{e_{ij}}|e_{ij} \in C_i\}) \\ &\preceq \mathbf{v} \preceq \max(\{\mathbf{e_{ij}}|e_{ij} \in C_i\})\} \\ \textbf{Box}(\mathcal{N}(C_i)) = \textbf{Box}&(C_i) + \textbf{Box}_{\mathcal{N}}\end{aligned}$$

The first operation represents mention m_i as a box, by taking the smallest box that contains the set C_i of candidate entities for m_i. This can be achieved by computing the dimension-wise minimum (maximum) of all entity embeddings in C_i to obtain the lower-left (upper-right) corner of the resulting box. The second operation takes m_i's box and produces the box containing its neighbors in the KG. Query2Box achieves this by representing $\textbf{Box}_{\mathcal{N}}$ via a center vector ψ and offset vector ω, both of which are learned parameters. The box of neighbors is then obtained by translating the center of m_i's box by ψ and adding the offset ω to its side.

Fig. 4 shows how these operations are used to disambiguate `Titanic` while exploiting the co-occurring mention `Cameron` and the KG structure. We take the box for `Cameron`, compute its neighborhood box, then intersect with the `Titanic` box. This intersection contains valid entities that can disambiguate `Titanic` *and* are connected to the entity for `Cameron`. For the actual score of each such entity, we take its distance to the center of the intersection box and convert it to a similarity score $Sim_{box}(\mathbf{m_i}, \mathbf{e_{ij}})$. We then linearly combine this with the BERT-based similarity measure: $\beta_{box} Sim_{box}(\mathbf{m_i}, \mathbf{e_{ij}}) + Sim_{cos}(\mathbf{m_i}, \mathbf{e_{ij}})$, where β_{box} is a hyper-parameter that adjusts the importance of the two scores. The approach described can be easily extended to more than two mentions.

4.2 Model

In this section, we describe how an EL algorithm composed of a disjunctive set of rules is reformulated into LNN representation for learning.

Entity Linking Rules are a restricted form of FOL rules comprising of a set of Boolean predicates connected via logical operators: conjunction ($\wedge$) and disjunction ($\vee$). A Boolean predicate has the form $f_k > \theta$, where $f_k \in \mathcal{F}$ is one of the feature functions, and θ can be either a user provided or a learned threshold in $[0, 1]$. Fig. 5(a) shows two example rules R_1 and R_2, where, for instance, $R_1(m_i, e_{ij})$ evaluates to `True` if both the predicate $jacc(m_i, e_{ij}) > \theta_1$ and $Ctx(m_i, e_{ij}) > \theta_2$ are `True`. Rules can be disjuncted together to form a larger EL algorithm, as the one shown in Fig. 5(b), which states that $Links(m_i, e_{ij})$ evaluates to `True` if any one of its rules evaluates to `True`. The $Links$ predicate is meant to store high-quality links between mention and candidate entities that pass the conditions of at least one rule. The EL algorithm also acts as a scoring mechanism. In general, there are many ways in which scores can computed. In a baseline implementation (no learning), we use the scoring function in Fig. 5(c), where rw$_i$ denote manually assigned rule weights, while fw$_i$ are manually assigned feature weights.

(a) <u>`EL Rules`</u>

$$R_1(m_i, e_{ij}) \leftarrow jacc(m_i, e_{ij}) > \theta_1 \wedge Ctx(m_i, e_{ij}) > \theta_2$$

$$R_2(m_i, e_{ij}) \leftarrow lev(m_i, e_{ij}) > \theta_3 \wedge Prom(m_i, e_{ij}) > \theta_4$$

(b) <u>`EL Algorithm`</u>

$$Links(m_i, e_{ij}) \leftarrow R_1(m_i, e_{ij}) \vee R_2(m_i, e_{ij})$$

(c) <u>`Scoring`</u>

$$s(m_i, e_{ij}) = \\ + \begin{pmatrix} rw_1 \times ((fw_1 \times jacc(m_i, e_{ij}) \times (fw_2 \times Ctx(m_i, e_{ij})) \\ rw_2 \times ((fw_3 \times jacc(m_i, e_{ij}) \times (fw_4 \times Ctx(m_i, e_{ij})) \end{pmatrix}$$

Figure 5: Example of entity linking rules and scoring.

An EL algorithm is an explicit and extensible description of the entity linking logic, which can be easily understood and manipulated by users. However, obtaining competitive performance to that of deep learning approaches such as BLINK (Wu et al., 2020) requires a significant amount of manual effort to fine tune the thresholds θ_i, the feature weights (fw$_i$) and the rule weights (rw$_i$).

