

A Musical Stylometry Study on the Music of Fanny Hensel and Felix Mendelssohn

A Thesis
Presented to
The Division of Mathematics and Natural Sciences
Reed College

In Partial Fulfillment
of the Requirements for the Degree
Bachelor of Arts

Emily Palmer

May 2018

Approved for the Division
(Mathematics)

Andrew Bray

Acknowledgements

I want to thank a few people.

Table of Contents

| | |
|--|---------------|
| Chapter 1: | 1 |
| 1.1 Introduction: | 1 |
| 1.2 Brief history of text classification | 1 |
| 1.3 Feature selection | 2 |
| 1.4 A Very Short Introduction to Music Theory | 2 |
| 1.5 Musical features | 3 |
| 1.6 Background on Variable and Feature Selection | 4 |
| 1.7 Previous research. | 5 |
| 1.8 Previous choices of features in applications | 6 |
| 1.9 Previous modeling in applications | 8 |
| 1.10 Fanny and Felix Mendelssohn | 9 |
| Chapter 2: About the data and conversion process | 13 |
| 2.1 Pieces used | 13 |
| 2.2 Optical music recognition | 13 |
| 2.3 .krn to R | 19 |
| Chapter 3: MuseR and features | 21 |
| 3.1 Importing data into R | 21 |
| 3.2 Features currently supported in museR | 22 |
| Chapter 4: About Classification Models | 25 |
| 4.1 Logistic Regression | 25 |
| 4.1.1 Lasso model selection | 26 |
| 4.2 Linear Discriminant Analysis | 26 |
| 4.3 Naive Bayes | 27 |
| 4.4 K- nearest - neighbor | 27 |
| 4.5 Random Forests | 28 |
| 4.6 Dimensionality Reduction: Principal component analysis (PCA) | 28 |
| 4.7 K-means | 29 |
| Chapter 5: Exploratory Data Analysis | 31 |
| 5.1 Key for feature abbreviations | 31 |
| 5.2 Bach and Mendelssohns | 31 |
| 5.3 Felix and Fanny | 37 |

| | |
|--|-----------|
| Chapter 6: Results | 43 |
| 6.1 Models Results - Bach/Mendelssohn | 43 |
| 6.2 Model fit Felix/Fanny | 44 |
| 6.3 Predictions for Disputed pieces: | 44 |
| Chapter 7: Discussion | 49 |
| 7.1 Discussion | 49 |
| Conclusion | 51 |
| 7.2 Future suggestions for musical stylometry research | 51 |
| References | 53 |

List of Tables

| | | |
|-----|---|----|
| 5.1 | Abbreviations for features and corresponding meanings. The features of Fanny and Felix begin with f. | 31 |
| 6.1 | Averaged 5-fold CV misclassification rates of 100 runs on the feature space and the first 11 principal components for each model. | 43 |
| 6.2 | Averaged 5-fold CV misclassification rates of 100 runs on the feature space and the first 11 principal components for each model. | 44 |
| 6.3 | Predicted composer for each disputed piece | 44 |
| 7.1 | Misclassification Rates for different models comparing Bach and Mendelssohns | 49 |
| 7.2 | Summary of misclassification rates for different models comparing Felix and Fanny | 50 |

List of Figures

| | | |
|------|--|----|
| 2.1 | Flowchart of the conversion process from physical copy to dataframe . | 14 |
| 2.2 | Score in physical form before conversion process has started | 15 |
| 2.3 | A score in MuseScore format after being read by an OMR like Photo-Score or Audiveris | 16 |
| 2.4 | A half middle C in musicXML encoding | 17 |
| 2.5 | How sheet music corresponds to kern files | 18 |
| 3.1 | Example musescore format excerpt | 21 |
| 3.2 | Example kern format excerpt of the above musescore | 22 |
| 3.3 | Example R data frame converted from the above MuseScore | 23 |
| 3.4 | Example of calculating proportion of melodic intervals | 23 |
| 5.1 | Density plot of each feature of Bach/Mendelssohn | 32 |
| 5.2 | Correlations between features of Bach/Mendelssohn | 32 |
| 5.3 | Biplot of the loading vectors of the first two principal components . . | 33 |
| 5.4 | Biplot of the first two principal components plotted with data points colored by composer. | 34 |
| 5.5 | Biplot of the second and third principal components plotted with data points colored by composer | 35 |
| 5.6 | Skree plot of the PCA's | 35 |
| 5.7 | KNN when $k = 2$ | 36 |
| 5.8 | KNN when $k = 3$ | 37 |
| 5.9 | Density plot of each feature of Fanny/Felix | 38 |
| 5.10 | Correlations between features of Fanny/Felix | 38 |
| 5.11 | PCA Loadings of the features for Fanny/Felix | 39 |
| 5.12 | Loadings and pieces for Fanny/Felix | 40 |
| 5.13 | PCA Felix/Fanny skree plot | 41 |
| 5.14 | K-means with $k = 2$ | 41 |
| 6.1 | Lasso penalties for each feature for changing lambda penalty values . | 44 |
| 6.2 | Cross-validated misclassification rates for different lambdas | 45 |
| 6.3 | Variable importance for a random forest model Bach/Mendelssohn . . | 45 |
| 6.4 | Lasso penalties for each feature for changing lambda penalty values for Fanny/Felix | 46 |
| 6.5 | Cross-validated misclassification rates for different lambdas | 46 |
| 6.6 | Cross-validated misclassification rates for different lambdas | 47 |

| | | |
|-----|---|----|
| 6.7 | Variable importance for a random forest model Fanny/Felix | 47 |
|-----|---|----|

Abstract

Classification for authorship of written classical music is a relatively infrequently explored field. In theory, composers leave behind unconscious (or perhaps conscious) signals that indicate a piece of music as their own. Ideally these signals or features can be extracted and used to build a model to predict the composer of a piece of music. For instance, the siblings Fanny Hensel and Felix Mendelssohn, although very similar in compositional style, likely have features that distinguish their music. It is speculated that there are at least six pieces that Fanny wrote that were published by Felix.

Low level features, including frequencies of chords and scale degrees were extracted from 31 lieder known to be written by Felix and 35 pieces known to be written by Fanny. These features were extracted using *museR*, an R package written by me*. Additionally, to check the validity of features used, 27 solo piano pieces from J.S. Bach's Well Tempered Clavier were used. A logistic lasso, k-nearest neighbors, random forests, naive Bayes, and linear discriminant analysis were performed on the feature space in order to classify the pieces. In addition, those same models were fit on the principal components. Comparing between Bach and Mendelssohn, the LDA model fit on the principal model resulted in the lowest misclassification rate (0.049). Between Felix and Fanny a logistic lasso model on the principal components resulted in the lowest misclassification rate. Finally the six disputed pieces were run through the models. As misclassification rates were so high on the models, we cannot trust the predictions for these disputed pieces.

Chapter 1

1.1 Introduction:

Treating text or music as data has challenges. Neither data nor music contains anything immediately analyzable by a computer. With the increasing availability of the internet and social media data, text classification has become an promising form of data analysis. While text classification problems are very frequent, similar methods that classify music have not been as commonly explored.

Feature extraction is an essential part in text or music classification. Unlike in a data set of numerical or categorical values, text and music must first go through a processing stage where features of interest can be extracted before any models can be fit.

For music, the extracted features are used to build a model that can correctly classify a likely composer for that piece of music. Studies that examine the style of a type of music or composer are known as musical stylometry studies. For musically trained humans, this might be an easy task. Some are able to automatically distinguish a piece composed by Bach or Mozart either by listening to a recording, or looking at sheet music. This becomes more difficult when composers are contemporaries. Mozart and Salieri might be distinguishable to a more discerning listener or scholar, but only with some difficulty. Harder still is when the identity of the composer piece of music is disputed. Examples of such disputed authorship exist throughout music history, most notably in the Renaissance.

In both text and music classification, we must create features that can be calculated that would give a signal to indicate some conscious or unconscious tendency of the composer that would make them distinguishable.

1.2 Brief history of text classification

The Federalist Papers were one of the earliest instances of text classification for determining authorship.(Mosteller & Wallace, 1964). The famous Federalist Papers were written under the pen name ‘Publius’. There are several of these disputed papers attributed to James Madison or Alexander Hamilton. The authors never admitted authorship, as some of the writings were contradictory to their later political positions (Adair, 1944). Prior to quantitative analysis, historians had often examined the papers

using styles of previously known writings of Madison and Hamilton. Their analysis was often partially based on the content of the letters, for example the existence of citing English history is a trait more common to Hamilton. (Ford & Bourne, 1897)

In contrast, Mosteller and Wallace (1964) used the frequency of words such as ‘and’, ‘by’, ‘from’, and ‘upon’, to fit a classifier that was trained on the writings on a set of known writings by each author. These unconscious indicators were able to differentiate between the two writers, and the model was able to identify the likely author of the disputed paper.

1.3 Feature selection

Text analysis, such as in the Federalist Papers, is accomplished by reading one word after another. Information in a piece of music, however, is read in a variety of ways. It can be read left to right note by note, but it can also be read vertically as the harmony, or the notes played together. Also in a piece with several instruments, both these processes happen at the same time for each instrument. There are also aspects that take place over large sections, such as phrasing, or cadential patterns. There are rules of counterpoint that are followed throughout the entire piece. Thus we need to find features that can be measured for each piece, or perhaps each measure or instrument, that can describe a certain piece of music. Then we must decide which features are those of rules and practices of classical music, and where the creativeness and individuality of a composer happens. As there are many different aspects of a piece, melodic changes, harmonies, etc., one can end up with many features to describe one piece. It is possible that the number of predictors (p) can be greater than the sample size (n), or that many features encode essentially the same thing. We thus need to figure a clever way to decrease our predictor space. This can happen in choosing what features we want to use to model, or by using a dimension reduction technique such as principal components analysis.

Most of the musical stylometry papers have focused on composers in the Renaissance (1400-1600), Baroque (1600-1750), and Classical (1730-1820) eras. The Mendelssohns were composing in the Romantic era (1780-1910). The focus of earlier papers may have been because composers in earlier eras followed rules of counterpoint more exactly, or perhaps had less “expressive” allowances for their composing, thus making features easier. There are also more pieces with doubtful authorship in those eras. In addition Computer Assisted Research in the Humanities (CCARH) has a large corpus of encoded music from these times. As will be described later, the initial step of coding the music is the most time consuming, having music in a format immediately conducive to analysis is very helpful.

1.4 A Very Short Introduction to Music Theory

Western sheet music is presented on a five line staff. The sound of a note depends on its pitch, which is a specific vibration of sound waves at a certain frequency. The vertical distance between notes (also known as an interval) depends on how many half

steps occur between two notes. There are 12 half steps in the western scale. Melodic intervals are defined as the number of half steps between two adjacent notes. Harmonic intervals are defined as the number of half steps between notes played at the same time. Chords occur when there are more than two notes being played simultaneously. The type of chord is also determined by the harmonic intervals it contains. Cadences are a type of chord progression, usually occurring at the end of a phrase and especially at the end of a piece. Musical notes are in the set $note \in \{A, B, C, D, E, F, G\}$. The value of a note can be changed up or down a half step, by adding a sharp or flat. The key signature of the piece indicates how many sharps or flats are normal for the piece. A piece's key can be major or minor, although the piece can internally modulate between the two. The scale degree of a note depends on how many notes it is away from the tonic, or key of the piece. The time signature of a piece determines the number of beats that are in each measure. Intervals and chords can be either dissonant or consonant. Consonant intervals sound nice to our ears, whereas dissonant intervals add a sense of tension or unease, which is used to shape the feel of the music. In addition chords and intervals can be minor or major. Minor intervals feel "sad", whereas major intervals feel "happy". These are also known as sonorities, and divided into three types: perfect intervals (1, 4, 5, and 8), imperfect intervals (3 and 6), and dissonant intervals (2 and 7).

For classical music there are some "rules" that most composers follow to some extent. These were followed more strictly in the Baroque era than the Romantic era. These rules are known as counterpoint. First species counterpoint is a type of counterpoint. One rule is that there are no parallel perfect consonances. Perfect consonances are octaves, fourths and fifths. Parallel indicates that they occur one after another. Another rule is that from one perfect consonance to another, the notes must proceed in contrary or oblique motion.

1.5 Musical features

Features calculated from music often closely follow music theory. In music, in addition to deciding the features, there is also the decision of the scope at which those features take place. They can be features for a given instrument, the entire piece, or each measure. Windowing techniques can be used where a "window" is created over a given number of bars or notes, and shifts across the whole piece. For each window, a feature is recorded. These can be overlapping windows by creating an "offset" of a number of beats or notes. Windowing produces more data, as instead of one feature for each piece, there is a feature for each window.

Musical features are often thought about as either high level, or low level. While the exact definition of each often varies, low level is often understood to be features such as note frequency, etc. High level features are more about a broader sense of the piece, including chord progressions, etc. High level features are often what music experts use in their analysis, whereas low-level features are more easily done with computer analysis. High level features often depend on the context of where the feature was extracted from. For example, cadences and modulations (brief changes in

key) throughout the piece are easily found when a human analyzes a piece of music, but are very hard to generalize as a feature. If one wants to use high-level features, they would first have to train the feature to see if it was performing accurately. For this reason, this paper focuses only on low level features, or features that are context independent.

Especially in research regarding text categorization, data sets have enormous numbers of variables. We use “feature” and “variable” interchangeably, with the exception when features are created from variables, and the distinction will be made in that case (Guyon & Elisseeff, 2003). There are several variable selection algorithms that select the “important” variables that could be included in models. If one included every variable that was extracted from a piece, the model would very likely be overfit. Thus some feature selection is important before fitting a model.

