Neutrino Oscillations in Matter

PhD Candidacy Exam

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Outline

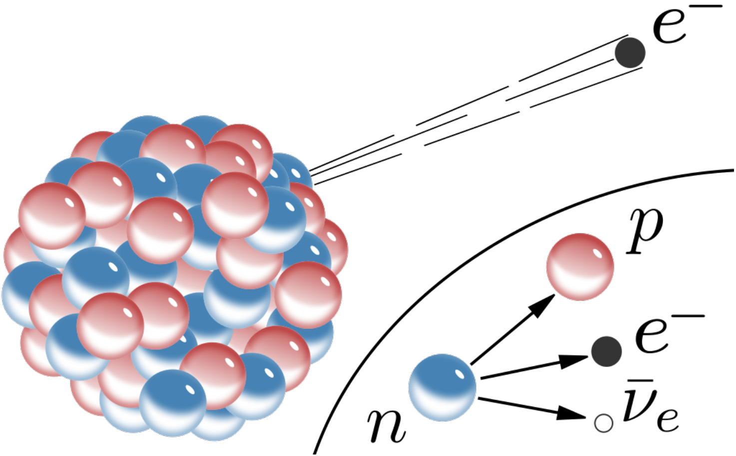
Overview

Neutrino Timeline

History of Neutrino (Partial)

- 1930 – Pauli, letter to ”Radioactive Ladies and Gentlemen”
- 1933 – Fermi, the name ”neutrino”
- 1956 – Reines & Cowan, first neutrino evidence
- 1957 – **Pontecorvo, theory of neutrino oscillations**
- 1968 – Homestake, first solar neutrino detection
- 1978 & 1985 – **Wolfenstein & Mikheyev & Smirnov, MSW effect**
- 1987 – Kamioka mine & Morton salt mine, SN1987A neutrino
- 1998 & 2001 – Super-Kamiokande & SNO, solar neutrino oscillations

What are Neutrinos?



Beta decay and antineutrino production. Source: Beta_Decay@Wikipedia

What are Neutrinos?

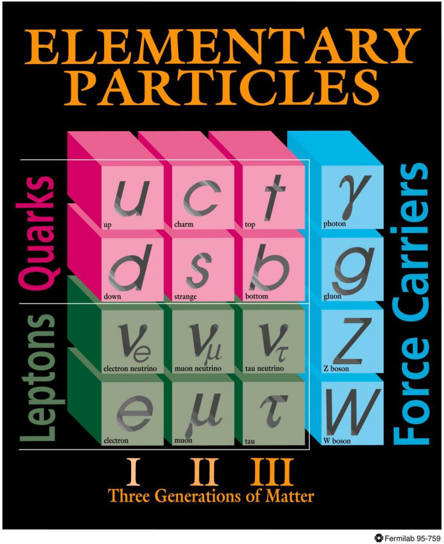
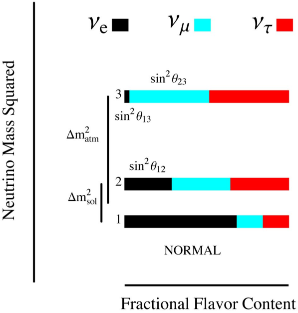


Table of elementary particles. Source: Fermilab

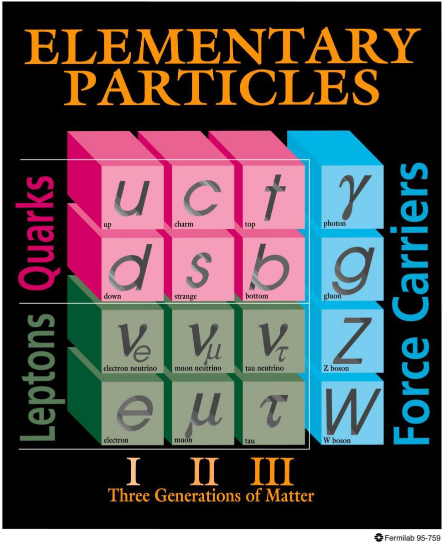
Neutrinos are

- Fermions,
- electrically neutral,
- **light**.



