PORTFOLIO 2

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PART A - The RLC Circuit

a) Get the homogenous solution to the ODE given m = 1, c = 40 and k = 144

$m = 1$
$c = 40$
$k = 144$

$$\frac{d^2x}{dt^2} + 40\frac{dx}{dt} + 144x = r(t)$$

Characteristic equation

$$\frac{d^2x}{dt^2} + 40\frac{dx}{dt} + 144x = 0$$

$$r^2 + 40r + 144 = 0$$

Solve Roots to Characteristic Equation

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \frac{-40 \pm \sqrt{(40)^2 - 4(1)(144)}}{2(1)}$$

$$r = -4 \text{ or } r = -36 \quad \text{(real and distinct)}$$

$$\therefore r(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

$$\boxed{x_H = C_1 e^{-4t} + C_2 e^{-36t}}$$

b) Find the particular solution given r(t) = 60cos(4pi t), 0 < t < 0.8

$r(t) = 60\boxed{\cos(4\pi t)}$, $0 \le t \le 0.8s$

RHS

find x_p

Find Particular Solution (x_p) using table

RHS $(r(t)) = \cos(4\pi t)$, given $x_p = A\cos(4\pi t) + B\sin(4\pi t)$

Sub x_p into ODE to solve unknown coefficients

$x_p' = 4\pi B\cos(4\pi t) - 4\pi A\sin(4\pi t)$

$x_p'' = -16\pi^2 A\cos(4\pi t) - 16\pi^2 B\sin(4\pi t)$

Sub in $\rightarrow \frac{d^2x}{dt^2} + 40\frac{dx}{dt} + 144x = r(t)$

$-16\pi^2 A\cos(4\pi t) - 16\pi^2 B\sin(4\pi t) + 40[4\pi B\cos(4\pi t) - 4\pi A\sin(4\pi t)]$
$+ 144[A\cos(4\pi t) + B\sin(4\pi t)] = 60\cos(4\pi t)$

Cos terms : $(-16\pi^2 A + 160\pi B + 144A)\cancel{\cos(4\pi t)} = 60\cancel{\cos(4\pi t)}$

$-13.91A + 160\pi B = 60 \ldots (1)$

Sin terms : $(-16\pi^2 B - 160\pi A + 144B)\cancel{\sin(4\pi t)} = 0$

$-13.91B - 160\pi A = 0 \ldots (2)$

Solving simultaneously

$-56.136\,(-13.91A + 160\pi B = 60)$
$-160\pi A - 13.91B = 0$

$\cancel{160\pi A} - 18164.05B = -2168.17$
$\cancel{-160\pi A} - 13.91B = 0$

$$\frac{-18177.96B}{-18177.96} = \frac{-2168.17}{-18177.96}$$

$\therefore B = 0.1193 \longrightarrow (2)$

$-160\pi A - 13.91(0.1193) = 0$
$-160\pi A - 1.659 = 0$

$-160\pi A = 1.659$

$\therefore A = -3.3\times10^{-3}$

So $x_p = -3.3\times10^{-3}\cos(4\pi t) + 0.1193\sin(4\pi t)$

$x = x_H + x_p$

$$\boxed{\therefore x = C_1e^{-4t} + C_2e^{-36t} - 3.3\times10^{-3}\cos(4\pi t) + 0.1193\sin(4\pi t)}$$

c) Get a specific solution for the ODE given x(0) = 0m, x'(0) = 0m/s

Apply initial conditions

$$x(0) = C_1 + C_2 - 3.3\times10^{-3} = 0$$
$$C_1 = 3.3\times10^{-3} - C_2$$

$$x(t) = 3.3\times10^{-3} - C_2 e^{-4t} + C_2 e^{-36t} - 3.3\times10^{-3}\cos(4\pi t) + 0.1193\sin(4\pi t)$$
$$x'(t) = -4(3.3\times10^{-3} - C_2)e^{-4t} - 36C_2 e^{-36t} + 0.04147\sin(4\pi t) + 1.49917\cos(4\pi t)$$
$$x'(0) = -0.0132 + 4C_2 - 36C_2 + 1.49917 = 0$$
$$\therefore C_2 = \frac{-1.48597}{-32} = 4.6\times10^{-2} \qquad \therefore C_1 = 3.3\times10^{-3} - 4.6\times10^{-2} = -0.0427$$

