A Musical Stylometry Study on Works by Fanny Hensel and Felix Mendelssohn

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Abstract

The use of quantitative methods for attribution of pieces of classical music to composers is a relatively underexplored field. In theory, composers leave behind unconscious (or perhaps conscious) signals that indicate a piece of music as their own. Ideally these signals or features can be extracted and used to build a model to predict the composer of a piece of music. For instance, the siblings Fanny Hensel and Felix Mendelssohn, although very similar in compositional style, likely have features that distinguish their music. It is speculated that there are at least six pieces that Fanny wrote that were published by Felix. This historical setting motivates the construction of a model to determine which sibling was the true author.

Low level features, including frequencies of chords and scale degrees, were extracted from 31 lieder known to be written by Felix and 35 pieces known to be written by Fanny. These features were extracted using museR, an R package written for the purpose of this thesis. Additionally, to check the validity of features used, 27 solo piano pieces from J.S. Bach's Well Tempered Clavier were also included in the analysis. Logistic lasso, $k-$nearest neighbors, random forests, naive Bayes, and linear discriminant analysis (LDA) were performed on the feature space in order to classify the pieces. In addition, those same models were fit on the principal components. The LDA model fit on the principal components comparing Bach and Mendelssohn resulted in the lowest misclassification rate (0.049). The logistic lasso model on the principal components between Felix and Fanny resulted in the lowest misclassification rate (0.45). Finaly, each model was used to predict the authorship of the the six disputed pieces. As misclassification rates were so high on the models, we cannot trust the predictions for these disputed pieces.

Chapter 1

1.1 Introduction

Treating text or music as data has challenges. Neither text nor music contains anything immediately analyzable by a computer. With the increasing availability of the internet and social media data, text classification has become a promising form of data analysis. While methods have been developed to attribute authorship of text, such methods have yet to be thoroughly explored for authorship of classical music.

Feature extraction is an essential part of text or music classification. Unlike in other applications where the data initially contains an informative set of numerical or categorical values, text and music must first go through a processing stage where features of interest can be extracted before any models can be fit.

Studies that examine the style of a type of music or composer using computation are known as musical stylometry studies. A subfield of musical stylometry is using the examined style to classify composer. These studies use extracted features to build a model that can classify a likely composer for that piece of music. For musically trained humans, classifying composer on their own might be an easy task. Some are able to automatically distinguish a piece composed by Bach or Mozart either by listening to a recording or looking at sheet music. Distinguishing composers by ear becomes more difficult when composers are contemporaries. Mozart and Salieri might be distinguishable to a more discerning listener or scholar, but only with some difficulty. When the identity of the composer is unknown or disputed, trying to classify the piece by ear becomes particularly challenging. Examples of disputed authorship exist throughout music history, most notably in the Renaissance era and earlier (Brinkman, Shanahan, & Sapp, n.d.).

In both text and music classification, we must first extract features from each piece that ideally signal some distinguishable tendency of the composer.

1.2 Brief history of text classification

The Federalist Papers were one of the earliest instances of text classification for determining authorship (Mosteller & Wallace, 1964). These famous 18th century documents were written under the pen name 'Publius'. Several of these disputed papers are attributed to either James Madison or Alexander Hamilton. Neither of them ever admitted authorship, as some of the writings were contradictory to their later political positions (Adair, 1944). Prior to quantitative analysis, historians had often examined the papers using styles of previously known writings of Madison and Hamilton. Their analysis was often partially based on the content of the papers, for example the existence of citing English history is a trait more common to Hamilton (Ford & Bourne, 1897).

In contrast, Mosteller and Wallace (1964) used the frequency of words such as 'and', 'by', 'from', and 'upon', to fit a classifier that was trained on the writings on a set of known writings by each author. These unconscious indicators were able to differentiate between the two writers, and the model was able to identify the likely author of the disputed paper.

1.3 Feature selection

Text analysis, such as in the Federalist Papers, is accomplished by reading one word after another. This often results in a vector where each entry is a word. Frequencies of word count, for example, can easily be done by counting all instances of word occurrence in the word vector. Information in a piece of music, however, is read in a variety of ways. It can be read left to right note by note, but it can also be read vertically as the harmony, or the notes played together. In a piece with several instruments, both these processes happen at the same time for each instrument. There are also aspects that take place over large sections, such as phrasing, or cadential patterns. There are rules of counterpoint that are followed throughout the entire piece. Thus we need to find features that can be measured for each piece, or perhaps each measure or instrument, that can describe a certain piece of music. Ideally, these features are not measuring certain rules and practices of classical music, but instead the creativeness and individuality of a composer. As there are many different aspects of a piece, melodic changes, harmonies, etc., one can end up with many features to describe one piece. It is possible that the number of features (p) can be greater than the sample size (n), or that many features encode essentially the same thing. We thus need to figure a clever way to decrease our feature space. This can happen in choosing what features we want to use to model, or by using a dimension reduction technique such as principal components analysis.

Most musical stylometry literature has focused on composers in the Renaissance (1400-1600), Baroque (1600-1750), and Classical (1730-1820) eras. The Mendelssohns were composing in the Romantic era (1780-1910). The focus of the literature may have been because composers in earlier eras followed rules of counterpoint more exactly, or perhaps had less "expressive" allowances for their composing, thus making it easier to define and extract features. There are also more pieces with doubtful authorship in those eras. In addition, Computer Assisted Research in the Humanities (CCARH) (C. S. Sapp, n.d.) has a large corpus of encoded music from these times. As will be described later, the initial step of coding the music is the most time consuming; having music in a format immediately conducive to analysis is very helpful.

1.4 A Very Short Introduction to Music Theory

Western sheet music is presented on a five line staff, where the position on the staff defines its sound. The sound of a note depends on its pitch, which is a specific vibration of sound waves at a certain frequency. Notes are in the set $note \in \{A\flat, A, A\sharp, B\flat, B, B\sharp, C\flat, C, C\sharp, D\flat, D, D\sharp, E\flat, E, E\sharp, F\flat, F, F\sharp, G\flat, G\, G\sharp\}$. The sharps and flats change the value of a note up or down a half step. To remove sharps of flats, a natural sign is used ($\natural$) There also exist double flats and double sharps. Some notes can be written differently, but sound the same, such as $F\flat$ and E. These are known as enharmonics.

Of the above possible notes, there are only twelve distinct notes (i.e. only twelve half steps in the western scale). The *key signature* of the piece indicates how many sharps or flats are normal for the piece. Each key is made up of seven different notes. A piece's key can be major or minor, although the piece can internally modulate between the two. The *scale degree* of a note depends on how many notes it is away from the note that represents the key of the piece. The *time signature* of a piece determines the number of beats that are in each measure. The vertical distance between notes (also known as an *interval*) depends on how many half steps occur between two notes. *Melodic intervals* are defined as the number of half steps between two adjacent notes. *Harmonic intervals* are defined as the number of half steps between notes played at the same time. *Chords* occur when there are more than two notes being played simultaneously. The type of chord is determined by the harmonic intervals it contains. Cadences are a type of chord progression, usually occurring at the end of a phrase and especially at the end of a piece. Intervals and chords can be either *dissonant* or *consonant*. Consonant intervals sound nice to our ears, whereas dissonant intervals add a sense of tension or unease, which is used to shape the feel of the music. In addition, chords and intervals can be minor or major. Minor intervals feel "sad", whereas major intervals feel "happy". These are also known as sonorities, and are divided into three types: perfect intervals (1, 4, 5, and 8), imperfect intervals (3 and 6), and dissonant intervals (2 and 7). There are some rules, known as counterpoint, that most composers follow to some extent. These were followed more strictly in the Baroque era than the Romantic era. First species counterpoint is a type of counterpoint that dictates there are no parallel octaves, fourths or fifths. Another rule is that when changing from one perfect consonance to another, the notes must proceed in contrary or oblique motion (Laitz, 2012).

1.5 Musical features

Features calculated from music often closely follow music theory. In addition to deciding the features, there is also the decision of the scope at which those features take place. The features can be for a given instrument, the entire piece, or each measure. Windowing techniques can be used where a "window" is created over a given number of bars or notes, and shifts across the whole piece. For each window, a feature is recorded. These can be overlapping windows by creating an "offset" of a number of beats or notes. Windowing produces more data, as instead of one feature for each piece, there is a feature for each window. Backer et.al used windowing techniques in a musical stylometry study on Bach and his contemporaries. They also used overlapping windowing over each entire composition to produce more data, using window of 30 bars to create a high enough number of fragments per piece and a low enough variance of the feature values between fragments (Backer & Kranenburg, 2005).

Musical features are often thought about as either high level or low level. While the exact definition of each often varies, low level is often understood to be features such as note frequency, etc. High level features are more about a broader sense of the piece, including chord progressions, etc. High level features are often what music experts use in their analysis, whereas low-level features are more easily extracted and analyzed by computer. High level features often depend on the context from which the feature was extracted. For example, cadences and modulations (brief changes in key) throughout the piece are easily found when a human analyzes a piece of music, but are very hard to generalize as a feature. If one wants to use high-level features, they would first have to train the feature to see if it was performing accurately. For this reason, this paper focuses only on low level features, or features that are context independent.

1.6 Background on Variable and Feature Selection

Before analysis is done or models are fit, numerous features are often extracted from the music without knowing a priori which ones will be helpful in identifying a composer's style. Especially in research regarding text categorization, data sets have enormous numbers of variables. We use "feature" and "variable" interchangeably, with the exception when features are created from variables, and the distinction will be made in that case (Guyon & Elisseeff, 2003). Fitting models with all of the features that are available can often lead to overfitting of the model. There are three main ways to deal with this problem: subset selection, shrinkage models, and dimensionality reduction. This section gives a broad overview of options for features selection. The methods used in this thesis will be described in further detail in chapter 4.

Subset selection methods choose which features to use in a model. There are many ways to do this and existing feature selection algorithms can help with this process. One idea to see which features are best for the model is to look at all possible subsets of the variables and see which model performs best. All possible subsets of variables is $2^p - 1$, where p is the number of features. For large p, this is often computationally impossible. Instead of fitting all possible subsets, stepwise selection strategies like forward selection and backward selection can be used.

Subset selection methods often use some sort of variable ranking tool to determine which variables are useful to include in the best subset. Variable ranking uses a score function to assign a score to each possible variable. It is tempting to only include variables that have a high score. However, this possibly leads to redundancy. In addition, variables that are not important by themselves can have a significant performance improvement when considered with other variables. Single variable classifiers rank the variable according to its individual predictive power. The predictive power can be measured in terms of error rate, or using the false positive or false negative rate. This classifier cannot distinguish variables that perfectly separate the data.

Shrinkage methods, in contrast, use all p features and constrain or shrink coefficients towards zero instead of choosing a subset of the features. Ideally, coefficients of features that are not as useful for the model will be shrunk towards zero, so they do not have a large effect on the model. Constraining features in this way often leads to a reduction in variance of the estimates. The most common shrinkage methods are ridge and lasso (Hoerl, 1959)(Tibshirani, 1996).

Both shrinkage methods and subset selection use the original variables to fit models. Dimension reduction methods transform the features space, and then fit models on the transformed variables. They are commonly used to attempt to encode enough of the information given in the original features in a smaller-dimensional space. This results in feature creation; using the recorded variables to create new features to fit the model. Principal components analysis (PCA) is a common dimension reduction techniques. Feature creation aims to achieve a good reconstruction of the data and to make succesful predictuions using the constucted features.

Once we have a model, we often want to know how “good” it is. For classification models, we often use the misclassification rate. To calculate this, we withhold a sample of our data as a testing set. We then fit the model on the training set, and then use the fitted model to predict classes on the testing set. The misclassification rate is then computed as the number of pieces in the testing set that were incorrectly classified divided by the total number of pieces. Cross validation involves fitting a model on a training set, and then using the fitted model to predict the responses in the testing set. In particular, k-fold cross validation involves splitting the data set into k different parts of equal size, and fitting a model on the the data set with one of the k parts withheld. The model is then tested on the withheld data. This is useful for model selection and determining the accuracy of the model. In this paper, as is common, we use $k = 5$. This results in a predicted misclassification rate with lower variance that other crossvalidation methods such as leave-one-out-error and validation sets.

1.7 Previous research

There have been numerous other applications of machine learning to music. These include many studies that are not specifically identifying a composer from sheet music. For example, a previous study used machine learning trained on a composer’s music to have the computer learn the style and compose a piece in a similar style (Papadopoulos & Wiggins, n.d.).

Musical stylometry has focused on two types of data: audio and sheet music. This analysis uses data in the form of sheet music. To predict a composer, a training data set of pieces of known composer is needed. Then a model can be fit to a testing data set to predict composer. If the fitted model shows good predictions, that model can be applied on the pieces of unknown authorship. Musical stylometry can resolve disputed authorship and detect distinguishing musical styles of composer, even if there are no disputed pieces. Musical stylometry can also look at the development of a composers musical style over time.

This paper draws its inspiration from four different studies that have classified composers by extracting features from sheet music. Initial studies in the 1970's started looking at ways to represent music in a way that could analyzed by computers. Recently, several previous papers have focused on Josquin des Prez. This is likely due to the fact that there is a large training and testing data set available in easily analyzable format provided by the Josquin Research Project ("The josquin research project," n.d.). In addition there are a number of pieces of disputed authorship that have been attributed to him.

Work by Brinkman et al. (Brinkman et al., n.d.) used machine learning to evaluate attribution of compositions by Josquin des Prez. They first compared music of Josquin to JS Bach's four part chorales. Many listeners could easily distinguish these two composers as they were separated by two centuries. They also compared Josquin to composers closer in time and style: Ockeghem and Du Fay, who are only one or two generations separated. These composers could likely be differentiated by experienced listeners of Renaissance music. Finally they compared Josquin's to his contemporaries: de Orto and La Rue.

Work by Backer et. al. looked at differences in style between J.S. Bach, Telemann, Handel, Haydn and Mozart and then examined a disputed piece: BWV 534 which is believed to be composed by either J.S. Bach, J.L. Krebs, or W.F. Bach (J.S. Bach's son) (Backer & Kranenburg, 2005).

The following two studies do not try to identify composers of unknown works, but instead try to extract features that can seperate differences in style, and also fit classifiers that can correctly assign composers based on their style.

Crerar used incipits from Vivaldi, Correlli, and Valentini, who could have been a student of Correlli. They used low level features, especially looking at intervalic movement. They also compared Bach, Haydn, and Mozart (Crerar, 1985).

Mearns et al. extracted high level musical features on J.S. Bach, Buxtehude, Ruggero, Vivaldi, Monteverdi, Corelli, and Frescobaldi. The idea was that they all adhered to the laws of counterpoint, and that they possibly still had distinct uses of the counterpoint. They used pieces across instrumental genres with the idea that stylistic counterpoint would remain constant across compositional type (Mearns, Tidhar, & Dixon, 2010).

1.8 Previous choices of features in applications

Deciding on and extracting features is the first step in analyzing music. Depending on the characteristics of the composer and time period, different features are useful. Often features are extracted en masse and then work is done later to determine which features are important or useful in identifying style.

In addition to deciding which features to use, one must also determine the scale at which the features should exist. There can be features for a given instrument, the entire piece, or each measure. Windowing techniques can be used where a window is created over a given number of bars or notes and the window moves through the whole piece. A feature is extracted and recorded for each window. Windows can be made to overlap by using an offset of a number of beats or notes. Windowing thus produces more data as there is a feature for each window instead of for the whole piece. There can thus be tens of windows in each piece.

Common types of features in music analysis include low level features of frequencies of notes, chords, etc. Also common are features that look at the fraction of the score that consisted of dissonant sonorities, as well as the fraction of bars that begin with a dissonant sonority. Other features include the type of intervals or consonances present in a piece: perfect consonance, imperfect consonances, and dissonance. In polyphonic pieces, the four types of motion, (parallel, similar, oblique, and contrary) can also be used as features. Crerar (Crerar, 1985) used pitch frequencies, after standardizing all pieces by transposing to the key of C, the first melodic interval of the piece, and note transition counts. Mearns used types of motion and consonances as features (Mearns et al., 2010).

