

**tags:** Methods1

**NOTE:**

**field:** Epidemiology Definition of Causation

**field:** Factor/variable  $X$  **causes** result  $Y$  if some cases of  $Y$  would not have occurred if  $X$  had been absent.

**NOTE:**

**field:** Sample variance

**field:**  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$

**NOTE:**

**field:** Population(s) of interest

**field:** The group to which you would like your answer to apply

**NOTE:**

**field:** Variable of Interest

**field:** A measurement that can be made on each individual/member of the population

**NOTE:**

**field:** Facts about Normal Distributions

**field:**

- If  $Z$  has a Normal(0,1) distribution then  $X = \sigma Z + \mu$  has a Normal( $\mu, \sigma^2$ ) distribution
- If  $X$  has a Normal( $\mu, \sigma^2$ ) distribution, then  $Z = \frac{X-\mu}{\sigma}$  has a Normal(0,1) distribution.
- If  $X$  has a Normal( $\mu_x, \sigma_x^2$ ) distribution, and  $Y$  has a Normal( $\mu_y, \sigma_y^2$ ) distribution, and  $X$  and  $Y$  are independent of each other, then  $X+Y \sim \text{Normal}(\mu_x + \mu_y, \sigma_x^2 + \sigma_y^2)$

**NOTE:**

**field:** Sample mean

**field:**  $\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i$

**NOTE:**

**field:** Sampling distribution for population  $Y \sim \text{Normal}(\mu, \sigma)$

**field:**  $N(\mu, \sigma^2/n)$

**NOTE:**

**field:** Variance (Expected value)

**field:**  $V(Y) = E[(X - E(X))^2] = E(X^2) - E[(X)]^2$

**NOTE:**

**field:** Covariance

**field:**  $Cov(X, Y) = E[(X - E(X))(Y - E(Y))]$

**NOTE:**

**field:** If  $X$  and  $Y$  are independent (covariance)

**field:** The covariance is 0

**NOTE:**

**field:** If  $Cov(X, Y) = 0$ , (independence)

**field:** Cannot say that  $X$  and  $Y$  are independent

**NOTE:**

**field:**  $Cov(X, X) =$

**field:**  $Var(X)$

**NOTE:**

**field:**  $X \sim N(\mu, \sigma^2)$

- $E(\bar{X}) =$

- $V(\bar{X}) =$

**field:**

- $E(\bar{X}) = \mu$

- $V(\bar{X}) = \sigma^2/n$

**NOTE:**

**field:** Central Limit Theorem (in words)

**field:** If the population distribution of a variable  $X$  has population mean  $\mu$  and finite population variance  $\sigma^2$ , then the sampling distribution of the sample mean becomes closer and closer to a Normal distribution as the sample size  $n$  increases:  $\bar{X} \sim N(\mu, \sigma^2/n)$

**NOTE:**

**field:** Central Limit Theorem (theoretical)

**field:** Let  $X_1, X_2, \dots, X_n$  be an iid sample from some population distribution  $F$  with mean  $\mu$  and variance  $\sigma^2 < \infty$ . Then as the sample size  $n \rightarrow \infty$ , we have

$$\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \rightarrow N(0, 1)$$

**NOTE:**

**field:**  $X \sim (\mu, \sigma^2)$

- $E(\bar{X}) =$

- $V(\bar{X}) =$

**field:**

- $E(\bar{X}) = \mu$

- $V(\bar{X}) = \sigma^2/n$

**NOTE:**

**field:** Reject  $H_0$  when  $H_0$  True

**field:** Type I error (false positive)

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**NOTE:**

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**field:** Type II error

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**field:** Fail to Reject  $H_0$  when  $H_0$  false

**NOTE:**

**field:** Significance level

**field:**  $\alpha$  the probability of a Type I error

**NOTE:**

**field:** Power (at  $\theta_1$ )

**field:** Probability of rejecting the null hypothesis when  $\theta_1$  is the truth

**NOTE:**

**field:** Test for data setting:  $X_1, X_2, \dots, X_n$  iid with sample mean  $\bar{X}$ , and known population variance  $\sigma^2$ , Null hypothesis  $\mu = \mu_0$

- Test name
- Test Statistic
- Test Reference Distribution
- Critical Value
  - Lower
  - Upper

- Two sided
- Confidence interval
- pvalue
  - upper:
  - lower:
  - two-sided
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:** z-test

- Test statistic:  $Z(\mu_0) = \frac{\bar{X} - \mu_0}{\sqrt{\sigma^2/n}}$
- Reference Distribution: Under  $H_0$ ,  $Z(\mu_0) \sim N(0, 1)$ 
  - Lower: Reject when  $Z(\mu_0) < z_\alpha = \text{qnorm}(\alpha)$
  - Upper: Reject when  $Z(\mu_0) > z_{1-\alpha} = \text{qnorm}(1-\alpha)$
  - Two sided: Reject when  $|Z(\mu_0)| > z_{1-\alpha/2} = \text{qnorm}(1 - \alpha/2)$
- Confidence interval:  $\bar{X} \pm z_{1-\alpha/2} \sqrt{\frac{\sigma^2}{n}}$
- pvalue:
  - upper:  $1 - \Phi(z) = 1 - \text{pnorm}(z)$
  - lower:  $\Phi(z) = \text{pnorm}(z)$
  - two-sided:  $2(1 - \Phi(|z|)) = 2*(1 - \text{pnorm}(\text{abs}(z)))$
- Consistent: Yes /Finite Sample Exact: Yes if  $X_i \sim N$ / Asymptotically Exact: Yes

**NOTE:**

**field:** Exactness (finite/asymptotic)

**field:** Under any setting for which the null hypothesis is true, is the actual rejection probability equal to the desired level  $\alpha$ ?

- Finite Sample Exact: for sample size  $n$  is  $P(\text{Reject } H_0) = \alpha$  when  $H_0$  is true?
- Asymptotic Exactness: As  $n \rightarrow \infty$  does  $P(\text{Reject } H_0) \rightarrow \alpha$  when  $H_0$  is true?

