# Neutrino Oscillations in Matter

PhD Candidacy Exam

Lei Ma
**Supervisor**: Huaiyu Duan

Department of Physics
UNM

April 19, 2016

# Outline

1. Introduction
   - History of Neutrinos
   - What are Neutrinos
   - Neutrino Oscillations
2. Matter Effect
   - Matter Interaction
   - MSW Effect
   - Stimulated Neutrino Oscillation
3. Understanding Stimulated Oscillations
   - Hamiltonian, and Basis
   - Single Frequency Matter Profile
4. Summary & Future Work

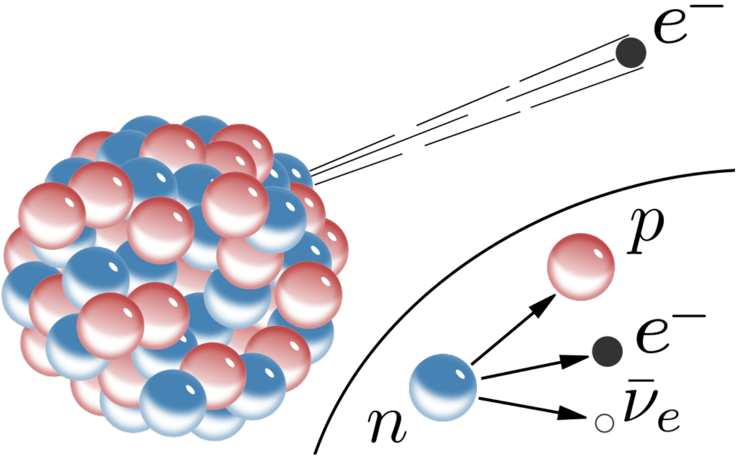
# Overview

# Neutrino Timeline

*History of Neutrino (Partial)*

- 1930 – Pauli, letter to ”Radioactive Ladies and Gentlemen”
- 1933 – Fermi, the name ”neutrino”
- 1956 – Reines & Cowan, first neutrino evidence
- 1957 – **Pontecorvo, theory of neutrino oscillations**
- 1968 – Homestake, first solar neutrino detection
- 1978 & 1985 – **Wolfenstein & Mikheyev & Smirnov, MSW effect**
- 1987 – Kamioka mine & Morton salt mine, SN1987A neutrino
- 1998 & 2001 – Super-Kamiokande & SNO, solar neutrino oscillations

# What are Neutrinos?




Beta decay and antineutrino production. Source: Beta_Decay@Wikipedia

# What are Neutrinos?

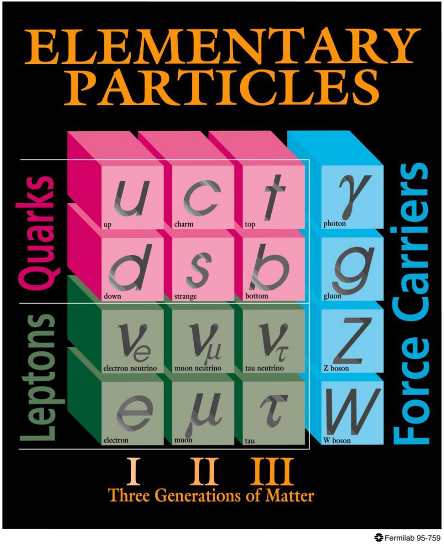
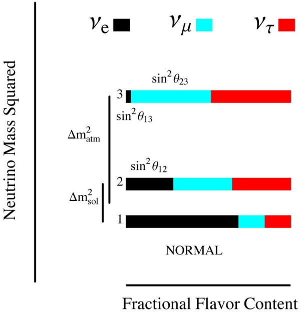



Table of elementary particles. Source: Fermilab

Neutrinos are

- Fermions,
- electrically neutral,
- **light**.




Adapted from Olga Mena & Stephen Parke, 2004

# WHAT ARE NEUTRINOS?

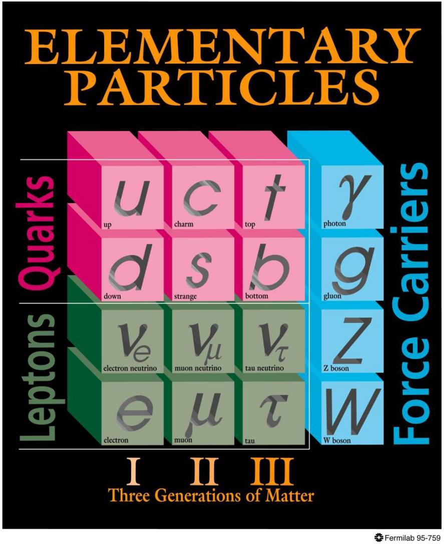
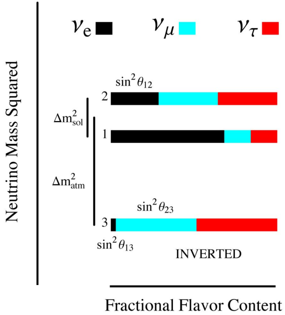



Table of elementary particles. Source: Fermilab

Neutrinos are

- Fermions,
- electrically neutral,
- **light**.



