tags: FromStatCheatsheet

NOTE:

field: CDF of Geometric (p)

field: $1-(1-p)^x$

NOTE:

field: CDF of Exponential(β)

field: $1-e^{-\frac{x}{\beta}}$

NOTE:

field:

- $P(\varnothing) =$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) =$
- $P(B) =$
- $P(\Omega) = \qquad P(\varnothing) =$
- $(\bigcup_n A_n) = \quad (\bigcap_n A_n) =$ DeMorgan

field:

- $P(\varnothing) = 0$
- $B = \Omega \cap B = (A \cup A^c) \cap B = (A \cap B) \cup (A^c \cap B)$
- $P(A^c) = 1 - P(A)$
- $P(B) = P(A \cap B) + P(A^c \cap B)$
- $P(\Omega) = 1 \qquad P(\varnothing) = 0$
- $(\bigcup_n A_n) = \bigcap_n A_n \quad (\bigcap_n A_n) = \bigcup_n A_n$ DeMorgan

NOTE:

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n A_n^c)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) =$
- $P(A \cap B^c) =$

field: Probability Set intersection

- $P(\bigcup_n A_n) = 1 - P(\bigcap_n \mathsf{A_n})$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  $\implies P(A \cup B) \leq P(A) + P(B)$
- $P(A \cup B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$
- $P(A \cap B^c) = P(A) - P(A \cap B)$

NOTE:

field: $P(A \cap B) =$ when A and Bindependent

field: $P(A \cap B) = P(A)P(B)$ when A and Bindependent

NOTE:

field:

$$P(A|B) =$$

field:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

NOTE:

field: Law of total probability

field: Law of total probability

$$P(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i) \quad \Omega = \cup_{i=1}^{n} A_i$$

$$P(B) = P(A \cup B) + P(A^c \cup B)$$

NOTE:

field: Bayes Theorem

field: Bayes Theorem

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)} \quad \Omega = \cup_{i=1}^{n} A_i$$

NOTE:

field: CDF Laws

field: CDF Laws

1. Nondecreasing: $x_1 < x_2 \implies F(x_1) \leq F(x_2)$
2. Limits: $\lim_{x \to -\infty} = 0$ and $\lim_{x \to \infty} = 1$
3. Right-Continuous $\lim_{y \to x^+} F(y) = F(x)$

NOTE:

field:

$$f_{y|x}(y|x) =$$

field:

$$f_{y|x}(y|x) = \frac{f(x,y)}{f_x(x)}$$

NOTE:

field: X, Y independent

- $P(X \leq x, Y \leq y) =$
- $f_{x,y}(x,y) =$

field: X, Y independent

- $P(X \leq x, Y \leq y) = P(X \leq x)P(Y \leq y)$
- $f_{x,y}(x,y) = f_x(x)f_y(y)$

NOTE:

field: Transformations $Z = \phi(X)$

- Discrete: $f_Z(z) =$
- Continuous: $F_Z(z) =$
- Cont, ϕ strictly monotone: $f_z(z)$

field: Transformations $Z = \phi(X)$

- Discrete:

$$f_Z(z) = P(\phi(X) = z) = P(X \in \phi^{-1}(z)) = \sum_{x \in \phi^{-1}(z)} f_x(x)$$

- Continuous (Method of CDF):

$$F_Z(z) = P(\phi(X) \leq z) = \int_{x:\phi(x) \leq z} f(x)dx$$

- Cont, ϕ strictly monotone: (Method of PDF) $f_z(z) = f_x(\phi^{-1}(z))|\frac{d}{dz}\phi^{-1}(z)|$

NOTE:

field: Rule of the Lazy Statistician: $E[g(x)] =$

field: Rule of the Lazy Statistician: $E[g(x)] = \int g(x) f_x(x) dx$

NOTE:

field: Expectation rules

- $E(c) =$
- $E(cX) =$
- $E(X + Y) =$
- $E(\phi(X)) =$

field: Expectation rules

- $E(c) = c$
- $E(cX) = cE(X)$
- $E(X + Y) = E(X) + E(Y)$
- $E(\phi(X)) \neq \phi(E(X))$

NOTE:

field: Conditional expectation

- $E(Y|X = x) =$
- $E(X) =$
- $E(Y + Z|X) =$
- $E(Y|X) = c \implies$

field: Conditional expectation

- $E(Y|X = x) = \int yf(y|x)dy$
- $E(X) = E(E(X|Y))$
- $E(Y + Z|X) = E(Y|X) + E(Z|X)$
- $E(Y|X) = c \implies Cov(X, Y) = 0$

NOTE:

field: Variance

- $V(X) = \sigma_x^2 =$
- $V(X + Y) =$
- $V\left[\sum_{i=1}^{n} X_i\right] =$

field: Variance

- $V(X) = \sigma_x^2 = E[(X - E(X))^2] = E(X^2) - E(X)^2$
- $V(X + Y) = V(X) + V(Y) + Cov(X, Y)$
- $V\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} V(X_i) + \sum_{i \neq j} Cov(X_i, X_j)$

NOTE:

field: Covariance

- $Cov(X, Y) =$
- $Cov(X, c) =$
- $Cov(Y, X) =$
- $Cov(aX, bY) =$
- $Cov(X + a, Y + b) =$
- $Cov\Bigg(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j \Bigg) =$

field: Covariance

- $Cov(X, Y) = E[(X - E(X)(Y - E(Y)))] = E(XY) - E(X)E(Y)$
- $Cov(X, c) = 0$
- $Cov(Y, X) = Cov(X, Y)$
- $Cov(aX, bY) = abCov(X, Y)$
- $Cov(X + a, Y + b) = Cov(X, Y)$
- $Cov\Bigg(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_j \Bigg) = \sum_{i=1}^{n} \sum_{j=1}^{m} Cov(X_i, Y_j)$

