

Option Value and Storefront Vacancy in New York City *

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Abstract

Why do retail vacancies persist for more than a year in some of the world’s highest-rent retail districts? To explain why retail vacancies last so long (16 months on average), we construct and estimate a dynamic, two-sided model of storefront leasing in New York City. The model incorporates key features of the commercial real estate industry: tenant heterogeneity, long lease lengths, high move-in costs, search frictions, and aggregate uncertainty in downstream retail demand. Consistent with the market norm in New York City, we assume that landlords cannot evict tenants unilaterally before lease expiration. However, tenants can exit leases early at a low cost, and nearly 55% of tenants with ten-year leases exit within five years. We estimate the model parameters using novel data on storefront occupancy and micro data on commercial leases. Move-in costs and heterogeneous tenant quality give rise to heterogeneity in match surplus, which generates option value for vacant landlords. Both features are necessary to explain long-run vacancy rates and the length of vacancy spells: in a counterfactual exercise, eliminating either move-in costs or tenant heterogeneity results in vacancy rates of close to zero. We then use the estimated model to quantify the impact of a retail vacancy tax on long-run vacancy rates, average rents, and social welfare. Vacancies would have to generate negative externalities of \$29.68 per square foot per quarter (about half of average rents) to justify a 1% vacancy tax on assessed property values.

1 Introduction

Why do retail vacancies persist in high-value urban areas, where holding a storefront vacant means forgoing quarterly rents of \$50 or even \$100 per square foot? In Manhattan, the average vacancy spell between 2015 and 2019 lasted 16 months, and only 50% of vacancies were filled after one

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year. We develop a structural model to test the hypothesis that a combination of heterogeneous tenant quality, high move-in costs, and search frictions are jointly necessary to explain long-run vacancy rates and long vacancy durations. High move-in costs take time for tenants to recoup, and generate long contractual lease lengths. In New York City, landlords cannot evict tenants who are compliant with their leases,¹ so they are highly selective when choosing a tenant with whom to contract. Tenants are heterogeneous in their willingness to pay for space, but search frictions prevent landlords from examining all potential tenants at once. This context incentivizes landlords to hold their spaces vacant until they encounter a tenant with a sufficiently high willingness to pay. In a counterfactual exercise, we show that relaxing or eliminating any of these market features would result in landlords filling vacancies much more quickly, driving long-run vacancy rates close to zero.

Downtown retail vacancy has not been widely studied in economics, primarily because of a lack of available data on vacancy rates, as well as individual store transitions into and out of vacancy.² We leverage a novel dataset which tracks storefront vacancy and occupancy at a high frequency with near-universal coverage of Manhattan storefronts. This allows us to compute vacancy rates for different neighborhoods with a high level of accuracy, as well as observe individual storefronts' transitions into and out of vacancy. We combine this storefront vacancy data with micro data on contractual rents and other lease information which is crowdsourced from brokers. To estimate our model, we extend our time series of neighborhood-level vacancy rates with vacancy rates reported by the New York City Office of the Comptroller. Our data is essential for this analysis because vacancy and commercial rent data for most cities is not widely available. City governments hold tax filings closely — New York has collected data on vacancy rates from tax filings since at least 2007, but did not make this information public until 2019³ — and real estate brokerage firms cover much smaller, more highly selected retail corridors.

We use our data to document several facts about the commercial real estate market. First, retail leasing markets have substantial heterogeneity on both sides: different tenants offer differentiated

¹Anecdotal evidence from many market participants suggests that lease renegotiation before expiration is extremely rare (the exception being the months following the onset of the COVID-19 pandemic, which occurred after our study period). This appears to be due to the fact that leases underlie the loans landlords make to tenants, which are then securitized and sold to investors.

²Leasing in suburban malls is comparatively better-studied; see Konishi and Sandfort (2003), Benjamin et al. (1992), Brueckner (1993) and Burayidi and Yoo (2021).

³2019 is also the year the city began its public storefront registry in 2019 under Local Law 157.

goods and services, and the highest and best use of each retail space may differ based on location, size, zoning restrictions, and other characteristics. Much of this heterogeneity is not attributable to characteristics observable to researchers, leading to unexplained rent variation. Second, leases in this market are long: 58% of retail leases in our sample have a contractual term of 10 years.⁴ However, most tenants exit prior to their contractual lease term: conditional on having a 10-year lease, 20% of tenants have exited after two years, and 54.8% of tenants have exited after five years.

We build and estimate a two-sided dynamic search-and-matching model which rationalizes long vacancies, rent dispersion, and early tenant exit using a combination of heterogeneous tenant quality (which is unobservable to the econometrician), high move-in costs, search frictions, and aggregate uncertainty in downstream retail demand. In the model, vacant landlords search for potential tenants, who vary in expected profitability and therefore willingness to pay for space. When landlords encounter tenants, they make take-it-or-leave-it rent offers for 10-year leases. If the tenant accepts the rent offer, they pay an up-front move-in cost and begin operating. Each period, all agents receive new information about downstream retail demand, and tenants choose to continue or exit. The model provides a clear interpretation of the evolution of option value over the course of the contractual relationship: landlords hold option value while they are vacant, and tenants hold it while the lease is in effect.

We estimate the model using simulated method of moments, and develop a novel method which allows us to handle unobservable heterogeneity in landlords' individual states (including their current tenant's quality). Previous search-and-matching models of this type either assume a steady-state environment so that the distribution of states in the market never changes (Brancaccio et al., 2020) or observe heterogeneity (Vreugdenhil, 2020).

Our model allows us to quantify the extent to which search frictions, move-in costs, tenant heterogeneity, and aggregate uncertainty contribute to long-run vacancy rates. This exercise confirms that search frictions, move-in costs, and tenant heterogeneity are all necessary to explain positive long-run vacancy rates. Collapsing tenant heterogeneity eliminates landlord option value entirely, leaving no reason for vacant landlords to hold out for future tenants. Removing move-in costs allows landlords to extract very high rents, leading to higher exit rates but also quicker filling of

⁴By comparison, residential leases usually have a term of one year, and office leases have a term of five to seven years.

vacancies. Aggregate uncertainty amplifies landlord option value: when downstream retail demand is high, tenants earn higher profits on average and landlords have more potential tenants to choose from. This generates more dispersion in long-run vacancy rates across markets than an environment without aggregate uncertainty.

Finally, we impose a counterfactual vacancy tax as a flow cost of vacancy for landlords and solve for the new vacancy rate, distribution of rents, and distribution of tenant quality in the market. This type of vacancy tax is currently under consideration in the New York State legislature.⁵ Proponents of the tax argue that landlords who hold storefronts vacant impose unnecessary costs on their neighborhoods by reducing local economic activity through underutilization of retail space, as well as posing a threat to neighborhood safety via a reduction in "eyes on the street" (Jacobs, 1961). They view the tax as a Pigouvian measure which would cause landlords to internalize the impacts of their vacancies on urban vibrancy. Retail vacancy taxes of this nature have been implemented over the last decade in Washington D.C., San Francisco, and Oakland, California.

We find that the proposed commercial vacancy tax would indeed encourage landlords to fill vacant spaces more quickly, reducing vacancy rates and retail rents. However, the tax would also distort the set of stores present, with lower-earnings stores arriving at opportune moments crowding out higher-earnings stores that might have arrived later. These lower-earnings stores are more likely to exit, increasing retail churn and reducing welfare.

Related literature. Methodologically, our paper grows out of the recent literature estimating dynamic search and matching models in various contexts. Dynamic search and matching models of labor markets (Mortensen and Pissarides, 1994; Hosios, 1990) have been recently adapted to and estimated in other settings, including taxis (Fr chet te et al., 2019), global shipping (Brancaccio et al., 2020), and oil and gas drilling (Vreugdenhil, 2020). We extend these models by allowing for one-sided early contract exit, as well as developing an estimation method for contexts in which agents' individual states are not fully observed.

Because landlords are unable to exit leases unilaterally once they have been signed, our work relates to the modern literature on the importance of uncertainty, adjustment costs, and irreversible investment in various settings. Economists have studied these forces in the context of individual firm investments (Dixit and Pindyck, 1994) as well as in business cycles (Christiano et al., 2005). In urban

⁵Senate Bill S2005/Assembly Bill A670.

contexts specifically, Capozza and Helsley (1990) examine the effect of uncertainty on equilibrium land prices and housing rents in a growing city, finding that uncertainty affects prices even when land owners are risk neutral. There is a growing literature quantifying the magnitude of these forces in particular industries, including shipping (Kalouptside, 2014), manufacturing (Caballero and Engel, 1999), and, most relevant to this paper, real estate (Bulan et al., 2009).

We also build upon the long history of papers on industry dynamics. This literature consists of many theoretical (Jovanovic, 1982; Ericson and Pakes, 1995) and empirical works (Pakes et al., 2007; Bajari et al., 2007; Aguirregabiria and Mira, 2010; Kalouptside, 2014), in a variety of settings. In retail, Jia (2008) studies the impact of Walmart entry on the exit of small firms, and Fang and Yang (2022) study an entry game of competing chains. We focus not on competition between retailers directly, but rather on how tenant entry and exit dynamics give rise to retail vacancy.

Finally, while growing literatures study the response of retail amenities to spatial differences in neighborhood demographics (Almagro and Domínguez-Iino, 2021; Couture and Handbury, 2020), gentrification (Su, 2022; Couture et al., 2021; Glaeser et al., 2020) and the rise of e-commerce (Quan and Williams, 2018), most sidestep the widespread phenomenon of storefront vacancy. Our work is not general equilibrium model of neighborhood choice with endogenous amenities, but is a partial equilibrium investigation into the market between firms and consumers which retailers must participate in before they can sell to final consumers in brick and mortar stores.

The paper proceeds as follows. Section 2 describes our data. Section 3 documents the key institutional details we build into our model, which we then present in section 4 presents the model. Section 5 estimates the model and reports the estimated parameters. In section 6, we quantify the relative strengths of the frictions in our model by counterfactually shutting them down one at a time and looking at the effect on long-run vacancy rates. Section 7 performs the counterfactual vacancy tax exercise. Our model is currently able to fit average vacancy rates but not the trend over 2007-2019; we explore a possible extension to our model that could help explain the trend in section 8. Section 9 concludes.

2 Data

We leverage a novel dataset which tracks storefront vacancy and occupancy at a high frequency with near-universal coverage of Manhattan storefronts. This dataset, constructed by mapping firm Live XYZ, explicitly records the location and duration of vacancies, as well as detailed information about tenants (when present). The Live XYZ dataset covers a limited time span, so we supplement it with vacancy rates from the New York City Comptroller’s Office. We gather information on rents and other contractual features from CompStak. Finally, to capture aggregate uncertainty in downstream retail demand, we add industry-level GDP data from the Bureau of Economic Analysis and the e-commerce share from the Census’s Monthly Retail Sales Report.

Data on commercial vacancy rates and rents has historically been very difficult for economists to obtain. Without our dataset, there are two natural places to obtain it: city governments and brokerage firms. City governments collect information on rent rolls in confidential tax filings, but do not make this information publicly available. Real estate brokerage firms publish semi-annual reports on high-rent retail markets, but these reports are not ideal for economic research. They focus exclusively on highly selected retail corridors rather than on the city as a whole. These reports tend to report on asking rather than taking rents, and track storefront availability rather than vacancy.⁶

2.1 Vacancy and occupancy data

We employ novel data from Live XYZ on the occupancy or vacancy status of the near-universe of Manhattan storefronts between 2016 and 2019. This dataset is key to our study of retail vacancy because data on neighborhood retail vacancy rates is generally not publicly available. In many cities, the government does not systematically collect vacancy data or (as in New York City’s case) does not make it available to the public. Real estate associations such as the Real Estate Board of New York publish quarterly market reports, but often report “availability” rather than vacancy, and only for specific retail corridors (for example, Fifth Avenue between 42nd Street and 49th Street) rather than entire neighborhoods (such as the Upper East Side) or zip codes. While many papers studying retail rely on store trackers (such as Infogroup, Yelp, or Google Reviews), those datasets do not record vacancies directly, and would have required us to infer vacancy from a lack of data on

⁶Available spaces are those for which landlords are actively looking for new tenants. The availability rate tends to be much higher than the actual vacancy rate because of direct store-to-store transitions.

occupancy. With the Live XYZ dataset, we do not have to infer that a storefront is vacant when data on a given store is not included.

The Live XYZ panel allows us to observe not only the vacancy rate in each period, but also the flow in to and out of vacancy, which are key moments we will attempt to match when we estimate our structural model. The dataset tracks detailed information about each storefront’s occupant, or lack thereof, over time, allowing us to construct a panel on storefront vacancy and occupancy at the quarterly level. Specifically, Live XYZ’s dataset consists of a sequence of changes to a storefront’s “state” over time. The state vector consists of an indicator for whether the storefront is occupied, under construction, or vacant; the tenant that occupies it, if there is one; and whether or not the tenant is operating, coming soon, closing soon, temporarily closed, or permanently closed. Each state is labeled with its start and end date. The dataset also records the industry of the tenant (for example, whether it is a restaurant or an apparel store), subcategory (for example, the type of cuisine a restaurant serves), and its parent chain if it has one. Live XYZ tracks changes in a storefront’s state by scraping individual store websites and Facebook pages, calling storefronts, and physically visiting store locations.

The main challenge with the Live XYZ dataset is that it covers a relatively short period of time: the end of 2016 through February 2022.⁷ Therefore, most of the tenants we observe are either installed in a space already by the time the dataset starts, are still in the space when the dataset ends, or both. To avoid selection bias as the dataset is build up, we only use Live XYZ to compute vacancy rates beginning in 2017Q1.

We report summary statistics on this dataset in table 1. We report the number of storefronts we observe in each community district we estimate our model for. We observe a total of 21,811 storefronts across our 8 neighborhoods, and the average vacancy rate across quarters and neighborhoods is 5.23%.

2.2 New York City Comptroller’s Report

We augment the Live XYZ occupancy data using zip code level data on retail vacancy rates from a city report on retail vacancy (Office of the New York City Comptroller, 2019). The report provides vacancy rates for each borough over the 2007-2017 period, as calculated from landlords’ annual

⁷The dataset actually begins in 2015, but rapidly adds observations through the end of 2016.

property tax filings. It provides the same statistic for select (but not all) zip codes.

2.3 Lease contract data

We combine our Live XYZ data with micro data on lease contracts from CompStak. These data allow us to observe contractual rents for different retail spaces and in different time periods. In addition to contractual rents, we observe a lease’s execution date (when the lease is signed), commencement date (when the tenant moves in) and expiration date, the identity of the tenant, and the address of the property. Our CompStak sample contains leases executed between 2005 and 2019.

The benefit of this data is that it gives us a broader picture of rents in a neighborhood than data reported by commercial real estate (CRE) agencies. CRE agencies often publish quarterly or semi-annual reports on the state of the leasing market, but they focus on relatively small and highly selected retail corridors. The Live XYZ data contain leases from the retail corridors that CRE brokers report on, but also contain leases for retail stores in more residential areas. CRE agencies also typically do not report on other contractual features, including commencement and expiration dates.