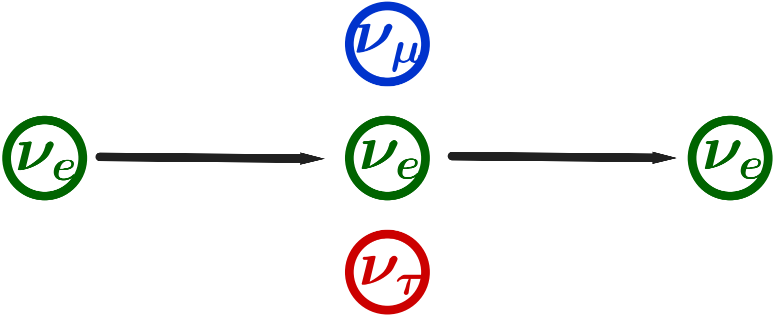

Adapted from Olga Mena & Stephen Parke, 2004

# What is Neutrino Oscillation?

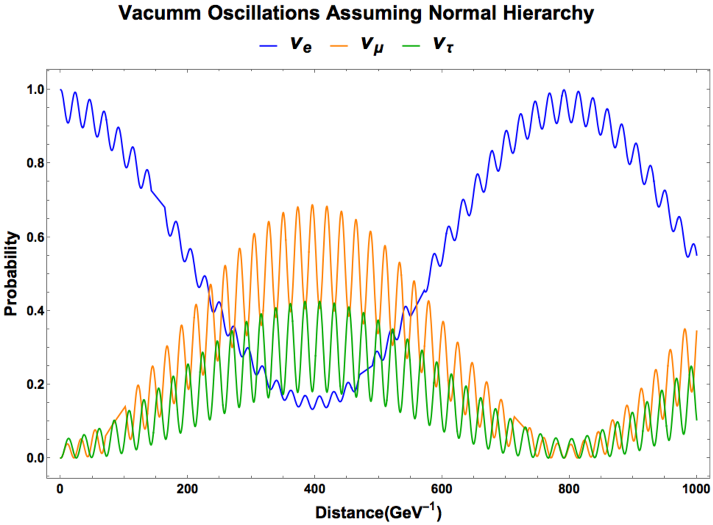
**Neutrino Oscillation**

||

**Neutrino Flavor Conversion**



# What is Neutrino Oscillation?

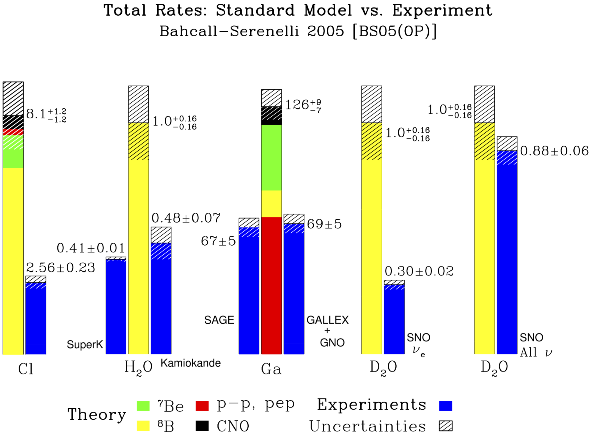



Survival probability

# Solar Neutrinos

## Solar Neutrinos

$$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$$



# Why Do Neutrinos Oscillate?

**Equation of Motion**

$$i\partial_x \Psi = \mathbf{H}\Psi$$

- Basis: Hamiltonian diagonalized basis/mass eigenbasis/propagation eigenbasis, $|\nu_1\rangle$ and $|\nu_2\rangle$.
- $$\mathbf{H} = -\frac{\omega_v}{2}\boldsymbol{\sigma_3}, \qquad \text{where } \omega_v = \frac{\delta m^2}{2E} = \frac{m_2^2 - m_1^2}{2E}.$$
- The system can be solved given initial condition of the wave function $(\langle\nu_1 \mid \Psi(0)\rangle, \langle\nu_2 \mid \Psi(0)\rangle)^T$,

$$\begin{pmatrix} \langle\nu_1 \mid \Psi(t)\rangle \\ \langle\nu_2 \mid \Psi(t)\rangle \end{pmatrix} = \begin{pmatrix} \langle\nu_1 \mid \Psi(0)\rangle \exp(\mathrm{i}\omega_v x/2) \\ \langle\nu_2 \mid \Psi(0)\rangle \exp(-\mathrm{i}\omega_v x/2) \end{pmatrix}$$

# Why Do Neutrinos Oscillate?

## Flavor basis

Neutrino wave function in flavor basis $(\psi_e, \psi_x)^T$ is related to state in energy eigenbasis $(\psi_1, \psi_2)^T$ through

$$\begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

## Hamiltonian H

Energy eigenbasis

$$\frac{\omega_v}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -\frac{\omega_v}{2}\boldsymbol{\sigma}_3$$

Flavor eigenbasis

$$\frac{\omega_v}{2} \begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix} = \frac{\omega_v}{2} \left(-\cos 2\theta_v \boldsymbol{\sigma}_3 + \sin 2\theta_v \boldsymbol{\sigma}_1\right)$$

# Why Do Neutrinos Oscillate?

## Flavor basis

Neutrino wave function in flavor basis $(\psi_e, \psi_x)^T$ is related to state in energy eigenbasis $(\psi_1, \psi_2)^T$ through

$$\begin{pmatrix}\psi_e \\ \psi_x\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$$