Features measuring stability are also popular. Stability can be measured for rhythmic changes or harmonic changes. Stability is computed by dividing the standard deviation of the lengths of the rhythm or note by the mean length of the notes. It is normalized in this way to be comparable over differing time signatures. Backer and Kranenburg (Backer & Kranenburg, 2005). chose to extract twenty features that include frequencies of intervals, and measures of stability, and entropy. The entropy of the probability of occurrence of chords was measured in two different ways, defining chords to be the same no matter what scale degree they are on, and distinguishing chords differently depending on scale degree. Next, they the calculated average number of active voices at one time, a measure of the voice density of the piece. Then for every interval, the duration of that interval was divided by the total duration of all intervals. Next, the total duration of parallel thirds, fourths, and sixths divided by the total duration of all intervals was measured. Finally, a measure of suspensions was calculated.

Brinkman et.al used both high-level and low-level features. The high-level features were 9-8 suspensions, oblique motion, contrary motion, similar motion and parallel motion. The low-level features were average melodic entropy, normalized pairwise variability index, and note-to-note transition probabilities (Brinkman et al., n.d.).

Other features that can be helpful for distinguishing Renaissance and Baroque composers look specifically at differences in counterpoint. Since most composers in that era generally followed the rules of counterpoint, these features are created to try to detect specific identifiable uses of counterpoint. These features include counterpoint movement types, dissonance distributions, parallel intervals of each kind, and vertical interval distributions. Mearns et al. calculated intervals for every subsequent pitch in each part. A weighted count of each interval was stored. The counts were weighted according to duration to account for the perception of use. The count vector was then normalized to account for the weighted interval content of the score. For movement types, Mearns et al. used ideas based on the rules of first species counterpoint. (Similar, oblique, parallel, and contrary). It does not make musical sense to compare every single adjacent group of notes as many of the note groups occurring at fractional metrical positions consist of notes held from the previous note group with the addition of a passing or neighbor note in another voice or voices. For this reason, only notes that occur at strong metrical positions, in this case on each beat of the bar, are used for contrapuntal evaluation. This is a relatively simplistic method of choosing structurally important notes. In Bach, for example, the majority of voice progressions take place at a closer level. The features measuring counterpoint rules include the nature of the approach to a perfect consonance, whether a dissonance is properly prepared (i.e. preceded by a consonant interval containing the same note which then becomes dissonant), whether a dissonance is correctly resolved downwards by step, details of parallel intervals, and the overall distribution of contrapuntal movements (oblique / contrary / similar / parallel / other). A feature for total vertical interval count is also used (Mearns et al., 2010).

1.9 Previous modeling and results in applications

Most of the previous research has included some kind of feature selection. Usually features are extracted before any analysis is done, and we don't know beforehand which features are distinguishing.

A modification of a forward selection (Floating Forward Selection) (Pudil, Novovičová, & Kittler, 1994) was used in Backer's (2005) study to extract features in order to identify distinguishing style between Bach, Handel, Telemann, Mozart, and Haydn, and then subsequently classify the authorship of BWV 535. Each composer was compared via creating bivariate comparisons of all possible class arrangements, ie (Bach)(Handel), (Bach)(Handel,Telemann), etc. The algorithm extracted features for each class arrangement that best distinguished the groups. A k-nearest neighbors classifier was fit using the features that the subset selection algorithm found useful. The model was successful in comparing Bach and others as well as each individual composer. The k-nearest neighbor classifier resulted in a leave-one-out-error of between 5% and 9%, for most class arrangements, and 24.3% between Haydn and Mozart, Decision trees were also used to interpret the features used in decision making of the different class arrangements. The decision trees used between 2 and 4 features depending on which class comparison was being analyzed. It resulted in a max of one classification error. To determine authorship of BWV 535, quadratic Bayesian classifier was trained to distinguish J.S. Bach, W.F. Bach and J.L Krebs. They again compare every possible class arrangement as potential composers. Their results showed that BWV 535 was likely composed by either W.F. Bach and his pupil, J.L. Krebs.

Principal components analysis (PCA) was used for dimension reduction for subsequent analysis on the music Josquin and his contemporaries (Brinkman et al., n.d.). Binary comparisons were used to compare composers. Although only two principal components accounted for most of the variance, in the comparison between Bach and Josquin, five principal components were used to account for more variance (85%). Between Josquin and his contemporaries, 27 principal components were used to account for 85% of the variance. This resulted in a relatively clear separation between Bach and Josquin when visualized on their first two principal components. Between Josquin and each of his contemporaries, the principal components did not have as much of an obvious separation. The transformed features from principal components analysis were used to train a classifier. First, a $k-$nearest neighbor classifier was used with $k = 6$. The training set consisted of 80% of the data, with 20% withheld as a testing set. The model correctly predicted pieces by de Orto 38.9% of the time, Josquin 60.6%, and La Rue, 80.6% Next, they trained a support vector machine classifier, which preformed similarly to the $k-$nearest neighbor classifier.

Mearns fit a classifier using a WEKA algorithm, as well as Naive Bayes and a Decision Tree was created that correctly predicted the composer 2/3 of the time (Mearns et al., 2010).

Crerar took a different approach, and instead of trying to fit a model that would classify composers, performed a chi-squared test to see if the features of the composers were in fact different. This would imply that the composers had different styles that could be distinguished using the features in the paper. The probability of the chi-squared values occurring were all less than 0.05, and thus the composers do indeed have different styles (Crerar, 1985).

1.10 Fanny and Felix Mendelssohn

Most musical stylometry analysis has focused on music of the Renaissance and Baroque period, as there are more questions of authorship in that period. As the Romantic period is much more modern in comparison, there are many more surviving records of original manuscripts that include the composer.

Felix Mendelssohn, often considered a prodigy akin to Mozart, was a prolific composer. Before he was fourteen years old, he had already written over 100 compositions. His lesser known sister Fanny Hensel née Mendelssohn was also a composer of incredible skill. The two were very close, for many years training and studying together. In their early education living in Berlin, Felix and Fanny received the same musical education. In their younger years, they were both instructed in piano by Madam Bigot, a famous pianist esteemed by Haydn and Beethoven. Beginning in 1818, Carl Friedrich Zelter, a somewhat removed student of Bach and the most influential Berlin musician of the time, began to teach them both composition. In addition to music, the children were tutored by some of the finest scholars in Berlin in subjects such as languages, history, and drawing. Goethe himself claimed that Fanny was “as gifted as Felix” (Tillard, 1996).

As Fanny grew up, her father started implying that she should focus her energy on the domestic sphere of her life. While the fact that she never became a world famous composer and performer is often attributed to the gender politics of her time, it is also likely due to her high class (Reich, 1991). Since the family had recently converted from Judaism to Christianity, and considering the anti-Semitic feelings of the time, the family did not want any other unusual characteristic such as a professional female composer to set them further apart from “polite” society.

Most of Fanny's available work are *lieder*, short pieces of voice accompanied by piano. They were accepted at the time as the more feminine, domestic compositions, acceptable for women to compose. Her brother moved on to more elaborate compositions such as operas, orchestral concertos and symphonies. Her father pressured Fanny to remain composing lieder (Todd, 2003).

> "Music will perhaps become his profession, while for you it can and must only be an ornament, never the root of your being and doing. We may therefore pardon him some ambition and desire to be acknowledged in a pursuit which appears very important to him,... while it does you credit that you have always shown yourself good and sensible in these matters; ... Remain true to these sentiments and to this line of conduct; they are feminine, and only what is truly feminine is an ornament to your sex."

Throughout their lives, Felix and Fanny maintained contact through letters until Fanny's death in 1847 and Felix's death shortly thereafter. These letters contain many instances of Felix asking for advice on his compositions.

Unlike Felix who conducted and performed piano and organ in some of Berlin's most esteemed concert halls, most of Fanny's performances were private, only performed in small circles of her friends and family at intimate parties. Similarly, although she was quite a prolific composer, under recommendation of her father Abraham Mendelssohn, and to a lesser extent Felix, Fanny did not publish her work until later in her life. In 1846 after her father's death and though her brother initially disapproved, she published her first collection of lieder. Many of Fanny's unpublished notebooks are in private collections and are inaccessible for study.

However, it is widely speculated that some of Fanny's work was published under her brother's name, especially three pieces each in his Op 8 and 9 lieder. Famously, when Felix met the Queen of England, she sang Felix's lied "Italien", and Felix had to admit that in fact, it was his sister that had written it. In a letter to Felix, Fanny admits:

> "I have just recently received a letter from Vienna, which contained basically nothing but the question of whether "On Wings of Song" was by me, and that I should really send a list of things that are running about in the world disguised, it seems that they aren't clever enough themselves to separate the wheat from the chaff." (Mace, 2013)

As she never made such a list, we are left to wonder if there are any other pieces of hers that have been published under her brothers name and reputation. It is suspected that most of Fanny's friends and family would have known at the time that she had composed the pieces published by Felix, as they would have heard her play those pieces at home and for private audiences. An article written in 1830 in the Harmonicon, an influential music journal in London, mentioned that not all the lieder in Felix's collection were written by him. The article also praises the work of Fanny, albeit in an odd way likely due to the time of publication.

> "I possess twelve published songs under Mr. Mendelssohn's name, which he wrote when a boy of fifteen. One of these appears in the musical

> annual, “Apollo’s Gift,” and deserves all the praise you have in your review bestowed on it. But the whole of the twelve are not by him: three of the best are by his sister, a young lady of great talents and accomplishments. I cannot refrain from mentioning Miss Mendelsshon’s name in connexion with these songs, more particularly when I see so many ladies without one atom of genius, coming forward to the public with their musical crudities, and, because these are printed, holding up their heads as if they were finished musicians. Miss Mendelssohn is a first-rate piano-forte player, of which you may form some idea when I mention that she can express the varied beauties of Beethoven’s extraordinary trio in Bb. She has not the wild energy of her brother; but possesses sufficient power and nerve for the accurate performance of Beethoven’s music. She is no superficial musician; she has studied the science deeply, and writes with the freedom of a master. Her songs are distinguished by tenderness, warmth, and originality: some which I heard were exquisite.”(Ayrton, 1830)

This project will use lieder of Fanny and Felix Mendelssohn in attempts to train a classifier to analyze the six disputed pieces. Most of the available work by Fanny are lieder. Felix also composed a good number of lieder. We will see if there is a determinable difference in style of these siblings who grew up very close and received mostly the same musical education. All other previous composership studies cannot be adjusted for training and upbringing, so it might be challenging to detect a difference.

Chapter 2

About the data and conversion process

2.1 Pieces used

The majority of the pieces used in this paper were lieder of Felix Mendelssohn and Fanny Hensel. Felix Mendelssohn composed many different styles of music, orchestral, piano, etc. Fanny Hensel in contrast has an available existing corpus of mostly lieder, although she did compose many works for solo piano and orchestra. Of Felix's music there were a total of 31 pieces: lieder of Op 8 (9 pieces), op 9 (9 pieces), Op 19.a (3 pieces), op 34 (1 piece), and 9 pieces published posthumously.

Of Fanny's music, a total of 35 pieces were used: 15 lieder were used from her lieder without name collection, 10 from her *Wo kommst du Her* collection, and 10 from an unnamed collection.

The six pieces published by Felix that are thought to be written by Fanny are Op8 no 2, 3 and 12, and Op 9, no 7, 10 and 12.

Data from JS Bach were also used to provide a comparison corpus that is expected to be very distinct from the work of the Mendelssohns. These data were available in Kern Score format from the Center for Computer Assisted Research in the Humanities. (CCARH). The pieces used were from the Well Tempered Clavier (WTC). These were written as training pieces and each collection contains 24 pieces with one in every possible key. Pieces from the Well Tempered Clavier were chosen as the data were more easily accessible (no scanning was required) and they were a similar format as the Mendelssohn songs, written as for solo piano (or harpsichord).

2.2 Optical music recognition

The vast majority of classical music scores are found solely in PDF or physical copies. Sheet music as a form of data requires a lengthy process of conversion before being able to be used in any analysis. Simply scanning the scores into, say, a PDF, gives no musical semantics and can only be viewed on screen or printed on paper. Thus, the two main steps in reading in data from sheet music are, first, using optical music

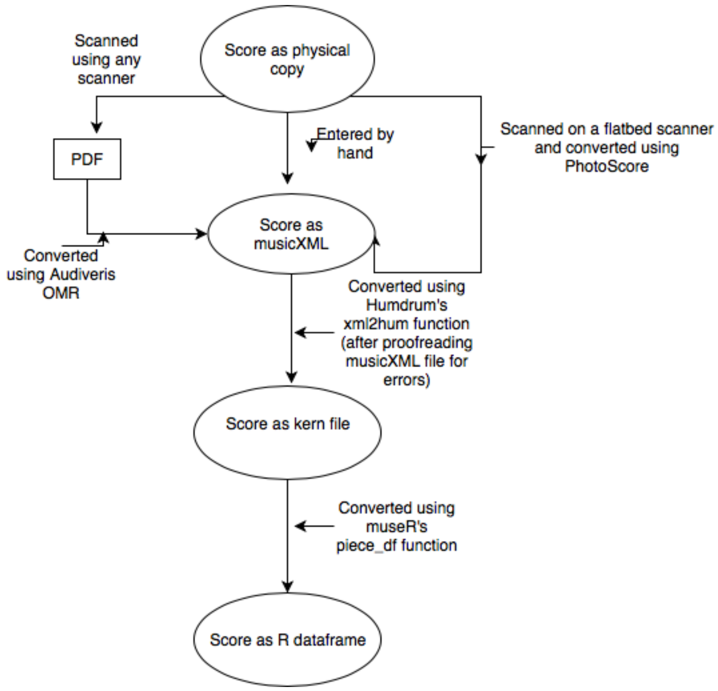

Score as physical copy
Scanned using any scanner
PDF
Entered by hand
Scanned on a flatbed scanner and converted using PhotoScore
Converted using Audiveris OMR
Score as musicXML
Converted using Humdrum's xml2hum function (after proofreading musicXML file for errors)
Score as kern file
Converted using museR's piece_df function
Score as R dataframe

Figure 2.1: Flowchart of the conversion process from physical score copy to dataframe in R.

recognition software to transform physical scores into digital formats, and second, to read the digital format in to R where subsequent analysis can be done. The scores used in this paper were obtained from physical copies available in the Reed music library. These scores were then scanned using software designed for optical music recognition (OMR).

Optical music recognition requires learning from graphical and textual information. The software must mainly pick up the locations of bar lines, notes, rests, slurs, dynamic markings, tempo markings, lyrics etc. Basic optical music recognition has been around since 1966.

Most commonly, the first step in optical music recognition is to remove the staff lines. The staff lines are critical, as they define the basis for the vertical definition distance of pitch, and the horizontal distance definition of rhythm. The staff gives a normalization that is helpful, essentially defining the size of what notes and rhythm look like (Doermann, Tombre, & others, 2014).

The next step is music symbol extraction and classification. These methods include template matching, where the object in question is compared to existing known musical symbols, simple operators, such as analysis of bounding boxes and projections, and joining graphical primitives, such as combining extracted objects such as notes, note heads, and note beams to connect them in a musically correct way to form chords. Other methods use statistical models for analyzing musical primitives (the objects the OMR is trying to classify) such as Neural Networks, Support Vector Machines, k-Nearest Neighbor, and Hidden Markov Models.