**NOTE:**

**field:** When is a test exact?

**field:**

- A test is FSE if the reference distribution is the true distribution of the test statistic  $T$  when  $H_0$  is true
- A test is AE if the reference distribution is the asymptotic distribution of the test statistic when  $H_0$  is true.
- (Distribution of p-values should be  $\text{Unif}(0,1)$ )

**NOTE:**

**field:** Consistency

**field:** When  $H_0$  is false (the alternative hypothesis is true), does the rejection probability (probability reject the null) tend to 1 as  $n \rightarrow \infty$ ?

**NOTE:**

**field:** Interpretation of Confidence intervals

**field:**  $(1 - \alpha)100\%$  of the time, intervals constructed in this manner will include  $\mu$

**NOTE:**

**field:** Test for data setting:  $X_1, X_2, \dots, X_n$  iid with sample mean  $\bar{X}$ , and unknown population variance, Null hypothesis  $\mu = \mu_0$

- Test name
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
  - upper:
  - lower:
  - two-sided
- Confidence interval
- pvalue
  - upper:
  - lower:
  - two-sided
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:**

- Test name: t-test
- Test Statistic:  $t(\mu_0) = \frac{\bar{X} - \mu_0}{\sqrt{s^2/n}}$
- Test Reference Distribution:  $t_{n-1}$
- Critical Value/ Rejection region
  - upper: Reject if  $t(\mu_0) > t_{(n-1), 1-\alpha} = \text{qt}(1 - \alpha, n-1)$
  - lower: Reject if  $t(\mu_0) < t_{n-1, \alpha}$
  - two sided: Reject if  $|t(\mu_0)| > t_{n-1, 1-\alpha/2}$
- Confidence interval:  $\bar{X} \pm t_{n-1, 1-\alpha/2} \sqrt{\frac{s^2}{n}}$



- pvalue, with  $t(\mu_0) = t$ , and pt representing the cdf of a t distribution
  - upper:  $1 - \text{pt}(t, n - 1)$
  - lower:  $\text{pt}(t, n-1)$
  - two-sided:  $2*(1 - \text{pt}(\text{abs}(t)), n-1)$
- Consistent Yes/Finite Sample Exact Yes if normal/ Asymptotically Exact Yes

**NOTE:**

**field:** Test for data setting  $Y_1, \dots, Y_n$  iid Bernoulli(p) (option 1), parameter of interest  $p$

- Test name
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
  - upper:
  - lower:
  - two-sided
- Confidence interval
- pvalue
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:** Test for data setting  $Y_1, \dots, Y_n$  iid Bernoulli(p), parameter of interest  $p$

- Test name: Exact Binomial Test (uses the distribution of the sum of Bern(p) RVs)
- Test Statistic:  $X = \sum_{i=1}^n Y_i = n\bar{Y}$

- Test Reference Distribution: Under  $H_0$  Binomial( $n, p_0$ )
- Critical Value/ Rejection region: Sometimes use randomized test
  - upper: Reject  $H_0$  for  $X \geq c$  for  $c$  such that  $P(X \geq c) \leq \alpha$
  - lower: Reject  $H_0$  for  $X \leq c$  for  $c$  such that  $P(X \leq c) \leq \alpha$
  - two-sided: Reject  $H_0$  for  $p_0(X) \leq c$  for  $c$  such that  $P_{H_0}(p_0(X) \leq c) \leq \alpha$ , where  $p_0(X)$  is  $P(X = x)$  under  $H_0$
- Confidence interval: Values that are not rejected
- pvalue: Sum of the probabilities that are less than or equal to the observed value (under the null hypothesis)
- Consistent/Finite Sample Exact/ Asymptotically Exact

**NOTE:**

**field:** Test for data setting  $Y_1, \dots, Y_n$ , parameter of interest:  $p$  iid Bernoulli( $p$ ) (option 2)

- Test name
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
  - upper:
  - lower:
  - two-sided
- Confidence interval
- pvalue
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:** Test for data setting  $Y_1, \dots, Y_n$ , parameter of interest:  $p$  iid Bernoulli( $p$ ) (option 2)

- Test name: Binomial  $z$ -test (Use when  $np_0 > 5$  and  $n(1 - p_0) > 5$ )
- Test Statistic:  $X = \sum_{i=1}^n Y_i = n\bar{Y}$ ,  $\hat{p} = X/n$ ,  $z(p_0) = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$  (score)
- Test Reference Distribution: Under  $H_0$ , Approximately  $X \sim N(np_0, np_0(1-p_0))$  and  $z(p_0) \sim N(0, 1)$
- Critical Value/ Rejection region
  - upper:  $z(p_0) > z_{1-\alpha}$
  - lower:  $z(p_0) < z_\alpha$
  - two-sided:  $|z(p_0)| > z_{1-\alpha/2}$
- Confidence interval: Uses wald interval (derived from t-test) (with  $z_w(p_0) = \frac{\hat{p} - p_0}{\sqrt{\hat{p}(1-\hat{p})/n}}$ )  $\hat{p} \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
- pvalue
  - upper:  $1 - \Phi(z(p_0)) = 1 - \text{pnorm}(z(p_0))$
  - lower:  $\Phi(z(p_0)) = \text{pnorm}(z)$
  - two-sided:  $2(1 - \Phi(|z(p_0)|)) = 2*(1 - \text{pnorm}(\text{abs}(z)))$
- Consistent: Yes/Finite Sample Exact: No/ Asymptotically Exact: Yes

**NOTE:**

**field:** Continuity correction for Binomial  $z$ -test

**field:** With  $X \sim \text{Binom}(n, p)$ , instead of  $P(X \leq x)$ , use  $P(W \leq x + 1/2)$  where  $W \sim N(np, np(1 - p))$

**NOTE:**

**field:** Data Setting:  $X_1, \dots, X_n$ , iid parameter of interest:  $M$  - median  
 $H_0 : M = M_0$  (option 1)

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
  - upper:
  - lower:
  - two-sided
- Confidence interval
- pvalue
  - upper:
  - lower:
  - two-sided
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:**