Rents are almost always quoted in nominal dollars per square foot, and are either constant over the lease term or include fixed step-ups at predetermined dates. The main rent variable we use in our analysis is “net effective rent”, which factors in both contractual rent increases and concessions the landlord grants to the tenant. Where reported, rent step-ups usually occur every 1 to 3 years, and are usually an increase in rent of a few percentage points. Landlord concessions take the form of either tenant improvements (payments the landlord agrees to make to help the tenant renovate the space) or months of free rent (time at the beginning of the lease when the tenant occupies the space without making rent payments). Leases are usually executed the quarter before commencement.

Although the CompStak dataset gives us a relatively broad view of contractual rents within a neighborhood, the data do have a few drawbacks. CompStak crowdsources lease information from commercial real estate brokers, which means our dataset is likely to reflect a selected sample of properties. Brokers are incentivized to share details from contracts they were involved with because doing so allows them to access more lease comparables themselves. This means our sample contains lease information primarily about deals that brokers were involved with and which are not so sensitive that brokers are unwilling to share information about them. Anecdotally, though we do

not have data on the volume of non-brokered deals in New York, the majority of lease transactions are mediated by brokers.

The crowdsourced nature of the CompStak dataset also means that our rent observations are likely to contain measurement error. For example, brokers do not always report the full contractual rent schedule or the lease concessions.

Although these data are imperfect, we believe they are the best data available for this analysis (short of the Real Property Income and Expense filings which the New York City Department of Finance holds very closely). From the CompStak microdata, we extract a time series of average rents from leases executed in each quarter. When we estimate our structural model, we will treat this time series as a moment to match.

Table 1 reports summary statistics from this dataset alongside the summary statistics for the Live XYZ dataset. We have a sample of 7,991 leases for properties located in our 8 community districts, all of which were executed between 2005 and 2019. Given that the CompStak data is a sample of leases while Live XYZ covers the near-universe of storefronts, we observe many fewer leases than storefronts in each neighborhood. We can see that some neighborhoods are better represented in our dataset than others: in particular, we observe the most leases in the three high-end retail districts: Midtown (which contains Times Square and Fifth Avenue), the Lower West Side (which contains SoHo and the Village), and the Upper East Side (which contains Madison Avenue). There is also some selection over time. Figure 2 shows the number of leases executed each quarter. CompStak itself entered in 2012, and the size of their dataset grows over time.

We note that, consistent with a standard notion of market clearing, there is a negative correlation between average rent and vacancy rates: the neighborhood with the highest average vacancy rate and the lowest average rent is the Lower East Side, while the Upper East Side has the highest rents and the second-lowest retail vacancy rate.

2.4 Downstream retail demand

To incorporate aggregate uncertainty in downstream retail demand, we use industry-level GDP from the Bureau of Economic Analysis (BEA). We also collect the brick-and-mortar share of retail sales from the Census’s Monthly Retail Sales report.

3 Institutional Detail

We document four main features of the commercial real estate leasing market that contribute to retail vacancy and will inform our structural model in section 4. First, commercial real estate leasing markets have heterogeneous agents on both sides, and there is reason to believe this market has substantial search frictions. Second, renovating a storefront for a new tenant is costly, and lease terms are long (10 years is most common) in order to create time to recoup these costs. Third, tenants can unilaterally exit leases more easily than landlords. This creates option value for the two parties at different times during their relationship: landlords have option value while vacant, while tenants have option value while the lease is in effect. Finally, during our sample period, the 90th percentile of rents is rapidly falling.

3.1 Heterogeneity and search frictions result in rent dispersion

Real estate markets in general are characterized by high search frictions.⁸ These are markets in which both landlords and tenants are heterogeneous, and finding a “good match” can be challenging. The real estate brokerage industry exists to help ameliorate these search frictions, and make substantial profits doing so.

Furthermore, rent dispersion across New York City retail leases is large, and cannot all be explained by observable characteristics of landlords or tenants. Figure 3 shows the histogram of real rents for each neighborhood in our sample, pooled across all periods. It shows that, in all of our neighborhoods, rents have a long right tail. While most tenants pay between \$10 and \$50 per square foot, in most neighborhood there is a small number who pay upwards of \$75 or even \$100 per square foot.

We use our matched sample of tenants to run a hedonic regression of rents on observable tenant and landlord characteristics. Table 2 shows the results of this regression. Column (1) includes transaction quarter fixed effects, column (2) adds tenant industry fixed effects at the 3-digit NAICS level, column (3) adds zoning fixed effects, and column (4) adds census tract fixed effects. As expected, statistical significance of the correlation between building-related characteristics and rents mostly vanish once zoning and census tract fixed effects are added. The most persistently significant

⁸Han and Strange (2015) reviews several models of search and matching models in housing markets.

coefficients are on the dummy for whether a store opens on to an avenue (a large thoroughfare running north-south, rather than the smaller east-west street) and on whether the tenant is a chain store. We also find that there appears to be a term premium (tenants pay more for longer leases) and a quantity discount (tenants who rent more square feet of space pay less per square foot).

A goal of our model (in section 4) will be to explain the rent variation remaining after controlling for these characteristics. We will explain this residual rent dispersion using search frictions and tenant heterogeneity.

3.2 High move-in costs necessitate long lease terms

The retail real estate market is characterized by very long-term leases, even relative to other commercial real estate markets. Figure 4 shows a histogram of lease lengths in our CompStak dataset. Nearly 60% of leases have a contractual term of 10 years. The second most common lease length is 15 years (accounting for about 12% of leases in our data), followed by 5 years (accounting for about 9% of leases). By comparison, most residential leases carry one year terms, and office leases usually have terms of five to seven years.

Retail leases carry long contractual terms in order to allow time for tenants to recoup high up-front move in costs. Storefronts are a key part of a store’s visual brand, so retailers are willing to invest in custom build-outs that help convey that image. This is especially the case in New York’s high-rent retail districts (such as Madison Avenue, Fifth Avenue, Times Square and SoHo) where large chains place their flagship establishments. Move-in costs can also include items other than renovations, such as specialized equipment (restaurants often rent their large appliances), permit fees, and advertising costs. Unfortunately, data on move-in costs is hard to obtain. We will treat them as a parameter in our structural model.

3.3 Asymmetric contract dissolution costs create option value

Because tenants can exit leases unilaterally at any time during the lease term while landlords are more constrained, tenants and landlords have option value at different points in their contractual relationship: the tenant has option value during the lease, while the landlord has option value while vacant. We seek to study the effect of a counterfactual vacancy tax on both the flow into vacancy (tenants’ exit decision) and the flow out of vacancy (landlords’ lease-up decision). In this section,

our goal is to characterize the nature of the option value that landlords and tenants are able to exercise, and how their option value contributes to vacancy.

Although lease terms are long, most tenants actually exit prior to their lease’s expiration date. Figure 5a plots a histogram of the tenant’s lease age at the time of exit. This plot is made from the sample of 3,248 stores we are able to match across the occupancy dataset and the leasing dataset, from which we observe 2,107 with 10-year leases and 462 exits over Live XYZ’s four years of observation.⁹ We can see that only about ten percent of the tenants in this sample actually exit at exactly the ten-year mark. A very small sample of tenants (composing less than 5% of the matched sample) appear to have renewed their leases, since we see them exit after their contractual lease term. By far the majority of tenants exit early. Exit is more common at the beginning of the lease; the most common age at exit is 2 to 2.5 years. Figure 5b shows that 20% of tenants have exited after two years, and 54.8% of tenants have exited after five years.

Tenants are able to exit early at low cost because of a standard lease clause called the “good guy guarantee.” This provision allows the tenant to provide the landlord with advance notice (usually about 3 months) that they will leave the space on a given date, called the surrender date. The tenant then pays all their rent obligations to the landlord through the surrender date, and vacates the space. All rent obligations after the surrender date are then cancelled. This clause is common because it benefits both parties: the tenant gains limited liability in the case of bankruptcy, while the landlord does not have to worry about evicting a bankrupt tenant and is able to fill the space again more quickly. Of course, the landlord does face a cost when the tenant exits (he stops earning rents, and has to find a new tenant to fill the space), but the ubiquity of the good guy guarantee suggests that on net landlords find it worthwhile to provide tenants with a cheap exit option rather than risk needing to evict a nonpaying tenant.

By contrast, it is costly in time and money for landlords to unilaterally dissolve a lease. To evict an unwilling tenant, they must go to court and show the tenant has broken the terms of the lease. The eviction process takes a long time, and lawyers are expensive. This inability to unilaterally dissolve an ongoing lease means that landlords are very selective when signing leases in the first place. It is often worth it for landlords to remain vacant for several quarters if there is a good chance

⁹CompStak dataset only gives us contractual lease length and not the ex post amount of time the store stays in the space. To determine ex post duration, we match leases to stores in the Live XYZ occupancy dataset, using name, address, and dates of occupancy.

that a high-paying tenant may come along later. The value of remaining vacant that results from long lease terms and landlords’ inability to unilaterally exit from the lease is what we will refer to as “landlord option value” for the remainder of the paper.

Because landlords cannot exit a lease whenever they choose, signing a lease is similar to making an irreversible investment under uncertainty, a la Dixit and Pindyck (1994). A central premise of the real options literature is that there is option value in waiting for new information to arrive, so capital owners often delay investment and/or require compensation for that option value in the form of higher rental rates. Bulan et al. (2009) show this mechanism is quantitatively important in real estate development; our goal is to illustrate a similar mechanism at work in leasing markets for existing buildings.

4 Model

In this section, we present our model of the storefront leasing market which incorporates the market features described in section 3: search frictions, move-in costs, tenant heterogeneity, endogenous tenant exit, and aggregate uncertainty. After estimating the model in section 5, we will show that all of the features in the model are necessary to generate observed long-run vacancy rates. We will evaluate the consequences of a counterfactual vacancy tax in section 7.

Our model is most similar to those of Vreugdenhil (2020) and Brancaccio et al. (2020), but we differ in several ways. Vreugdenhil specifies a two-sided search and matching model with observable heterogeneity on both sides of the market, and seeks to explain a pro-cyclical assortative matching pattern. We assume that there is unobservable heterogeneity on one side of the market, and focus on recovering the distribution of the unobservable types. Our model also explains endogenous early contract dissolution, which (as we documented in the previous section) is an important feature of the commercial real estate market that is not observed in other markets in which search-and-matching models of this type are usually estimated. For example, the shipping model of Brancaccio et al. (2020) assumes that traveling ships arrive at their destination with a fixed, exogenous probability each period. Labor market search-and-matching models often assume that job arrangements are at will. Our model of endogenous contract dissolution is relevant for other settings with long contract terms where market participants cannot commit to a contract. This is a prominent feature of

life insurance markets (Hendel and Lizzeri, 2003) as well as many consumer lending markets (for example, residential mortgages and auto loans).

4.1 Environment

We model the formation and dissolution of leases between landlords and heterogeneous tenants within a neighborhood (which we refer to as a "market"). Within a market, landlords are homogeneous, but tenants differ in quality. Each tenant needs only a single storefront to operate their business. Time is discrete and infinite-horizon. There is no information asymmetry between landlords and tenants, and all leases have a contractual term of T periods. We assume that the set of storefronts is fixed.¹⁰

Every period prior to lease expiration, tenants choose whether to continue operating or exit the market. Tenants base this decision on the realizations of two stochastic state variables: downstream retail demand g_t (which is constant across all tenants and follows a known Markov process), and an opportunity cost ϕ_{it} (which is drawn iid for all tenants each period from a known distribution). If the tenant continues, they earn gross profits (a function of the aggregate state and the tenant's quality), pay rent, and continue to the next period. If the tenant chooses to exit, they cease operating, and earn their opportunity cost. Exiting tenants remain obligated to pay the current period's rent under the good guy guarantee. When a tenant's lease ends, either by endogenous exit or by termout, the tenant exits forever.

At the beginning of each period, each landlord is either vacant or has an incumbent tenant i . When the landlord is vacant, the lease is in its final period before expiration, or i has invoked the good guy guarantee, the landlord draws a new potential tenant n from a fixed, exogenous distribution of potential tenants. Observing n 's type, the landlord either makes n a take-it-or-leave-it rent offer for a lease of fixed length T , or rejects them. If n accepts the rent offer, they move in, sink move-in costs and begin paying rent in period $t + 1$.

The idiosyncratic state variable of a landlord includes its contractual rent due this period, r , and the age of its current lease, j . If the landlord is vacant, then $j_{lt} = \text{vacant}$ and $r_{lt} = 0$.

¹⁰Davidoff (2010) notes that there is almost no vacant land left in Manhattan. Though existing buildings could be redeveloped, especially if zoning restrictions were relaxed, we abstract from entry and exit of storefronts in this model.

The aggregate state follows an AR(1) process:¹¹

$$g_t - \mu_g = \rho_g(g_{t-1} - \mu_g) + \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0, \sigma_g^2) \quad (1)$$

4.2 Payoffs and Behavior

Tenant flow payoffs Each tenant i is endowed with a type θ_i which is drawn from an exogenous distribution F^θ after the tenant enters the market and is fixed for their lifetime. For estimation, we assume F^θ is lognormal with parameters μ_θ and σ_θ .

Let $s_{it} = (j_{it}, r_{it}, g_t)$ denote tenant i 's state in period t . j_{it} is the tenant's lease age in period t , r_{it} is the rent the tenant owes in period t , and g_t is the aggregate state in period t . j_{it} and r_{it} evolve deterministically: rent is simply fixed over the lease's term and j_{it} increases by 1 in period $t + 1$ if the tenant chooses not to exit in period t .

Per-period tenant gross profits are a multiplicative function of the aggregate tenant profitability state g_t , and tenant's quality θ_i . The deterministic portion of gross profits is given by

$$\pi(g_t; \theta_i) = \theta_i g_t \quad (2)$$

In Appendix A we microfound this multiplicative functional form with a model of downstream retail in which consumers have CES utility over varieties and retailers engage in Cournot competition. In this leasing model, θ corresponds to a function of retailer costs and consumer preferences across varieties in the CES demand model. θ is increasing in consumer preferences for a tenant's own good, and decreasing in the tenant's own marginal costs. Variation in our model's aggregate state g corresponds to pure demand shocks in the CES retail demand model.¹²

Existing tenant continuation/exit payoffs Incumbent tenant i observes the current period's aggregate state g_t and opportunity cost ϕ_{it} and chooses whether to continue or exit. If they stay in, they earn net profits (gross profits minus rent), continue to the next period, and repeat the process again. We denote tenant i 's value function when entering state s_{it} by $W(s_{it}; \theta_i)$.

¹¹Note that this AR(1) process can be re-written as $g_t = \delta + \rho_g g_{t-1} + \varepsilon_t$ where $\mu_g = \frac{\delta}{1-\rho}$. We choose to express the transition of the aggregate state in terms of μ_g since μ_g is the long-run mean of g_t .

