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Robust output synchronization of heterogeneous nonlinear agents in uncertain networks

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This paper investigates the global robust output synchronization problem for a class of nonlinear multi-agent systems. In the considered setup, the controlled agents are heterogeneous and with both dynamic and parametric uncertainties, the controllers are incapable of exchanging their internal states with the neighbors, and the communication network among agents is defined by an uncertain simple digraph. The problem is pursued via nonlinear output regulation theory and internal model based design. For each agent, the input-driven filter and the internal model compose the controller, and the decentralized dynamic output feedback control law is derived by using backstepping method and the modified dynamic high-gain technique. The theoretical result is applied to output synchronization problem for uncertain network of Lorenz-type agents.

1. Introduction

Controlling a group of agents and rendering their outputs to common reference trajectory is one fundamental objective in many control problems for multi-agent systems. In recent years, the researches on output synchronization problem have been expanded from linear into nonlinear scenario. For different kinds of nonlinear multi-agent systems under various settings of network topologies, certain notable concepts and design methodologies have been proposed to achieve output synchronization, for instance, the concepts of passivity and dissipativity [5,18], the cyclic-small-gain theorem [17], the pinning control method [9,30,35], and the references therein. Besides, other design issues regarding the engineering applications have also been discussed, such as the input saturation [20–22], the unknown control direction [16], the switching network topology and the bounded synchronization, [18,19,25–27], to name only a few.

It is noticeable that the classical output regulation theory and the internal model principle also exhibit their great potentials in accomplishing such kind of problems. In the output regulation theory, the reference trajectory and external disturbance are governed by an exogenous dynamics known as the exosystem, which plays the same role as the virtual leader in the leader-following format. For nonlinear multi-agent systems, the so-called cooperative or distributed output regulation have been studied for agents of several standard nonlinear forms, e.g., the normal form [19,23], the output feedback form [6–8,31,32,34], the lower triangular form [11,24,36], etc. Furthermore, in the leaderless format, the internal model based design also provides certain feasibility to synchronize nonlinear multi-agent systems, c.f. [13,29]. The results therein exploit the importance of embedding the internal model as part of the controller to achieve synchronization.

In this paper, the global robust output synchronization problem is considered for a class of nonlinear multi-agent systems where the heterogeneous agents are in output feedback form with different relative degrees, and the communication network is specified by an uncertain digraph. Provided that there exists a directed spanning tree with the virtual leader as the root, the dynamic output feedback controller comprising the input-driven filter and internal model can be designed, and the global robust output synchronization is achieved which admits the bounds of parametric uncertainties for both agents and network and the bound of external disturbances are not known a priori.

Our main contributions are two folds: 1) The control design is independent of any explicit or implicit quantitative information of the communication network among agents, i.e., since the network is completely unknown, quantitative information regarding the size of the corresponding digraph, the weights of the edges, so as the underlying adjacency matrix or the Laplacian matrix is not used to achieve the control law. 2) The control is of decentralized type so that it overcomes the restriction that no information exchange is allowed among controllers. Also, the design avoids the fully collaborative manner and simplifies most of the procedures to perform w.r.t. each agent separately. In turn, it provides certain feasibility to cope with heterogeneous agents with different relative degrees.

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The rest of the paper is organized as follows. In Section 2, we formulate the global robust output synchronization problem. In Section 3, the problem is pursued via nonlinear output regulation theory and the internal model based design. In Section 4, the theoretical result is applied to an output synchronization problem for Lorenz-type agents in uncertain networks. And the paper is concluded in Section 5.

Notations: Given a column vector x_i, $x_{i,j}$ denotes its j-th component. For column vectors $x_i \in \mathbb{R}^{n_i}$, $\mathrm{col}(x_1, \ldots, x_n) \triangleq \left[x_1^\top, \ldots, x_n^\top\right]^\top \in \mathbb{R}^{n_1+\cdots+n_n}$. In the similar manner, a collection of vector-valued functions f_i is denoted by $\mathrm{col}(f_1, \ldots, f_n)$. $A \triangleq [a_{ij}] \in \mathbb{R}^{n\times m}$ denotes a matrix with elements a_{ij}. I_n denotes an identity matrix of size n. $D \triangleq \mathrm{diag}[d_1, \ldots, d_n]$ denotes a diagonal matrix with d_i being its i-th diagonal element.

In the following context, any parameter that can be arbitrarily chosen is referred to as a free parameter. For brevity, the arguments of a function are omitted when no ambiguity occurs.

2. Problem formulation

Consider a group of heterogeneous nonlinear agents $\mathcal{S}_i$, $i = 1, \ldots, n$,

$$\mathcal{S}_i:\quad \begin{aligned} \dot{z}_i &= f_i(z_i, y_i, v, w),\\ \dot{x}_{i,j} &= g_{i,j}(z_i, y_i, v, w) + x_{i,j+1}, \quad j = 1, \ldots, r_i,\\ y_i &\triangleq x_{i,1}, \quad u_i \triangleq x_{i,r_i+1}. \end{aligned} \tag{1}$$

Agent $\mathcal{S}_i$ is in nonlinear output feedback form with relative degree r_i, $r_i \geq 1$. $\mathrm{col}(z_i, x_i)$ is the state, $z_i \in \mathbb{R}^{n_i}$, $n_i \geq 0$, $x_i = \mathrm{col}(x_{i,1}, \ldots, x_{i,r_i}) \in \mathbb{R}^{r_i}$, where z_i represents the the dynamic uncertainty, i.e., the unmodeled dynamics. $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the control input and performance output, respectively. $w \in \mathbb{W}$ represents the parametric uncertainty, $v \in \mathbb{V}$ describes the external disturbance, where $\mathbb{W} \subset \mathbb{R}^{n_w}$ and $\mathbb{V} \subset \mathbb{R}^{n_v}$ are compact sets with unknown bounds.

Compared with the controlled agents $\mathcal{S}_i$, it is assumed that the reference trajectory y_0 is generated by the following autonomous agent,

$$\mathcal{S}_0:\quad \dot{v} = A_0 v, \quad y_0 = g_0(v, w). \tag{2}$$

where $\mathcal{S}_0$ is neutrally stable, i.e., all eigenvalues of A_0 are semi-simple with zero real parts. Consequently, $\mathcal{S}_0$ is forward complete and invariant w.r.t. any compact set $\mathbb{V}$, i.e., for any $v(0) \in \mathbb{V}$, it is certain that $v(t) \in \mathbb{V}$, $\forall\, t \in \mathbb{R}^+$.

It is also assumed that all functions in (1)–(2) are smooth and satisfying $f_i(0, 0, 0, w) = 0$, $g_{i,j}(0, 0, 0, w) = 0$ and $g_0(0, w) = 0$, thus the origin is the equilibrium point of $\mathcal{S}_i$.

For the given system $\{\mathcal{S}_0, \mathcal{S}_1, \ldots, \mathcal{S}_n\}$, the underlying communication can be characterized by a weighted digraph $\bar{\mathcal{G}}$. If $\mathcal{S}_i$ receives information provided by $\mathcal{S}_j$, then there exists a directed edge $<j, i>$ with unknown weight $\bar{a}_{ij} > 0$, otherwise $\bar{a}_{ij} = 0$. Correspondingly, the weighted adjacency matrix of $\bar{\mathcal{G}}$ is defined by $\bar{\mathcal{A}} \triangleq [\bar{a}_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$, and the weighted Laplacian matrix is defined by $\bar{\mathcal{L}} \triangleq [\bar{l}_{ij}]$, where $\bar{l}_{ii} = \sum_{j=0}^{n} \bar{a}_{ij}$ for $i = 0, 1, \ldots, n$, and $\bar{l}_{ij} = -\bar{a}_{ij}$, $i \neq j$. Based on $\bar{\mathcal{G}}$, the measurement output e_i^m and the tracking error e_i for agent $\mathcal{S}_i$ are defined as follows,

$$e_i^m = \sum_{j=0}^{n} \bar{a}_{ij}(y_i - y_j), \quad e_i = y_i - y_0, \quad i = 1, \ldots, n. \tag{3}$$

Note that $\bar{\mathcal{L}}$ can be partitioned as $\bar{\mathcal{L}} = \left[\begin{array}{c|c} 0 & 0\cdots 0 \\ \hline \bar{l} & \mathcal{L} \end{array}\right]$, where $\mathcal{L} \triangleq [l_{i,j}] \in \mathbb{R}^{n\times n}$ and $\bar{l} = \mathrm{col}(\bar{l}_{10}, \ldots, \bar{l}_{n0})$, so (3) can be alternatively written into the following compact form,

$$e^m = \mathcal{L}e, \quad e^m = \mathrm{col}(e_1^m, \ldots, e_n^m), \quad e = \mathrm{col}(e_1, \ldots, e_n). \tag{4}$$

The global robust output synchronization problem is formulated as follows: design the following dynamic output feedback control law for each controlled agent $\mathcal{S}_i$,

$$\dot{\chi}_i = \alpha_i(\chi_i, e_i^m), \quad u_i = \gamma_i(\chi_i, e_i^m), \tag{5}$$

such that for any initial conditions $\mathrm{col}\big(z_i(0), x_i(0)\big) \in \mathbb{R}^{n_i+r_i}$, any $v \in \mathbb{V}$, $w \in \mathbb{W}$, the states of the closed-loop system composed of (1) and (5) are bounded, and $\lim_{t\to+\infty} e_i = 0$, i.e., the performance output y_i tracks the reference trajectory y_0 asymptotically.