- Test name: Sign Test
- Test Statistic:  $Y_i = I(X_i < M_0)$  ,  $\hat{p}_{M_0} = \frac{\sum Y_i}{n}$  (proportion of observations less than or equal to hypothesized median)
- Test Reference Distribution: Normal distribution: with  $p_0 = .5$
- Critical Value/ Rejection region:  $z = \frac{\hat{p}_{M_0} - p_0}{\sqrt{p_0(1-p_0)/n}}$ 
  - upper:  $z > z_{1-\alpha}$
  - lower:  $z < z_\alpha$
  - two-sided:  $|z| > z_{1-\alpha/2}$

- Confidence interval: cant use the binomial proportion CI Set of values of  $M_0$  that wouldn't be rejected at level  $\alpha$

$$\left(\frac{n - z_{1-\alpha/2\sqrt{n}}}{2}\right)^{th} \text{ Smallest Observation, } \left(\frac{n - z_{1-\alpha/2\sqrt{n}}}{2}\right)^{th} \text{ Smallest Observation}$$

- pvalue (binomial test on proportion)
  - upper:  $1 - \Phi(z(p_0)) = 1 - \text{pnorm}(z(p_0))$
  - lower:  $\Phi(z(p_0)) = \text{pnorm}(z)$
  - two-sided:  $2(1 - \Phi(|z(p_0)|)) = 2*(1 - \text{pnorm}(\text{abs}(z)))$
- If there are ties: remove all observations equal to  $M_0$ , then test prop of observations  $< M_0$  given not equal to  $M_0$  is .5
- Consistent: yes/Finite Sample Exact: No / Asymptotically Exact: yes

#### NOTE:

**field:** Data Setting:  $X_1, \dots, X_n$ , iid parameter of interest:  $M$  - median  
 $H_0 : M = M_0$  (option 2)

- Test name:
- Procedure:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
  - upper:
  - lower:
  - two-sided
- Confidence interval
- pvalue
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:** Data Setting:  $X_1, \dots, X_n$ , iid parameter of interest:  $M$  - median  
 $H_0 : M = M_0$  (option 1)

- Test name: Wilcoxon signed-rank test (require symmetry assumption)  
 - equivalently a test of the mean - Tests the pseudo-median
- Procedure: testing  $c_0$  is the center (median)
  - Calculate distance of each observation from  $c_0$
  - Rank observations by the distance (abs value) from  $c_0$
- Test Statistic:  $S$  sum of the ranks that correspond to observations larger than  $c_0$ ,  $Z = \frac{S - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \sim N(0, 1)$
- Test Reference Distribution:
  - Exact p-value - assume each rank has the same chance of being assigned to observations above or below  $c_0$  - all possible ways to assign the ranks
  - Normal approximation to the null distribution  $S \sim N\left(\frac{n(n+1)}{4}, \frac{n(n+1)(2n+1)}{24}\right)$
- Critical Value/ Rejection region
  - upper:
  - lower:
  - two-sided
- Confidence interval
- pvalue - Same as for Normal
- Consistent Yes under symmetry assumption /Finite Sample Exact No/  
 Asymptotically Exact Yes (under symmetry assumption)

**NOTE:**

**field:** Pseudomedian

**field:** Median of the distribution of sample means from samples of size 2

**NOTE:**

**field:** Data Setting:  $X_1, \dots, X_n$ , iid  $N(\mu, \sigma^2)$  parameter of interest:  $\sigma^2 = \text{Var}(X)$ , sample variance:  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ ,  $H_0 : \sigma^2 = \sigma_0^2$

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval
- pvalue
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:** Data Setting:  $X_1, \dots, X_n$ , iid  $N(\mu, \sigma^2)$  parameter of interest:  $\sigma^2 = \text{Var}(X)$ , sample variance:  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ ,  $H_0 : \sigma^2 = \sigma_0^2$

- Test name:  $\chi^2$  for Population Variance
- Test Statistic  $X(\sigma_0) = \frac{(n-1)s^2}{\sigma_0^2}$
- Test Reference Distribution: Under  $H_0 : X(\sigma_0) = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi_{n-1}^2$
- Critical Value/ Rejection region
  - $\sigma^2 > \sigma_0^2$  Reject  $H_0$  for  $X(\sigma_0^2) > \chi_{n-1}^2(1 - \alpha)$
  - $\sigma^2 < \sigma_0^2$  Reject  $H_0$  for  $X(\sigma_0^2) < \chi_{n-1}^2(\alpha)$
  - $\sigma^2 \neq \sigma_0^2$  Reject  $H_0$  for  $X(\sigma_0^2) > \chi_{n-1}^2(1 - \alpha/2)$  or  $X(\sigma_0) < \chi_{n-1}^2(\alpha/2)$
- Confidence interval

$$\left( \frac{(n-1)s^2}{\chi_{n-1}^2(1 - \alpha/2)}, \frac{(n-1)s^2}{\chi_{n-1}^2(\alpha/2)} \right)$$

- pvalue

- $\sigma^2 > \sigma_0^2$ :  $p = 1 - pchisq(X(\sigma_0)^2, n - 1)$
- $\sigma^2 < \sigma_0^2$ :  $p = pchisq(X(\sigma_0^2), n - 1)$
- $\sigma^2 \neq \sigma_0^2$ :  $p = 2 \min(1 - pchisq(X(\sigma_0^2), n - 1), pchisq(X(\sigma_0^2)), n - 1)$

- Consistent/Finite Sample Exact/ Asymptotically Exact

**NOTE:**

**field:** Data Setting:  $X_1, \dots, X_n$ , iid parameter of interest:  $\sigma^2 = Var(X)$ ,  
sample variance:  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ ,  $H_0 : \sigma^2 = \sigma_0^2$

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval
- pvalue
- Consistent/Finite Sample Exact/ Asymptotically Exact

**field:** Data Setting:  $X_1, \dots, X_n$ , iid parameter of interest:  $\sigma^2 = Var(X)$ ,  
sample variance:  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$ ,  $H_0 : \sigma^2 = \sigma_0^2$