$$\boxed{\begin{aligned}\therefore x &= -0.0427e^{-4t} + 4.6\times10^{-2}e^{-36t} - 3.3\times10^{-3}\cos(4\pi t) + 0.1193\sin(4\pi t)\ [m]\\ x' &= 0.1708e^{-4t} - 1.656e^{-36t} + 0.04147\sin(4\pi t) + 1.49917\cos(4\pi t)\ [m]\end{aligned}}$$

d) Sam is out riding at Mount Coot-tha. upon landing a jump, the force pressing into the frame is defined as r(t) = 600cos(e^(2t)), 0 < t < 0.1s

i) Show that 600cos(e^(2t)) can be approximated by the linearisation r(t) = 324.18 - 1009t, when centred at 0.

$$r(t) = 600\cos e^{2t} \qquad 0 \le t \le 0.1s$$

Show that $600\cos(e^{2t})$ can be approximated by the linearisation $r(t) = 324.18 - 1009t$, when centred at zero.

$$L(x) = f(a) + f'(a)(x-a)$$

$$r(t) = 600\cos(e^{2t}) \qquad r(0) = 600\cos(1)$$
$$r'(t) = -1200e^{2t}\cdot\sin(e^{2t}) \qquad r'(0) = -1200\sin(1)$$

$$L(t) = 600\cos(1) - 1200\sin(1)(t+0)$$
$$\boxed{L(t) = 600\cos(1) - 1200t\sin(1)}$$

ii) Using the approximation of r(t), find the new specific solution where x(0) = 0m, x'(0) = 11.11m/s

$x(0) = 0m \qquad x'(0) = 11.11\ m/s$

Given : $RHS = 324.18 - 1009t \quad , \; x_p = b_0 + b_1 t$

$x'_p = b_1 \qquad -b_0 - b_1 t = 324.18 - 1009t$

$x''_p = 0 \qquad b_0 = -324.18 \;,\; b_1 = 1009t$

$\therefore y_p = 1009t - 324.18$

$x = x_H + x_p$

$x(t) = C_1 e^{-4t} + C_2 e^{-36t} + 1009t - 324.18$

$x'(t) = -4C_1 e^{-4t} - 36C_2 e^{-36t} + 1009$

$x(0) = C_1 + C_2 - 324.18 = 0$

$C_1 = 324.18 - C_2 \quad \ldots (1)$

$x'(0) = -4(324.18 - C_2)e^{-4(0)} - 36C_2 e^{-36(0)} + 1009 = 11.11$

$-1296.72 + 4C_2 - 36C_2 + 1009 = 11.11$

$-32C_2 - 287.72 = 11.11$

$C_2 = \dfrac{298.83}{-32} \qquad \therefore C_2 = -9.338$

Sub (2) → (1)

$C_1 = 324.18 + 9.338 = 333.518$

$$\boxed{\begin{aligned} x &= 333.518e^{-4t} - 9.338^{-36t} + 1009t - 324.18 \\ x' &= -1334.072e^{-4t} + 336.168e^{-36t} + 1009 \end{aligned}}$$

check $x(0) = 0$

$x'(0) \approx 11.096\ m/s$

$\approx 11.1\ m/s$

Note: All problems need to be solved by hand. It is strongly recommended that you check your solutions in MATLAB with the *dsolve* function, although correct hand solutions by themselves will receive full marks.

```
syms y(t) %must define y and t as symbolic variables, where y is a function of t
dy = diff(y); %defining first order derivative, y'(t)
d2y = diff(y,2); %defining second order derivative, y''(t)
dsolve(d2y + 40*dy + 144*y == 0) % Question a)
```

ans $= C_1 e^{-4t} + C_2 e^{-36t}$

PART B - Anchor Motor

a) Solve the ODE by hand and plot your answer if initially the anchor is stationary and takes 5 minutes to be raised. The mass of the anchor is 500kg and the lifting force is a constant 1000N.