It is well comprehended that the network topology plays an essential role in achieving the aforementioned output synchronization problem. To make our statement self-sustained, we propose the following assumption.

Assumption 1. The digraph $\bar{\mathcal{G}}$ is simple[1] and contains a directed spanning tree with node 0 as the root. ■

Remark 1. Assumption 1 implies that all controlled agents $\mathcal{S}_i$ are reachable from $\mathcal{S}_0$. It is quite a standard and necessary assumption that provides the "minimal connectedness" of the static network topology to synchronize all agents' outputs, see Remark 2.2, 3.2 in [34] for detailed discussions.

Notice that $\mathcal{L}$ in (4) is an uncertain matrix since the size and the weights a_{ij} are not known a priori. However, it is fully recognized that $\mathcal{L}$ is indeed a non-singular $\mathcal{M}$-matrix under Assumption 1. And for any non-singular $\mathcal{M}$-matrix, there always exists a positive diagonal matrix $D \triangleq \mathrm{diag}[d_1, \ldots, d_n]$ such that

$$D\mathcal{L} + \mathcal{L}^\top D = Q, \tag{6}$$

for some positive definite matrix Q [1]. The existence of matrix D ensures that $(\delta D)\mathcal{L} + \mathcal{L}^\top(\delta D)$ is also a positive definite matrix for any constant $\delta > 0$. ■

3. Output synchronization via output regulation theory

In this section, the aforementioned global robust output synchronization problem is pursued via nonlinear output regulation theory. Some basic assumptions regarding the output regulation theory are proposed first. Then the dynamic controller is designed which consists of the input-driven filter and internal model, and the output synchronization problem is converted into a decentralized stabilization problem. The relevant stabilization problem is further achieved via certain nonlinear control techniques.

3.1. Basic assumptions

Assumption 2. For each $\mathcal{S}_i$, there exists a smooth function $\mathbf{z}_i(v, w)$: $\mathcal{R}^{n_v} \times \mathcal{R}^{n_w} \mapsto \mathcal{R}^{n_i}$ satisfying $\mathbf{z}_i(0, w) = 0$, such that, for any $v \in \mathbb{V}$, $w \in \mathbb{W}$, $\frac{\partial \mathbf{z}_i(v,w)}{\partial v} A_0 v = f_i\big(\mathbf{z}_i(v, w), g_0(v, w), v, w\big)$. ■

Assumption 2 provides partial solution of the regulator equation associated with (1) and (2), and the rest can be obtained as follows: denote $\mathbf{x}_{i,1}(v, w) = g_0(v, w)$, then $\mathbf{x}_{i,j+1}(v, w) = \frac{\partial \mathbf{x}_{i,j}(v,w)}{\partial v} A_0 v - g_{i,j}\big(\mathbf{z}_i(v, w), g_0(v, w), v, w\big)$, $j = 1, \ldots, r_i$, $\mathbf{u}_i(v, w) = \mathbf{x}_{i,r_i+1}(v, w)$. Let $\mathbf{x}_i(v, w) = \mathrm{col}\big(\mathbf{x}_{i,1}(v, w), \ldots, \mathbf{x}_{i,r_i}(v, w)\big)$, then $\mathrm{col}\big(\mathbf{z}_i(v, w), \mathbf{x}_i(v, w)\big)$ and $\mathbf{u}_i(v, w)$ are termed as the steady-state state and steady-state input of $\mathcal{S}_i$, respectively, and they characterize the steady-state

[1] When $\bar{\mathcal{G}}$ contains no self-loops or parallel directed edges, it is called simple.

information of the controlled agent when synchronization achieves.

Assumption 3. The steady-state input $\mathbf{u}_i(v, w)$ is a polynomial in v with coefficients depend on w. ■

Assumption 3 is purposed to facilitate the further design of internal model.

3.2. Dynamic controller design

For controlled agent (1) with relative degree $r_i \geq 2$, the following reduced-order input-driven filter is designed w.r.t. the x_i dynamics,

$$\dot{\zeta}_i = A_i\zeta_i + B_iu_i, \quad \zeta_i \in \mathbb{R}^{r_i-1}, \tag{7}$$

where (A_i, B_i) is in controllability canonical form, i.e. $\dot{\zeta}_{i,j} = -a_{i,j}\zeta_{i,1} + \zeta_{i,j+1}$, $j = 1, \ldots, r_i - 1$, and $\zeta_{i,r_i} = u_i$. The design parameters $a_{i,j}$ are chosen so that A_i is Hurwitz. In case that agent is with relative degree $r_i = 1$, the input-driven filter is not needed, see Remark 4 for treatment.

Due to the linear structure of (7), it is convenient to show that the steady-state state $\boldsymbol{\zeta}_i(v, w)$ satisfies $\frac{\partial\boldsymbol{\zeta}_i(v,w)}{\partial v}A_0v = A_i\boldsymbol{\zeta}_i(v, w) + B_i\mathbf{u}_i(v, w)$, where $\boldsymbol{\zeta}_i(v, w) = \mathrm{col}\left(\boldsymbol{\zeta}_{i,1}(v, w), \ldots, \boldsymbol{\zeta}_{i,r_i-1}(v, w)\right)$. Meanwhile, the error dynamics $\bar{x}_{i,j} = x_{i,j+1} - \zeta_{i,j} - a_{i,j}x_{i,1}$ can be compactly written as follows,

$$\dot{\bar{x}}_i = A_i\bar{x}_i + \bar{g}_i(z_i, y_i, v, w), \tag{8}$$

where $\bar{x}_i = \mathrm{col}(\bar{x}_{i,1}, \ldots, \bar{x}_{i,r_i-1})$, $\bar{g}_i(z_i, y_i, v, w) = \mathrm{col}(\bar{g}_{i,1}, \ldots, \bar{g}_{i,r_i-1})$, $\bar{g}_{i,j} = (-a_{i,j}\,a_{i,1} + a_{i,j+1})x_{i,1} + (-a_{i,j}\,g_{i,1} + g_{i,j+1})$, and $a_{i,r_i} = 0$. Clearly, the steady-state state of the error dynamics is $\bar{\mathbf{x}}_{i,j}(v, w) = \mathbf{x}_{i,j+1}(v, w) - \boldsymbol{\zeta}_{i,j}(v, w) - a_{i,j}\mathbf{x}_{i,1}(v, w)$.

Now it is ready to design the internal model. Notice that under Assumption 3, all $\boldsymbol{\zeta}_{i,j}(v, w)$ are polynomials in v with coefficients depend on w, then w.r.t. $\boldsymbol{\zeta}_{i,1}(v, w)$, there exist some known constants $c_{i,k}$, $k = 0, 1, \ldots, s_i - 1$, so that

$$\boldsymbol{\zeta}_{i,1}^{(s_i)}(v, w) = c_{i,s_i-1}\boldsymbol{\zeta}_{i,1}^{(s_i-1)}(v, w) + \cdots + c_{i,1}\boldsymbol{\zeta}_{i,1}^{(1)}(v, w) + c_{i,0}\boldsymbol{\zeta}_{i,1}(v, w), \tag{9}$$

where $\boldsymbol{\zeta}_{i,1}^{(0)}(v, w) = \boldsymbol{\zeta}_{i,1}(v, w)$, $\boldsymbol{\zeta}_{i,1}^{(k)}(v, w) = \frac{\partial\boldsymbol{\zeta}_{i,1}^{(k-1)}(v,w)}{\partial v}A_0v$, c.f. Proposition 6.14 in [12].