- Test name: Asymptotic  $t$ -test for population variance
- Test Statistic:  $Y = (X_i - \bar{X})^2 t(\sigma_0^2) = \frac{Y - \frac{n-1}{n} \sigma_0^2}{\sqrt{s_y^2/n}} \rightarrow N(0, 1)$
- Test Reference Distribution  $\frac{\frac{n-1}{n} s^2 - \frac{n-1}{n} \sigma^2}{\sqrt{Var(\frac{n-1}{n} s^2)}} = \frac{\bar{Y} - \frac{n-1}{n} \sigma^2}{\sqrt{Var(\bar{Y})}} \rightarrow N(0, 1)$ , so we can use t-test
- Critical Value/ Rejection region
  - upper: Reject if  $t(\sigma_0^2) > t_{(n-1), 1-\alpha} = qt(1 - \alpha, n-1)$
  - lower: Reject if  $t(\sigma_0^2) < t_{n-1, \alpha}$



- two sided: Reject if  $|t(\sigma_0^2)| > t_{n-1, 1-\alpha/2}$
- Confidence interval:  $\bar{X} \pm t_{n-1, 1-\alpha/2} \sqrt{\frac{s^2}{n}}$
- pvalue, with  $t(\mu_0) = t$ , and pt representing the cdf of a t distribution
  - upper:  $1 - \text{pt}(t, n - 1)$
  - lower:  $\text{pt}(t, n-1)$
  - two-sided:  $2*(1 - \text{pt}(\text{abs}(t)), n-1)$

**NOTE:**

**field:** Test for data setting  $X_1, \dots, X_n$  iid from population distribution  $F$ .  
 Test  $H_0 : F = F_0$

- Test name:
- Process
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region

**field:** Test for data setting  $X_1, \dots, X_n$  iid from population distribution  $F$ .  
 Test  $H_0 : F = F_0$

- Test name: Kolmogorov-Smirnov Test
- Process
- Test Statistic:  $D(F_0) = \sup_x |\hat{F}(x) - F_0(x)|$ , where  $\hat{F}(x) = \frac{1}{n} \sum_{i=1}^n 1(X_i \leq x)$  is the empirical cdf and  $F_0(x)$  is the null hypothesis cdf (maximum values of difference between empirical and null)
- Test Reference Distribution: Kolmogorov distribution
- Critical Value/ Rejection region: Reject for large values of  $\sqrt{n}D(F_0)$
- Note the one sided version does not have an easy interpretation

## NOTE:

**field:** Data setting:  $X_1, \dots, X_n$  iid from discrete distribution. Test fit of distribution

- Test name:
- Process
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region

**field:** Data setting:  $X_1, \dots, X_n$  iid from discrete distribution. Test fit of distribution

- Test name:  $\chi^2$  goodness of fit test, test for discrete distributions
- Process: Test the underlying population distribution is  $P(X = x) = p_0(x)$ , where  $\hat{p}(x) = \frac{1}{n} \sum_{i=1}^n 1(X_i = x)$ 
  - Let  $j = 1, \dots, k$  the different categories that  $X_i$  can take
  - Let  $O_j$  be the observed number of observations that belong to category  $j$
  - Let  $E_j = np_0(j)$  be the expected number of observations that would belong to category  $j$  if the null hypothesis were true
- Test Statistic:  $X(p_0) = \sum_x \frac{n(\hat{p}(x) - p_0(x))^2}{p_0(x)} = \sum_{j=1}^k \frac{(O_j - E_j)^2}{E_j}$
- Test Reference Distribution: Under  $H_0$ ,  $X(p_0) \rightarrow \chi_{k-1}^2$
- Critical Value/ Rejection region: Reject for large values of  $X(p_0)$  -  
Reject  $H_0$  for  $X(p_0) > \chi_{k-1}^2(1 - \alpha)$
- Note: Null hypothesis doesn't completely specify the distribution, just the family of distributions with perhaps unknown parameters
  - Estimate the parameters

- Use null distribution with estimated parameter values for  $E_j$
- Compute  $\chi^2$  test statistic
- Compare to  $\chi^2_{k-d-1}$  distribution where  $k$  = number of categories,  $d$  = number of estimated parameters

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n, Y_1, \dots, Y_m$  iid with known  $\sigma_x, \sigma_y$ . Estimate  $d$

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval
- p-value

**field:** Data setting  $X_1, \dots, X_m, Y_1, \dots, Y_n$  iid with known  $\sigma_x, \sigma_y$ . Estimate  $d$ ,

- Test name: 2 sample  $z$  test
- Test Statistic:  $z(d_0) = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{\frac{\sigma_x^2}{m} + \frac{\sigma_y^2}{n}}}$
- Test Reference Distribution: Under  $H_0$ ,  $z(d_0) \sim N(0, 1)$
- Critical Value/ Rejection region
  - Lower:  $d \leq d_0$  Reject when  $z(d_0) < z_\alpha = \text{qnorm}(\alpha)$
  - Upper:  $d \geq d_0$  Reject when  $z(d_0) > z_{1-\alpha} = \text{qnorm}(1-\alpha)$
  - Two sided:  $d \neq d_0$  Reject when  $|z(d_0)| > z_{1-\alpha/2} = \text{qnorm}(1 - \alpha/2)$
- Confidence interval:

$$(\bar{X} - \bar{Y}) \pm z(1 - \frac{\alpha}{2}) \sqrt{\frac{\sigma_x^2}{m} + \frac{\sigma_y^2}{n}}$$

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n, Y_1, \dots, Y_m$  iid with unknown but equal  $\sigma_x, \sigma_y$  Estimate  $d$

- Test name:
- Estimate of  $\sigma_x^2 = \sigma_y^2$
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval
- When not equal