1.6 Background on Variable and Feature Selection

Before analysis is done or models are fit, numerous features are often extracted from the music without knowing a priori which ones will be helpful in identifying a composer’s style. Fitting models with all of the features that are available can often lead to overfitting of the model. There are three main ways to deal with this problem: subset selection, shrinkage models, and dimensionality reduction. This section gives a broad overview of options for features selection. The methods used in this thesis will be described in further detail in chapter 4.

Subset selection methods choose which features to use in a model. There are many ways to do this and existing feature selection algorithms can help with this process. One idea to see which features are best for the model is to look at all possible subsets of the variables and see which model performs best. All possible subsets of variables is $2^p - 1$, where p is the number of features. For large p , this is often computationally impossible. Instead of fitting all possible subsets, strategies like best-first, branch and bound, simulated annealing, and genetic algorithms can help with the computational difficulties.

Subset selection methods often use some sort of variable ranking tool to determine which variables are useful to include in the best subset. Variable ranking uses a score function to assign a score to each possible variable. It is a computationally efficient method and is robust against overfitting as it introduces bias but may result in less variance. It is tempting to only include variables that have a high score. However, this possibly leads to redundancy. In addition, variables that are not important by themselves can have a significant performance improvement when considered with other variables. Single variable classifiers rank the variable according to its individual predictive power. The predictive power can be measured in terms of error rate, or using the false positive or false negative rate. This classifier cannot distinguish variables that perfectly separate the data.

Shrinkage methods, instead of picking which features to use, use all p features in the model. They instead use techniques that constrain the coefficient estimates of the model used. Ideally, coefficients of features that are not as useful for the

model will be shrunk towards zero, so they do not have a large effect on the model. Constraining features in this way often leads to a reduction in variance. The most common shrinkage methods are ridge regression and the lasso.

The two methods mentioned above, shrinkage methods and subset selection both use the original variables. Dimension reduction methods transform the features space, and then fit models on the transformed variables. It is commonly used to combine enough of the information given in the features in a smaller dimensional space. This results in feature creation; using the recorded variables to create new features to fit the model on. Common dimension reduction techniques include Principal components analysis (PCA) and linear discriminant analysis (LDA). Goals of feature creations include that we can achieve a good reconstruction of the data and that we can be successful in making our predictors.

Once we have a model, we often want to know how “good” it is. For classification, we often use the misclassification rate. To calculate this, we withhold a sample of our data as a testing set. We then fit the model on the training set, and then use the fitted model to predict classes on the testing set. The misclassification rate is then computed as the number of pieces in the testing set that were incorrectly classified divided by the total number of pieces. Cross validation involves fitting a model on a training set, and then using the fitted model to predict the responses in the testing set. k -fold cross validation involves splitting the data set into k different parts of equal size, and fitting a model on the the data set with one of the k parts withheld. The model is then tested on the withheld data. This is useful for model selection and determining the accuracy of the model. In this paper, as is common, we use $k = 5$. This results in a predicted misclassification rate with lower variance.

1.7 Previous research.

There have been numerous other applications of machine learning to music. These include many studies that are not specifically identifying a composer from sheet music. For example, a previous study used machine learning trained on a composer’s music to have the computer learn the style and compose a piece in a similar style. (Papadopoulos & Wiggins, n.d.)

Research on musical stylometry has focused on two types of data: audio and sheet music. This analysis uses data in the form of sheet music. To predict a composer, a training data set of pieces of known composer is needed. Then a model can be fit to a testing data set to predict composer. If the fitted model shows good predictions, that model can be used on the pieces of unknown authorship. Musical stylometry can be used to resolve disputed authorship, but also to detect distinguishing musical styles of composer, even if there are no disputed pieces. This same process can also be used to look at the development of a composers musical style over time.

This paper draws its inspiration from four different studies that have looked at the question of classifying composers by extracting features from sheet music. Initial studies in the 1970’s started looking at ways to represent music in a way that could analyzed by computers. Recently, several previous papers have focused on Josquin des

Prez. This is likely due to the fact that there is a large training and testing data set available in easily analyzable format provided by the Josquin Research Project (“The josquin research project,” n.d.). In addition there are a number of pieces of disputed authorship that have been attributed to him.

Work by Brinkman et al. (Brinkman, Shanahan, & Sapp, n.d.) uses machine learning to evaluate attribution of compositions by Josquin des Prez. They first compared music of Josquin and JS Bach’s four part chorales. Many listeners could easily distinguish these two composers as they were separated by two centuries. They also compared Josquin to composers closer in time and style: Ockeghem and Du Fay, who are only one or two generations separated. These composers could likely be differentiated by experienced listeners of Renaissance music. Finally they compared Josquin’s to his contemporaries: de Orto and La Rue to find differences in style.

Work by Backer et. al.(2005) looked at differences in style between J.S. Bach, Telemann, Handel, Haydn and Mozart and then examined a disputed piece: BWV 534 which is believed to be composed by J.S. Bach, J.L. Krebs, or W.F. Bach (J.S. Bach’s son). (Backer & Kranenburg, 2005)

The following two studies do not try to identify composers of unknown works, but instead try to fit classifiers that can correctly predict composers based on differences in style.

Crerar (Crerar, 1985) used incipits from Vivaldi, Corelli, and Valentini, who could have been a student of Corelli. They used low level features, especially looking at intervallic movement. They also compared Bach, Haydn, and Mozart.

Mearns et. al. (Mearns, Tidhar, & Dixon, 2010) extracted high level musical features on J.S. Bach, Buxtehude, Ruggero, Vivaldi, Monteverdi, Corelli, and Frescobaldi. The idea was that they all adhered to the laws of counterpoint, and that they possibly still had distinct uses of the counterpoint. They used pieces across instrumental genres with the idea that stylistic counterpoint would remain constant across compositional type.

1.8 Previous choices of features in applications

Deciding on and extracting features of music is the first step of analysis. Depending on the characteristics of the composer and time period different features would be useful. Often features are extracted en masse and then work is done later to determine which features are important or useful in identifying style.

In addition to what kinds of features, there is also the question of the scale of those features. They can be features for a given instrument, the entire piece, or each measure. Also windowing techniques can be used where a “window” is created over a given number of bars or notes, and moves through the whole piece. For each window, a feature is recorded. These can be overlapping windows by creating an “offset” of a number of beats or notes. This produces more data, as instead of one feature for each piece, there is a feature for each window, and there can be tens of windows in each piece.

Common types of features used before in music analysis are: Frequencies or

fractions of notes, chords, etc. are a common low-level feature. These include the fraction of the score that consisted of dissonant sonorities, as well as the fraction of bars that begin with a dissonant sonority. Other features include the type of intervals or consonances present in a piece: perfect consonance, imperfect consonances, and dissonance. In polyphonic pieces, the four types of motion, (parallel, similar, oblique, and contrary) can also be used as features. Crerar (Crerar, 1985) used pitch frequencies, after standardizing all pieces by transposing to the key of C, the first melodic interval of the piece, and note transition counts.

Features measuring “stability” are also popular. Stability is computed by dividing the standard deviation of the lengths of the fragment by the mean length of the fragments. It is normalized in this way to be comparable over differing time signatures. These types of features were used by Backer et. al. (Backer & Kranenburg, 2005). They also used overlapping windowing over each entire composition to produce more data. They chose a window of 30 bars to create a high enough number of fragments per piece and a low enough variance of the feature values between fragments. They chose to extract 20 features including features of fractions and measuring stability, and entropy. They computed the entropy of the probability of occurrence chords. They measured this two different ways, defining chords to be the same no matter what scale degree they are on, and distinguishing chords differently depending on scale degree. Next the entropy was calculated given the probability of each pitch in the score. Entropy was calculated by $-\sum_{i=1}^N p_i \log p_i$ where p_i is the probability of occurrence, and N is the total number. Next they the calculated average number of active voices at one time which represents the voice density of the piece. Then for every interval, the duration of that interval was divided by the total duration of all intervals. Next the total duration of parallel thirds, fourths, and sixths divided by the total duration of all intervals was measured. Finally a measure of suspensions was found.

Brinkman et.al (Brinkman et al., n.d.) used both high-level and low-level features. The high level features were 9-8 suspensions, oblique motion, contrary motion, similar motion and parallel motion. The low level features were average melodic entropy, normalized pairwise variability index, and note-to-note transition probabilities.

Other features that can be helpful for distinguishing Renaissance and Baroque composers look specifically at differences in counterpoint. Since most composers in that era generally followed the rules of counterpoint, these features are created to try to detect specific identifiable uses of counterpoint. These features include counterpoint movement types, dissonance distributions, parallel intervals of each kind, and vertical interval distributions. Mearns used such features (Mearns et al., 2010). They calculated intervals for every subsequent pitch in each part. A weighted count of each interval was stored. They were weighted according to duration to account for the perception of use. The count vector was then normalized to account for the weighted interval content of the score. For movement types, they used ideas based on the rules of first species counterpoint. (Similar, oblique, parallel, and contrary). It does not make musical sense to compare every single adjacent group of notes. Many of the note groups occurring at fractional metrical positions consist of notes held from the previous note group with the addition of a passing or neighbor note in another

voice or voices. For this reason, only notes that occur at strong metrical positions, in this case on each beat of the bar, are used for contrapuntal evaluation. This is a relatively simplistic method of choosing structurally important notes. In Bach, for example, the majority of voice progressions take place at a closer level. The features for counterpoint method include the nature of the approach to a perfect consonance, whether a dissonance is properly prepared (i.e. preceded by a consonant interval containing the same note which then becomes dissonant), whether a dissonance is correctly resolved downwards by step, details of parallel intervals, and the overall distribution of contrapuntal movements (oblique / contrary / similar / parallel / other). A feature for total vertical interval count is also used.

1.9 Previous modeling in applications

Most of the previous research has needed to do some kind of feature selection. Usually features are extracted before any analysis is done, and we don't know before hand which features are distinguishing.

A modification of a forward selection (Floating Forward Selection (Pudil, Novovičová, & Kittler, 1994) was used in Backer's (2005) study to extract features in order to identify distinguishing style between Bach, Handel, Telemann, Mozart, and Haydn, and then subsequently classify the authorship of BWV 535. Each composer was compared via creating comparisons of all possible class arrangements, ie (Bach)(Handel), (Bach)(Handel,Telemann), etc. The algorithm extracted features for each class arrangement that distinguished the groups the best. A k -nearest neighbors classifier was fit using the features that the subset selection algorithm found useful. The model was successful in comparing Bach and others as well as each individual composer. The k -nearest neighbor classifier resulted in a leave-one-out-error of between 5% and 9%, for most class arrangements, and 24.3% between Hayden and Mozart. Decision trees were also used to interpret the features used in decision making of the different class arrangements. The decision trees used between 2 and 4 features depending on which class comparison was being analyzed. It resulted in a max of one classification error. To determine authorship of BWV 535, quadratic Bayesian classifier was trained to distinguish J.S. Bach, W.F. Bach and J.L. Krebs. They again compare every possible class arrangement as potential composers. Their results showed that BWV 535 was likely composed by either W.F. Bach and his pupil, J.L. Krebs.

Principal components analysis (PCA) was used for dimension reduction for subsequent analysis on the music Josquin and his contemporaries. (Brinkman et al., n.d.) Binary comparisons were used to compare composers. Although only two principal components accounted for most of the variance, in the comparison between Bach and Josquin, five principal components were used to account for more variance (85%). Between Josquin and his contemporaries, 27 principal components were used to account for 85% of the variance. This resulted in a relatively clear separation between Bach and Josquin when visualized on their first two principal components. Between Josquin and each of his contemporaries, the principal components did not have as

much of an obvious separation. The transformed features from principal components analysis were used to train a classifier. First a k -nearest neighbor classifier was used with $k = 6$. The training set consisted of 80% of the data, with 20% withheld as a testing set. The model correctly predicted pieces de Orto 38.9% of the time, Josquin 60.6%, La Rue, 80.6% Next they trained a support vector machine classifier, which performed similarly to the k -nearest neighbor classifier.

Mearns fit a classifier using a WEKA algorithm, as well as Naive Bayes and a Decision Tree was created that correctly predicted composer 2/3 of the time. (Mearns et al., 2010)

Crerar (Crerar, 1985) took a different approach, and instead of trying to fit a model that would classify composers, performed a chi-squared test to see if the groups were in fact different. This would imply that the composers had different styles that could be distinguished using the features in the paper. The probability of the chi-squared values occurring were all less than 0.05, and thus the composers do indeed have different styles.

1.10 Fanny and Felix Mendelssohn

Most musical stylometry analysis has focused on music of the Renaissance and Baroque period, as there are more questions of authorship in that period. As the Romantic period is much more modern in comparison, there are many more surviving records of original manuscripts that include the composer.

Felix Mendelssohn, often considered a prodigy akin to Mozart, was a prolific composer. Before he was fourteen years old, he had already written over 100 compositions.

His lesser known sister Fanny Hensel née Mendelssohn was also a composer of incredible skill. The two were very close, for many years training and studying together. In their early education living in Berlin, Felix and Fanny received the same musical education, first piano lessons by Madam Bigot, a famous pianist esteemed by Haydn and Beethoven. Beginning in 1818, Carl Friedrich Zelter, a somewhat removed student of Bach and the most influential Berlin musician of the time, began to teach them both composition. In addition to music, the children were tutored by some of the finest scholars in Berlin in subjects such as languages, history, and drawing. Goethe himself claimed that Fanny was “as gifted as Felix” (Tillard, 1996).

As Fanny grew up, her father started implying that she should focus her energy on the domestic sphere of her life. While the fact that she never became a world famous composer and performer is often attributed to the gender politics of her time, it is also likely due to her high class. (Reich, 1991) Especially considering the anti-Semitic feelings of the time, and since the family had recently converted from Judaism to Christianity, the family did not want any other unusual characteristic such as a professional female composer to set them further apart from “polite” society.

