

Hessian:

$$X_{pq \times np}^t \Delta_{npq \times np} A_{np \times np}^{-1/2} R_{np \times np}^{-1} A_{np \times np}^{-1/2} \Delta_{npq \times np} X_{np \times pq}$$

Where

- $\Delta_i = \left(\frac{\partial \boldsymbol{\mu}_i}{\partial \boldsymbol{\beta}} \right)_{pq \times p}^t = \frac{1}{\alpha_{i0}^2} (\alpha_{i0} \text{diag}(\alpha_i) - \alpha_i \alpha_i^t) \otimes \mathbf{x}_i$
- $\Delta = \text{blockdiag}(\Delta_i)$
- $A_{jj_i} = 1/\sqrt{v}$, where v is the variance of a Dirichlet RV for a given α_{ij}
- $A = \text{blockdiag}(A_i)$
- R : working correlation matrix.

After Week 3 meeting:

The GEE equations (gradient) (in block matrix form) are:

$$\Delta V_i^{-1}(\mathbf{Y}_i - \boldsymbol{\mu}_i) = \mathbf{0}$$

and the hessian is

(Because we are using fisher scoring and the expectation of $(Y_i - \mu_i) = 0$)