tags: FromStatCheatsheet

NOTE:

field: CDF of Geometric (p)

field: $1-(1-p)^x$

NOTE:

field: CDF of Exponential(β)

field: $1-e^{-\frac{x}{\beta}}$

NOTE:

field:

- $P(\varnothing) =$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) =$
- $P(B) =$
- $P(\Omega) = \qquad P(\varnothing) =$
- $(\bigcup_n A_n) = \quad (\bigcap_n A_n) =$ DeMorgan

field:

- $P(\varnothing) = 0$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) = 1 - P(A)$
- $P(B) = P(A \cap B) + P(A^c \cap B)$
- $P(\Omega) = 1 \qquad P(\varnothing) = 0$
- $(\bigcup_n A_n) = \bigcap_n A_n \quad (\bigcap_n A_n) = \bigcup_n A_n$ DeMorgan

NOTE:

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n A_n^c)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) =$
- $P(A \cap B^c) =$

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n \mathsf{A_n})$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  $\implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$
- $P(A \cap B^c) = P(A) - P(A \cap B)$

NOTE:

field: $P(A \cap B) =$ when A and Bindependent

field: $P(A \cap B) = P(A)P(B)$ when A and Bindependent

NOTE:

field:

$$P(A|B) =$$

field:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

NOTE:

field: Law of total probability

field: Law of total probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i) \quad \Omega = \cup_{i=1}^{n} A_i$$

NOTE:

field: Bayes Theorem

field: Bayes Theorem

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \quad \Omega = \cup_{i=1}^{n} A_i$$

NOTE:

field: CDF Laws

field: CDF Laws

1. Nondecreasing: $x_1 < x_2 \implies F(x_1) \leq F(x_2)$
2. Limits: $\lim_{x\to-\infty} = 0$ and $\lim_{x\to\infty} = 1$
3. Right-Continuous $\lim_{y\to x^+} F(y) = F(x)$

NOTE:

field:

$$f_{y|x}(y|x) =$$

field:

$$f_{y|x}(y|x) = \frac{f(x,y)}{f_x(x)}$$

NOTE:

field: X, Y independent

- $P(X \leq x, Y \leq y) =$
- $f_{x,y}(x,y) =$

field: X, Y independent

- $P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$
- $f_{x,y}(x,y) = f_x(x)f_y(y)$

NOTE:

field: Transformations $Z = \phi(X)$

- Discrete: $f_Z(z) =$
- Continuous: $F_Z(z) =$
- Cont, ϕ strictly monotone: $f_z(z)$

field: Transformations $Z = \phi(X)$

- Discrete:

$$f_Z(z) = P(\phi(X) = z) = P(X \in \phi^{-1}(z)) = \sum_{x \in \phi^{-1}(z)} f_x(x)$$

- Continuous (Method of CDF):

$$F_Z(z) = P(\phi(X) \leq z) = \int_{x:\phi(x) \leq z} f(x)dx$$

- Cont, ϕ strictly monotone: (Method of PDF) $f_z(z) = f_x(\phi^{-1}(z))|\frac{d}{dz}\phi^{-1}(z)|$

NOTE:

field: Rule of the Lazy Statistician: $E[g(x)] =$

field: Rule of the Lazy Statistician: $E[g(x)] = \int g(x) f_x(x) dx$

NOTE:

field: Expectation rules

- $E(c) =$
- $E(cX) =$
- $E(X + Y) =$
- $E(\phi(X)) =$

field: Expectation rules

- $E(c) = c$
- $E(cX) = cE(X)$
- $E(X + Y) = E(X) + E(Y)$
- $E(\phi(X)) \neq \phi(E(X))$

NOTE:

field: Conditional expectation

- $E(Y|X = x) =$
- $E(X) =$
- $E(Y + Z|X) =$
- $E(Y|X) = c \implies$

field: Conditional expectation

- $E(Y|X = x) = \int yf(y|x)dy$
- $E(X) = E(E(X|Y))$
- $E(Y + Z|X) = E(Y|X) + E(Z|X)$
- $E(Y|X) = c \implies Cov(X, Y) = 0$