Adapted from Olga Mena & Stephen Parke, 2004

What are Neutrinos?



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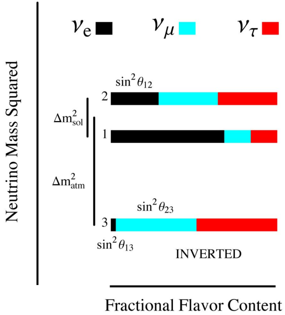


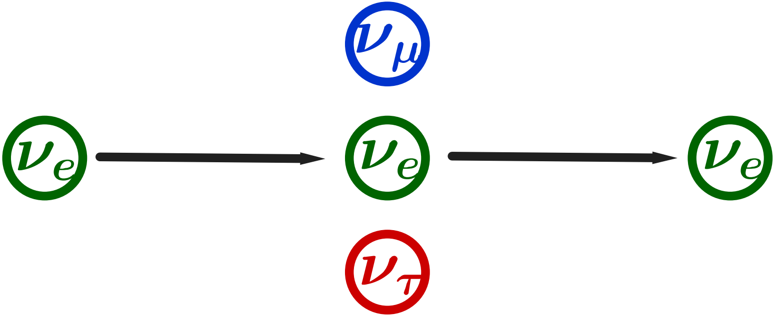
Adapted from Olga Mena & Stephen Parke, 2004

What is Neutrino Oscillation?

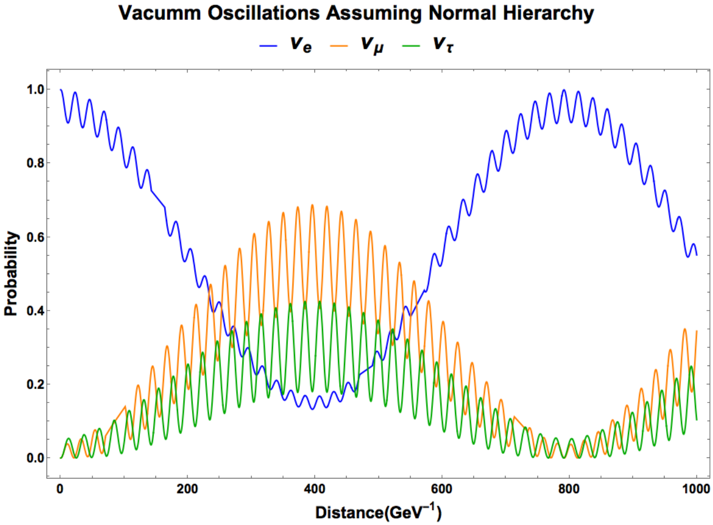
Neutrino Oscillation

||

Neutrino Flavor Conversion



What is Neutrino Oscillation?

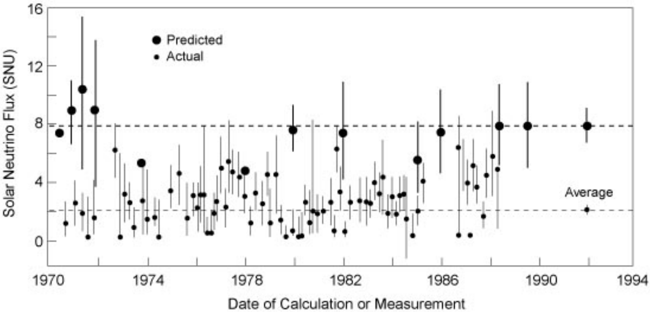


Survival probability

Solar Neutrino Problem

Solar Neutrinos

pp chain, ^{7}Be, ^{8}B, pep, CNO



Chlorine detector (Homestake experiment) results and theory predictions.
SNU: 1 event for 10^{36} target atoms per second.

Why Do Neutrinos Oscillate?

