PORTFOLIO 1

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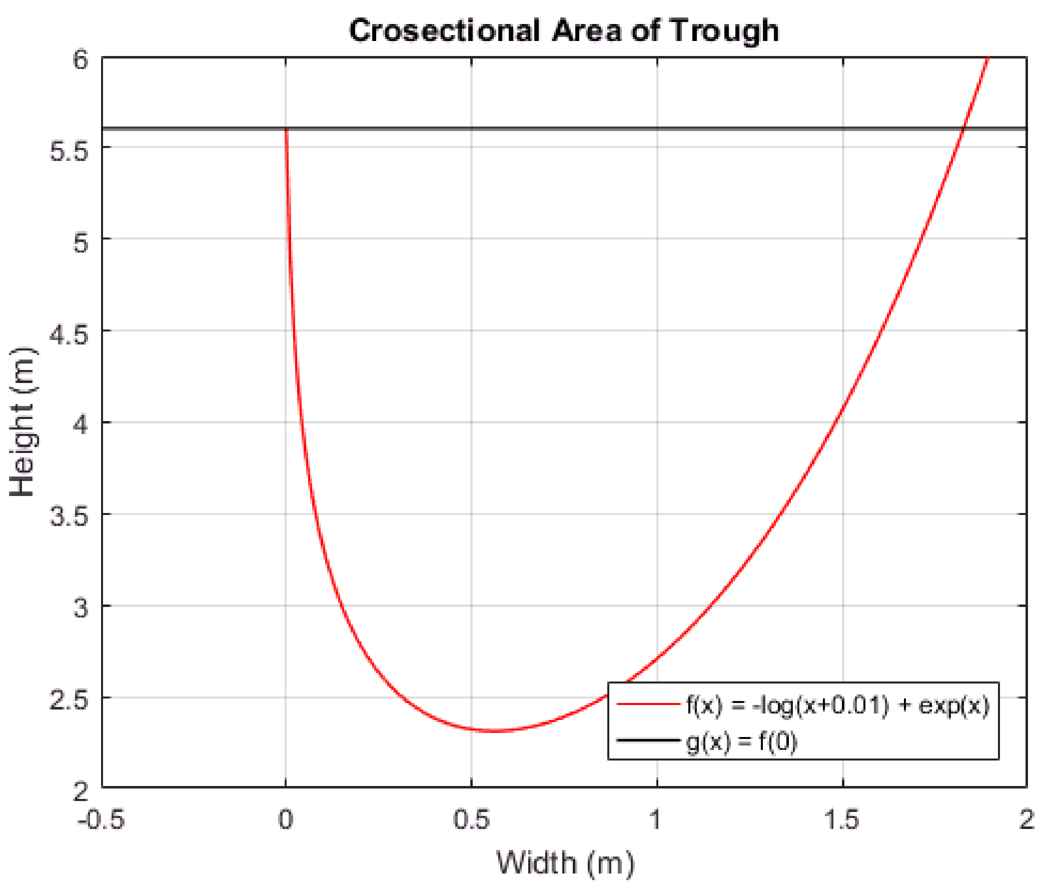
PART A

1a) Using MATLAB, plot the cross sectional area of the trough.

```
x = [0:0.01:2];
g = 0; % Variable for g(x)
f = -log(x + 0.01) + exp(x); % function of f(x)
f2 = -log(g + 0.01) + exp(g); % function of g(x)
plot(x,f,'red','linewidth',1)
axis([-0.5 2 2 6])
hold on
plot([-2 2],[f2 f2],'black','linewidth',1)
title('Crosectional Area of Trough')
xlabel('Width (m)')
ylabel('Height (m)')
legend('f(x) = -log(x+0.01) + exp(x)','g(x) = f(0)','location','southeast')
grid on;
```