NOTE:

field: Variance

- $V(X) = \sigma_x^2 =$
- $V(X + Y) =$
- $V\left[\sum_{i=1}^{n} X_i\right] =$

field: Variance

- $V(X) = \sigma_x^2 = E[(X - E(X))^2] = E(X^2) - E(X)^2$
- $V(X + Y) = V(X) + V(Y) + Cov(X, Y)$
- $V\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} V(X_i) + \sum_{i \neq j} Cov(X_i, X_j)$

NOTE:

field: Covariance

- $Cov(X, Y) =$
- $Cov(X, c) =$
- $Cov(Y, X) =$
- $Cov(aX + bY) =$
- $Cov(X + a, Y + b) =$
- $Cov\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) =$

field: Covariance

- $Cov(X,Y) = E[(X - E(X)(Y - E(Y)))] = E(XY) - E(X)E(Y)$
- $Cov(X,c) = 0$
- $Cov(Y,X) = Cov(X,Y)$
- $Cov(aX + bY) = abCov(X,Y)$
- $Cov(X + a, Y + b) = Cov(X,Y)$
- $Cov\left(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j\right) = \sum_{i=1}^{n} \sum_{j=1}^{m} Cov(X_i, Y_j)$

NOTE:

field: Correlation: $\rho(X,Y)$

field: Correlation: $\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

NOTE:

field: Conditional Variance

- $V(Y|X) =$
- $V(Y) =$

field: Conditional Variance

- $V(Y|X) = E\left[(Y - E(Y|X))^2|X\right] = E(Y^2|X) - E(Y|X)^2$
- $V(Y) = E(V(Y|X)) + V(E(Y|X))$

tags: distributionrelationships

NOTE:

field:

$$X_1, \ldots, X_n \sim iidN(0,1)$$
$$\sum X_i \overset{?}{\sim}$$

field:

$$X_1, \ldots, X_n \sim iidN(0,1)$$
$$\sum X_i \sim N(0,n)$$

NOTE:

field:

$$X_1, \ldots, X_n \sim iidN(\mu_i, \sigma_i^2)$$
$$\sum X_i \overset{?}{\sim}$$

field:

$$X_1, \ldots, X_n \sim iidN(\mu_i, \sigma_i^2)$$
$$\sum X_i \sim N(\sum \mu_i, \sum \sigma_i^2)$$

NOTE:

field:

$$X \sim N(\mu, \sigma^2)$$
$$aX + b \overset{?}{\sim}$$

field:

$$aX + Y \sim N(a\mu + b, a^2\sigma^2)$$

NOTE:

field: $X \sim Binom(1, p) \overset{?}{\sim}$

field: $X \sim Bern(p)$

NOTE:

field: $X \sim NegBinom(1, p) \overset{?}{\sim}$

field: $X \sim Geom(p)$

NOTE:

field: $X \sim Gamma(1, \theta) \overset{?}{\sim}$

field: $X \sim Exp(\theta)$

NOTE:

field: $X \sim Exp(\theta) \overset{?}{\sim}$

field: $X \sim Gamma(1, \theta)$

NOTE:

field: $X \sim Gamma(v/2, 1/2) \overset{?}{\sim}$

field: $X \sim \chi^2(v)$

NOTE:

field:

$$X \sim \chi^2(v) \overset{?}{\sim}$$

field:

$$X \sim Gamma(v/2, 1/2)$$

NOTE:

field:

$$X \sim \chi^2(2) \overset{?}{\sim}$$

field:

$$X \sim exp(2)$$

NOTE:

field:

$$X \sim Weibull(1, \beta) \overset{?}{\sim}$$

field:

$$X \sim Exp(\beta)$$

NOTE:

field: $X_1, X_2 \sim \chi^2(v_i)$ independent $\frac{X_1/v_1}{X_2/v_2}$

field:

$$\frac{(X_1/v_1)}{(X_2/v_2)} \sim F(v_1, v_2)$$

NOTE:

field:

$$X \sim beta(1,1) \overset{?}{\sim}$$

field:

$$X \sim Unif(0,1)$$

NOTE:

field:

$$X \sim Unif(0,1) \overset{?}{\sim}$$

field:

$$X \sim beta(1,1)$$

NOTE:

field: Special case of t

$$X \sim t(1) \overset{?}{\sim}$$

field:

$$X \sim Caucy(0,1)$$

NOTE:

field: Scaled Gamma

$$X \sim Gamma(\alpha,\beta), Y = aX \overset{?}{\sim}$$

field:

$$Y \sim Gamma(\alpha, a\beta)$$

NOTE:

field: Scaled Exponential

$$X \sim Exp(\lambda), Y = aX \overset{?}{\sim}$$

field:

$$Y \sim Exp(a\lambda)$$

NOTE:

field: Sum of Exponential, equal rate $X_i \sim Exp(\lambda), Y = \sum X_i$

field:

$$Y \sim Gamma(n, \lambda)$$

NOTE:

field:

$$X \sim Exp(\lambda), Y = e^{-x}$$

field:

$$Y \sim Beta(\lambda, 1)$$

NOTE:

field: Min of Exponential

$$X_1, \ldots, X_n Exp(\lambda_i), Y = \min(X_i) \stackrel{?}{\sim}$$

field: $Y \sim exp(\sum \lambda_i)$

NOTE:

field: Min of Uniform

$$X_i \sim Unif(0,1), Y = \lim n \min(X_i) \stackrel{?}{\sim}$$

field:

$$Y \sim Exp(1)$$

NOTE:

field:

$$X \sim Beta(\alpha, \beta), Y = (1 - X)$$

field:

$$Y \sim Beta(\beta, \alpha)$$

NOTE:

field: $X \sim F_X(X), Y = F_X^{-1}(X)$

field: $Y \sim Unif(0,1)$

NOTE:

field: $X \sim N(\mu, \sigma^2), Y = e^X$

field: $Y \sim lognormal(\mu, \sigma^2)$

NOTE:

field: $X \sim exp(\beta), Y = X^{1/z}$

field: $Y \sim Weibull(z, \beta)$

NOTE:

field: Square of Normal $X \sim N(0, 1), Y = X^2$

field: $Y \sim \chi^2(1)$

NOTE:

field: Square of t $X \sim t(v), Y = X^2$

field: $Y \sim F(1, v)$

NOTE:

field: Sum of Poisson $X_i \sim Poisson(\mu_i) Y = \sum X_i$

field: $Y \sim Poisson(\sum \mu_i)$

NOTE:

field: Sum of Gamma $X_i \sim Gamma(\alpha_i, \beta), Y = \sum X_i$

field: $Y \sim Gamma(\sum \alpha_i, \beta)$

NOTE:

field: Sum of independent Chi-squared $X_i \sim \chi^2(v_i) Y = \sum X_i$

field: $Y \sim \chi^2(\sum v_i)$

NOTE:

field: X, Y independent $X, Y \sim N(0,1), X/Y$

field: $X/Y \sim Cauchy(0,1)$

NOTE:

field: $X_1, X_2 \sim gamma(\alpha_i, 1)$ independent, $\frac{X_1}{X_1+X_2}$

field:

$$\frac{X_1}{X_1 + X_2} \sim beta(\alpha_1, \alpha_2)$$

NOTE:

field: $X_1, X_2 \sim gamma(\alpha_i, \beta_i)$ independent, $\frac{\beta_2 X_1}{\beta_2 X_1 + \beta_1 X_2}$

field:

$$\frac{\beta_2 X_1}{\beta_2 X_1 + \beta_1 X_2} \sim beta(\alpha_1, \alpha_2)$$

NOTE:

field: X, Y independent $exp(\mu)$ $X - Y$

field: $X - Y \sim$ double exponential$(0, \mu)$

NOTE:

field: Inverted Gamma $X \sim Gamma(\alpha, \beta)$ $Y = 1/X$

tags:

NOTE:

field: Bernoulii$(p), E(X) =, V(X) =$

Bernoulii$(p), E(X) = p, V(X) = p(1-p)$
tags:

NOTE:

field: Discrete Uniform $N, E(X) =, V(X) =$

field: Discrete Uniform $N, E(X) = \frac{N+1}{2}, V(X) = \frac{(N+1)(N-1)}{12}$

NOTE:

field: Cauchy$(\theta, \sigma), E(X) =, V(X) =$

field: Cauchy$(\theta, \sigma), E(X) = na, V(X) = na$

NOTE:

field: Double Exponential$(\mu, \sigma), E(X) =, V(X) =$

field: Double Exponential$(\mu, \sigma), E(X) = \mu, V(X) = 2\sigma^2$

NOTE:

field: F$(v_1, v_2), E(X) =, V(X) =$

field: $\mathrm{F}(v_1, v_2), E(X) = \frac{v_1}{v_2-2}, V(X) = 2\left(\frac{v_2}{v_2-2}\right)^2 \frac{(v_1+v_2-2)}{v_1(v)2-4}$