Most of Fanny’s available work are lieder, short pieces of voice accompanied by piano. They were accepted at the time as the more feminine, domestic compositions, acceptable for women to compose. Her brother moved on to more elaborate compositions such as operas, orchestral concertos and symphonies. Her father pressured

Fanny to remain composing *Lieder*. (Todd, 2003)

“Music will perhaps become his profession, while for you it can and must only be an ornament, never the root of your being and doing. We may therefore pardon him some ambition and desire to be acknowledged in a pursuit which appears very important to him, . . . while it does you credit that you have always shown yourself good and sensible in these matters; . . . Remain true to these sentiments and to this line of conduct; they are feminine, and only what is truly feminine is an ornament to your sex.”

Throughout their lives, Felix and Fanny maintained contact through letters until Fanny’s death in 1847 and Felix’s death shortly thereafter. These letters contain many instances of Felix asking for advice on his compositions.

Unlike Felix who conducted and performed piano and organ in some of Berlin’s most esteemed concert halls, most of Fanny’s performances were private, only performed in small circles of her friends and family at intimate parties. Similarly, although she was quite a prolific composer, under recommendation of her father Abraham Mendelssohn, and to a lesser extent Felix, Fanny did not publish her work until later in her life. In 1846 after her father’s death and though her brother disapproved, she published her first collection of *Lieder*. Many of Fanny’s unpublished notebooks are in private collections and are inaccessible for study.

However, it is widely speculated that some of Fanny’s work was published under her brother’s name, especially three pieces each in his Op 8 and 9 *lieder*.

Famously, when Felix met the Queen of England, she sang Felix’s lied “*Italien*”, and Felix had to admit that in fact, it was his sister that had written it. In a letter to Felix, Fanny admits:

“I have just recently received a letter from Vienna, which contained basically nothing but the question of whether “*On Wings of Song*” was by me, and that I should really send a list of things that are running about in the world disguised, it seems that they aren’t clever enough themselves to separate the wheat from the chaff.” (Mace, 2013)

As she never made such a list, we are left to wonder if there are any other pieces of hers that have been published under her brother’s name and reputation. It is suspected that most of Fanny’s friends and family would have known at the time that she had composed the pieces published by Felix. An article written in the *Harmonicon* in 1830, an influential music journal in London, mentioned that all the lied in Felix’s collection were not written about him:

" I possess twelve published songs under Mr. Mendelssohn’s name, which he wrote when a boy of fifteen. One of these appears in the musical annual, “*Apollo’s Gift*,” and deserves all the praise you have in your review bestowed on it. But the whole of the twelve are not by him: three of the best are by his sister, a young lady of great talents and accomplishments. I cannot refrain from mentioning Miss Mendelsshon’s name in connexion with these songs, more particularly when I see so many ladies without one

atom of genius, coming forward to the public with their musical crudities, and, because these are printed, holding up their heads as if they were finished musicians. Miss Mendelssohn is a first-rate piano-forte player, of which you may form some idea when I mention that she can express the varied beauties of Beethoven's extraordinary trio in Bb, beginning *She has not the wild energy of her brother; but possesses sufficient power and nerve for the accurate performance of Beethoven's music. She is no superficial musician; she has studied the science deeply, and writes with the freedom of a master. Her songs are distinguished by tenderness, warmth, and originality: some which I heard were exquisite.*"(Ayrton, 1830)

This project will use lieder of Fanny and Felix Mendelssohn. Most of the available work by Fanny are lieder, of which Felix also composed a great deal. We will see if there is a determinable difference in style of these siblings who grew up very close and received mostly the same musical education. All other previous composership studies cannot be adjusted for training and upbringing, so it might be challenging to detect a difference.

Chapter 2

About the data and conversion process

2.1 Pieces used

The majority of the pieces used in this paper were lieder of Felix Mendelssohn and Fanny Hensel. Felix Mendelssohn composed many different styles of music, orchestral, piano, etc. Fanny Hensel in contrast has an available existing corpus of mostly lieder, although she did compose many works for solo piano and orchestra. Of Felix's music there were a total of 31 pieces: lieder of Op 8 (9 pieces), op 9 (9 pieces), Op 19.a (3 pieces), op 34 (1 piece), and 9 pieces published posthumously.

Of Fanny's music, a total of 35 pieces were used: 15 lieder were used from her lieder without name collection, 10 from her *Wo kommst du Her* collection, and 10 from an unnamed collection.

The six pieces published by Felix that are thought to be written by Fanny are Op8 no 2, 3 and 12, and Op 9, no 7, 10 and 12.

Data from JS Bach were also used. These data were available in Kern Score format from the Center for Computer Assisted Research in the Humanities. (CCARH). The pieces used were from the Well Tempered Clavier (WTC). These were written as training pieces and each collection contains 24 pieces with one in every possible key. Pieces from the Well Tempered Clavier were chosen as the data were more easily accessible (no scanning was required) and they were a similar format as the Mendelssohn songs, written as for solo piano (or harpsichord).

2.2 Optical music recognition

The vast majority of classical music is found solely in PDF or physical copies. Sheet music as a form of data requires a lengthy process of conversion before being able to be used in any analysis. Simply scanning the scores into, say, a PDF, gives no musical semantics and can only be viewed on screen or printed on paper. Thus, the two main steps in reading in data from sheet music are: first using optical music recognition software to transform physical scores into digital formats, and second, to read the

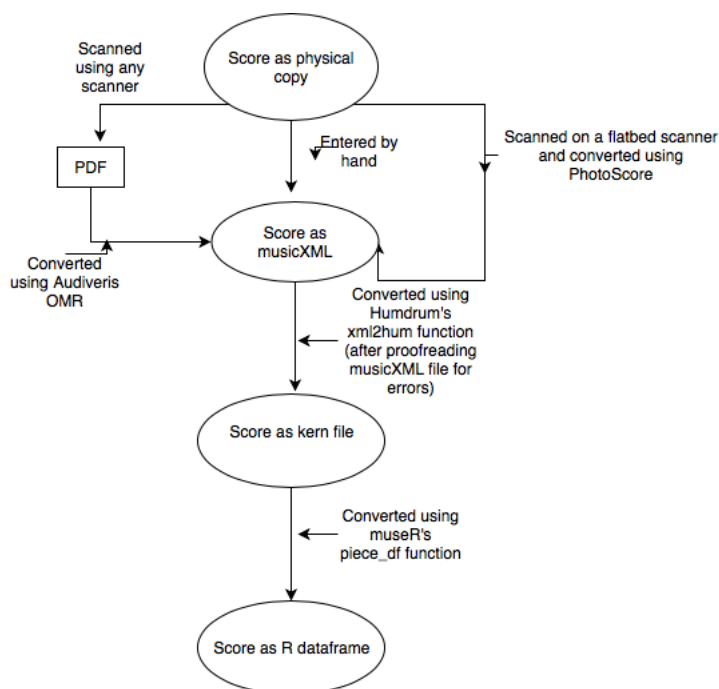


Figure 2.1: Flowchart of the conversion process from physical copy to dataframe

digital format in to R where subsequent analysis can be done. The scores used in this paper were obtained from physical copies available in the Reed music library. These scores were then scanned using software designed for optical music recognition (OMR).

Optical music recognition requires learning from graphical and textual information. The main things the software must pick up are the locations of bar lines, notes, rests, slurs, dynamic markings, tempo markings, lyrics etc. Basic optical music recognition has been around since 1966.

Most commonly, the first step in optical music recognition is to remove the staff lines. The staff lines are critical, as they define the basis for the vertical definition distance of pitch, and the horizontal distance definition of rhythm. The staff gives a normalization that is helpful, essentially defining the size of what notes and rhythm look like. Staff removal methods include projections, histograms, run lengths, candidates assemblage, contour tracking, and graph path search. (Doermann, Tombre, & others, 2014)

The next step is music symbol extraction and classification. These methods include template matching, where the object in question is compared to existing known musical symbols, simple operators, such as analysis of bounding boxes and projections, joining graphical primitives, such as combining extracted objects such as notes, note heads, and note beams to connect them in a musically correct way to form chords etc. Other methods use statistical models for analyzing musical primitives (the objects the OMR is trying to classify) such as Neural Networks, Support Vector Machines, k-Nearest Neighbor, and Hidden Markov Models.

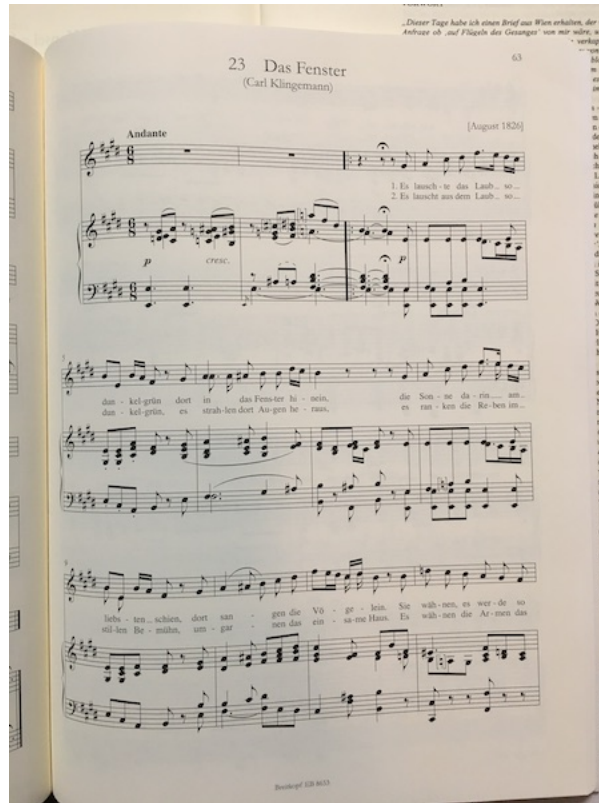


Figure 2.2: Score in physical form before conversion process has started

The next step OMR performs is syntactical analysis and validation. This step essentially uses defined grammars describing the organization of music notation in terms of music symbols. This makes the classification problem simpler, as there are existing rules and relationships between musical symbols.

The two OMR softwares used in this paper were PhotoScore and Audiveris. Each has its own benefits and issues. Photo Score works by scanning the physical score on a flatbed scanner at a high resolution. It then uses OMR techniques to output a musicXML file that can be read in by most music composing software, such as Sibelius, Finale or Muse Score.

Audiveris in contrast works by inputting a high resolution PDF and then uses OMR techniques to output a music XML file. Often, high enough resolution PDFs do not exist, so the physical scores must also be scanned by any garden variety scanner. MusicXML is commonly used as a format for digital music, as it is conducive to representing sheet music and music notation, and it can be transferable to many different music software. Muse Score was chosen to be the music software for viewing digital scores, as it is a free software that can read MusicXML.

After being scanned by PhotoScore and read into Muse Score, each piece was proof-read and corrected. This involves looking through every piece line by line for each bar to “spell check” the digital version. PhotoScore did a good job recognizing notes, but often had issues recognizing rhythms, and had issues keeping the structure of the piece. Often in the scanning process clefs or bar lines were not found, causing PhotoScore to

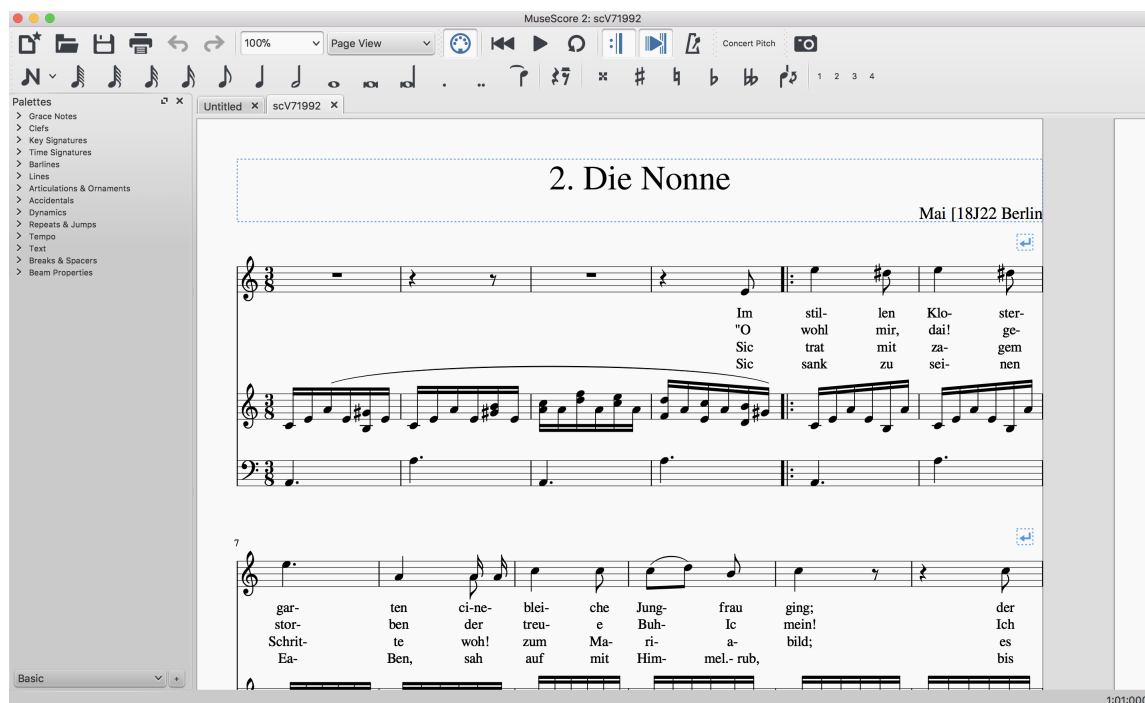


Figure 2.3: A score in MuseScore format after being read by an OMR like PhotoScore or Audiveris

output every staff on one line. Using PhotoScore, there were on average approximately 6 minor issues per piece, although there were on occasion major structural issues. Audiveris often added extra beats to measures. It also always identified a bass clef as a baritone clef.