LNN Reformulation. To facilitate learning of the thresholds and weights in an EL algorithm, we map the Boolean-valued logic rules into the LNN formalism, where the LNN constructs – LNN-$\vee$ (for logical OR) and LNN-$\wedge$ (for logical AND) – allow for continuous real-valued numbers in $[0, 1]$. As described in Sec. 3.2, LNN-$\wedge$ and LNN-$\vee$ are a weighted real-valued version of the classical logical operators, where a hyperparameter α is used as a proxy for 1. Each LNN operator produces a value in $[0, 1]$ based on the values of the inputs, their weights and bias β. Both the weights and β are learnable parameters. The score of each link is based on the score that the LNN operators give, with an added complication related to how we score the feature functions. To illustrate, for the EL rules in Fig. 5, the score of a link is computed as:

$$s(m_i, e_{ij}) = \\ \text{LNN-}\vee \left(\begin{array}{l} \text{LNN-}\wedge \begin{pmatrix} TL(jacc(m_i, e_{ij}), \theta_1), \\ TL(Ctx(m_i, e_{ij}), \theta_2) \end{pmatrix}, \\ \\ \text{LNN-}\wedge \begin{pmatrix} TL(lev(m_i, e_{ij}), \theta_3), \\ TL(Prom(m_i, e_{ij}), \theta_4) \end{pmatrix} \end{array} \right)$$

Dataset	Train		Test	
	|Q|	|E|	|Q|	|E|
LC-QuAD 1.0 (Trivedi et al., 2017)	4,000	6,823	1000	1,721
QALD-9 (Usbeck et al., 2018)	408	568	150	174
WebQSP$_{\text{EL}}$ (Li et al., 2020)	2974	3,237	1603	1,798

Table 2: Characteristics of the datasets.

Here the top-level LNN-$\vee$ represents the disjunction $R_1 \vee R_2$, while the two inner LNN-$\wedge$ capture the rules R_1 and R_2 respectively. For the feature functions with thresholds, a natural scoring mechanism would be to use $\texttt{score}(f > \theta) = f$ if $f > \theta$ else 0, which filters out the candidates that do not satisfy the condition $f > \theta$, and gives a non-zero score for the candidates that pass the condition. However, since this is a step function which breaks the gradient flow through a neural network, we approximate it via a smooth function $TL(f, \theta)$ $= f \cdot \sigma(f - \theta)$, where σ is Sigmoid function and θ is the learnable threshold that is generated using σ, i.e., $\theta = \sigma(\gamma)$, to ensure that it lies in $[0, 1]$.

Training. We train the LNN formulated EL rules over the labeled data and use a margin-ranking loss over all the candidates in C_i to perform gradient descent. The loss function $\mathcal{L}(m_i, C_i)$ for mention m_i and candidates set C_i is defined as

$$\sum_{e_{in} \in C_i \setminus \{e_{ip}\}} \max(0, -(s(m_i, e_{ip}) - s(m_i, e_{in})) + \mu)$$

Here, $e_{ip} \in C_i$ is a positive candidate, $C_i \setminus \{e_{ip}\}$ is the set of negative candidates, and μ is a margin hyper parameter. The positive and negative labels are obtained from the given labels L_i (see Fig. 3).

Inference. Given mention m_i and candidate set C_i, similar to training, we generate features for each mention-candidate pair (m_i, e_{ij}) in the feature generation step. We then pass them through the learned LNN network to obtain final scores for each candidate entity in C_i as shown in Fig. 3.

5 Evaluation

We first evaluate our approach w.r.t performance & extensibility, interpretability and transferability. We also discuss the training and inference time. At last, we compare RuleEL and LNN-EL approaches with Logistic Regression.

Datasets. As shown in Tab. 2, we consider three short-text QA datasets. LC-QuAD and QALD-9 are datasets comprising of questions (Q) over DBpedia together with their corresponding SPARQL queries. We extract entities (E) from SPARQL queries and manually annotate mention spans. WebQSP$_{\text{EL}}$ dataset (Li et al., 2020) comprises of both mention spans and links to the correct entity. Since the target KG for WebQSP is Wikidata, we translate each Wikidata entity to its DBpedia counterpart using DBpedia Mappings[7]. In addition, we discard mentions that link to DBpedia concepts (e.g., `heaviest player` linked to `dbo:Person`) and mentions m_i with empty result (i.e., $C_i = \phi$) or all not-link labels (i.e, $\forall l_{ij} \in L_i, l_{ij} = 0$)[8].

Baselines. We compare our approach to (1) *BLINK* (Wu et al., 2020), the current state-of-the-art on both short-text and long-text EL, (2) three BERT-based models - (a) *BERT*: both mention and candidate entity embeddings are obtained via BERT$_{base}$ pre-trained encoder, similar to (Gillick et al., 2019), (b) *BERTWiki*: mention embeddings are obtained from BERT$_{base}$, while candidate entity is from pretrained Wiki2Vec (Yamada et al., 2020), (c) *Box*: BERTWiki embeddings finetuned with Query2Box embeddings (see Sec. 4.1).