Under condition (9), an intermediate uncontrolled dynamics known as the steady-state generator exists,

$$\dot{\tau}_i = \Phi_i\tau_i, \quad \boldsymbol{\zeta}_{i,1}(v, w) = \Gamma_i\tau_i, \tag{10}$$

where $\Phi_i = \left[\begin{array}{c|c} c_i & \begin{array}{c} I_{s_i-1} \\ \hdashline 0\cdots0 \end{array} \end{array}\right]$, $\Gamma_i = [1, 0, \ldots, 0]$, $c_i = \mathrm{col}(c_{i,s_i-1}, \ldots, c_{i,0})$.

Based on the (10), the internal model can be designed correspondingly,

$$\dot{\eta}_i = M_i\eta_i + N_i\zeta_{i,1}, \quad u_{im} = \Gamma_i\eta_i, \tag{11}$$

where $M_i = \Phi_i - N_i\Gamma_i$, $N_i = m_i + c_i$, and the designed parameters $m_i = \mathrm{col}(m_{i,s_i-1}, \ldots, m_{i,0})$ are chosen so that M_i is Hurwitz.

Remark 2. It can be verified that when $\eta_i = \tau_i$ and $\zeta_{i,1} = \boldsymbol{\zeta}_{i,1}(v, w)$, (11) reduces to (10), i.e., $M_i\tau_i + N_i\boldsymbol{\zeta}_{i,1}(v, w) = (M_i + N_i\Gamma_i)\tau_i = \Phi_i\tau_i$. So (11) is the internal model with output $\zeta_{i,1}$, c.f., Definition 6.6 in [12].

It is important to mention that internal model (11) is capable of reproducing not only the steady-state information $\boldsymbol{\zeta}_{i,1}(v, w)$, but $\boldsymbol{\zeta}_{i,2}(v, w), \ldots, \boldsymbol{\zeta}_{i,r_i}(v, w)$ as well, where $\boldsymbol{\zeta}_{i,r_i}(v, w) = \mathbf{u}_i(v, w)$. In fact, as indicated in [33], denote $\beta_{i,j}(\tau_i) = \boldsymbol{\zeta}_{i,j}(v, w)$, then by (7) and (10), it shows,

$$\beta_{i,1}(\tau_i) = \Gamma_i\tau_i,$$
$$\beta_{i,j}(\tau_i) = \frac{\partial\beta_{i,j-1}(\tau_i)}{\partial\tau_i}\Phi_i\tau_i + a_{i,j-1}\,\beta_{i,1}(\tau_i), \quad j = 2, \ldots, r_i. \tag{12}$$

So (11) is the internal model with output $\beta_{i,j}(\eta_i)$, $j = 1, \ldots, r_i$. ■

Now for the controlled agent (1), replace $\mathrm{col}(x_{i,2}, \ldots, x_{i,r_i})$ with ζ_i, and lump the the error dynamics and the internal model together, it gives,

$$\begin{aligned}
\dot{z}_i &= f_i(z_i, y_i, v, w),\\
\dot{\bar{x}}_i &= A_i\bar{x}_i + \bar{g}_i(z_i, y_i, v, w),\\
\dot{\eta}_i &= M_i\eta_i + N_i\zeta_{i,1},\\
\dot{x}_{i,1} &= \zeta_{i,1} + g_{i,1}^*(\bar{x}_{i,1}, z_i, y_i, v, w),\\
\dot{\zeta}_i &= A_i\zeta_i + B_iu_i,
\end{aligned} \tag{13}$$

where $g_{i,1}^* = \bar{x}_{i,1} + a_{i,1}x_{i,1} + g_{i,1}(z_i, y_i, v, w)$.

Performing the following coordinate and input transformations on (13),

$$\begin{aligned}
&\tilde{z}_i = z_i - \mathbf{z}_i(v, w), \quad \tilde{x}_i = \bar{x}_i - \bar{\mathbf{x}}_i(v, w), \quad \tilde{\eta}_i = \eta_i - \tau_i - N_ie_i,\\
&e_i = x_{i,1} - \mathbf{x}_{i,1}(v, w), \quad \tilde{\zeta}_{i,j} = \zeta_{i,j} - \beta_{i,j}(\eta_i), \quad j = 1, \ldots, r_i, \quad \tilde{\zeta}_{i,r_i} = u_i,
\end{aligned} \tag{14}$$

it yields,

$$\begin{aligned}
\dot{\tilde{z}}_i &= \tilde{f}_i(\tilde{z}_i, e_i, v, w),\\
\dot{\tilde{x}}_i &= A_i\tilde{x}_i + \tilde{g}_i(\tilde{z}_i, e_i, v, w),\\
\dot{\tilde{\eta}}_i &= M_i\tilde{\eta}_i + M_iN_ie_i - N_i\tilde{g}_{i,1}^*(\tilde{x}_i, \tilde{z}_i, e_i, v, w),\\
\dot{e}_i &= \tilde{\zeta}_{i,1} + \Gamma_i\tilde{\eta}_i + \Gamma_iN_ie_i + \tilde{g}_{i,1}^*(\tilde{x}_i, \tilde{z}_i, e_i, v, w),\\
\dot{\tilde{\zeta}}_{i,j} &= -a_{i,j}\tilde{\zeta}_{i,1} + \tilde{\zeta}_{i,j+1} - \frac{\partial\beta_{i,j}(\eta_i)}{\partial\eta_i}N_i\tilde{\zeta}_{i,1}, \quad j = 1, \ldots, r_i - 1,
\end{aligned} \tag{15}$$

where $\tilde{f}_i = f_i\left(\tilde{z}_i + \mathbf{z}_i(v, w), e_i + \mathbf{x}_{i,1}(v, w), v, w\right) - f_i\left(\mathbf{z}_i(v, w), \mathbf{x}_{i,1}(v, w), v, w\right)$, $\tilde{g}_i = \bar{g}_i\left(\tilde{z}_i + \mathbf{z}_i(v, w), e_i + \mathbf{x}_{i,1}(v, w), v, w\right) - \bar{g}_i\left(\mathbf{z}_i(v, w), \mathbf{x}_{i,1}(v, w), v, w\right)$, $\tilde{g}_{i,1}^* = \tilde{x}_{i,1} + a_{i,1}e_i + g_{i,1}\left(\tilde{z}_i + \mathbf{z}_i(v, w), e_i + \mathbf{x}_{i,1}(v, w), v, w\right) - g_{i,1}\left(\mathbf{z}_{i,1}(v, w), \mathbf{x}_{i,1}(v, w), v, w\right)$, $\tilde{\zeta}_{i,r_i} = \tilde{u}_i$, and $\frac{\partial\beta_{i,j}(\eta_i)}{\partial\eta_i}N_i$ are some known constants under (12).

It can be verified that when $(\tilde{z}_i, \tilde{x}_i, \tilde{\eta}_i, e_i, \tilde{\zeta}_i) = 0$, all smooth functions in (15) are vanishing at zero for any $v \in \mathbb{V}$, $w \in \mathbb{W}$, so the origin is the equilibrium point of (15). This implies the output synchronization problem for (13), or equivalently, for (1), is converted into the stabilization problem for (15), i.e., provided any control law $\tilde{u}_i(e_i^m, \tilde{\zeta}_{i,1}, \ldots, \tilde{\zeta}_{i,r_i-1})$ that stabilizes (15), then the control law $u_i = \tilde{u}_i\left(e_i^m, \tilde{\zeta}_{i,1} + \beta_{i,1}(\eta_i), \ldots, \tilde{\zeta}_{i,r_i-1} + \beta_{i,r_i-1}(\eta_i)\right) + \beta_{i,r_i}(\eta_i)$ achieves output synchronization for (1). Particularly, both the measurement output and the internal states of the dynamic controller are available for feedback back design, which makes stabilization problem for (15) more tractable.

3.3. Main result

To achieve robust stabilization, one more assumption is proposed w.r.t. (15).