Unfortunately, the scanning process is very lengthy and time consuming, as the scanning often gives a large number of mistakes. Often the score must be then scanned again. In addition the proof-reading process is lengthy. One must check each note and theme for errors against the original score, and change the afflicted notes using Muse Score. The corrected score must then be output as a musicXML file. In addition, there were some pieces that PhotoScore or Audiveris had a hard time reading. These pieces were then entered into MuseScore by hand and then proofread.

MusicXML on its own is not conducive to converting into a data frame as representing the single half note middle C looks like this: We then need to convert into a format more easily readable into R. The Kern Score music format is much more easily readable. It has clearly expressed time signature, bar, beat and musical voicing information (Mearns et al., 2010). Because of this, it is also more conducive to being read into an R data frame. The below picture shows how a basic piece of music corresponds to a .krn file. Kern files are organized with columns each representing one staff of music. Each line of a Kern file represents one note of one value of a time base. The time base for a Kern file is based on the smallest (shortest) rhythm value of a note found in a piece. For example, if a piece was in 4/4 and there were sixteenth notes present there would be 16*4 rows for each measure. The “attack” of each note

```

<?xml version="1.0" encoding="UTF-8" standalone="no"?>
<!DOCTYPE score-partwise PUBLIC
"-//Recordare//DTD MusicXML 0.5 Partwise//EN"
"http://www.musicxml.org/dtds/partwise.dtd">
<score-partwise>
  <part-list>
    <score-part id="P1">
      <part-name>Music</part-name>
    </score-part>
  </part-list>
  <part id="P1">
    <measure number="1">
      <attributes>
        <divisions>1</divisions>
        <key>
          <fifths>0</fifths>
        </key>
        <time>
          <beats>4</beats>
          <beat-type>4</beat-type>
        </time>
        <clef>
          <sign>G</sign>
          <line>2</line>
        </clef>
      </attributes>
      <note>
        <pitch>
          <step>C</step>
          <octave>4</octave>
        </pitch>
        <duration>4</duration>
        <type>whole</type>
      </note>
    </measure>
  </part>
</score-partwise>

```

Figure 2.4: A half middle C in musicXML encoding

is the only note printed, the following time while the note is held is represented with dots in the remaining rows until a new note is sounded for that staff. The pitch of each note is represented by the letters a through g. The case (lower or upper) as well as the repetition (c or ccc) represents which octave the pitch occurs. Any accidental is represented with a #, -, or n symbol. Each instrument/staff in a piece is represented using one (or more) columns called splines. For example, most lieder consist of voice and piano. There are thus three splines, one for voice, one for the treble clef staff of the piano, and one for the bass clef of the piano. In addition, there are splines that contain the text for the voice for the corresponding notes. This was not of interest to the musical classification problem, so these splines were removed. Chords are represented by multiple notes on the same line. For example, if there was a half note C major triad followed by a quarter note D flat, it would be represented as

```

2c 2e 2g
4d- . .

```

In addition, there is a lot of information about the appearance of the piece, stem direction etc for notes, but this was decided as not important for determining style, so it was removed.

We convert to kern format by using Humdrum's function `xml2hum` that converts a musicXML file into a kern file. Humdrum is a computational music software used to analyze music. It is a command line tool that has many functions for music analysis. The Kern file type can be read much more easily into R. Compared to above, the code for a single middle c whole note would be :

```

**kern
*clefG2
*k[c]
*M4/4

```

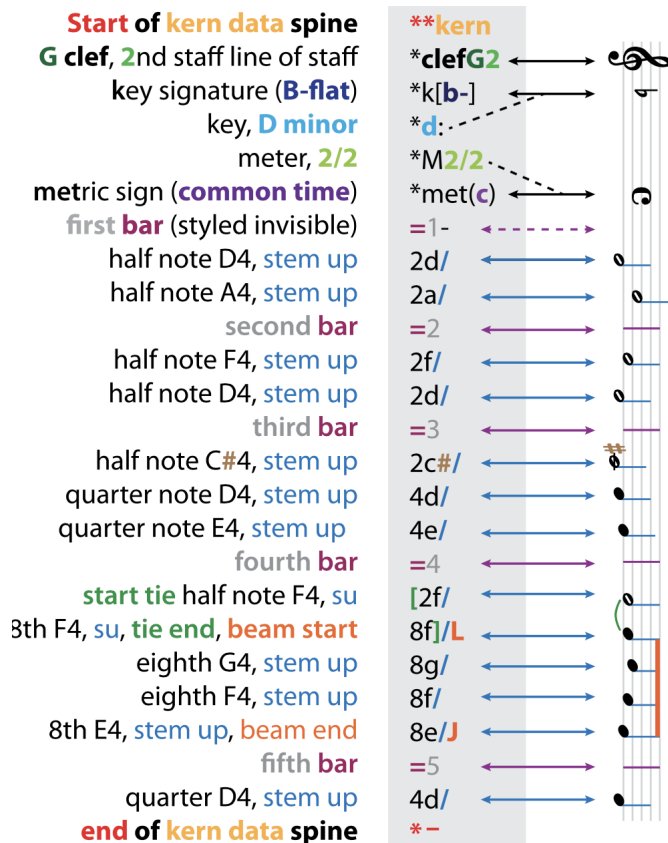


Figure 2.5: How sheet music corresponds to kern files

=1-
1c/

The `import_xml_files.sh` file goes through the process of converting scores from musicXML to .krn. Each spline needs to be individually converted using `xml2hum`. The individual splines are then converted into the same time base (there are issues when the bass line only has half notes and the soprano line has a lot of 32nds). This conversion essentially adds dots as placements so that the splines can line up correctly by measure and beat.

The CCARH has a large data base containing work mostly Baroque and Renaissance composers already in the Kern format, which is where the Bach data came from.

The files that were scanned (ie all pieces by Felix and Fanny) need to be separated into separate file for each staff, ie a separate file for each instrument. In addition, since we are focused on musical style, the text of the pieces is removed in this stage. In the case of analyzing lieder, each piece always has two or three files. These files consist of voice, piano right hand, and piano left hand. This is necessary to avoid the bugs in `xml2hum` that have issues when staves don't necessarily match up as a result of the conversion process. This is often caused by an inconsistency in time base.

2.3 .krn to R

Once we have .krn files to represent each piece we use regular expressions to extract key information. For scanned music (Felix and Fanny music), there are as many files as there are staves, usually three. MuseR's `krn2df()` and `piece_df()` functions read in .krn files and output a data frame in R for each piece. First the data in .krn format are read in line by line using R's `readLines()` function. This takes every line of the .krn file and converts it into a vector. Each entry contains the rhythm value and note value for all notes in that line. If there are multiple notes played at the same time, they are all in one line. The notes are separated by splitting up the string by spaces. This converts a single string representing one Kern line to multiple strings each representing one note (or dot placeholder) for one Kern line. Then each entry is separated out into the theme and note value for each note. Each line contains the following columns: the measure the note occurs in, the rhythm value for the note (for example 4), note name, octave inclusive (for example cc), note name (octave exclusive)[C sharp]. In addition, for the whole piece the key signature and meter are recorded as columns. If there are 3 splines and each spline has at most one note at a time, there would thus be $3 + 3 * 3 = 12$. If there are 3 splines and one of the splines has at most 3 values, that is equivalent to having 5 total splines there are then $3 + 3 * 5 = 18$ columns.

A lot of data included in the .krn files are not necessary. For example, we assume that whether or not a note has a stem up or stem down offers no help in classifying composer style, so this information is removed when converting to an R data frame.

Inspired by the .krn file type, each row of the R data frame contains one time base value. For a given piece, the time base represents the shortest note duration value. For example, if the shortest note a piece contained was a sixteenth note, the time base would be 16. Each measure then would contain 16 rows. This results in many rows of NA for certain instruments, when a note is still being voiced, but it is not the instance of the note being attacked.

Chapter 3

MuseR and features

To the best of my knowledge, there is currently no package of R that has been built to analyze sheet music. There are existing packages (such as `tuneR`) that examine audio formats of music. The intention of this thesis was to create a package, `museR`, that imports sheet music in the proper form (musicXML or Kern) and does all of the analysis using R.

3.1 Importing data into R

MuseR is equipped to import data in the Kern format. The functions for converting these files are `kern2df()` and `piece_df()`. These are most usefully in the form of individual splines. This allows for naming the columns according to instrument. Below we have a short piece appearing as it would look from MuseScore. This piece would

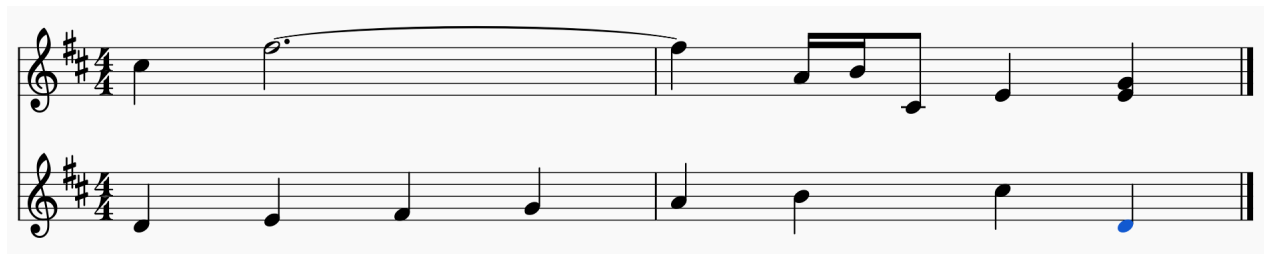


Figure 3.1: Example musescore format excerpt

have the following Kern format representation: Using `museR`'s `piece_df()` function to create the following data frame. Data in R are commonly expressed as data frames. Expressing music as a data frame has challenges, as music cannot be easily expressed in rectangular form. Music as a data structure is very rugged. When expressing it in rectangular form, there needs to be placeholder or padding entries to account for the nonrectangularness. `MuseR`'s `piece_df()` works by using regular expressions to extract note and rhythm information. It uses `NA` and `.` values to indicate empty spaces and duration respectively.

```

1  **kern  **kern  **kern
2  *staff3 *staff2 *staff1
3  *I"Bass *I"Voice *I"Voice
4  =1- =1- =1-
5  *clefF4 *clefG2 *clefG2
6  *k[f#c#] *k[f#c#] *k[f#c#]
7  *M4/4 *M4/4 *M4/4
8  4D\ 4d/ 4cc#\
9  4E\ 4G\ 4e/ [2.ff#\
10 4F#\ 4A\ 4f#/ .
11 4F#\ 4g/ .
12 =2 =2 =2
13 1G 4a/ 4ff#\]
14 . 4b\ 16a/LL
15 . . 16b/J
16 . . 8c#/J
17 . 4cc#\ 4e/
18 . 4d/ 4dd/ 4e/ 4g/
19 == == ==
20 *- *- *-

```

Figure 3.2: Example kern format excerpt of the above musescore

3.2 Features currently supported in museR

Melodic intervals

Melodic intervals, or the interval between two successive notes, are found using the `mel_ints()` function. It is currently only equipped to look at melodic intervals for the top note of each staff. In this context, it is most commonly used for analyzing melodic intervals of the voice. The function first extracts the top line of any instrument, and then outputs the proportion of each melodic interval happening over the whole piece. There are 12 possible intervals that are counted (ignoring augmented and diminished): unison, m2, M2, m3, M3, p4, tt, p5, m6, M6, m7, M7. `mel_ints()` outputs a vector of the proportion of each of the intervals. For example if this function was run on the above piece, the melodic top line intervals would be: $\{(f, c), (c, d), (d, f)\} = \{p4, M2, m3\}$, which would output the proportion vector $(0, 1/3, 1/3, 0, 0, 1/3, 0, 0, 0, 0, 0)$.

If we are interested in the types of melodic intervals, we can use the `consonance()` to examine the proportion of consonant (perfect, imperfect, dissonant) intervals over the piece. This function works by calling `mel_ints()` and then adding up the perfect, imperfect, and dissonant intervals proportions.

| | piece _key 1 | piece _meter 1 | piece _measure 1 | piece _r.v 1 | piece _r.n 1 | piece _n.o 1 | piece _n.n 1 | piece _r.v 2 | piece _r.n 2 | piece _n.o 2 | piece _n.n 2 |
|----|--------------------|----------------------|------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 1 | *k[f#c#] | *M4/4 *M4/4 | 1 | 4 | Quarter note | d | D | 4 | Quarter note | cc# | C# |
| 2 | *k[f#c#] | *M4/4 *M4/4 | 1 | 4 | Quarter note | e | E | 2. | Tbd | ff# | F# |
| 3 | *k[f#c#] | *M4/4 *M4/4 | 1 | 4 | Quarter note | f# | F# | NA | NA | NA | NA |
| 4 | *k[f#c#] | *M4/4 *M4/4 | 1 | 4 | Quarter note | g | G | NA | NA | NA | NA |
| 5 | *k[f#c#] | *M4/4 *M4/4 | 2 | 4 | Quarter note | a | A | 4 | Quarter note | ff# | F# |
| 6 | *k[f#c#] | *M4/4 *M4/4 | 2 | 4 | Quarter note | b | B | 16 | Tbd | a | A |
| 7 | *k[f#c#] | *M4/4 *M4/4 | 2 | NA | NA | NA | NA | 16 | Tbd | b | B |
| 8 | *k[f#c#] | *M4/4 *M4/4 | 2 | NA | NA | NA | NA | 8 | Tbd | c# | C# |
| 9 | *k[f#c#] | *M4/4 *M4/4 | 2 | 4 | Quarter note | cc# | C# | 4 | Quarter note | e | E |
| 10 | *k[f#c#] | *M4/4 *M4/4 | 2 | 4 | Quarter note | d | D | 4 | Quarter note | dd | D |

Figure 3.3: Example R data frame converted from the above MuseScore

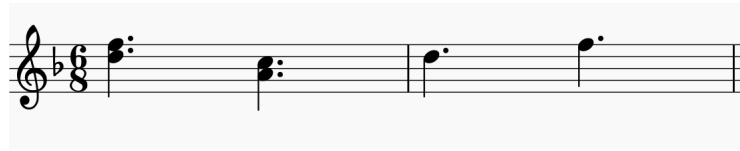


Figure 3.4: Example of calculating proportion of melodic intervals

Density

The `beat_density()` function analysis the average and standard deviation of density of each measure in the piece. It is called “beat” density as it only accounts for the instance a note starts. For example if a measure consisted of a single whole note it would be only counted once even though it is voiced the entire measure.