Furthermore, we use the following variants of our approach: (4) *RuleEL*: a baseline rule-based EL approach with manually defined weights and thresholds, (5) *LogicEL*: a baseline approach built on RuleEL where only the thresholds are learnable, based on product t-norm (see Sec. 3.2), (6) *LNN-EL*: our core LNN-based method using non-embedding features plus SpaCy, and (7) *LNN-EL$_{ens}$*: an ensemble combining core LNN-EL with additional features from existing EL approaches, namely BLINK and Box (we consider Box, as it outperforms BERT and BERTWiki on all datasets). Detailed rule templates are provided in App. A.3.

Setup. All the baselines are trained for 30 epochs, except for BLINK which we use as a zero-shot approach. For BERT approaches, we use BERT$_{\text{base}}$ as pretrained model. We used two Nvidia V100 GPUs with 16GB memory each. We perform hyper-parameter search for margin μ and learning rates in the range $[0.6, 0.95]$, $[10^{-5}, 10^{-1}]$ respectively.

5.1 Results

Overall Performance. As seen in Tab. 3, among logic-based approaches, LNN-EL outperforms LogicEL and RuleEL, showing that parameterized real-valued LNN learning is more effective than the non-parameterized version with t-norm (LogicEL) and the manually tuned RuleEL. BERT-based ap-

[7] `http://mappings.dbpedia.org/`

[8] Please check our arXiv version to access the datasets.

Model	LC-QuAD			QALD-9			WebQSP$_{\text{EL}}$		
	Precision	Recall	F1	Precision	Recall	F1	Precision	Recall	F1
BLINK	87.04	87.04	87.04	89.14	89.14	89.14	92.15	92.05	92.10
BERT	57.14	63.09	59.97	55.46	61.11	58.15	70.26	72.15	71.20
BERTWiki	66.96	73.85	70.23	55.46	61.11	58.15	81.11	83.29	82.19
Box	67.31	74.32	70.64	68.91	75.93	72.25	81.53	83.72	82.61
RuleEL	79.82	80.10	79.96	81.55	75.15	78.22	76.56	74.55	75.54
LogicEL	86.68	86.48	86.58	83.05	83.05	83.05	82.60	82.58	82.59
LNN-EL	87.74	87.54	87.64	88.52	88.52	88.52	85.11	85.05	85.08
LNN-EL$_{ens}$	**91.10**	**90.90**	**91.00**	**91.38**	**91.38**	**91.38**	**92.17**	**92.08**	**92.12**

Table 3: Performance comparison of purely neural approaches with our neuro-symbolic variants.

proaches (BERT, BERTWiki, Box) that are trained on the QA datasets underperform the logic-based approaches, which incorporate finer-grained features. BLINK (also a BERT-based approach, but trained on the entire Wikipedia) is used as zero-shot approach and achieves SotA performance (when not counting the LNN-EL variants). The core LNN-EL version is competitive with BLINK on LC-QuAD and QALD-9, despite being a rule-based approach. Furthermore, LNN-EL$_{ens}$, which combines the core LNN-EL with both BLINK and Box features, easily beats BLINK on LC-QuAD and QALD-9 and slightly on WebQSP$_{\text{EL}}$. We also observe similar patterns of LNN-EL and BLINK model performance on metric Recall@k in Tab. 4.

Dataset	Model	R@5	R@10	R@64
LC-QuAD	BLINK	94.69	96.01	96.92
	LNN-EL	93.66	94.39	97.56
	LNN-EL$_{ens}$	97.07	97.20	97.68
QALD-9	BLINK	93.39	93.39	94.29
	LNN-EL	92.72	95.94	98.04
	LNN-EL$_{ens}$	94.63	94.63	95.48
WebQSP$_{\text{EL}}$	BLINK	97.40	97.64	98.61
	LNN-EL	93.54	95.12	96.59
	LNN-EL$_{ens}$	96.34	96.59	96.95

Table 4: Top K performance of LNN-EL models

Extensibility. Here, we inspect empirically how a multitude of EL features coming from various black-box approaches can be combined in a principled way with LNN-EL, often leading to an overall better performance than the individual approaches. A detailed ablation study of the core LNN-EL version can be found in App. A.2. As seen in Tab. 5, approaches like BERTWiki and Box which in isolation underperform compared to LNN-EL, help boost the latter's performance if they are included as predicates. Similarly, LNN-EL which has comparable performance to BLINK, can accommodate the latter's score to produce better performance (see LNN-EL$_{+\texttt{BLINK}}$). We also note that adding features is not a guarantee to improve performance, as LNN-EL$_{ens}$ (which includes both BLINK and Box) slightly underperforms LNN-EL$_{+\texttt{BLINK}}$ on WebQSP$_{\text{EL}}$. For such cases, the interpretability of LNN-EL (discussed next) can help users select the right features based on their relative importance.