¹²We do not separately estimate landlord marginal costs and consumer preference parameters because we lack data on retail sales or profits.

The conditional value of choosing to continue on lease is

$$W^{continue}(j_{it}, r_{it}, g_t; \theta_i) = \pi(g_t; \theta_i) - r_{it} + \beta \mathbb{E}_t[W(j_{it} + 1, r_{it}, g_{t+1}; \theta_i) \mid g_t] \quad (3)$$

If the tenant chooses to exit, they immediately cease operations, pay rent r_{it} , and exit, receiving a continuation payoff of 0:

$$W^{exit}(j_{it}, r_{it}, g_t, \phi_{it}; \theta_i) = -r_{it} + \phi_{it} \quad (4)$$

In the final period of the lease, there is not time for the tenant to invoke the good guy guarantee, so the tenant has no exit choice. They simply operate and receive flow profits, so their terminal value is

$$W(T_{it}, r_{it}, g_t; \theta_i) = \pi(g_t; \theta_i) - r_{it} \quad (5)$$

For all ages $j < T$ (prior to the final period of the lease), the tenant's value $W(s_{it}, \phi_{it}; \theta_i)$ is the value of choosing between staying in and exiting:

$$W(s_{it}, \phi_{it}; \theta_i) = \max\{W^{continue}(s_{it}; \theta_i), W^{exit}(s_{it}, \phi_{it}; \theta_i)\} \quad (6)$$

The tenant's continuation value of staying operational is given by the expected value of making the same choice at period $t + 1$, where the expectation is taken over the $t + 1$ draws of the aggregate state and opportunity costs:

$$\mathbb{E}_t W(s_{i,t+1}, \phi_{i,t+1}; \theta_i) = \mathbb{E}_{g_{t+1}, \phi_{i,t+1}} [\max\{W^{continue}(s_{i,t+1}; \theta_i), W^{exit}(s_{i,t+1}, \phi_{i,t+1}; \theta_i)\} \mid g_t] \quad (7)$$

Existing tenant's continuation/exit probabilities In state s_{it} , tenant i will choose to exit if and only if their current opportunity cost draw exceeds a threshold $\phi^*(s_{it}; \theta)$:

$$\phi_{it} > \pi(g_t, \theta_i) + \beta \mathbb{E}_t[W(s_{i,t+1}) \mid s_{it}] \equiv \phi^*(s_{it}; \theta_i) \quad (8)$$

so his exit probability is

$$p^x(s_{it}; \theta_i) = 1 - F_j^\phi(\phi^*(s_{it}; \theta_i)) \quad (9)$$

where F_j^ϕ is the cdf of the opportunity cost distribution.

We index the opportunity cost distribution by the lease age j in order to mimic the empirical distribution of tenants' age at exit in figure 5a. In our model, tenants' continuation value functions are decreasing in their lease age: the older the lease is, the fewer future periods the tenant has remaining in which to earn profits. If we assumed opportunity costs were drawn from the same distribution every period regardless of age, tenants would only exit at the end of their leases, in contrast with the empirical distribution of lease ages at the time of tenant exit which we observe in figure 5a.

Specifically, we assume that when a lease is of age j , the tenant draws their opportunity cost from an Exponential distribution with a mean equal to

$$\sigma_\phi(T - j) \quad (10)$$

so the mean opportunity cost is decreasing in lease age.

The empirical distribution of tenants' age at exit is consistent with Jovanovic (1982)'s theory of industry evolution: tenants learn about their own quality over time, and the high-quality tenants survive while the low-quality tenants fail. However, a model like Jovanovic's is difficult to estimate without data on firm sales or profits, which we lack. Therefore, rather than modeling θ_i as a learning process, we stick with persistent but fixed unobservable tenant heterogeneity, and allow the opportunity cost distribution to vary with tenant age.

Potential tenant's participation constraint If the tenant accepts a rent offer, they will move in at the beginning of the following period (sinking their move-in cost m) and then begin a lease in its first period. The tenant's value of accepting rent offer r in period t (for move-in in period $t + 1$) is therefore given by

$$W^{accept}(r, g_t; \theta_i) = \beta \left(-m + \mathbb{E}_t[W(1, r, g_{t+1}; \theta_i) \mid g_t] \right) \quad (11)$$

and we normalize the tenant's value of rejecting a rent offer to 0.

We can characterize the tenant's value function and behavior using the following propositions, whose proofs can be found in Appendix B.

Proposition 4.1. *The tenant value function $W(j, r, g; \theta)$ is strictly decreasing in rent r for all j, g, θ . The exit probability $p^x(j, r, g)$ is strictly increasing in r for all $j < T, g, \theta$.*

Landlord payoffs We now turn to describing the landlord's payoffs and behavior. In the following description of landlord actions and payoffs, we use i to refer to a landlord's incumbent tenant, and n to refer to a new potential tenant drawn during the period.

A landlord l who is on lease has state $s_{lt} = (j_{lt}, r_{lt}, g_t, \theta_{i(l,t)})$, where (as for the tenant) j_{lt} is the age of the current lease (or an indicator for vacancy), r_{lt} is the contractual rent owed to l in period t , g_t is the aggregate state in period t , and $\theta_{i(l,t)}$ is the type of l 's incumbent tenant i at time t . We note that while θ is a fixed type for the tenant, since landlords have multiple tenants over time, they treat $\theta_{i(l,t)}$ as a state variable which evolves deterministically over the course of a lease and only is uncertain when they are searching for a new tenant.

In all periods in which the landlord has a tenant, the landlord earns contractual rent as a flow payoff. Their continuation value depends on the tenant's continue/exit decision $\gamma(s_{lt})$. If the tenant continues, the landlord's continuation value is simply the discounted expected value of a lease one period older, with the same rent and the same tenant. If the tenant exits (either because their lease expires or the tenant endogenously exits), then the landlord searches, receiving the value of searching given the current value of the aggregate state ($U(g_t)$, which we define below). The landlord's value function when in state s_{lt} can thus be written:

$$V(s_{lt}) = r_{lt} + (1 - p^x(s_{lt}; \theta_{i(l,t)})) \left(\beta \mathbb{E}_t[V(s_{l,t+1}) \mid s_{lt}] \right) + p^x(s_{lt}; \theta_{i(l,t)}) U(g_t) \quad (12)$$

Upon reaching the final period T of a lease with a tenant, the tenant pays the contractual rent and the landlord searches for a new tenant. Therefore the value of a lease in its final period is simply equal to the flow rent payment plus the value of searching:

$$V(T, r_{lt}, g_t, \theta_{i(l,t)}) = r_{lt} + U(g_t) \quad (13)$$

Once a searching landlord has observed a new potential tenant's move-in cost, they choose rent to maximize the value of a first-period lease with that tenant subject to the tenant's participation constraint. Therefore, the value of accepting a tenant of quality θ is:

$$\begin{aligned} V^{accept}(g_t, \theta) &= \max_r \beta \cdot \mathbb{E}_t[V(1, r, g_{t+1}, \theta) \mid g_t] \\ \text{subject to } & W^{accept}(r, g_t, \theta) \geq 0 \end{aligned} \quad (14)$$

Let $r^*(g_t, \theta)$ denote the solution to this problem.

If a searching landlord rejects a tenant, they simply repeat the search process again in the next period. Searching is costless and carries no flow payoff, so landlords who can search always do. Therefore, the conditional value of rejecting a tenant for a vacant landlord is:

$$V^{reject}(g_t) = 0 + \beta \cdot \mathbb{E}_t[U(g_{t+1}) \mid g_t] \quad (15)$$

where the expectation is taken over next period's aggregate state.

Matching function and the value of searching Landlords search whenever they enter the period vacant ($j_{lt} = \text{vacant}$), when their incumbent tenant's lease is expiring ($j_{lt} = T$), or the incumbent tenant has announced their intention to exit this period ($\phi_{i,t} > \phi^*(s_{it})$). We model search very simply: in each period in which the landlord searches, they match with a tenant drawn randomly from the exogenous tenant type distribution with probability p^m . We allow p^m to depend on the aggregate state, and assume it is given by the following functional form:

$$p^m(g_t) = \frac{\exp(\lambda_0 + \lambda_g(g_t - \mu_t)/\sigma_g)}{1 + \exp(\lambda_0 + \lambda_g(g_t - \mu_t)/\sigma_g)} \quad (16)$$

The λ_0 parameter adjusts the average probability of drawing a tenant each period. When λ_0 goes to infinity, the probability of drawing a tenant each period goes to 1; when it goes to negative infinity, the probability of drawing a tenant each period goes to 0.

The λ_g parameter induces variation in search frictions across levels of the aggregate state. If λ_g is positive, then it will be easier for landlords to find a tenant when the aggregate state is high, and harder to match when the aggregate state is low (and vice versa if λ_g is negative). There are two ways to interpret a positive value of λ_g . One interpretation is that more retailers enter and look

for retail space when the aggregate state is high. The other is that search frictions are lower when the aggregate state is high. Since we do not observe searching tenants before they lease a space, we cannot distinguish between the two interpretations.

The λ_g parameter also adjusts the degree of dispersion in V^{reject} and U across levels of the aggregate state. When $\lambda_g = 0$, the landlord's outside option is highest in the highest aggregate state and lowest in the lowest aggregate state. This is because landlords can command higher rents when the aggregate state is high and tenants earn higher profits. Even though the aggregate state is likely to mean-revert during the term of the lease, the more that mean-reversion can be discounted, the higher the rent that landlords can extract. When $\lambda_g > 0$, all else equal, V^{reject} increases for the highest levels of the aggregate state and decreases for the lowest levels of the aggregate state. This increases the landlord's outside option in the good state, and for large enough λ_g s, can generate procyclical vacancy.

If the landlord matches with a tenant, they observe θ_n and choose to either reject n outright or make them a rent offer. Regardless of the landlord's decision about n , they earn flow payoffs r_i according to the contract with their incumbent tenant i (if they have one) and then i exits.

The value of searching, before observing θ_n , is therefore

$$U(g_t) = p^m(g_t)\mathbb{E}_\theta \left[\max\{V^{reject}(g_t), V^{accept}(g_t, \theta)\} \right] + (1 - p^m(g_t))V^{reject}(g_t) \quad (17)$$

Landlord leaseup policy The landlord accepts a tenant of type θ whenever $V^{accept}(g_t; \theta) \geq V^{reject}(g_t)$. The leaseup probability is therefore given by

$$p^l(g_t) = Pr(V^{accept}(g_t; \theta) \geq V^{reject}(g_t)) \quad (18)$$

4.3 Equilibrium

Equilibrium is defined by a set of rents, lease-up probabilities, and exit probabilities such that landlords accept only tenants who are preferable to vacancy (conditional on the aggregate state), rents maximize $V^{accept}(g, \theta_i)$; tenants exit only when $\phi_{it} > \phi^*(s_{it}; \theta_i)$, and all agents have rational expectations.

There is a key distinction between policies which are optimal for individual agents and market-

level equilibrium outcomes. While individual agents' policies depend only on the current state (and expectations over future states conditional on the current state), some aggregate equilibrium objects depend on the composition of agents in the market, which depends on the history of states. Throughout the paper, we will use the term "probability" to refer to an individual agent and "rate" to refer to an aggregate quantity. So, for example, we use "exit probability" to refer to an individual tenant's probability of exit in a given state. We will use the term "exit rate" to refer to the share of incumbent tenants who exit in a given period. We will use bold text to indicate aggregate quantities in mathematical notation.

For example, individual tenant exit probabilities ($p^x(s_t)$) are a function of the current state s_t only. However, the overall market exit rate (the share of incumbent tenants who exit in a given period) is obtained by integrating $p^x(s_t)$ over the distribution of tenants in the market, conditional on the aggregate state g :

$$\mathbf{p}^x_t = \int_{j,r,\theta} p^x(j, r, g; \theta) dF_t(j, r, \theta) \quad (19)$$

where $F_t(j, r, \theta)$ is the joint distribution of lease age, contractual rents, and tenant quality at time t . This joint distribution depends on the last T periods worth of history: there may still be tenants in the market who signed leases T periods ago, and the share of leases of each age j which survived until period t depends on the entire path of the aggregate state from period $t - j$ until t .

The vacancy rate is also a non-Markov equilibrium object. The vacancy rate at the beginning of period t depends on the vacancy rate in period $t - 1$, the share of vacant landlords who signed leases in $t - 1$, and the share of occupied landlords in period $t - 1$ whose tenants exited and who failed to sign new leases. Specifically, the transition of the vacancy rate is given by

$$\nu_t = \nu_{t-1} - \mathbf{p}^l(g_{t-1})\nu_{t-1} + (1 - \nu_{t-1})(1 - \mathbf{p}^l(g_t))\mathbf{p}^x_t \quad (20)$$

That our model generates a non-Markov vacancy rate is a choice: we could expand the state space to include the T most recent values of the aggregate state. However, this dramatic expansion of the the size of the state space would make our model much more difficult to solve.

4.4 Solving the Model

Because our model is a combination of two single-agent dynamic optimization problems (one each for landlords and tenants), we are able to solve the model sequentially. The key idea is that once rent has been set in the contract, the tenant’s exit policy does not depend on the landlord at all. We can therefore solve the tenant’s and landlord’s problems sequentially.

We first discretize the state space. We discretize the aggregate state g into 10 bins, and allow for 40 rent values and 40 tenant types. We compute discrete approximations to the tenant type distribution given the parameters, and compute the aggregate state transition matrix. We then solve each tenant’s value and policy functions for each state $s = (j, r, g)$ and each type θ by backward induction from the end of the lease. Finally, we perform a contraction mapping to solve the landlord’s value function, taking tenant behavior as given. We start the contraction mapping with a guess of the value of searching, $U(g)$, for each value of g . Given that guess, we backward-induct the value of a lease with each tenant type θ at each rent r , from the final period ($j = T$) back to its first period ($j = 1$). Then we find the rent which maximizes the landlord’s value of a lease in its first period for each tenant type, subject to that type’s participation constraint. Finally, we update the guess of $U(g)$ and repeat until convergence.

4.5 Identification

We conclude this section with some intuition about the identification of the model parameters. In estimation, we will recover the parameters associated with the transition of the aggregate state $(\rho_g, \mu_g, \sigma_g)$, the tenant type distribution $(\mu_\theta, \sigma_\theta)$, the opportunity cost distribution (σ_ϕ) , and the matching function (λ_0, λ_g) . The parameters associated with the aggregate state are identified from its transition over time. The mean of the tenant type distribution is identified from average rents. The shape parameter of the tenant type distribution is identified from landlords’ lease-up probabilities. The opportunity cost parameter σ_ϕ is identified from exit rates. λ_0 is also identified from average lease-up rates. The matching parameter λ_g is identified by variation in the lease-up probability across levels of the aggregate state.