## Hamiltonian $\mathbf{H}$

Energy eigenbasis

$$\frac{\omega_\nu}{2}\begin{pmatrix}-1 & 0 \\ 0 & 1\end{pmatrix} = -\frac{\omega_\nu}{2}\boldsymbol{\sigma_3}$$

Flavor eigenbasis

$$\frac{\omega_\nu}{2}\begin{pmatrix}-\cos 2\theta_\nu & \sin 2\theta_\nu \\ \sin 2\theta_\nu & \cos 2\theta_\nu\end{pmatrix} = \frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu\boldsymbol{\sigma_3} + \sin 2\theta_\nu\boldsymbol{\sigma_1}\right)$$

# Nature of Neutrino Oscillation

**Survival Probability**

$$P(|\nu_e\rangle \rightarrow |\nu_x\rangle) = \sin^2(2\theta_\nu)\frac{1 - \cos{(\omega_\nu x)}}{2}$$

Mixing angle $\rightarrow$ Flavor oscillation amplitude
Eigenenergies $\rightarrow$ Oscillation frequency

# Overview

# Matter Interaction

PLACEHOLDER

SHOULD ADD IN WHY MATTER INTERACTION IS LIKE THIS.

# Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis
($\omega_v = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_v}{2}\begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix}} + \boxed{\sqrt{2}G_F n_e(x)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2}G_F n_e(x)$

# Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis
($\omega_v = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_v}{2}\begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix}} + \boxed{\frac{\sqrt{2}G_F n_e(x)}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2}G_F n_e(x)$

# Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis
($\omega_\nu = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu \boldsymbol{\sigma_3} + \sin 2\theta_\nu \boldsymbol{\sigma_1}\right)} + \boxed{\frac{\sqrt{2}G_F n_e(x)}{2}\boldsymbol{\sigma_3}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2}G_F n_e(x)$

# Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis ($\omega_\nu = \delta m^2/2E$):

$$\mathbf{H} = \frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu \boldsymbol{\sigma_3} + \sin 2\theta_\nu \boldsymbol{\sigma_1}\right) + \frac{\lambda(x)}{2}\boldsymbol{\sigma_3}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2} G_F n_e(x)$

**Hamiltonian with Matter Potential**

$$\mathbf{H} = \frac{\lambda(x) - \omega_\nu \cos 2\theta_\nu}{2}\boldsymbol{\sigma_3} + \frac{\omega_\nu \sin 2\theta_\nu}{2}\boldsymbol{\sigma_1}$$

# MSW Effect

**Hamiltonian in Vacuum**

$$\mathbf{H} = \frac{\omega_v \cos 2\theta_v}{2}\boldsymbol{\sigma_3} + \frac{\omega_v \sin 2\theta_v}{2}\boldsymbol{\sigma_1}$$

**Hamiltonian with Matter Potential**

$$\mathbf{H} = \frac{\omega_m(x)\cos 2\theta_m(x)}{2}\boldsymbol{\sigma_3} + \frac{\omega_m(x)\sin 2\theta_m(x)}{2}\boldsymbol{\sigma_1},$$

where

$$\omega_m(x) = \omega_v\sqrt{\left(\frac{\lambda}{\omega_v} - \cos 2\theta_v\right)^2 + \sin^2 2\theta_v},$$

$$\tan 2\theta_m(x) = \sin 2\theta_v / \left(\cos 2\theta_v - \frac{\lambda(x)}{\omega_v}\right).$$

# Constant Matter Density

**Transition Probability for Constant Matter Density**

$$P(|\nu_e\rangle \to |\nu_x\rangle) = \frac{1}{2}\sin^2(2\theta_m)\left[1 - \cos\left(\omega_m x\right)\right]$$

where

$$\omega_m(x) = \omega_v \sqrt{\left(\frac{\lambda}{\omega_v} - \cos 2\theta_v\right)^2 + \sin^2 2\theta_v},$$

$$\tan 2\theta_m(x) = \sin 2\theta_v / \left(\cos 2\theta_v - \frac{\lambda(x)}{\omega_v}\right).$$

# Constant Matter Density

**Transition Probability for Constant Matter Density**

$$P(|\nu_e\rangle \to |\nu_x\rangle) = \frac{1}{2}\sin^2(2\theta_m)\left[1 - \cos\left(\omega_m x\right)\right]$$

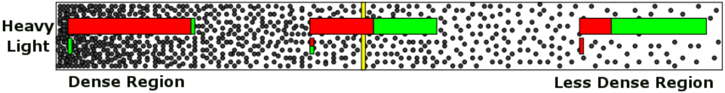
- Oscillation amplitude depends on
- Oscillation angular frequency is

$$\omega_m \geq \omega_v \sin 2\theta_v$$

# MSW Effect

$$\mathbf{H} = \frac{\lambda(x) - \omega_\nu \cos 2\theta_\nu}{2}\boldsymbol{\sigma_3} + \frac{\omega_\nu \sin 2\theta_\nu}{2}\boldsymbol{\sigma_1}$$

$$\begin{pmatrix}\nu_e \\ \nu_x\end{pmatrix} = \begin{pmatrix}\cos\theta_m(x) & \sin\theta_m(x) \\ -\sin\theta_m(x) & \cos\theta_m(x)\end{pmatrix}\begin{pmatrix}\nu_{1m} \\ \nu_{2m}\end{pmatrix}$$




Yellow bar is the resonance point. Smirnov, 2003.