NOTE:

field: Correlation: $\rho(X, Y)$

field: Correlation: $\rho(X, Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

NOTE:

field: Conditional Variance

- $V(Y|X) =$
- $V(Y) =$

field: Conditional Variance

- $V(Y|X) = E\big[(Y - E(Y|X))^2|X\big] = E(Y^2|X) - E(Y|X)^2$
- $V(Y) = E(V(Y|X)) + V(E(Y|X))$

tags: distributionrelationships

NOTE:

field: $X \sim Gamma(a, b)$ $P(X \leq X) =$

field: $X \sim Gamma(a, b)$ $P(X \leq X) = P(Y \geq a)$ Where $Y \sim Pois(x/b)$

NOTE:

field:

$$X_1, \ldots, X_n \sim iidN(0, 1)$$
$$\sum X_i \overset{?}{\sim}$$

field:

$$X_1, \ldots, X_n \sim iidN(0, 1)$$
$$\sum X_i \sim N(0, n)$$

NOTE:

field:

$$X_1, \ldots, X_n \sim iidN(\mu_i, \sigma_i^2)$$
$$\sum X_i \overset{?}{\sim}$$

field:

$$X_1, \ldots, X_n \sim iidN(\mu_i, \sigma_i^2)$$
$$\sum X_i \sim N(\sum \mu_i, \sum \sigma_i^2)$$

NOTE:

field:

$$X \sim N(\mu, \sigma^2)$$
$$aX + b \overset{?}{\sim}$$

field:

$$aX + Y \sim N(a\mu + b, a^2\sigma^2)$$

NOTE:

field: $X \sim Binom(1, p) \overset{?}{\sim}$

field: $X \sim Bern(p)$

NOTE:

field: $X \sim NegBinom(1, p) \overset{?}{\sim}$

field: $X \sim Geom(p)$

NOTE:

field: $X \sim Gamma(1, \theta) \overset{?}{\sim}$

field: $X \sim Exp(\theta)$

NOTE:

field: $X \sim Exp(\theta) \overset{?}{\sim}$

field: $X \sim Gamma(1, \theta)$

NOTE:

field: $X \sim Gamma(v/2, 1/2) \overset{?}{\sim}$

field: $X \sim \chi^2(v)$

NOTE:

field:

$$X \sim \chi^2(v) \overset{?}{\sim}$$

field:

$$X \sim Gamma(v/2, 1/2)$$

NOTE:

field:

$$X \sim \chi^2(2) \overset{?}{\sim}$$

field:

$$X \sim exp(2)$$

NOTE:

field:

$$X \sim Weibull(1, \beta) \overset{?}{\sim}$$

field:

$$X \sim Exp(\beta)$$

NOTE:

field: $X_1, X_2 \sim \chi^2(v_i)$ independent $\frac{X_1/v_1}{X_2/v_2}$

field:

$$\frac{(X_1/v_1)}{(X_2/v_2)} \sim F(v_1, v_2)$$

NOTE:

field:

$$X \sim beta(1,1) \overset{?}{\sim}$$

field:

$$X \sim Unif(0,1)$$

NOTE:

field:

$$X \sim Unif(0,1) \overset{?}{\sim}$$

field:

$$X \sim beta(1,1)$$

NOTE:

field: Special case of t

$$X \sim t(1) \overset{?}{\sim}$$

field:

$$X \sim Caucy(0,1)$$

NOTE:

field: Scaled Gamma

$$X \sim Gamma(\alpha, \beta), Y = aX \overset{?}{\sim}$$

field:

$$Y \sim Gamma(\alpha, a\beta)$$

NOTE:

field: Scaled Exponential

$$X \sim Exp(\lambda), Y = aX \overset{?}{\sim}$$

field:

$$Y \sim Exp(a\lambda)$$

NOTE:

field: Sum of Exponential, equal rate $X_i \sim Exp(\lambda), Y = \sum X_i$

field:

$$Y \sim Gamma(n, \lambda)$$

NOTE:

field:

$$X \sim Exp(\lambda), Y = e^{-x}$$

field:

$$Y \sim Beta(\lambda, 1)$$

NOTE:

field: Min of Exponential

$$X_1, \ldots, X_n Exp(\lambda_i), Y = \min(X_i) \overset{?}{\sim}$$

field: $Y \sim exp(\sum \lambda_i)$

NOTE:

field: Min of Uniform

$$X_i \sim Unif(0,1), Y = \lim n \min(X_i) \overset{?}{\sim}$$

field:

$$Y \sim Exp(1)$$

NOTE:

field:

$$X \sim Beta(\alpha, \beta), Y = (1 - X)$$

field:

$$Y \sim Beta(\beta, \alpha)$$

NOTE:

field: $X \sim F_X(X), Y = F_X^{-1}(X)$

field: $Y \sim Unif(0,1)$

NOTE:

field: $X \sim N(\mu, \sigma^2), Y = e^X$

field: $Y \sim lognormal(\mu, \sigma^2)$

NOTE:

field: $X \sim exp(\beta), Y = X^{1/z}$

field: $Y \sim Weibull(z, \beta)$

NOTE:

field: Square of Normal $X \sim N(0, 1), Y = X^2$

field: $Y \sim \chi^2(1)$

NOTE:

field: Square of t $X \sim t(v), Y = X^2$

field: $Y \sim F(1, v)$

NOTE:

field: Sum of Poisson $X_i \sim Poisson(\mu_i) Y = \sum X_i$

field: $Y \sim Poisson(\sum \mu_i)$

NOTE:

field: Sum of Gamma $X_i \sim Gamma(\alpha_i, \beta), Y = \sum X_i$

field: $Y \sim Gamma(\sum \alpha_i, \beta)$

NOTE:

field: Sum of independent Chi-squared $X_i \sim \chi^2(v_i) Y = \sum X_i$

field: $Y \sim \chi^2(\sum v_i)$

NOTE:

field: X, Y independent $X, Y \sim N(0,1), X/Y$

field: $X/Y \sim Cauchy(0,1)$

NOTE:

field: $X_1, X_2 \sim gamma(\alpha_i, 1)$ independent, $\frac{X_1}{X_1+X_2}$

field:

$$\frac{X_1}{X_1+X_2} \sim beta(\alpha_1, \alpha_2)$$

NOTE:

field: $X_1, X_2 \sim gamma(\alpha_i, \beta_i)$ independent, $\frac{\beta_2 X_1}{\beta_2 X_1+\beta_1 X_2}$

field:

$$\frac{\beta_2 X_1}{\beta_2 X_1+\beta_1 X_2} \sim beta(\alpha_1, \alpha_2)$$

NOTE:

field: X, Y independent $exp(\mu)$ $X - Y$

field: $X - Y \sim$ double exponential$(0, \mu)$

NOTE:

field: Inverted Gamma $X \sim Gamma(\alpha, \beta)$ $Y = 1/X$

tags:

NOTE:

field: Bernoulii(p), $E(X) =, V(X) =$

field: Bernoulii(p), $E(X) = p, V(X) = p(1-p)$

NOTE:

field: Discrete Uniform $N, E(X) =, V(X) =$

field: Discrete Uniform $N, E(X) = \frac{N+1}{2}, V(X) = \frac{(N+1)(N-1)}{12}$

NOTE:

field: Cauchy(θ, σ), $E(X) =, V(X) =$

field: Cauchy(θ, σ), $E(X) = na, V(X) = na$

NOTE:

field: Double Exponential(μ, σ), $E(X) =, V(X) =$

field: Double Exponential(μ, σ), $E(X) = \mu, V(X) = 2\sigma^2$

NOTE:

field: F(v_1, v_2), $E(X) =, V(X) =$

field: F(v_1, v_2), $E(X) = \frac{v_1}{v_2-2}, V(X) = 2\left(\frac{v_2}{v_2-2}\right)^2 \frac{(v_1+v_2-2)}{v_1(v)2-4}$

NOTE:

field: Mean and Variance for Distributions not on bible (but in CB)

- Double Exponential$(\mu, \sigma), E(X) =, V(X) =$
- F$(v_1, v_2), E(X) =, V(X) =$
- Logistic$(\mu, \beta), E(X) =, V(X) =$
- Lognormal$(\mu, \sigma^2), E(X) =, V(X) =$
- Pareto$(\alpha, \beta), E(X) =, V(X) =$
- t$(v), E(X) =, V(X) =$
- Weibull$(\gamma, \beta), E(X) =, V(X) =$

field: Mean and Variance. for Distributions not on bible (but in CB)

- Logistic$(\mu, \beta), E(X) = \mu, V(X) = \frac{\phi^2\beta^2}{3}$
- Lognormal$(\mu, \sigma^2), E(X) = e^{\mu+(\sigma^2/2)})V(X) = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$
- Pareto$(\alpha, \beta), E(X) = \frac{\beta\alpha}{\beta-1}, V(X) = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)}$
- t$(v), E(X) = 0, V(X) = \frac{v}{v-2}$
- Weibull$(\gamma, \beta), E(X) = \beta^{1/\gamma}\Gamma(1+1/\gamma), V(X) = \beta^{2/\gamma}(\Gamma(1+2/\gamma) - \Gamma^2(1+1/\gamma))$

tags: Theory1

NOTE:

field:

	replace	no replacement
number of trials		
Draw till nth success		

field:

	replace	no replacement
number of trials	Binom	Hypergeometric
Draw till nth success	Nbinom	Negative hypergeometric

NOTE:

field: Plug uniform into inverse CDF

field: Get cdf

NOTE:

field: Sample Space

field: The set, S, of all possible outcomes of a particular experiment is called the *sample space* for the experiment.

NOTE:

field: Event

field: An *event* is any collection of possible outcomes of an experiment, that is, any subset of S (including S itself).

NOTE:

field: Union

field: $A \cup B = \{x : x \in A \text{ or } x \in B\}$

NOTE:

field: Intersection

field: $A \cap B = \{x : x \in A \text{ and } x \in B\}$

NOTE:

field: Complementation

field: $A^c = \{x : x \notin A\}$

NOTE:

field: Commutativity

$$A \cup B =$$
$$A \cap B =$$

field: Commutativity

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

NOTE:

field: Associativity

$$A \cup (B \cup C) =$$
$$A \cap (B \cap C) =$$

field: Associativity

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

NOTE:

field: Distributive Laws

$$A \cap (B \cup C) =$$
$$A \cup (B \cap C) =$$

field: Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

NOTE:

field: DeMorgan's Laws

$$(A \cup B)^c =$$

$$(A \cap B)^c =$$

field: DeMorgan's Laws

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

NOTE:

field: Disjoint

field: Disjoint: Two events A and B are disjoint (or mutually exclusive) if $A \cap B = \emptyset$

NOTE:

field:

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) =$$

field:

$$P(A_1)P(A_2|A_1)P(A_3|A_1A_2)\dots P(A_n|A_1\cdots A_{n-1})$$

NOTE:

field:

$$P(A,B,C) =$$

field:

$$P(A,B,C) = P(A)P(B|A)P(C|A,B)$$

NOTE:

field:

$$P(A\cup B\cup C) =$$

field:

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

NOTE:

field: Pairwise disjoint

field: Two Events A_1, A_2 are pairwise disjoint (or mutually exclusive) if $A_i \cap A_j = \emptyset$ for all $i \neq j$

NOTE:

field: Partition

field: If $A_1, A_2, \ldots$ are pairwise disjoint and $\cup_{i=1}^{\infty} A_i = S$, then the collection $A_1, A_2, \ldots$ forms a partition of S.