**field:** Data setting  $X_1, \dots, X_n, Y_1, \dots, Y_m$  iid with unknown  $\sigma_x, \sigma_y$ . Estimate  $d$

- Test name: Equal variance 2-sample t-test
- Note: Estimate of  $\sigma_x^2 = \sigma_y^2 = s_p^2 = \frac{\sum_{i=1}^m (X_i - \bar{X})^2 + \sum_{i=1}^n (Y_i - \bar{Y})^2}{(m-1) + (n-1)} = \frac{(m-1)s_x^2 + (n-1)s_y^2}{(m+n-2)}$   
(weighted average of the two sample variances )
- Test Statistic:  $t(d_0) = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{s_p^2(\frac{1}{m} + \frac{1}{n})}}$
- Test Reference Distribution: For Normal populations, under  $H_0$ :  $t(d_0) \sim t_{m+n-2}$
- Critical Value/ Rejection region
  - $d > d_0$  Reject  $H_0$  for  $t_e(d_0) > t_{m+n-2}(1 - \alpha)$
  - $d < d_0$  Reject  $H_0$  for  $t_e(d_0) < t_{m+n-2}(\alpha)$
  - $d \neq d_0$  Reject  $H_0$  for  $|t_e(d_0)| > t_{m+n-2}(1 - \alpha/2)$
- Confidence interval  $(\bar{X} - \bar{Y}) \pm t_{m+n-2}(1 - \frac{\alpha}{2}) \sqrt{s_p^2(\frac{1}{m} + \frac{1}{n})}$
- When not equal:

- Expected value of Estimated variance is larger than it should be when the smaller sample comes from the population with smaller variance - the test statistic will be closer to zero than it should be, and rejection rates will be smaller - Less power - more conservative
- Expected value of Estimated variance is smaller than it should be when smaller sample comes from the population with the larger variance - test statistic will have a larger absolute value than it should an rejection rates will be larger - more power - anti conservative

**NOTE:**

**field:** Data setting  $X_1, \dots, X_m, Y_1, \dots, Y_n$  iid with unknown not equal equal  $\sigma_x, \sigma_y$  Estimate  $d$

- Test name:
- Estimate of  $Var(\bar{X} - \bar{Y})$
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval
- Compare to equal variance

**field:** Data setting  $X_1, \dots, X_m, Y_1, \dots, Y_n$  iid with unknown not equal equal  $\sigma_x, \sigma_y$  Estimate  $d$

- Test name: Unequal variance 2 sample t-test
- Estimate of  $Var(\bar{X} - \bar{Y}) = \frac{s_x^2}{m} + \frac{s_y^2}{n}$
- Test Statistic:  $t_U(d_0) = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{\frac{s_x^2}{m} + \frac{s_y^2}{n}}}$

- Test Reference Distribution: If the two distributions are Normal, there is not an exact distribution for the test statistic - Use Welch-Satterthwaite approximation: Estimate degrees of freedom

$$v = \frac{\left(\frac{s_x^2}{m} + \frac{s_y^2}{n}\right)^2}{\frac{s_x^4}{m^2(m-1)} + \frac{s_y^4}{n^2(n-1)}}$$

$\min(m-1, n-1) \leq v \leq m+n-2$  Under  $H_0$   $t_u(d_0)$  approx  $\sim t_v$

- Critical Value/ Rejection region: same as t-test
- Confidence interval:  $(\bar{X} - \bar{Y}) \pm t_v(1 - \frac{\alpha}{2})\sqrt{\frac{s_x^2}{m} + \frac{s_y^2}{n}}$
- Compare to equal variance:
  - For unequal sample sizes with unequal population variances, equal variance t-test does not have correct calibration
  - When samples sizes are equal both test statistics are the same, but degrees of freedom differ
  - When equal variance assumption is true, equal variance has slightly better power, and very slightly better calibration (more exact )

## NOTE:

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ ,  $X_i$  not independent  $Y_i$ ,  $(X_1, Y_1), \dots, (X_n, Y_n)$  iid  $F_{XY}$   $Cov(X_i, Y_i) = \sigma_{XY}$ ,  $Cov(X_i, Y_j) = 0$   
Estimate  $d = \mu_x - \mu_y$ ,  $\sigma_x^2, \sigma_y^2, \sigma_{XY}$  known

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ ,  $X_i$  not independent  $Y_i$ ,  $(X_1, Y_1), \dots, (X_n, Y_n)$  iid  $F_{XY}$   $Cov(X_i, Y_i) = \sigma_{XY}$ ,  $Cov(X_i, Y_j) = 0$ . Estimate  $d = \mu_x - \mu_y$ ,  $\sigma_x^2, \sigma_y^2, \sigma_{XY}$  known

- Test name: Paired z-test
- Test Statistic:  $z(d_0) = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{n} - 2\frac{\sigma_{XY}}{n}}} = \frac{\bar{D} - d_0}{\sqrt{\frac{\sigma_D^2}{n}}}$
- Test Reference Distribution: Under  $H_0$ ,  $z(d_0)$  approx  $\sim N(0, 1)$
- Critical Value/ Rejection region: Same as normal
- Confidence interval :

$$(\bar{X} - \bar{Y}) \pm z(1 - \frac{\alpha}{2}) \sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{n} - 2\frac{\sigma_{XY}}{n}} = \bar{D} \pm z(1 - \alpha/2) \sqrt{\frac{\sigma_D^2}{n}}$$

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ ,  $X_i$  not independent  $Y_i$ ,  $(X_1, Y_1), \dots, (X_n, Y_n)$  iid  $F_{XY}$   $Cov(X_i, Y_i) = \sigma_{XY}$ ,  $Cov(X_i, Y_j) = 0$  Estimate  $d = \mu_x - \mu_y$ ,  $\sigma_x^2, \sigma_y^2, \sigma_{XY}$  unknown

- Test name:
- Estimate of  $\sigma_{XY}$
- Estimate of  $Var(\bar{X} - \bar{Y})$
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ ,  $X_i$  not independent  $Y_i$ ,  $(X_1, Y_1), \dots, (X_n, Y_n)$  iid  $F_{XY}$   $Cov(X_i, Y_i) = \sigma_{XY}$ ,  $Cov(X_i, Y_j) = 0$   
Estimate  $d = \mu_x - \mu_y$ ,  $\sigma_x^2, \sigma_y^2, \sigma_{XY}$  unknown