The next step OMR performs is syntactical analysis and validation. This step

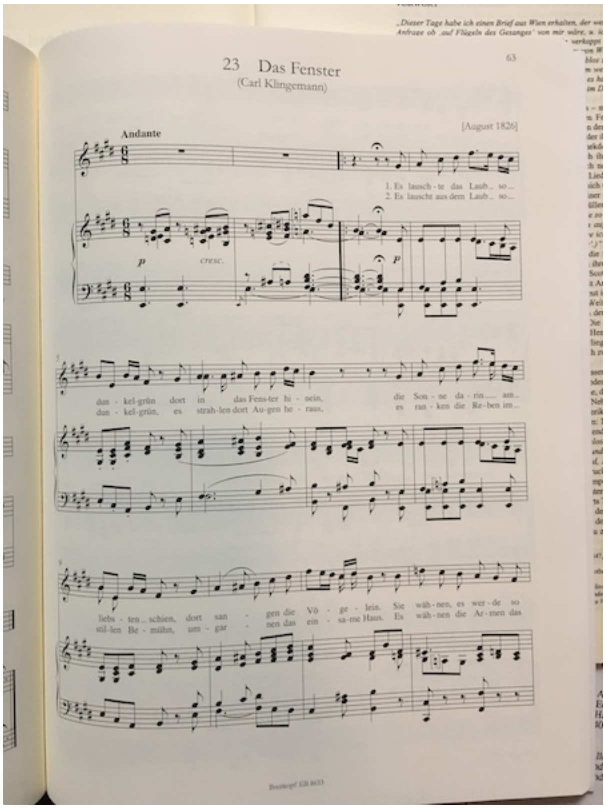

23 Das Fenster
(Carl Klingemann)
63
[August 1826]
Andante
1. Es lausch-te das Laub... so...
2. Es lauscht aus dem Laub... so...
p
cresc.
p
dun-kel-grün dort in das Fens-ter hi-nein, die Son-ne da-rin..... am...
dun-kel-grün, es strah-len dort Au-gen he-raus, es ran-ken die Re-ben im...
liebs-ten... schien, dort san-gen die Vö-ge-lein. Sie wäh-nen, es wer-de so
stil-len Be-mühn, um-gar-nen das ein-sa-me Haus. Es wäh-nen die Ar-men das
Breitkopf EB 8653

Figure 2.2: Score in physical form before conversion process has started.

essentially uses defined grammars describing the organization of music notation in terms of music symbols. This makes the classification problem simpler, as there are existing rules and relationships between musical symbols.

The two OMR softwares used in this paper were PhotoScore and Audiveris. Each has its own benefits and issues. PhotoScore works by scanning the physical score on a flatbed scanner at a high resolution. It then uses OMR techniques to output a musicXML file that can be read in by most music composing software, such as Sibelius, Finale or MuseScore.

Audiveris, in contrast, works by inputting a high resolution PDF and then uses OMR techniques to output a music XML file. Often, high enough resolution PDFs do not exist, so the physical scores must also be scanned by any garden variety scanner. MusicXML is commonly used as a format for digital music, as it is conducive to representing sheet music and music notation, and it can be transferable to many different music software. Muse Score was chosen to be the music software for viewing digital scores as it is a free software that can read MusicXML.

After being scanned by PhotoScore and read into MuseScore, each piece was proof-read and corrected. This involves looking through every piece line by line for each bar to “spell check” the digital version. PhotoScore did a good job recognizing notes, but often had issues recognizing rhythms, and had issues keeping the structure of the piece. Often in the scanning process clefs or bar lines were not found, causing PhotoScore to output every staff on one line. In contrast, Audiveris often added extra

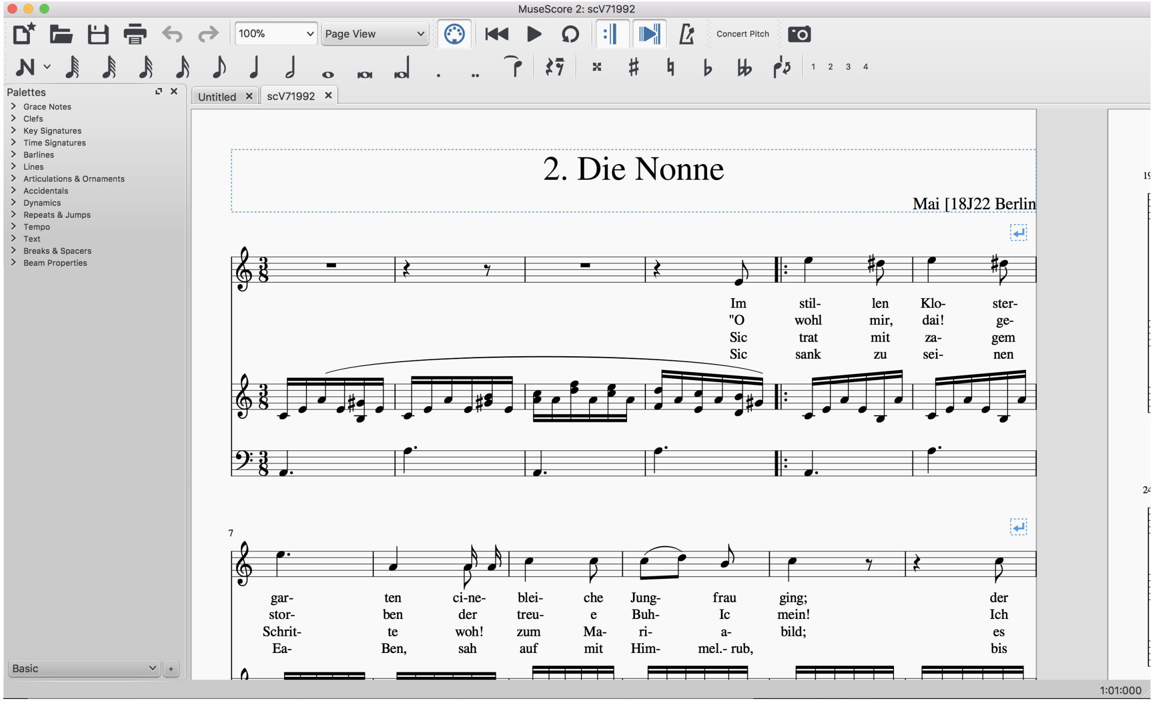
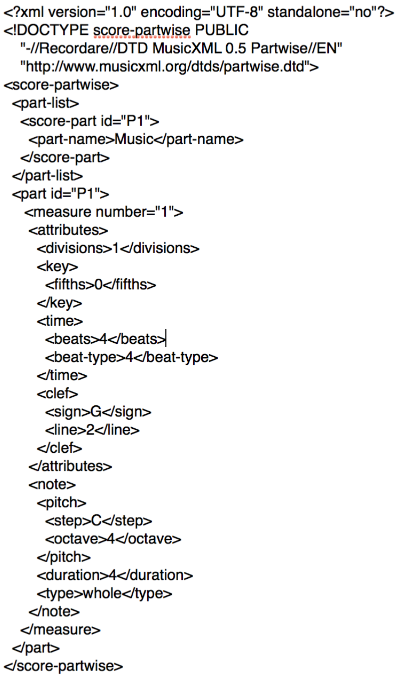


Figure 2.3: A score in MuseScore format after being read by an OMR like PhotoScore or Audiveris.

beats to measures, and assigned notes to the wrong staff. It also always identified a bass clef as a baritone clef.

Unfortunately, the scanning process is very lengthy and time consuming, as the scanning often gives a large number of mistakes. Often the score must be then scanned again. In addition, the proof-reading process is lengthy. One must check each note and rhythm for errors against the original score, and change the incorrect notes using MuseScore. The corrected score must then be re-outputted as a musicXML file. In addition, there were some pieces that PhotoScore or Audiveris had a hard time reading. These pieces were then entered into MuseScore by hand and then proofread.

MusicXML on its own is not conducive to converting into a data frame as representing the single half note middle C is represented in Figure 2.4. We then need to convert into a format more easily readable into R. The Kern Score music format is much more easily readable. It has clearly expressed time signature, bar, beat and musical voicing information (Mearns et al., 2010). Because of this, it is also more conducive to being read into an R data frame. Figure 2.5 (Huron, 1994) shows how a basic piece of music corresponds to a Kern file. Kern files are organized with columns each representing one staff of music. Each line of a Kern file represents one note of one value of a time base. The time base for a Kern file is based on the smallest (shortest) rhythm value of a note found in a piece. For example, if a piece was in 4/4 and there were sixteenth notes present there would be 16*4 rows for each measure. The “attack” of each note is the only note printed, the following time while the note is held is represented with dots in the remaining rows until a new note is sounded for



```
<?xml version="1.0" encoding="UTF-8" standalone="no"?>
<!DOCTYPE score-partwise PUBLIC
   "-//Recordare//DTD MusicXML 0.5 Partwise//EN"
   "http://www.musicxml.org/dtds/partwise.dtd">
<score-partwise>
 <part-list>
  <score-part id="P1">
   <part-name>Music</part-name>
  </score-part>
 </part-list>
 <part id="P1">
   <measure number="1">
    <attributes>
     <divisions>1</divisions>
     <key>
      <fifths>0</fifths>
     </key>
     <time>
      <beats>4</beats>
      <beat-type>4</beat-type>
     </time>
     <clef>
      <sign>G</sign>
      <line>2</line>
     </clef>
    </attributes>
    <note>
     <pitch>
      <step>C</step>
      <octave>4</octave>
     </pitch>
     <duration>4</duration>
     <type>whole</type>
    </note>
  </measure>
 </part>
</score-partwise>
```

Figure 2.4: A half middle C in musicXML encoding

that staff. The pitch of each note is represented by the letters a through g. The case (lower or upper) as well as the repetition (c or ccc) represents which octave the pitch occurs. Any accidental is represented with a ♯, -, or n symbol. Each instrument/staff in a piece is represented using one (or more) columns called splines. For example, most lieder consist of voice and piano. There are thus three splines, one for voice, one for the treble clef staff of the piano, and one for the bass clef of the piano. In addition, there are splines that contain the text for the voice for the corresponding notes. This was not of interest to the musical classification problem, so these splines were removed. Chords are represented by multiple notes on the same line. For example, if there was a half note C major triad followed by a quarter note D flat, it would be represented as

```
2c 2e 2g
4d- . .
```

In addition, there is a lot of information about the appearance of the piece, stem direction etc. for notes, but these factors were decided as not important for determining style, so it was removed.

We convert to kern format by using Humdrum's function `xml2hum` that converts a musicXML file into a kern file. Humdrum is a computational music software used to analyze music. It is a command line tool that has many functions for music analysis. The Kern file type can be read much more easily into R. Compared to above, the code for a single middle c whole note would be :

```
**kern
*clefG2
```

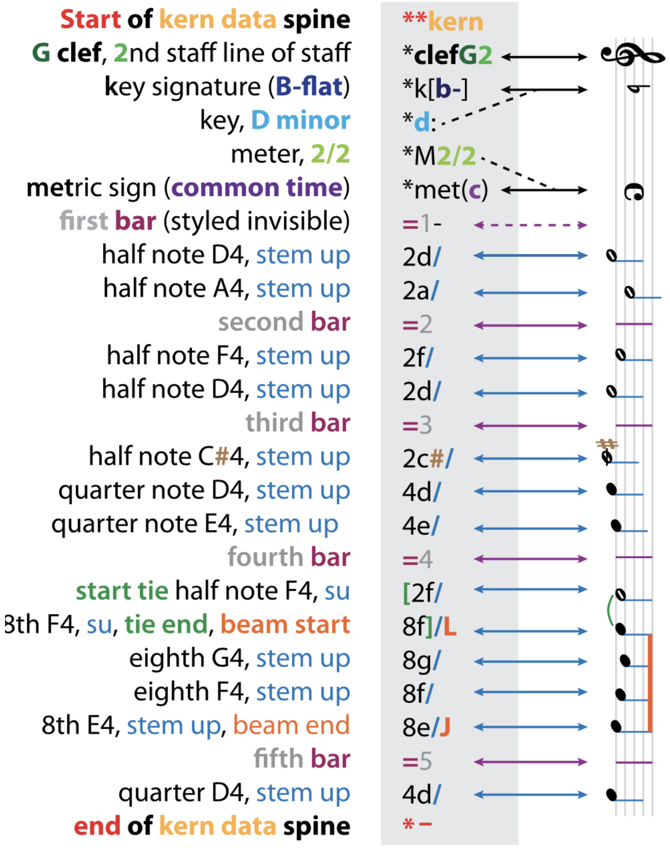


Figure 2.5: How sheet music corresponds to kern files.

```
*k[c]
*M4/4
=1-
1c/
```

The import_xml_files.sh file goes through the process of converting scores from musicXML to .krn. Each spine needs to to be individually converted using xml2hum. The individual spines are then converted into the same time base (there are issues when the bass line only has half notes and the soprano line has a lot of 32nds). This conversion essentially adds dots as placements so that the spines can line up correctly by measure and beat.

The CCARH has a large data base containing work mostly Baroque and Renaissance composers already in the Kern format, which is where the Bach data came from.

The files that were scanned (i.e. all pieces by Felix and Fanny) need to be separated into separate files for each staff, ie a separate file for each instrument. In addition, since we are focused on musical style, the text of the pieces is removed in this stage. In the case of analyzing lieder, each piece always has two or three files. These files consist of voice, piano right hand, and piano left hand. This is necessary to avoid the bugs in `xml2hum` that have issues when staffs don't necessarily match up as a result of the conversion process. This is often caused by an inconsistency in time base.

2.3 Kern to R

Once we have Kern files to represent each piece we use regular expressions to extract key information. For scanned music (Felix and Fanny music), there are as many files are there are staffs, usually three. MuseR's `krn2df()` and `piece_df()` functions read in Kern files and output a data frame in R for each piece. First the data in Kern format are read in line by line using R's `readLines()` function. This takes every line of the Kern file and converts it into a vector. Each entry contains the rhythm value and note value for all notes in that line. If there are multiple notes played at the same time, they are all in one line. The notes are separated by splitting up the string by spaces. This converts a single string representing one Kern line to multiple strings each representing one note (or dot placeholder) for one Kern line. Then each entry is separated out into the theme and note value for each note. Each line contains the following columns: the measure the note occurs in, the rhythm value for the note (for example 4), note name, octave inclusive (for example cc), note name (octave exclusive)(for example C♯). In addition, for the whole piece the key signature and meter are recorded as columns. If there are 3 splines and each spline has at most one note at a time, there would thus be $3 + 3 \times 3 = 12$. If there are 3 splines and one of the splines has at most 3 values, that is equivalent to having 5 total splines there are then $3 + 3 \times 5 = 18$ columns.

A lot of data included in the Kern files are not necessary. For example, we assume that whether or not a note has a stem up or stem down offers no help in classifying composer style, so this information is removed when converting to an R data frame.

Inspired by the Kern file type, each row of the R data frame contains one time base value. For a given piece, the time base represents the shortest note duration value. For example, if the shortest note a piece contained was a sixteenth note, the time base would be 16. Each measure then would contain 16 rows. This results in many rows of NA for certain instruments, when a note is still being voiced, but it is not the instance of the note being attacked.

Chapter 3

MuseR and features

To the best of my knowledge, there is currently no package in R that has been built to analyze sheet music. There are existing packages (such as tuneR) that examine audio formats of music. The intention of this thesis was to create a package, museR, that imports sheet music in the proper form (musicXML or Kern) and does all of the analysis using R.

3.1 Importing data into R

MuseR is equipped to import data in the Kern format. The functions for converting these files are `kern2df()` and `piece_df()`. These are most usefully in the form of individual splines. This allows for naming the columns according to instrument. Figure 3.1 shows short example piece appearing as it would look from MuseScore. This piece would have the Kern format representation as shown in Figure 3.2. We use museR's `piece_df()` function to create the data frame shown in Figure 3.3. Data in R are commonly expressed as data frames. Music as a data structure is very rugged. Expressing music as a data frame has challenges, as music cannot be easily expressed in rectangular form. When expressing it in rectangular form, placeholder or padding entries must be added to account for the nonrectangularness. MuseR's `piece_df()` works by using regular expressions to extract note and rhythm information. It uses NA and . values to indicate empty spaces and duration respectively.