Assumption 4. There exists a C^1 function $V_{i0}(\tilde{z}_i)$, such that along the trajectory of $\tilde{z}_i$ dynamics in (15), the following holds for any $v \in \mathbb{V}$, $w \in \mathbb{W}$,

$$\underline{\alpha}_{i0}(\|\tilde{z}_i\|) \leq V_{i0}(\tilde{z}_i) \leq \bar{\alpha}_{i0}(\|\tilde{z}_i\|), \quad \dot{V}_{i0}(\tilde{z}_i) \leq -\|\tilde{z}_i\|^2 + p_i\gamma_{i0}(|e_i|),$$

where $\underline{\alpha}_{i0}$, $\bar{\alpha}_{i0}$ are some $\mathcal{K}_\infty$ functions, γ_{i0} is a known class $\mathcal{K}$ function[2] satisfying $\lim\sup_{s\to 0^+}\frac{\gamma_{i0}(s)}{s^2} < \infty$, and $p_i > 0$ is the unknown constant. ■

Remark 3. Assumption 4 guarantees the $\tilde{z}_i$ dynamics is robust ISS w.r.t. state $\tilde{z}_i$ and input e_i. The unknown constant p_i is introduced due to the unknown bounds of $\mathbb{V}$ and $\mathbb{W}$. Under Assumption 4, by using changing supply function technique, for any smooth function $\Delta'_i(\tilde{z}_i) > 0$, there exists another C^1 function V'_{i0} satisfying $\underline{\alpha}'_{i0}(\|\tilde{z}_i\|) \le V'_{i0}(\tilde{z}_i) \le \bar{\alpha}'_{i0}(\|\tilde{z}_i\|)$, $\dot{V}'_{i0}(\tilde{z}_i) \le -\Delta'_i(\tilde{z}_i)\|\tilde{z}_i\|^2 + p'_i\gamma'_{i0}(e_i)e_i^2$, where $\underline{\alpha}'_{i0}, \bar{\alpha}'_{i0}$ are some class $\mathcal{K}_\infty$ functions, $\gamma'_{i0} \ge 1$ is a known smooth function, $p'_i > 0$ is the unknown constant. See Assumption 2.4 and Corollary 2.3 in [4] for detailed discussions. ■

Denote the inverse dynamics of (15) by $Z_i = \mathrm{col}(\tilde{z}_i, \tilde{x}_i, \tilde{\eta}_i)$, and put (15) into the following compact form,

$$\begin{aligned}
\dot{Z}_i &= h_{i0}(Z_i, e_i, v, w),\\
\dot{e}_i &= h_{i1}(Z_i, e_i, v, w) + \tilde{\zeta}_{i,1},\\
\dot{\tilde{\zeta}}_{i,j} &= -\tilde{a}_{i,j}\tilde{\zeta}_{i,1} + \tilde{\zeta}_{i,j+1}, \quad j = 1, \dots, r_i - 1,
\end{aligned} \tag{16}$$

where $\tilde{\zeta}_{i,r_i} = \tilde{u}_i$, and

$$\begin{aligned}
h_{i0} &= \begin{bmatrix} \tilde{f}_i(\tilde{z}_i, e_i, v, w) \\ A_i\tilde{x}_i + \tilde{g}_i(\tilde{z}_i, e_i, v, w) \\ M_i\tilde{\eta}_i + M_iN_ie_i - N_i\tilde{g}^*_{i,1}(\tilde{x}_i, \tilde{z}_i, e_i, v, w) \end{bmatrix},\\
h_{i1} &= \Gamma_i\tilde{\eta}_i + \Gamma_iN_ie_i + \tilde{g}^*_{i,1}(\tilde{x}_{i,1}, \tilde{z}_i, e_i, v, w),\\
\tilde{a}_{i,j} &= a_{i,j} + \frac{\partial\beta_{i,j}(\eta_i)}{\partial\eta_i}N_i.
\end{aligned}$$

Since both A_i and M_i are designed to be Hurwitz, it is easy to show that the inverse dynamics Z_i is in fact robust ISS w.r.t. state Z_i and input e_i. By using changing supply function technique again and recalling the relation $e^m = \mathcal{L}e$, it gives the following proposition.

Proposition 1. *Under Assumption 4, for any smooth function $\Delta_i(Z_i) > 0$, there exists a C^1 functions $V_{i1}(Z_i)$, such that along the trajectory of Z_i in (16), the following holds for any $v \in \mathbb{V}$, $w \in \mathbb{W}$,*

$$\begin{aligned}
\underline{\alpha}_{i1}(\|Z_i\|) &\le V_{i1}(Z_i) \le \bar{\alpha}_{i1}(\|Z_i\|),\\
\dot{V}_{i1}(Z_i) &\le -\Delta_i(Z_i)\|Z_i\|^2 + \hbar_{i1}\varphi_{i1}(e_i^m)(e_i^m)^2,
\end{aligned} \tag{17}$$

where $\underline{\alpha}_{i1}$ and $\bar{\alpha}_{i1}$ are some $\mathcal{K}_\infty$ functions, $\varphi_{i1}(s_i) \ge 1$ is a known smooth function, and $\hbar_{i1}$ is the unknown constant. ■

To accomplish the robust stabilization problem for (16), the backstepping design method combined with the modified dynamic high-gain technique is used.

In each design step, consider $\tilde{\zeta}_{i,j}$ as the virtual control, the corresponding control law $\pi_{i,j}$ is given by (18), and $\bar{\zeta}_{i,j} = \tilde{\zeta}_{i,j} - \pi_{i,j}$ is the error variables. Also, let $\bar{\zeta}_{i,r_i} = 0$, then $\tilde{u}_i = \pi_{i,r_i}$,

$$\begin{aligned}
\pi_{i,1} &= -k_{i1}\,\mu_i(e_i^m), \quad \dot{k}_{i1} = \nu_i(e_i^m),\\
\pi_{i,2} &= \tilde{a}_{i,1}\bar{\zeta}_{i,1} - k_{i2}\bar{\zeta}_{i,1} - \left(\frac{\partial\pi_{i,1}}{\partial e_i^m}\right)^2\bar{\zeta}_{i,1} + \frac{\partial\pi_{i,1}}{k_{i1}}\dot{k}_{i1}, \quad \dot{k}_{i2} = \bar{\zeta}_{i,1}^2,\\
\pi_{i,j} &= \tilde{a}_{i,j-1}\bar{\zeta}_{i,1} - \bar{\zeta}_{i,j-2} - \bar{\zeta}_{i,j-1} - \left(\frac{\partial\pi_{i,j-1}}{\partial y_i}\right)^2\bar{\zeta}_{i,j-1}\\
&\quad + \frac{\partial\pi_{i,j-1}}{\partial k_{i1}}\dot{k}_{i1} + \frac{\partial\pi_{i,j-1}}{\partial k_{i2}}\dot{k}_{i2} + \sum_{\ell=1}^{j-2}\frac{\partial\pi_{i,j-1}}{\partial\tilde{\zeta}_{i,\ell}}\dot{\tilde{\zeta}}_{i,\ell}, \ j = 3, \dots, r_i.
\end{aligned} \tag{18}$$

In (18), μ_i: $\mathbb{R}\mapsto\mathbb{R}$, ν_i: $\mathbb{R}\mapsto\mathbb{R}^+$ are the design functions taking the form $\mu_i(s) = \rho_i(s)s$, $\nu_i(s) = \varrho_i(s)s^2$, where $\rho_i(s) \ge 1$, $\varrho_i(s) \ge 1$ are smooth functions, $\mu_i(s)$ is strictly increasing, see (26)–(28), and k_{i1}, k_{i2} are the dynamic high-gains with initial condition $k_{i1}(0) \ge 1$. Since $\nu_i(s_i)$ is non-negative, so under (18), $k_{i1} \ge 1$.

The main result of the paper is given as follows.

Theorem 1. *Under Assumptions 1–4, the decentralized control law*

$$\begin{aligned}
\dot{\eta}_i &= M_i\eta_i + N_i\zeta_{i,1}, \quad \dot{\zeta}_i = A_i\zeta_i + B_iu_i,\\
u_i &= \pi_{i,r_i} + \beta_{i,r_i}(\eta_i), \quad i = 1, \dots, n,
\end{aligned} \tag{19}$$

solves the global robust output synchronization problem. ■

Proof. As indicated in Section 3.2, we will prove Theorem 1 by showing that the control law $\tilde{u}_i(e_i^m, \tilde{\zeta}_{i,1}, \dots, \tilde{\zeta}_{i,r_i-1}) = \pi_{i,r_i}$ solves the robust stabilization problem for (16).