In the above example, the first measure would have a beat density of 4, and the second measure would have a beat density of 2.

Major_minor

For most musical analysis, the key of the piece is important in determining chords, etc. The key is based on the key signature, which is always given in a Kern file. Kern files from CCARH have the key of the piece given, but scanned files do not. For kern files from CCARH, `Major_minor()` extracts the key given by the Kern file. for scanned files, `Major_minor()` identifies the two options for key given the key signature. For example, if there was a key signature with one sharp, the options would be G Major or E minor. The tonic for each option is identified, and then the count of instances of both choices for tonic is made. The key is determined by which of the options for tonic has the higher count.

Prop scale degree

Once the key of a piece is determined, the proportion of each scale degree is calculated. The scale degrees consist of : Tonic, Supertonic, Mediant, Submediant, Dominant, Submediant, and Leading tone. In the example above, if we assume the key is F major,

the tonic has a tonic scale degree proportion of $2/6$, the mediant $1/6$, dominant, $1/6$, submediant, $2/6$.

Length

We determine a measure for the length of a piece by how many measures the piece has.

Chapter 4

About Classification Models

Classification problems attempt to divide observations (features) into groups (composer) based on similarities in the observations. A classifier is some function f that maps a vector of input features $x \in X$ to a composer $y \in Y$ or possibly a probability of a composer $P(Y = y) \in [0, 1]$. The notation used in this chapter is inspired by *The Elements of Statistical Learning* (Friedman, Hastie, & Tibshirani, 2001) and *An Introduction to Statistical Learning* (James, Witten, Hastie, & Tibshirani, 2013). Our features space X is an $n \times p$ matrix, where n is the size of our data, and p is the number of predictors. Each X_i is vector of values for a certain feature. x_{ij} denotes the i^{th} values of the j^{th} feature. This X contains the information of the extracted features that hopefully will be useful in determining the composer of the piece. If the features are different enough between the composer, ie, that the features encode some sense of unconscious (or conscious) use that differs between composer, we can then build and fit models that can use how the features differ between composers. These models can both explain the relationship and difference of the features between composers, and use the use the way the fitted model explains the differences to predict the composer of a piece if we know the same features for that piece.

In addition to the features for each piece, each piece has a composer (response), known or unknown, that we denote by Y . The i^{th} piece has composer Y_i where $i \in 1, \dots, n$. In our case we have $Y \in \{\text{Fanny, Felix, Bach}\}$, or more generally, $y \in \{\text{list of composers}\}$. Since the options for composer are in a discrete set, we can divide the input features space into different groups, or regions, that are labeled according to the classification of composer a model assigns or predicts.

4.1 Logistic Regression

Knowing the class posteriors $P(Y = k|X)$, or the probability that a piece has a certain composer, given the features of that piece, gives us an optimal classification. Logistic regression directly models $P(Y = k|X)$ by using the logistic function. The idea is to model the posterior probabilities of each of the K classes as linear functions in x and requiring that the probabilities sum to 1. As is often the case, we use logistic regression to model a binary response: where there are two options for composer.

Thus in the case of a binary response we can use an indicator function with coding 0/1. We then name $p(X) = P(Y = 1|X)$. The model has the form:

$$\log \left(\frac{p(X)}{1 - p(X)} \right) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p$$

where which can be written as:

$$p(X) = \frac{e^{\beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p}}{1 + e^{\beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p}}$$

We estimate the regression coefficients by using maximum likelihood. This results in coefficient estimates such that for the predicted probability $\hat{p}(x_i)$ for the composer of each piece corresponds as closely as possible to the observed composer. The log-likelihood for N observations is:

$$\ell(\beta) = \sum_{i=1}^N \left\{ y_i \log p(x_i; \beta) + (1 - y_i) \log(1 - p(x_i; \beta)) \right\} = \sum_{i=1}^N \left\{ y_i \beta^T x_i - \log(1 + e^{\beta^T x_i}) \right\}$$

We then choose β to maximize $\ell(\beta)$

4.1.1 Lasso model selection

The Lasso penalty is a shrinkage method proposed by Robert Tibshirani in 1996. Lasso regression works by giving a penalty to regression coefficients. It can be somewhat equivalent to performing variable selection, as for high enough penalties (or λ), coefficients shrink to zero. It is often used in linear regression, but can be expanded to logistic regression and other generalized linear models. For logistic regression, the lasso works by choosing coefficients β_λ^L that minimize

$$\ell(\beta) + \lambda \sum_{j=1}^p \beta_j$$

where $\ell(\beta)$ is the log-likelihood function for logistic regression. This is equivalent to maximizing $\ell(\beta)$ subject to $\sum_{j=1}^p |\beta_j| < s$ (Tibshirani, 1996)

4.2 Linear Discriminant Analysis

Whereas logistic regression involves directly modeling $P(Y = k|X = x)$, linear discriminant analysis (LDA), estimates these values less directly by using Bayes Theorem. When classes are well separated or if n is small and the distribution of predictors is approximately normal, logistic regression estimates can be unstable, which is not the case for LDA. LDA is also popular when there are more than two response cases.

To perform LDA we must first model the distribution of each of the features that make up X in each of the response classes ($P(X = x|Y)$). We notate $f_k(X) = P(X =$

$x|Y = k$) as the class-conditional density of X in class $Y = k$. We denote the prior for class k , π_k , or the probability that a chosen observation is from the k^{th} class. We have that $\sum_{k=1}^K \pi_k = 1$. Using Bayes' theorem to calculate $P(Y = k|X)$ gives us the following:

$$P(Y = k|X = x) = \frac{f_k(x)\pi_k}{\sum_{l=1}^K f_l(x)\pi_l}$$

We thus must have a model to find $f_k(x)$. Different discriminant analysis techniques do this different ways. LDA assumes a multivariate Gaussian density, given by:

$$f_k(x) = \frac{1}{(2\pi)^{p/2} |\Sigma_k|^{1/2}} e^{-\frac{1}{2}(x-\mu_k)^T \Sigma_k^{-1} (x-\mu_k)}$$

, where μ_k is the mean parameter for the k th class and Σ_k is the covariance matrix for the k th class.

In addition, LDA assumes that the covariance matrix is equal for every k : $\Sigma_k = \Sigma \forall k$. Other discriminant models do not make this assumption. We also assume $\hat{\pi}_k = N_k/N$ where N_k is the number of class - k observations,

Using the formula for $P(Y = k|X = x)$ as stated above, we can use LDA's assumption of $f_k(X = x)$ which results in the discriminant function:

$$\delta_k(x) = x^T \Sigma^{-1} \mu_k - \frac{1}{2} \mu_k^T \Sigma^{-1} \mu_k + \log \pi_k$$

These functions are known as *linear discriminant functions*. We predict the class by finding the maximum value of the discriminant functions of all k .

4.3 Naive Bayes

The Naive Bayes classifier is often used for musical classification as it is good when the dimension p of the features space is large. Like LDA, it also involves modeling $P(Y = y|X = x)$ by using assumptions of the form of $f_i(X)$ and using Bayes Theorem. Naive Bayes makes the (naive) assumption that all the features are independent for a given class i , ie that

$$f_i(X) = \prod_{k=1}^p f_{ik}(X_k)$$

. These marginals are often estimated by using a Gaussian distribution, ie that

$f_{ik}(X_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} e^{-\frac{(x-\mu_k)^2}{2\sigma_k^2}}$ In practice the independence assumption is not the case, but the model still performs surprisingly well in practice when this assumption does not hold.

4.4 K- nearest - neighbor

Another method for classification is k -nearest neighbor methods. It uses observations from a training set of the data. Then for an new observation in a testing set at point

$x_0 = x_1 \dots x_p$, it finds the K closest points in the training set, and classifies x_0 as the majority vote of the responses for the K neighbors. Euclidean distance is often used as the metric for closeness, although other methods exist. Euclidean distance is defined as $d = |x_{(i)} - x_0|$. After the K nearest neighbors are found, the predicted class for x_0 is assigned to be the mode of the classes of the neighbors.

4.5 Random Forests

Tree-based methods are another form of classifier. Tree based methods involve segmenting the predictor space into smaller regions. To classify a point x_0 , we take the mode of the classes of training set observations in the smaller region where x_0 lies, and assign the class of x_0 as the mode. To create the tree, we recursively split the predictor space into two smaller regions at the point where there is high node purity. The Gini index is often used to measure this, defined as $G = \sum_{k=1}^K \hat{p}_{mk}(1 - \hat{p}_{mk})$, where \hat{p}_{mk} is the proportion of observations in the m th region of the k th class. If the Gini index is small, this indicates that a region contains mostly observations from a single class.

Random forests offer an improvement over trees in general. They create a “forest” of decision trees on bootstrapped training samples. In each of the trees in the forest, a random sample of m predictors are chosen to create the tree. For random forests, $m = \sqrt{p}$ typically. Because strong predictors are often chosen for the first split of the tree, choosing only a random sample of the predictors to build the tree makes the trees in a Random forest significantly different from each other. This choice helps in decorrelating the trees.

4.6 Dimensionality Reduction: Principal component analysis (PCA)

Principal component analysis (PCA) transforms the features space into a lower dimensional representation. It chooses the transformed features to have maximal variance and be mutually uncorrelated.

Principal component analysis can be useful when the predictors are correlated. We suspect many of our features are correlated, due to certain patterns in music, as well as the way we created our features. These relationships are caused by similarity in the features, and from music theory rules. (For example, if there is a high frequency of first scale degrees, we might expect a high frequency of chords that include the first scale degree. Another example, if we had a high frequency of seventh scale degrees, we would expect them to resolve to the first scale degree.)

Principal component analysis is also helpful when there are many predictors, and we want to deal with a smaller dimension of predictor space.

As an unsupervised method, PCA can inform about latent meta variables.

Used in supervised methods, the transformed features from PCA can be used to fit models instead of the original features space.

Principal components transforms the feature space. If our original features are X_1, X_2, \dots, X_p , we transform the features to Z_1, Z_2, \dots, Z_M , where $M < p$. Each Z_i is a linear combination of the original predictors, ie, $Z_m = \sum_{j=1}^p \phi_{jm} X_j$, for constants $\phi_{1m}, \phi_{2m}, \dots, \phi_{pm}$ for $m = 1, \dots, M$. Given an $n \times p$ data set \mathbf{X} where x_{ij} is the i^{th} instance of the j^{th} feature, we solve for the m^{th} principal component loading vector $\phi_m = \phi_{1m}, \phi_{2m}, \dots, \phi_{pm}$ that solves the optimization problem:

$$\max_{\phi_{1m}, \dots, \phi_{pm}} \left\{ \frac{1}{n} \sum_{i=1}^n \left(\sum_{j=1}^p \phi_{jm} x_{ij} \right)^2 \right\}$$

, where the ϕ s are subject to $\sum_{j=1}^p \phi_{jm}^2 = 1$. Our principal components are then calculated as $z_{im} = \sum_{j=1}^p \phi_{jm} x_{ij}$

The loadings of the first principal component, ϕ_1 thus determine the direction in the feature space with the most variance, Z_1 , or the scores of the first principal component is then a new feature in our transformed feature space. We continue calculating Z_i , where each following Z_i has the maximal variance in a direction uncorrelated to the previous principal components.

Before PCA is performed, we center all features to have mean zero, as the scale of some features are not the same, which will lead to issues in the loadings, as the features with higher scales would automatically have the higher variance.

We can observe the proportion of variance explained by each principal component. This is usually visualized in a skree plot. We can use this information to decide how many principal components to use.

4.7 K-means

K-means is a form of unsupervised learning where we only use the features without the associated class of composer. When given a *K* or number of clusters, the algorithm assigns each piece to a cluster. This is used to see if there are any latent groupings in the feature space. If composers are differentiable with their features, we might expect that *K* means would differentiate the clusters similar to the difference in composer. *K*-means clustering works by trying to minimize the within-cluster variation $W(C_k)$ for each cluster C_k . Often we define the within-cluster variation by squared Euclidean distance, so

$$W(C_k) = \frac{1}{|C_k|} \sum_{i, i' \in C_k} \sum_{j=1}^p (x_{ij} - x_{i'j})^2$$

where $|C_k|$ is the number of observations in the k th cluster.

Chapter 5

Exploratory Data Analysis

5.1 Key for feature abbreviations

Table 5.1: Abbreviations for features and corresponding meanings. The features of Fanny and Felix begin with f.

| name | meaning |
|--------|--|
| dens_. | Density of notes, mean and standard deviation |
| cons_. | Type of consonance, imp(imperfect), dis(dissonant), perf(perfect) |
| rf. | Rhythm frequency, in 2(half), 2d(dotted half), 4 (quarter), 4d(dotted quarter), 8(eighth), 8d(dotted eighth), 16(sixteenth) ,32(thirty second) |
| sf_. | Scale degree frequency, in 1 to 8 |
| len | Length of the piece |

5.2 Bach and Mendelssohns

Figure 5.1 shows density plots for each of the predictors used. We can see some difference of that feature being used by composer in the density features, and some of the rhythmic frequency features, especially use of eighth notes and sixteenth notes. The peaks for Bach are mostly narrower than those for Mendelssohn. This could be caused by We suspect the difference in density is somewhat caused by the type of piece of Bach and Mendelssohn. We might expect that solo pieces, like those in the Bach data set would have lower densities then for lieder, like in the Mendelssohn data set.