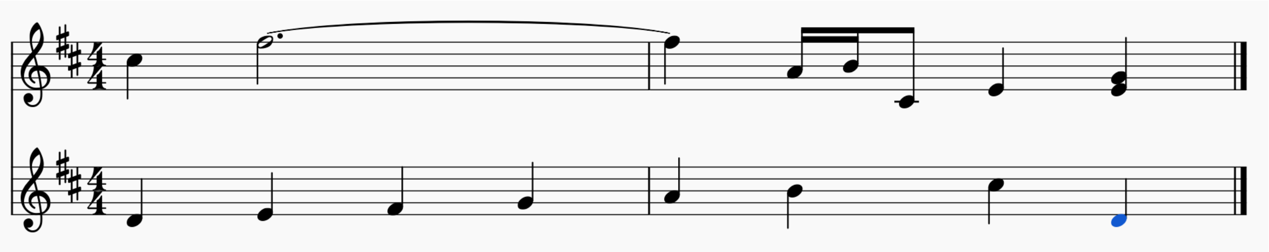
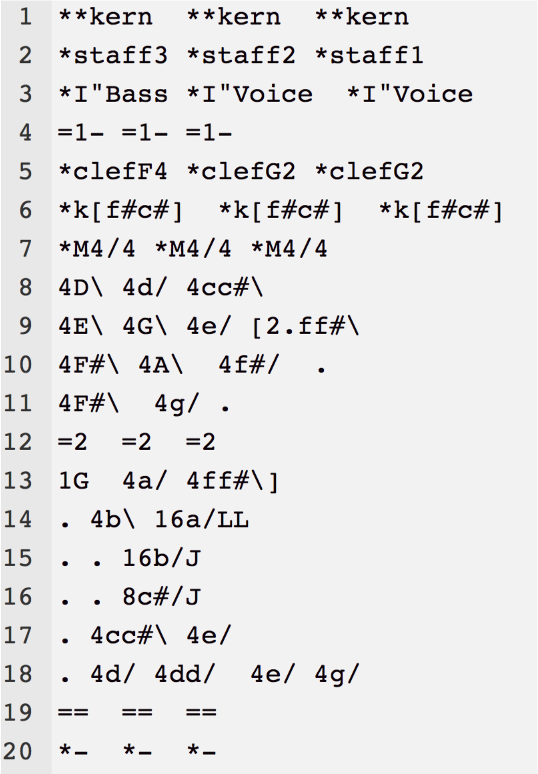


Figure 3.1: Example MuseScore format excerpt



```
**kern  **kern  **kern
*staff3 *staff2 *staff1
*I"Bass *I"Voice  *I"Voice
=1- =1- =1-
*clefF4 *clefG2 *clefG2
*k[f#c#]  *k[f#c#]  *k[f#c#]
*M4/4 *M4/4 *M4/4
4D\ 4d/ 4cc#\
4E\ 4G\ 4e/ [2.ff#\
4F#\ 4A\  4f#/  .
4F#\  4g/ .
=2  =2  =2
1G  4a/ 4ff#\]
. 4b\ 16a/LL
. . 16b/J
. . 8c#/J
. 4cc#\ 4e/
. 4d/ 4dd/  4e/ 4g/
==  ==  ==
*-  *-  *-
```

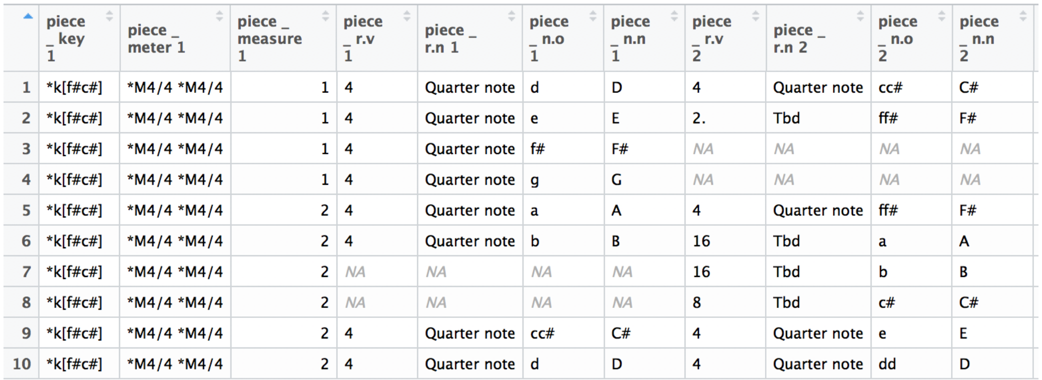
Figure 3.2: Example kern format excerpt of the above MuseScore

3.2 Features currently supported in museR

Melodic intervals

Melodic intervals, or the interval between two successive notes, are found using the `mel_ints()` function. It is currently only equipped to look at melodic intervals for the top note of each staff. In this context, it is most commonly used for analyzing melodic intervals of the voice. The function first extracts the top line of any instrument, and then outputs the proportion of each melodic interval happening over the whole piece. There are 12 possible intervals that are counted (ignoring augmented and diminished): unison, m2, M2, m3, M3, p4, tt, p5, m6, M6, m7, M7. `mel_ints()` outputs a vector of the proportion of each of the intervals. For example if this function was run on the above piece, the melodic top line intervals would be: $\{(f,c),(c,d),(d,f)\} = \{p4, M2, m3\}$, which would output the proportion vector $(0, 1/3, 1/3, 0, 0, 1/3, 0, 0, 0, 0, 0, 0)$.

If we are interested in the types of melodic intervals, we can use `consonance()` to examine the proportion of consonant (perfect, imperfect, dissonant) intervals over the piece. This function works by calling `mel_ints()` and then adding up the perfect, imperfect, and dissonant intervals proportions.



	piece _ key 1	piece _ meter 1	piece _ measure 1	piece _ r.v 1	piece _ r.n 1	piece _ n.o 1	piece _ n.n 1	piece _ r.v 2	piece _ r.n 2	piece _ n.o 2	piece _ n.n 2
1	*k[f#c#]	*M4/4 *M4/4	1	4	Quarter note	d	D	4	Quarter note	cc#	C#
2	*k[f#c#]	*M4/4 *M4/4	1	4	Quarter note	e	E	2.	Tbd	ff#	F#
3	*k[f#c#]	*M4/4 *M4/4	1	4	Quarter note	f#	F#	NA	NA	NA	NA
4	*k[f#c#]	*M4/4 *M4/4	1	4	Quarter note	g	G	NA	NA	NA	NA
5	*k[f#c#]	*M4/4 *M4/4	2	4	Quarter note	a	A	4	Quarter note	ff#	F#
6	*k[f#c#]	*M4/4 *M4/4	2	4	Quarter note	b	B	16	Tbd	a	A
7	*k[f#c#]	*M4/4 *M4/4	2	NA	NA	NA	NA	16	Tbd	b	B
8	*k[f#c#]	*M4/4 *M4/4	2	NA	NA	NA	NA	8	Tbd	c#	C#
9	*k[f#c#]	*M4/4 *M4/4	2	4	Quarter note	cc#	C#	4	Quarter note	e	E
10	*k[f#c#]	*M4/4 *M4/4	2	4	Quarter note	d	D	4	Quarter note	dd	D

Figure 3.3: Example R data frame converted from the above MuseScore

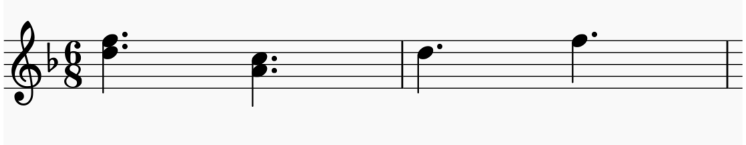


Figure 3.4: Example of calculating proportion of melodic intervals

Density

The `beat_density()` function analyzes the average and standard deviation of density of each measure in the piece. The function is named "beat" density as it only accounts for the instance a note starts. For example if a measure consisted of a single whole note it would be only counted once even though it is voiced the entire measure.

In the above example, the first measure would have a beat density of 4, and the second measure would have a beat density of 2.

Major_minor

For most musical analysis, the key of the piece is important in determining chords, etc. The key is based on the key signature, which is always given in a Kern file. Kern files from CCARH have the key of the piece given, but scanned files do not. For kern files from CCARH, `Major_minor()` extracts the key given by the Kern file. for scanned files, `Major_minor()` identifies the two options for key given the key signature. For example, if there was a key signature with one sharp, the options would be G Major or E minor. The tonic for each option is identified, and then the count of instances of both choices for tonic is made. The key is determined by which of the options for tonic has the higher count.

Scale degree frequencies

Once the key of a piece is determined, the proportion of each scale degree is calculated. The scale degrees consist of: Tonic, Supertonic, Mediant, Submediant, Dominant, Submediant, and Leading tone. In the example above, if we assume the key is F major, the tonic has a tonic scale degree proportion of 2/6, the mediant 1/6, dominant, 1/6, submediant, 2/6.

Frequency of rhythm use

We determine the frequency using `rhythm_freq()` to find the most common types of rhythm: half notes, dotted half notes, quarter notes, dotted quarter notes, eighth notes, dotted eight notes, sixteenth notes, and thirty second notes.

Length

We use `length_measures()` as a feature for the length of a piece. It is calculated by finding how many measures the piece has.

Chapter 4

About Classification Models

Classification methods attempt to divide observations (features) into groups (composer) based on similarities in the observations. A classifier is some function f that maps a vector of input features $x \in X$ to a composer $y \in Y$ or possibly a probability of a composer $P(Y = y) \in [0, 1]$. The notation used in this chapter is inspired by *The Elements of Statistical Learning* (Friedman, Hastie, & Tibshirani, 2001) and *An Introduction to Statistical Learning* (James, Witten, Hastie, & Tibshirani, 2013).

Our feature space X is an $n \times p$ matrix, where n is the size of our data, and p is the number of predictors. Each X_i is vector of values for a certain feature. x_{ij} denotes the i^{th} values of the j^{th} feature. This X contains the information of the extracted features that hopefully will be useful in determining the composer of the piece. If the features are different enough between the composers, i.e., that the features encode some difference of unconscious (or conscious) style, we can then build and fit models that leverage this difference. These models can both explain the relationship and difference of the features between composers, and use the way the fitted model explains the differences to predict the composer of a piece if we know the same features for that piece.

In addition to the features for each piece, each piece has a composer (response), known or unknown, that we denote by Y. The i^{th} piece has composer Y_i where $i \in 1, \dots, n$ In our case we have $Y \in \{\text{Fanny}, \text{Felix}, \text{Bach}\}$, or more generally, $y \in \{\text{list of composers}\}$. Since the options for composer are in a discrete set, we can divide the input features space into different groups, or regions, that are labeled according to the classification of composer a model assigns or predicts.

4.1 Supervised Methods

Supervised methods involve learning about data with knowledge of the response or composer. Classification models rely on supervised techniques to predict composers. These models are built using the observations with their associated response.

4.1.1 Logistic Regression

Knowing the conditional probability $P(Y = k|X)$, or the probability that a piece has a certain composer, given the features of that piece, results in an optimal classification. Logistic regression directly models $P(Y = k|X)$ by using the logistic function. The idea is to model the posterior probabilities of each of the K classes as linear functions in x and requiring that the probabilities sum to 1. As is often the case, we use logistic regression to model a binary response: where there are two options for composer. Thus in the case of a binary response we can use an indicator function with coding 0/1. We then name $p(X) = P(Y = 1|X)$ The model has the form:

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p$$

where which can be written as:

$$p(X) = \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_p X_p}}$$

We estimate the regression coefficients, β_i, by using maximum likelihood. This results in coefficient estimates such that for the predicted probability $\hat{p}(x_i)$ for the composer of each piece corresponds as closely as possible to the observed composer. The log-likelihood for N observations is:

$$\ell(\beta) = \sum_{i=1}^{N}\Big\{y_i \log p(x_i;\beta) + (1-y_i)\log(1-p(x_i;\beta))\Big\} = \sum_{i=1}^{N}\Big\{y_i\beta^T x_i - \log(1+e^{\beta^T x_i})\Big\}$$

We then choose β to maximize $\ell(\beta)$.

Lasso model selection

The Lasso penalty is a shrinkage method proposed by Robert Tibshirani in 1996. Lasso regression works by giving a penalty to the magnitude of regression coefficients. The intercept is not included in the penalty. It can be somewhat equivalent to performing variable selection, as for high enough penalties (or λ), coefficients shrink to zero. It is often used in linear regression, but can be expended to logistic regression and other generalized linear models. For logistic regression, the lasso works by choosing coefficients β_λ^L that minimize

$$\ell(\beta) + \lambda\sum_{j=1}^{p}|\beta_j|$$

where $\ell(\beta)$ is the log-likelihood function for logistic regression. This is equivalent to maximizing $\ell(\beta)$ subject to $\sum_{j=1}^{p}|\beta_j| < s$ (Tibshirani, 1996).

4.1.2 Linear Discriminant Analysis

Whereas logistic regression involves directly modeling $P(Y = k|X = x)$, linear discriminant analysis (LDA), estimates these values less directly by using Bayes

Theorem. When classes are well separated or if n is small and the distribution of predictors is approximately normal, logistic regression estimates can be unstable, which is not the case for LDA. LDA is also popular when there are more than two response cases.

To perform LDA we must first model the distribution of each of the features that make up X in each of the response classes, $P(X = x|Y)$. We denote $f_k(X) = P(X = x|Y = k)$ as the class-conditional density of X in class $Y = k$. We denote the prior for class k, π_k, or the probability that a chosen observation is from the k^{th} class. We have that $\sum_{k=1}^{K} \pi_k = 1$. Using Bayes' theorem to calculate $P(Y = k|X)$ gives us the following:

$$P(Y = k|X = x) = \frac{f_k(x)\pi_k}{\sum_{l=1}^{K} f_l(x)\pi_l}$$

We thus must have a model to find $f_k(x)$. Different discriminant analysis techniques do this different ways. LDA assumes a multivariate Gaussian density, given by:

$$f_k(x) = \frac{1}{(2\pi)^{p/2}|\mathbf{\Sigma}_k|^{1/2}} e^{-\frac{1}{2}(x-\mu_k)^T \mathbf{\Sigma}_k^{-1}(x-\mu_k)},$$

where μ_k is the mean parameter for the kth class and Σ_k is the covariance matrix for the kth class.

In addition, LDA assumes that the covariance matrix is equal for every k: $\mathbf{\Sigma}_k = \mathbf{\Sigma}$ $\forall$ k. Other discriminant models do not make this assumption. We also assume $\hat{\pi}_k = N_k/N$ where N_k is the number of class - k observations,

Using the formula for $P(Y = k|X = x)$ as stated above, we can use LDA's assumption of $f_k(X = x)$ which results in the discriminant function:

$$\delta_k(x) = x^T \mathbf{\Sigma}^{-1}\mu_k - \frac{1}{2}\mu_k^T \mathbf{\Sigma}^{-1}\mu_k + \log \pi_k$$

These functions are known as *linear discriminant functions.* We predict the class by finding the maximum value of the discriminant functions of all k.

4.1.3 Naive Bayes

The Naive Bayes classifier is often used for musical classification as it is good when the dimension p of the features space is large. Like LDA, it also involves modeling $P(Y = y|X = x)$ by using assumptions of the form of $f_i(X)$ and using Bayes Theorem. Naive Bayes makes the (naive) assumption that all the features are independent for a given class i,

$$f_i(X) = \prod_{k=1}^{p} f_{ik}(X_k).$$

These marginals are often estimated by using a Gaussian distribution, ie that $f_{ik}(X_k) = \frac{1}{\sqrt{2\pi\sigma_k^2}} e^{\frac{(x-\mu_k)^2}{2\sigma_k^2}}$. In practice, the independence assumption is not the case, but the model still performs surprisingly well.

4.1.4 k-nearest neighbor

Another method for classification is k-nearest neighbors. It uses observations from a training set of the data. Then for a new observation in a testing set at point $x_0 = x_1 \dots x_p$, it finds the k closest points in the training set, and classifies x_0 as the majority vote of the responses for the K neighbors. Euclidean distance is often used as the metric for closeness, although other methods exist. Euclidean distance is defined as $d = |x_{(i)} - x_0|$. After the K nearest neighbors are found, the predicted class for x_0 is assigned to be the mode of the classes of the neighbors.