NOTE:

field: Mean and Variance for Distributions not on bible (but in CB)

- Double Exponential$(\mu, \sigma), E(X) =, V(X) =$
- $\mathrm{F}(v_1, v_2), E(X) =, V(X) =$
- Logistic$(\mu, \beta), E(X) =, V(X) =$
- Lognormal$(\mu, \sigma^2), E(X) =, V(X) =$
- Pareto$(\alpha, \beta), E(X) =, V(X) =$
- $\mathrm{t}(v), E(X) =, V(X) =$
- Weibull$(\gamma, \beta), E(X) =, V(X) =$

field: Mean and Variance. for Distributions not on bible (but in CB)

- Logistic$(\mu, \beta), E(X) = \mu, V(X) = \frac{\phi^2\beta^2}{3}$
- Lognormal$(\mu, \sigma^2), E(X) = e^{\mu+(\sigma^2/2)})V(X) = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$
- Pareto$(\alpha, \beta), E(X) = \frac{\beta\alpha}{\beta-1}, V(X) = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$
- $\mathrm{t}(v), E(X) = 0, V(X) = \frac{v}{v-2}$
- Weibull$(\gamma, \beta), E(X) = \beta^{1/\gamma}\Gamma(1+1/\gamma), V(X) = \beta^{2/\gamma}(\Gamma(1+2/\gamma) - \Gamma^2(1+1/\gamma))$

tags: CasellaCh1

NOTE:

field: Sample Space

field: The set, S, of all possible outcomes of a particular experiment is called the *sample space* for the experiment.

NOTE:

field: Event

field: An *event* is any collection of possible outcomes of an experiment, that is, any subset of S (including S itself).

NOTE:

field: Union

field: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

NOTE:

field: Intersection

field: $A \cap B = \{x : x \in A \text{ and } x \in B\}$

NOTE:

field: Complementation

field: $A^c = \{x : x \notin A\}$

NOTE:

field: Commutativity

$$A \cup B =$$

$$A \cap B =$$

field: Commutativity

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

NOTE:

field: Associativity

$$A \cup (B \cup C) =$$
$$A \cap (B \cap C) =$$

field: Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

NOTE:

field: Distributive Laws

$$A \cap (B \cup C) =$$
$$A \cup (B \cap C) =$$

field: Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

NOTE:

field: DeMorgan's Laws

$$(A \cup B)^c =$$

$$(A \cap B)^c =$$

field: DeMorgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

NOTE:

field: Disjoint

field: Disjoint: Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \emptyset$

NOTE:

field: Pairwise disjoint

field: Two Events A_1, A_2 are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \emptyset$ for all $i \neq j$

NOTE:

field: Partition

field: If $A_1, A_2, \ldots$ are pairwise disjoint and $\cup_{i=1}^{\infty} A_i = S$, then the collection $A_1, A_2, \ldots$ forms a partition of S.

NOTE:

field: Sigma Algebra

field: A collection of subsets of S is called a sigma algebra (or Borel field), denoted by $\mathcal{B}$, if it satisfies the following three properties:

1. $\emptyset \in \mathcal{B}$ (the empty set is an element of $\mathcal{B}$)
2. If $A \in \mathcal{B}$, then $A^c \in \mathcal{B}$ ($\mathcal{B}$ is closed under complementation)
3. If $A_1, A_2, \ldots \in \mathcal{B}$, then $\cup_{i-1}^{\infty} A_i \in \mathcal{B}\mathcal{B}$ is closed under countable unions)

NOTE:

field: Probability Function / Kolmogorov Axioms

field: Given a sample space S and an associated sigma algebra $\mathcal{B}$, a probability function is a function P with domain $\mathcal{B}$ that satisfies:

1. $P(A) \geq 0$ for all $A \in \mathcal{B}$
2. $P(S) = 1$
3. If $A_1, A_2, \ldots \mathcal{B}$ are pairwise disjoint, then $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$ (Axiom of Countable Additivity)

NOTE:

field: If $A \in \mathcal{B}$ and $B \in \mathcal{B}$ are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

Axiom of Finite Additivity

field: If $A \in \mathcal{B}$ and $B \in \mathcal{B}$ are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