Equation of Motion

$$i\partial_x \Psi = \mathbf{H}\Psi$$

- Basis: Hamiltonian diagonalized basis/mass eigenbasis/propagation eigenbasis, $|\nu_1\rangle$ and $|\nu_2\rangle$.
- $$\mathbf{H} = -\frac{\omega_\nu}{2}\boldsymbol{\sigma_3}, \qquad \text{where } \omega_\nu = \frac{\delta m^2}{2E} = \frac{m_2^2 - m_1^2}{2E}.$$
- The system can be solved given initial condition of the wave function $(\langle \nu_1 \mid \Psi(0)\rangle, \langle \nu_2 \mid \Psi(0)\rangle)^T$,

$$\begin{pmatrix} \langle \nu_1 \mid \Psi(t)\rangle \\ \langle \nu_2 \mid \Psi(t)\rangle \end{pmatrix} = \begin{pmatrix} \langle \nu_1 \mid \Psi(0)\rangle \exp\left(\mathrm{i}\omega_\nu x/2\right) \\ \langle \nu_2 \mid \Psi(0)\rangle \exp\left(-\mathrm{i}\omega_\nu x/2\right) \end{pmatrix}$$

Why Do Neutrinos Oscillate?

Flavor basis

Neutrino wave function in flavor basis $(\psi_e, \psi_x)^T$ is related to state in energy eigenbasis $(\psi_1, \psi_2)^T$ through

$$\begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Hamiltonian H

Energy eigenbasis	Flavor eigenbasis
$\frac{\omega_\nu}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -\frac{\omega_\nu}{2}\boldsymbol{\sigma}_3$	$\frac{\omega_\nu}{2}\begin{pmatrix} -\cos 2\theta_\nu & \sin 2\theta_\nu \\ \sin 2\theta_\nu & \cos 2\theta_\nu \end{pmatrix} = \frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu \boldsymbol{\sigma}_3 + \sin 2\theta_\nu \boldsymbol{\sigma}_1\right)$

Why Do Neutrinos Oscillate?

Flavor basis

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$$\begin{pmatrix}\psi_e \\ \psi_x\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$$

Hamiltonian $\mathbf{H}$

Energy eigenbasis

$$\frac{\omega_\nu}{2}\begin{pmatrix}-1 & 0 \\ 0 & 1\end{pmatrix} = -\frac{\omega_\nu}{2}\boldsymbol{\sigma_3}$$

Flavor eigenbasis

$$\frac{\omega_\nu}{2}\begin{pmatrix}-\cos 2\theta_\nu & \sin 2\theta_\nu \\ \sin 2\theta_\nu & \cos 2\theta_\nu\end{pmatrix} = \frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu\boldsymbol{\sigma_3} + \sin 2\theta_\nu\boldsymbol{\sigma_1}\right)$$

Nature of Neutrino Oscillation

Transition Probability

$$P(|\nu_e\rangle \rightarrow |\nu_x\rangle) = \sin^2(2\theta_\nu)\frac{1-\cos(\omega_\nu x)}{2}$$

Mixing angle $\rightarrow$ Flavor oscillation amplitude
$(m_2^2 - m_1^2)/2E \rightarrow$ Oscillation frequency

Overview

Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis
$(\omega_\nu = \delta m^2/2E)$:

$$\mathbf{H} = \boxed{\frac{\omega_\nu}{2}\begin{pmatrix} -\cos 2\theta_\nu & \sin 2\theta_\nu \\ \sin 2\theta_\nu & \cos 2\theta_\nu \end{pmatrix}} + \boxed{\sqrt{2} G_F n_e(x) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2} G_F n_e(x)$

Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis
($\omega_v = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_v}{2}\begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix}} + \boxed{\frac{\sqrt{2}G_F n_e(x)}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2}G_F n_e(x)$

Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis ($\omega_\nu = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu \boldsymbol{\sigma_3} + \sin 2\theta_\nu \boldsymbol{\sigma_1}\right)} + \boxed{\frac{\sqrt{2} G_F n_e(x)}{2}\boldsymbol{\sigma_3}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2} G_F n_e(x)$

Matter Interaction

Hamiltonian with Matter Interaction in Flavor Basis
($\omega_\nu = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_\nu}{2}\left(-\cos 2\theta_\nu \boldsymbol{\sigma_3} + \sin 2\theta_\nu \boldsymbol{\sigma_1}\right)} + \boxed{\frac{\lambda(x)}{2}\boldsymbol{\sigma_3}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2} G_F n_e(x)$