For presentation convenience, denote $\Delta_{i1} = \left(\sum_{j=1}^n l_{ij}\pi_{j,1}\right)^2$, $\Delta_{i2} = \left(\sum_{j=1}^n l_{ij}\bar{h}_{j1}\right)^2$, $\mu_i^*(e_i^m) = k_{i1}\mu_i(e_i^m)$, $\mu^* = \mathrm{col}(\mu_1^*, \dots, \mu_n^*)$, and under (18), rewrite the dynamics of e_i and the error variables $\bar{\zeta}_{i,j}$ as follows,

$$\begin{aligned}
\dot{e}_i &= \bar{h}_{i1} + \pi_{i,1}, \quad \bar{h}_{i1} = h_{i1}(Z_i, e_i, v, w) + \bar{\zeta}_{i,1},\\
\dot{\bar{\zeta}}_{i,1} &= -\tilde{a}_{i,1}\bar{\zeta}_{i,1} + \bar{\zeta}_{i,2} + \pi_{i,2} - \frac{\partial\pi_{i,1}}{\partial k_{i1}}\dot{k}_{i1} - \frac{\partial\pi_{i,1}}{\partial e_i^m}\sum_{j=1}^n l_{ij}\left(\bar{h}_{j1} + \pi_{j,1}\right),\\
\dot{\bar{\zeta}}_{i,j-1} &= -\tilde{a}_{i,j-1}\bar{\zeta}_{i,1} + \bar{\zeta}_{i,j} + \pi_{i,j} - \frac{\partial\pi_{i,j-1}}{\partial k_{i1}}\dot{k}_{i1} - \frac{\partial\pi_{i,j-1}}{\partial k_{i2}}\dot{k}_{i2}\\
&\quad - \frac{\partial\pi_{i,j-1}}{\partial e_i^m}\sum_{\ell=1}^n l_{ij}\left(\bar{h}_{\ell 1} + \pi_{\ell,1}\right) - \sum_{\ell=1}^{j-2}\frac{\partial\pi_{i,j-1}}{\partial\tilde{\zeta}_{i,\ell}}\dot{\tilde{\zeta}}_{i,\ell}, \quad j = 3, \dots, r_i.
\end{aligned} \tag{20}$$

Define $V_1 = \sum_{i=1}^n V_{i1}(Z_i)$, it follows from Proposition 1 that,

$$\dot{V}_1 \le \sum_{i=1}^n\left(-\Delta_i(Z_i)\|Z_i\|^2 + \hbar_{i1}\varphi_{i1}(e_i^m)(e_i^m)^2\right). \tag{21}$$

Define $V_2 = \sum_{i=1}^n V_{i2}(e_i^m)$, where $V_{i2}(e_i^m) = d_ik_{i1}\int_0^{e_i^m}\mu_i(s)ds$, d_i is the i-th diagonal element of D as in (6). Notice that $d_i > 0$, $k_{i1} \ge 1$, $\mu_i(s) = \rho_i(s)s$ is strictly increasing, so we claim that V_2 is positive definite and radially unbounded, and $\int_0^{e_i^m}\mu_i(s)ds \le \mu_i(e_i^m)(e_i^m)$.

Along the trajectory of e_i in (20), and by $e^m = \mathcal{L}e$, it shows,

$$\begin{aligned}
\dot{V}_2 &= \sum_{i=1}^n\left(d_ik_{i1}\mu_i(e_i^m)\sum_{j=1}^n l_{ij}\left(\bar{h}_{j1} + \pi_{j,1}\right)\right) + \sum_{i=1}^n\left(d_i\nu_i(e_i^m)\int_0^{e_i^m}\mu_i(s)ds\right)\\
&\le -\mu^* D\mathcal{L}\mu^* + \sum_{i=1}^n\frac{\varepsilon_{i2}}{2}(\mu_i^*)^2 + \sum_{i=1}^n\frac{d_i^2}{2\varepsilon_{i2}}\Delta_{i2} + \sum_{i=1}^n d_i\nu_i(e_i^m)\mu_i(e_i^m)e_i^m\\
&\le \frac{-\lambda_{\min} + \varepsilon_{i2}}{2}\|\mu_i^*\|^2 + \sum_{i=1}^n\frac{d_i^2}{2\varepsilon_{i2}}\Delta_{i2} + \sum_{i=1}^n d_i\nu_i(e_i^m)\mu_i(e_i^m)e_i^m,
\end{aligned} \tag{22}$$

where $\varepsilon_{i2} > 0$ are free parameters, $\lambda_{\min}$ is the minimal eigenvalue of $D\mathcal{L} + \mathcal{L}^{\top}D$. Due to the possible choices of D as indicated in Remark 1, $\lambda_{\min}$ is also a free parameter.

For the $\bar{\zeta}_i$ dynamics, define $U_{i,1} = \frac{1}{2}\bar{\zeta}_{i,1}^2$, and it yields, $\dot{U}_{i,1} = \bar{\zeta}_{i,1}$

[2] A continuous function α: $[0, a)\mapsto[0, \infty)$ is of class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. α is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and in addition, $a = \infty$ and $\lim_{r\to\infty}\alpha(r) \to \infty$, c.f. Definition 4.2 in [14].

$\left(\bar{\zeta}_{i,2} - k_{i2}\bar{\zeta}_{i,1} - \left(\frac{\partial \pi_{i,1}}{\partial e_i^m}\right)^2 \bar{\zeta}_{i,1} - \frac{\partial \pi_{i,1}}{\partial e_i^m} \sum_{j=1}^n l_{ij}(\bar{h}_{j1} + \pi_{j,1})\right) \leq \bar{\zeta}_{i,1}\bar{\zeta}_{i,2} - k_{i2}\bar{\zeta}_{i,1}^2$ $+\frac{1}{2}(\Delta_{i1} + \Delta_{i2})$. Recursively, define $U_{i,\ell} = U_{i,\ell-1} + \frac{1}{2}\bar{\zeta}_{i,\ell}^2$, $\ell = 2, \ldots, r_i - 1$, and recall that $\bar{\zeta}_{i,r_i} = 0$, it is possible to show, $\dot{U}_{i,r_i-1} \leq -k_{i2}\bar{\zeta}_{i,1}^2 - \sum_{\ell=2}^{r_i-1} \bar{\zeta}_{i,\ell}^2 + \frac{r_i - 1}{2}(\Delta_{i1} + \Delta_{i2})$.

Define $V_3 = \sum_{i=1}^n U_{i,r_i-1}$ and denote $\bar{\zeta}_i^* = \mathrm{col}(\bar{\zeta}_{i,2}, \ldots, \bar{\zeta}_{i,r_i-1})$, it shows,

$$\dot{V}_3 \leq \sum_{i=1}^n \left(-k_{i2}\bar{\zeta}_{i,1}^2 - \| \bar{\zeta}_i^* \|^2 + \frac{r_i - 1}{2}(\Delta_{i1} + \Delta_{i2}) \right). \tag{23}$$

For Δ_{i1} in (23), due to the fact that l_{ij} are unknown, it is possible to show $\sum_{i=1}^n \frac{r_i - 1}{2}\Delta_{i1} \leq \sum_{i=1}^n \delta_{i1}(\mu_i^*)^2$ for some unknown constants $\delta_{i1} > 0$. Then there exists free parameter $K_2 > 0$, together with free parameters $\lambda_{\min}$ and ε_{i2} in (22), such that $K_2\frac{-\lambda_{\min} + \varepsilon_{i2}}{2} + \delta_{i1} \leq -2$.

For Δ_{i2} in (22) and (23), since $\bar{h}_{i1} = h_{i1}(Z_i, e_i, v, w) + \bar{\zeta}_{i,1}$, it can be shown $\sum_{i=1}^n \left(\frac{K_2 d_i^2}{2\varepsilon_{i2}} + \frac{r_i - 1}{2} \right)\Delta_{i2} \leq \sum_{i=1}^n \left(\delta_{i2}\bar{\zeta}_{i,1}^2 + \delta_{i3}\Delta_{i3}(Z_i)\| Z_i \|^2 + \hbar_{i2}\varphi_{i2}(e_i^m)(e_i^m)^2 \right)$, where $\Delta_{i3} \geq 1$, $\varphi_{i2} \geq 1$ are known smooth functions, and δ_{i2}, δ_{i3}, $\hbar_{i2}$ are unknown constants.