NOTE:

field: Sigma Algebra

field: A collection of subsets of S is called a sigma algebra (or Borel field), denoted by $\mathcal{B}$, if it satisfies the following three properties:

1. $\emptyset \in \mathcal{B}$ (the empty set is an element of $\mathcal{B}$)
2. If $A \in \mathcal{B}$, then $A^c \in \mathcal{B}$ ($\mathcal{B}$ is closed under complementation)
3. If $A_1, A_2, \ldots \in \mathcal{B}$, then $\cup_{i-1}^{\infty} A_i \in \mathcal{B}\mathcal{B}$ is closed under countable unions)

NOTE:

field: Probability Function / Kolmogorov Axioms

field: Given a sample space S and an associated sigma algebra $\mathcal{B}$, a probability function is a function P with domain $\mathcal{B}$ that satisfies:

1. $P(A) \geq 0$ for all $A \in \mathcal{B}$
2. $P(S) = 1$
3. If $A_1, A_2, \ldots \mathcal{B}$ are pairwise disjoint, then $P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$ (Axiom of Countable Additivity)

NOTE:

field: If $A \in \mathcal{B}$ and $B \in \mathcal{B}$ are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

Axiom of Finite Additivity

field: If $A \in \mathcal{B}$ and $B \in \mathcal{B}$ are disjoint, then

$$P(A \cup B) = P(A) + P(B)$$

NOTE:

field: Properties of probability functions

1. $P(\emptyset) =$
2. $P(A)$
3. $P(A^c) =$

field: Properties of probability functions

1. $P(\emptyset) = 0$
2. $P(A) \leq 1$
3. $P(A^c) = 1 - P(A)$

NOTE:

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(B \cap A^c) =$$

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(B \cap A^c) = P(B) - P(A \cap B)$$

NOTE:

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(A \cup B) =$$

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

NOTE:

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then if $A \subset B$ then

field: If P is a probability function and A and B are any sets in $\mathcal{B}$, then if $A \subset B$ then $P(A) \leq P(B)$

NOTE:

field: Bonferroni's Inequality

$$P(A \cap B)$$

field: Bonferroni's Inequality:

$$P(A \cap B) \geq P(A) + P(B) - 1$$

NOTE:

field: If P is a probability function, then for any partition $C_1, C_2, \ldots P(A) =$

field: If P is a probability function, then for any partition $C_1, C_2, \ldots P(A) = \sum_{i=1}^{\infty} P(A \cap C_i)$

NOTE:

field: Boole's Inequality

$$P(\cup_{i=1}^{\infty} A_i)$$

field: If P is a probability function,

$$P(\cup_{i=1}^{\infty} A_i) \leq \sum_{i=1}^{\infty} P(A_i) \text{ for any sets } A_1, A_2, \ldots$$

NOTE:

field: Fundamental Theorem of Counting

field: If a job consists of k separate tasks, the ith of which can be done in n_i ways, $i = 1, \ldots, k$, then the entire job can be done in $n_1 \times n_2 \times \cdots, \times n_k$ ways.

NOTE:

field: Ordered without replacement: number of arrangements of size r from n objects

field:

$$\frac{n!}{(n-r)!}$$

eg lottery with $n = 44$ choices for $r = 6$ values, cant use same number twice, order matters

NOTE:

field: Unordered without replacement: number of arrangements of size r from n objects

field:

$$\binom{n}{r} = \frac{n!}{r!(n-r!)}$$

eg lottery with $n = 44$ choices for $r = 6$ values, cant use same number twice, order does not matter (Use ordered without replacement and divide by redundant orderings)

NOTE:

field: Ordered with replacement: number of arrangements of size r from n objects

field: Ordered with replacement: number of arrangements of size r from n objects

$$n^r$$

eg lottery with $n = 44$ choices for $r = 6$ values, can use same number twice, order matters

NOTE:

field: Unordered with replacement: number of arrangements of size r from n objects

field: Unordered with replacement: number of arrangements of size r from n objects

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

eg lottery with $n = 44$ choices for $r = 6$ values, can use same number twice, order does not matters

NOTE:

field: Number of arrangements of size r from n objects

	Without Replacement	With replacement
Ordered		
Unordered		

field: Number of arrangements of size r from n objects

	Without Replacement	With replacement
Ordered	$\frac{n!}{(n-r)!}$	n^r
Unordered	$\binom{n}{r}$	$\binom{n+r-1}{r}$

NOTE:

field: Binomial Coefficient $\binom{n}{r}$

field: Binomial Coefficient

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

NOTE:

field:

$$P(A|B) =$$

field:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

NOTE:

field: Statistically independent $P(A \cap B) =$

field: Statistically independent $P(A \cap B) = P(A)P(B)$

NOTE:

field: If A and B are independent events, what else is independent?

field:

- A and B^c
- A^c and B
- A^c and B^c

NOTE:

field: Mutually independent

field: A collection of events $A_1, \ldots, A_n$ are mutually independent for any subcollection $A_{i1}, \ldots, A_{ik}$, we have

$$P((\cap_{j=1}^{k} A_{ij})) = \prod_{j=1}^{k} P(A_{ij})$$

NOTE:

field: Random variable

field: A random variable is a function from a sample space S into the real numbers

NOTE:

field: Definition of a pdf

field: A function $f_X(x)$ is a pdf (or pmf) of a random variable X if and only if

1. $f_x(x) \geq 0$ for all x
2. $\sum_x f_x(x) = 1$ or $\int_{-\infty}^{\infty} f_x(x)dx = 1$

