tags: FromStatCheatsheet

NOTE:

field: CDF of Geometric (p)

field: $1-(1-p)^x$

NOTE:

field: CDF of Exponential(β)

field: $1-e^{-\frac{x}{\beta}}$

NOTE:

field:

- $P(\varnothing) =$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) =$
- $P(B) =$
- $P(\Omega) = \qquad P(\varnothing) =$
- $(\bigcup_n A_n) = \quad (\bigcap_n A_n) =$ DeMorgan

field:

- $P(\varnothing) = 0$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) = 1 - P(A)$
- $P(B) = P(A \cap B) + P(A^c \cap B)$
- $P(\Omega) = 1 \qquad P(\varnothing) = 0$
- $(\bigcup_n A_n) = \bigcap_n A_n \quad (\bigcap_n A_n) = \bigcup_n A_n$ DeMorgan

NOTE:

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n A_n^c)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) =$
- $P(A \cap B^c) =$

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n \mathsf{A_n})$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  $\implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$
- $P(A \cap B^c) = P(A) - P(A \cap B)$

NOTE:

field: $P(A \cap B) =$ when A and Bindependent

field: $P(A \cap B) = P(A)P(B)$ when A and Bindependent

NOTE:

field:

$$P(A|B) =$$

field:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

NOTE:

field: Law of total probability

field: Law of total probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i) \quad \Omega = \cup_{i=1}^{n} A_i$$

$$P(B) = P(A \cup B) + P(A^c \cup B)$$

NOTE:

field: Bayes Theorem

field: Bayes Theorem

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \quad \Omega = \cup_{i=1}^{n} A_i$$

NOTE:

field: CDF Laws

field: CDF Laws

1. Nondecreasing: $x_1 < x_2 \implies F(x_1) \leq F(x_2)$
2. Limits: $\lim_{x \to -\infty} = 0$ and $\lim_{x \to \infty} = 1$
3. Right-Continuous $\lim_{y \to x^+} F(y) = F(x)$

NOTE:

field:

$$f_{y|x}(y|x) =$$

field:

$$f_{y|x}(y|x) = \frac{f(x,y)}{f_x(x)}$$

NOTE:

field: X, Y independent

- $P(X \leq x, Y \leq y) =$
- $f_{x,y}(x,y) =$

field: X, Y independent

- $P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$
- $f_{x,y}(x,y) = f_x(x)f_y(y)$

NOTE:

field: Transformations $Z = \phi(X)$

- Discrete: $f_Z(z) =$
- Continuous: $F_Z(z) =$
- Cont, ϕ strictly monotone: $f_z(z)$

field: Transformations $Z = \phi(X)$

- Discrete:

$$f_Z(z) = P(\phi(X) = z) = P(X \in \phi^{-1}(z)) = \sum_{x \in \phi^{-1}(z)} f_x(x)$$

- Continuous (Method of CDF):

$$F_Z(z) = P(\phi(X) \leq z) = \int_{x:\phi(x) \leq z} f(x)dx$$

- Cont, ϕ strictly monotone: (Method of PDF) $f_z(z) = f_x(\phi^{-1}(z))|\frac{d}{dz}\phi^{-1}(z)|$

NOTE:

field: Rule of the Lazy Statistician: $E[g(x)] =$

field: Rule of the Lazy Statistician: $E[g(x)] = \int g(x) f_x(x) dx$

NOTE:

field: Expectation rules

- $E(c) =$
- $E(cX) =$
- $E(X + Y) =$
- $E(\phi(X)) =$

field: Expectation rules

- $E(c) = c$
- $E(cX) = cE(X)$
- $E(X + Y) = E(X) + E(Y)$
- $E(\phi(X)) \neq \phi(E(X))$

NOTE:

field: Conditional expectation

- $E(Y|X=x) =$
- $E(X) =$
- $E(Y+Z|X) =$
- $E(Y|X) = c \implies$

field: Conditional expectation

- $E(Y|X=x) = \int y f(y|x) dy$
- $E(X) = E(E(X|Y))$
- $E(Y+Z|X) = E(Y|X) + E(Z|X)$
- $E(Y|X) = c \implies Cov(X,Y) = 0$

NOTE:

field: Variance

- $V(X) = \sigma_x^2 =$
- $V(X+Y) =$
- $V\left[\sum_{i=1}^n X_i\right] =$

field: Variance

- $V(X) = \sigma_x^2 = E[(X - E(X))^2] = E(X^2) - E(X)^2$
- $V(X+Y) = V(X) + V(Y) + Cov(X,Y)$
- $V\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n V(X_i) + \sum_{i \neq j} Cov(X_i, X_j)$