4.1.5 Random Forests

Tree-based methods are another form of classifier. Tree based methods involve segmenting the predictor space into smaller regions that are similar in their response. To classify a point x_0, we take the mode of the classes of training set observations in the smaller region where x_0 lies, and assign the class of x_0 as the mode. To create the tree, we recursively split the predictor space into two smaller regions, where the split point is known as a node. Nodes with high node purity are choices where the two resulting regions have mostly the same class in the region. We chose a good split based on the split with highest node purity. The Gini index is often used to measure node purity, defined as $G = \sum_{k=1}^{K} \hat{p}_{mk}(1 - \hat{p}_{mk})$, where $\hat{p}_{mk}$ is the proportion of observations in the mth region of the kth class. If the Gini index is small, this indicates that a region contains mostly observations from a single class.

Random forests offer an improvement over trees in general. They create a “forest” of decision trees on bootstrapped training samples. In each of the trees in the forest, a random sample of m predictors are chosen to create the tree. For random forests, $m = \sqrt{p}$ typically. Because strong predictors are often chosen for the first split of the tree, choosing only a random sample of the predictors to build the tree makes the trees in a random forest significantly different from each other. This choice helps in decorrelating the trees. To predict the class of a piece, we then take the majority vote of the class predicted for each tree in the random forest.

4.2 Unsupervised methods

Unsupervised methods are methods where the response is not used. The methods involve learning structures in the data without any labeled response or composer. Some unsupervised methods, such as PCA, can be used for later models that rely on supervised methods.

4.2.1 Principal component analysis (PCA)

Principal component analysis (PCA) transforms the features space into a lower dimensional representation. It chooses the transformed features to have maximal variance and be mutually uncorrelated.

Principal component analysis can be useful when the predictors are correlated. We suspect many of our features are correlated, due to certain patterns in music, as well as the way we created our features. These relationships are caused by similarity in the features, and from music theory rules. For example, if there is a high frequency of first scale degrees, we might expect a high frequency of chords that include the first scale degree. Another example, if we had a high frequency of seventh scale degrees, we would expect them to resolve to the first scale degree.

Principal component analysis is also helpful when there are many predictors, and we want to deal with a smaller dimension of predictor space. To do this, we choose to use fewer principal components than features, and as principal components are chosen in a way to maximize the variability in the data, so including fewer principal components than features still ideally gives a good representation of the initial data. Used in supervised methods, the transformed features from PCA can be used to fit models instead of the original features.

As an unsupervised method, PCA can inform about latent meta variables. Meta variables are features included in the data that aren't specifically measured by individual features. PCA explores these by giving similar weights to features that are correlated with each other. Thus original features that have similar PCA scores have similar interpretability.

Principal components transforms the feature space. If our original features are $X_1, X_2, \ldots, X_p$, we transform the features to $Z_1, Z_2, \ldots, Z_M$, where $M \leq p$. Each Z_i is a linear combination of the original predictors, ie, $Z_m = \sum_{j=1}^{p} \phi_{jm} X_j$, for constants $\phi_{1m}, \phi_{2m}, \ldots, \phi_{pm}$ for $m = 1, \ldots, M$. Given an $n \times p$ data set $\mathbf{X}$ where x_{ij} is the i^{th} instance of the j^{th} feature, we solve for the m^{th} principal component loading vector $\phi_m = \phi_{1m}, \phi_{2m}, \ldots, \phi_{pm}$ that solves the optimization problem:

$$\max_{\phi_{1m}, \ldots, \phi_{pm}} \left\{ \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{p} \phi_{jm} x_{ij} \right)^2 \right\},$$

where the ϕs are subject to $\sum_{j=1}^{p} \phi_{jm}^2 = 1$. Our principal components are then calculated as $z_{im} = \sum_{j=1}^{p} \phi_{jm} x_{ij}$.

The loadings of the first principal component, ϕ_1 thus determine the direction in the feature space with the most variance, Z_1, or the scores of the first principal component is then a new feature in our transformed feature space. We continue calculating Z_i, where each following Z_i has the maximal variance in a direction uncorrelated to the previous principal components.

Before PCA is performed, we center and scale all features to have mean zero and standard deviation one, as initially the scale of some features are not the same. This would lead to issues in the loadings, as the features with higher scales would automatically have the higher variance.

We can observe the proportion of variance explained by each principal component. This is usually visualized in a skree plot. The number of the principal component is plotted on the x-axis, and the percentage of variance which that principal component accounts for is on the y-axis. We can use this information to decide how many principal components to use. We often choose the cut-off principal component at an "elbow", or where the decrease in variance explained by an additional principal component starts to decrease.

4.2.2 K-means

K-means is a form of unsupervised learning where we only use the features without the associated class of composer. When given a K or number of clusters, the algorithm assigns each piece to a cluster. This is used to see if there are any latent groupings in the feature space. If composers are differentiable by their features, we might expect that K means would differentiate the clusters similar to the difference in composer. K-means clustering minimizing the within-cluster variation $W(C_k)$ for each cluster C_k. Often we define the within-cluster variation by squared Euclidean distance, so

$$W(C_k) = \frac{1}{|C_k|} \sum_{i,i' \in C_k} \sum_{j=1}^{p} (x_{ij} - x_{i'j})^2$$

where $|C_k|$ is the number of observations in the kth cluster.

Chapter 5

Exploratory Data Analysis

Before fitting any classifiers, we first want to perform exploratory data analysis to examine visually any patterns or separations between variables and composer. This is done by looking at individual density plots of each variable for each composer, looking at correlation structures in the features, and using PCA as an unsupervised method to gain insight into the feature space. We examine two cases, features for Bach/not Bach, where not Bach is the pieces of Fanny Hensel and Felix Mendelssohn combined. We denote this as Bach/Mendelssohn. We also examine the features for Fanny/Felix.

5.1 Key for feature abbreviations

Table 5.1: Abbreviations for features and corresponding meanings. The features of Fanny and Felix are the same but begin with f.

name	meaning
dens_.	Density of notes: mean and standard deviation
cons_.	Type of melodic intervals: imp(imperfect), dis(dissonant), perf(perfect)
rf.	Rhythm frequency: 2(half), 2d(dotted half), 4 (quarter), 4d(dotted quarter), 8(eighth), 8d(dotted eighth), 16(sixteenth) ,32(thirty second)
sf_.	Scale degree frequency: 1 to 8
len	Length of the piece

5.2 Bach and Mendelssohns

Figure 5.1 shows density plots for each of the predictors used. We can see some difference of that feature being used by composer in the density features, and some of the rhythmic frequency features, especially use of eighth notes and sixteenth notes. The peaks for Bach are mostly narrower than those for the Mendelssohns. One can suspect the difference in density is partially caused by the type of piece of Bach and Mendelssohn. We might expect that solo pieces, such as those in the Bach data set would have lower densities then for lieder, as in the Mendelssohn data set, which also include voice.

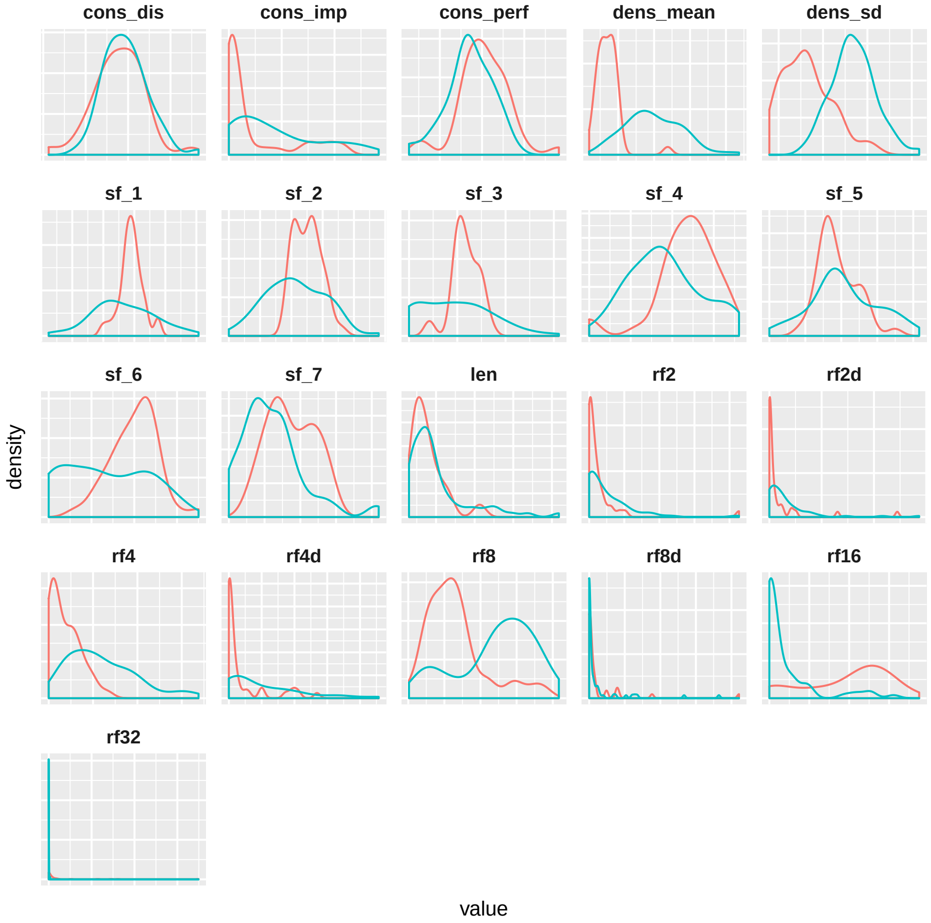


Figure 5.1: Density plot for each feature of Bach/Mendelssohn. Red represents Bach and blue represents the Mendelssohns.

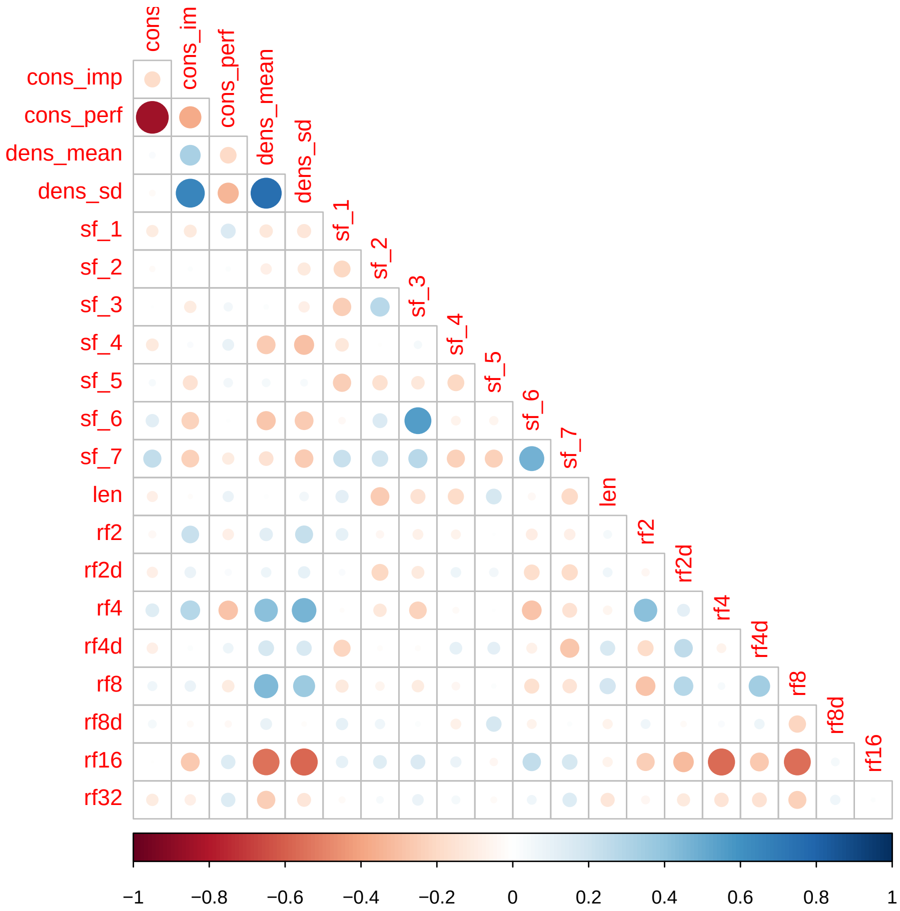


Figure 5.2: Correlations between features of Bach/Mendelssohn.

Figures 5.2 is a visualization of the correlation matrix. The size of the circle represents the absolute value of correlation between the two predictors. We can see higher correlations between perfect consonances and dissonant consonances. This is to be expected, as a higher frequency of perfect consonances necessarily means a lower frequency of dissonant consonances. We also see high negative correlations between frequency of sixteenth notes and density, and frequency of eight notes. In addition we see high positive correlations between frequency of second, third, sixth, and seventh scale degree. It is likely that these negative correlations are due to structures in music theory or sensible melodic compositional technique.

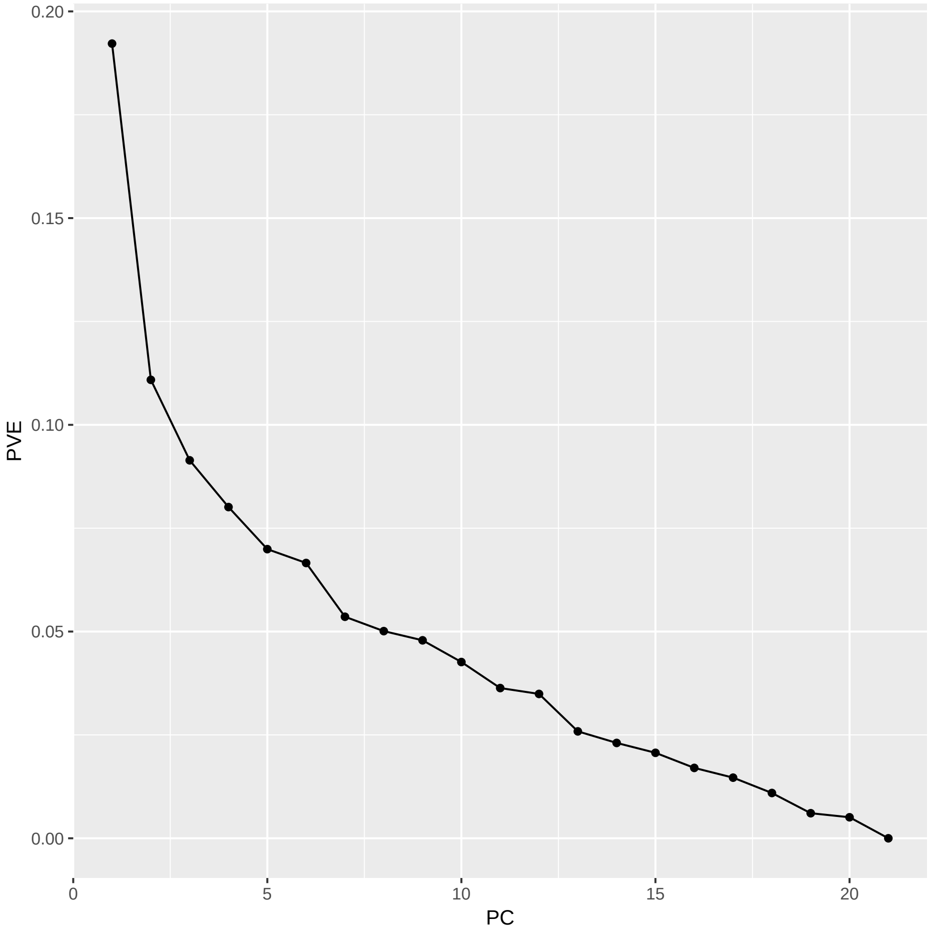


Figure 5.3: Skree plot of the PCA's.

Figure 5.3 shows the skree plot of the principal components of Bach and Mendelssohn. For each principal component used (shown on the x-axis), we have the corresponding percentage of variance explained (y-axis). When using PCA for dimension reduction, we often look for an elbow in the skree plot. This is done to choose the smallest number of principal components needed to explain a sizable amount of the variation in the feature space. There is a possible elbow at the second principal component. This indicates that we might be able to perform analysis using only the first two principal components. However, especially considering that our data are not very separable, we likely would likely want to use more information in our model. As a similar result occurred for Brinkman, in our analysis we will use the principal components that account for 85% of the variance. In the Bach/Mendelssohn case this corresponds to using 11 principal components.