Hamiltonian with Matter Potential

$$\mathbf{H} = \frac{\lambda(x) - \omega_\nu \cos 2\theta_\nu}{2}\boldsymbol{\sigma_3} + \frac{\omega_\nu \sin 2\theta_\nu}{2}\boldsymbol{\sigma_1}$$

MSW Effect

Hamiltonian in Vacuum

$$\mathbf{H} = \frac{\omega_\nu \cos 2\theta_\nu}{2}\boldsymbol{\sigma_3} + \frac{\omega_\nu \sin 2\theta_\nu}{2}\boldsymbol{\sigma_1}$$

Hamiltonian with Matter Potential

$$\mathbf{H} = \frac{\omega_m(x)\cos 2\theta_m(x)}{2}\boldsymbol{\sigma_3} + \frac{\omega_m(x)\sin 2\theta_m(x)}{2}\boldsymbol{\sigma_1},$$

where

$$\begin{aligned} \omega_m(x) &= \omega_\nu\sqrt{\left(\frac{\lambda}{\omega_\nu} - \cos 2\theta_\nu\right)^2 + \sin^2 2\theta_\nu}, \\ \tan 2\theta_m(x) &= \sin 2\theta_\nu / \left(\cos 2\theta_\nu - \frac{\lambda(x)}{\omega_\nu}\right). \end{aligned}$$

Constant Matter Density

Transition Probability for Constant Matter Density

$$P(|\nu_e\rangle \to |\nu_x\rangle) = \frac{1}{2}\sin^2(2\theta_m)\left[1 - \cos\left(\omega_m x\right)\right]$$

where

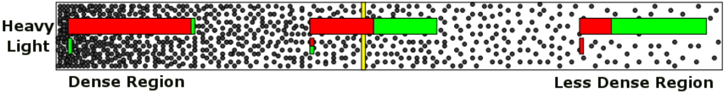
$$\omega_m(x) = \omega_v\sqrt{\left(\frac{\lambda}{\omega_v} - \cos 2\theta_v\right)^2 + \sin^2 2\theta_v},$$

$$\tan 2\theta_m(x) = \sin 2\theta_v / \left(\cos 2\theta_v - \frac{\lambda(x)}{\omega_v}\right).$$

MSW Effect

$$\mathbf{H} = \frac{\lambda(x) - \omega_\nu \cos 2\theta_\nu}{2}\boldsymbol{\sigma_3} + \frac{\omega_\nu \sin 2\theta_\nu}{2}\boldsymbol{\sigma_1}$$

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta_m(x) & \sin\theta_m(x) \\ -\sin\theta_m(x) & \cos\theta_m(x) \end{pmatrix} \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$



Yellow bar is the resonance point. Smirnov, 2003.

Parametric Effect

Parametric Effect

Parametric Effect, Parametric Resonance?

Length Scales

Characteristic Scales

- Vacuum problem: only one length scale

$$l_\nu \sim \frac{1}{\omega_\nu}$$

- Constant matter profile λ_0:

$$l_\nu, \quad l_m \sim \frac{1}{\omega_m}$$

- Varying matter profile $\lambda(x) = \lambda_0 \sin(kx)$,

$$l_\nu, \quad l_m, \quad l_k \sim \frac{1}{k}$$

MSW Resonance

$$l_\nu \sim l_m \cos 2\theta_\nu$$

Length Scales

$$\omega_m = \omega_\nu \sqrt{\left(\frac{\lambda}{\omega_\nu} - \cos 2\theta_\nu\right)^2 + \sin^2 2\theta_\nu}$$

Characteristic Scales

- Vacuum problem: only one length scale

$$l_\nu \sim \frac{1}{\omega_\nu}$$

- Constant matter profile λ_0:

$$l_\nu, \quad l_m \sim \frac{1}{\omega_m}$$

- Varying matter profile $\lambda(x) = \lambda_0 \sin(kx)$,

$$l_\nu, \quad l_m, \quad l_k \sim \frac{1}{k}$$

Matching of l_m, and l_k?

