

tags: FromStatCheatsheet

NOTE:

field: CDF of Geometric (p)

field: $1 - (1 - p)^x$

NOTE:

field: CDF of Exponential(β)

field: $1 - e^{-\frac{x}{\beta}}$

NOTE:

field:

- $P(\emptyset) =$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) =$
- $P(B) =$
- $P(\Omega) = \quad P(\emptyset) =$
- $(\bigcup_n A_n) = \quad (\bigcap_n A_n) = \quad \text{DEMORGAN}$

field:

- $P(\emptyset) = 0$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) = 1 - P(A)$
- $P(B) = P(A \cap B) + P(A^c \cap B)$
- $P(\Omega) = 1 \quad P(\emptyset) = 0$
- $(\bigcup_n A_n) = \bigcap_n A_n \quad (\bigcap_n A_n) = \bigcup_n A_n \quad \text{DEMORGAN}$

NOTE:

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n A_n^c)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) =$
- $P(A \cap B^c) =$

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n A_n^c)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $\implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$
- $P(A \cap B^c) = P(A) - P(A \cap B)$

NOTE:

field: $P(A \cap B) =$ when A and B independent

field: $P(A \cap B) = P(A)P(B)$ when A and B independent

NOTE:

field:

$$P(A|B) =$$

field:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

NOTE:

field: Law of total probability

field: Law of total probability

$$P(B) = \sum_{i=1}^n P(B|A_i)P(A_i) \quad \Omega = \cup_{i=1}^n A_i$$

NOTE:

field: Bayes Theorem

field: Bayes Theorem

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^n P(B|A_j)P(A_j)} \quad \Omega = \cup_{i=1}^n A_i$$

NOTE:

field: CDF Laws

field: CDF Laws

1. Nondecreasing: $x_1 < x_2 \implies F(x_1) \leq F(x_2)$
2. Limits: $\lim_{x \rightarrow -\infty} F(x) = 0$ and $\lim_{x \rightarrow \infty} F(x) = 1$
3. Right-Continuous $\lim_{y \rightarrow x^+} F(y) = F(x)$

NOTE:

field:

$$f_{y|x}(y|x) =$$

field:

$$f_{y|x}(y|x) = \frac{f(x, y)}{f_x(x)}$$

NOTE:

field: X, Y independent

- $P(X \leq x, Y \leq y) =$
- $f_{x,y}(x, y) =$

field: X, Y independent

- $P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$
- $f_{x,y}(x, y) = f_x(x)f_y(y)$

NOTE:

field: Transformations $Z = \phi(X)$

- Discrete: $f_Z(z) =$
- Continuous: $F_Z(z) =$
- Cont, ϕ strictly monotone: $f_z(z)$

field: Transformations $Z = \phi(X)$

- Discrete:

$$f_Z(z) = P(\phi(X) = z) = P(X \in \phi^{-1}(z)) = \sum_{x \in \phi^{-1}(z)} f_x(x)$$

- Continuous (Method of CDF):

$$F_Z(z) = P(\phi(X) \leq z) = \int_{x: \phi(x) \leq z} f(x) dx$$

- Cont, ϕ strictly monotone: (Method of PDF) $f_z(z) = f_x(\phi^{-1}(z)) \left| \frac{d}{dz} \phi^{-1}(z) \right|$

NOTE:

field: Rule of the Lazy Statistician: $E[g(x)] =$

field: Rule of the Lazy Statistician: $E[g(x)] = \int g(x)f_x(x)dx$

NOTE:

field: Expectation rules

- $E(c) =$
- $E(cX) =$
- $E(X + Y) =$
- $E(\phi(X)) =$

field: Expectation rules

- $E(c) = c$
- $E(cX) = cE(X)$
- $E(X + Y) = E(X) + E(Y)$
- $E(\phi(X)) \neq \phi(E(X))$

NOTE:

field: Conditional expectation

- $E(Y|X = x) =$
- $E(X) =$
- $E(Y + Z|X) =$
- $E(Y|X) = c \implies$

field: Conditional expectation

- $E(Y|X = x) = \int yf(y|x)dy$
- $E(X) = E(E(X|Y))$
- $E(Y + Z|X) = E(Y|X) + E(Z|X)$
- $E(Y|X) = c \implies Cov(X, Y) = 0$