where $m = 500\,kg$

$f_m(t) = 1000\ N$

$t = 5\ mins$

Characteristic equation :

$$\frac{dv}{dt} + \frac{100}{500} v = \frac{1000}{500}$$

$$\frac{dv}{dt} + \frac{1}{5} v = 2$$

$$\frac{dv}{dt} + P(t) v = Q(t)$$

$$P(t) = \frac{1}{5} \quad , \quad Q(t) = 2$$

$$R(t) = e^{\int P(t)\,dt}$$

$$= e^{\int \frac{1}{5}\,dt}$$

$$= e^{\frac{1}{5}t}$$

$$y = \frac{1}{R(t)} \int Q(t) R(t)\,dt$$

$$y = \frac{1}{e^{\frac{1}{5}t}} \int 2e^{\frac{1}{5}t}\,dt$$

$$= e^{-\frac{1}{5}t}\left(10e^{\frac{1}{5}t} + C_1\right)$$

$$= 10e^{-\frac{t}{5}+\frac{t}{5}} + C_1 e^{-\frac{t}{5}}$$

$$= 10e^{0} + C_1 e^{-\frac{t}{5}}$$

$$\boxed{V = 10 + C_1 e^{-\frac{t}{5}}}$$

Apply initial Conditions :

Solve for C_1 ,

$$v(0) = 0$$

$$0 = 10 + C_1$$

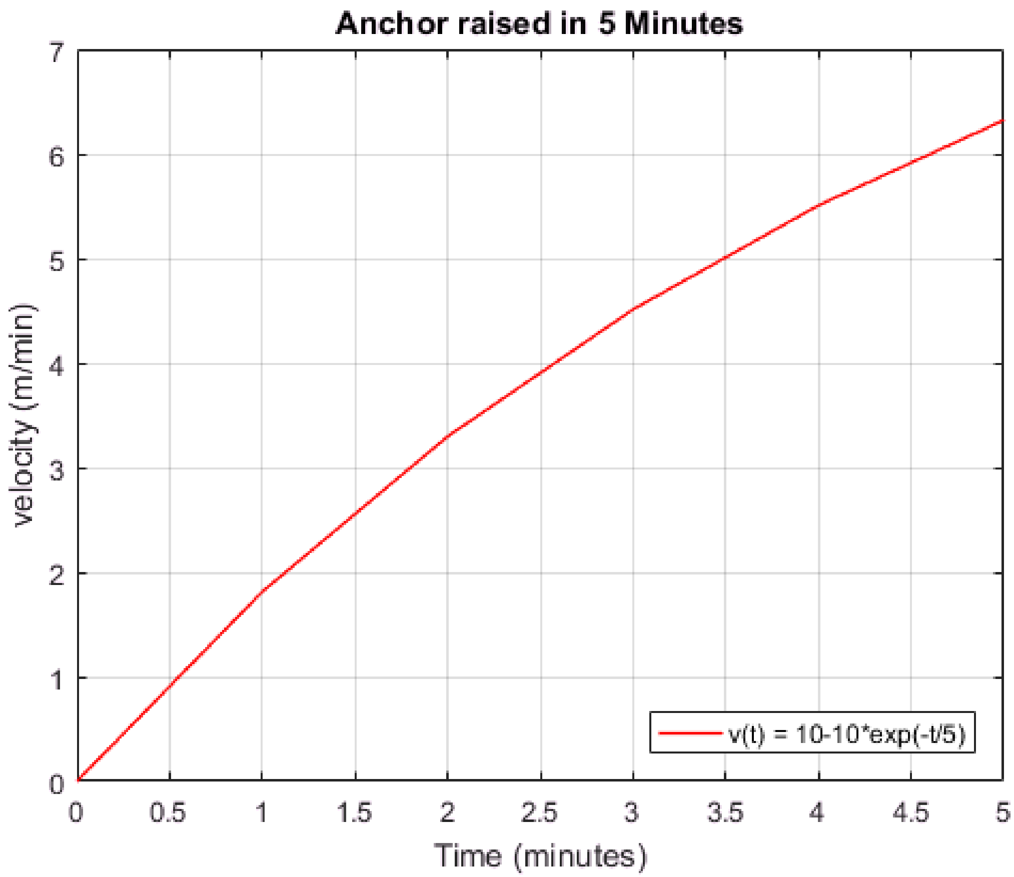
$$\therefore \quad C_1 = -10$$

$$\boxed{v(t) = 10 - 10e^{-\frac{t}{5}}}$$

Given t = 5 minutes ,

$$v(5) = 6.321 \text{ m/min}$$

```
x = 0:1:5;
y = 10-10*exp(-x/5); % function of v(t)
plot(x,y,'red','linewidth',1)
title('Anchor raised in 5 Minutes')
xlabel('Time (minutes)')
ylabel('velocity (m/min)')
legend('v(t) = 10-10*exp(-t/5)','location','southeast')
grid on;
```



b) A more accurate way to model the motor is with this ODE. Show that the simple ODE is just the accurate ODE linearised at v = 0.