NOTE:

field: Covariance

- $Cov(X, Y) =$
- $Cov(X, c) =$
- $Cov(Y, X) =$
- $Cov(aX, bY) =$
- $Cov(X + a, Y + b) =$
- $Cov\Bigg(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j \Bigg) =$

field: Covariance

- $Cov(X, Y) = E[(X - E(X)(Y - E(Y)))] = E(XY) - E(X)E(Y)$
- $Cov(X, c) = 0$
- $Cov(Y, X) = Cov(X, Y)$
- $Cov(aX, bY) = abCov(X, Y)$
- $Cov(X + a, Y + b) = Cov(X, Y)$
- $Cov\Bigg(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j \Bigg) = \sum_{i=1}^{n} \sum_{j=1}^{m} Cov(X_i, Y_j)$

NOTE:

field: Correlation: $\rho(X, Y)$

field: Correlation: $\rho(X, Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

NOTE:

field: Conditional Variance

- $V(Y|X) =$
- $V(Y) =$

field: Conditional Variance

- $V(Y|X) = E\big[(Y - E(Y|X))^2|X\big] = E(Y^2|X) - E(Y|X)^2$
- $V(Y) = E(V(Y|X)) + V(E(Y|X))$

tags: UndergradTextbook

NOTE:

field: Law of total probability $k = 2$ (using conditional probability)

field: $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$

NOTE:

field: Bayes formula in terms of law of total probability,

field: $P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B^c)P(B^c)}$

NOTE:

field: $P(A \text{ and } B)$

field: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

NOTE:

field: Events A and B are independent if

field: $P(A|B) = P(A)$ equivalently $P(A \text{ and } B) = P(A)P(B)$

NOTE:

field: Poisson setting

field: The Poisson setting arises in the context of discrete counts of events that occur over space or time with the small probability and where successive events are independent

Eg: 2 on average calls a minute, X is number of calls a minute, $X \sim Pois$

NOTE:

field: Poisson approximation of binomial distribution

field: Suppose $X \sim Binom(n, p)$, $Y \sim Pois(\lambda)$. If $n \to \infty$, and $p \to 0$, in such a way that $np \to \lambda > 0$, then for all k, $P(X = k) \to P(Y = k)$. The Poisson distribution with parameter $\lambda = np$ serves as a good approiximation for the binomial distribution when n is large and p is small.

NOTE:

field: $E(f(X, Y))$ when X, Y are discrete

field: $E(f(X, Y)) = \sum_x \sum_y f(x, y)P(X = x, Y = y)$

NOTE:

field: If X, Y are independent, then $f(X), g(Y)$

field: are also independent

NOTE:

field: If X, Y independent, $E(XY) =, E(f(X)g(Y)) =$

field: If X, Y independent, $E(XY) = E(X)E(Y), E(f(X)g(Y)) = E(f(X))E(g(Y))$

NOTE:

field: Sum of independent discrete random variables X, Y: $P(X+Y=k)$

field: $P(X+Y=k) = \sum_i P(X=i)P(Y=k-i)$

NOTE:

field: $V(X) = 0$

field: If and only if X is a constant

NOTE:

field: $E(I_A) =, V(I_A)$ Where I_A is an indicator function

field: $E(I_A) = P(A), V(I_A) = P(A)P(A^c)$

NOTE:

field: For discrete jointly distributed random variables,

$$P(X=y|X=x) =$$

field: For discrete jointly distributed random variables,

$$P(X=y|X=x) = \frac{P(X=x, Y=y)}{P(X=x)}$$

NOTE:

field: For discrete random variables $E(Y|X=x) =$

field: For discrete random variables $E(Y|X=x) = \sum_y yP(Y=y|X=x)$

NOTE:

field: Problem solving strategy for expected value of counting

field: Use indicator functions for each trial , where $X = \sum I$ and use linearity of expectation

NOTE:

field: $P(X > s+t|X > t)$ for geometric, exponential

field: $P(X > s+t|X > t) = P(X > s)$

NOTE:

field: Distribution for: A bag of N balls which conatins r red balls and $N-r$ blue balls, X is number of red balls in a sample of size n taken without replacement.

field: Hypergeometric.