NOTE:

field: Variance

- $V(X) = \sigma_x^2 =$
- $V(X + Y) =$
- $V\left[\sum_{i=1}^n X_i\right] =$

field: Variance

- $V(X) = \sigma_x^2 = E[(X - E(X))^2] = E(X^2) - E(X)^2$
- $V(X + Y) = V(X) + V(Y) + Cov(X, Y)$
- $V\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n V(X_i) + \sum_{i \neq j} Cov(X_i, X_j)$

NOTE:

field: Covariance

- $Cov(X, Y) =$
- $Cov(X, c) =$
- $Cov(Y, X) =$
- $Cov(aX + bY) =$
- $Cov(X + a, Y + b) =$
- $Cov\left(\sum_{i=1}^n X_i, \sum_{j=1}^m Y_j\right) =$

field: Covariance

- $Cov(X, Y) = E[(X - E(X))(Y - E(Y)))] = E(XY) - E(X)E(Y)$
- $Cov(X, c) = 0$
- $Cov(Y, X) = Cov(X, Y)$
- $Cov(aX + bY) = abCov(X, Y)$
- $Cov(X + a, Y + b) = Cov(X, Y)$
- $Cov\left(\sum_{i=1}^n X_i, \sum_{j=1}^m Y_j\right) = \sum_{i=1}^n \sum_{j=1}^m Cov(X_i, Y_j)$

NOTE:

field: Correlation: $\rho(X, Y)$

field: Correlation: $\rho(X, Y) = \frac{Cov(X, Y)}{\sqrt{V(X)V(Y)}}$

NOTE:

field: Conditional Variance

- $V(Y|X) =$
- $V(Y) =$

field: Conditional Variance

- $V(Y|X) = E[(Y - E(Y|X))^2|X] = E(Y^2|X) - E(Y|X)^2$
- $V(Y) = E(V(Y|X)) + V(E(Y|X))$

tags: distributionrelationships

NOTE:

field:

$$X_1, \dots, X_n \sim iidN(0, 1) \\ \sum X_i \stackrel{?}{\sim}$$

field:

$$X_1, \dots, X_n \sim iidN(0, 1) \\ \sum X_i \sim N(0, n)$$

NOTE:

field:

$$X_1, \dots, X_n \sim iidN(\mu_i, \sigma_i^2) \\ \sum X_i \stackrel{?}{\sim}$$

field:

$$X_1, \dots, X_n \sim iidN(\mu_i, \sigma_i^2) \\ \sum X_i \sim N(\sum \mu_i, \sum \sigma_i^2)$$

NOTE:

field:

$$X \sim N(\mu, \sigma^2) \\ aX + b \stackrel{?}{\sim}$$

field:

$$aX + Y \sim N(a\mu + b, a^2\sigma^2)$$

NOTE:

field: $X \sim \text{Binom}(1, p) \stackrel{?}{\sim}$

field: $X \sim \text{Bern}(p)$

NOTE:

field: $X \sim \text{NegBinom}(1, p) \stackrel{?}{\sim}$

field: $X \sim \text{Geom}(p)$

NOTE:

field: $X \sim \text{Gamma}(1, \theta) \stackrel{?}{\sim}$

field: $X \sim \text{Exp}(\theta)$

NOTE:

field: $X \sim \text{Exp}(\theta) \stackrel{?}{\sim}$

field: $X \sim \text{Gamma}(1, \theta)$

NOTE:

field: $X \sim \text{Gamma}(v/2, 1/2) \stackrel{?}{\sim}$

field: $X \sim \chi^2(v)$

NOTE:

field:

$$X \sim \chi^2(v) \stackrel{?}{\sim}$$

field:

$$X \sim \text{Gamma}(v/2, 1/2)$$

NOTE:

field:

$$X \sim \chi^2(2) \stackrel{?}{\sim}$$

field:

$$X \sim \exp(2)$$

NOTE:

field:

$$X \sim \text{Weibull}(1, \beta) \stackrel{?}{\sim}$$

field:

$$X \sim \text{Exp}(\beta)$$

NOTE:

field: $X_1, X_2 \sim \chi^2(v_i)$ independent $\frac{X_1/v_1}{X_2/v_2}$

field:

$$\frac{(X_1/v_1)}{(X_2/v_2)} \sim F(v_1, v_2)$$

NOTE:

field:

$$X \sim \beta(1,1) \stackrel{?}{\sim}$$

field:

$$X \sim Unif(0,1)$$

NOTE:

field:

$$X \sim Unif(0,1) \stackrel{?}{\sim}$$

field:

$$X \sim \beta(1,1)$$

NOTE:

field: Special case of t

$$X \sim t(1) \stackrel{?}{\sim}$$

field:

$$X \sim Cauchy(0,1)$$

NOTE:

field: Scaled Gamma

$$X \sim Gamma(\alpha, \beta), Y = aX \stackrel{?}{\sim}$$

field:

$$Y \sim \textit{Gamma}(\alpha, a\beta)$$

NOTE:

field: Scaled Exponential

$$X \sim \textit{Exp}(\lambda), Y = aX \stackrel{?}{\sim}$$

field:

$$Y \sim \textit{Exp}(a\lambda)$$

NOTE:

field: Sum of Exponential, equal rate $X_i \sim \textit{Exp}(\lambda), Y = \sum X_i$

field:

$$Y \sim \textit{Gamma}(n, \lambda)$$

NOTE:

field:

$$X \sim \textit{Exp}(\lambda), Y = e^{-x}$$

field:

$$Y \sim \textit{Beta}(\lambda, 1)$$

NOTE:

field: Min of Exponential

$$X_1, \dots, X_n \text{Exp}(\lambda_i), Y = \min(X_i) \stackrel{?}{\sim}$$

field: $Y \sim \text{exp}(\sum \lambda_i)$

NOTE:

field: Min of Uniform

$$X_i \sim \text{Unif}(0, 1), Y = \lim n \min(X_i) \stackrel{?}{\sim}$$

field:

$$Y \sim \text{Exp}(1)$$

NOTE:

field:

$$X \sim \text{Beta}(\alpha, \beta), Y = (1 - X)$$

field:

$$Y \sim \text{Beta}(\beta, \alpha)$$

NOTE:

field: $X \sim F_X(X), Y = F_X^{-1}(X)$

field: $Y \sim \text{Unif}(0, 1)$

NOTE:

field: $X \sim N(\mu, \sigma^2), Y = e^X$

field: $Y \sim \text{lognormal}(\mu, \sigma^2)$

NOTE:

field: $X \sim \exp(\beta), Y = X^{1/z}$

field: $Y \sim \text{Weibull}(z, \beta)$

NOTE:

field: Square of Normal $X \sim N(0, 1), Y = X^2$

field: $Y \sim \chi^2(1)$

NOTE:

field: Square of t $X \sim t(v), Y = X^2$

field: $Y \sim F(1, v)$

NOTE:

field: Sum of Poisson $X_i \sim \text{Poisson}(\mu_i) Y = \sum X_i$

field: $Y \sim \text{Poisson}(\sum \mu_i)$

NOTE:

field: Sum of Gamma $X_i \sim \text{Gamma}(\alpha_i, \beta), Y = \sum X_i$

field: $Y \sim \text{Gamma}(\sum \alpha_i, \beta)$

NOTE:

field: Sum of independent Chi-squared $X_i \sim \chi^2(v_i) Y = \sum X_i$

field: $Y \sim \chi^2(\sum v_i)$

NOTE:

field: X, Y independent $X, Y \sim N(0, 1), X/Y$

field: $X/Y \sim Cauchy(0, 1)$

NOTE:

field: $X_1, X_2 \sim gamma(\alpha_i, 1)$ independent, $\frac{X_1}{X_1 + X_2}$

field:

$$\frac{X_1}{X_1 + X_2} \sim beta(\alpha_1, \alpha_2)$$

NOTE:

field: $X_1, X_2 \sim gamma(\alpha_i, \beta_i)$ independent, $\frac{\beta_2 X_1}{\beta_2 X_1 + \beta_1 X_2}$

field:

$$\frac{\beta_2 X_1}{\beta_2 X_1 + \beta_1 X_2} \sim beta(\alpha_1, \alpha_2)$$

NOTE:

field: X, Y independent $exp(\mu) X - Y$

field: $X - Y \sim double\ exponential(0, \mu)$

NOTE:

field: Mean and Variance for Distributions not on bible (but in CB)