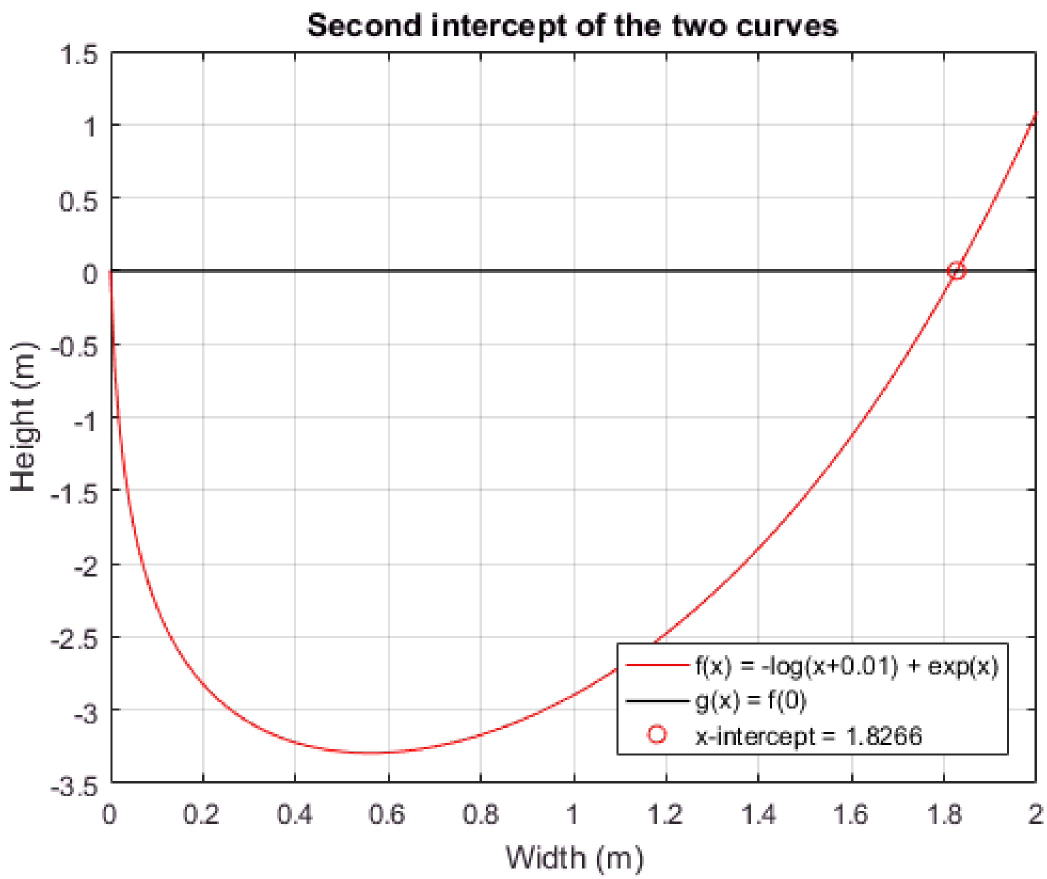
1b) Using MATLAB's fzero function, find the second intercept of two curves.

```
% fzero function
```

```
x = [0:0.01:2];
f = @(x)-log(x + 0.01) + exp(x) - 5.605;
f2 = 0; % function of g(x)
zero = fzero(f,2)
```

```
zero = 1.8266
```

```
plot(x,f(x),'red');
hold on
plot([0 2],[f2 f2],'black','linewidth',1)
plot(zero,f(zero),'ro')
title('Second intercept of the two curves')
xlabel('Width (m)')
ylabel('Height (m)')
legend('f(x) = -log(x+0.01) + exp(x)','g(x) = f(0)','x-intercept = 1.8266'...
,'location','southeast')
grid on;
```



1c) By hand, calculate the cross sectional area of the trough, and then calculate the volume of the trough.

Cross sectional area of the trough

Given :

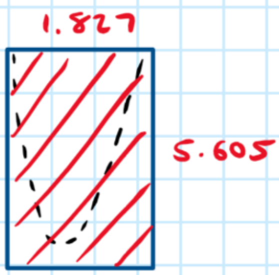
$$f(x) = -\ln(x + 0.01) + e^{x}$$

$$g(x) = f(0) = 5.605$$

x range = 0 to 1.827

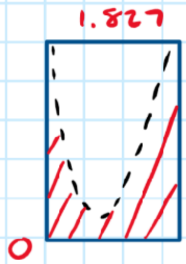
y range = 0 to 5.605

① Area of rectangle = $x \cdot y$



$$A = 1.827 \times 5.605$$
$$= 10.240 \text{ m}^2$$

(2) Area of curve $= \int_a^b f(x)\,dx$



$b = 1.827$
$a = 0$

$$\int_0^{1.827} -\ln(x+0.01) + e^x \, dx$$

Simplify:

$$\int_0^{1.827} e^x - \ln\left(x + \frac{1}{100}\right) dx$$

$$= \int e^x \, dx - \int \ln\left(\frac{100x+1}{100}\right) dx$$

Solving: $\int \ln\left(\frac{100x+1}{100}\right) dx$

$u = \frac{100x+1}{100} \rightarrow dx = du$

$$= \int \ln(u)\, du$$

$$= u\ln(u) - u$$

$$= \frac{(100x+1)\ln\left(\frac{100x+1}{100}\right)}{100} - \frac{100x+1}{100}$$