Thus, define $V = V_1 + K_2 V_2 + V_3$, and let $\Delta_i(Z_i) \geq \delta_{i3}\Delta_{i3}(Z_i) + 1$ in (21), then by (21)–(23), it gives

$$\begin{aligned}\dot{V} = \sum_{i=1}^n \Big(&-\| Z_i \|^2 + \hbar_{i1}\varphi_{i1}(e_i^m)(e_i^m)^2 - 2(\mu_i^*)^2 + K_2 d_i \nu_i(e_i^m)\mu_i(e_i^m)e_i^m \\ &- k_{i2}\bar{\zeta}_{i,1}^2 - \| \bar{\zeta}_i^* \|^2 + \delta_{i2}\bar{\zeta}_{i,1}^2 + \hbar_{i2}\varphi_{i2}(e_i^m)(e_i^m)^2 \Big).\end{aligned} \tag{24}$$

Further, define $V_k = \sum_{i=1}^n V_{ik}$, where $V_{ik} = \frac{1}{2}(k_{i1} - \bar{k}_{i1})^2 + \frac{1}{2}(k_{i2} - \bar{k}_{i2})^2$, $\bar{k}_{i1}$, $\bar{k}_{i2}$ are free parameters, it yields,

$$\dot{V}_k = \sum_{i=1}^n \left((k_{i1} - \bar{k}_{i1})\nu_i(e_i^m) + (k_{i2} - \bar{k}_{i2})\bar{\zeta}_{i,1}^2 \right). \tag{25}$$

Choose free parameter $\bar{k}_{i2} \geq \delta_{i2} + 1$, then within (24) and (25), $-k_{i2}\bar{\zeta}_{i,1}^2 - \| \bar{\zeta}_i^* \|^2 + \delta_{i2}\bar{\zeta}_{i,1}^2 + (k_{i2} - \bar{k}_{i2})\bar{\zeta}_{i,1}^2 \leq -\| \bar{\zeta}_i \|^2$, where $\bar{\zeta}_i = \mathrm{col}(\bar{\zeta}_{i,1}, \ldots, \bar{\zeta}_{i,r_i-1})$.

Consider the term $K_2 d_i \nu_i(e_i^m)\mu_i(e_i^m)e_i^m$ in (24). By Young's Inequality, it shows that $K_2 d_i \mu_i(s)s \leq \frac{(\delta K_2 d_i)^8}{8} + \frac{7}{8}\left(\frac{\mu_i(s)s}{\delta} \right)^{\frac{8}{7}}$ for any constant $\delta > 0$. Thus, $K_2 d_i \nu_i(e_i^m)\mu_i(e_i^m)(e_i^m) \leq \delta_{i4}\nu_i(e_i^m) + \epsilon_{i4}\nu_i(e_i^m)\left(\mu_i(e_i^m)e_i^m \right)^{\frac{8}{7}}$, where ϵ_{i4} is a free parameter, and δ_{i4} is the unknown constant.

Confine the design functions $\nu_i(s)$ and $\mu_i(s)$ such that

$$\nu_i(s)\left(\mu_i(s)s \right)^{\frac{8}{7}} \leq \left(\mu_i(s) \right)^2, \quad \text{i. e.,} \ \varrho_i(s)(s)^{\frac{16}{7}} \leq \left(\rho_i(s) \right)^{\frac{6}{7}}, \tag{26}$$

since $k_{i1} > 0$ by design, then $\mu_i(s) \leq k_{i1}\mu_i(s)$ and $\left(\mu_i(s) \right)^2 \leq \left(\mu_i^*(s) \right)^2$, so $K_2 d_i \nu_i(e_i^m)\mu_i(e_i^m)(e_i^m) \leq \delta_{i4}\nu_i(e_i^m) + \epsilon_{i4}\left(\mu_i^*(e_i^m) \right)^2$.

Consider the term $k_{i1}\nu_i(e_i^m)$ in (25). Confine the design functions $\nu_i(s)$ and $\mu_i(s)$ such that

$$\nu_i(s) \leq \mu_i^2(s), \quad \text{i. e.,} \ \varrho_i(s) \leq \left(\rho_i(s) \right)^2, \tag{27}$$

it gives, $k_{i1}\nu_i(e_i^m) \leq k_{i1}(\mu_i(e_i^m))^2 \leq (\mu_i^*)^2$.

Further, choose the design function $\nu_i(e_i^m)$ such that

$$\nu_i(s) \geq \left(\varphi_{i1}(s) + \varphi_{i2}(s) \right)(s)^2, \quad \text{i. e.,} \ \varrho_i(s) \geq \varphi_{i1}(s) + \varphi_{i2}(s), \tag{28}$$

and choose the free parameters $\bar{k}_{i1} \geq 1 + \delta_{i4} + \hbar_{i1} + \hbar_{i2}$, $\epsilon_{i4} < 1$, then by (24)–(28), it yields, $-2(\mu_i^*)^2 + K_2 d_i \nu_i(e_i^m)\mu_i(e_i^m)e_i^m + \hbar_{i1}\varphi_{i1}(e_i^m)(e_i^m)^2$ $+ \hbar_{i2}\varphi_{i2}(e_i^m)(e_i^m)^2 + (k_{i1} - \bar{k}_{i1})\nu_i(e_i^m) \leq -2(\mu_i^*)^2 + \epsilon_{i4}(\mu_i^*)^2 + (\mu_i^*)^2$ $-\nu_i^2(e_i^m) \leq -\nu_i^2(e_i^m)$.

Finally, define the Lyapunov function $V + V_k$, it gives,

$$\dot{V} + \dot{V}_k \leq \sum_{i=1}^n \left(-\| Z_i \|^2 - \| \bar{\zeta}_i \|^2 - \nu_i^2(e_i^m) \right).$$

By LaSalle-Yoshizawa Theorem, the origin is the globally asymptotically stable equilibrium point of the closed-loop system, and $\lim_{t\to\infty} e_i^m = 0$, which indicates that $\lim_{t\to\infty} e_i = 0$. The proof is completed. □

Remark 4. When there exists some controlled agents $\mathcal{S}_i$ with relative degree $r_i = 1$, there's no need to utilize the reduced-order input-driven filter. Instead, under Assumption 3, it is evident that $\mathbf{u}_i(v, w)$ satisfies the similar condition as (9), thus the internal model can be designed of the form $\dot{\eta}_i = M_i\eta_i + N_i u_i$, $u_{im} = \Gamma_i\eta_i$. Moreover, under the coordinates and input transformations $\tilde{z}_i = z_i - \mathbf{z}_i(v, w)$, $\tilde{\eta}_i = \eta_i - \tau_i - N_i e_i$, $e_i = x_{i,1} - \mathbf{x}_{i,1}(v, w)$, it gives,

$$\dot{\tilde{z}}_i = \tilde{f}_i(z_i, e_i, v, w),$$

$$\dot{\tilde{\eta}}_i = M_i\tilde{\eta}_i + M_i N_i e_i - N_i\tilde{g}_{i,1}^*(\tilde{z}_i, e_i, v, w),$$

$$\dot{e}_i = \tilde{u}_i + \Gamma_i\tilde{\eta}_i + \Gamma_i N_i e_i + \tilde{g}_{i,1}^*(\tilde{z}_i, e_i, v, w),$$

where $\tilde{g}_{i,1}^* = g_{i,1}(\tilde{z}_i + \mathbf{z}_i, e_i + \mathbf{x}_{i,1}, v, w) - g_{i,1}(\mathbf{z}_i, \mathbf{x}_{i,1}, v, w)$. (29) is in the form of (16) by viewing $Z_i = \mathrm{col}(\tilde{z}_i, \tilde{\eta}_i)$ as the inverse dynamics, and the control law $\dot{\eta}_i = M_i\eta_i + N_i u_i$, $u_i = \pi_{i,1} + \Gamma_i\eta_i$ is applicable to solve the problem.

3.4. Discussions

It is worth mentioning that the key ingredient in the proof of Theorem 1 is the construction of V_{i2}. The utilization of such type of Lyapunov function in variant forms is vital to the stability analysis under the directed network topology. This fact has been fully recognized in the recent publications, c.f., [2,24,28,31,32,34].