# Parametric Effect

**Parametric Effect**

Parametric Effect, Parametric Resonance?

# Length Scales

## Characteristic Scales

- Vacuum problem: only one length scale

$$l_\nu \sim \frac{1}{\omega_\nu}$$

- Constant matter profile $\lambda_0$:

$$l_\nu, \quad l_m \sim \frac{1}{\omega_m}$$

- Varying matter profile $\lambda(x) = \lambda_0 \sin(kx)$,

$$l_\nu, \quad l_m, \quad l_k \sim \frac{1}{k}$$

## MSW Resonance

$$l_\nu \sim l_m \cos 2\theta_\nu$$

# Length Scales

$$\omega_m = \omega_\nu \sqrt{\left(\frac{\lambda}{\omega_\nu} - \cos 2\theta_\nu\right)^2 + \sin^2 2\theta_\nu}$$

## Characteristic Scales

- Vacuum problem: only one length scale

$$l_\nu \sim \frac{1}{\omega_\nu}$$

- Constant matter profile $\lambda_0$:

$$l_\nu, \quad l_m \sim \frac{1}{\omega_m}$$

- Varying matter profile $\lambda(x) = \lambda_0 \sin(kx)$,

$$l_\nu, \quad l_m, \quad l_k \sim \frac{1}{k}$$

Matching of $l_\nu$, and $l_k$?

# Stimulated Neutrino Oscillations

## Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

## Basis

Background matter basis: Hamiltonian is diagonalized with only background matter profile $\lambda_0$,

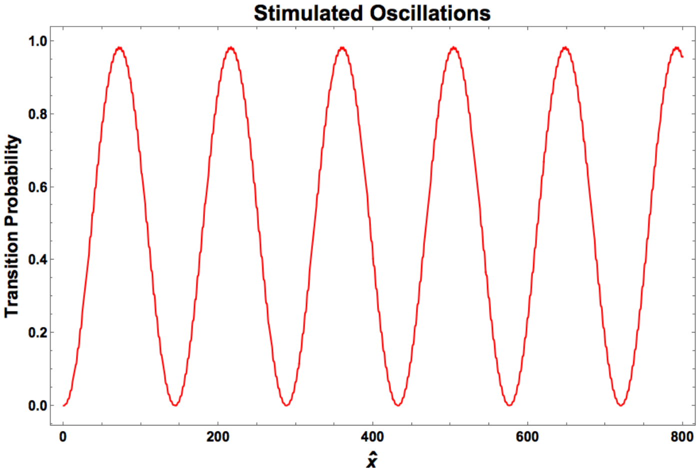
$$\mathbf{H} = -\frac{\omega_m}{2}\boldsymbol{\sigma_3}.$$

## Hamiltonian

$$\mathbf{H} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

# Stimulated Neutrino Oscillations

Kneller, J. P., McLaughlin, G. C., & Patton, K. M. (2013). J. Phys. G: Nucl. Part. Phys. **40** (2013) 055002.




Stimulated oscillation

# Stimulated Neutrino Oscillations

## Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

## Basis

Backgroun ... lized with
only backg ...

$\delta\lambda(x)$ is the problem here.

- Varying background eigenenergy
- $\cdots$

## Hamiltonian

$$\mathbf{H} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

# Overview

# Understanding Stimulated Oscillations

## A Better Basis

Remove the position dependence of the diagonalized part of the Hamiltonian BY CHOOSING A NEW BASIS!

New basis where the wave function $(\psi_{p1}, \psi_{p2})^T$ is related to the wave function in background matter basis through

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{-i\eta(x)} \end{pmatrix} \begin{pmatrix} \psi_{p1} \\ \psi_{p2} \end{pmatrix},$$

where

$$\eta(x) - \eta(0) = -\frac{\omega_m}{2} x + \frac{\cos 2\theta_m}{2} \int_0^x \delta\lambda(\tau) d\tau.$$

## Transition Probability in Background Basis

$$P_{1\to 2}(x) = \left| e^{i\eta} \psi_{p2}(x) \right|^2 = \left| \psi_{p2}(x) \right|^2.$$

# UNDERSTANDING STIMULATED OSCILLATIONS

- Hamiltonian in Background Matter Basis

$$\mathbf{H_0} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m \boldsymbol{\sigma_1}.$$

- Hamiltonian in New Basis

$$\mathbf{H_p} = -\frac{\delta\lambda(x)}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

# Understanding Stimulated Oscillations

- Hamiltonian in Background Matter Basis

$$\mathbf{H_0} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

- Hamiltonian in New Basis

$$\mathbf{H_p} = -\frac{\delta\lambda(x)}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

**Rabi Oscillation**

Rabi oscillation Hamiltonian

$$\begin{pmatrix} E_1 & W \\ W^* & E_2 \end{pmatrix},$$

where $W$ is periodic, e.g., $e^{ikx}$.