- Bernoulli(p), $E(X) =$, $V(X) =$
- Discrete Uniform N , $E(X) =$, $V(X) =$
- Cauchy(θ, σ), $E(X) =$, $V(X) =$
- Double Exponential(μ, σ), $E(X) =$, $V(X) =$
- F(v_1, v_2), $E(X) =$, $V(X) =$
- Logistic(μ, β), $E(X) =$, $V(X) =$
- Lognormal(μ, σ^2), $E(X) =$, $V(X) =$
- Pareto(α, β), $E(X) =$, $V(X) =$
- t(v), $E(X) =$, $V(X) =$
- Weibull(γ, β), $E(X) =$, $V(X) =$

field: Mean and Variance for Distributions not on bible (but in CB)

- Bernoulli(p), $E(X) = p$, $V(X) = p(1 - p)$
- Discrete Uniform N , $E(X) = \frac{N+1}{2}$, $V(X) = \frac{(N+1)(N-1)}{12}$
- Cauchy(θ, σ), $E(X) = na$, $V(X) = na$
- Double Exponential(μ, σ), $E(X) = \mu$, $V(X) = 2\sigma^2$
- F(v_1, v_2), $E(X) = \frac{v_1}{v_2-2}$, $V(X) = 2\left(\frac{v_2}{v_2-2}\right)^2 \frac{(v_1+v_2-2)}{v_1(v)2-4}$
- Logistic(μ, β), $E(X) = \mu$, $V(X) = \frac{\phi^2\beta^2}{3}$
- Lognormal(μ, σ^2), $E(X) = e^{\mu+(\sigma^2/2)}$, $V(X) = e^{2(\mu+\sigma^2)} - e^{2\mu+2\sigma^2}$
- Pareto(α, β), $E(X) = \frac{\beta\alpha}{\beta-1}$, $V(X) = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$
- t(v), $E(X) = 0$, $V(X) = \frac{v}{v-2}$
- Weibull(γ, β), $E(X) = \beta^{1/\gamma}\Gamma(1+1/\gamma)$, $V(X) = \beta^{2/\gamma}(\Gamma(1+2/\gamma) - \Gamma^2(1+1/\gamma))$

tags: CasellaCh1

NOTE:

field: Sample Space

field: The set, S , of all possible outcomes of a particular experiment is called the *sample space* for the experiment.

NOTE:

field: Event

field: An *event* is any collection of possible outcomes of an experiment, that is, any subset of S (including S itself).

NOTE:

field: Union

field: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

NOTE:

field: Intersection

field: $A \cap B = \{x : x \in A \text{ and } x \in B\}$

NOTE:

field: Complementation

field: $A^c = \{x : x \notin A\}$

NOTE:

field: Commutativity

$$A \cup B =$$

$$A \cap B =$$

field: Commutativity

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

NOTE:

field: Associativity

$$A \cup (B \cup C) =$$

$$A \cap (B \cap C) =$$

field: Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$A \cap (B \cap C) = (A \cap B) \cap C$$

NOTE:

field: Distributive Laws

$$A \cap (B \cup C) =$$

$$A \cup (B \cap C) =$$

field: Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

NOTE:

field: DeMorgan's Laws

$$(A \cup B)^c =$$

$$(A \cap B)^c =$$

field: DeMorgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

NOTE:

field: Disjoint

field: Disjoint: Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \emptyset$

NOTE:

field: Pairwise disjoint

field: Two Events A_1, A_2 are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \emptyset$ for all $i \neq j$

NOTE:

field: Partition

field: If A_1, A_2, \dots are pairwise disjoint and $\cup_{i=1}^{\infty} A_i = S$, then the collection A_1, A_2, \dots forms a partition of S .