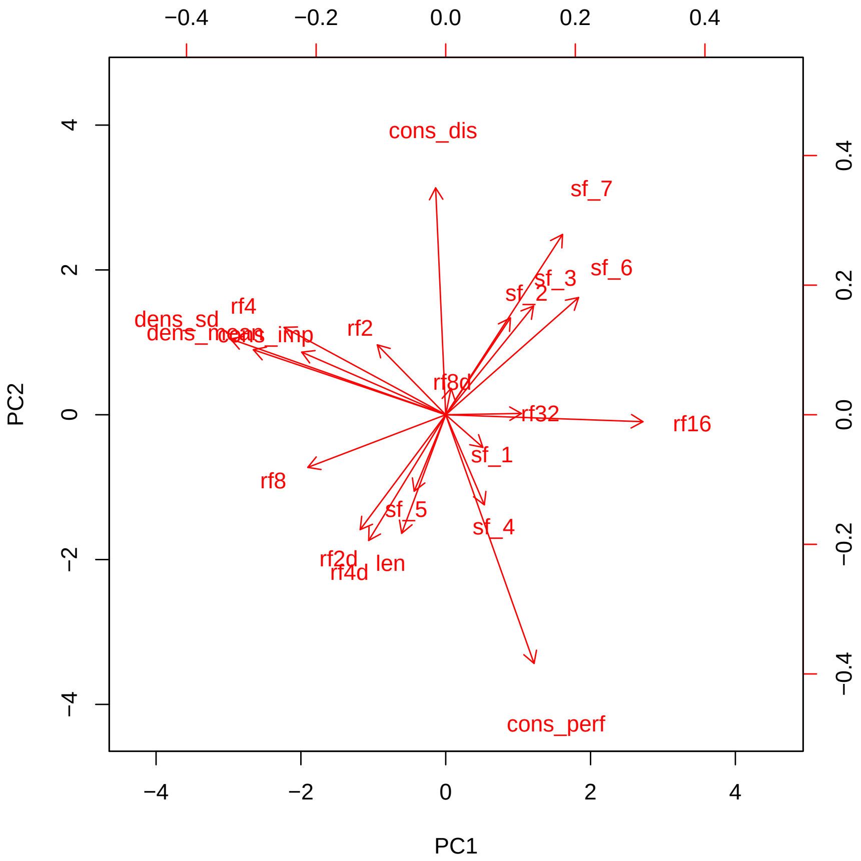


Figure 5.4: Biplot of the loading vectors of the first two principal components.

The biplot in Figure 5.4 shows the loading vectors plotted on the first two principal components. The loading vectors of features seem to arrange into approximately three groups. The features: perfect consonances of melodic intervals of the voice, frequency of the first scale degree, frequency of the fourth scale degree are all grouped together. Similarly, frequencies for the 7th, 6th, 2nd, and 3rd are grouped together. In addition we have perfect consonances and imperfect consonances on two sides of the second principal component. It appears that the first principal component seems to encode tempo/rhythm. Faster rhythmic notes, like sixteenth notes and thirty second notes have opposite loadings than slower rhythmic notes. The second principal component appears to encodes harmonic structure, especially differences in consonances.

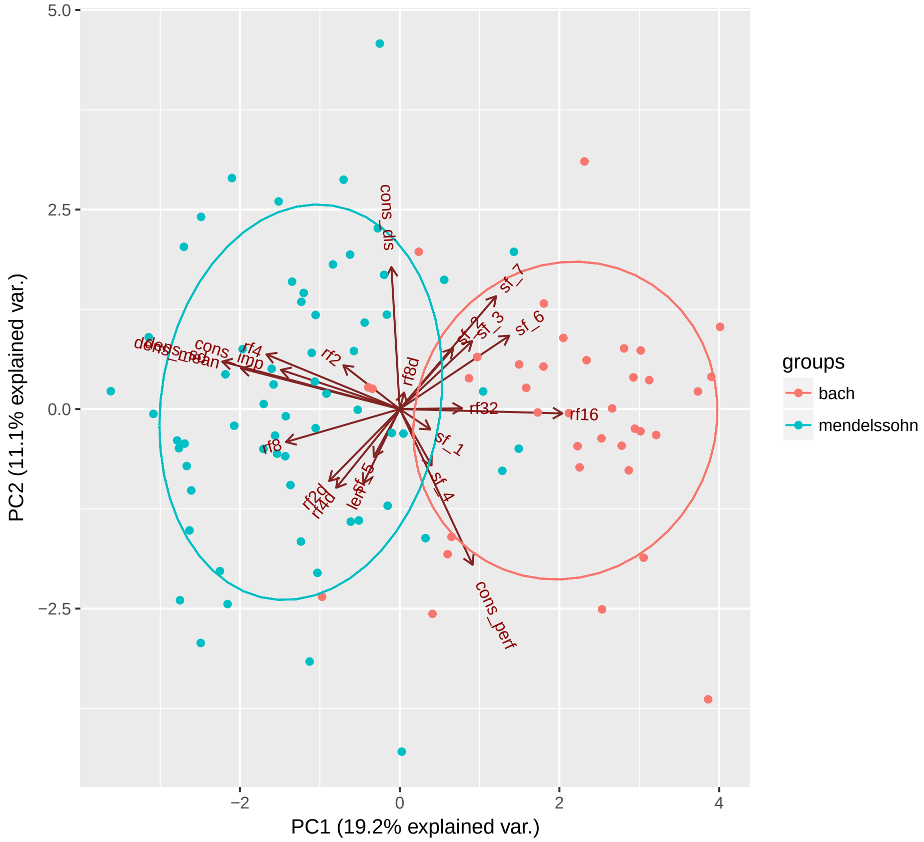


Figure 5.5: Biplot of the first two principal components plotted with data points colored by composer.

Figure 5.5 shows the same loadings of the principal components, but with the addition of points representing each piece graphed by their first two principal components. Each piece is colored by composer. The ellipses represent a 95% concentration area, or where 95% of the data lie. We can see that the ellipses only overlap slightly, indicating that pieces by Bach and Mendelssohn usually have different values in the first and second principal component. Figure 5.6 shows the pieces plotted on the second and third principal component and there is much less separation. The third principal component also appears to encode rhythmic information. We have that dotted rhythmic frequencies have opposite loadings on the third principal component to their corresponding non dotted rhythm. For example, frequencies of eight notes have an opposite loading on the third principal component to the frequency of dotted eighth notes.

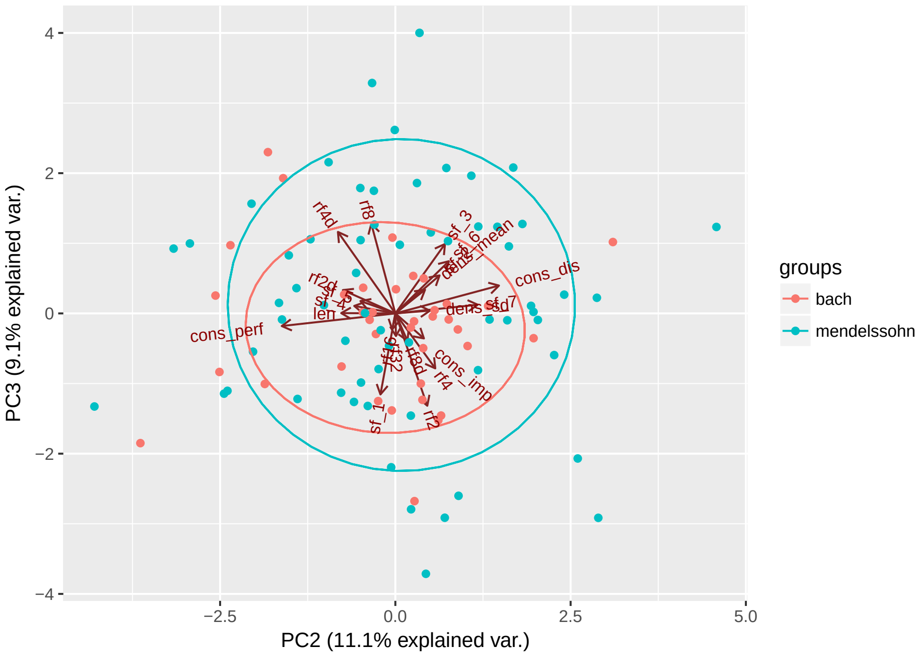


Figure 5.6: Biplot of the second and third principal components plotted with data points colored by composer.

We can also use K-means clustering to see if there are any apparent groups in the data. When we run K-means on the first 11 principal components of Bach and Mendelssohn with K = 2 (since we are comparing two classes), we get the results we see in Figure 5.7. The coloring of the points indicate which cluster the K-means algorithm assigned to each piece. The label of each piece indicates the actual composer. We can see that for the most part, for pieces with high or low scores on the first principal component, the clustering preserves the composer. For pieces with middle scores of the first PC, there is a good number of pieces with a cluster assignment not consistent with composer.

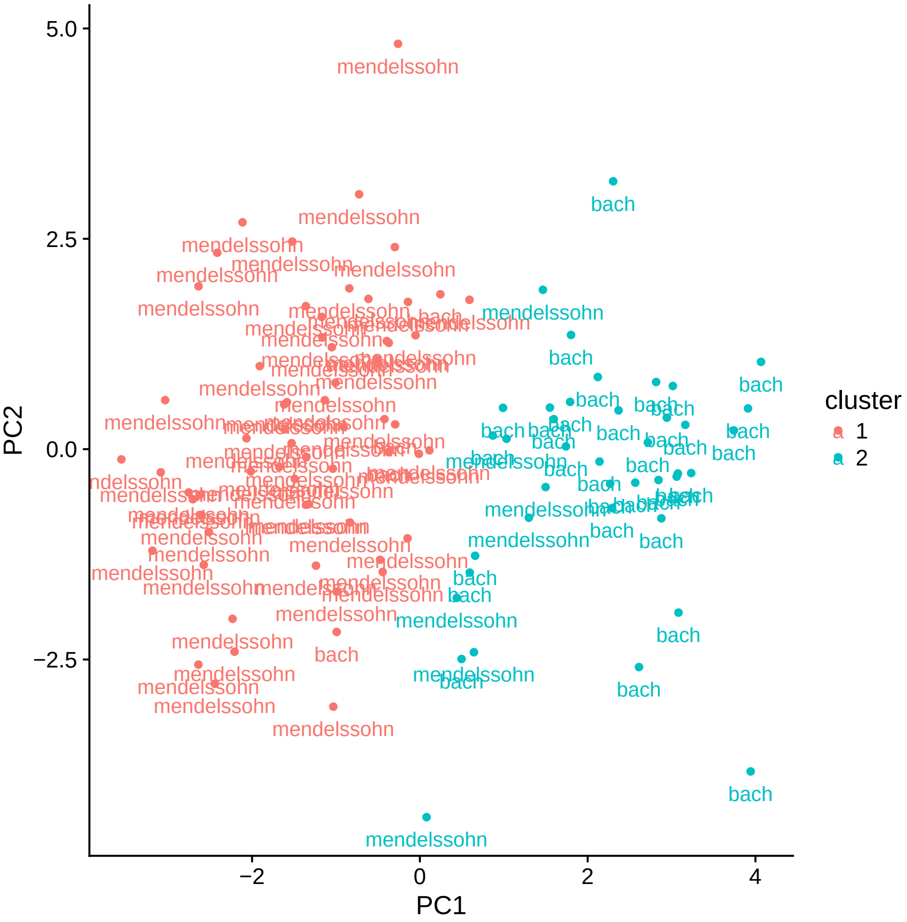


Figure 5.7: KNN when k = 2.

We can also examine when $K = 3$, indicating that the K-means algorithm should look for three clusters. This is reasonable as the Mendelssohn set is made up of Fanny and Felix, so there are actually three composers. The clusters seem to be mostly assigned by the value of the first principal component. For the most part, pieces by Bach are assigned to the third cluster, although there are a couple in the other two clusters.

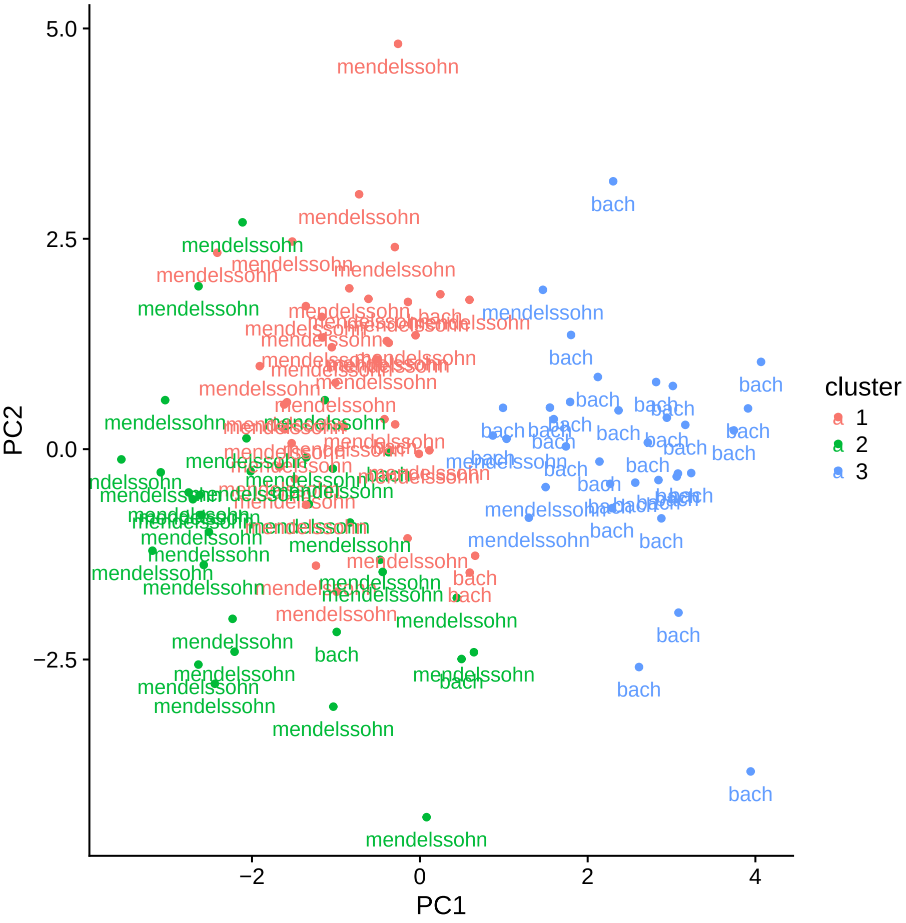


Figure 5.8: KNN when $k = 3$.

5.3 Felix and Fanny

As we can see from density distributions in Figure 5.9, most of the features used have very little separation between the two composers. Most of the density distributions for features from pieces of Fanny and Felix overlap completely and have nearly identical peaks. There is a small difference in peak for the frequency of use of the first scale degree, with Fanny using it more often. The lack of difference in features between the two composers is not very encouraging for later fitting of models.