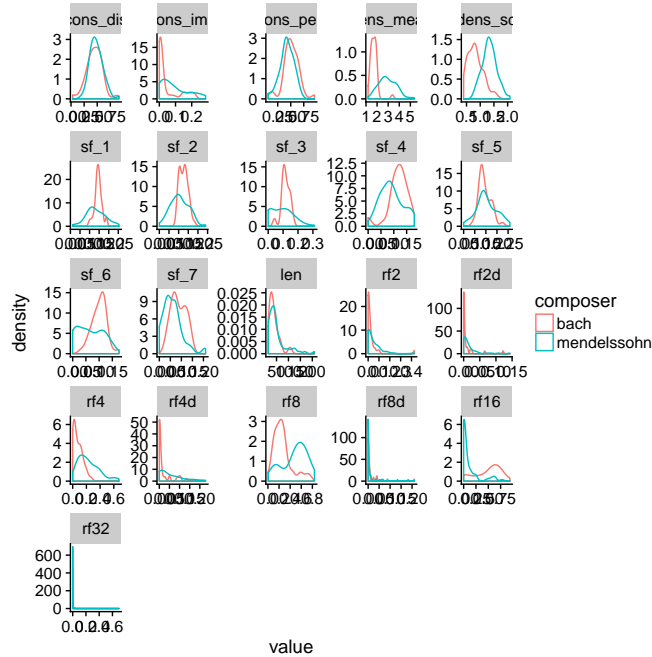


Figure 5.1: Density plot of each feature of Bach/Mendelssohn

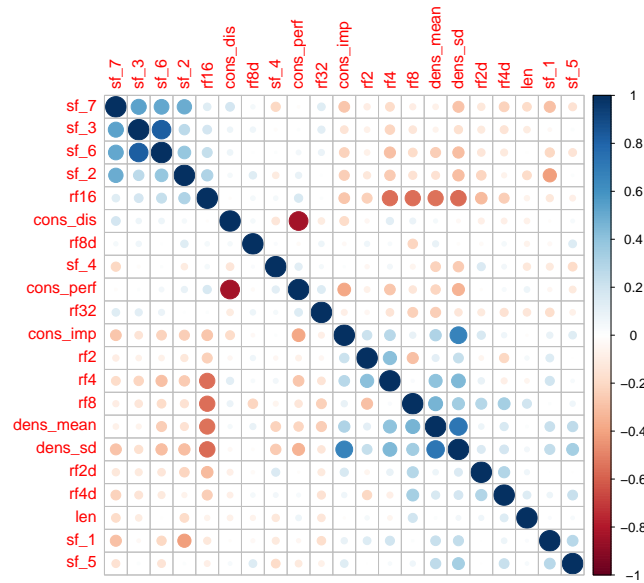


Figure 5.2: Correlations between features of Bach/Mendelssohn

Figure 5.2 shows a visualization of the correlation matrix. The larger the circle represents a larger absolute value of correlation between the two predictors. We can see higher correlations between perfect consonances and dissonant consonances. This is to be expected. We also see high negative correlations between frequency of sixteenth

notes and density, and frequency of eight notes. In addition we see high positive correlations between frequency of second, third, sixth, and seventh scale degree. There is likely some musical explanation to this.

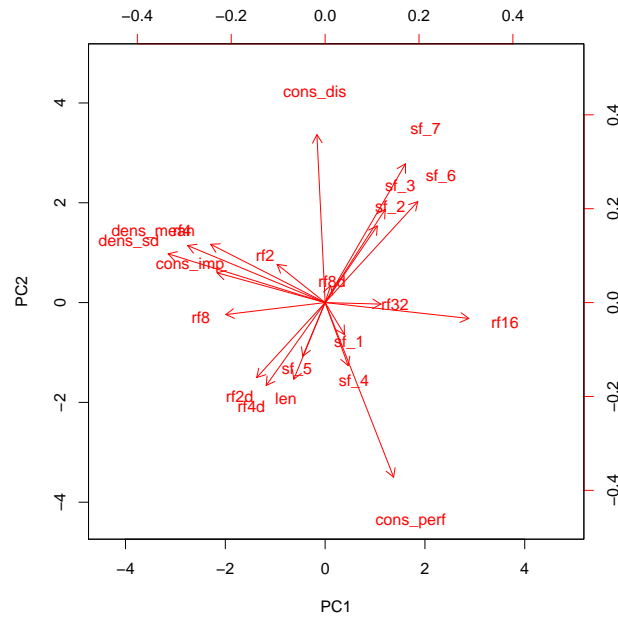


Figure 5.3: Biplot of the loading vectors of the first two principal components

The biplot in Figure 5.3 shows the loading vectors plotted on the first two principal components. The loading vectors of features seem to arrange into approximately three groups. The features: perfect consonances of melodic intervals of the voice, frequency of the first scale degree, frequency of the fourth scale degree are all grouped together. Similarly, frequencies for the 7th, 6th, 2nd, and 3rd are grouped together. In addition we have perfect consonances and imperfect consonances on two sides of the second principal component. This leads me to believe that the second principal component encodes some sense of use of consonant notes. The first principal seems to encode some sense of rhythm. Faster rhythmic notes, like sixteenth notes and thirty second notes have opposite loadings than slower rhythmic notes.

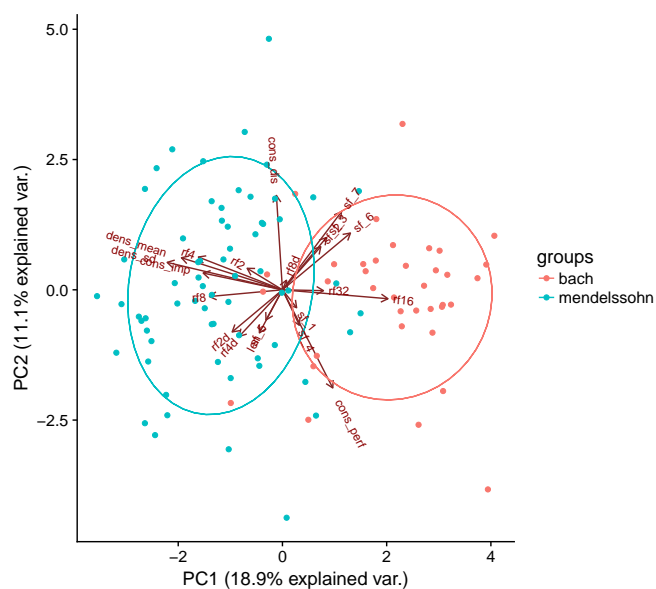


Figure 5.4: Biplot of the first two principal components plotted with data points colored by composer.

Figure 5.4 shows the same loadings of the principal components, but with the addition of points representing each piece graphed by their first two principal components. Each piece is colored by composer. The ellipses represent a 95% concentration area, or where 95% of the data lie. We can see that the ellipses only overlap slightly, indicating that pieces by Bach and Mendelssohn usually have different values in the first and second principal component. Figure 5.5 shows the pieces plotted on the second and third principal component and there is much less separation. There also does not seem to be any obvious musical interpretation of the loadings.

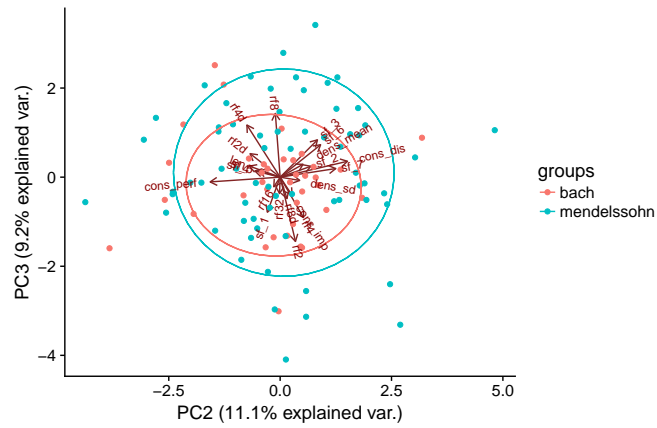


Figure 5.5: Biplot of the second and third principal components plotted with data points colored by composer

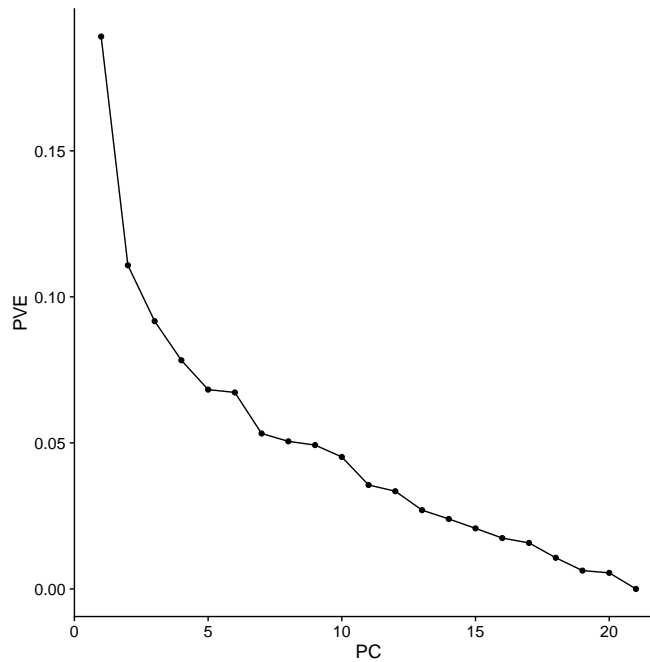


Figure 5.6: Skree plot of the PCA's

Figure 5.6 shows the skree plot of the PCs of Bach and Mendelssohn. For each principal component used (shown on the x-axis), we have the corresponding percentage of variance explained (y-axis). Often when using PCA for dimensionality, we often

look for an elbow in the skree plot. This is done to choose the smallest number of principal components needed to explain a sizable amount of the variation in the feature space. There is a possible elbow at the second principal component. This indicates that we might be able to perform analysis using only the first two principal components. However, especially considering that our data are not very separable, we likely would likely want to use more information in our model. As a similar result occurred for Brinkman, in our analysis we will use the principal components that account for 85% of the variance. In the Bach/Mendelssohn case this corresponds to using 11 principal components.

We can also use K-means clustering to see if there are any apparent groups in the data. When we run K-means on the first 11 principal components of Bach and Mendelssohn with $K = 2$ (since there are two composers), we get the results we see in Figure 5.7. The coloring of the points indicate which cluster the K-means algorithm identified each piece as. The label of each piece indicates the actual composer. We can see that for the most part, for pieces with high or low scores on the first principal component, the clustering preserves the composer. For pieces with middle scores of the first PC, there is a good number of pieces with a cluster assignment not consistent with composer.

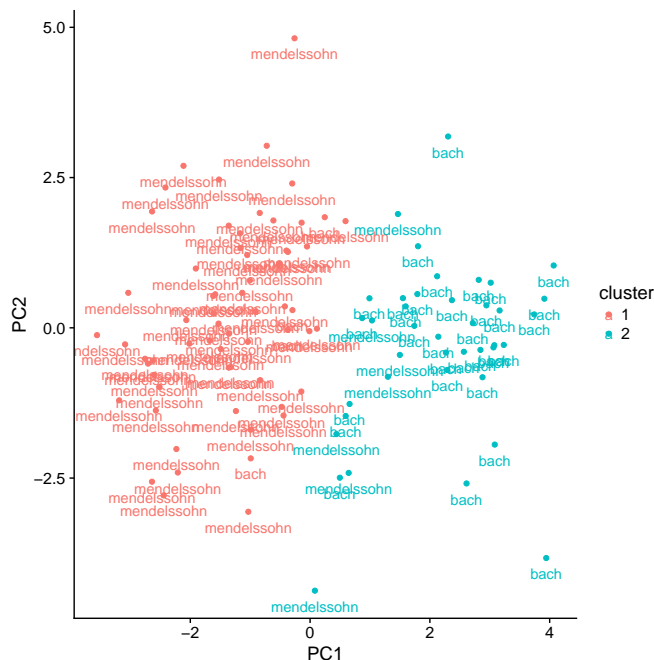
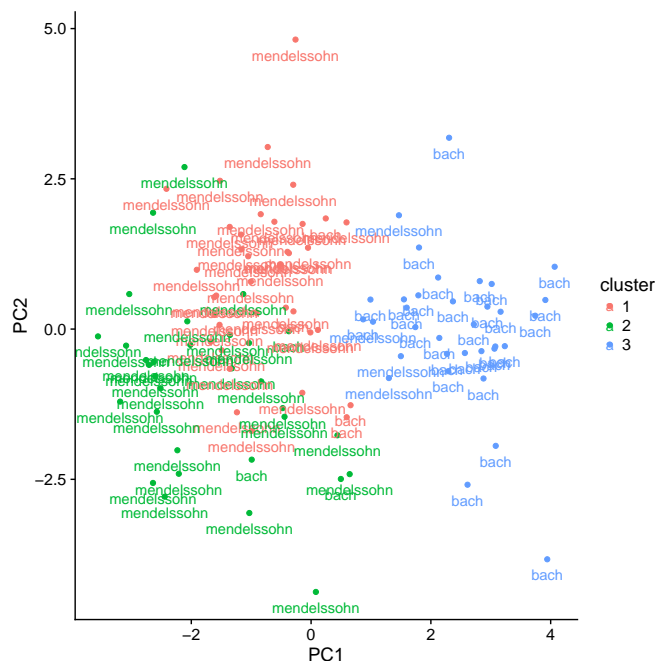


Figure 5.7: KNN when $k = 2$

We can also examine when $K = 3$, as the Mendelssohn set is made up of Fanny and Felix, so there are actually three composers. (fix labeling)

Figure 5.8: KNN when $k = 3$

5.3 Felix and Fanny

As we can see from density distributions in Figure 5.9, most of the features used have very little separation between the two composers. This is not very encouraging for later fitting of models.