# Understanding Stimulated Oscillations

- Hamiltonian in Background Matter Basis

$$\mathbf{H_0} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

- Hamiltonian in New Basis

$$\mathbf{H_p} = -\frac{\delta\lambda(x)}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

**Off-diagonal Term in Our System**

$$e^{2i\eta(x)},$$

where

$$\eta(x) - \eta(0) = -\frac{\omega_m}{2}x + \frac{\cos 2\theta_m}{2}\int_0^x \delta\lambda(\tau)d\tau.$$

# Single Frequency Matter Profile

**Matter Profile**

$$\lambda(x) = \lambda_0 + A\sin(kx),$$

which leads to

$$\eta(x) = -\frac{\omega_m}{2}x - \frac{\cos 2\theta_m}{2}\frac{A}{k}\cos(kx).$$

**Hamiltonian in New Basis**

$$-\frac{\delta\lambda(x)}{2}\begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

The system can be (approximately) solved using Jacobi-Anger expansion

$$e^{iz\cos(\Phi)} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{in\Phi}.$$

# Single Frequency Matter Profile

## Scaled Quantities

Characteristic scale: $\omega_m$

- $\hat{A} = A/\omega_m$
- $\hat{k} = k/\omega_m$
- $\hat{x} = \omega x$

# Single Frequency Matter Profile

## Rotation Wave Approximation

The off-diagonal element of Hamiltonian

$$\hat{h} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\hat{B}_n e^{i(n\hat{k}-1)\hat{x}},$$

where $\hat{B}_n = -(-i)^n n\hat{k} \tan 2\theta_m J_n(\hat{A}\cos 2\theta_m/\hat{k})$.
$(n\hat{k}-1) \rightarrow$ perturbation frequency in matter profile for each component.

## Which $n$ to Choose

RWA: small $(n\hat{k}-1) \rightarrow$ important term
Find integer $n_0 = \text{Round}\left[1/\hat{k}\right]$ that minimizes $n\hat{k}-1$.

# Single Frequency Matter Profile

## Transition Probability

RWA $\rightarrow$ analytically solvable equation

$$P_{1\rightarrow 2} = \frac{|\Gamma/2|^2}{q^2}\sin^2\left(\frac{q}{2}x\right),$$

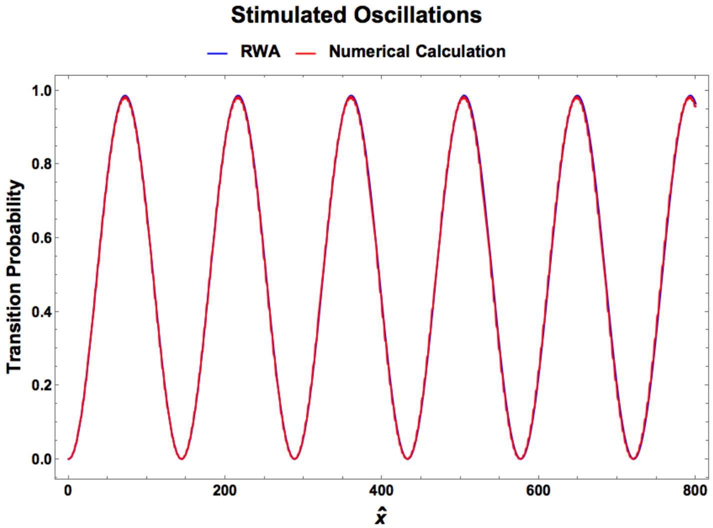
where

$$q^2 = |\Gamma/2|^2 + (n_0\hat{k}-1)^2, \qquad \text{frequency of oscillations}$$

$$\Gamma = \left|\hat{B}_{n_0}\right|, \qquad \text{width of resonance } (nk \text{ as parameter})$$

$$n_0 = \text{Round}\left[1/\hat{k}\right]$$

# Single Frequency Matter Profile




RWA works. $\hat{A} = 0.1$, $\hat{k} = 0.995$, $\theta_m = \pi/6$

# Single Frequency Matter Profile

## Why Does RWA Work?

Hamiltonian off-diagonal element amplitude

$$\hat{B}_n \propto nJ_n(n\,\mathrm{sech}\,\alpha) \sim \sqrt{n}\frac{e^{-n(\alpha-\tanh\alpha)}}{\sqrt{2\pi\tanh\alpha}}, \quad \text{for large } n$$

where $\mathrm{sech}\,\alpha = \hat{A}\cos 2\theta_m$.
$\Rightarrow$

$$\Gamma \propto \hat{B}_n \propto \frac{e^{-n(\alpha-\tanh\alpha)}}{\sqrt{2\pi n_0 \tanh\alpha}}$$

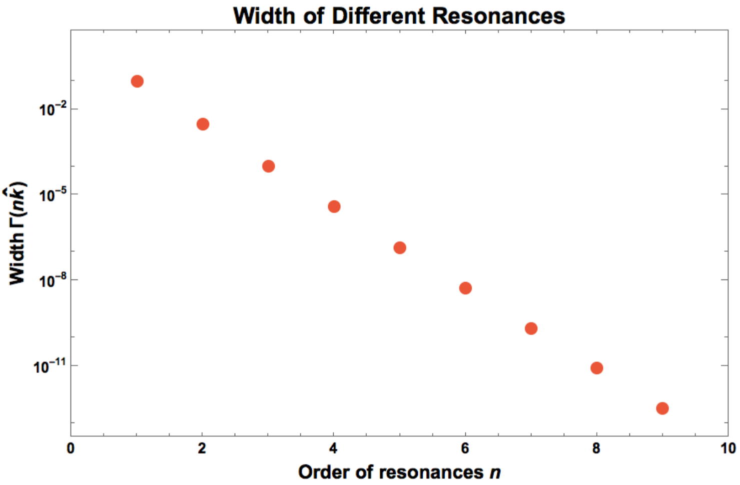
Small perturbation $\Rightarrow$ Small $\hat{A}$ $\Rightarrow$ Large $\alpha$ $\Rightarrow$ Drops fast at large $n$.