Dataset	LNN-EL	LNN-EL +BLINK	LNN-EL +BERTWiki	LNN-EL +Box	LNN-EL$_{ens}$
LC-QuAD	87.64	90.24	88.23	89.05	**91.00**
QALD-9	88.52	90.96	86.41	88.52	**91.38**
WebQSP$_{\text{EL}}$	85.08	**92.32**	91.70	91.44	92.12

Table 5: F1 scores of LNN-EL with additional features coming from various black-box EL approaches.

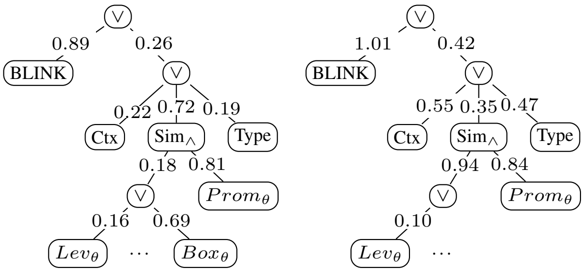


Figure 6: Feature weights of two models LNN-EL$_{ens}$(left) and LNN-EL$_{+\texttt{BLINK}}$ (right) on WebQSP$_{\text{EL}}$

Interpretability. Unlike black-box models, rule-based approaches provide the capability to inspect the model, specifically on how the features impact performance. This inspection can help in dropping or adjusting features that are detrimental. For instance, consider our case of LNN-EL$_{+\texttt{BLINK}}$ and LNN-EL$_{ens}$ trained on WebQSP$_{\text{EL}}$ dataset, where we observed that LNN-EL$_{ens}$'s performance is inferior to LNN-EL$_{+\texttt{BLINK}}$ even though the former model has more features. A human expert can find insights into this behavior by looking at the feature weights in each model. In Fig. 6 (left), the disjunction tree with the *Box* feature is given a low weight of 0.26, thus discounting some of the other useful features in the same tree. Removal of the Box feature leads to a re-weighting of the features in the model; the modified disjunction tree (Fig. 6 (left)) has now a weight of 0.42. Such visualization can help the rule designer to judiciously select features to combine towards building a performant model.

Transferability. To study the transferability aspect, we train LNN-EL on one dataset and evaluate

Train	Test		
	LC-QuAD	QALD-9	$\text{WebQSP}_{\text{EL}}$
LC-QuAD	<u>87.64</u>	86.41	78.90
QALD-9	85.58	<u>88.52</u>	83.06
$\text{WebQSP}_{\text{EL}}$	80.95	87.25	<u>85.08</u>

Table 6: F1 scores of LNN-EL in transfer settings.

the model on the other two, without any finetuning. We use the core LNN-EL variant for this, but similar properties hold for the other variants. Tab. 6 shows F1 scores on different train-test configurations, with diagonal (underlined numbers) denoting the F1 score when trained and tested on the same dataset. We observe that LNN-EL transfers reasonably well, even in cases where training is done on a very small dataset. For example, when we transfer from QALD-9 (with only a few hundred questions to train) to $\text{WebQSP}_{\text{EL}}$, we obtain an F1-score of 83.06 which is within 2 percentage points of the F1-score when trained directly on $\text{WebQSP}_{\text{EL}}$. We remark that the zero-shot BLINK by design has very good transferability and achieves F1 scores of 87.04, 89.14, 92.10 on LC-QuAD, QALD-9, $\text{WebQSP}_{\text{EL}}$ respectively. However, BLINK is trained on the entire Wikipedia, while LNN-EL needs much less data to achieve reasonable transfer performance.

Training and inference time. To study the efficiency of our method, we compute the average training and inference time per epoch for 5 runs for both *LNN-EL* and *LNN-EL*$_{ens}$ models. We want to remark that our proposed model has at most 72 parameters which are several orders of magnitude smaller than the black box models like BLINK. Thus, we observe that the training and inference times of the proposed model are both in the range of 10s of seconds (Table 7 and 8).

training time	*LNN-EL*	*LNN-EL*$_{ens}$
QALD	3.50±0.30	4.21±0.09
$\text{WebQSP}_{\text{EL}}$	16.50±1.08	28.39±0.19
LC-QuAD	40.62±1.05	44.13±1.54

Table 7: average training time per epoch across 5 runs (in seconds)

Comparison with Logistic Regression. To show the power of FOL for entity linking, we also compare LNN-EL against logistic regression (LR) with MarginRankingLoss in Table 9. We feed the same features that LNN-EL uses into LR and find that LNN-EL outperform LR on all three benchmark datasets. After integrating BLINK feature, LR_{blink}'s performance increases dramatically compared with LR but still underperforms its counterpart LNN-EL_{ens}. Apart from performance, our LNN-EL framework is capable of providing human interpretable rules, whereas the LR approach only provides an weighted average of features.