Solving: $\int e^x \, dx \therefore = e^x$

$$\int e^x \, dx - \int \ln\left(\frac{100x+1}{100}\right) dx$$

$$= -\frac{(100x+1)\ln\left(\frac{100x+1}{100}\right)}{100} + e^x + \frac{100x+1}{100}$$

$$= -\frac{(100x+1)\ln\left(\frac{100x+1}{100}\right)}{100} + e^x + x + C$$

or $-\left(x+\frac{1}{100}\right)\ln\left(x+\frac{1}{100}\right) + e^x + x + C$

$$\left[-\left(x+\frac{1}{100}\right)\ln\left(x+\frac{1}{100}\right) + e^x + x\right]_0^{1.827}$$

$= 6.925 - 1.046$

$= 5.879 \text{ m}^2$

③ Total Area of the Trough = Rectangle − Curve

$\therefore 10.240 - 5.879 = 4.361 \text{ m}^2$

④ Volume of the Trough = Area × length

Given:

Area $= 4.361 \text{m}^2$ $\qquad \therefore V = 4.361 \times 5$

length $= 5\text{m}$ $\qquad = 21.805 \text{m}^3$

2a) In your own words, explain what numerical integration is (reference any sources you use).

Integration is commonly used for finding area under curves, but there are many functions out there that are difficult to solve just by integrating (whether it's definite integrals or functions that only exist as table of values or measurements), hence, it's best to use numerical integrations instead. In general, numerical integration is a method that computes approximate solutions for any definite integrals by using numerical techniques. The term is often called ‘quadrature’, which is based off the ‘Newton-Cotes formulas’. There are many methods for numerical integration, such as the ‘Trapezoidal rule’, ‘Simpson’s rule’, ‘Gaussian quadrature’, and ‘Romberg’s method’. Algorithms also support numerical integration, and are known to be implemented in programming languages such as Wolfram, Matlab, C++, C, and many more.

Source:

- Mathworld.wolfram.com. (n.d.). *Numerical Integration -- from Wolfram MathWorld*. [online] Available at: http://mathworld.wolfram.com/NumericalIntegration.html [Accessed 6 Aug. 2017].
- En.wikibooks.org. (n.d.). *Numerical Methods/Numerical Integration - Wikibooks, open books for an open world*. [online] Available at: https://en.wikibooks.org/wiki/Numerical_Methods/Numerical_Integration [Accessed 18 Aug. 2017].

2c) Use your code to compute the volume of the trough in question 1. Compare the two solutions and comment on any differences you observe.

```
f = @(x) -log(x + 0.01) + exp(x);
b = 1.827;
L = 5;

[vol] = trough_volume_n9934731(f,b,L)
```

```
vol = 21.8081
```

The function’s answer is slightly different to the hand drawn solution. By evaluating the code, we can observe that the answer spews out 21.808m^3, compared to the original answer of 21.805m^3, being approximately 3*10^-3 numbers away from the original answer. This could mean Matlab's numerical method is more accurate than using integration.

2b) Write an m-file that finds the volume of a trough with a cross sectional area defined by a function f(x) and the horzontal line segment given by g(x) = f(0) from x = 0 to the second interception point at x = b. The length of the trough is L meters. The first line of the m-file should be (replace numbers with your student number)

function vol = trough_volume_n1234567(f,b,L)

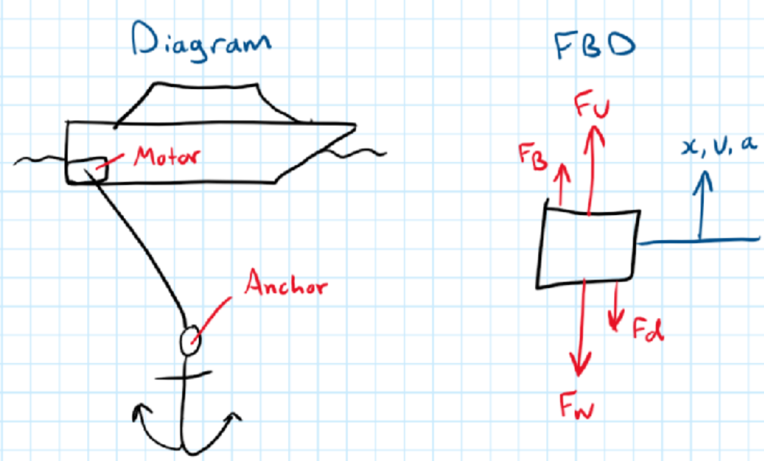
```
function [vol] = trough_volume_n9934731(f,b,L)
%TROUGH_VOLUME_N9934731(f,b,L) calculates the volume of the trough given
%the crosectional area defined by the function f(x) (f), the second
%interception point x = b (b), and the length of the trough in metres (L)
%as inputs.

height = f(0);
area = height * b;
area_total = area - integral(f,0,b);
vol = area_total*L;

end
```

2d) What are the limitations to your code?