Stimulated Neutrino Oscillations

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

Basis

Background matter basis: Hamiltonian is diagonalized with only background matter profile λ_0,

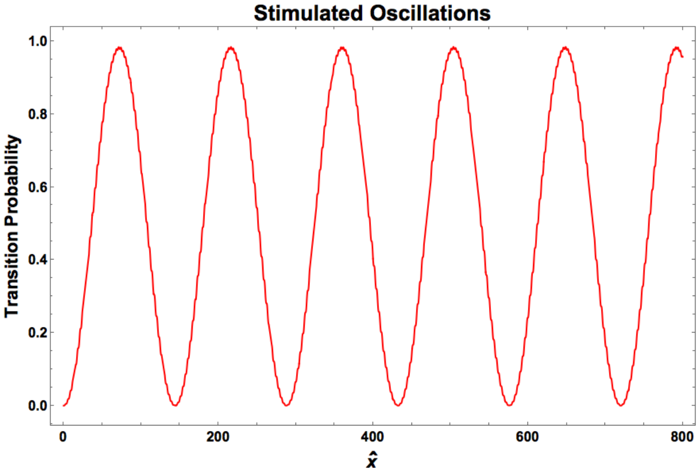
$$\mathbf{H} = -\frac{\omega_m}{2}\boldsymbol{\sigma_3}.$$

Hamiltonian

$$\mathbf{H} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

Stimulated Neutrino Oscillations

Kneller, J. P., McLaughlin, G. C., & Patton, K. M. (2013). J. Phys. G: Nucl. Part. Phys. **40** (2013) 055002.



Stimulated oscillation. $\delta\lambda(x) = A\sin(kx)$

Stimulated Neutrino Oscillations

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

Basis

Backgroun … lized with
only backg…

$\delta\lambda(x)$ is the problem here.

- Varying background eigenenergy
- $\cdots$

Hamiltonian

$$\mathbf{H} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

Overview

Understanding Stimulated Oscillations

A Better Basis

Remove the position dependence of the diagonalized part of the Hamiltonian BY CHOOSING A NEW BASIS!

New basis where the wave function $(\psi_{p1}, \psi_{p2})^T$ is related to the wave function in background matter basis through

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{i\eta(x)} \end{pmatrix} \begin{pmatrix} \psi_{p1} \\ \psi_{p2} \end{pmatrix},$$

where

$$\eta(x) - \eta(0) = -\frac{\omega_m}{2}x + \frac{\cos 2\theta_m}{2} \int_0^x \delta\lambda(\tau) d\tau.$$

Transition Probability in Background Basis

$$P_{1\to 2}(x) = \left| e^{i\eta} \psi_{p2}(x) \right|^2 = \left| \psi_{p2}(x) \right|^2.$$

Understanding Stimulated Oscillations

- Hamiltonian in Background Matter Basis

$$\mathbf{H_0} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

- Hamiltonian in New Basis

$$\mathbf{H_p} = -\frac{\delta\lambda(x)}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

Rabi Oscillation

Rabi oscillation Hamiltonian

$$\begin{pmatrix} E_1 & W \\ W^* & E_2 \end{pmatrix},$$

where W is periodic, e.g., e^{ikx}.

UNDERSTANDING STIMULATED OSCILLATIONS

- Hamiltonian in Background Matter Basis

$$\mathbf{H_0} = \frac{1}{2}\left(-\omega_m + \delta\lambda(x)\cos 2\theta_m\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_m\boldsymbol{\sigma_1}.$$

- Hamiltonian in New Basis

$$\mathbf{H_p} = -\frac{\delta\lambda(x)}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

Off-diagonal Term in Our System

$$e^{2i\eta(x)},$$

where

$$\eta(x) - \eta(0) = -\frac{\omega_m}{2}x + \frac{\cos 2\theta_m}{2}\int_0^x \delta\lambda(\tau)d\tau.$$

Single Frequency Matter Profile

Matter Profile

$$\lambda(x) = \lambda_0 + A\sin(kx),$$

which leads to

$$\eta(x) = -\frac{\omega_m}{2}x - \frac{\cos 2\theta_m}{2}\frac{A}{k}\cos(kx).$$