inference time	*LNN-EL*	*LNN-EL*$_{ens}$
QALD	0.21±0.01	1.47±0.08
$\text{WebQSP}_{\text{EL}}$	1.54±0.09	19.29±0.79
LC-QuAD	1.57±0.10	12.96±0.39

Table 8: average inference time across 5 runs (in seconds)

Dataset	Model	Precision	Recall	F1
LC-QuAD	*RuleEL*	79.82	80.10	79.96
	LogisticRegression	87.04	86.83	86.93
	LogisticRegression$_{blink}$	90.50	90.30	90.40
	LNN-EL	87.74	87.54	87.64
	LNN-EL$_{ens}$	91.10	90.90	91.00
QALD-9	*RuleEL*	81.55	75.15	78.22
	LogisticRegression	84.73	84.73	84.73
	LogisticRegression$_{blink}$	88.94	88.94	88.94
	LNN-EL	88.52	88.52	88.52
	LNN-EL$_{ens}$	91.38	91.38	91.38
$\text{WebQSP}_{\text{EL}}$	*RuleEL*	76.56	74.55	75.54
	LogisticRegression	83.39	83.33	83.36
	LogisticRegression$_{blink}$	89.33	89.28	89.31
	LNN-EL	85.11	85.05	85.08
	LNN-EL$_{ens}$	92.17	92.08	92.12

Table 9: Compare Logistic Regression approaches against RuleEL and LNN-EL approaches.

6 Conclusion

We introduced LNN-EL, a neuro-symbolic approach for entity linking on short text. Our approach complements human-given rule templates through neural learning and achieves competitive performance against SotA black-box neural models, while exhibiting interpretability and transferability without requiring a large amount of labeled data. While LNN-EL provides an extensible framework where one can easily add and test new features in existing rule templates, currently this is done manually. A future direction is to automatically learn the rules with the optimal combinations of features.

Acknowledgements

We thank Ibrahim Abdelaziz, Pavan Kapanipathi, Srinivas Ravishankar, Berthold Reinwald, Salim Roukos and anonymous reviewers for their valuable inputs and feedback.

References

Arvind Arasu, Michaela Götz, and Raghav Kaushik. 2010. On active learning of record matching packages. In *Proceedings of the 2010 ACM SIGMOD International Conference on Management of Data*, SIGMOD '10, page 783–794, New York, NY, USA. Association for Computing Machinery.

Stephen H. Bach, Matthias Broecheler, Bert Huang, and Lise Getoor. 2017. Hinge-loss markov random fields and probabilistic soft logic. *J. Mach. Learn. Res.*, 18(1):3846–3912.

Razvan Bunescu and Marius Pasca. 2006. Using encyclopedic knowledge for named entity disambiguation. In *Proceesings of the 11th Conference of the European Chapter of the Association for Computational Linguistics (EACL-06)*, pages 9–16, Trento, Italy.

S. Chaudhuri, Bee-Chung Chen, V. Ganti, and R. Kaushik. 2007. Example-driven design of efficient record matching queries. In *VLDB*.

Silviu Cucerzan. 2007. Large-scale named entity disambiguation based on Wikipedia data. In *Proceedings of the 2007 Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning (EMNLP-CoNLL)*, pages 708–716, Prague, Czech Republic. Association for Computational Linguistics.

Marina Danilevsky, Kun Qian, Ranit Aharonov, Yannis Katsis, Ban Kawas, and Prithviraj Sen. 2020. A survey of the state of explainable AI for natural language processing. In *Proceedings of the 1st Conference of the Asia-Pacific Chapter of the Association for Computational Linguistics and the 10th International Joint Conference on Natural Language Processing*, pages 447–459, Suzhou, China. Association for Computational Linguistics.

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2019. Bert: Pre-training of deep bidirectional transformers for language understanding.

F. Esteva and L. Godo. 2001. Monoidal t-norm based logic: Towards a logic for left-continuous t-norms. *Fuzzy Sets and Systems*.

Richard Evans and Edward Grefenstette. 2018. Learning explanatory rules from noisy data. *JAIR*.

Paolo Ferragina and Ugo Scaiella. 2012. Fast and accurate annotation of short texts with wikipedia pages. *IEEE Softw.*, 29(1):70–75.

Octavian-Eugen Ganea and Thomas Hofmann. 2017. Deep joint entity disambiguation with local neural attention. In *Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing*, pages 2619–2629, Copenhagen, Denmark. Association for Computational Linguistics.

Lise Getoor and Ben Taskar. 2007. *Introduction to Statistical Relational Learning (Adaptive Computation and Machine Learning)*. The MIT Press.