NOTE:

field: Sigma Algebra

field: A collection of subsets of S is called a sigma algebra (or Borel field), denoted by \mathcal{B} , if it satisfies the following three properties:

1. $\emptyset \in \mathcal{B}$ (the empty set is an element of \mathcal{B})
2. If $A \in \mathcal{B}$, then $A^c \in \mathcal{B}$ (\mathcal{B} is closed under complementation)
3. If $A_1, A_2, \dots \in \mathcal{B}$, then $\cup_{i=1}^{\infty} A_i \in \mathcal{B}$ (\mathcal{B} is closed under countable unions)

NOTE:

field: Probability Function / Kolmogorov Axioms

field: Given a sample space S and an associated sigma algebra \mathcal{B} , a probability function is a function P with domain \mathcal{B} that satisfies:

1. $P(A) \geq 0$ for all $A \in \mathcal{B}$
2. $P(S) = 1$
3. If $A_1, A_2, \dots \in \mathcal{B}$ are pairwise disjoint, then $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$
(Axiom of Countable Additivity)

NOTE:

field: Theorem 1.2.6

field: Let $S = \{s_1, \dots, s_n\}$ be a finite set. Let \mathcal{B} be any sigma algebra of subsets of S . Let p_1, \dots, p_n be nonnegative numbers that sum to 1. For any $A \in \mathcal{B}$, define $P(A)$ by

$$P(A) = \sum_{\{i:s_i \in A\}} p_i$$

(The sum over an empty set is defined to be 0) Then P is a probability function on \mathcal{B} . This remains true if $S = \{s_1, s_2, \dots\}$ is a countable set.

NOTE:

field: Axiom of Finite Additivity

field: If $A \in \mathcal{B}$ and $B \in \mathcal{B}$ are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

NOTE:

field: Properties of probability functions

field: Properties of probability functions

1. $P(\emptyset) = 0$
2. $P(A) \leq 1$
3. $P(A^c) = 1 - P(A)$

NOTE:

field: If P is a probability function and A and B are any sets in \mathcal{B} , then

$$P(B \cap A^c) =$$

field: If P is a probability function and A and B are any sets in \mathcal{B} , then

$$P(B \cap A^c) = P(B) - P(A \cap B)$$

NOTE:

field: If P is a probability function and A and B are any sets in \mathcal{B} , then $P(A \cup B) =$

field: If P is a probability function and A and B are any sets in \mathcal{B} , then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

NOTE:

field: If P is a probability function and A and B are any sets in \mathcal{B} , then if $A \subset B$ then

field: If P is a probability function and A and B are any sets in \mathcal{B} , then if $A \subset B$ then $P(A) \leq P(B)$

NOTE:

field: Bonferroni's Inequality

field: Bonferroni's Inequality:

$$P(A \cap B) \geq P(A) + P(B) - 1$$

NOTE:

field: If P is a probability function, then for any partition C_1, C_2, \dots $P(A) =$

field: If P is a probability function, then for any partition C_1, C_2, \dots $P(A) = \sum_{i=1}^{\infty} P(A \cap C_i)$

NOTE:

field: Boole's Inequality

field: If P is a probability function,

$$P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i) \text{ for any sets } A_1, A_2, \dots$$

NOTE:

field: Fundamental Theorem of Counting

field: If a job consists of k separate tasks, the i th of which can be done in n_i ways, $i = 1, \dots, k$, then the entire job can be done in $n_1 \times n_2 \times \dots \times n_k$ ways.

NOTE:

field: Ordered without replacement: outcomes of choosing r from n objects

field:

$$\frac{n!}{(n-r)!}$$

NOTE:

field: Unordered without replacement chose r from n objects

field: Unordered without replacement chose r from n objects

$$\binom{n}{r}$$

NOTE:

field: Ordered with replacement: choose r from n objects:

field: Ordered with replacement: choose r from n objects:

$$n^r$$

NOTE:

field: Unordered without replacement choose r from n objects

field: Unordered without replacement choose r from n objects:

$$\binom{n}{r}$$

NOTE:

field: Unordered with replacement choose r from n objects

field: Unordered with replacement choose r from n objects:

$$\binom{n+1-1}{r}$$

NOTE:

field: Binomial Coefficient $\binom{n}{r}$

field: Binomial Coefficient

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$