In the settings of our problem, the only requirement of the network is that the digraph $\bar{\mathcal{G}}$ contains a directed spanning tree with node 0 as the root. Other quantitative information of the network is not known a prior, e.g., the size of the network or the weights of the edges. This is an obvious relaxation of assumption regarding the network topology. However, it also means that any control law that explicitly or implicitly relies on the quantitative information of the network is not applicable. For instance, the distributed observer based designs as in [10,19,34] fail in the current case since they depend on the weights of edges, or equivalently, l_{ij} of $\mathcal{L}$, which are unknown as we claimed in Remark 1. Moreover, as shown in the proof of Theorem 1, both Δ_{i1} and Δ_{i2} contains uncertain parameters l_{ij} without known bounds, this fact implies that the static control gain as in [24,31,32] is incapable of action.

As a matter of fact, we consider the quantitative information of the uncertain network as sort of parametric uncertainties, which are indeed represented by the unknown matrix $\mathcal{L}$. And we utilize the dynamic high-gain technique to tackle the problem. By the designed control law (19), it is possible to rearrange the communication network topology among the given agents, see the example in next section, or even introduce more controlled agents into the system without redesign. In the latter case, the newly added agents are required to satisfy certain structure constraints comparing with the existing ones. The preliminary results on this topic is fascinating and it deserves our further investigation.

The design method provided herein also inherits one essential feature which motivates our study of distributed observer based design in [34], that is, to avoid the fully collaborative design manner such as "block backstepping". Specifically, by using fully collaborative design, the virtual control laws for every controlled agents are determine together in every design steps, thus all agents are required to have the same relative degrees, c.f. [7,8,23,24]. On the contrary, our partially collaborative design manage to implement most of the design for each controlled agent separately. It provides certain feasibility to handle the case that heterogeneous controlled agents are with different relative degrees, and reduces the computational difficulty to achieve the final control law, see Remark 3.4 in [34] for discussions. Also, the designed control law is of decentralized type rather than distributed type, so it simplifies the engineering application by avoiding communications between controllers.

4. Applications

In this section, we consider an output synchronization problem for heterogeneous agents of Lorenz-type in an uncertain network.

The agents in the form of Hyperchaotic Lorenz System [15] are described by the output feedback system with relative degree $r_i = 2$,

$$\dot{z}_{i1} = -35z_{i1} + 35x_{i1}, \quad \dot{z}_{i2} = -3z_{i2} + z_{i1}x_{i1},$$
$$\dot{x}_{i1} = x_{i2} + 7z_{i1} - z_{i1}z_{i2} + 12x_{i1}, \quad \dot{x}_{i2} = u_i - w_i z_{i1}. \tag{29}$$

where the input-driven filter $\dot{\zeta}_{i,1} = -\zeta_{i,1} + u_i$ is designed first.

The agents in the form of Generalized Lorenz System [3] are described by the output feedback system with relative degree $r_i = 1$,

$$\dot{z}_{i1} = -z_{i1} + x_{i1}, \quad \dot{z}_{i2} = -z_{i2} + \frac{-w_i + 1}{2} z_{i1}^2,$$
$$\dot{x}_{i1} = u_i + 6z_{i1} - 5z_{i1}z_{i2} - \frac{w_i + 1}{2} z_{i1}^3. \tag{30}$$

The reference trajectory y_0 is generated by a harmonic oscillator,

$$\dot{v}_1 = v_2, \quad \dot{v}_2 = -v_1, \quad y_0 = w_0 v_1, \tag{31}$$

In (29)–(31), w_0 and w_i are unknown constants, and y_0 is actually a sinusoidal signal with unknown amplitude and phase.

It can be verified that both Assumptions 2 and 3 are satisfied. Particularly, both $\zeta_{i,1}(v, w)$ for the input-driven filter of (29) and $\mathbf{u}_i(v, w)$ for (30) are taking the form $\chi(v, w) = \bar{w}_1 v_1 + \bar{w}_2 v_2 + \bar{w}_3 v_1^3 + \bar{w}_4 v_1^2 v_2 + \bar{w}_5 v_1 v_2^2 + \bar{w}_6 v_2^3$, where $\bar{w}_i$ are unknown constants. It can be verified that along the trajectory of (31), $\chi(v, w)$ satisfies condition (9), i.e., $\chi^{(4)}(v, w) = -10\chi^{(2)}(v, w) - 9\chi^{(0)}(v, w)$. Thus, w.r.t. $\zeta_{i,1}(v, w)$ and $\mathbf{u}_i(v, w)$, the steady-state input generators can be derived, and internal models can be designed w.r.t. (29) and (30),

$$\dot{\eta}_i = M_i \eta_i + N_i \zeta_{i,1}, \tag{32a}$$

$$\dot{\eta}_i = M_i \eta_i + N_i u_i, \tag{32b}$$

where

$$M_i = \begin{bmatrix} -4 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad N_i = \begin{bmatrix} 4 \\ -4 \\ 4 \\ -8 \end{bmatrix}.$$

For (29) and (30), attaching the designed input-driven filter and the internal models, after coordinate and the input transformations, it can be verified that the resulting dynamics are satisfying Assumption 4.

Finally, according to Theorem 1, the design functions can be derived: $\varrho_i = 1 + (e_i^m)^6$, $\rho_i = 2\left(1 + (e_i^m)^{10}\right)$. So for agents (29), the control law is given as: $\dot{\zeta}_{i,1} = -\zeta_{i,1} + u_i$, $\dot{\eta}_i = M_i \eta_i + N_i \zeta_{i,1}$, $u_i = \pi_{i,2} + \eta_{i,1} + \eta_{i,2}$. And for agents (30), the control law is given as:

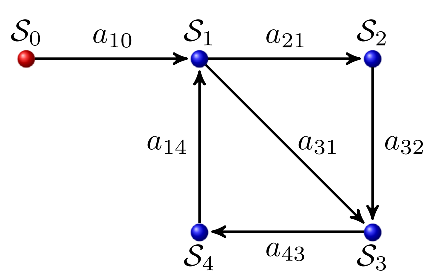

Fig. 1. The Network Topology: Network I (Left); Network II (Right).

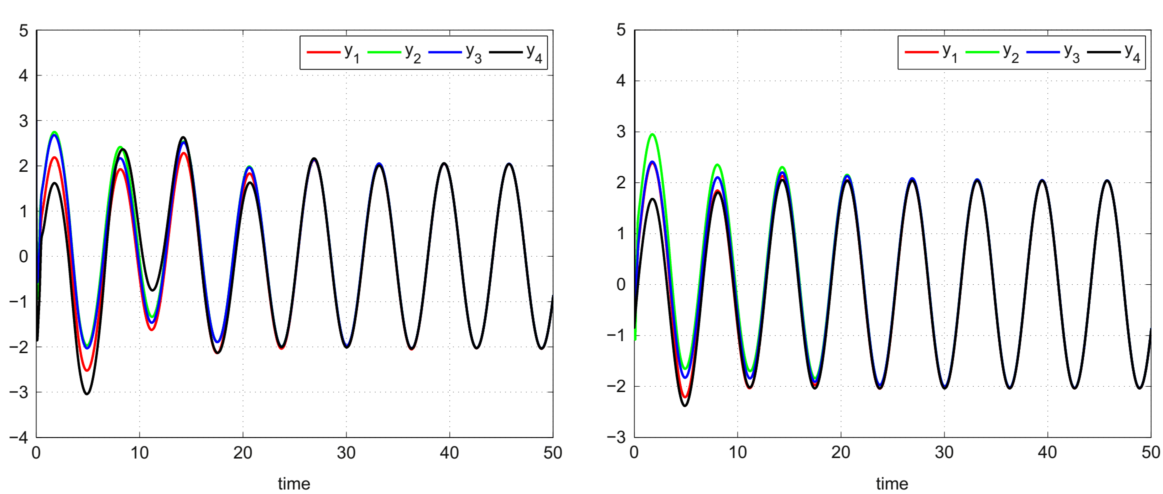

Fig. 2. Tracking Performance (0–50 s): Network I (Left); Network II (Right).