NOTE:

field: Distribution for modeling arrival time

field: Exponential

NOTE:

field: $E(g(X,Y)) =$ (continuous)

field: $E(g(X,Y)) = \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} g(x,y)f(x,y)dxdy$

NOTE:

field: $Cov(X, Y) =$ (integration)

field: $Cov(X, Y) = \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} (x - E(X))(y - E(Y)) dxdy$

NOTE:

field: Problem solving strategies for functions of random variables

field:

- Methods of cdf: $Y = g(X)$, find cdf $P(Y \leq y) = P(g(X) \leq y) = P(X \leq g^{-1}(y))$
- For finding $P(X < Y)$, set up integrals that cover
- For finding probabilities of independent uniform random variables, use geometric (area) properties

NOTE:

field: Quantile

field: If X is a continuous random variable, then the pth quantile is is the number q that satisfies $P(X \leq q) = p/100$

NOTE:

field: Poisson process

field: Times between arrivals are modeled as iid exponential random variables with parameter $\lambda = 1/\beta$. Let N_t be the number of arrivals up to time t. Then $N_t \sim Pois(\lambda t)$

NOTE:

field: Conditional density function $f_{Y|X}(y|x) =$

field: $f_{Y|X}(y|x) = \frac{f(x,y)}{f_x(x)}$

NOTE:

field: Continuous bayes formula

field: $f_{X|Y}(x|y) = \frac{f_{Y|X}(y|x)f_x(x)}{\int_{t=-\infty}^{\infty} f_{Y|X}(y|t)f_x(t)dt}$

NOTE:

field: Conditional expectation for continuous random variables $E(Y|X = x)$

field: $E(Y|X = x) = \int_y y f_{Y|X}(y|x)dy$

NOTE:

field: Law of total expectation

field: $E(Y) = E(E(Y|X))$

NOTE:

field: Properties of conditional expectation

- $E(aY + bZ|X) =$
- $E(g(Y)|X = x) =$
- If X, Y independent, $E(Y|X) =$
- If $Y = g(X)$, then $E(Y|X) =$

field: Properties of conditional expectation

- $E(aY + bZ|X) = aE(Y|X) + bE(Z|X)$
- $E(g(Y)|X = x) = \int_y g(y) f_{Y|X}(y|x) dy$
- If X, Y independent, $E(Y|X) = E(Y)$
- If $Y = g(X)$, then $E(Y|X) = Y$

NOTE:

field: Law of total probability, continuous

field: $P(A) = \int_{-\infty}^{\infty} P(A|X = x) f_x(x) dx$

NOTE:

field: Conditional variance $V(Y|X = x)$

field:

$$V(Y|X = x) = \sum_y (y - E(Y|X = x))^2 P(Y = y|X = x)$$

discrete

$$V(Y|X = x) = \int_y (y - E(Y|X = x))^2 f_{Y|X}(y|x) dy$$

continuous

NOTE:

field: Properties of conditional variance

- $V(Y|X = x) =$
- $V(aY + b|X = x) =$
- If Y, Z independent, $V(Y + Z|X = x) =$

field: Properties of conditional variance

- $V(Y|X=x) = E(Y^2|X=x) - (E(Y|X=x))^2$
- $V(aY+b|X=x) = a^2V(Y|X=x)$
- If Y, Z independent, $V(Y+Z|X=x) = V(Y|X=x) + V(Z|X=x)$

NOTE:

field: $P(X \geq \epsilon)$

field: $P(X \geq \epsilon) \leq E(X)/\epsilon$ (Markov's Inequality)

NOTE:

field: $P(|X-\mu| \geq \epsilon)$

field: $P(|X-\mu| \geq \epsilon) \leq \sigma^2/\epsilon^2$ (Chebyshev's inequality, if mean and variance finite)

NOTE:

field: $P(\lim_{n\to\infty} S_n/n = \mu) =$

field: $P(\lim_{n\to\infty} S_n/n = \mu) = 1$ (Strong law of large numbers)

tags: Calculus

NOTE:

field: $\int_0^\infty e^{-x^2/2} =$

field: $\int_0^\infty e^{-x^2/2} = \sqrt{\pi/2}$

NOTE:

field: $\int_0^\infty x^{a-1}e^{-x/b} =$

field: $\int_0^\infty x^{a-1}e^{-x/b} = \Gamma(a)b^a$

NOTE:

field: $\int_0^1 x^{a-1}(1-x)^{b-1} =$

field: $\int_0^1 x^{a-1}(1-x)^{b-1} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$