- Test name: Paired Data t-test
- Estimate of  $\sigma_{XY} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$
- Estimate of  $Var(\bar{X} - \bar{Y}) = \frac{s_d^2}{n} = \frac{s_x^2}{n} + \frac{s_y^2}{n} - 2\frac{s_{XY}}{n}$
- Test Statistic:  $t(d_0) = \frac{(\bar{X} - \bar{Y}) - d_0}{\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{n} - 2\frac{s_{XY}}{n}}} = \frac{\bar{D} - d_0}{\sqrt{\frac{s_D^2}{n}}}$
- Test Reference Distribution: If differences are Normal (note X,Y Normal does not imply Differences are normal unless X,Y are jointly multivariate-normal) Under  $H_0$ ,  $t(d_0) \sim t_{n-1}$  (exact distribution)
- Critical Value/ Rejection region Same as t
- Confidence interval

$$(\bar{X} - \bar{Y}) = t_{n-1}(1 - \frac{\alpha}{2})\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{n} - 2\frac{s_{XY}}{n}} = \bar{D} \pm t_{n-1}(1 - \frac{\alpha}{2})\sqrt{\frac{s_D^2}{n}}$$

- Equivalent to a one sample - t-test on the differences

## NOTE:

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
Test  $H_0 : p_x - p_y = 0$

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval



**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
 Test  $H_0 : p_x - p_y = 0$

- Test name: Binomial proportions two-sample z-test
- Test Statistic:

$$z = \frac{\hat{p}_x - \hat{p}_y}{\sqrt{\hat{p}_c(1 - \hat{p}_c(\frac{1}{m} + \frac{1}{n}))}}$$

$$\text{Where } \hat{p}_c = \frac{m\hat{p}_x + n\hat{p}_y}{m+n} = \frac{b+d}{N}$$

- Test Reference Distribution: Under  $H_0 : z$  approx  $\sim N(0, 1)$
- Critical Value/ Rejection region: Same as regular 2-sample
- Confidence interval:

$$\hat{p}_x - \hat{p}_y \pm z_{1-\alpha/2} \sqrt{\left(\frac{\hat{p}_x(1 - \hat{p}_x)}{m} + \frac{\hat{p}_y(1 - \hat{p}_y)}{n}\right)}$$

**NOTE:**

**field:** Multinomial sampling

**field:** Collection of random samples, recording what group they are in: Can estimate  $P(X = x|G = g)$ , where  $G$  is the group

**NOTE:**

**field:** Two-Sample Binomial sampling

**field:** Sample  $m$  units from group 1 and  $n$  units from group 2

**NOTE:**

**field:**  $P(X = x|G = g)$  with binomial sampling

**field:** Cannot estimate

**NOTE:**

**field:**  $P(X = x|G = g)$  with multinomial sampling

**field:** Can estimate

**NOTE:**

**field:**  $E(g(T)) =$

**field:**  $E(g(T)) \neq g(E(T))$

**NOTE:**

**field:** Reason for performing transformations on data

**field:** Some tests are FSE only when population distribution is Normal (otherwise the methods are asymptotically exact), requiring a large  $n$ . Transformations that improve approximation of normality make Normal-based methods perform more exactly

**NOTE:**

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
Test  $H_0 : p_x - p_y = 0$  (Association/independent/relationship)

- Test name:
- Test Statistic
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
 Test  $H_0 : p_x - p_y = 0$  (Association/independent/relationship)

- Test name: Pearson's Chi-squared Test
- Test Statistic:  $X = \sum_{i,j \in \{1,2\}} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$  Where  $O_{ij} = n_{ij}$  and  $E_{ij} = \frac{R_i C_j}{N}$
- Test Reference Distribution: Under  $H_0$   $X \sim \chi_1^2$
- Critical Value/ Rejection region: Reject for  $X > \chi_1^2(1 - \alpha)$
- Note: Equal to to sided z-test for binomial proportions:  $X = z^2$

**NOTE:**

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
 Test  $H_0 : p_x = p_y$  (No association between response variable  $X$  and grouping variable  $G$ )

- Test name:
- Test Statistic:
- pvalue
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
 Test  $H_0 : p_x = p_y$  (No association between response variable  $X$  and grouping variable  $G$ )

- Test name: Fisher's Exact Test (of homogeneity of proportions)
- Test Statistic: Probability of observed table conditioning on margins:  
 Compute all tables with the same margin totals:  $\frac{\binom{C_2}{O_{12}} \binom{C_1}{O_{11}}}{\binom{N}{R_1}}$

- pvalue: Sum of probability of all tables more extreme than observed table More Extreme:
  - $p_x > p_y$  More extreme = larger  $O_{12}$
  - $p_x < p_y$  More extreme = smaller  $O_{12}$
  - $p_x \neq p_y$  More extreme = less likely table

**NOTE:**

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
Test  $H_0 : p_x = p_y$  Binomial sampling scheme

- Test name:
- Test Statistic:
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval

**field:** Data setting  $X_1, \dots, X_m$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  
Test  $H_0 : p_x = p_y$  Binomial sampling scheme

- Test name: Log Odds - test  $H_0 : \omega = 1$
- Test Statistic:  $\hat{\omega} = \frac{ad}{bc}$ ,  $z = \frac{\log(\hat{\omega})}{\sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}}$
- Test Reference Distribution  $\log(\hat{\omega})$  approx  $\sim N(\log(\omega), \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d})$ ,  
 $z$  approx  $\sim N(0, 1)$
- Critical Value/ Rejection region
- Confidence interval  $(\hat{\omega} e^{-z(1-\frac{\alpha}{2})\sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}}, \hat{\omega} e^{z(1-\frac{\alpha}{2})\sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}})$
- :  $\omega > 1, p_1 > p_2$ ,  $\omega = 1, p_1 = p_2$ , small  $p_1, p_2$ ,  $\omega = p_1/p_2$  = relative risk

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  $X, Y$  not independent (paired) test proportions equal in groups (equally likely/probability)