NOTE:

field: (Theorem) Let X have cdf $F_X(x)$, let $Y = g(X)$

1. If g is an increasing function on X, $F_Y(y) =$ for $y \in Y$
2. If g is a decreasing function on X and X is a continuous random variable, $F_y(y) =$ for $y \in Y$

field: (Theorem) Let X have cdf $F_X(x)$, let $Y = g(X)$

1. If g is an increasing function on X, $F_Y(y) = F_X(g^{-1}(y))$ for $y \in Y$
2. If g is a decreasing function on X and X is a continuous random variable, $F_y(y) = 1 - F_X(g^{-1}(y))$ for $y \in Y$

NOTE:

field: Method of pdf

field: Conditions:

1. g is a monotone function
2. $f_X(x)$ is continuous on X
3. $g^{-1}(y)$ has a continuous derivative

Let X have pdf $f_x(x)$ and let $Y = g(Y)$

$$f_Y(y) = f_x(g^{-1}(y))\Big|\frac{d}{dy}g^{-1}(y)$$

NOTE:

field: (Theorem) Let X have cdf $F_X(x)$, let $Y = g(X)$

- If g is an increasing function, $F_Y(y) =$
- If g is a decreasing function, and X is a continuous random variable, $F_Y(y) =$

field: (Theorem) Let X have cdf $F_X(x)$, let $Y = g(X)$

- If g is an increasing function, $F_Y(y) = F_x(g^{-1}(y))$
- If g is a decreasing function, and X is a continuous random variable, $F_Y(y) = 1 - F_X(g^{-1}(y))$

NOTE:

field: eg: $X \sim Unif(0,1)$, $Y = -log(X)$ $F_Y(y) =$

field: $F_Y(y) = 1 - F_x(g^{-1}(y)) = 1 - F_X(e^{-y}) = 1 - e^{-y}$

NOTE:

field: X is a continuous random variable. For $y > 0$, $F_Y(y) =$

field:

$$\begin{aligned}
F_Y(y) &= P(Y \leq y) \\
&= P(X^2 \leq y) \\
&= P(-\sqrt{y} \leq X \leq \sqrt{y}) \\
&= P(X \leq \sqrt{y}) - P(X \leq -\sqrt{y}) \\
&= F_X(\sqrt{y}) - F_X(-\sqrt{y})
\end{aligned}$$

NOTE:

field: Pdf of $F_X(g(X))$, where $Y = g(X)$

field: Chain rule: $f_Y(y) = g'(y)f(g(y))$

NOTE:

field: Method of pdf if g is not monotone all entire domain

field: $f_Y = \sum f_x(g_i^{-1}(y))|\frac{d}{dy}g_i^{-1}(y)|$ $y \in Y$, 0 otherwise
eg: $Y = X^2$,

NOTE:

field: $P(Y \leq y)$ when $Y = F_X(x)$

field:

$$\begin{aligned} P(Y \leq y) &= P(X \leq F_x^{-1}(y)) \\ &= F_X(F_X^{-1}(y)) \\ &= y \end{aligned}$$

Y is uniformly distributed

NOTE:

field: $M_x(t) =$ (discrete)

field: $M_x(t) = E(e^{tX}) = \sum_x e^{tX}P(X)$ (discrete)

NOTE:

field: $M_x(t) =$ (continuous)

field: $M_x(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tX}f_x(x)dx$ (continuous)

NOTE:

field: $E(X^n) =$

field: $E(X^n) = M_x^n(0) = \frac{d^n}{dt^n} M_x(t)|_{t=0}$

NOTE:

field: $M(aX + b)(t) =$

field: $M(aX + b)(t) = e^{bt} M_x(at)$

NOTE:

field: If $E(X^n)$ exists then...

field: If $E(X^n)$ exists then $E(X^m)$ exists for $m \leq n$

NOTE:

field: If X_i are independent and $Y = a_1X_1 + \cdots + a_nX_n + b$, then $M_Y(t) =$

field: If X_i are independent and $Y = a_1X_1 + \cdots + a_nX_n$, then $M_Y(t) = e^{bt} \prod_{i=1}^{n} M_{X_i}(a_i t)$

NOTE:

field: Example of using MGF for finding expected value: MGF gamma: $(\frac{1}{1-\beta t})^\alpha$: $E(X) =$

field: $E(X) = \frac{\alpha\beta}{(1-\beta t)^{\alpha+1}}|_{t=0} = \alpha\beta$

NOTE:

field: Using MGF to relate distributions: MGF exp $= (1 - \beta t)^{-1}$

field: $Y = \sum X_i$ is gamma as MGF gamma is $(1 - \beta t)^{-\alpha}$

NOTE:

field: First step in transforming a RV

field: Determine support

NOTE:

field: nth Moment of X

field: $E(X^n)$

NOTE:

field: nth central moment of X

field: $E(X-\mu)^n$

NOTE:

field: $(a+b)^n =$

field: $(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} a^x b^{n-x}$

NOTE:

field: $\sum_{x=0}^{n} \binom{n}{x} a^x b^{n-x} =$

field: $(a+b)^n$

NOTE:

field: N balls r red $N-r$ green. Select n balls. Probability that y are red?

field: Hypergeometric distribution(N, r, n)

NOTE:

field: Hypergeometric distribution description (N, r, n)

field: N is total balls, r is number red balls, n is number balls selected.

NOTE:

field: Negative binomial description

field: Number of Bernoulli trials required to get a fixed number of successes. r being the rth success

NOTE:

field: Geometric description

field: Modeling waiting time. X is the trial at which the first success occurs.