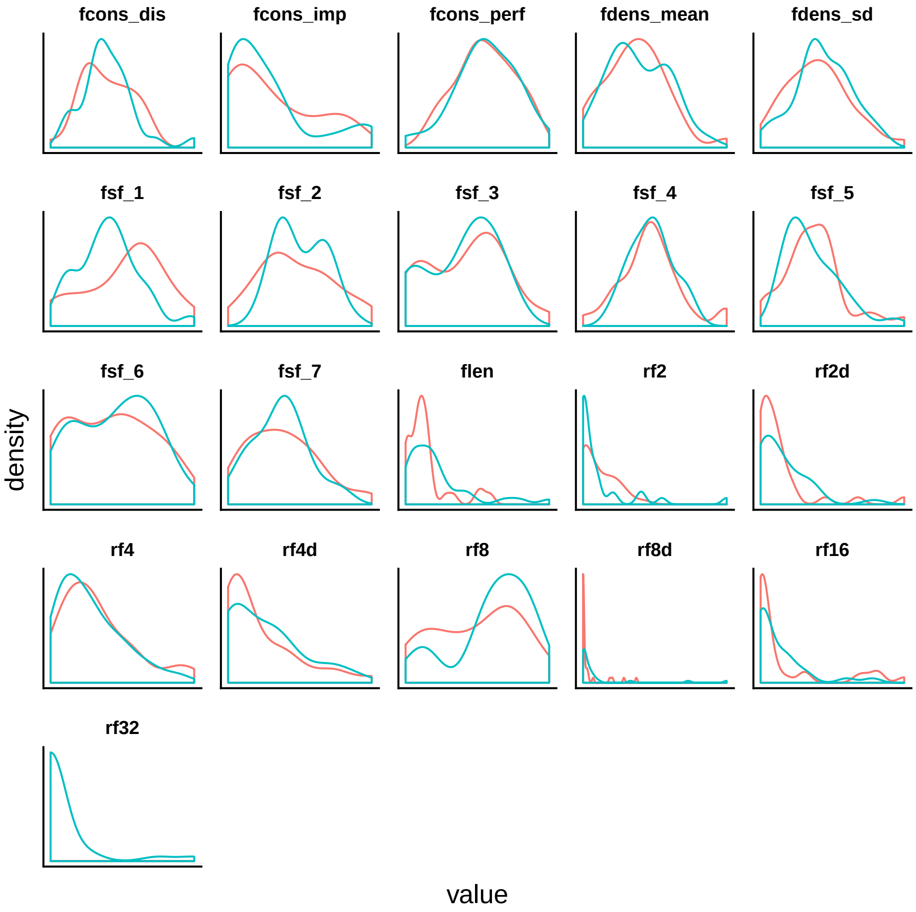


Figure 5.9: Density plot of each feature of Fanny/Felix. Blue represents Fanny and red represents Felix.

The correlations of the Fanny/Felix features are shown in Figure 5.10. The rhythmic features do not seem to have as much correlation as the Bach/Mendelssohn features. We still see higher correlations for frequencies of types of melodic consonances, and frequencies of scale degrees 2, 3, 6, and 7.

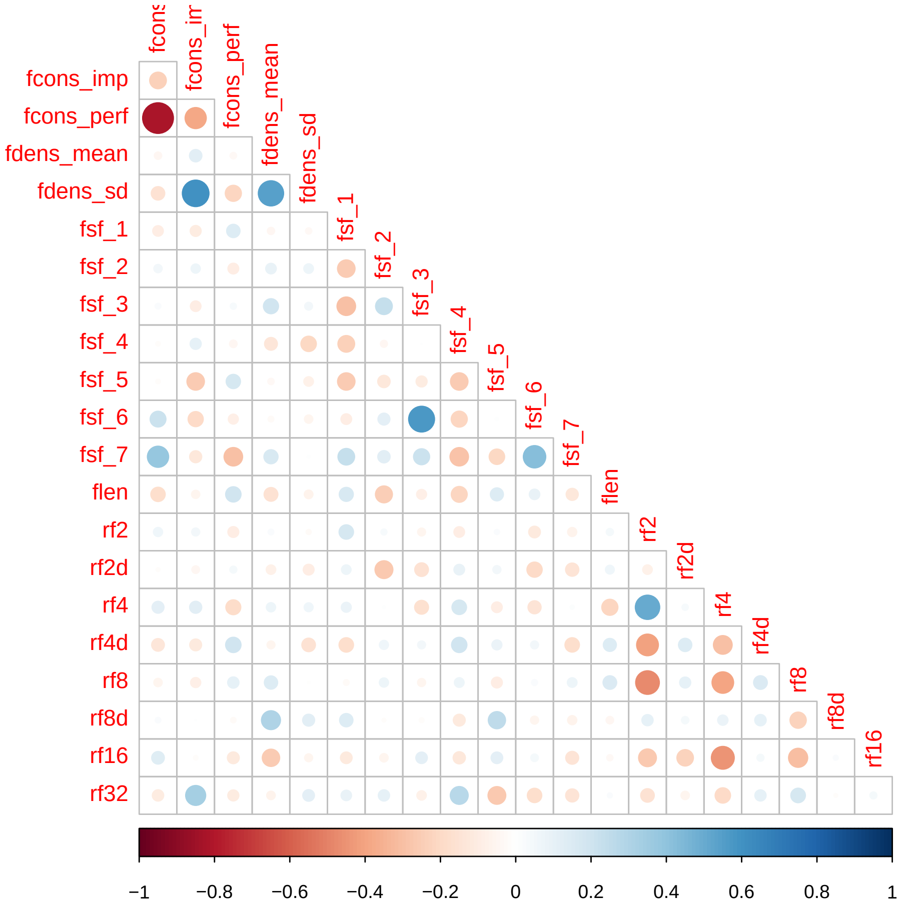


Figure 5.10: Correlations between features of Fanny/Felix.

Figure 5.11 shows the skree plot for Fanny/Felix. It can be argued that there is an elbow at principal component thirteen, but it is not that clear. Using 11 principal components again accounts for 85% of the variance, so we will use 11 PCs in fitting our classifiers as well.

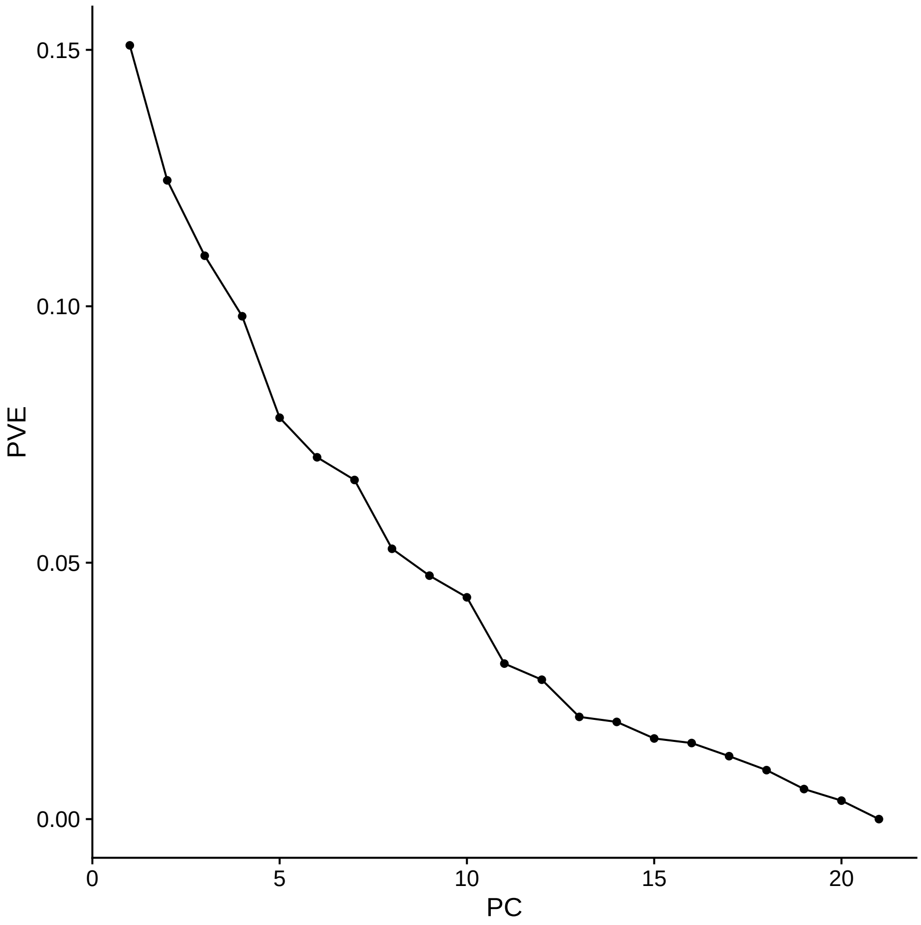


Figure 5.11: PCA Felix/Fanny skree plot.

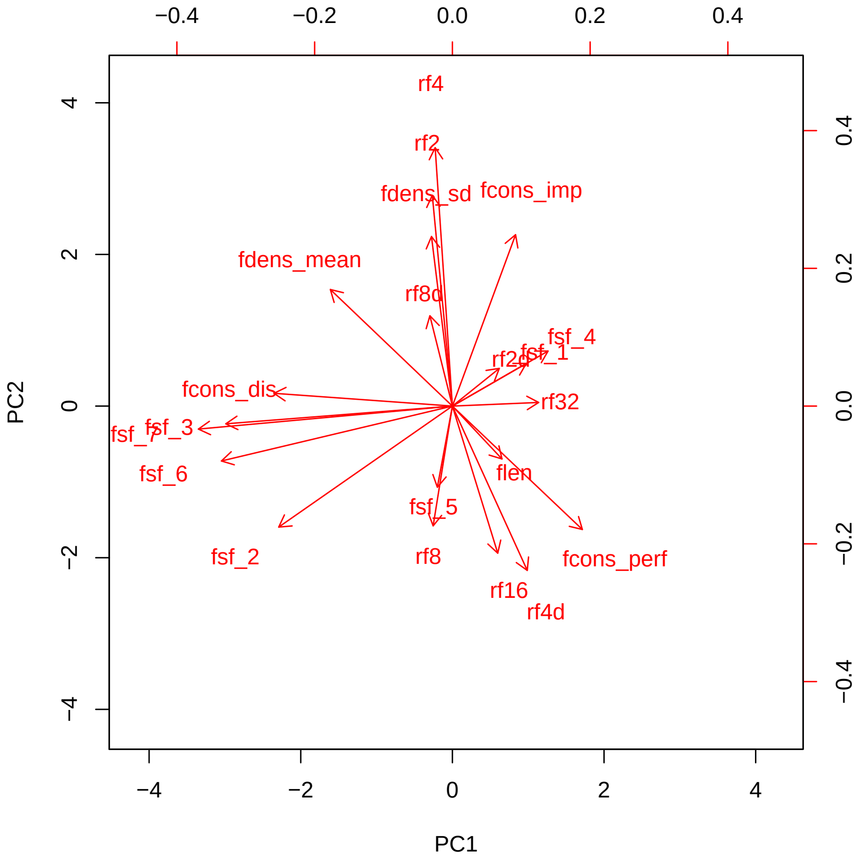


Figure 5.12: PCA Loadings of the features for Fanny/Felix.

Figure 5.12 shows the loadings of the principal components of the features of Fanny and Felix. In contrast to the biplot for Bach/Mendelssohn, it seems like the first principal component encodes harmonic aspects of music, and second principal component encodes rhythm. The loadings for scale degrees 1, 5 and perfect consonant intervals are in opposite direction on the second principal component to scale degrees 2, 7 and dissonant intervals. Also we see that frequencies for faster rhythmic values have opposite loadings to those of slower notes on the second principal component.

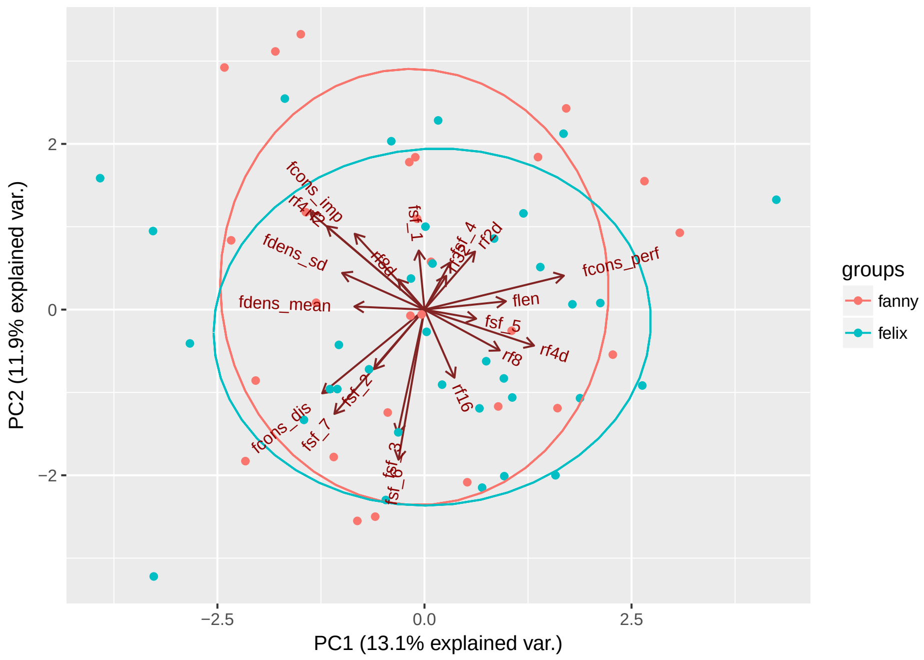


Figure 5.13: Loadings and pieces for Fanny/Felix.

When the pieces colored by composer are plotted on the biplot as shown in Figure 5.13, we do not see as much separation between the Fanny and Felix pieces as we did for Bach/Mendelssohn. The 95% ellipses almost completely overlap. This lack of separation for the two groups might lead to issues in creating a model to classify.

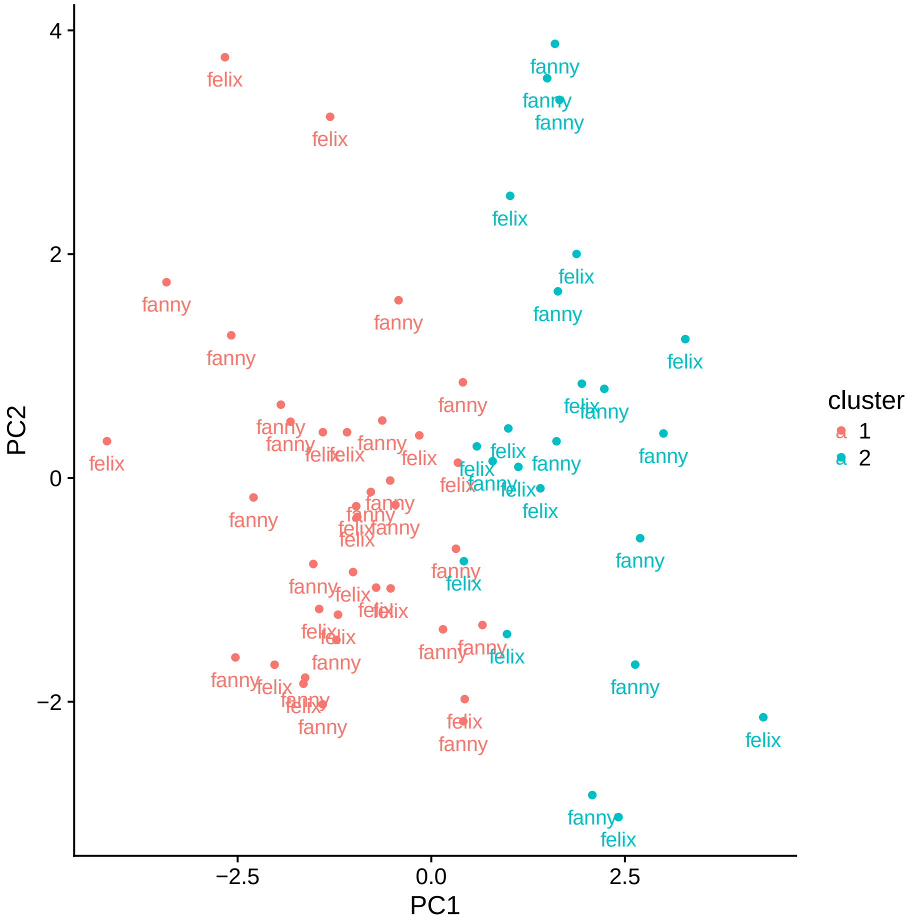


Figure 5.14: K-means with k = 2.