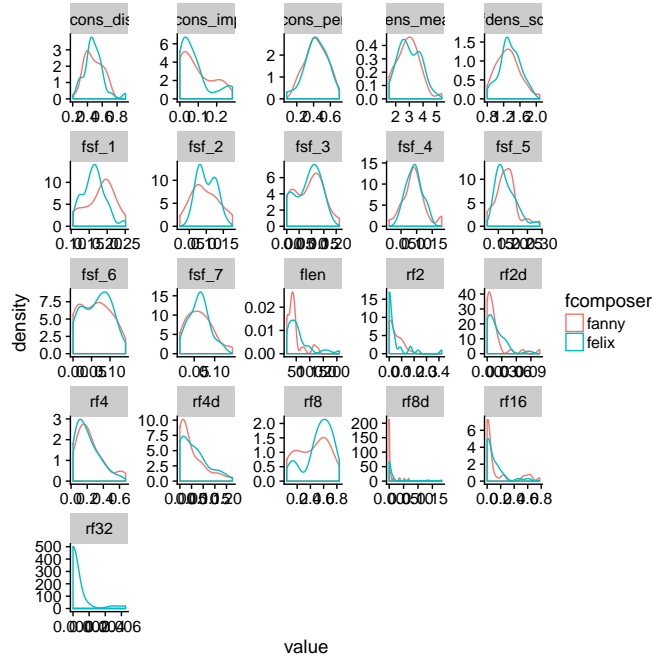


Figure 5.9: Density plot of each feature of Fanny/Felix

The correlations of the Fanny/Felix features are shown in Figure 5.10. The features appear to not be as correlated as the Bach/Mendelssohn features with the exception of frequencies of types of melodic consonances, and frequencies of scale degrees 2, 3, 6, and 7.

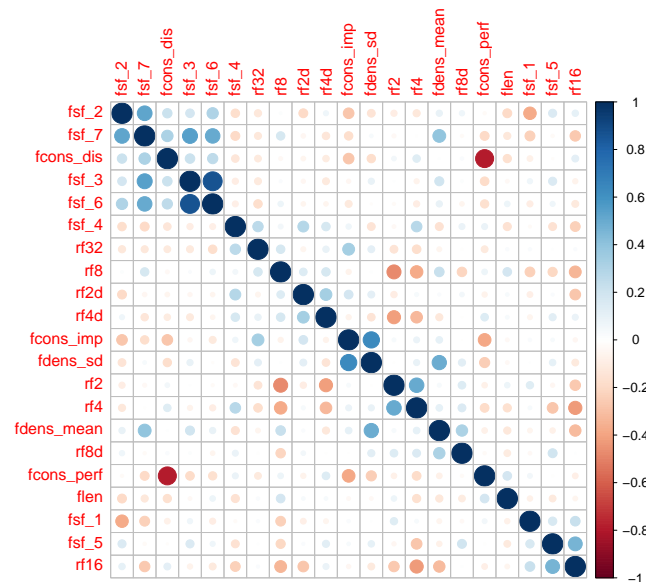


Figure 5.10: Correlations between features of Fanny/Felix

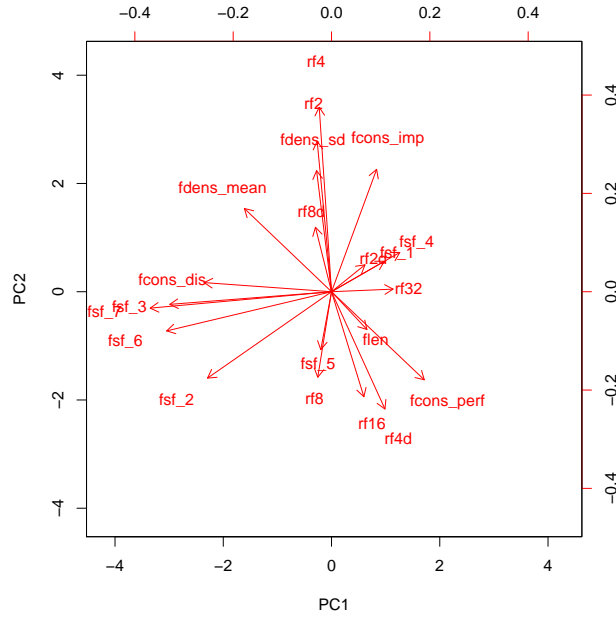


Figure 5.11: PCA Loadings of the features for Fanny/Felix

Figure 5.11 shows the loadings of the principal components of the features of Fanny and Felix. Again we see that the loadings for scale degrees 1 and 5 and perfect consonant intervals are in opposite direction on the second principal component to scale degrees 2, 7 and dissonant intervals. Also we see again that frequencies for faster rhythmic values have opposite loadings to those of slower notes.

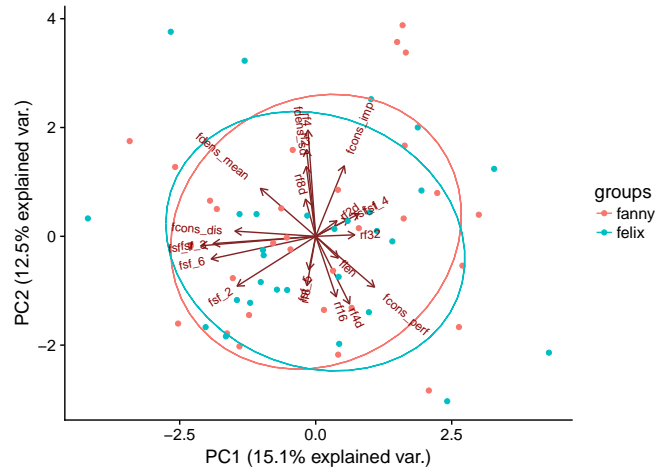


Figure 5.12: Loadings and pieces for Fanny/Felix

When the pieces are plotted on the biplot as shown in Figure 5.12, we see that there is not that much separation between the Fanny pieces and the Felix pieces. This might lead to issues in creating a model to classify.

Figure 5.13 shows the skree plot for Fanny/Felix. It can be argued that there is an elbow at principal component 13, but it is not that clear. Using 11 PCs accounts again for 85% of the variance, so we will use 11 PCs in models as well.

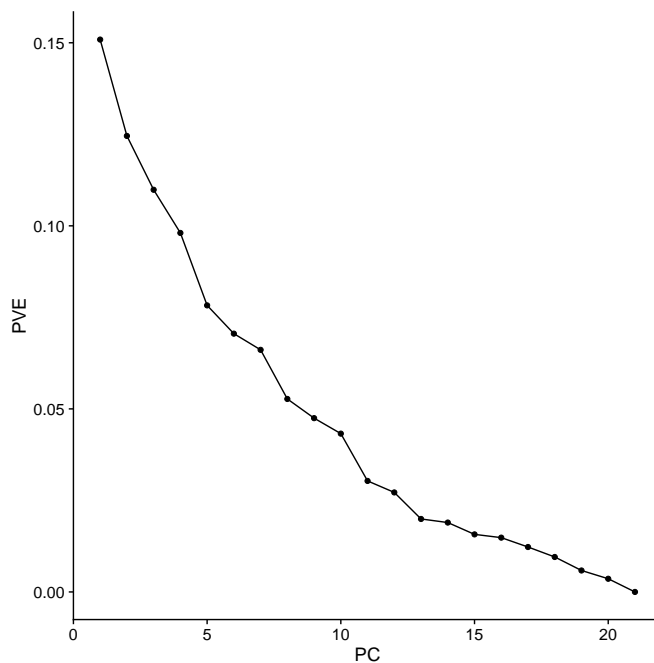


Figure 5.13: PCA Felix/Fanny skree plot

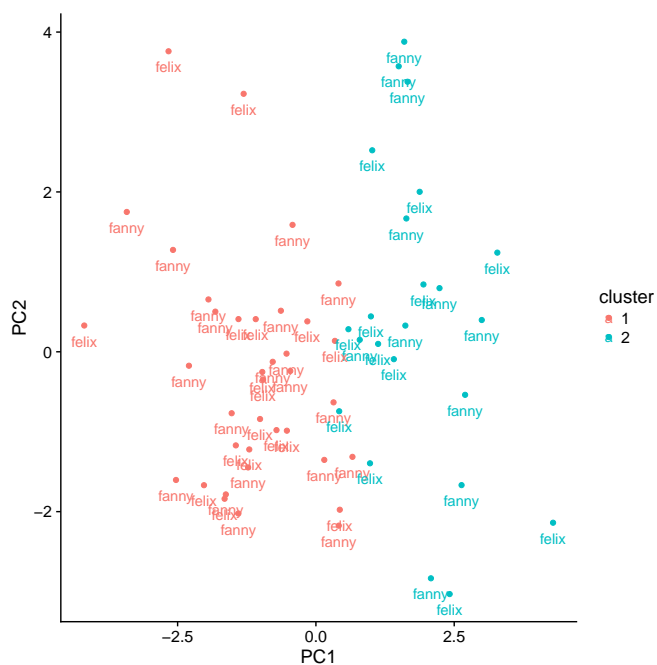
Figure 5.14: K-means with $k = 2$

Figure 5.14 shows the results of K-means with $K = 2$. We see that in the two clusters, there are many pieces labeled of both composer.

Chapter 6

Results

6.1 Models Results - Bach/Mendelssohn

Table 6.1: Averaged 5-fold CV misclassification rates of 100 runs on the feature space and the first 11 principal components for each model.

| Model | misclass. rate - features | misclass. rate - 11 PCs |
|---------------|---------------------------|-------------------------|
| Logistic | 0.078 | 0.097 |
| LDA | 0.092 | 0.049 |
| KNN | 0.097 (K=9) | 0.098 (K = 9) |
| Naive Bayes | 0.103 | 0.112 |
| Random forest | 0.058 | 0.111 |

Table 6.1 shows the averaged 5-fold cross validated misclassification rates over 100 runs. For the KNN case, the features were scaled and centered first.

Figure 6.1 shows the misclassification rates corresponding to different values of $\log(\lambda)$ in the logistic lasso model. As the $\log(\lambda)$ increases, the coefficients of each of the features start approaching zero. They do this at different times and rates. Figure 6.2 shows coefficient values for each feature for varying $\log(\lambda)$ values. It appears that the features for density and frequency of 16th note rhythms stay non-zero the longest. A 5-fold cross validation fit found that a $\log(\lambda)$ value of -6.2 produced the lowest misclassification rate, represented by the left most dotted line. A model fit using that lambda value resulted in a misclassification rate of 0.078. We also fit a lasso logistic model on the first 11 principal components. This resulted in an optimal $\log(\lambda)$ of -3.8, and a resulting misclassification rate of 0.097 Figure X shows the variable importance rankings as a mean decrease in the Gini index. The density features are the most important in deciding splits of the trees, followed by frequency of sixteenth notes and frequency of second scale degree.

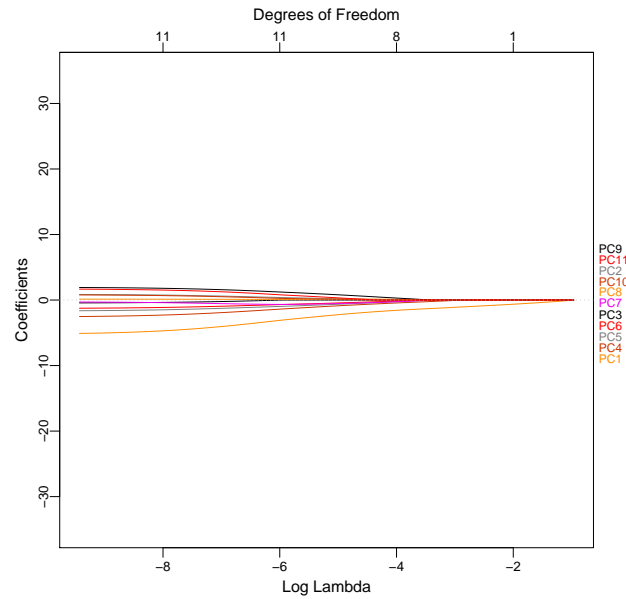


Figure 6.1: Lasso penalties for each feature for changing lambda penalty values

6.2 Model fit Felix/Fanny

Table 6.2: Averaged 5-fold CV misclassification rates of 100 runs on the feature space and the first 11 principal components for each model.

| Model | misclass. rate - features | avg. misclass. rate - 11 PCs |
|---------------|---------------------------|------------------------------|
| Logistic | 0.517 | 0.45 |
| LDA | 0.518 | 0.505 |
| KNN | 0.498 | 0.467 |
| Naive Bayes | 0.502 | 0.521 |
| Random forest | 0.535 | 0.571 |

Table X shows all the misclassification rates of each model. Unfortunately most of the misclassification rates are above 0.5, indicating that random guessing would do better at predicting the composer. Figure X shows the coefficient estimates for varying log lambda penalties for a logistic lasso model fit on the feature space. The frequency of the first scale degree appears to remain non-zero for the longest time. Figure X shows the misclassification rates for varying values of $\log(\lambda)$. Most rates are well above 0.5 (the error for a coin toss), although at $\log(\lambda)$ of -2.8 we have the lowest misclassification error of 0.517.

When fit on the first eleven principal components, we have a lower misclassification rate. Figure X shows the misclassification errors for varying $\log(\lambda)$. At -3.21 we have the lowest misclassification rate. Figure X shows the variable importance used in random forests. We can see that frequency of the first and second scale degree as well as the length of the piece are useful features in fitting the model. However, the scale of the Mean decrease in Gini is a lot smaller than the one for Bach/Mendelssohn, so these features are likely not as helpful in distinguishing the composers.