Hamiltonian in New Basis

$$-\frac{\delta\lambda(x)}{2}\begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix}$$

The system can be (approximately) solved using Jacobi-Anger expansion

$$e^{iz\cos(kx)} = \sum_{n=-\infty}^{\infty} i^n J_n(z) e^{inkx}.$$

Single Frequency Matter Profile

Scaled Quantities

Characteristic scale: ω_m

- $\hat{A} = A/\omega_m$
- $\hat{k} = k/\omega_m$
- $\hat{x} = \omega x$

Single Frequency Matter Profile

Rotation Wave Approximation

The off-diagonal element of Hamiltonian

$$\hat{h} = \sum_{n=-\infty}^{\infty} \frac{1}{2} \hat{B}_n e^{i(n\hat{k}-1)\hat{x}},$$

where $\hat{B}_n = -(-i)^n n\hat{k} \tan 2\theta_m J_n(\hat{A} \cos 2\theta_m / \hat{k})$.
$(n\hat{k} - 1) \rightarrow$ perturbation frequency in matter profile for each component.

Which n to Choose

RWA: small $(n\hat{k} - 1) \rightarrow$ important term

Find integer $n_0 = \text{Round}\left[1/\hat{k}\right]$ that minimizes $n\hat{k} - 1$.

Single Frequency Matter Profile

Transition Probability

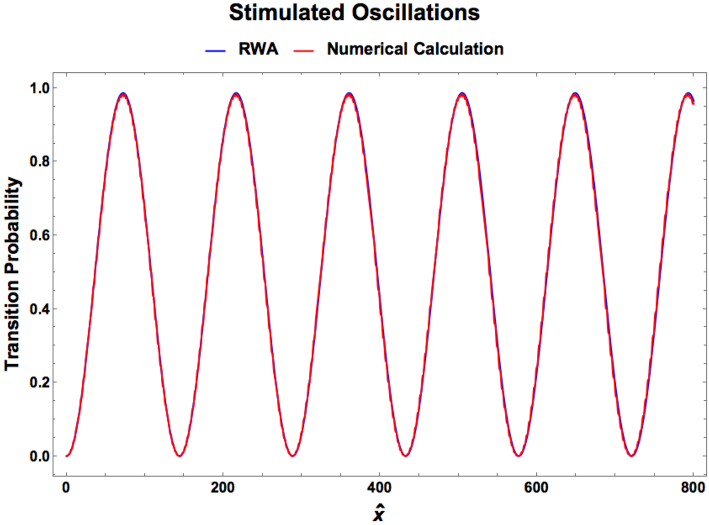
RWA $\rightarrow$ analytically solvable equation

$$P_{1\rightarrow 2} = \frac{|\Gamma/2|^2}{q^2} \sin^2\left(\frac{q}{2}x\right),$$

where

$$q^2 = \left|\Gamma/2\right|^2 + (n_0\hat{k} - 1)^2, \qquad \text{frequency of oscillations}$$
$$\Gamma = \left|\hat{B}_{n_0}\right|, \qquad \text{width of resonance (} nk \text{ as parameter)}$$
$$n_0 = \text{Round}\left[1/\hat{k}\right]$$

SINGLE FREQUENCY MATTER PROFILE



RWA works. $\hat{A} = 0.1$, $\hat{k} = 0.995$, $\theta_m = \pi/6$

Single Frequency Matter Profile

Why Does RWA Work?

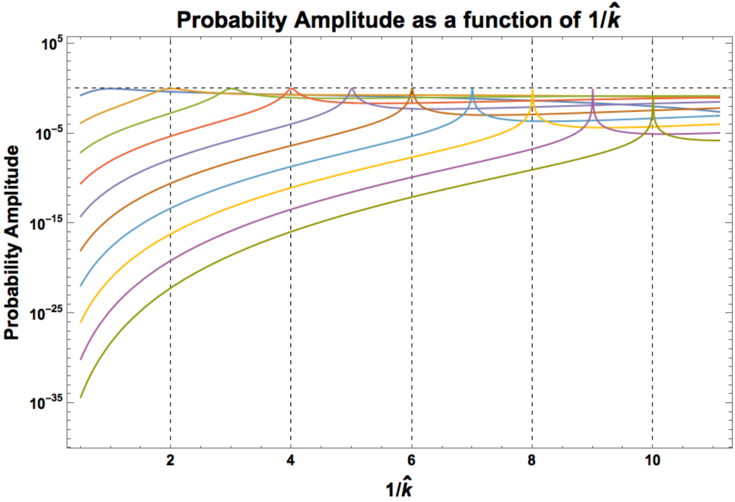
$$J_n(n\,\mathrm{sech}\,\alpha) \sim \frac{e^{-n(\alpha - \tanh\alpha)}}{\sqrt{2\pi n \tanh\alpha}}, \quad \text{for large } n$$