NOTE:

field: Properties of probability functions

1. $P(\emptyset) =$
2. $P(A)$
3. $P(A^c) =$

field: Properties of probability functions

1. $P(\emptyset) = 0$
2. $P(A) \leq 1$
3. $P(A^c) = 1 - P(A)$

NOTE:

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(B \cap A^c) =$$

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(B \cap A^c) = P(B) - P(A \cap B)$$

NOTE:

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(A \cup B) =$$

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

NOTE:

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then if $A \subset B$ then

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then if $A \subset B$ then $P(A) \leq P(B)$

NOTE:

field: Bonferroni's Inequality

$$P(A \cap B)$$

field: Bonferroni's Inequality:

$$P(A \cap B) \geq P(A) + P(B) - 1$$

NOTE:

field: If P is a probability function, then for any partition $C_1, C_2, \ldots P(A) =$

field: If P is a probability function, then for any partition $C_1, C_2, \ldots P(A) = \sum_{i=1}^{\infty} P(A \cap C_i)$

NOTE:

field: Boole's Inequality

$$P(\cup_{i=1}^{\infty} A_i)$$

field: If P is a probability function,

$$P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i) \text{ for any sets } A_1, A_2, \ldots$$

NOTE:

field: Fundamental Theorem of Counting

field: If a job consists of k separate tasks, the ith of which can be done in n_i ways, $i = 1, \ldots, k$, then the entire job can be done in $n_1 \times n_2 \times \cdots, \times n_k$ ways.

NOTE:

field: Ordered without replacement: outcomes of choosing r from n objects

field:

$$\frac{n!}{(n-r)!}$$

NOTE:

field: Unordered without replacement chose r from n objects

field: Unordered without replacement chose r from n objects

$$\binom{n}{r}$$

NOTE:

field: Ordered with replacement: choose r from n objects:

field: Ordered with replacement: choose r from n objects:

$$n^r$$

NOTE:

field: Unordered without replacement choose r from n objects

field: Unordered without replacement choose r from n objects:

$$\binom{n}{r}$$

NOTE:

field: Unordered with replacement choose r from n objects

field: Unordered with replacement choose r from n objects:

$$\binom{n+1-1}{r}$$

NOTE:

field: Binomial Coefficient $\binom{n}{r}$

field: Binomial Coefficient

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

NOTE:

field:

$$P(A|B) =$$

field:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

NOTE:

field: Statistically independent $P(A \cap B) =$

field: Statistically independent $P(A \cap B) = P(A)P(B)$

NOTE:

field: If A and B are independent events, what else is independent?

field:

- A and B^c
- A^c and B
- A^c and B^c

NOTE:

field: Mutually independent

field: A collection of events $A_1, \ldots, A_n$ are mutually independent for any subcollection $A_{i1}, \ldots, A_{ik}$, we have

$$P((\cap_{j=1}^{k} A_{ij})) = \prod_{j=1}^{k} P(A_{ij})$$

NOTE:

field: Random variable

field: A random variable is a function from a sample space S into the real numbers

NOTE:

field: Definition of a pdf

field: A function $f_X(x)$ is a pdf (or pmf) of a random variable X if and only if

1. $f_x(x) \geq 0$ for all x
2. $\sum_x f_x(x) = 1$ or $\int_{-\infty}^{\infty} f_x(x)dx = 1$

tags: CasellaCh2

NOTE:

field: (Theorem) Let X have cdf $F_X(x)$, let $Y = g(X)$

1. If g is an increasing function on X, $F_Y(y) =$ for $y \in Y$
2. If g is a decreasing function on X and X is a continuous random variable, $F_y(y) =$ for $y \in Y$

field: (Theorem) Let X have cdf $F_X(x)$, let $Y = g(X)$

1. If g is an increasing function on X, $F_Y(y) = F_X(g^{-1}(y))$ for $y \in Y$
2. If g is a decreasing function on X and X is a continuous random variable, $F_y(y) = 1 - F_X(g^{-1}(y))$ for $y \in Y$

NOTE:

field: Method of pdf

field: Conditions:

1. g is a monotone function
2. $f_X(x)$ is continuous on X
3. $g^{-1}(y)$ has a continuous derivative

Let X have pdf $f_x(x)$ and let $Y = g(Y)$

$$f_Y(y) = f_x(g^{-1}(y))\Big|\frac{d}{dy}g^{-1}(y)$$