We again run K-means to identify clusters in the data. Ideally, pieces would be clustered if they shared the same composer, even though the K-means algorithm does not have access to the composer information. Figure 5.14 shows the results of K-means with K = 2. Both clusters contain pieces written by Fanny and Felix. This indicates that when clusters exist in the data, they do not correspond naturally to composer identification.

Chapter 6

Classification Results

The following section presents the results of fitting five different models for classficiation on the original feature space as well as the first 11 principal components to account for 85% of the variance. This was done as there are many features that measure aspects of the same thing, i.e. there are seven different features measuring aspects of scale degree use where each scale degree can individually have interesting musical interpretations.

The five classifiers were chosen as they were used previously in the literature for similar problems (KNN, Naive Bayes and Random forests), or are popular models for classification (Logistic, LDA). For the logistic classifier we used lasso as a shrinkage method.

Each model (10 total) was tested for predictive accuracy by finding the 5−fold cross-validated misclassification rate. The cross-validated misclassification rate showed high of variance depending on the random fold assigned, so the process was repeated 100 times, and the rate was averaged.

6.1 Models Results - Bach/Mendelssohn

Table 6.1: Averaged 5-fold CV misclassification rates for each model from 100 runs on the feature space and on the first 11 PCs.

Model	misclass. rate - features	misclass. rate - 11 PCs
Logistic	0.078	0.097
LDA	0.092	0.049
KNN (K = 9)	0.097	0.098
Naive Bayes	0.103	0.112
Random forest	0.058	0.111

Table 6.1 shows the averaged 5-fold cross-validated misclassification rates over 100 runs. For the KNN case, the features were scaled and centered first. Most models have misclassification rates round 10%. The best model was found to be linear discriminant analysis run on the principal components.

Figure 6.1 shows coefficient values for each feature for varying log(λ) values. λ corresponds to the restriction penalty. Higher values of log(λ) correspond to a higher penalty, resulting in coefficient estimates that shrink to zero. The cross-validated lasso logistic fit chose log(λ) $= -6.2$ to have the lowest misclassification rate. At that value, most features besides the third and fifth scale degrees and frequency of eighth note are non-zero. We have that scale degree two and mean density features have low coefficient estimates at this point, but are non-zero. Overall, for all λ penalties, it appears that the features for density and frequency of 16th note rhythms stay non-zero the longest, implying these features are more useful features for distinguishing the composer.

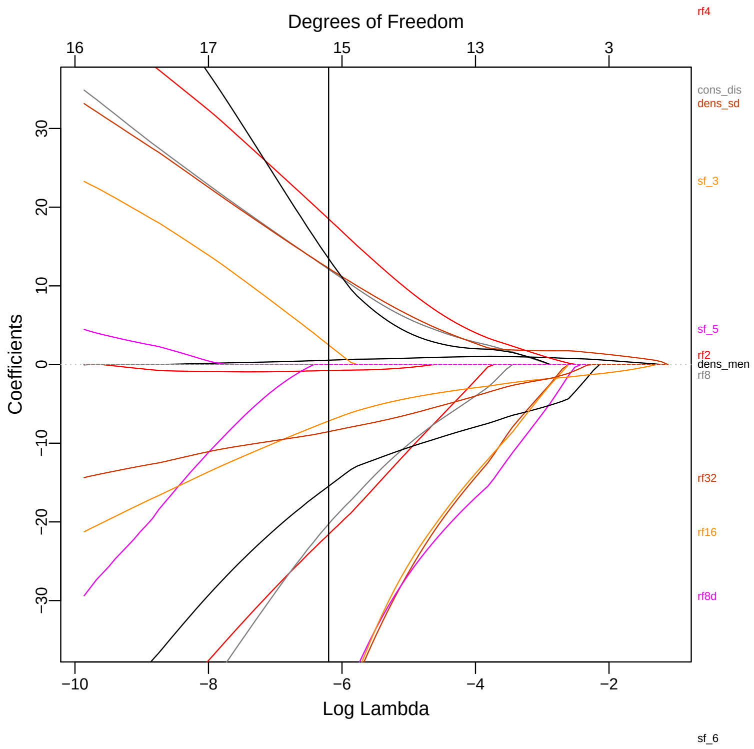


Figure 6.1: Bach/Mendelssohn lasso logistic regression coefficients for changing λ penalty values. The vertical line represents the value of best λ.

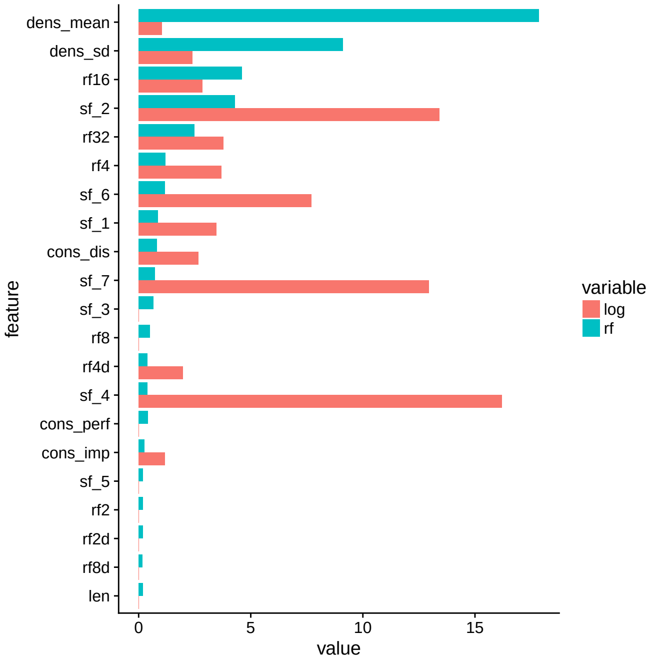


Figure 6.2: Random forest variable importance and logistic regression coefficients and best λ Bach/Mendelssohn.

Figure 6.2 shows the variable importance rankings as a mean decrease in the Gini index for the random forest model. It also shows the absolute value of the logistic regression coefficients at the best value for λ. Note that we look at the relative height of the bars here to determine importance. We can see that while the random forest does not view frequencies of dotted quarter notes to be a very useful predictor, the logistic model views it as very important. The density features are the most important in deciding splits of the trees, although these values do not have high coefficients in the logistic. Both models agree that the last seven variables are not important. They were shrunk to zero in the logistic lasso model.

6.2 Model fit Felix/Fanny

Table 6.2: Averaged 5-fold CV misclassification rates of 100 runs on the feature space and on the first 11 principal components for each model.

Model	misclass. rate - features	avg. misclass. rate - 11 PCs
Logistic	0.467	0.45
LDA	0.518	0.505
KNN	0.498	0.467
Naive Bayes	0.502	0.521
Random forest	0.535	0.571

Table 6.2 shows all the misclassification rates of each model. Most of the misclassification rates are above 0.5, indicating that random guessing would perform better at predicting the composer. A logistic lasso classification model fit on the principal components performed best, with a misclassification rate of 0.45.

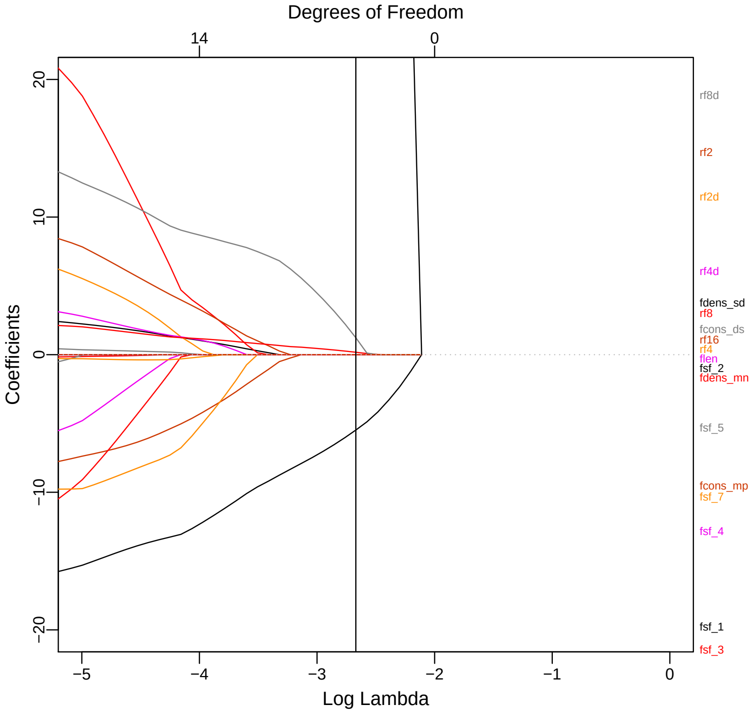


Figure 6.3: Fanny/Felix lasso logistic regression coefficients for changing λ penalty values. The vertical line represents the value of best λ.

Figure 6.3 shows the coefficient estimates for varying $\log(\lambda)$ penalties for a logistic lasso model fit on the feature space. The frequency of the first scale degree appears to remain non-zero for the longest time. At $\log(\lambda)$ of -2.67 we get the best λ fit. At this $\log(\lambda)$ value we have most of the coefficients shunk to zero besides frequency of thirty second notes and dotted eighth notes, and frequency of the tonic (first scale degree).

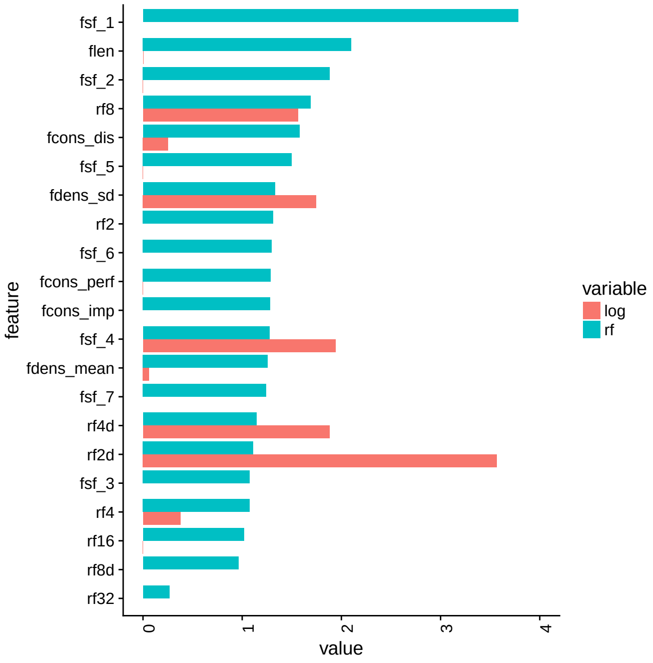


Figure 6.4: Random forest variable importance and logistic regression coefficients and best λ Fanny/Felix.

Figure 6.4 shows the variable importance used in random forests and the logistic regression coefficients at the best λ value (-2.67). At the best λ value, only nine of the coefficients of logistic regression are non-zero. The features that the random forest model found useful did not correspond very well to the features remaining non-zero in the logistic lasso model.

6.3 Predictions for disputed pieces

Since we have such high misclassification rates using each model, the following predictions are likely not accurate. Even so, we do see that most of the pieces have a higher proportion of predictions for Fanny than Felix. Table 6.3 shows the predicted classification for each piece for each model.

Table 6.3: Predicted composer for each disputed piece

Model	Op.8 2	Op.8 3	Op.8 12	Op.9 7	Op.9 10	Op.9 12
logistic-lasso	fanny	fanny	felix	fanny	felix	fanny
LDA	fanny	fanny	fanny	felix	felix	fanny
KNN	felix	fanny	felix	felix	fanny	felix
Naive Bayes	fanny	fanny	fanny	fanny	fanny	fanny
Random Forest	felix	felix	felix	fanny	fanny	felix

6.4 Discussion

On very basic low-level features, consisting mostly of frequencies of notes, intervals and chords, most models comparing Bach to the Mendelssohns do relatively well. This is likely due to the decent separation of the features encoding density. We likely see so much separation because the Bach data are for solo piano and the Mendelssohn data have an additional instrument, thus making the piece automatically more dense.

On the other hand, models fit to compare Felix and Fanny did not do as well. They are only very slightly better than random guessing. This could be because there is no true difference between Fanny and Felix in the features we extracted, i.e., the features extracted are not good enough to pick up any existing signal. On the other hand, it is certainly believable that Fanny and Felix could have very similar unconscious signals in their writing. They were trained together, and did critique each other's work extensively. The other possibility is that there are more pieces by Fanny snuck into Felix's published work, leading to overlap in the extracted features. These features did accurately predict composer for the previous comparison (Bach/Mendelssohn), but perhaps composers so similar in style cannot be differentiated using these features. The included features only encode very basic aspects of music, and are more based on frequencies than how music actually seems to differ in style to a listener, such as features for uses of melodic phrasing.

Running the classfication models on the first eleven principal components resulted in a slight improvement for two of the models in the Bach/Mendelssohn comparison, and a slight improvement for three of the models in the Fanny/Felix comparison. For LDA, at least, we know LDA is a model that is sensitive to collinearity, and running on the principal components helps with this. For the models where no improvement was made using the principal components, it is possible that those models did better with more information included. Perhaps we should have chosen more principal components to account for more explanation of variance to be used in models.

Even though our classifiers for Felix/Fanny did not perform very well, it does seem like there is a good amount of musical interpretation contained in the features used. The first three principal components of the features seem to correspond to harmonic meaning, rhythmic meaning, and dotted vs not dotted notes.

Conclusion

Fitting classifiers using low-level features composed mostly of frequencies of notes and rhythms was able to differentiate style between Bach and Fanny and Felix Mendelssohn. However classifiers using these low-level features were not successful in differentiating between Fanny and Felix. Of the ten models fit on pieces by Fanny and Felix, only 4/10 did better than random guessing. The $k-$nearest neighbor classifier and lasso logistic performed the best. Thus the low-level features used in this study were not able to measure style differences between Fanny and Felix. This is likely either to the fact that these features were not able to pick an existing signal, or that Fanny and Felix have no discernible difference in style, although the first is more likely. Most of the previous studies regarding composer identification were able to distinguish composer. The work by Brinkman when comparing Josquin to contemporaries did have substantial error in assigning composers although not as bad as our classifiers performed, except for de Orto. Josquin and his contemporaries however, did not grow up together or receive identical music training, like Fanny and Felix did. Also analyzing music in the Romantic era has not been done before, and perhaps different kinds of features need to be created to train a successful classifier.

6.5 Future suggestions for research and expansion of museR

Most suggestions for future work lie in expanding the capabilities for museR. One issue to be improved upon is increasing the ease of importing data. The scanning/OMR step is incredibly time consuming, which lead to a limited number of pieces being used in this analysis. Running this same analysis with more data could also lead to better results and lower errors, or at least errors with smaller variance. While museR cannot directly help with speeding up the scanning process, museR could be expanded to be able to import music in the musicXML format in order to not need to rely on Humdrum's conversion program. Future versions of museR will ideally be able to extract high level features. These are features that take into account the context of the music from where the feature is extracted. These features could include more harmonic features, such as chord progressions, and more chord analysis. These high level features are what humans generally pick up on when distinguishing style. In addition, in the future, museR should be able to implement windowing techniques for extracting features. These features could still be low-level features, but they might have more meaning in determining style as they are on a smaller scale instead of averaged across the entire piece.

In addition, it would be interesting to perform this same analysis on music of different types of pieces. We might expect unconscious signals to exist in the same way in symphonies as well as lieder, or perhaps not. Considering features that are similar between symphonic work and solo piano works is a challenging problem in of itself. Also, including another composer in this study, perhaps a contemporary of Felix and Fanny, or at least a composer in the Romantic era would also be helpful in checking the validity of the features. Also, it would be interesting to examine composers throughout their career. Composers, Dvorak for example, changed drastically their career. Maybe composers have different aspects of style when they first started composing than when they were older. Ideally, museR could be used to pick up on these differences.

Appendix

How to download museR

The following commands download `museR` from GitHub.

```r
library(devtools)
install_github("empalmer/Thesis/museR")
```

How to access code used in analysis

The R code used in this analysis can be found at `https://github.com/empalmer/Thesis/Models_EDA`

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