The move-in cost parameter m is formally identified by the assumption that tenant profits covary with the aggregate state, but that the tenant type distribution itself is fixed over time. This co-

variation of profits with the aggregate state means a tenant with a given quality θ may be accepted at some levels of the aggregate state and rejected at others. However, we are currently having trouble estimating m separately from μ_θ . We therefore calibrate m and test the sensitivity of the model to changing the calibration.

5 Estimation

We estimate a discrete approximation to the continuous model described in section 4. Estimation proceeds in 2 stages. In the first stage, we estimate the aggregate state process by maximum likelihood. As we saw in the model section, this process is the primary driver of changes in tenant profits, and thus landlords' leasing policies, over time. In the second stage, conditional on the estimated aggregate state process, we recover the remaining model parameters: the distribution of unobserved tenant heterogeneity (μ_θ and σ_θ), the parameter of the scrap distribution σ_ϕ , and the matching function parameters λ_0 and λ_g .

Market Definition We estimate the model separately by neighborhood. Our 8 neighborhoods correspond to Manhattan community districts 1 through 8. Community districts are large, geographically contiguous neighborhoods that are represented by community boards. New York City has 59 Community Districts across all 5 boroughs, ranging in population from 50,000 to 200,000 residents. We focus on the 8 community districts in Manhattan south of 110th street, since these are the areas in which we have sufficient occupancy and leasing data to compute our moments each quarter. While our 8 community districts do not cover the whole city, they do make up the areas with the densest and most valuable retail space. While we would like to estimate the model at a more spatially disaggregated level, the number of leases we observe per market in each quarter begins to rapidly decline as we split the markets into smaller and smaller neighborhoods.

5.1 Estimating the aggregate transition process

The aggregate state variable in the model, g , is meant to represent exogenous downstream retail demand. To construct our empirical aggregate state variable, we select the industry-level GDP series corresponding to our tenants' NAICS categories and average them. Our tenants fall into three main

categories: retail trade, consumer services, and what we call "business services." The retail trade category corresponds to NAICS codes beginning with 44 and 45, including for example furniture, hardware, apparel, grocery and hobby stores. Consumer services corresponds to tenants who provide hospitality, entertainment, repair, or personal services. It includes NAICS codes beginning with 7 and 8, such as restaurants, salons, spas, and shoe repair stores. Finally, many of our tenants fall in a category we call business services, which includes banks, real estate agencies, doctors' offices, and lawyers' offices. This final group of tenants fall in NAICS codes beginning with 5 and 6.

Before taking the average of GDP across industries, we adjust the BEA's industry-level GDP measure for the goods industry in order to account for the growth in e-commerce over our sample period. We compute the national, quarterly, brick-and-mortar share from the Monthly Retail Sales report, and then apply it to each quarter's goods GDP measure.

Our aggregate state variable requires a few modifications before it can be brought to our structural model. Our model is stationary, so we first remove the trend from our aggregate state measure. We then convert the level of real GDP to per-square-foot units using a two-step process. First, we use establishment counts by NAICS code from the County Business Patterns to obtain output per establishment. Then, we use the average square footage of a storefront in each category in our CompStak sample to compute average output per square foot.

Using national data for our aggregate state variable constrains our estimation by ruling out variation in aggregate uncertainty across neighborhoods. For example, retailers in the Financial District and on the Upper West Side face the same g at all times and have the same expectations about the future evolution of g . However, we feel this assumption is not terribly restrictive because of the multiplicative structure of tenant gross profits. The mean and variance of quality θ varies freely across neighborhoods, so tenants in different markets face systematically different flow profit distributions. Similarly, the unobservable type distribution absorbs level differences between our aggregate state measure (GDP per square foot) and tenant profits ($\pi(g, \theta)$ in the model).

We estimate the parameters of the AR(1) process governing the aggregate state using maximum likelihood. The estimated parameters are reported in table 3, and the empirical and fitted series are shown in figure 6. The aggregate state is highly persistent, with an estimated persistence parameter of 0.95. The average value of the aggregate state is \$333.75 per square foot. Our sample period contains a substantial business cycle corresponding to the Great Recession. The aggregate state

only recovers to its mean value in about 2015, just 2 years before the Live XYZ dataset begins.

5.2 Estimating remaining model parameters

We estimate the remaining parameters separately for each market by matching the simulated method of moments. Specifically, for each market, we recover the parameters governing the distribution of unobserved tenant heterogeneity (μ_θ and σ_θ), the scrap value distribution (σ_ϕ), and the matching function (λ_0 and λ_g).

We estimate the parameters using simulated method of moments, and match four aggregate moments over up to 60 quarters each. Our first moment is the average contractual rents for leases signed in each quarter. We construct this moment from the sample of leases in our leasing dataset, which contains leases signed between 2005Q1 and 2019Q4. We are therefore able to match average contractual rents over this entire 60-quarter period.

Our second moment is the vacancy rate in each quarter, which is simply the number of vacant storefronts in the market divided by the total number of storefronts. We concatenate the time series of vacancy rates reported by the Comptroller’s office with the time series of vacancy rates that we calculate from the Live XYZ dataset. The property tax filings ask landlords to report on vacancy rates as of January 5, so we assume the reported vacancy rates correspond to first-quarter vacancy rates, and linearly interpolate to estimate vacancy rates in the intermediate quarters for 2007-2017. There are 2 community districts for which we do not observe vacancy rates for any contained zip codes. For these zip codes, we substitute Manhattan’s overall vacancy rate.

We construct exit and lease-up rates from our occupancy dataset. The comprehensiveness of this data is what allows us to compute these moments at all, but this dataset covers a relatively short period of time (2017Q1 through 2019Q4). We define the exit rate as the share of incumbent tenants who exit in each quarter. We define the lease-up rate as the share of searching landlords who sign leases in a given quarter. We assume that a storefront has leased up in period $t - 1$ if a new tenant appears in period t , and assume that they are searching in t if they are either vacant or their incumbent tenant exits during period t .

Finally, we match the correlations of the aggregate state with each of the moments described above: the exit rate, leaseup rate, vacancy rate, and average rents.

Our moment condition corresponding to period t simply takes the difference between the model-

predicted moments and the observed moments. To construct the GMM moment condition, we stack the period-level moment conditions into a vector, and then append the correlation moments. Our estimated parameters are those that minimize the weighted mean squared error of the model-predicted moments. We weight each moment by the reciprocal of the number of quarters for which it is observed. For example, we observe 60 quarters of average rent for each market, so the average rent moments each receive a weight of $1/60$. Since we are estimating only 5 parameters for each market, the model is over-identified.

Initial Condition The main challenge we address in our estimation is a variant of the initial conditions problem of Heckman (1981). In order to match moments, we need to start our model simulation at some initial distribution of individual landlord states. This state vector includes the aggregate state g , as well as (for occupied storefronts) the age of the lease j , the contractual rent r , and the tenant’s type θ . Because we do not observe the distribution of individual states at the beginning of our sample period, we use our model to simulate it. Our approach is similar to those taken by Pakes (1986) and Ho and Lee (2022), but we adapt it for aggregate uncertainty.

While we do not observe landlords’ full individual state vectors, we do observe the aggregate state g beginning in 2005Q1. We therefore construct an algorithm to draw paths of landlord states which are consistent with the observed aggregate state from 2005Q1-2019Q4.

First, given a draw of the parameters, we simulate the model for many periods, starting all landlords as vacant. The purpose of this simulation is to reach and explore the recurrent class of landlord states (which Ericson and Pakes (1995) show exists). From this simulation, we compute the long-run distribution of individual landlord states. Next, we use the estimated aggregate state parameters (ρ_g, σ_g , and μ_g) to simulate a large number of pre-period aggregate state paths that all end at the the observed aggregate value in 2005Q1. For each simulated aggregate state path, we assume there is a fixed number of landlords, and draw their initial states from the long-run distribution of individual landlord states, conditional on the aggregate state. From this initial state, we simulate the model forward following each aggregate state path, transitioning in 2005Q1 from following the simulated path to the observed path of the aggregate state.

5.3 Results

The estimated parameters for each market are reported in table 4.

As discussed in section 4.5, move-in costs m are formally identified, but we are having trouble estimating them separately from μ_θ . For now, we calibrate the value of m and show the model’s sensitivity to changing the calibration in figure 7. In our model, m captures not just the fixed cost of renovating a space, but also any up-front investment involved in starting their store. Market participants have told us that renovation costs are often upwards of several hundred dollars per square foot. To account for additional costs of starting a retail business, we calibrate move-in costs at \$650 per square foot.

Table 4 shows that in most markets, the unconditional probability that searching landlords get to draw a tenant is 94% or higher. We do not interpret this as evidence that search frictions are not strong; rather, we believe that landlords are in reality able to inspect multiple (but finite) potential tenants per quarter. If landlords could inspect an infinite number of potential tenants each quarter, they would not need to wait for a tenant with high θ to arrive.

The Upper East Side is the only exception, where the probability of matching with a tenant in any given quarter is only 32%. The model fit in general for the Upper East Side is poor, because its average rents are so much higher than any other market in our data. We believe this is due to the fact that these tenants are not small businesses serving local residents, but rather are large (usually luxury) chain tenants who view their flagship Madison Avenue stores as a status symbol that they are willing to operate at a loss. Though we lack data on renewal rates, we believe that most Madison Avenue stores have occupied their spaces for a long time and renewed their leases multiple times. Our model does not account for this behavior, leading to poor fit in this particular market.

6 Quantifying the Sources of Vacancy

In this section, we perform a series of counterfactual exercises with our structural model to quantify the degree to which each market feature (search frictions, move-in costs, tenant heterogeneity, and aggregate uncertainty) contributes to long-run retail vacancy. To determine relative importance of each feature, we scale the parameters associated with each market feature to reduce the intensity

of its effects. We find that search frictions, move-in costs, and tenant heterogeneity are jointly necessary to generate positive long-run vacancy rates.

In our first exercise, we relax search frictions by allowing searching landlords to draw a tenant with probability 1 every period, regardless of the level of the aggregate state. In the model, this happens when the parameter λ_0 goes to infinity and the parameter λ_g is zero. Since in most of our markets, we estimate the probability of drawing a tenant to be 1 or close to 1, we do not expect significant changes in this counterfactual relative to the baseline. However, we do not interpret these estimates as suggesting that this market has weak search frictions - rather, we see it as evidence that we should relax our model to allow landlords to inspect multiple (but finite) tenants per period.

In our second exercise, we set move-in costs m to zero. This has two main effects. First, this relaxes all tenants' participation constraints, allowing landlords to extract much more rent from tenants with a given value of θ . Second, when move-in costs are zero, there are no longer any negative-surplus tenants. This occurs because tenant types are distributed lognormally (so tenant types θ and gross flow profits $\pi(\theta, g) = \theta g$ are bounded below by zero). Therefore, all tenants will generate non-negative profits and therefore non-negative surplus. Landlords may still reject some tenants because they would rather wait for a higher-quality tenant draw, but this is never driven by the fact that the tenant is ex ante negatively profitable. If tenant types followed a distribution with negative values of θ in its support, then setting move-in costs to zero would have a less dramatic effect on the share of positive-surplus tenants.

In our third exercise, we shut down landlord option value by removing tenant heterogeneity. Intuitively, there is no reason to wait for a better tenant to come along if all tenants have the same expected profitability before moving in to a space. For this counterfactual exercise, we change the tenant type distribution to a point mass on the average tenant type for each market (so $\theta = \exp(\mu_\theta + \frac{\sigma_\theta^2}{2})$ for all tenants).

Finally, we repeat all three of these exercises in an environment with no aggregate uncertainty. Specifically, we simulate the model under the assumption that the aggregate state stays at its mean value forever. Removing aggregate uncertainty has several effects. First, it completely shuts down variation in tenant profits over time. This means that the only uncertainty tenants face each period comes from their idiosyncratic opportunity cost draws. Furthermore, there is no longer any dispersion in landlords' outside options across periods. This means that, unlike in the baseline

model, landlords have the same accept/reject threshold in every period.

Results from each exercise are presented in table 5. For each counterfactual, we report the mean, 25th percentile, and 75th percentile of long-run vacancy rates across markets. The left-hand panel shows the results from counterfactuals with aggregate uncertainty, while the right-hand panel removes aggregate uncertainty. Baseline long-run vacancy rates (those predicted at the estimated parameters for each market) are shown in the first three columns of the first row.

The interquartile range of baseline long-run vacancy rates is 7.45-7.96%. Removing aggregate uncertainty has small effects on long-run vacancy rates: the 25th percentile of long-run vacancy rates stays the same, but the 50th and 75th percentiles fall slightly. Aggregate uncertainty has a small effect because, in most markets, landlords' accept/reject thresholds are not very sensitive to the aggregate state at the estimated parameters. As discussed, reducing search frictions has a small effect because the estimated search frictions are all close to 1.

Eliminating move-in costs and tenant heterogeneity both have strong effects. Setting move-in costs to zero reduces the median long-run vacancy rate from 7.79% to 1.45%. This large effect size comes from two mechanisms. First, setting move-in costs to zero means that all tenants are capable of generating positive surplus. When move-in costs are high, some tenants are poor quality and thus will not earn back move-in costs in expectation over the course of their tenure. Reducing move-in costs to zero means that these tenants now generate surplus. Second, landlords are very sensitive to changes in the move-in cost because they fully extract the increase in surplus associated with the reduction of this cost. When move-in costs get low enough, rents are set by the landlord's first order condition rather than the tenant's move-in costs. The effect is likely to be more moderated in a model in which prices are set by Nash bargaining between landlords and tenants, where tenants have some bargaining power.

Collapsing tenant heterogeneity so that all tenants have the mean type for their market does one of two things to the long-run vacancy rate. If the mean tenant type generates positive surplus, the vacancy rate falls to nearly 0% (since the estimated search frictions are small, landlords are almost always able to fill their space immediately and transition straight from one tenant to another without going through a period of vacancy). If move-in costs are sufficiently high, then the mean tenant in some neighborhoods actually will generate ex ante negative surplus. In these neighborhoods, long-run vacancy rates go to 100%. This results in a 75th percentile long-run vacancy rate of 28%.

Removing move-in costs and heterogeneity are rather dramatic changes, and we can glean some insight by tracing out what happens to the long-run vacancy rate as we vary the magnitude of move-in costs and the variance of tenant types more slowly. In figure 7, we plot the long-run vacancy rate as we scale the size of the frictions between the "no friction" counterfactual and the estimated model. This plot shows our results for the Midtown community district; other community districts show qualitatively similar patterns. On the x axis, we scale "friction intensity" from 0 (where the market feature is eliminated, as in table 5) to 1 (the estimated level). At the far right, where the friction intensity is 1, we plot the long-run average vacancy rate at the estimated model parameters. Moving from right to left along the x -axis, we reduce the magnitude of the friction relative to the estimated model, until we arrive at a version of the model where the friction is completely eliminated.