The code's objective is to calculate the volume of a trough. These limitations are strictly based on the function's display of extra information about the trough's details, for the only output of this function is volume (vol). For example, the area and the height of the trough, should be included in the output of the function for coders to see and verify. It also does not grant the input for custom heights, for the function will automatically set the height for us, given that height = f(0). Moreover, the function f(x) must always have a lower limit of 0 to work; from x = 0 to x = b.

PART B

a) Draw a FBD of this system and consider only the vertical forces. You can treat the bouyancy force as a constant as long as the anchor is fully submerged and the drag will behave similarily to air resistance.



Vertical Forces :

Weight → $W = mg$ (F_W)

Buoyancy → (On anchor) (F_B)

$B = \rho_W V_A g = mg \frac{\rho_A}{\rho_W}$

ρ_W = Water density

V_A = fluid Volume (anchor)

Motor Force (up) → (F_U)

Drag → $F_d = \frac{1}{2} \rho_W v^2 A D$ (F_d)

V = anchor velocity

ρ_W = density (water)

A = Cross-Sectional Area

D = drag coefficient

b) derive an ODE for the vertical position of the anchor. Express your final answer in standard form.

$$\sum F = ma$$

$$F_B + F_U - F_w - F_d = ma$$

$$mg\frac{\rho_w}{\rho_a} + F_U - mg - \frac{1}{2}\rho_w V^2 AD = ma$$

$$\frac{mg\frac{\rho_w}{\rho_a}}{m} + \frac{F_U}{m} - \frac{mg}{m} - \frac{\frac{1}{2}\rho_w V^2 AD}{m} = \frac{m x''}{m}$$

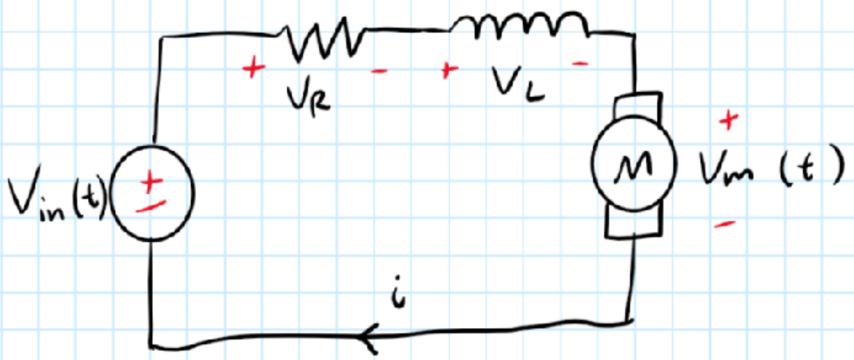
$$g\frac{\rho_w}{\rho_a} + \frac{F_U}{m} - g - \frac{\rho_w v^2 AD}{m} = x''$$

$$x'' + \left(\frac{\rho_w AD}{m}\right)v^2 = g\left(\frac{\rho_w}{\rho_a} - 1\right) + \frac{F_U}{m}$$

Let $k_1 = \dfrac{\rho_w AD}{m}$ and $k_2 = g\left(\dfrac{\rho_w}{\rho_a} - 1\right) + \dfrac{F_U}{m}$

$$\boxed{x'' + k_1 v^2 = k_2}$$

c) Derive an ODE for the voltage of the resistor in the circuit . Express you final answer in standard form.



$V_R = iR$

$V_L = L\frac{di}{dt}$

$\sum V_{loop} = 0$ w.r.t (t)

$$V_{in}(t) - V_R(t) - V_L(t) - V_m(t) = 0$$

$$\frac{V_{in}(t)}{L} = \frac{iR(t)\cdot R}{L} + \frac{L\frac{di}{dt}(t)}{L} + \frac{V_m(t)}{L}$$

$$\frac{V_{in}(t)}{L} = \frac{R}{L}i(t) + \frac{di}{dt} + \frac{V_m(t)}{L}$$

$$\frac{di}{dt} + \frac{R}{L}i(t) = \frac{V_{in}(t) - V_m(t)}{L}$$

Voltage across resistor $V_R(t) = iR(t) \times R$

$$L\frac{di}{dt}(t) + V_R(t) = V_{in}(t) - V_m(t)$$

$$\boxed{V_R(t) = V_{in}(t) - V_m(t) - L\frac{di}{dt}(t)}$$