STA 237 A—TIME SERIES ANALYSIS HOMEWORK SERIES 1

To be submitted Thursday, April 10 in class

Problem 1: [Stationarity] Let $(Z_t\colon t \in \mathbb{Z})$ be a sequence of independent zero mean normal random variables with variance σ^2 and let a, b and c be constants. Which of the following processes are weakly and/or strictly stationary? For each weakly stationary process specify the mean and the ACVF.

(a) $X_t = a + bZ_t + cZ_{t-1}$;

(b) $X_t = Z_t \cos(ct) + Z_{t-1} \sin(ct)$;

(c) $X_t = a + bZ_0$;

(d) $X_t = Z_t Z_{t-1}$.

Problem 2: [U.S. Population] Download the file `population.xls` from the course website. It contains the size of the population in the U.S.A. at ten-year intervals from 1790 to 2000.

(a) Plot the data;

(b) Assuming the model $X_t = m_t + Z_t$, $E[Z_t] = 0$, fit a polynomial trend $\hat{m}_t$ to the data;

(c) Plot the residuals $\hat{Z}_t = X_t - \hat{m}_t$. Comment on the quality of the fitted model;

(d) Use the fitted model to predict the population size in 2010 and 2020 (using predicted noise values of zero).

Problem 3: [Projection Theorem] If $\mathcal{M}$ is a closed subspace of a Hilbert space $\mathcal{H}$ and $x \in \mathcal{H}$, prove that

$$\min_{y \in \mathcal{M}} \|x - y\| = \max\left\{|\langle x, z\rangle|\colon z \in \mathcal{M}^{\perp}, \|z\| = 1\right\},$$

where $\mathcal{M}^{\perp}$ is the orthogonal complement of $\mathcal{M}$.

Problem 4: [Prediction Equations] If $X_t = Z_t - \theta Z_{t-1}$, where $|\theta| < 1$ and $(Z_t\colon t \in \mathbb{Z})$ is a sequence of uncorrelated random variables, each with mean 0 and variance σ^2, show by checking the prediction equations that the best mean square predictor of X_{n+1} in $\overline{\mathrm{sp}}(X_j\colon j \leq n)$ is

$$\hat{X}_{n+1} = -\sum_{j=1}^{\infty} \theta^j X_{n+1-j}.$$

What is the mean squared error of $\hat{X}_{n+1}$?