The red line shows how the long-run vacancy rate varies as we change the variance of the tenant type distribution, holding the mean constant. At the far left, when the friction intensity is 0, all tenants have the mean tenant type. At the far right, when the friction intensity is 1, tenant types are drawn from the estimated lognormal distribution. In between, when the friction intensity is α , tenant types are drawn from a lognormal distribution with the same mean as the estimated distribution, but with a variance of α times the estimated variance. This is a fairly monotonic relationship, though our plot shows some noise: as the variance of tenant types increases, the vacancy rate rises because landlords have more option value from waiting.

The blue line shows the long-run vacancy rate when we vary only the move-in cost, and we find that while move-in costs are positive, the long-run vacancy rate traces a U shape. When $m = 0$, the long-run vacancy rate is nearly zero. Under the hood, landlords are charging exorbitant rents and tenants fail often and very early in their leases. However, landlords are able to replace tenants very quickly, so the long-run vacancy rate is low. When move-in costs are positive but are small (when "friction intensity" is about 0.1), landlords still charge very high rents, but are not able to fill vacancies as fast since the share of tenants that generate positive surplus is lower. For intermediate levels of m (when "friction intensity" is about 0.25), rents become more moderated and turnover falls, reducing the long-run vacancy rate. As m approaches its calibrated value, however, rents remain moderated but the share of tenants who generate positive surplus falls, so the long-run vacancy rate climbs.

The green line shows the combined effects of varying both move-in costs and tenant heterogeneity

together.

7 Vacancy Tax Counterfactual

Given the parameter estimates from section 5, we impose a counterfactual vacancy tax (a flow cost of vacancy for landlords) and solve for the new vacancy rate, distribution of rents, and distribution of unobserved tenant profitability conditional on entry. We find that the vacancy tax reduces the vacancy rate and average rents, but distorts the retail mix towards tenants with lower profitability and increases tenant churn. We also use our model to infer the size of the externality implied by the proposed vacancy tax, under some assumptions.

A commercial vacancy tax is currently being debated by the New York State Senate as State Senate Bill S2005 (Jackson, 2021). This bill was originally introduced in the 2019-2020 legislative session, but was tabled for several years during the COVID-19 pandemic. It proposes to tax vacant commercial storefronts in New York City an amount equal to one percent of the assessed value of the property including the vacant storefront. We use our model to predict what would happen to long-run vacancy rates, average rents, and welfare if New York State's proposed vacancy tax were to go into effect.

Why is the state legislature considering imposing a vacancy tax? Policymakers, journalists, and residents often argue that vacancy imposes negative externalities on pedestrians. They argue that it is therefore appropriate to implement a Pigouvian tax which forces landlords to internalize the effect of the vacancy. Many residents think of vacant storefronts as an eyesore or a waste of valuable real estate. Some are concerned that higher retail vacancy poses a threat to neighborhood safety via a reduction in "eyes on the street" (Jacobs, 1961), though there is mixed empirical evidence on whether real estate vacancy is actually associated with increased crime. Chang and Jacobson (2017) find that a short-term mass closing of medical marijuana dispensaries in Los Angeles lead to an immediate increase in crime near those stores. They find similar results for temporary restaurant closures due to health code violations. However, in a study of urban vibrancy and crime in Philadelphia, Humphrey et al. (2020) find that neighborhoods with more vacant land have higher crime rates, but that crimes tend not to occur at vacant properties themselves.

Because of the market features we model (search frictions, move-in costs, tenant heterogeneity,

and aggregate uncertainty), the presence of vacant storefronts is not necessarily evidence of inefficiency in the leasing market. In fact, it may even be socially optimal for landlords to exercise their option value in the interest of signing long-term tenants who will best meet local retail demand. Landlords' exercise of option value is inefficient only if vacancies impose an externality on other market participants (tenants, landlords, or consumers).

New York City is not the only city which has proposed a vacancy tax on some types of real estate in recent years, though the structure of the tax varies widely across municipalities. Washington, D.C. has had a vacancy tax of \$5 per \$100 of assessed value on vacant commercial and residential properties since 2011. In March 2020, a supermajority of San Francisco voters approved a tax on commercial storefronts that remain vacant for more than 182 days. The tax went into effect on January 1, 2022, and is calculated based on a building's street frontage and how long the property has been vacant.¹³ Oakland, California levies a fixed tax of \$3000 per vacant property containing a ground-floor commercial retail vacancy. Vancouver, British Columbia introduced a residential vacancy tax in 2017. In 2021, the Vancouver "Empty Homes Tax" was 3% of assessed taxable value.

7.1 Incorporating the vacancy tax in the model

We incorporate the vacancy tax into our model by adding a flow cost τ of rejecting a tenant to equation 15 when the landlord is vacant:

$$V^{reject}(s_t, g_t) = -\tau \times \mathbf{1}(\text{vacant}_t) + \beta \cdot \mathbb{E}_t[U(g_{t+1}) \mid g_t] \quad (21)$$

Intuitively, a positive value of τ reduces the landlord's outside option, so the tax makes landlords less selective and reduces average rents. Specifically, the tax decreases the landlord's accept/reject threshold $\theta^*(g_t)$ for each value of the aggregate state and increases the probability that a searching tenant signs a lease. Since the marginal tenants are lower quality, their participation constraint binds at lower values of rent. Rents offered to inframarginal tenants are unchanged, because rents hold all tenants to their individual participation constraints. Therefore, average contractual rents fall relative to a world with no vacancy tax.

¹³The tax rate is \$250 per foot of frontage in the first year, \$500 in the second year of vacancy, and \$1000 if the vacancy lasts for 3 or more years.

The vacancy tax decreases vacancy at the cost of higher churn among retail tenants. For a small tax, the marginally accepted tenants are more likely to exit in any given period than the inframarginal tenants because they are lower quality. As the tax gets larger, the marginal tenants are offered such low rents that they exit slightly less frequently, but overall churn remains elevated relative to the no-tax environment.

7.2 Vacancy and Rent Effects

In this section, we quantify the degree to which the proposed tax would reduce long-run vacancy rates and rents.

The parameter τ represents a constant tax in dollars per square foot, while the proposed tax is 1% of the assessed value of the property. For each market, we therefore set τ equal to 1 percent of the landlord's expected value of searching. We feel the value of searching is the appropriate quantity from our model to use proxy for assessed value, since New York City's assessed values for commercial properties are based primarily on the rents landlords report on their annual Real Property Income and Expense filings. When vacant, landlords are by definition not earning any rent, so we expect their property's assessed values to fall relative to periods in which they are occupied. We could instead use the average assessed value of buildings in each neighborhood, but New York City's publicly available data on assessed property values does not identify which buildings contain vacant storefronts.

We next simulate the model given the proposed tax to determine the quantitative impact of the tax on long-run vacancy rates and average rents. We first simulate the model for a many periods, in order to reach the recurrent class. For each level of the aggregate state g , we compute the average vacancy rate and average rent for each lease signed across all periods in which the aggregate state was g . Finally, we arrive at the long-run average outcomes by averaging the expected outcomes conditional on g over the long-run stationary distribution of g .

We show the effects of the vacancy tax on long-run vacancy rates and average rents in table 6. Long-run vacancy rates fall by 0.23 percentage points on average, or about four percent. Average rents fall by 0.44 percent. This is driven entirely by the change in landlords' acceptance threshold: the marginally accepted tenants are lower quality and so can only afford lower rents, but higher quality tenants are still held to their outside options and thus pay the same rent regardless of

whether the vacancy tax is in effect or not.

In table 7, we show the correlation of the effects of the tax with observable neighborhood characteristics. Since we only have eight neighborhoods, few of these correlations are statistically significant.

7.3 Welfare Effects

We can also use our structural model to infer the size of the externality implied by the proposed vacancy tax. To do this, we must first define a welfare function. The surplus associated with a lease in our model is given by the discounted ex post gross profits generated by the tenant, plus the scrap value they receive upon exit, less move-in costs. Mathematically, this can be written

$$\mathcal{S}(\{g_t\}, \theta_i) = \underbrace{\sum_{t=t_i^1}^{t_i^x} \beta^t \pi(g_t, \theta_i)}_{\text{profits}} + \underbrace{\beta^{t_i^x} \mathbb{E}[\phi \mid \text{exit at age } t_i^x - t_i^1]}_{\text{scrap values}} - \underbrace{\beta^{t_i^1} m}_{\text{move in costs}} \quad (22)$$

where t_i^1 denotes the period that tenant i enters the market and t_i^x denotes the period that i exits.

We assume a social welfare function which is the discounted expected value of the surplus from all future leases (starting from some initial date $t = 0$), less an externality e of vacancy each period, expressed on a per-square-foot basis. This welfare function is given by

$$\mathcal{W}(e; \tau) = \left(\sum_i S(g_t, \theta_i; \tau) \right) - \left(\sum_{t=0}^{\infty} \beta^t L \nu_t(\tau) \times e \right) \quad (23)$$

The parameter e multiplies the per-period vacant square footage ($L \nu_t$).

We back out e for each market under the assumption that the proposed vacancy tax is the welfare-maximizing Pigouvian tax. Under this assumption, the actual externality per storefront e^* is given by

$$\tau^{proposed} = \underset{\tau}{\operatorname{argmax}} W(e^*; \tau) \quad (24)$$

We compute welfare by simulating our model for many periods, for different simulated paths of the aggregate state. For each simulation, we keep track of the surplus generated by each lease, and

the vacancy rate in each period. We then compute welfare for each simulation, and average over all simulations.

We estimate e^* by computing welfare for many combinations of e and τ . For each value of e , we find the value $\tau(e)$ which maximizes $\mathcal{W}(e)$. We can then interpolate, for any proposed tax $\tau^{proposed}$, the e^* for which τ is welfare-maximizing.

7.4 Results

Table 6 presents the effect of the proposed vacancy tax on long-run average vacancy rates and rents, as well as the implied externality associated with the tax. We find that on average, to justify a vacancy tax of 1% of assessed values, a vacant storefront would have to impose an externality of \$29.68 per square foot, or about half of observed average rents.

In table 7, we examine the correlation of the outcomes from table 6 with neighborhood characteristics. Since we only have 8 neighborhoods, we do not expect any of these correlations to be statistically significant, and indeed, most are not. However, the direction of each correlation is in line with our expectations. The long-run vacancy rate and average rents appear to decrease by more in neighborhoods where the baseline long-run vacancy rate and rents are higher. The implied externality is generally higher in neighborhoods with higher vacancy tax levels, higher baseline vacancy and higher baseline rents.

8 Explaining Rising Vacancy

Our estimated model cannot explain the rise in vacancy that occurred between 2007 and 2019, or why rents did not adjust to "clear the market" and return vacancy to its previous level over this time. At the estimated parameters, the model predicts that vacancy in all of our markets is counter-cyclical: when the aggregate state is low, vacancy rates should be high. Over the 2007-2019 period, this appears not to have been the case. The New York City Office of the Comptroller reports that retail vacancy has been increasing over the 2007-2017 period (Office of the New York City Comptroller, 2019). The real puzzle is that average real rents did not adjust enough to clear the market or return the vacancy rate to a lower level.

Though rent dispersion is large over our entire sample period, the ratio of the 90th percentile

to the 50th percentile of rents for leases signed in a given quarter has been trending downward over time. In figure 8, we plot the smoothed time series of the 50th and 90th percentiles of rents alongside their ratio. The x axis shows the quarter of lease execution, and the y axis shows quarterly rent per square foot. Both the 50th and 90th percentiles fall by nearly 50% over our sample period, but for the 90th percentile this decline is larger in dollar terms: the 50th percentile falls from \$51/sqft to \$28/sqft in 2017 dollars, while the 90th percentile falls from about \$143/sqft to \$72/sqft. This drives a downward trend in the ratio of the 90th to the 50th percentile of rents.

One way to explain the simultaneously rising vacancy rates and falling 90/50 rent ratio with our model is that the distribution of tenant quality (F^θ) is changing over time, but that landlords update their expectations of the tenant type distribution with a lag. Since landlords' potential tenants are drawn stochastically every period, we think this is a reasonable assumption: when average rents are lower on average, landlords may have a hard time distinguishing whether they got unlucky draws of tenant type or whether the underlying distribution itself has actually changed. We implement a counterfactual to explore the degree to which the F^θ distribution would need to change over time in order to generate the pattern of rising vacancies that we observe over our sample period. Specifically, we suppose that in 2015, the tenant type distribution changes in a specific way: we redistribute a share γ of the mass above the 90th percentile of the original quality distribution evenly across the types below the 90th percentile. However, we assume that landlords continue to behave under the belief that the tenant type distribution is unchanged. We choose γ to minimize the squared difference between the observed and predicted vacancy rates. We think of this as a calibration exercise, and this exercise results in a calibrated γ equal to 0.775.

As a proof of concept, in figure 9, we show the predicted vacancy rate on the Lower West Side.¹⁴ We plot the empirical vacancy path, as well as predicted vacancy for $\gamma = 1$ (the baseline model) and $\gamma = 0.775$ (the value we calibrate as described in the previous paragraph). We are able to generate a faster increase in vacancy with $\gamma < 1$.

To us, this counterfactual exercise indicates a direction for future work: exploring the reasons why the underlying distribution of tenant types may be changing over time. We feel that a fruitful direction may to incorporate the changing share of chain stores into our analysis, though we would

¹⁴The Lower West Side contains SoHo, a neighborhood whose rising vacancy rates received a lot of press attention in 2017 and 2018.

first like to look for additional evidence that landlords learn about the tenant type distribution (and maybe other state variables, such as the aggregate state g) over time. Rents above the 90th percentile are mostly being paid by large chain retailers in the high-end retail corridors on 5th Avenue, Madison Avenue, and in SoHo. These high-paying tenants include luxury clothing and accessory brands, as well as lower-end stores like pharmacies and sandwich chains. Many of these chains are reducing their number of stores over this time period. Our Live XYZ dataset shows a net decline of 137 chain stores (corresponding to a decline of 2.23 percentage points in the chain share of all storefronts) south of 110th Street between January 1, 2017 and December 31, 2019. During this time period, retailers are also rapidly building their online brands. Their flagship Manhattan stores may be less important for branding than they had been in the pre-internet era.

9 Conclusion

In this paper, we leverage novel data to begin to understand the key market forces that drive long-run retail vacancy rates and rents. We show that the asymmetry of landlords' and tenants' ability to commit to long-term leases, combined with tenant heterogeneity, up-front move-in costs, and search frictions, create option value for different market participants at different times, and that this option value fluctuates with the business cycle. We use our model to investigate the potential impacts of commercial vacancy taxes, a much-discussed urban policy. We view our work as a starting point for future work on the dynamics of commercial real estate leasing markets.

