

Define a bilinear map $E : \mathcal{V} \times \mathcal{V} \rightarrow \mathcal{V}$ with the following properties:

- i. Bilinear property: $E(ax + by, z) = aE(\mathbf{x}, \mathbf{z}) + bE(\mathbf{y}, \mathbf{z})$, $E(\mathbf{x}, ay + bz) = aE(\mathbf{x}, \mathbf{y}) + bE(\mathbf{x}, \mathbf{z})$ where $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathcal{V}$ and $a, b \in \mathbb{F}$.
- ii. Antisymmetric property: $E(\mathbf{x}, \mathbf{y}) = -E(\mathbf{y}, \mathbf{x})$.
- iii. Orthogonal property: $g(E(\mathbf{x}, \mathbf{y}), \mathbf{x}) = g(E(\mathbf{x}, \mathbf{y}), \mathbf{y}) = 0$ where $g : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{F}$ is the non-degenerate, bilinear form that defines the inner product between two vectors.