NOTE:

field: Location-scale family for $f(x)$

field: $1/\sigma f((x-\mu)/\sigma)$

NOTE:

field: Given X give the mean and variance for the location-scale random $Y = 1/\sigma f((y-\mu)/\sigma)$ variable

field: $E(Y) = \sigma E(X) + \mu$, $V(Y) = \sigma^2 V(X)$

NOTE:

field: $X \sim Pois(\lambda)$ $P(X = x + 1) =$

field: $X \sim Pois(\lambda)$ $P(X = x + 1) = \frac{\lambda}{x+1} P(X = x)$

NOTE:

field: $f(y|x) =$

field: $f(y|x) = \frac{f(x,y)}{f_x(x)}$

NOTE:

field: $E(g(Y)|x) =$

field: $E(g(Y)|x) = \int_{-\infty}^{\infty} g(y)f(y|x)dy$

NOTE:

field: Example of calculating donditional pdfs $f(x, y) = e^{-y}, 0 < x < y < \infty$. $f(y|x) =$

field:

$$f_x(x) = \int_{-\infty}^{\infty} f(x, y)dy$$
$$= e^{-x}$$

$$\begin{aligned} f(y|x) &= \frac{f(x, y)}{f_x(x)} \\ &= \frac{e^{-y}}{e^{-x}} \text{ if } y > x \\ &= \frac{0}{e^{-x}} \text{ if } y \leq x \end{aligned}$$

NOTE:

field: Let (X, Y) be given as $f(x, y)$. Then X and Y are independent if

field: Let (X, Y) be given as $f(x, y)$. Then X and Y are independent if there exist functions $g(x), h(y)$ such that $f(x, y) = g(x)h(y)$ (factorization - don't need to compute marginals)

NOTE:

field: Let X, Y be independent. Then $E(g(X)h(Y)) =$

field: $E(g(X)h(Y)) = (E(g(X)))(E(h(Y)))$
example: $E(X^2Y) = E(X^2)E(Y)$

NOTE:

field: X, Y independent
$Z = X + Y$
$M_Z(t) =$

field: $M_Z(t) = M_x(t)M_Y(t)$

NOTE:

field: Method of pdf bivariate

field: $f_{u,v}(u, v) = f_{x,y}(h_1(u, v), h_2(u, v))|J|$
Where $|J| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$
and $u = g_1(x, y), v = g_2(x, y)$ and $x = h_1(x, y), y = h_2(x, y)$

NOTE:

field: X, Y independent, $g(X)$ a function only of X and $h(Y)$ a function only of Y. Then

field: $g(X)$ and $g(Y)$ are independent.

NOTE:

field: Correlation

field: $\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$

NOTE:

field: m independent trials, each trial resulting in one of n outcomes, with probabilities $p_1, \dots, p_n$. X_i is the count of the number of times the ith outcome occured in the m trials.

field: Multinomial distribution $f(x_1, \dots, x_n) = \frac{m!}{x_1! \cdots x_n!} p_1^{x_i} \cdots p_n^{x_n}$

NOTE:

field: $|E(XY)| \leq$ (Cauchy-Schwartz)

field: $|E(XY)| \leq E(|XY|) \leq (E(|X|^2))^{1/2}(E(|Y|^2))^{1/2}$

NOTE:

field: $E(g(X)) \geq$ where g is a convex function

field: $E(g(X)) \geq g(E(X))$ where g is a convex function (Jensen's inequlity)

NOTE:

field: Ranking of types of means

field: $\mu_{\text{harmonic}} \leq \mu_{\text{geometric}} \leq \mu_{\text{arithmetic}}$ By Jensens inequality (using logs)

NOTE:

field: Linear transformations of multivariate normal $X \sim N(\vec{\mu}, \Sigma)$
$A\vec{X} + \vec{b}$

field: $A\vec{X} + \vec{b} \sim N(A\vec{\mu} + \vec{v}, A\Sigma A^t)$

NOTE:

field: $X \sim N(\vec{\mu}, \Sigma)$

$\vec{X}_a | \vec{X}_b \sim$

field: $\vec{X}_a | \vec{X}_b \sim N\big(\vec{\mu_a} + \Sigma_{ab}\Sigma_{bb}^{-1}(\vec{x}_b - \vec{\mu}_b), \Sigma_{ba} - \Sigma_{ab}\Sigma_{bb}^{-1}\Sigma_{ba}$

ex: $(X_1, X_2, X_3), \vec{\mu} = (1,2,3)^t, \Sigma = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ $X1, X_3 | X_2 = 1$
$a = \{1,3\}, b = \{2\}$
$\mu_a = (1,3)^t, \mu_b = 1$
$\Sigma_{aa} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \Sigma_{ab} = (1,1)^t$

NOTE:

field: (X, Y) multinomial
$aX + bY \sim$

field: $aX + bY \sim N(a\mu_x + b\mu_y, a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\rho\sigma_x\sigma_y)$

NOTE:

field: (X, Y) multinomial
$Y | X \sim$

field: $Y | X \sim N(\mu_y + \rho\frac{\sigma_y}{\sigma_x}(x - \mu_x), \sigma_Y^2(1 - \rho^2))$

NOTE:

field: CDF for Max order statistic

field: $(F(x))^n$

NOTE:

field: PDF for Max order statistic

field: $n(F(x))^{n-1}f(x)$

NOTE:

field: CDF for Min order statistic

field: $1-(1-F(x))^n$

NOTE:

field: PDF for Min order statistic

field: $n(1-F(x))^{n-1}f(x)$

NOTE:

field: CDF for kth order statistic

field: $F_{(k)}(x)=\sum_{j=k}^{n}\binom{n}{j}(F(x))^j(1-F(x))^{n-j}$

NOTE:

field: PDF for kth order statistic

field: $f_{(k)}(x)=k\binom{n}{k}f(x)F(x)^{k-1}(1-F(x))^{n-k}$

tags: UndergradTextbook

NOTE:

field: Law of total probability $k = 2$ (using conditional probability)

field: $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$

NOTE:

field: Bayes formula in terms of law of total probability,

field: $P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|B^c)P(B^c)}$

NOTE:

field: $P(A \text{ and } B)$

field: $P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$

NOTE:

field: Events A and B are independent if

field: $P(A|B) = P(A)$ equivalently $P(A \text{ and } B) = P(A)P(B)$

NOTE:

field: Poisson setting

field: The Poisson setting arises in the context of discrete counts of events that occur over space or time with the small probability and where successive events are independent

Eg: 2 on average calls a minute, X is number of calls a minute, $X \sim Pois$

NOTE:

field: Poisson approximation of binomial distribution

field: Suppose $X \sim Binom(n, p)$, $Y \sim Pois(\lambda)$. If $n \to \infty$, and $p \to 0$, in such a way that $np \to \lambda > 0$, then for all k, $P(X = k) \to P(Y = k)$. The Poisson distribution with parameter $\lambda = np$ serves as a good approiximation for the binomial distribution when n is large and p is small.