Given Accurate ODE : $\frac{dv}{dt} + \frac{100}{m} v + \frac{2}{m} v^2 = \frac{F_m(t)}{m}$

Characteristic equation :

$$\frac{dv}{dt} + \frac{100}{500} v + \frac{2}{500} v^2 = \frac{1000}{500}$$

$$\frac{dv}{dt} + \frac{1}{5} v + \frac{1}{250} v^2 = 2$$

Simple ODE → ① when $V = 0$
Accurate ODE → ②

$$\frac{dv}{dt} + \frac{1}{5} v = 2 \quad , \quad \frac{dv}{dt} = 2$$

$$\frac{dv}{dt} + \frac{1}{5} v + \frac{1}{250} v^2 = 2 \quad , \quad \frac{dv}{dt} = 2$$

∴ Simple ODE is just an accurate ODE linearised @ $V = 0$

c) Solve the simple and accurate ODEs using ode45, and plot the answers in the same figure.

Using ODE45 to plot:

Simple ODE

$$\boxed{\frac{dv}{dt} = 2 - \frac{1}{5}v}$$

Accurate ODE

$$\boxed{\frac{dv}{dt} = 2 - \frac{1}{5}v - \frac{1}{250}v^2}$$

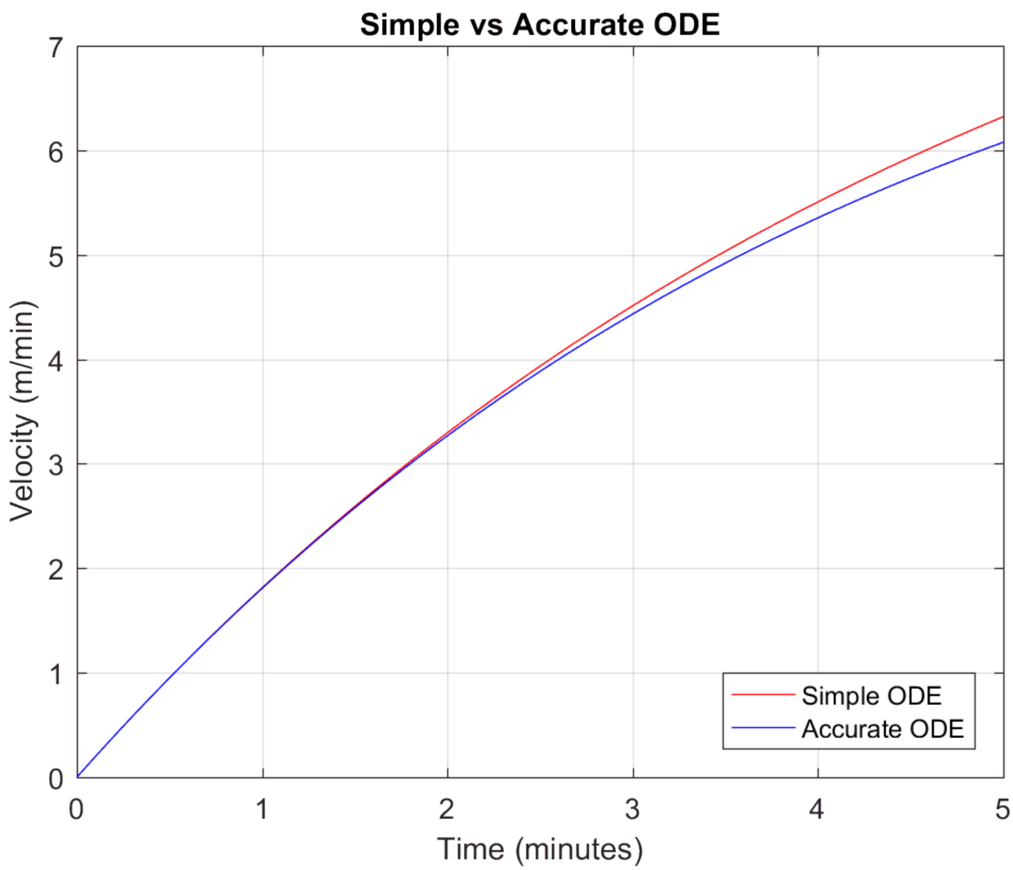
```matlab
ODEFUN = @(t,y) 2 - 1/5*y; %v' equation for simple ODE
TSPAN = [0 5]; %time (minutes)
Y0 = 0; %this is the initial condition for v
[tout,yout] = ode45(ODEFUN,TSPAN,Y0);
plot(tout,yout , 'red')

hold on

ODEFUN = @(t,y) 2 - 1/5*y - 1/250*y^2; %v' equation for accurate ODE
TSPAN = [0 5]; %time (minutes)
Y0 = 0; %this is the initial condition for v
[tout,yout] = ode45(ODEFUN,TSPAN,Y0);
plot(tout,yout , 'blue')

title('Simple vs Accurate ODE')
xlabel('Time (minutes)')
ylabel('Velocity (m/min)')
legend('Simple ODE','Accurate ODE','location','southeast')
grid on;

hold off
```