- Test name:
- Test Statistic:
- Test Reference Distribution
- Critical Value/ Rejection region
- Confidence interval

**field:** Data setting  $X_1, \dots, X_n$  iid Bernoulli( $p_x$ ),  $Y_1, \dots, Y_n$  iid Bernoulli( $p_y$ ),  $X, Y$  not independent (paired), test proportions equal in groups (equally likely/probability)

- Note, requires a table that keeps track of the pairs
- Test name: McNemar's Test
- Test Statistic:  $z = \frac{b-c}{\sqrt{b+c}}$
- Test Reference Distribution:  $z \sim N(0, 1)$ ,  $z^2 \sim \chi_1^2$
- Critical Value/ Rejection region
- Confidence interval
- Note equivalent to performing a paired t-test on the differences:

$$t = \frac{b - c}{\sqrt{\frac{n}{n-1} \left( b + c - \frac{(b-c)^2}{40} \right)}}$$

compare to  $t_{n-1}$

**NOTE:**

**field:** Data setting:  $n$  observations, record Group 1 and Group 2, where each group takes on  $j$  2 values, Test if there is an association between the groups

- Test name:
- Test Statistic:
- Test Reference Distribution

**field:** Data setting:  $n$  observations, record Group 1 ( $r$  values) and Group 2 ( $c$ ) values, Test if there is an association between the groups

- Test name: Pearsons  $\chi^2$
- Test Statistic:  $X = \sum_{i=1}^r \sum_{j=1}^c \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$ , where  $E_{ij} = \frac{n_i n_j}{N}$
- Test Reference Distribution: Under  $H_0$ ,  $X$  approx  $\sim \chi^2_{(r-1)(c-1)}$
- Note not FSE, but performance is good if  $E_{ij} > 5$

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test  $m_x = m_y$  (medians )

- Test name:
- Process
- pvalue
- Test Statistic:
- Test Reference Distribution

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test  $m_x = m_y$  (medians )

- Test name: Wilcoxon Rank-Sum (Mann-Whitney U-test)
- Note this is only a test of medians only if just additive effect - G1 is just a shift from G2 (shape and scale must be same ) (but then just the same as a test of mean, 10th percentile, min,  $F_x = F_y$  etc )
- If No additive assumption - test of  $H_0 : P(X > Y) = .5$
- Process:
  - Combine samples
  - Rank the observations in combined sample from smallest to largest (1 to  $n + m$ )
  - Add ranks of the smaller group
- pvalue: Calculate using permutations: Count number of permutations that lead to a R value more extreme than observed out of total permutations ( $\binom{n+m}{m}$ )
- Test Statistic:  $R$  sum of the ranks, or  $z = \frac{R - \frac{m(m+n+1)}{2}}{\sqrt{\frac{mn(m+n+1)}{12}}}$
- Test Reference Distribution: If there was no difference between two populations, then each rank has equal chance of being assigned to group 1 (belongs to  $X$ :  $p = \frac{m}{n+m}$ ) Normal approximation:  $R \sim N(\frac{m(m+n+1)}{2}, \frac{mn(m+n+1)}{12})$ ,  $z \sim N(0, 1)$
- Notes: If ranks, assign ranks, and then average ranks of tied values
- Continuity correction to normal distribution: add .5 to R if lower probability, subtract .5 from R if upper probability (ie  $1 - \text{pnorm}()$ )
- Not consistent test unless under additive assumption. IS consistent test of  $H_0 : P(X > Y) = .5$

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test  $m_x = m_y$  (medians )

- Test name:
- Process
- Test statistic:

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test  $m_x = m_y$  (medians )

- Test name: Mood's Test for Equality of Population Medians
- Process:
  - Find combined sample median  $\hat{m}$
  - Calculate  $\hat{p}_x =$  proportion of  $X$ s greater than  $\hat{m}$ ,  $\hat{p}_y$ , proportion of  $Y$ s greater than  $\hat{m}$
  - Conduct two sample binomial z-test( Pearsons chi-squared test) or Fisher's exact test
  - Test statistic:

$$z = \frac{\hat{p}_x - \hat{p}_y}{\sqrt{\hat{p}_c(1 - \hat{p}_c(\frac{1}{m} + \frac{1}{n}))}}$$

$$\text{Where } \hat{p}_c = \frac{m\hat{p}_x + n\hat{p}_y}{m+n} = \frac{b+d}{N}$$

**NOTE:**

**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test some statistic  $W$

- Test name:
- Process



**field:** Data setting  $X_1, \dots, X_n$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test some statistic  $W$

- Test name: Permutation test
- Process: Permute group labels across observations and recalculate statistic for each permutation to create permutation distribution - calculate p-values using the permutation distribution
- Performance: Many settings (like medians equal), will not reject correctly (even in large samples) if the medians are equal, but the distributions differ
- Permutation hypothesis is that the observations from the two populations are exchangeable (ie same population distributions, not just equal medians )

**NOTE:**

**field:** Data setting: Estimate value of nuisance parameter

**field:**

- Test name: Bootstrap
- Process: Since the empirical distribution function converges to the true distribution function, we can use samples from the empirical distribution to approximate how samples from the true distribution would behave.
- Confidence interval:  $100(\alpha/2)$  largest resampled statistic  $100(1-(\alpha/2))$  largest resampled statistic

**NOTE:**

**field:** Data setting: Data setting  $X_1, \dots, X_m$  iid  $N$ ,  $Y_1, \dots, Y_n$  iid  $N$ .  $H_0 : \sigma_x^2 = \sigma_y^2$  or  $H_0 \sigma_x^2 / \sigma_y^2 = r$

**field:** Data setting: Data setting  $X_1, \dots, X_m$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ .  $H_0 : \sigma_x^2 = \sigma_y^2$  or  $H_0 \sigma_x^2 / \sigma_y^2 = r$