Daniel Gillick, Sayali Kulkarni, Larry Lansing, Alessandro Presta, Jason Baldridge, Eugene Ie, and Diego Garcia-Olano. 2019. Learning dense representations for entity retrieval.

Riccardo Guidotti, Anna Monreale, Salvatore Ruggieri, Franco Turini, Fosca Giannotti, and Dino Pedreschi. 2018. A survey of methods for explaining black box models. *ACM Computing Surveys*.

Mauricio Hernández, Georgia Koutrika, Rajasekar Krishnamurthy, Lucian Popa, and Ryan Wisnesky. 2013. Hil: A high-level scripting language for entity integration. In *Proceedings of the 16th International Conference on Extending Database Technology*, EDBT '13, page 549–560, New York, NY, USA. Association for Computing Machinery.

Johannes Hoffart, Stephan Seufert, Dat Ba Nguyen, Martin Theobald, and Gerhard Weikum. 2012. Kore: Keyphrase overlap relatedness for entity disambiguation. In *Proceedings of the 21st ACM International Conference on Information and Knowledge Management*, CIKM '12, page 545–554, New York, NY, USA. Association for Computing Machinery.

Johannes Hoffart, Mohamed Amir Yosef, Ilaria Bordino, Hagen Fürstenau, Manfred Pinkal, Marc Spaniol, Bilyana Taneva, Stefan Thater, and Gerhard Weikum. 2011. Robust disambiguation of named entities in text. In *Proceedings of the 2011 Conference on Empirical Methods in Natural Language Processing*, pages 782–792, Edinburgh, Scotland, UK. Association for Computational Linguistics.

Belinda Z. Li, Sewon Min, Srinivasan Iyer, Yashar Mehdad, and Wen-tau Yih. 2020. Efficient one-pass end-to-end entity linking for questions. In *EMNLP*.

Lajanugen Logeswaran, Ming-Wei Chang, Kenton Lee, Kristina Toutanova, Jacob Devlin, and Honglak Lee. 2019. Zero-shot entity linking by reading entity descriptions. In *Proceedings of the 57th Annual Meeting of the Association for Computational Linguistics*, pages 3449–3460, Florence, Italy. Association for Computational Linguistics.

Pablo N Mendes, Max Jakob, Andrés García-Silva, and Christian Bizer. 2011. Dbpedia spotlight: shedding light on the web of documents. In *Proceedings of the 7th international conference on semantic systems*, pages 1–8.

Rada Mihalcea and Andras Csomai. 2007. Wikify! linking documents to encyclopedic knowledge. In *Proceedings of the Sixteenth ACM Conference on Conference on Information and Knowledge Management*, CIKM '07, page 233–242, New York, NY, USA. Association for Computing Machinery.

Stephen Muggleton. 1996. Learning from positive data. In *Worshop on ILP*.

D. Nguyen, Johannes Hoffart, M. Theobald, and G. Weikum. 2014. Aida-light: High-throughput named-entity disambiguation. In *LDOW*.

Jeffrey Pennington, Richard Socher, and Christopher D. Manning. 2014. Glove: Global vectors for word representation. In *Empirical Methods in Natural Language Processing (EMNLP)*, pages 1532–1543.

J. Pujara and L. Getoor. 2016. Generic statistical relational entity resolution in knowledge graphs. *ArXiv*, abs/1607.00992.

Lev Ratinov, Dan Roth, Doug Downey, and Mike Anderson. 2011. Local and global algorithms for disambiguation to Wikipedia. In *Proceedings of the 49th Annual Meeting of the Association for Computational Linguistics: Human Language Technologies*, pages 1375–1384, Portland, Oregon, USA. Association for Computational Linguistics.

Hongyu Ren, Weihua Hu, and Jure Leskovec. 2020. Query2box: Reasoning over knowledge graphs in vector space using box embeddings. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net.

Matthew Richardson and Pedro Domingos. 2006. Markov logic networks. *Mach. Learn.*, 62(1–2):107–136.

Ryan Riegel, Alexander Gray, Francois Luus, Naweed Khan, Ndivhuwo Makondo, Ismail Yunus Akhalwaya, Haifeng Qian, Ronald Fagin, Francisco Barahona, Udit Sharma, Shajith Ikbal, Hima Karanam, Sumit Neelam, Ankita Likhyani, and Santosh Srivastava. 2020. Logical neural networks. *arXiv*.

Ahmad Sakor, Isaiah Onando Mulang', Kuldeep Singh, Saeedeh Shekarpour, Maria Esther Vidal, Jens Lehmann, and Sören Auer. 2019. Old is gold: Linguistic driven approach for entity and relation linking of short text. In *Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies, Volume 1 (Long and Short Papers)*, pages 2336–2346, Minneapolis, Minnesota. Association for Computational Linguistics.