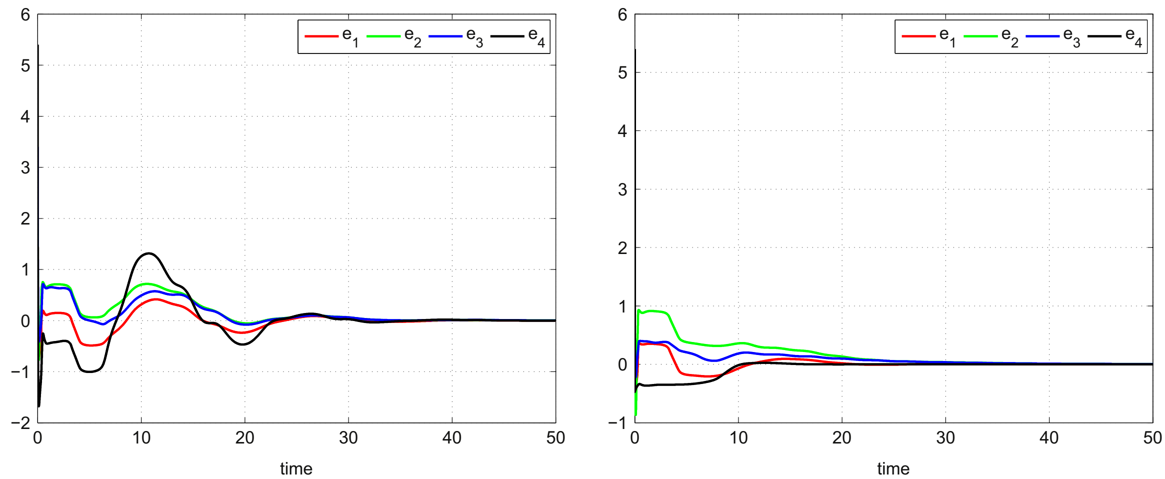

Fig. 3. Tracking Error (0–50 s): Network I (Left); Network II (Right).

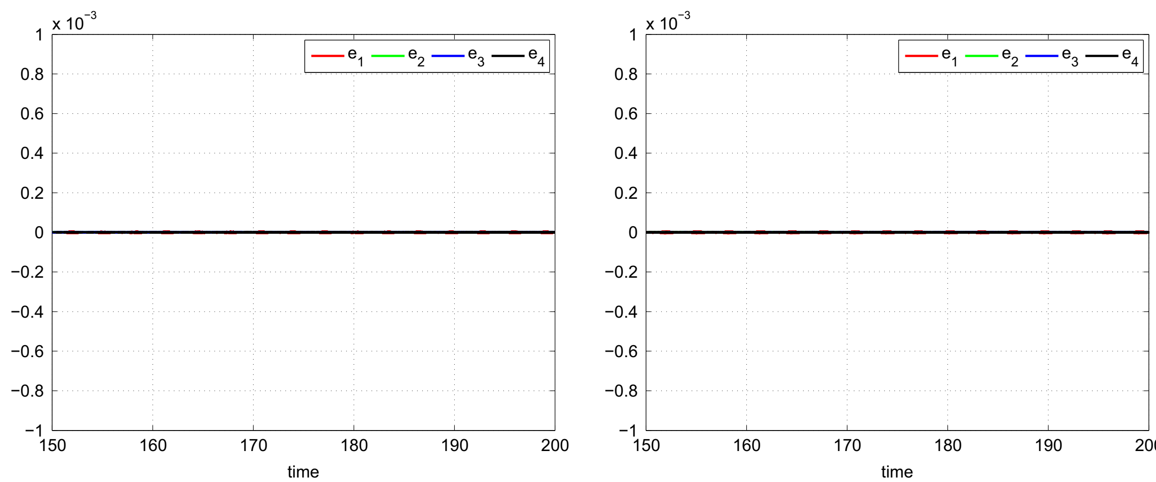

Fig. 4. Tracking Error (150–200 s): Network I (Left); Network II (Right).

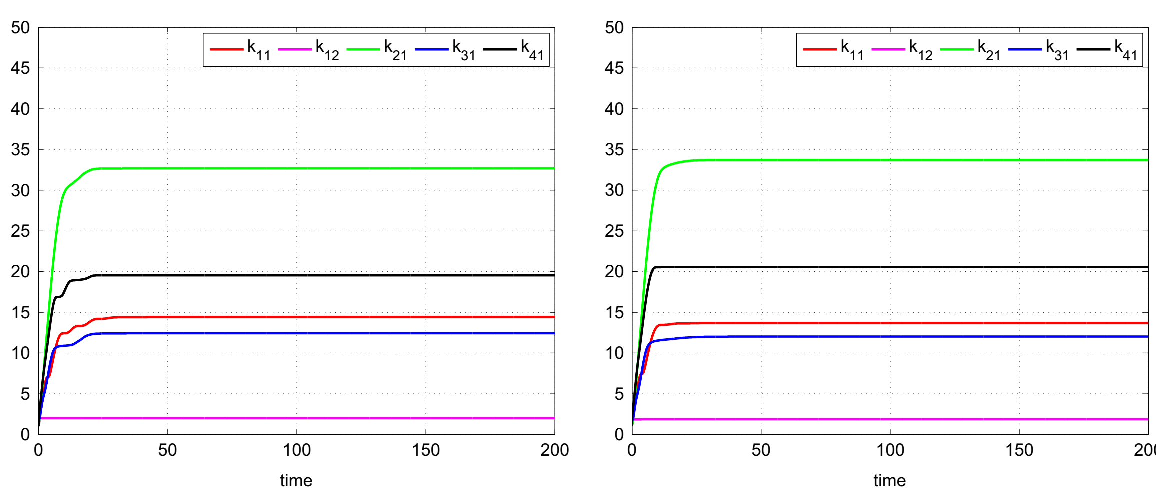

Fig. 5. Dynamic High-gain (0–200 s): Network I (Left); Network II (Right).

$\dot{\eta}_i = M_i\eta_i + N_iu_i,\ u_i = \pi_{i,1} + \eta_{i,1}$.

We consider two networks which are illustrated in Fig. 1. In both networks, $\mathcal{S}_0$ represents the harmonic oscillator (31), $\mathcal{S}_1$ is the agent in the form of Hyperchaotic Lorenz System (29), and $\mathcal{S}_2$-$\mathcal{S}_4$ are the agents in the form of Generalized Lorenz System (30).

In simulation, the unknown weights a_{ij} are chosen to be $a_{10} = 3$, $a_{21} = 2,\ a_{32} = 1$ for both networks, $a_{31} = 2,\ a_{43} = 1,\ a_{14} = 1$ for network I, and $a_{40} = 3,\ a_{34} = 2,\ a_{13} = 1$ for network II. The uncertain parameters of the agents are chosen to be $w_0 = 2,\ w_1 = 3,\ w_2 = 2,$

$w_3 = 1$, $w_4 = -0.5$. The initial conditions for agents $\mathcal{S}_i$ are chosen to be $(-0.2, 1)$, $(3, -2, 1, -1)$, $(4.5, -3, 1.5)$, $(-2, 4, 3)$, $(3, 6, 5)$, respectively. The initial conditions for controllers are chosen to be $\eta_1(0) = \eta_2(0) = \eta_3(0) = \eta_4(0) = 0$, $\zeta_1(0) = 0$, and $k_{11}(0) = k_{12}(0) = k_{21}(0) = k_{31}(0) = k_{41}(0) = 1$.

Without changing the designed control laws, we perform the simulation on both networks for 200 s. The simulation results on tracking performance, tracking error and dynamic high-gain are illustrated in Figs. 2–5.

5. Conclusion

This paper investigates the global robust output synchronization problem for a class of heterogeneous nonlinear multi-agent systems in uncertain networks. The existence of uncertainty in the network topology, the heterogeneity of the uncertain controlled agents, and the inability to exchange controllers' internal states present a number of challenges to the problems.

We solve the problem via nonlinear output regulation theory and internal model based design. For each controlled agent, the designed input-driven filter and the internal model compose the dynamic controller. After certain coordinate and input transformations, the problem is converted into a robust stabilization problem. Then the decentralized dynamic output feedback control law is derived to achieve robust feedback stabilization. Particularly, the modified dynamic high-gain technique is utilized to cope with the uncertainty of the network and the unknown bounds of both parametric uncertainty and external disturbance of the agents. We also use the output synchronization problem of Lorenz-type agents for illustration.

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