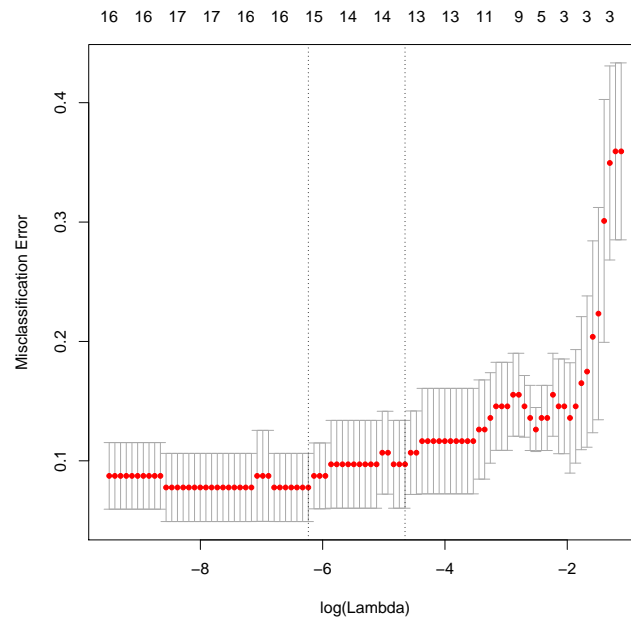


Figure 6.2: Cross-validated misclassification rates for different lambdas

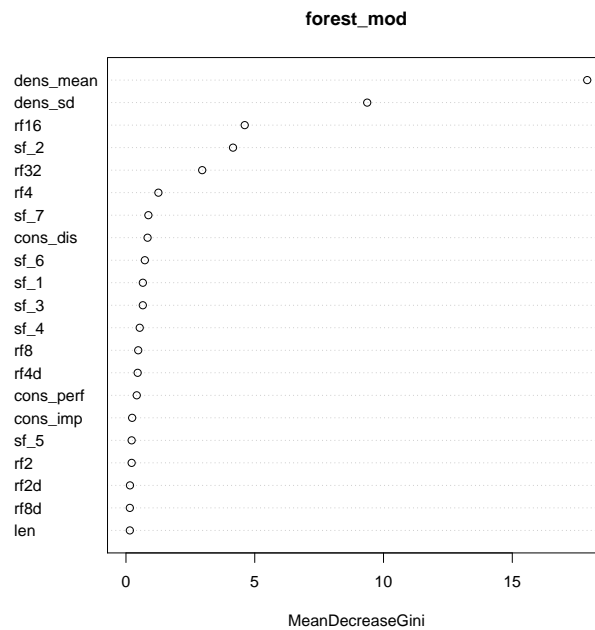


Figure 6.3: Variable importance for a random forest model Bach/Mendelssohn

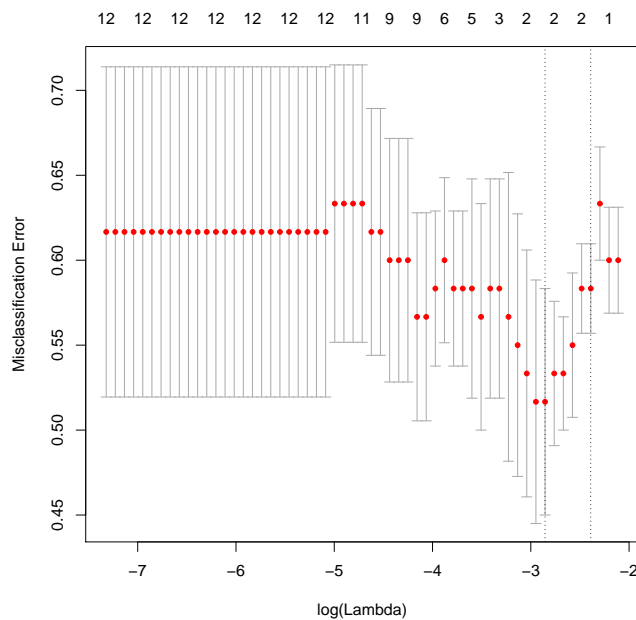


Figure 6.4: Lasso penalties for each feature for changing lambda penalty values for Fanny/Felix

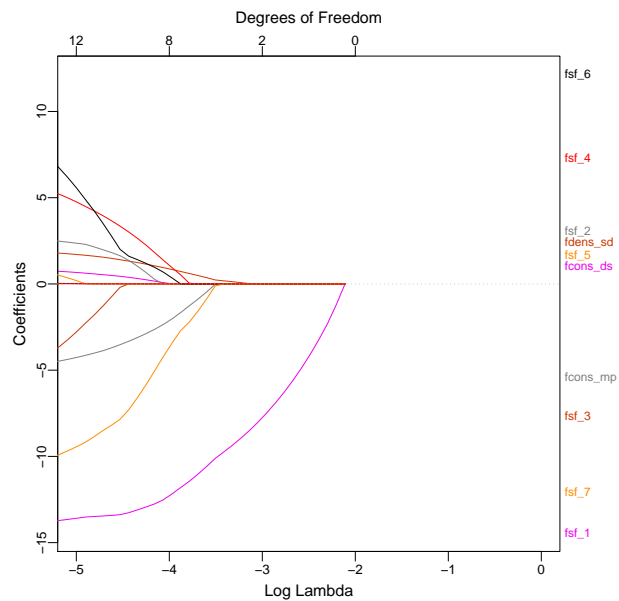


Figure 6.5: Cross-validated misclassification rates for different lambdas

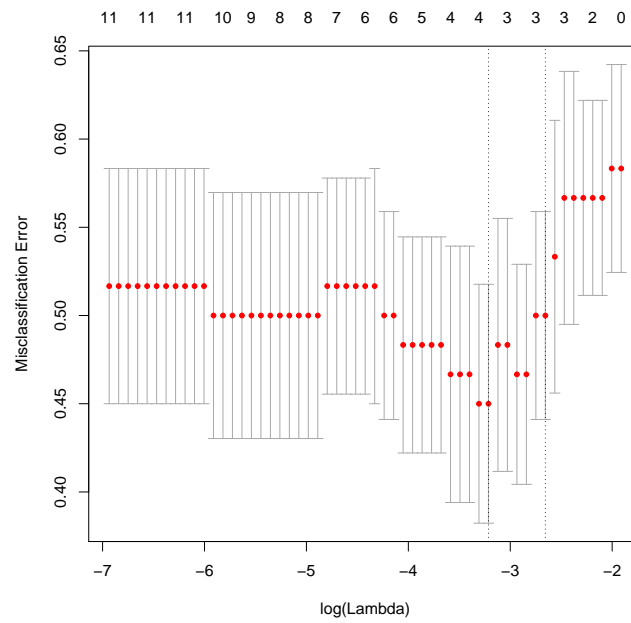


Figure 6.6: Cross-validated misclassification rates for different lambdas

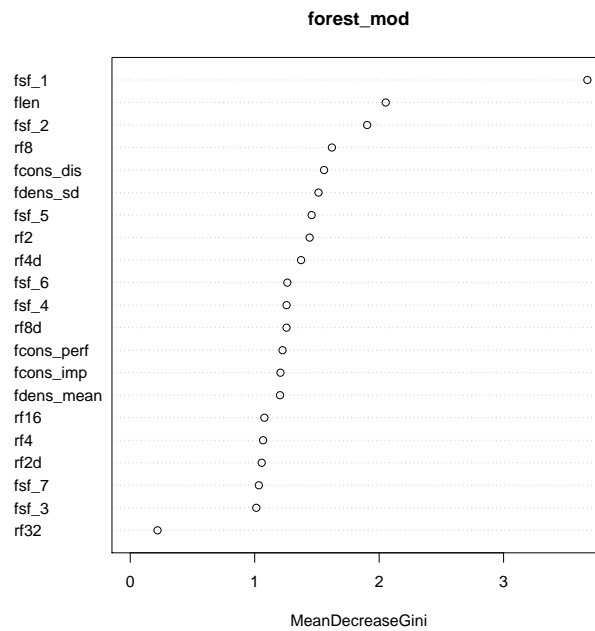


Figure 6.7: Variable importance for a random forest model Fanny/Felix

Chapter 7

Discussion

7.1 Discussion

On very basic low-level features, consisting mostly of frequencies of notes, intervals and chords, most models comparing Bach to the Mendelssohn do relatively well. This is likely due to the decent separation of the features encoding density. We likely see so much separation because the Bach data is for solo piano and the Mendelssohn data has an additional instrument, thus making the piece automatically more dense.

Table 7.1: Misclassification Rates for different models comparing Bach and Mendelssohns

| Model | misclass. rate - features | misclass. rate - 11 PCs |
|---------------|---------------------------|-------------------------|
| Logistic | 0.078 | 0.097 |
| LDA | 0.092 | 0.049 |
| KNN | 0.097 (K=9) | 0.098 (K = 9) |
| Naive Bayes | 0.103 | 0.112 |
| Random forest | 0.058 | 0.111 |

On the other hand, models fit to compare Felix and Fanny did not do as well. They are only very slightly better than random guessing. This could be because there is no true difference between Fanny and Felix in the features we extracted, ie. the features extracted are not good enough to pick up any existing signal. On the other hand, it is certainly believable that Fanny and Felix could have very similar unconscious signals in their writing. They were trained together, and did critique each others work extensively. The other possibility is that there are more works by Fanny snuck into Felix's published work, leading to overlap in the extracted features. We do know these features can accurately predict composer because as above for Bach and Mendelssohn, but perhaps composers so similar in style cannot be differentiated using these features. The included features only encode very basic aspects of music, and are more based on frequencies than how music acutally seems to differr in style to a listener.

Table 7.2: Summary of misclassification rates for different models comparing Felix and Fanny

| Model | misclass. rate - features | avg. misclass. rate - 11 PCs |
|---------------|---------------------------|------------------------------|
| Logistic | 0.517 | 0.45 |
| LDA | 0.518 | 0.505 |
| KNN | 0.498 | 0.467 |
| Naive Bayes | 0.502 | 0.521 |
| Random forest | 0.535 | 0.571 |

Conclusion

Fitting classifiers using low-level features composed mostly of frequencies of notes and rhythms was able to differentiate style between Bach and Fanny and Felix Mendelssohn. However classifiers using these low-level features were not successful in differentiating between Fanny and Felix.

7.2 Future suggestions for musical stylometry research

Most suggestions for future work lie in expanding the capabilities for museR. The first lies in increasing the ease of importing data. The scanning/OMR step is incredibly time consuming, which lead to a limited number of pieces being used in this analysis. While museR cannot help with this issue, museR could be expanded to be able to import music in the musicXML format in order to not need to rely on Humdrum's conversion program.

In addition, it would be interesting to perform this analysis on music of different forms, as we might expect unconscious signals to exist in the same way in symphonies as well as lieder.

Also it would be interesting to examine composers throughout their career. Maybe they had different aspects when they first started composing than when they were older.

Mostly, future versions of museR would be able to extract high level features. These are features that take into account the context of the feature in the music. These include more harmonic features, such as chord progressions, and more chord analysis.

Included will be the vignette for museR

Also a piratical how to guide for converting music into R

References

- Adair, D. (1944). The authorship of the disputed federalist papers. *The William and Mary Quarterly: A Magazine of Early American History*, 98–122.
- Ayrton, W. (1830). *The harmonicon* (Vol. 8). W. Pinnock.
- Backer, E., & Kranenburg, P. van. (2005). On musical stylometry—a pattern recognition approach. *Pattern Recognition Letters*, 26(3), 299–309.
- Brinkman, A., Shanahan, D., & Sapp, C. (n.d.). Musical stylometry, machine learning, and attribution studies: A semi-supervised approach to the works of josquin.
- Crerar, M. (1985). Elements of a statistical approach to the question of authorship in music. *Computers and the Humanities*, 19(3), 175–182.
- Doermann, D., Tombre, K., & others. (2014). *Handbook of document image processing and recognition*. Springer.
- Ford, P. L., & Bourne, E. G. (1897). The authorship of the federalist. *The American Historical Review*, 2(4), 675–687. Retrieved from <http://www.jstor.org/stable/1833983>
- Friedman, J., Hastie, T., & Tibshirani, R. (2001). *The elements of statistical learning* (Vol. 1). Springer series in statistics New York.
- Guyon, I., & Elisseeff, A. (2003). An introduction to variable and feature selection. *Journal of Machine Learning Research*, 3(Mar), 1157–1182.
- James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). *An introduction to statistical learning* (Vol. 112). Springer.
- Mace, A. R. (2013). *Fanny hensel, felix mendelssohn bartholdy, and the formation of the “mendelssohnian” style* (PhD thesis).
- Mearns, L., Tidhar, D., & Dixon, S. (2010). Characterisation of composer style using high-level musical features. In *Proceedings of 3rd international workshop on machine learning and music* (pp. 37–40). ACM.
- Mosteller, F., & Wallace, D. (1964). *Inference and disputed authorship: The federalist*. Addison-Wesley.
- Papadopoulos, G., & Wiggins, G. (n.d.). AI methods for algorithmic composition: A

survey, a critical view and future prospects.

Pudil, P., Novovičová, J., & Kittler, J. (1994). Floating search methods in feature selection. *Pattern Recognition Letters*, 15(11), 1119–1125.

Reich, N. B. (1991). The power of class: Fanny hensel. *Mendelssohn and His World*, 86–99.

The josquin research project. (n.d.). *The Josquin Research Project*. Retrieved from <http://josquin.stanford.edu/>

Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, 267–288.

Tillard, F. (1996). *Fanny mendelssohn*. Hal Leonard Corporation.

Todd, R. L. (2003). *Mendelssohn: A life in music*. Oxford University Press.