NOTE:

field: $E(f(X, Y))$ when X, Y are discrete

field: $E(f(X, Y)) = \sum_x \sum_y f(x, y) P(X = x, Y = y)$

NOTE:

field: If X, Y are independent, then $f(X), g(Y)$

field: are also independent

NOTE:

field: If X, Y independent, $E(XY) =, E(f(X)g(Y)) =$

field: If X, Y independent, $E(XY) = E(X)E(Y), E(f(X)g(Y)) = E(f(X))E(g(Y))$

NOTE:

field: Sum of independent discrete random variables X, Y: $P(X + Y = k)$

field: $P(X + Y = k) = \sum_i P(X = i) P(Y = k - i)$

NOTE:

field: $V(X) = 0$

field: If and only if X is a constant

NOTE:

field: $E(I_A) =, V(I_A)$ Where I_A is an indicator function

field: $E(I_A) = P(A), V(I_A) = P(A)P(A^c)$

NOTE:

field: For discrete jointly distributed random variables,

$$P(X = y|X = x) =$$

field: For discrete jointly distributed random variables,

$$P(X = y|X = x) = \frac{P(X = x, Y = y)}{P(X = x)}$$

NOTE:

field: For discrete random variables $E(Y|X = x) =$

field: For discrete random variables $E(Y|X = x) = \sum_y yP(Y = y|X = x)$

NOTE:

field: Problem solving strategy for expected value of counting

field: Use indicator functions for each trial , where $X = \sum I$ and use linearity of expectation

NOTE:

field: $P(X > s + t|X > t)$ for geometric, exponential

field: $P(X > s + t|X > t) = P(X > s)$

NOTE:

field: Distribution for: A bag of N balls which conatins r red balls and $N - r$ blue balls, X is number of red balls in a sample of size n taken without replacement.

field: Hypergeometric.

NOTE:

field: Distribution for modeling arrival time

field: Exponential

NOTE:

field: $E(g(X, Y)) =$ (continuous)

field: $E(g(X, Y)) = \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} g(x, y) f(x, y) dx dy$

NOTE:

field: $Cov(X, Y) =$ (integration)

field: $Cov(X, Y) = \int_{y=-\infty}^{\infty} \int_{x=-\infty}^{\infty} (x - E(X))(y - E(Y)) dx dy$

NOTE:

field: Problem solving strategies for functions of random variables

field:

- Methods of cdf: $Y = g(X)$, find cdf $P(Y \leq y) = P(g(X) \leq y) = P(X \leq g^{-1}(y))$
- For finding $P(X < Y)$, set up integrals that cover
- For finding probabilities of independent uniform random variables, use geometric (area) properties

NOTE:

field: Quantile

field: If X is a continuous random variable, then the pth quantile is is the number q that satisfies $P(X \leq q) = p/100$

NOTE:

field: Poisson process

field: Times between arrivals are modeled as iid exponential random variables with parameter $\lambda = 1/\beta$. Let N_t be the number of arrivals up to time t. Then $N_t \sim Pois(\lambda t)$

NOTE:

field: Conditional density function $f_{Y|X}(y|x) =$

field: $f_{Y|X}(y|x) = \frac{f(x,y)}{f_x(x)}$

NOTE:

field: Continuous bayes formula

field: $f_{X|Y}(x|y) = \frac{f_{Y|X}(y|x)f_x(x)}{\int_{t=-\infty}^{\infty} f_{Y|X}(y|t)f_x(t)dt}$

NOTE:

field: Conditional expectation for continuous random variables $E(Y|X = x)$

field: $E(Y|X = x) = \int_y y f_{Y|X}(y|x) dy$

NOTE:

field: Law of total expectation

field: $E(Y) = E(E(Y|X))$

NOTE:

field: Properties of conditional expectation

- $E(aY + bZ|X) =$
- $E(g(Y)|X = x) =$
- If X, Y independent, $E(Y|X) =$
- If $Y = g(X)$, then $E(Y|X) =$

field: Properties of conditional expectation

- $E(aY + bZ|X) = aE(Y|X) + bE(Z|X)$
- $E(g(Y)|X = x) = \int_y g(y) f_{Y|X}(y|x) dy$
- If X, Y independent, $E(Y|X) = E(Y)$
- If $Y = g(X)$, then $E(Y|X) = Y$

NOTE:

field: Law of total probability, continuous

field: $P(A) = \int_{-\infty}^{\infty} P(A|X = x) f_x(x) dx$

NOTE:

field: Conditional variance $V(Y|X = x)$

field:

$$V(Y|X = x) = \sum_y (y - E(Y|X = x))^2 P(Y = y|X = x)$$

discrete

$$V(Y|X = x) = \int_y (y - E(Y|X = x))^2 f_{Y|X}(y|x) dy$$

continuous

NOTE:

field: Properties of conditional variance

- $V(Y|X = x) =$
- $V(aY + b|X = x) =$
- If Y, Z independent, $V(Y + Z|X = x) =$

field: Properties of conditional variance

- $V(Y|X = x) = E(Y^2|X = x) - (E(Y|X = x))^2$
- $V(aY + b|X = x) = a^2 V(Y|X = x)$
- If Y, Z independent, $V(Y + Z|X = x) = V(Y|X = x) + V(Z|X = x)$

