# Computational Many Body Physics: Part 1
# Percolation Theory

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**Abstract**

This project studies site percolation on a two-dimensional square lattice using Monte Carlo simulations. Clusters are identified using the Hoshen–Kopelman algorithm, and quantities such as the weighted average cluster size, percolation strength, and percolation rate are measured as functions of the site occupation probability and system size. The emergence of a percolating cluster is observed, and an estimate of the critical percolation probability is obtained.

## 1 Introduction

Percolation theory provides a simple yet powerful framework to study connectivity and phase transitions in disordered systems. In this work, we numerically investigate site percolation on a two-dimensional square lattice and analyze how global connectivity emerges as the occupation probability is varied.

## 2 Model and Methodology

### 2.1 Site Percolation Model

We consider an $L \times L$ square lattice where each site is independently occupied with probability $p$ and empty with probability $1 - p$. Nearest-neighbour connectivity (up, down, left, right) is used to define clusters.

### 2.2 Cluster Identification

Clusters are identified using the Hoshen–Kopelman algorithm, which efficiently labels connected components in a single scan of the lattice using a union–find data structure.

### 2.3 Percolation Criterion

A configuration is said to percolate if there exists a cluster that connects the top and bottom edges of the lattice. The label of the percolating cluster is recorded when such a connection exists.

## 3 Measured Quantities

### 3.1 Weighted Average Cluster Size

The weighted average cluster size is defined as

$$S(p) = \frac{\sum_s s^2 n_s}{\sum_s s n_s}, \tag{1}$$

where $n_s$ is the number of clusters of size $s$. If the system percolates, the percolating cluster is excluded from the sum.

### 3.2 Percolation Strength

The percolation strength is defined as

$$P(p) = \frac{1}{L^2} \langle s_{\text{perc}} \rangle, \tag{2}$$

where $s_{\text{perc}}$ is the size of the percolating cluster. If the system does not percolate, $P(p) = 0$.

### 3.3 Percolation Rate

The percolation rate $\Pi(p)$ is defined as the probability that a randomly generated configuration percolates.

## 4 Results

### 4.1 Dependence on Occupation Probability

Figures 1 and 2 show the percolation strength and percolation rate as functions of the occupation probability $p$ for system sizes $L = 8, 16, 32$.

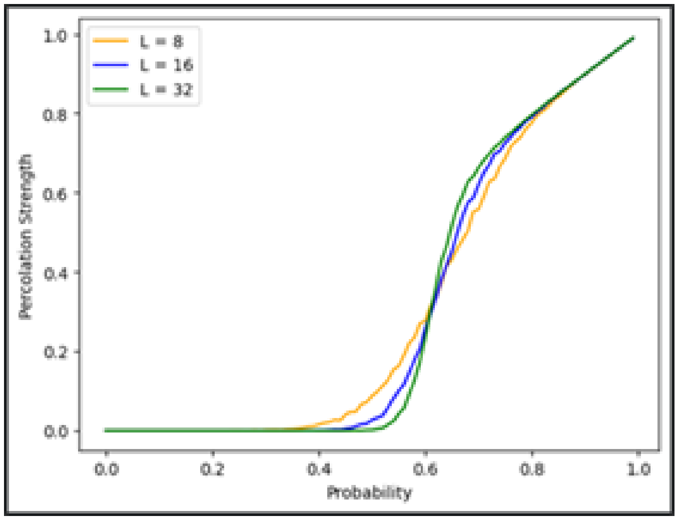


Figure 1: Percolation strength as a function of occupation probability.

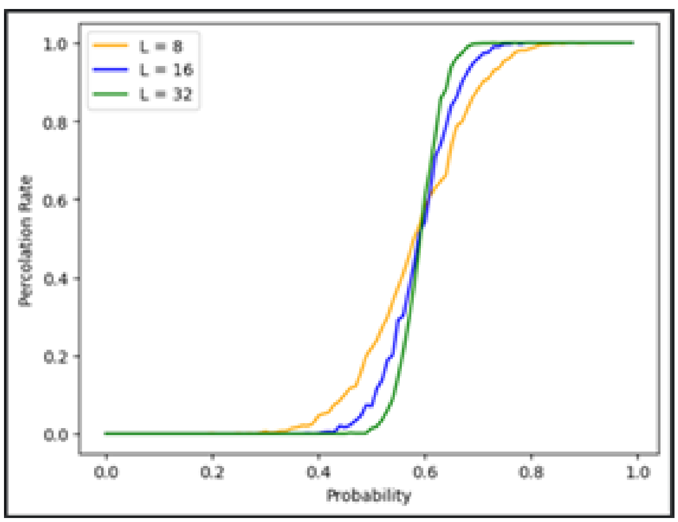


Figure 2: Percolation rate as a function of occupation probability.

### 4.2 Finite-Size Effects

As the system size increases, the transition from the non-percolating to the percolating phase becomes sharper, indicating the presence of a phase transition in the thermodynamic limit.

### 4.3 Estimate of the Critical Probability

An estimate of the critical probability is obtained by locating the occupation probability at which the percolation rate is closest to 0.5. The estimated value approaches the known result for two-dimensional site percolation, $p_c \approx 0.5927$, as the system size increases.

## 5 Conclusion

We performed a numerical study of site percolation on a two-dimensional square lattice. Using cluster identification and Monte Carlo averaging, we measured several percolation observables and observed a clear phase transition. The estimated critical probability agrees well with the known theoretical value.

## References


[1] J. Hoshen and R. Kopelman, "Percolation and cluster distribution. I. Cluster multiple labeling technique and critical concentration algorithm," *Phys. Rev. B*, **14**, 3438–3445 (1976).

[2] L. Levitov, "Notes on Percolation Theory," MIT Course Notes (8.334).