# Single Frequency Matter Profile

**PLACEHOLDER**

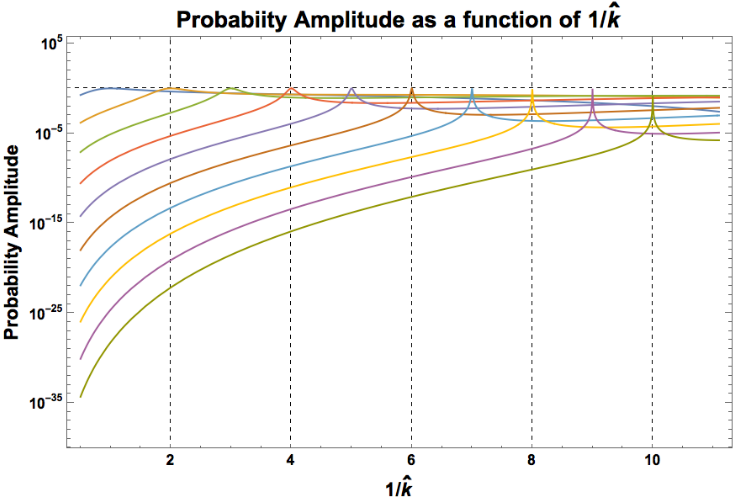
NEED A GRAPH TO EXPLAIN WHY FOR A SPECIFIC n WE ONLY TAKE THE ONE THAT IS MOST CLOSE TO RESONANCE!!!

# Single Frequency Matter Profile




Width of the resonances for $A = 0.1$, $\theta_m = \pi/5$. Each width is a parameter of $n\hat{k}$.

# SINGLE FREQUENCY MATTER PROFILE




Transition probability amplitude as a function of $1/\hat{k}$ (the phase is $(n - 1/\hat{k})$). The colors represent different orders of $n_0$.

# Two-frequency Matter Profile

**Matter Profile**

$$\lambda(x) = \lambda_0 + \delta\lambda(x), \quad \delta\lambda(x) = A_1 \sin(k_1 x) + A_2 \sin(k_2 x).$$

# Two-frequency Matter Profile

$$\hat{h} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\hat{B}_n e^{i(n\hat{k}-1)\hat{x}},$$

## Hamiltonian Off-diagonal Element

Apply Jacobi-Anger expansion,

$$\hat{h} = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \frac{1}{2}\hat{B}_{n_1,n_2}(\hat{k}_1,\hat{k}_2) e^{i(n_1\hat{k}_1+n_2\hat{k}_2-1)\hat{x}},$$

where

$$\begin{aligned} &\hat{B}_{n_1,n_2}(\hat{k}_1,\hat{k}_2) \\ =& -(-i)^{n_1+n_2}(n_1\hat{k}_1+n_2\hat{k}_2) J_{n_1}\left(\frac{\hat{A}_1\cos 2\theta_m}{\hat{k}_1}\right) J_{n_2}\left(\frac{\hat{A}_2\cos 2\theta_m}{\hat{k}_2}\right) \end{aligned}$$

# Two-frequency Matter Profile $\hat{h} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\hat{B}_n e^{i(n\hat{k}-1)\hat{x}}$,

## Hamiltonian Off-diagonal Element

Apply Jacobi-Anger expansion,

$$\hat{h} = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \frac{1}{2}\hat{B}_{n_1,n_2}(\hat{k}_1,\hat{k}_2) e^{i(n_1\hat{k}_1+n_2\hat{k}_2-1)\hat{x}},$$

where

$$\begin{aligned}&\hat{B}_{n_1,n_2}(\hat{k}_1,\hat{k}_2)\\ =& -(-i)^{n_1+n_2}(n_1\hat{k}_1+n_2\hat{k}_2) J_{n_1}\left(\frac{\hat{A}_1\cos 2\theta_m}{\hat{k}_1}\right) J_{n_2}\left(\frac{\hat{A}_2\cos 2\theta_m}{\hat{k}_2}\right)\end{aligned}$$

Which terms are important?

# Two-frequency Matter Profile

## Resonance Lines

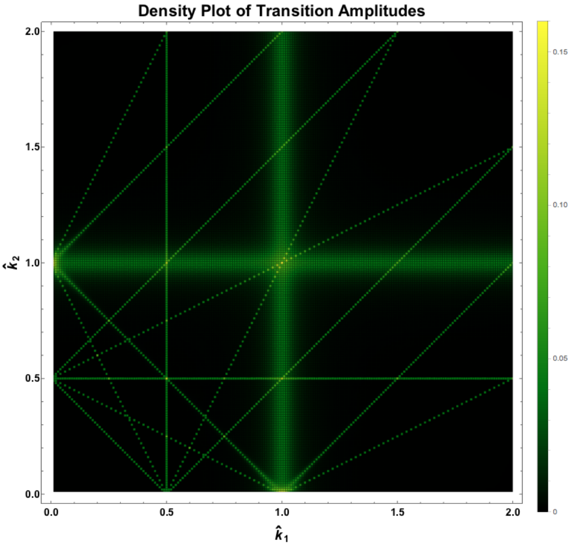
There are still resonances, i.e., (almost) zero phases, on lines

$$n_{1,0}\hat{k}_1 + n_{2,0}\hat{k}_2 - 1 = 0$$

in $\{\hat{k}_1, \hat{k}_2\}$ plane. $\Rightarrow$ Resonance width for each point on resonance lines.