NOTE:

field: $P(X \geq \epsilon)$

field: $P(X \geq \epsilon) \leq E(X)/\epsilon$ (Markov's Inequality)

NOTE:

field: $P(|X-\mu| \geq \epsilon)$

field: $P(|X-\mu| \geq \epsilon) \leq \sigma^2/\epsilon^2$ (Chebyshev's inequality, if mean and variance finite)

NOTE:

field: $P(\lim_{n\to\infty} S_n/n = \mu) =$

field: $P(\lim_{n\to\infty} S_n/n = \mu) = 1$ (Strong law of large numbers)

tags: Calculus

NOTE:

field: $\int_0^\infty e^{-x^2/2} =$

field: $\int_0^\infty e^{-x^2/2} = \sqrt{\pi/2}$

NOTE:

field: $\int_0^\infty x^{a-1}e^{-x/b} =$

field: $\int_0^\infty x^{a-1}e^{-x/b} = \Gamma(a)b^a$

NOTE:

field: $\int_0^1 x^{a-1}(1-x)^{b-1} =$

field: $\int_0^1 x^{a-1}(1-x)^{b-1} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$

NOTE:

field: $\log(x) = y, x =$

field: $\log(x) = y, x = e^y$

NOTE:

field: $\lim_{x\to\infty}(1 + \frac{a}{x})^x =$

field: $\lim_{x\to\infty}(1 + \frac{a}{x})^x = e^a$

NOTE:

field: $\lim_{x\to\infty}(1 + \frac{a}{x})^x = e^a$

field: $\lim_{x\to\infty}(1 + \frac{a}{x})^x =$

NOTE:

field: $\frac{d}{dx}f(g(x)) =$

field: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain rule)

NOTE:

field: $\frac{d}{dx}\int_a^x f(t)dt =$

field: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$ (fundamental theorem of calculus)

NOTE:

field: $\int_a^b udv =$
ex: $\int xe^{-x}$

field: $\int_a^b udv = uv|_a^b - \int_a^b vdu$
ex: $u = x, dv = e^{-x}, du = dx, v = -e^{-x}$

$$\begin{aligned}\int xe^{-x} &= -xe^{-x} + \int e^{-x} \\ &= -xe^{-x} - e^{-x} + c\end{aligned}$$

NOTE:

field: $\sum_{k=0}^{\infty} \frac{x^k}{k!} =$

field: $\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$

NOTE:

field: $e^x =$

field: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

NOTE:

field: $\sum_{k=0}^{\infty} x^k =$

field: $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ for $|x| < 1$

NOTE:

field: $\sum_{k=0}^{n} x^k =$

field: $\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$ for $x \neq 1$

NOTE:

field: $\lim_{x\to-\infty} e^{-x} =$

field: $\lim_{x\to-\infty} e^{-x} = \infty$

NOTE:

field: $\lim_{x\to\infty} e^{-x} =$

field: $\lim_{x\to-\infty} e^{-x} = 0$

NOTE:

field:

$$(fg)' =$$

field:

$$(fg)' = f'g + g'f$$

(product rule)

NOTE:

field: $\frac{d}{dx}x^n =$

field: $\frac{d}{dx}x^n = nx^{n-1}$

NOTE:

field: $\frac{d}{dx}a^x =$

field: $\frac{d}{dx}a^x = a^x ln(a)$

NOTE:

field: $\frac{d}{dx}ln(x) =$

field: $\frac{d}{dx}ln(x) = \frac{1}{x}$

NOTE:

field: $\frac{d}{dx}(f(x))^n =$

field: $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1}f'(x)$

NOTE:

field: $\frac{d}{dx}ln(f(x)) =$

field: $\frac{d}{dx}ln(f(x)) = \frac{f'(x)}{f(x)}$

NOTE:

field: $\frac{d}{dx}e^{f(x)} =$

field: $\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$

NOTE:

field: $\int x^n =$

field: $\int x^n = \frac{1}{n+1}x^{n+1}$

NOTE:

field: $\int \frac{1}{x} =$

field: $\int \frac{1}{x} = ln(|x|)$

NOTE:

field: $\int \frac{1}{ax+b} =$

field: $\int \frac{1}{ax+b} = \frac{1}{a}ln(|ax+b|)$

NOTE:

field: $\int e^{cx} =$

field: $\int e^{cx} = \frac{1}{c}e^{cx}$

NOTE:

field: $\int xe^{-cx^2} =$

field: $\int xe^{-cx^2} = -\frac{1}{2c}e^{-cx^2}$

NOTE:

field: U substitution:
example; $\int_1^2 5x^2\cos(x^3)$

field: $\int_a^b f(g(x))g'(x) = \int_{g(a)}^{g}(b)f(u)du$
Where $u = g(x), du = g'dx$
Ex: $u = x^3, du = 3x^2, x^2du = 1/3du$ $\int_1^2 5x^2\cos(x^3) = \int_1^8 5/3\cos(u)du$

NOTE:

field: $\Gamma(a) =$

field: $\int_0^\infty t^{a-1}e^{-t}dt$

NOTE:

field: $\int_0^\infty t^{a-1}e^{-t}dt$

field: $= \Gamma(a)$

NOTE:

field: $\Gamma(a+1) =$

field: $\Gamma(a+1) = a\Gamma(a)$

NOTE:

field: $\Gamma(n) =$

field: $\Gamma(n) = (n-1)!$ (for n an integer)

NOTE:

field: $\Gamma(1/2) =$

field: $\Gamma(1/2) = \sqrt{\pi}$

NOTE:

field: $\Gamma(1) =$

field: $\Gamma(1) = 1$