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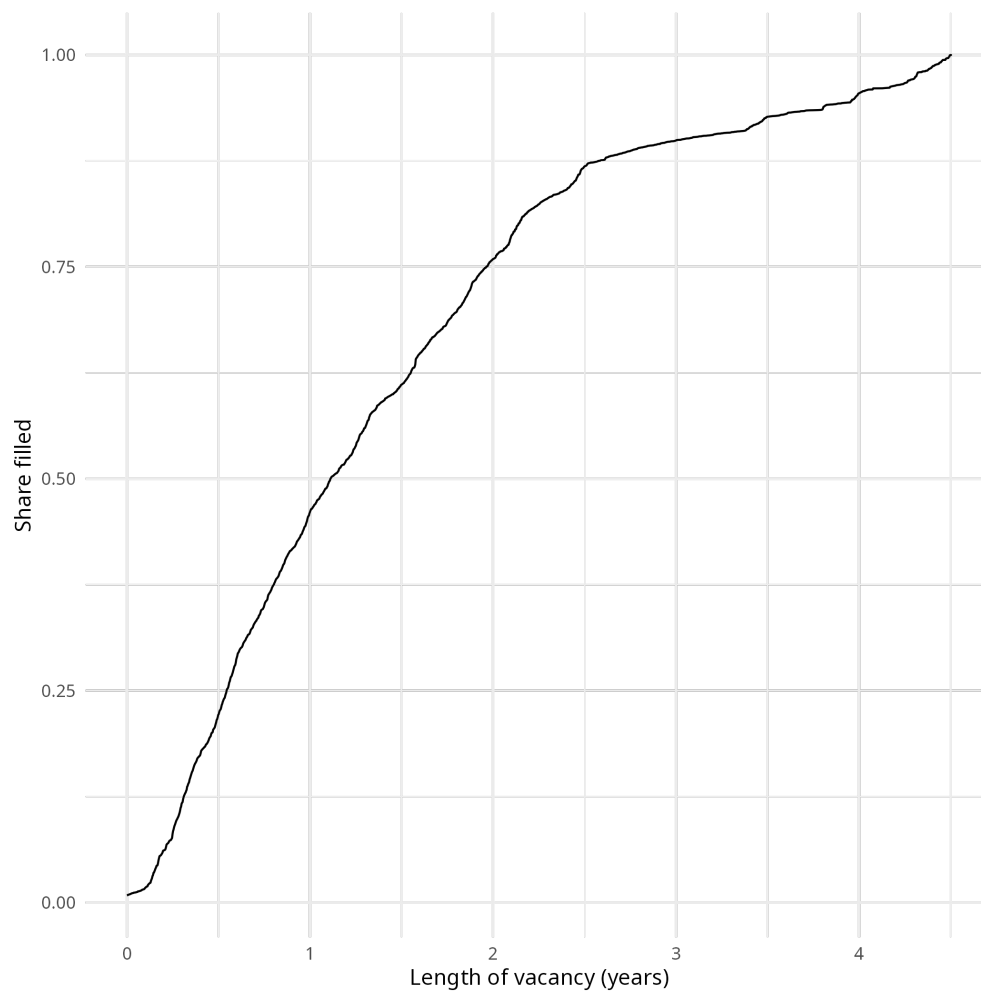
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Figure 1: Empirical Cumulative Distribution of Vacancy Durations



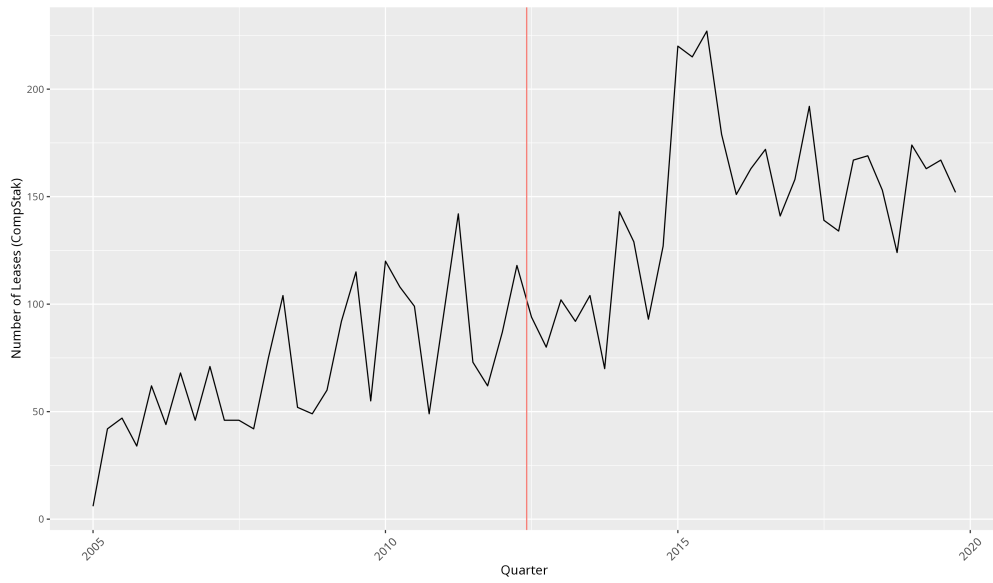
Note: Here we plot the empirical cumulative distribution of vacancy lengths in our Live XYZ dataset. This dataset is censored, and here we include all vacancies, including those storefronts which were vacant at the time they are first observed and those storefronts which were still vacant at the time they were last observed. The earliest observations in the Live XYZ dataset are from late 2015, and the last observations were in March 2020. We therefore do not observe any vacancies that last longer than four and a half years. If anything, therefore, we under-estimate the duration of the longest-lived storefront vacancies.

Table 1: Summary Statistics by Community District

Community District	Rent (\$/sqft)	Vacancy (%)	Total # Storefronts	Total # Leases
Financial District	35.21	5.64	1433	645
Lower West Side	60.47	6.37	3740	1778
Lower East Side	31.68	5.39	3381	406
Midtown West	39.16	5.08	2207	658
Midtown	69.26	5.32	3954	2565
Midtown East	38.48	3.77	1918	533
Upper West Side	54.99	4.76	1888	505
Upper East Side	101.80	4.67	3290	901
Weighted Average	60.88	5.23		
Total			21811	7991

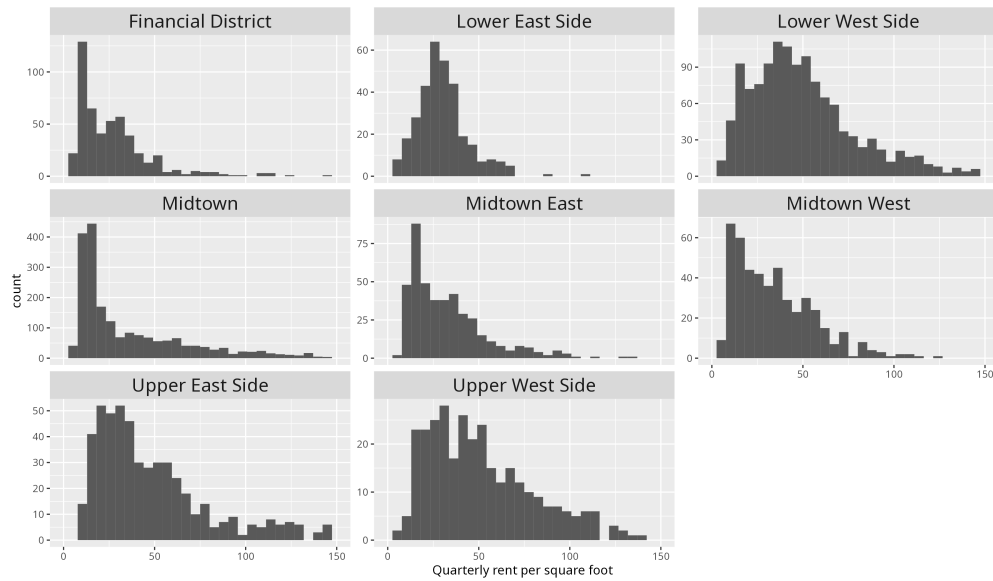
Note: We report summary statistics by neighborhood. From the CompStak dataset, we report the average quarterly rent per square foot and the total number of leases we observe from each market. Vacancy rates for 2007Q1-2017Q1 period come from the New York City Comptroller’s report on retail vacancy; from 2017Q2 onward we compute vacancy rates from the Live XYZ dataset. The Comptroller’s report provides vacancy rates at an annual frequency (corresponding to the first quarter of the year); we linearly interpolate to fill in quarterly vacancy rates in missing quarters. We also report the number of unique storefronts and leases observed in each market. To compute the average rent across neighborhoods, we weight each neighborhood’s average rent by the number of leases we observe in that market. Similarly, we weight market-level vacancy rates by the number of storefronts in each market.

Figure 2: CompStak Leases Executed Per Quarter



Note: We plot the number of leases in the CompStak dataset that were executed in each quarter. The vertical line indicates the date CompStak was founded. CompStak’s dataset is composed of transactions reported by commercial real estate brokers. Brokers are incentivized to report transactions because sharing information allows them to learn more about transactions they were not involved with. However, we want to be wary of leases reported prior to CompStak’s entry, since brokers who report these transactions may be selected.

Figure 3: Rent Distribution By Community District



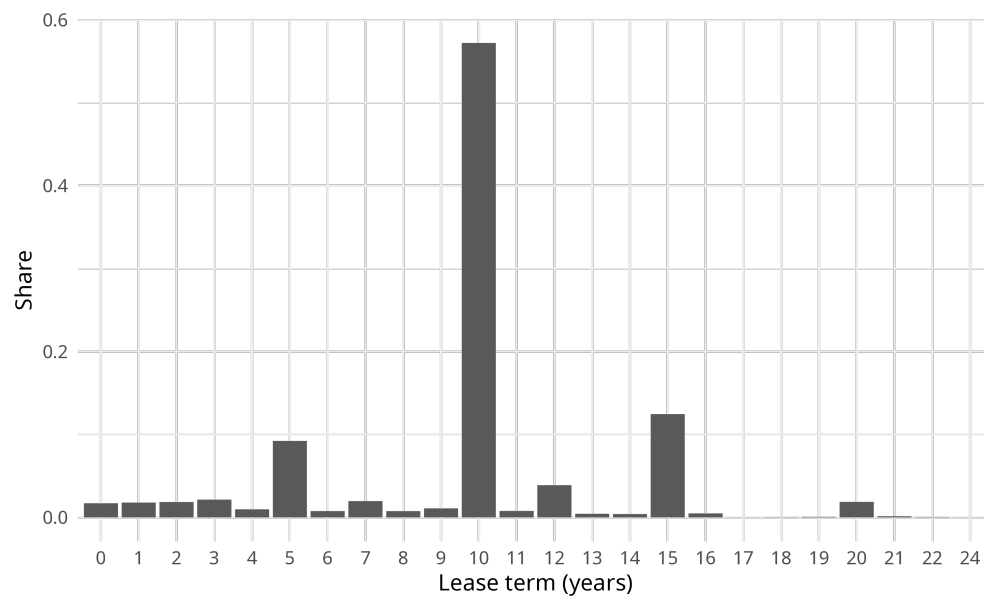
Note: We plot the distribution of real quarterly net effective rents (expressed in dollars per square foot) for each market from CompStak. Each observation is a lease, and we pool all leases observed over our whole 2005-2019 sample period.

Table 2: Correlates of Rent

Dependent Variable	(1) log(rent/sqft)	(2) log(rent/sqft)	(3) log(rent/sqft)	(4) log(rent/sqft)
Log Transaction Sqft	-0.20*** (0.02)	-0.22*** (0.01)	-0.23*** (0.02)	-0.23*** (0.02)
Log Median Income	0.22*** (0.04)	0.14*** (0.03)	0.12*** (0.03)	-0.05 (0.05)
Log Years Since Built	0.08** (0.02)	0.02 (0.02)	0.001 (0.02)	-0.02 (0.02)
Avenue Address	0.23*** (0.04)	0.24*** (0.03)	0.23*** (0.03)	0.24*** (0.04)
Log Building Frontage	-0.06 (0.03)	-0.06* (0.03)	-0.06** (0.02)	-0.03 (0.03)
Log Building Floors	0.17*** (0.04)	0.14*** (0.03)	0.06 (0.03)	0.02 (0.04)
Residential Share	-0.48*** (0.09)	-0.33*** (0.08)	-0.13 (0.07)	-0.07 (0.06)
Office Share	-0.38*** (0.10)	-0.20* (0.08)	-0.11 (0.08)	-0.07 (0.07)
Special Purpose District	0.16*** (0.03)	0.19*** (0.03)	0.03 (0.04)	0.13 (0.07)
Chain	0.36*** (0.04)	0.24*** (0.03)	0.24*** (0.03)	0.21*** (0.03)
Lease Term Years	0.0005 (0.005)	0.02*** (0.005)	0.02*** (0.004)	0.02*** (0.004)
<i>Fixed Effects</i>				
Transaction Quarter	Yes	Yes	Yes	Yes
Tenant Industry	No	Yes	Yes	Yes
Zoning	No	No	Yes	Yes
Census Tract	No	No	No	Yes
Observations	2,630	2,630	2,630	2,630
R2	0.22518	0.40147	0.52022	0.62460
Within R2	0.20193	0.20548	0.14973	0.15012