W. Shen, J. Wang, and J. Han. 2015. Entity linking with a knowledge base: Issues, techniques, and solutions. *IEEE Transactions on Knowledge and Data Engineering*, 27(2):443–460.

Avirup Sil, Ernest Cronin, Penghai Nie, Yinfei Yang, Ana-Maria Popescu, and Alexander Yates. 2012. Linking named entities to any database. In *Proceedings of the 2012 Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning*, pages 116–127, Jeju Island, Korea. Association for Computational Linguistics.

Parag Singla and Pedro Domingos. 2006. Entity resolution with markov logic. In *Proceedings of the Sixth International Conference on Data Mining*, ICDM '06, page 572–582, USA. IEEE Computer Society.

Fabian M Suchanek, Gjergji Kasneci, and Gerhard Weikum. 2007. Yago: a core of semantic knowledge. In *Proceedings of the 16th international conference on World Wide Web*, pages 697–706.

Priyansh Trivedi, Gaurav Maheshwari, Mohnish Dubey, and Jens Lehmann. 2017. Lc-quad: A corpus for complex question answering over knowledge graphs. In *The Semantic Web – ISWC 2017*, pages 210–218, Cham. Springer International Publishing.

Ricardo Usbeck, Ria Hari Gusmita, Axel-Cyrille Ngonga Ngomo, and M. Saleem. 2018. 9th challenge on question answering over linked data (qald-9) (invited paper). In *Semdeep/NLIWoD@ISWC*.

Denny Vrandečić and Markus Krötzsch. 2014. Wikidata: A free collaborative knowledgebase. *Commun. ACM*, 57(10):78–85.

Jiannan Wang, Tim Kraska, Michael J. Franklin, and Jianhua Feng. 2012. Crowder: Crowdsourcing entity resolution. *Proc. VLDB Endow.*, 5(11):1483–1494.

Ledell Wu, Fabio Petroni, Martin Josifoski, Sebastian Riedel, and Luke Zettlemoyer. 2020. Zero-shot entity linking with dense entity retrieval. In *EMNLP*.

Ikuya Yamada, Akari Asai, Jin Sakuma, Hiroyuki Shindo, Hideaki Takeda, Yoshiyasu Takefuji, and Yuji Matsumoto. 2020. Wikipedia2Vec: An efficient toolkit for learning and visualizing the embeddings of words and entities from Wikipedia. In *Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing: System Demonstrations*, pages 23–30. Association for Computational Linguistics.

Fan Yang, Zhilin Yang, and William W Cohen. 2017. Differentiable learning of logical rules for knowledge base reasoning. In *NeurIPS*.

A Supplementary

A.1 t-norm and t-conorm

While linear classifiers, decision lists/trees may also be considered interpretable, rules expressed in first-order logic (FOL) form a much more powerful, closed language that offer semantics clear enough for human interpretation and a larger range of operators facilitating the expression of richer models. To learn these rules, neuro-symbolic AI substitutes conjunctions (disjunctions) with differentiable t-norms (t-conorms) (Esteva and Godo, 2001). However, since it does not have any learnable parameters, this behavior cannot be adjusted, which limits how well it can model the data. For example, while linear classifiers such as logistic regression can only express a (weighted) sum of features which is similar to logic's disjunction ($\vee$) operator, logic also contains other operators including, but not limited to, conjunction ($\wedge$), and negation ($\neg$).

As opposed to inductive logic programming (Muggleton, 1996) and statistical relational learning (Getoor and Taskar, 2007), neuro-symbolic AI utilizes neural networks to learn rules. Towards achieving this, the first challenge to overcome is that classical Boolean logic is non-differentiable and thus, not amenable to gradient-based optimization (e.g., backpropagation). To address this, neuro-symbolic AI substitutes conjunctions (disjunctions) with differentiable t-norms (t-conorms) (Esteva and Godo, 2001). For example, product t-norm, used in multiple neuro-symbolic rule-learners (Evans and Grefenstette, 2018; Yang et al., 2017), is given by $x \wedge y \equiv xy$, where $x, y \in [0, 1]$ denote input features in real-valued logic. Product t-norm agrees with Boolean conjunction at the extremities, i.e., when x, y are set to 0 (`false`) or 1 (`true`). However, when $x, y \in [0, 1] \setminus \{0, 1\}$, its behavior is governed by the product function. More importantly, since it does not have any learnable parameters, this behavior cannot be adjusted, which limits how well it can model the data.

A.2 Ablation Study

To understand the roles of eac rule in LNN-EL, we also conduct ablation study on the largest benchmark dataset LC-QuAD (see Table 10). We observe that Context is the most performant rule alone. Although PureName rule is behind the other two alone, PureName + Context improves the performance of Context by 1%. Meanwhile, Context + Type only improves Context's performance by 0.05%. Interestingly, the combination of three rules performs slightly worse than PureName + Context by 0.35%. These results show that Type rule is less important among the three rules. To be consistent with the RuleEL system, we apply "PureName + Context + Type" setting for LNN-EL in our experiments.