d) Comment on the accuracy of the simple ode solution, assuming the accurate ODE solution is 100% accurate. Also compare the hand solution from Qa to the numerical solution in Qc for the simple ODE.

Assuming that Accurate ODE is 100% accurate, the Simple ODE has a higher velocity value than the Accurate ODE along the time span , reducing its accuracy when coming to the end of the graph.

According to 'yout' for both ODES when t = 5 minutes, the Simple ODE acquired a velocity of 6.3212 m/min just like the hand drawn solution in question a),while the Accurate ODE acquired a velocity of 6.0765 m/min. This means the Simple ODE is 96% accurate, coming off at 0.2447 away from the Accurate ODE value.

e) Solve the accurate ODE by hand using separation. *Note: this problem is challenging, and will require partial fractions to solve an integral.

$$\frac{dv}{dt} + \frac{1}{5}v + \frac{1}{250}v^2 = 2$$

$$N(v) \cdot v' = M(t)$$

$$\frac{dv}{\boxed{dt}} = \left[2 - \frac{1}{5}v - \frac{1}{250}v^2\right]$$

$$dv = 2 - \frac{1}{5}v - \frac{1}{250}v^2 \, dt$$

$$\int \frac{1}{2 - \frac{1}{5}v - \frac{1}{250}v^2} \, dv = \int 1 \, dt$$

$$" \qquad = t + C_1$$

Integrating dv

Simplify :

$$\int \frac{1}{2 - \frac{v}{5} - \frac{v^2}{250}} \, dv = \int \frac{250}{-v^2 - 50v + 500} \, dv$$

$$= 250 \int \frac{1}{-v^2 - 50v + 500} \, dv$$

Complete the square:

$$= 250 \int \frac{1}{-(v+25)^2 + 1125}\, dv$$

let $u = v + 25$ $\quad dv \to du$

$$= 250 \int \frac{1}{-u^2 + 1125}\, du$$

let $u = 15\sqrt{5}\, w$ $\quad du \to dw$

$$= 250 \int \frac{1}{15\sqrt{5}\,(-w^2+1)}\, dw$$

$$= 250 \cdot \frac{1}{15\sqrt{5}} \int \frac{1}{-w^2+1}\, dw$$

$$= 250 \cdot \frac{1}{15\sqrt{5}} \left(\frac{\ln|w+1|}{2} - \frac{\ln|w-1|}{2} \right)$$

Substitute $w = \frac{u}{15\sqrt{5}}$, $u = V + 25$

$$= 250 \cdot \frac{1}{15\sqrt{5}} \left(\frac{\ln\left|\frac{V+25}{15\sqrt{5}} + 1\right|}{2} - \frac{\ln\left|\frac{V+25}{15\sqrt{2}} - 1\right|}{2} \right)$$

$$= \frac{5\sqrt{5}}{3} \left(\ln\left|\frac{V+25}{15\sqrt{5}} + 1\right| - \ln\left|\frac{V+25}{15\sqrt{5}} - 1\right| \right) + C_2$$