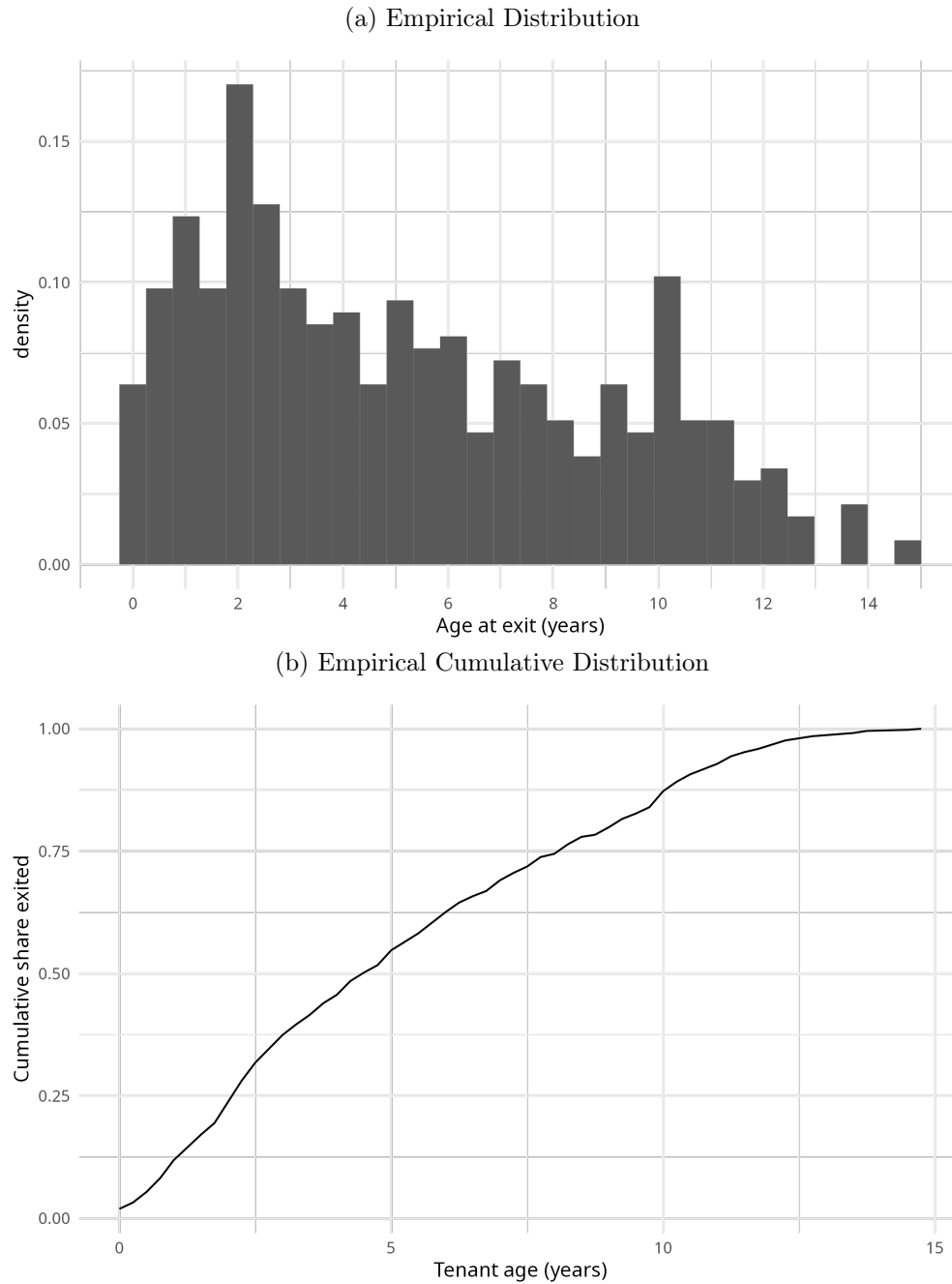
Note: We regress log monthly rent per square foot on observable landlord and tenant characteristics using OLS. Our sample is the set of leases from the CompStak dataset that we can match to tenants active in the Live XYZ dataset, so that we can include industry fixed effects and a dummy for whether the door is on a street (a small east-west side street) or an avenue (a major north-south thoroughfare). Standard errors are clustered at the transaction quarter and census tract level. Transaction square footage refers to the total amount of space being rented by the tenant. Median income is the income of the census tract in 2016 from the American Community Survey. Residential share and office share refer to the share of the entire building floor space devoted to residential and office uses, respectively, as reported by New York City's Primary Land Use Tax Lot Output (PLUTO) dataset. Special purpose districts have additional, unique zoning rules that vary on a case-by-case basis.

Figure 4: Contractual Lease Term Distribution



Note:

Figure 5: Lease Age at Exit



Note: These figures are constructed from the sample of 462 tenants for whom we are able to match their lease in CompStak to their exit date in the Live XYZ dataset.

Figure 6: Aggregate State Path

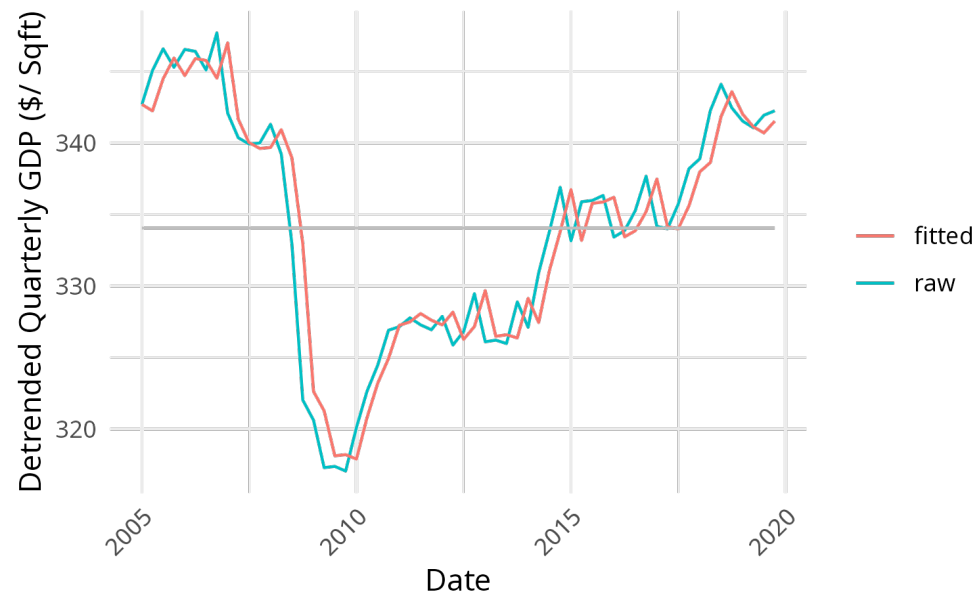


Table 3: Aggregate State Transition Parameter Estimates

Parameter	Estimate
ρ_g	0.95 (0.04)
μ_g	333.75 (6.65)
σ_g^2	6.8

Table 4: Model Parameter Estimates

Community District	σ_ϕ	μ_θ	σ_θ	m	λ_0	λ_g	Pr(draw tenant)
Financial District	2.21	-1.82	0.18	650	10.51	0.18	1.00
Lower West Side	2.19	-1.55	0.25	650	2.84	0.07	0.94
Lower East Side	2.43	-1.80	0.16	650	6.27	0.29	1.00
Midtown West	1.87	-1.68	0.17	650	15.64	0.13	1.00
Midtown	1.92	-1.53	0.28	650	8.17	-0.05	1.00
Midtown East	1.96	-1.70	0.17	650	21.04	0.25	1.00
Upper West Side	2.27	-1.60	0.23	650	12.09	0.17	1.00
Upper East Side	3.18	-0.86	0.16	650	-0.78	-0.16	0.32

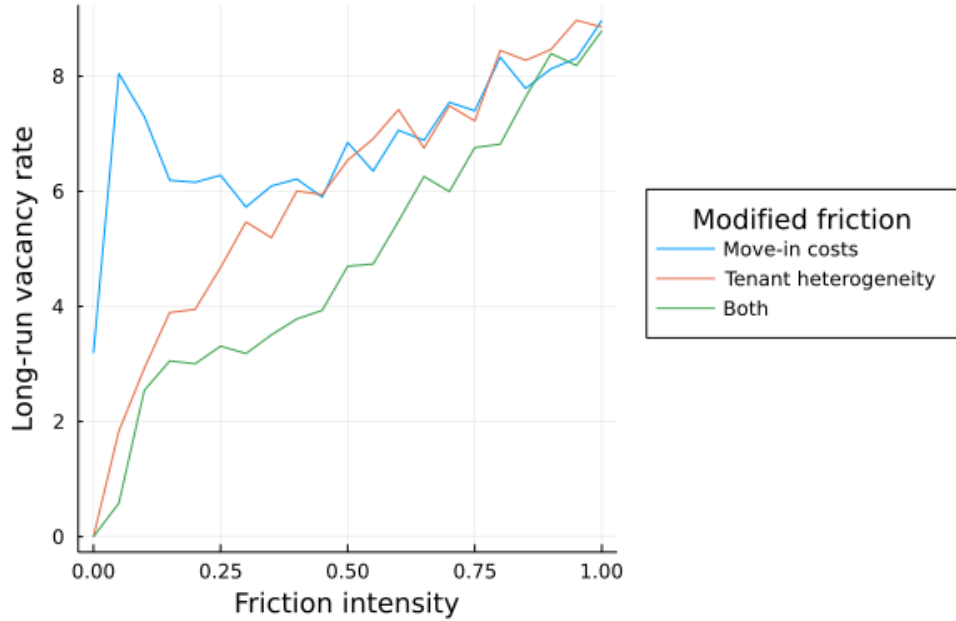
Estimated parameters for each of our markets. μ_θ and σ_θ are the parameters of the lognormal distribution from which tenant types are drawn. σ_ϕ governs the mean of tenant opportunity costs; a tenant of age j has an average opportunity cost of $\sigma_\phi \times (T - j)$. Move-in costs m are calibrated to \$650 per square foot. λ_0 and λ_g are parameters of the matching function. Standard errors are forthcoming.

Table 5: Long-Run Vacancy Rate Across Counterfactuals and Neighborhoods

	With Aggregate Uncertainty			No Aggregate Uncertainty		
	p25	p50	p75	p25	p50	p75
Baseline	7.45	7.79	7.96	7.45	7.63	7.89
Move-in costs = 0	0.99	1.45	2.43	0.00	0.03	0.53
$\Pr(\text{draw tenant when search}) = 1$	7.31	7.54	7.77	7.28	7.53	7.75
No tenant heterogeneity	0.00	0.01	28.16	0.00	0.00	0.00

We report the 25th, 50th, and 75th percentile of long-run average vacancy rates across our 8 neighborhoods, under each counterfactual scenario. Long-run average vacancy rates corresponding to our estimated model are shown in the first three columns of the first row.

Figure 7: Effect of Scaling Tenant Heterogeneity On Long-Run Vacancy Rate



Note: We plot the long-run vacancy rate as we vary the size of the move-in cost and tenant heterogeneity. These results are for Midtown, but other neighborhoods are qualitatively similar.

Table 6: Effect of Vacancy Tax on Long-Run Moments

Community District	Vacancy Tax τ (\$/sqft)	Δ Vacancy Rate (p.p.)	Δ Average Rent (%)	Implied Externality (\$/vacant sqft)
Financial District	0.98	-0.08	0.00	11.19
Lower West Side	1.93	-0.09	-0.16	36.88
Lower East Side	0.96	-0.08	-0.23	18.97
Midtown West	1.29	-0.04	0.00	16.43
Midtown	2.17	-0.25	-0.50	42.58
Midtown East	1.24	-0.12	-0.25	30.00
Upper West Side	1.74	-0.11	-0.18	35.79
Upper East Side	3.48	-1.05	-2.20	45.56
Average	1.72	-0.23	-0.44	29.68

This table reports the effects of a 1% tax on vacant assessed values for each of our 8 neighborhoods. We proxy for assessed value of vacant properties using the value of search in the model, $\mathbb{E}[U(g)]$. The levels of tax and the implied externality of vacancy are reported in dollars per square foot at a quarterly level. Relative to the estimated model, we report the percentage point change in the vacancy rate and the percentage change in average rents.

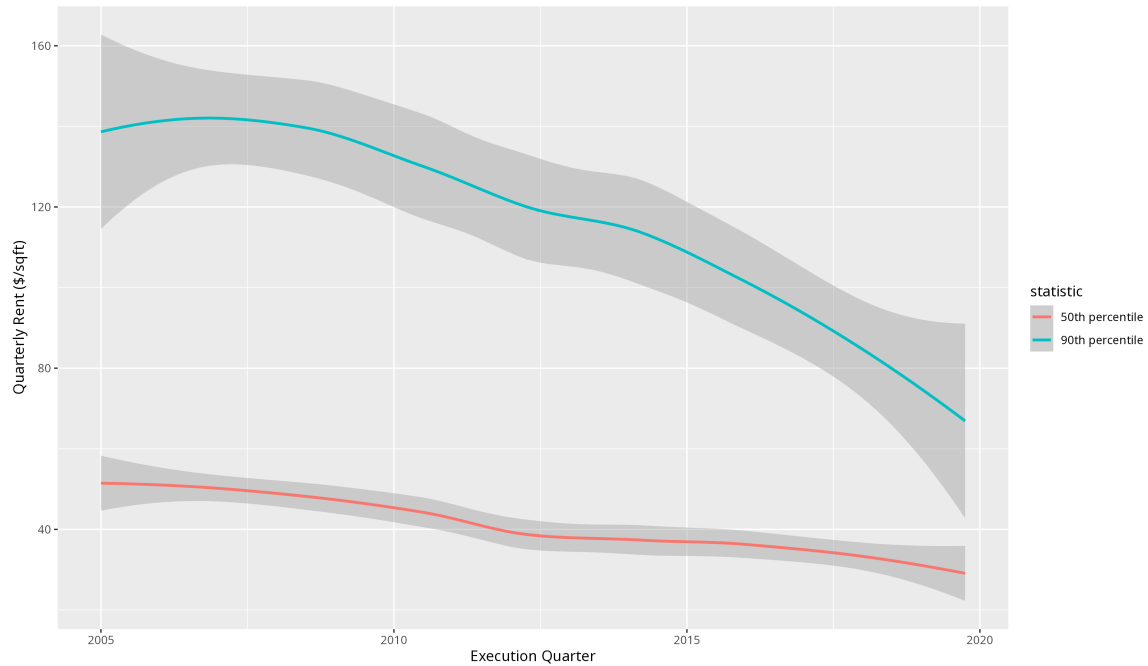
Table 7: Correlation of Vacancy Tax Outcomes with Neighborhood Characteristics

Correlate	Δ Vacancy Rate	Δ Average Rent	ImpliedExternality
Vacancy tax level	-0.9 (<0.01)	-0.9 (<0.01)	0.84 (<0.01)
Baseline long-run vacancy rate	-0.2 (0.63)	-0.17 (0.69)	0.04 (0.92)
Baseline average rent	-0.9 (<0.01)	-0.9 (<0.01)	0.84 (<0.01)
High-end retail district	-0.57 (0.14)	-0.58 (0.13)	0.78 (0.02)

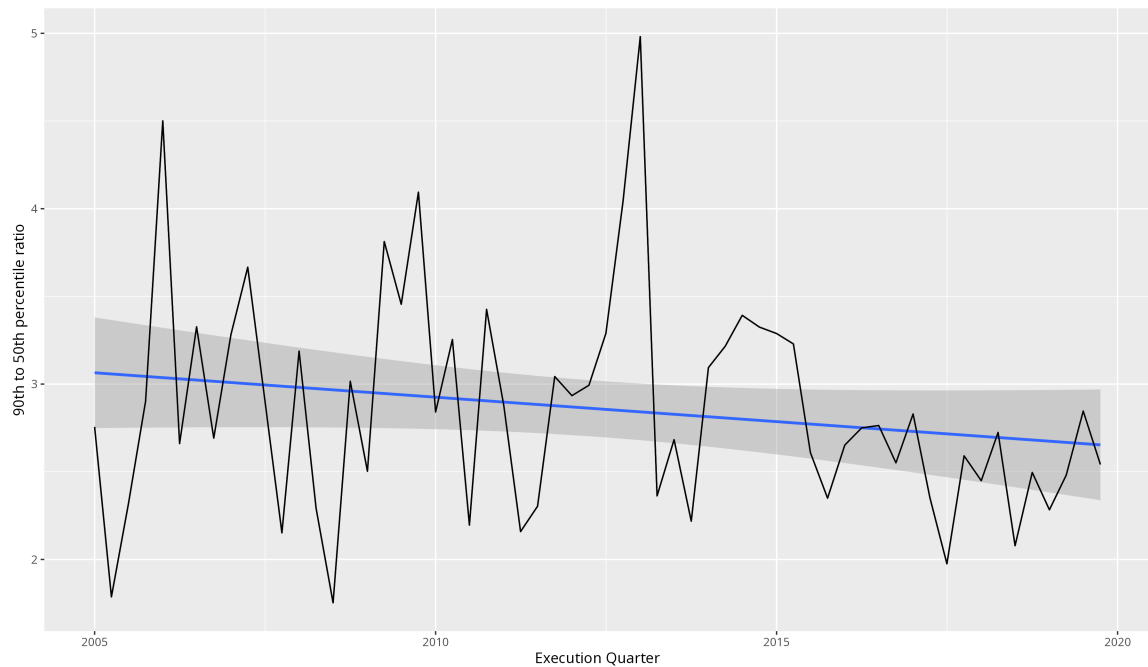
This table reports the correlation between the consequences of the proposed vacancy tax and observable community district characteristics. p-values are reported in parentheses. Since we have only 8 community districts, we do not expect any of these correlations to be statistically significant - indeed, most are not.