NOTE:

field: $\log(x) = y, x =$

field: $\log(x) = y, x = e^y$

NOTE:

field: $\lim_{x\to\infty}(1+\frac{a}{x})^x =$

field: $\lim_{x\to\infty}(1+\frac{a}{x})^x = e^a$

NOTE:

field: $\lim_{x\to\infty}(1+\frac{a}{x})^x = e^a$

field: $\lim_{x\to\infty}(1+\frac{a}{x})^x =$

NOTE:

field: $\frac{d}{dx}f(g(x)) =$

field: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain rule)

NOTE:

field: $\frac{d}{dx}\int_a^x f(t)dt =$

field: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$ (fundamental theorem of calculus)

NOTE:

field: $\int_a^b udv =$
ex: $\int xe^{-x}$

field: $\int_a^b udv = uv|_a^b - \int_a^b vdu$
ex: $u = x, dv = e^{-x}, du = dx, v = -e^{-x}$

$$\begin{aligned}\int xe^{-x} &= -xe^{-x} + \int e^{-x} \\ &= -xe^{-x} - e^{-x} + c\end{aligned}$$

NOTE:

field: $\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

field: $\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$

NOTE:

field: $e^x =$

field: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

NOTE:

field: $\sum_{k=0}^{\infty} x^k =$

field: $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ for $|x| < 1$

NOTE:

field: $\sum_{k=0}^{n} x^k =$

field: $\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$ for $x \neq 1$

NOTE:

field: $\lim_{x\to-\infty} e^{-x} =$

field: $\lim_{x\to-\infty} e^{-x} = \infty$

NOTE:

field: $\lim_{x\to\infty} e^{-x} =$

field: $\lim_{x\to-\infty} e^{-x} = 0$

NOTE:

field:

$$(fg)' =$$

field:

$$(fg)' = f'g + g'f$$

(product rule)

NOTE:

field: $\frac{d}{dx}x^n =$

field: $\frac{d}{dx}x^n = nx^{n-1}$

NOTE:

field: $\frac{d}{dx}a^x =$

field: $\frac{d}{dx}a^x = a^x ln(a)$

NOTE:

field: $\frac{d}{dx}ln(x) =$

field: $\frac{d}{dx}ln(x) = \frac{1}{x}$

NOTE:

field: $\frac{d}{dx}(f(x))^n =$

field: $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1}f'(x)$

NOTE:

field: $\frac{d}{dx}ln(f(x)) =$

field: $\frac{d}{dx}ln(f(x)) = \frac{f'(x)}{f(x)}$

NOTE:

field: $\frac{d}{dx}e^{f(x)} =$

field: $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

NOTE:

field: $\int x^n =$

field: $\int x^n = \frac{1}{n+1}x^{n+1}$

NOTE:

field: $\int \frac{1}{x} =$

field: $\int \frac{1}{x} = ln(|x|)$

NOTE:

field: $\int \frac{1}{ax+b} =$

field: $\int \frac{1}{ax+b} = \frac{1}{a} ln(|ax+b|)$

NOTE:

field: $\int e^{cx} =$

field: $\int e^{cx} = \frac{1}{c} e^{cx}$

NOTE:

field: $\int xe^{-cx^2} =$

field: $\int xe^{-cx^2} = -\frac{1}{2c} e^{-cx^2}$

NOTE:

field: U substitution:
example; $\int_1^2 5x^2 \cos(x^3)$

field: $\int_a^b f(g(x))g'(x) = \int_{g(a)}^g (b) f(u) du$
Where $u = g(x), du = g' dx$
Ex: $u = x^3, du = 3x^2, x^2 du = 1/3 du$ $\int_1^2 5x^2 \cos(x^3) = \int_1^8 5/3 \cos(u) du$

NOTE:

field: $\Gamma(a) =$

field: $\int_0^\infty t^{a-1}e^{-t}dt$

NOTE:

field: $\int_0^\infty t^{a-1}e^{-t}dt$

field: $= \Gamma(a)$

NOTE:

field: $\Gamma(a+1) =$

field: $\Gamma(a+1) = a\Gamma(a)$

NOTE:

field: $\Gamma(n) =$

field: $\Gamma(n) = (n-1)!$ (for n an integer)

NOTE:

field: $\Gamma(1/2) =$

field: $\Gamma(1/2) = \sqrt{\pi}$

NOTE:

field: $\Gamma(1) =$

field: $\Gamma(1) = 1$