Finalising Equation :

$$\frac{5\sqrt{5}}{3} \left(\ln\left|\frac{V+25}{15\sqrt{5}} + 1\right| - \ln\left|\frac{V+25}{15\sqrt{5}} - 1\right| \right) + \overset{-C_2}{\cancel{C_2}} = t + C_1 - C_2$$

let $C_3 = C_1 - C_2$

$$= \frac{5\sqrt{5}}{3} \left(\ln\left|\frac{V+25}{15\sqrt{5}} + 1\right| - \ln\left|\frac{V+25}{15\sqrt{5}} - 1\right| \right) = t + C_3$$

$$= 5\sqrt{5} \left(\ln\left|\frac{V+25}{15\sqrt{5}} + 1\right| - \ln\left|\frac{V+25}{15\sqrt{5}} - 1\right| \right) = 3(t + C_3)$$

$$\boxed{V = - \frac{5\sqrt{5}\left(-3e^{\frac{3(t+C_3)}{5\sqrt{5}}} + \sqrt{5}\, e^{\frac{3(t+C_3)}{5\sqrt{5}}} - 3 - \sqrt{5}\right)}{e^{\frac{3(t+C_3)}{5\sqrt{5}}} - 1}}$$

PART C - The RLC circuit part 2

a) Show that the circuit can be modelled as an ODE: di^2/dt^2 + R/L di/dt + 1/LC *i = 1/L dVs/dt

KVL : $-V_s + V_c + V_R + V_L = 0 \ldots \quad (1)$

KCL : same current throughout

Resistor : $V = iR \quad \ldots \quad (2)$

Inductor : $V_L = L\dfrac{di}{dt} \quad \ldots (3)$

Capacitor : $i = C\dfrac{dv_c}{dt}$

$$V_c = \frac{1}{C}\int i\,dt \quad \ldots \quad (4)$$

Sub (2), (3), (4) into (1)

$$-V_s + \frac{1}{C}\int i\,dt + Ri + L\frac{di}{dt} = 0$$

$$-\frac{dV_s}{dt} + \frac{1}{C}i + R\frac{di}{dt} + L\frac{d^2 i}{dt^2} = 0$$

$$-\frac{1}{L}\frac{dV_s}{dt} + \frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

$$\boxed{\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{1}{L}\frac{dV_s}{dt}}$$

b) Convert the 2nd order ODE to a system of two first order ODEs (in matrix form)

$$\boxed{\text{Voltage Source: } \frac{dV_s}{dt} = 0}$$

let $i_1 = i'$ and $i_2 = i''$

so $i = i_1$, $i_2 = i'$, $i'' = i_2'$

$$i'' + \frac{R}{L} i' + \frac{1}{LC} i = 0$$

$$i_2' + \frac{R}{L} i_2 + \frac{1}{LC} i_1 = 0$$

$$i_2' = -\frac{R}{L} i_2 - \frac{1}{LC} i_1 \quad \dots (2)$$

let $i_1 = i_2$

$$i_1' = 0 i_1 + i_2 \quad \dots (1)$$

Describing in matrix form

$$y' = A\underset{\sim}{y}$$

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

Given $R = 20000\,\Omega$, $L = 0.01\text{ H}$

$C = 5 \times 10^{-5}\text{ F}$

Can verify with matrix multiplication

$$\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2000000 & -2000000 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} i_1' \\ i_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2000000 & -2000000 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}}$$

c) Solve the eigenvalues of the matrix A

$$A - \lambda I = \begin{pmatrix} -\lambda & 1 \\ -2000000 & -2000000 - \lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (-\lambda)(-2000000 - \lambda) - (1)(-2000000)$$

$(ad - bc)$

$$= \lambda^2 + 2000000\lambda + 2000000$$

$$\lambda^2 + 2\times10^6 \lambda + 2\times10^6 = 0$$

Solve for λ :

$$\frac{-2\times10^6 \pm \sqrt{(2\times10^6)^2 - 4(1)(2\times10^6)}}{2(1)}$$

$$\boxed{\lambda_1 = -1.000000505 \qquad \lambda_2 = -1999999}$$

d) Solve the corresponding eigenvectors for the matrix A

$$(A - \lambda I)\underset{\sim}{c} = 0$$

For $\lambda_1 = -1.000000505$

$$\begin{pmatrix} 1.000000505 & 1 \\ -2000000 & -1999999 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Equation 1 :

$$1.000000505\, c_1 + c_2 = 0$$

$$\therefore c_2 = -1.000000505\, c_1$$

$$\therefore \underset{\sim}{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ -1.000000505\, c_1 \end{pmatrix} = \cancel{c_1} \begin{pmatrix} 1 \\ -1.000000505 \end{pmatrix}$$