Figure 8: 50th and 90th Percentile of Contractual Rents

(a) 90th and 50th Percentiles



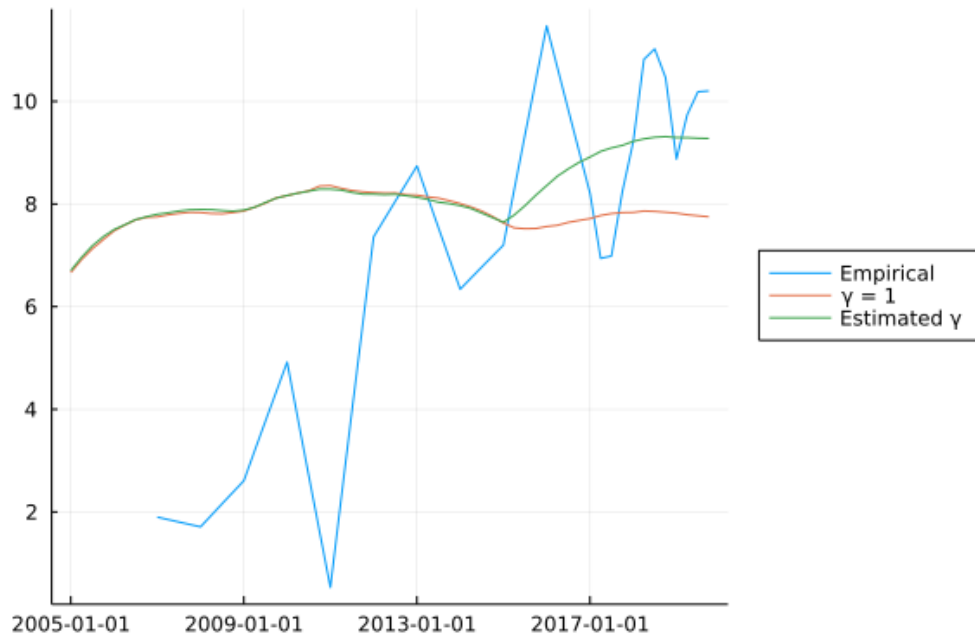
(b) Ratio



Source: CompStak

Note: In panel (a), we plot the 90th and 50th percentile of real rents for leases signed each quarter (in 2017 dollars). We smooth the series using a local polynomial regression, and the confidence interval for each series is shown in grey. In panel (b), we plot the ratio of 90th percentile to 50th percentile of rents for new leases in each quarter. The black plots the raw ratio, while the blue line shows a linear trend.

Figure 9: Exploration of adaptive landlord preferences



We explore whether unexpected changes to the tenant type distribution can allow us to match the trend in the vacancy rate for the Lower West Side. The blue line shows the empirical vacancy rate, and the red line shows the predicted vacancy rate path at our estimated parameter values. We assume that the tenant type distribution changed once, in 2015, and that landlords were unaware of this change. The post-2015 tenant type distribution redistributes a 0.775 share of the mass above the original distribution's 90th percentile evenly across the types below the 90th percentile. The green line shows the predicted vacancy rates under this assumption, and clearly matches the post-2015 path of the vacancy rate much better. We view this exercise as suggestive in nature only.

A Microfounding Tenant Profits

We assume a representative consumer with CES utility across the retail tenants i within a market.

Each retailer offers a unique variety of retail goods or services:

$$U = \left(\sum_i \alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{1-\sigma}} \text{ subject to } \sum_i p_i x_i = B \quad (25)$$

where x_i is the quantity of variety i , p_i is the price of variety i , σ is the elasticity of substitution, B is the budget constraint, and α_i is a preference parameter for variety i .

We derive consumer demand in the usual way and obtain the standard demand curve:

$$x_a^{\text{demand}} = \frac{B p_a^{-\sigma}}{\sum_i \frac{\alpha_i}{\alpha_a} p_i^{1-\sigma}} \quad (26)$$

Tenants engage in Cournot competition. Specifically, the tenant producing variety a chooses a quantity x_a to produce to maximize static profits, given marginal costs c_a and a vector x_{-a} of quantities produced by all other varieties:

$$\pi_a(x_a, x_{-a}) = \max_{x_a} x_a \cdot (p_a(x_a, x_{-a}) - c_a) \quad (27)$$

The resulting supply curve is

$$x_a^{\text{supply}} = \left(\frac{B}{\sum_i \frac{\alpha_i}{\alpha_a} p_i^{1-\sigma}} \right) \left(\frac{1}{\sigma^\sigma (p_a - c_a)^\sigma} \right) \quad (28)$$

Setting demand (26) and supply (28) equal to each other allows us to solve for equilibrium prices and quantities. We obtain the familiar Cournot markup formula

$$p_a^* = \frac{\sigma}{\sigma - 1} c_a \quad (29)$$

and plug this into the supply curve to get equilibrium quantities:

$$x_a^* = \frac{B}{\left(\frac{\sigma}{\sigma-1}\right) c_a + P_{-a} \left(\frac{\sigma}{\sigma-1}\right)^\sigma c_a^\sigma} \quad (30)$$

Finally, we solve for equilibrium static flow profits by plugging equilibrium prices and quantities

into the profit function for variety a :

$$\pi_a^* = \frac{B}{\sigma + Ac_a^{\sigma-1}} \quad (31)$$

where $A = \sigma P_{-a} \sigma^{1-\sigma} (\sigma - 1)^{1-\sigma}$.

In the structural leasing model of section 4, tenant quality θ_a represents $\frac{1}{\sigma + Ac_a^{\sigma-1}}$. The aggregate state variable g in the structural model corresponds to consumer budgets B . We therefore assume in the structural model that tenant profits take a multiplicative form: $\pi(g, \theta) = g\theta$.

B Model Proofs

Claim: $W(j, r, g, \varepsilon; \theta)$ is strictly decreasing in r for all $j, g, \varepsilon, \theta$.

Proof. Consider two arbitrary rent values, r and r' , such that $r' > r$.

For lease age $j = T$, we have

$$W(T, r, g, \varepsilon; \theta) = \pi(g, \theta) + \varepsilon - r > \pi(g, \theta) + \varepsilon - r' = W(T, r', g, \varepsilon; \theta)$$

for all g, θ, ε .

Suppose for lease age $j + 1$, $W(j, r, g, \varepsilon; \theta) > W(j, r', g, \varepsilon; \theta)$ for $r' > r$ and all g, θ, ε .

Then taking expectations over ε and g , we get

$$\begin{aligned} \mathbb{E}_{\varepsilon', g'}[W(j+1, r, g', \varepsilon', \theta) \mid g] &= \int_{g', \varepsilon'} W(j+1, r, g', \varepsilon'; \theta) dF(\varepsilon', g' \mid g) \\ &> \int_{g', \varepsilon'} W(j+1, r', g', \varepsilon'; \theta) dF(\varepsilon', g' \mid g) \\ &= \mathbb{E}_{\varepsilon', g'}[W(j+1, r', g', \varepsilon', \theta) \mid g] \end{aligned}$$

We now back up to compare the conditional values of exiting and staying in the lease in period j at rents r and r' . Since $-r > -r'$ and $\mathbb{E}_{\varepsilon', g'}[W(j+1, r, g', \varepsilon', \theta) \mid g] > \mathbb{E}_{\varepsilon', g'}[W(j+1, r', g', \varepsilon', \theta) \mid g]$, we have

$$\begin{aligned}
W^{continue}(j, r, g, \varepsilon; \theta) &= \pi(g, \theta) + \varepsilon - r + \beta \mathbb{E}_{\varepsilon', g'}[W(j+1, r, g', \varepsilon', \theta) \mid g] \\
&> \pi(g, \theta) + \varepsilon - r' + \beta \mathbb{E}_{\varepsilon', g'}[W(j+1, r', g', \varepsilon', \theta) \mid g] \\
&= W^{continue}(j, r', g, \varepsilon; \theta)
\end{aligned}$$

and

$$W^{exit}(j, r, g, \varepsilon; \theta) = -r > -r' = W^{exit}(j, r', g, \varepsilon; \theta)$$

Since $W^{continue}(j, r, g, \varepsilon; \theta) > W^{continue}(j, r', g, \varepsilon; \theta)$ and $W^{exit}(j, r, g, \varepsilon; \theta) > W^{exit}(j, r', g, \varepsilon; \theta)$,

$$\begin{aligned}
W(j, r, g, \varepsilon; \theta) &= \max\{W^{continue}(j, r, g, \varepsilon; \theta), W^{exit}(j, r, g, \varepsilon; \theta)\} \\
&> \max\{W^{continue}(j, r', g, \varepsilon; \theta), W^{exit}(j, r', g, \varepsilon; \theta)\} \\
&= W(j, r', g, \varepsilon; \theta)
\end{aligned}$$

□

Claim: $\varepsilon^*(j, r, g; \theta)$ and $p^x(j, r, g; \theta)$ are strictly increasing in r for all $j < T, g, \varepsilon, \theta$.

Proof. We know from the previous proof that $\mathbb{E}_{g, \varepsilon}[W(j, r, g, \varepsilon; \theta)] > \mathbb{E}_{g, \varepsilon}[W(j, r', g, \varepsilon; \theta)]$ for $r' > r$.

Therefore

$$\begin{aligned}
\varepsilon^*(j, r', g; \theta) &= -\pi(g, \theta) - \beta \mathbb{E}_{g, \varepsilon}[W(j+1, r', g, \varepsilon; \theta)] \\
&> -\pi(g, \theta) - \beta \mathbb{E}_{g, \varepsilon}[W(j+1, r, g, \varepsilon; \theta)] = \varepsilon^*(j, r, g, \theta)
\end{aligned}$$

So $\varepsilon^*(j, r, g, \theta)$ is strictly increasing in r . As long as the CDF of ε is strictly increasing, then the tenant's exit probability $p^x(j, r, g; \theta)$ is increasing in r for all j, g, θ .

□

Claim: $W(j, r, g, \varepsilon; \theta)$ is weakly increasing in θ for all $j, r, g > 0, \varepsilon$.

Proof. Consider two different tenant qualities θ and θ' such that $\theta' > \theta$.

In the terminal lease period

$$\begin{aligned}
W(T, r, g, \varepsilon; \theta') &= \pi(g, \theta') + \varepsilon - r \\
&= g\theta' + \varepsilon - r \\
&> g\theta + \varepsilon - r \\
&= \pi(g, \theta) + \varepsilon - r \\
&= W(T, r, g, \varepsilon; \theta)
\end{aligned}$$

Suppose for lease age $j + 1$ we know that $W(j + 1, r, g, \varepsilon; \theta') \geq W(j + 1, r, g, \varepsilon; \theta)$ for $\theta' > \theta$. Then taking expectations over ε and g , we get

$$\begin{aligned}
\mathbb{E}_{\varepsilon', g'}[W(j + 1, r, g', \varepsilon', \theta') \mid g] &= \int_{g', \varepsilon'} W(j + 1, r, g', \varepsilon', \theta') dF(\varepsilon', g' \mid g) \\
&\geq \int_{g', \varepsilon'} W(j + 1, r, g', \varepsilon', \theta) dF(\varepsilon', g' \mid g) \\
&= \mathbb{E}_{\varepsilon', g'}[W(j + 1, r, g', \varepsilon', \theta) \mid g]
\end{aligned}$$

We now back up to compare the conditional values of exiting and staying in the lease in period j for tenants of types θ and θ' . W^{exit} doesn't depend on θ , but $W^{continue}$ does and is strictly increasing in θ since $\pi(g, \theta)$ is strictly increasing in θ :

$$\begin{aligned}
W^{continue}(j, r, g, \varepsilon; \theta') &= \pi(g, \theta') + \varepsilon - r + \beta \mathbb{E}_{\varepsilon', g'}[W(j + 1, r, g', \varepsilon', \theta') \mid g] \\
&> \pi(g, \theta) + \varepsilon - r + \beta \mathbb{E}_{\varepsilon', g'}[W(j + 1, r, g', \varepsilon', \theta) \mid g] \\
&= W^{continue}(j, r, g, \varepsilon; \theta)
\end{aligned}$$

Since $W^{continue}(j, r, g, \varepsilon; \theta') > W^{continue}(j, r, g, \varepsilon; \theta)$ and $W^{exit}(j, r, g, \varepsilon; \theta') = W^{exit}(j, r, g, \varepsilon; \theta)$,

$$\begin{aligned}
W(j, r, g, \varepsilon; \theta') &= \max\{W^{continue}(j, r, g, \varepsilon; \theta'), W^{exit}(j, r, g, \varepsilon; \theta')\} \\
&\geq \max\{W^{continue}(j, r, g, \varepsilon; \theta), W^{exit}(j, r, g, \varepsilon; \theta)\} \\
&= W(j, r, g, \varepsilon; \theta)
\end{aligned}$$

□

Claim: $\varepsilon^*(j, r, g; \theta)$ and $p^x(j, r, g; \theta)$ are strictly decreasing in θ for all $j < T, r, g > 0, \varepsilon$.

Proof. We know from the previous proof that $\mathbb{E}_{g, \varepsilon}[W(j, r, g, \varepsilon; \theta')] \geq \mathbb{E}_{g, \varepsilon}[W(j, r, g, \varepsilon; \theta)]$ for $\theta' > \theta$.

We also know that for $g > 0$, $\pi(g, \theta) > \pi(g, \theta')$. Therefore:

$$\begin{aligned}\varepsilon^*(j, r, g; \theta) &= -\pi(g, \theta) - \beta \mathbb{E}_{g, \varepsilon}[W(j+1, r, g, \varepsilon; \theta)] \\ &> -\pi(g, \theta') - \beta \mathbb{E}_{g, \varepsilon}[W(j+1, r, g, \varepsilon; \theta')] = \varepsilon^*(j, r, g, \theta')\end{aligned}$$

So $\varepsilon^*(j, r, g, \theta)$ is strictly decreasing in θ . As long as the CDF of ε is strictly increasing, then the tenant's exit probability $p^x(j, r, g; \theta)$ is decreasing in θ for all j, g, r .

□