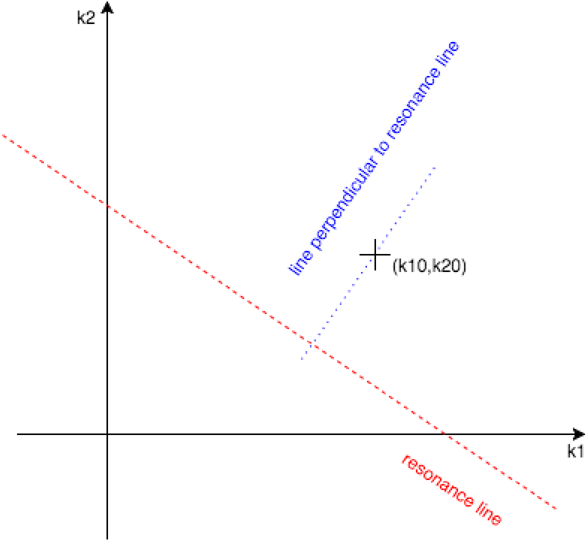
# Two-frequency Matter Profile

$$\hat{h} = \sum_{n_1}\sum_{n_2}\frac{1}{2}\hat{B}_{n_1,n_2}(\hat{k}_1,\hat{k}_2)e^{i(n_1\hat{k}_1+n_2\hat{k}_2-1)\hat{x}},$$




Density plot of transition amplitudes calculated using only one term out of the whole summation in Hamiltonian. $n_1, n_2 \in [-2, 2]$

# Two-frequency Matter Profile




Resonance line, distance to resonance, and width

# Two-frequency Matter Profile

## Width

$$\Gamma_2 = \frac{\hat{B}_{n_1,n_2}(\hat{k}_{1,\text{intercept}}, \hat{k}_{2,\text{intercept}})}{\sqrt{n_1^2 + n_2^2}}.$$