$\Rightarrow$

$$\Gamma \propto \hat{B}_n \propto \frac{e^{-n(\alpha - \tanh\alpha)}}{\sqrt{2\pi n \tanh\alpha}}$$

Small perturbation $\Rightarrow$ Small $\hat{A}$ $\Rightarrow$ Large α $\Rightarrow$ Drops fast at large n.

Single Frequency Matter Profile



Transition probability amplitude as a function of $1/\hat{k}$ (the phase is $(n - 1/\hat{k})$). The colors represent different orders of n_0.

Two-frequency Matter Profile

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x), \quad \delta\lambda(x) = A_1 \sin(k_1 x) + A_2 \sin(k_2 x).$$

Two-frequency Matter Profile $\hat{h} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\hat{B}_n e^{i(n\hat{k}-1)\hat{x}}$,

Hamiltonian Off-diagonal Element

Apply Jacobi-Anger expansion,

$$\hat{h} = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \frac{1}{2}\hat{B}_{n_1,n_2}(\hat{k}_1, \hat{k}_2) e^{i(n_1\hat{k}_1 + n_2\hat{k}_2 - 1)\hat{x}},$$

where

$$\begin{aligned} &\hat{B}_{n_1,n_2}(\hat{k}_1, \hat{k}_2) \\ =& -(-i)^{n_1+n_2}(n_1\hat{k}_1 + n_2\hat{k}_2) J_{n_1}\left(\frac{\hat{A}_1 \cos 2\theta_m}{\hat{k}_1}\right) J_{n_2}\left(\frac{\hat{A}_2 \cos 2\theta_m}{\hat{k}_2}\right) \end{aligned}$$

Which terms are important?

Two-frequency Matter Profile

Resonance Lines

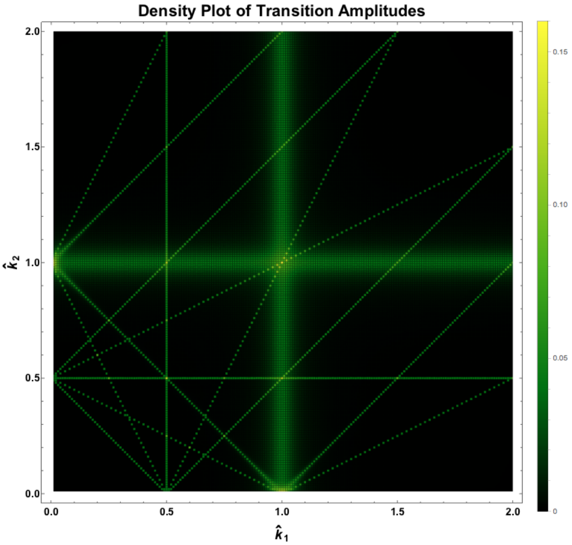
There are still resonances, i.e., (almost) zero phases, on lines

$$n_{1,0}\hat{k}_1 + n_{2,0}\hat{k}_2 - 1 = 0$$

in $\{\hat{k}_1, \hat{k}_2\}$ plane. $\Rightarrow$ Resonance width for each point on resonance lines.