Dataset	Precision	Recall	F1
PureName	76.03	75.83	75.93
+ Context	88.09	87.89	87.99
+ Type	87.74	87.54	87.64
+ Type	81.46	81.26	81.36
Context	87.04	86.83	86.93
+ Type	87.09	86.88	86.98
Type	87.04	86.83	86.93

Table 10: LNN-EL Ablation Analysis on LC-QuAD

Besides, we also compare the transferability between LNN-EL and LR in Table 11. LNN-EL outpeform LR in 4 out of 6 transferability tests, demonstrating LNN-EL has superior transferability.

A.3 LNN-EL Rules

In our experiments, we explore the following modules, implemented in PyTorch.

Name Rule:

$$\begin{aligned} R_{name} \leftarrow\ & [f_{jacc}(m_i, e_{ij}) > \theta_1 \vee f_{lev}(m_i, e_{ij}) > \theta_2 \\ & \vee f_{jw}(m_i, e_{ij}) > \theta_3 \vee f_{spacy}(m_i, e_{ij}) > \theta_4] \\ & \wedge f_{prom}(m_i, e_{ij}) \end{aligned}$$

Context Rule:

$$\begin{aligned} R_{ctx} \leftarrow\ & [f_{jacc}(m_i, e_{ij}) > \theta_1 \vee f_{lev}(m_i, e_{ij}) > \theta_2 \\ & \vee f_{jw}(m_i, e_{ij}) > \theta_3 \vee f_{spacy}(m_i, e_{ij}) > \theta_4] \\ & \wedge f_{ctx}(m_i, e_{ij}) > \theta_5 \\ & \wedge f_{prom}(m_i, e_{ij}) \end{aligned}$$

Type Rule:

$$\begin{aligned} R_{type} \leftarrow\ & [f_{jacc}(m_i, e_{ij}) > \theta_1 \vee f_{lev}(m_i, e_{ij}) > \theta_2 \\ & \vee f_{jw}(m_i, e_{ij}) > \theta_3 \vee f_{spacy}(m_i, e_{ij}) > \theta_4] \\ & \wedge f_{type}(m_i, e_{ij}) > \theta_5 \\ & \wedge f_{prom}(m_i, e_{ij}) \end{aligned}$$

Blink Rule:

$$\begin{aligned} R_{blink} \leftarrow\ & [f_{jacc}(m_i, e_{ij}) > \theta_1 \vee f_{lev}(m_i, e_{ij}) > \theta_2 \\ & \vee f_{jw}(m_i, e_{ij}) > \theta_3 \vee f_{spacy}(m_i, e_{ij}) > \theta_4] \\ & \wedge f_{blink}(m_i, e_{ij}) \end{aligned}$$

Train	Test		
	LC-QuAD	QALD-9	$\text{WebQSP}_{\text{EL}}$
LC-QuAD	<u>86.93</u>	84.73	76.72
QALD-9	87.14	<u>84.73</u>	80.03
$\text{WebQSP}_{\text{EL}}$	83.42	86.83	<u>83.59</u>

Table 11: F1 scores of LR in transfer settings.

Box Rule:

$$\begin{aligned} R_{box} \leftarrow\ & [f_{jacc}(m_i, e_{ij}) > \theta_1 \vee f_{lev}(m_i, e_{ij}) > \theta_2 \\ & \vee f_{jw}(m_i, e_{ij}) > \theta_3 \vee f_{spacy}(m_i, e_{ij}) > \theta_4] \\ & \vee f_{box}(m_i, e_{ij}) > \theta_5 \end{aligned}$$

BERT Rule:

$$\begin{aligned} R_{bert} \leftarrow\ & [f_{jacc}(m_i, e_{ij}) > \theta_1 \vee f_{lev}(m_i, e_{ij}) > \theta_2 \\ & \vee f_{jw}(m_i, e_{ij}) > \theta_3 \vee f_{spacy}(m_i, e_{ij}) > \theta_4] \\ & \vee f_{bert}(m_i, e_{ij}) > \theta_5 \end{aligned}$$

LNN-EL:

$$R_{LNN-EL} \leftarrow R_{name} \vee R_{ctx} \vee R_{type}$$

LNN-EL$_{+\texttt{BLINK}}$:

$$R_{LNN-EL_{+\texttt{BLINK}}} \leftarrow R_{LNN-EL} \vee R_{blink}$$

LNN-EL$_{ens}$:

$$R_{LNN-EL_{ens}} \leftarrow R_{LNN-EL} \vee R_{blink} \vee R_{box}$$