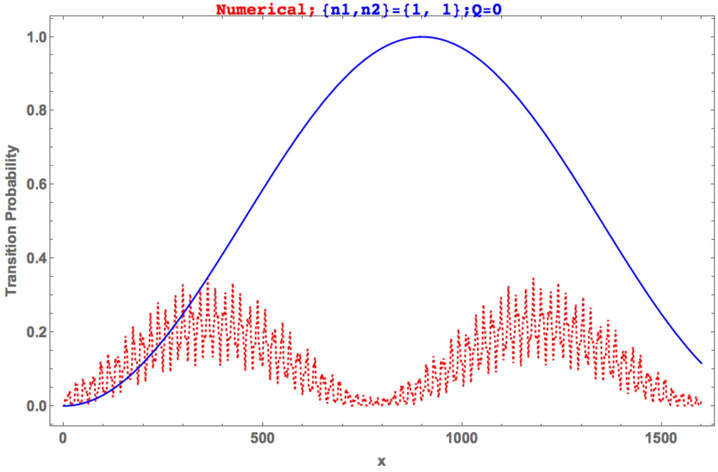
## Distance to Resonance Line

$$d = \frac{|n_1\hat{k}_{10} + n_2\hat{k}_{20} - 1|}{\sqrt{n_1^2 + n_2^2}}.$$

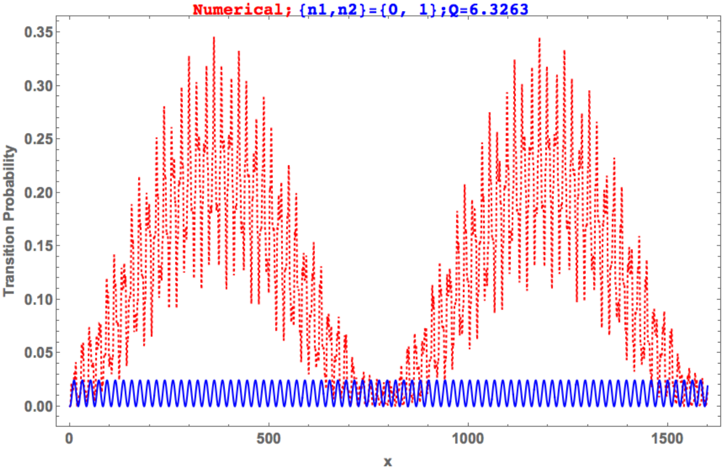
## Distance to Resonance Width Ratio

$$Q_2 = \frac{d}{\Gamma_2}.$$

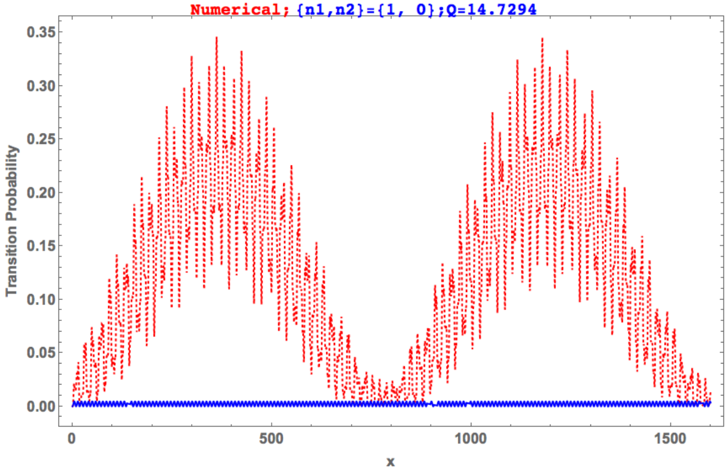
# Two-frequency Matter Profile



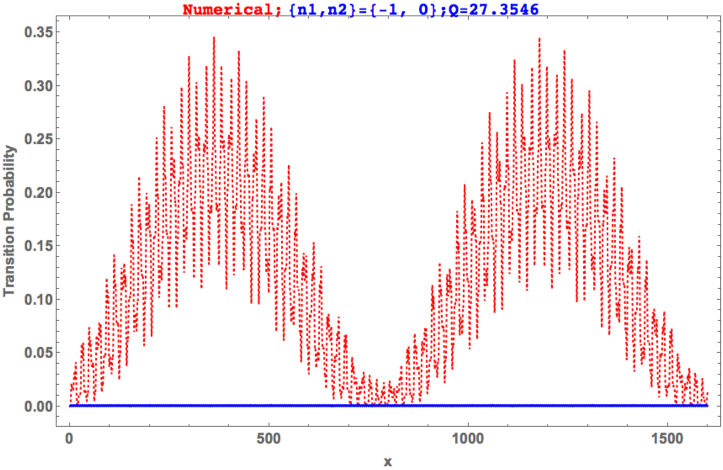
# Two-frequency Matter Profile



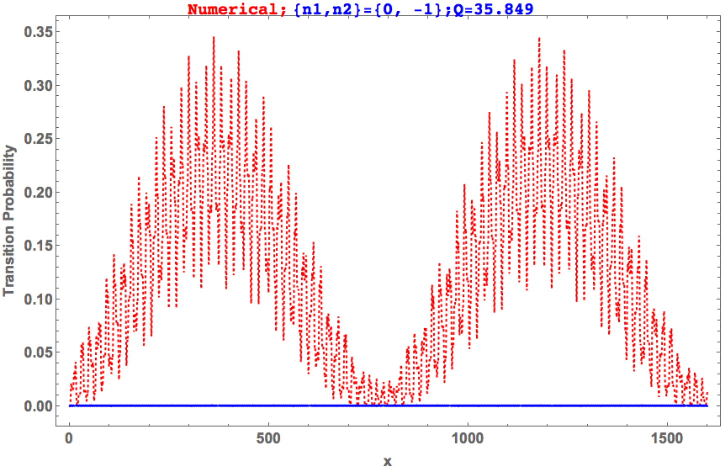
# Two-frequency Matter Profile



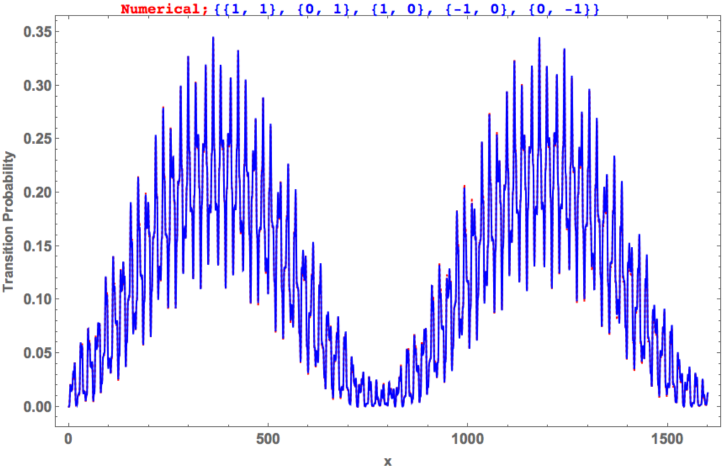
# Two-frequency Matter Profile



# Two-frequency Matter Profile



# Two-frequency Matter Profile



# Overview

# Summary & Future Work

- Matter interaction depends on matter profile
- Single frequency periodic matter profile can be approximated using lowest order of RWA.
- Two frequency periodic matter profile can be calculated using some lowest orders of RWA.

- Multi-frequency matter profile

# Acknowledgement

# Why Do Neutrinos Oscillate?

- this term blablabla

$$X = A + B + \boxed{C} + D + \boxed{E}$$

- Transversal acceleration
- Centripetal acceleration

# Why Do Neutrinos Oscillate?

- this term blablabla

$$X = A + B + \boxed{C} + D + \boxed{E}$$

- Transversal acceleration
- Centripetal acceleration

# Why Do Neutrinos Oscillate?

- this term blablabla

$$X = A + B + \boxed{C} + D + \boxed{E}$$

- Transversal acceleration
- Centripetal acceleration

# Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.
The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.
**metropolis** will automatically turn off slide numbering and progress bars for slides in the appendix.

# References I