- Test name: F
- Recall  $s_x^2 = \frac{1}{n-1} \sum_{i=1}^m (X_i - \bar{X})^2$
- Note that  $\frac{(m-1)s_x^2}{\sigma_x^2} \sim \chi_{m-1}^2$ ,  $\frac{(n-1)s_y^2}{\sigma_y^2} \sim \chi_{n-1}^2$ ,
- Test Statistic:  $F(r) = \frac{s_x^2/\sigma_x^2}{s_y^2/\sigma_y^2} = \frac{s_x^2}{s_y^2} \frac{1}{r}$
- Test Reference Distribution: Under  $H_0 : F(r) \sim F_{m-1, n-1}$
- Critical Value/ Rejection region
  - $\sigma_x^2/\sigma_y^2 > r$  Reject for  $F(r) > F_{m-1, n-1}(1 - \alpha)$
  - $\sigma_x^2/\sigma_y^2 > r$  Reject for  $F(r) > F_{m-1, n-1}(\alpha)$
  - $\sigma_x^2/\sigma_y^2 \neq r$  Reject for  $F(r) > F_{m-1, n-1}(1 - \alpha/2)$  or  $F(r) < F_{m-1, n-1}(\alpha/2)$
- Performance: Not Well if underlying population is not normal: Not FSE or AE (but is consistent) - don't use if population is not normal

**NOTE:**

**field:** Data setting: Data setting  $X_1, \dots, X_m$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ .  $H_0 : \sigma_x^2 = \sigma_y^2$

- Test name:
- Process:
- Interpretation
- Assumptions

**field:** Data setting: Data setting  $X_1, \dots, X_m$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ .  $H_0 : \sigma_x^2 = \sigma_y^2$

- Test name: Levene's Test
- Process:
  - Construct new variables:
    - \*  $U_i = |X_i - \text{med}(X)|$  or  $(X_i - \text{med}(X))^2$  or  $|X_i - \bar{X}|$  or  $(X_i - \bar{X})^2$
    - \*  $V_i = |Y_i - \text{med}(Y)|$  or  $(Y_i - \text{med}(Y))^2$  or  $|Y_i - \bar{Y}|$  or  $(Y_i - \bar{Y})^2$
  - Perform two-sample  $t$  test on  $U_i$  and  $V_i$  (use Welch)
- Interpretation: If last option used, can be a test in difference in population variances
- Assumptions:
  - Independence
  - Large sample sizes, so t-test assumptions are met
- Note: dont use as a test to determine which t-test version to use

#### NOTE:

**field:** Data setting: Data setting  $X_1, \dots, X_m$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test  $H_0 : F_x = F_y$

- Test name
- Test statistic

**field:** Data setting: Data setting  $X_1, \dots, X_m$  iid  $F_x$ ,  $Y_1, \dots, Y_n$  iid  $F_y$ . Test  $H_0 : F_x = F_y$

- Test name: Two-sample Kolmogorov-Smirnov Test
- Test statistic:  $D = \sup_x |\hat{F}_x(x) - \hat{F}_y(y)|$  ie the largest distance between the empirical CDF for  $X$  and  $Y$
- Reject for large values of  $\sqrt{\frac{mn}{m+n}}$
- Only for continuous distributions, for discrete distributions, use Pearsons  $\chi^2$

**NOTE:**

**field:** Multiple 2x2 tables under  $k$  different conditions  $p_{xj} = P(X = 1 \text{ in Table } j), p_{yj} = P(Y = 1 \text{ in Table } j)$   $H_0 : p_{xj} = p_{yj}$  for all  $j$

**field:**

- Test name: Mantel-Haenszel Test
- Test statistic:  $\omega_j = \frac{p_{xj}(1-p_{xj})}{p_{yj}(1-p_{yj})}, H_0 : \omega_j = 1$  for all  $j$

$$E(n_{X1j}) = \mu_{X1j} = \frac{n_{X \cdot j} n_{\cdot 1j}}{n_{\cdot j}}, V(n_{X1j}) = \sigma_{X1j}^2 = \frac{n_{X \cdot j} n_{Y \cdot j} n_{\cdot 1j} n_{\cdot 0j}}{n_{\cdot j}^2 (n_{\cdot j} - 1)}$$

$$C = \frac{[\sum_j (n_{X1j} - \mu_{X1j})]^2}{\sum_j \sigma_{X1j}^2}$$

- Under  $H_0$   $C \sim \chi^2(1)$
- Assumes the odds-ratios are the same in all  $k$  tables

**NOTE:**

**field:** Sample 1:  $X_{1,1}, \dots, X_{1n_1}$  from population 1 with mean  $\mu_1$ , Sample 2:  $X_{2,1}, \dots, X_{2n_2}$  from population 2 with mean  $\mu_2, \dots$  Sample M:  $X_{M,1}, \dots, X_{Mn_M}$  from population M with mean  $\mu_M$

- Independence within and between groups
- Populations (approximately ) normal
- Equal variances

**field:**

- Test name: ANOVA
- Estimate of common variance  $s_p = \frac{(n_1-1)s_1^2 + \dots + (n_M-1)s_M^2}{(n_1-1) + \dots + (n_M-1)}$
- Could use two-sample-t test on two population means

- Could test are population means 1 through M equal to each other?
- Compare the variability between groups to the variability within groups
- Sum of squares within groups:

$$SSW = (n - M)s_p^2 = \sum_{i=1}^{n_1} (X_{1i} - \bar{X}_1)^2 + \cdots + \sum_{i=1}^{n_M} (X_{Mi} - \bar{X}_M)^2$$

degrees of freedom:  $n - M$

- Sum of squares total

$$SST = \sum_{i=1}^{n_1} (X_{1,i} - \bar{X})^2 + \cdots + \sum_{i=1}^{n_M} (X_{M,i} - \bar{X})^2$$

degrees of freedom:  $n - 1$

- Sum of squares between groups:  $SSB = SST - SSW = \sum_{j=1}^M n_j (\bar{X}_j - \bar{X})^2$  df:  $(n - 1) - (n - M) = M - 1$
- Test statistic:

$$F = \frac{MSB}{MSW} = \frac{SSB/(M - 1)}{SSW/(n - M)}$$

- Reference distribution: Under  $H_0$ ,  $F \sim F_{M-1, n-M}$