For $\lambda_2 = -1999999$

$$\begin{pmatrix} 1999999 & 1 \\ -2000000 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Equation 2 :

$$1999999 \; c_1 + c_2 = 0$$

$$\therefore c_2 = -1999999 \; c_1$$

$$\therefore \underset{\sim}{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} c_1 \\ -1999999 \; c_1 \end{pmatrix} = \cancel{c_1} \begin{pmatrix} 1 \\ -1999999 \end{pmatrix}$$

$$\boxed{\begin{matrix} \lambda_1 = -1.000000505 \, , & \underset{\sim}{c_1} = \begin{pmatrix} 1 \\ -1.000000505 \end{pmatrix} \\ \lambda_2 = -1999999 \, , & \underset{\sim}{c_2} = \begin{pmatrix} 1 \\ -1999999 \end{pmatrix} \end{matrix}}$$

e) Verify your answers for c) and d) using the eig function in MATLAB

```
A = [0 1;-2000000 -2000000];
[eigenvectors,eigenvalues] = eig(A)
```

```
eigenvectors =
   0.707106604409697  -0.000000500000250
  -0.707106957963353   0.999999999999875

eigenvalues =
   1.0e+06 *

  -0.000001000000500                   0
                   0  -1.999998999999500
```

f) Solve the system of first order ODEs, given that i(0) = 10A, and i'(0) = 0. Compare your answer to part b) from the week 4 exercise.

Write General Solution:

$$\underset{\sim}{y} = k_1 C_1 e^{\lambda_1 t} + k_2 C_2 e^{\lambda_2 t}$$

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ -1.000000505 \end{pmatrix} e^{-1.000000505t} + k_2 \begin{pmatrix} 1 \\ -1999999 \end{pmatrix} e^{-1999999t}$$

Apply initial conditions

$i(0) = 10A$, $i'(0) = 0$

$$y(0) = \begin{pmatrix} 10 \\ 0 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ -1.000000505 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ -1999999 \end{pmatrix}$$

row 1 : $10 = k_1 + k_2$ ... (1)

row 2 : $0 = -1.000000505\,k_1 - 1999999\,k_2$ ...(2)

Solving simultaneously

let $k_1 = 10 - k_2$

$$0 = -1.000000505\,(10 - k_2) - 1999999\,k_2$$

$$0 = -10.00000505 + 1.000000505 k_2 - 1999999\,k_2$$

$$0 = -10.00000505 - 1999998\,k_2$$

$$\therefore k_2 = \frac{10.00000505}{-1999998} = -0.000005$$

$$\therefore k_1 = 10 + 0.000005$$

$$= 10.000005$$

$$\therefore \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = 10.000005 \begin{pmatrix} 1 \\ -1.000000505 \end{pmatrix} e^{-1.000000505t} - 0.000005 \begin{pmatrix} 1 \\ -1999999 \end{pmatrix} e^{-1999999t}$$

row 1 : $i_1 = 10.000005 e^{-1.000000505t} - 0.000005 e^{-1999999t}$ [A]

row 2 : $i_2 = -10.00001005 e^{-1.000000505t} + 9.999995 e^{-1999999t}$ [A]

Given $i(0) = 10A$, $i'(0) = 0$

$i_1(0) = 10$ ✓

$i_2'(0) \simeq 0$ ✓

$$\boxed{\begin{aligned} i(t) &= 9.999995\, e^{-1.000000505t} + 0.000005\, e^{-1999999t} \\ i'(t) &= -10.00001005\, e^{-1.000000505t} + 9.999995\, e^{-1999999t} \end{aligned}}$$

Looking back at part b) from week 4 porfolio exercise, we had to get a specific solution for an ODE, given that x(0) = 0m and x'(0) = m/s.

The result was a long equation that combined the homogenous and specific solution together to obtain a general solution in order to solve for the initial conditions. To get the initial condition however, we must find the unknown coefficients (C1 & C2) to verify if the condition is true. Unknown coefficients rely on initial conditions to give out the full completed equation.

In this case, we only need to use the general equation specifically for eigenvalues and eigenvectors, and plug in the values into the variables to get two equations. This equation lets us first solve two unknown coefficients (k1 & k2) simultaneously which in the end gives us the final equation, allowing us to check the initial conditions without the need to solve any unknowns. Furthermore, the latest equation is smaller, accurate, and much more neater to use than the week 4 part b) answer.