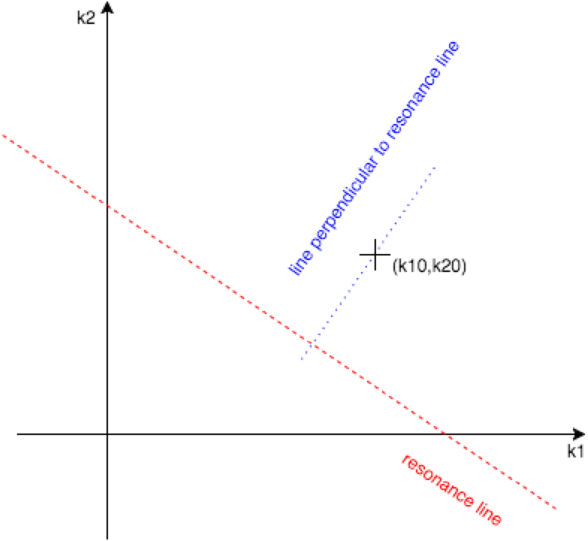
Two-frequency Matter Profile

$$\hat{h} = \sum_{n_1}\sum_{n_2} \frac{1}{2}\hat{B}_{n_1,n_2}(\hat{k}_1,\hat{k}_2)e^{i(n_1\hat{k}_1+n_2\hat{k}_2-1)\hat{x}},$$



Density plot of transition amplitudes calculated using only one term out of the whole summation in Hamiltonian. $n_1, n_2 \in [-2, 2]$

Two-frequency Matter Profile



Resonance line, distance to resonance, and width

Two-frequency Matter Profile

Width

$$\Gamma_2 = \frac{\hat{B}_{n_1,n_2}(\hat{k}_{1,\mathrm{intercept}}, \hat{k}_{2,\mathrm{intercept}})}{\sqrt{n_1^2 + n_2^2}}.$$

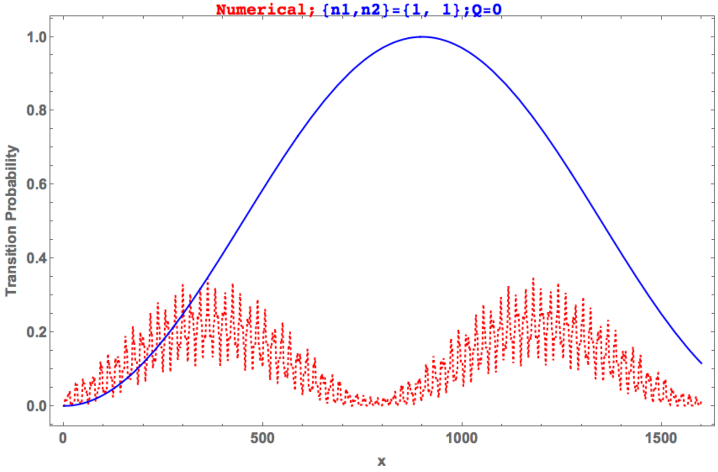
Distance to Resonance Line

$$d = \frac{|n_1\hat{k}_{10} + n_2\hat{k}_{20} - 1|}{\sqrt{n_1^2 + n_2^2}}.$$

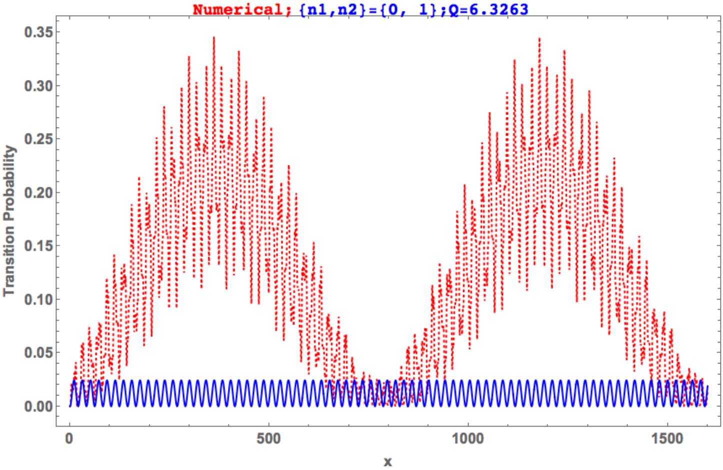
Distance to Resonance Width Ratio

$$Q_2 = \frac{d}{\Gamma_2}.$$

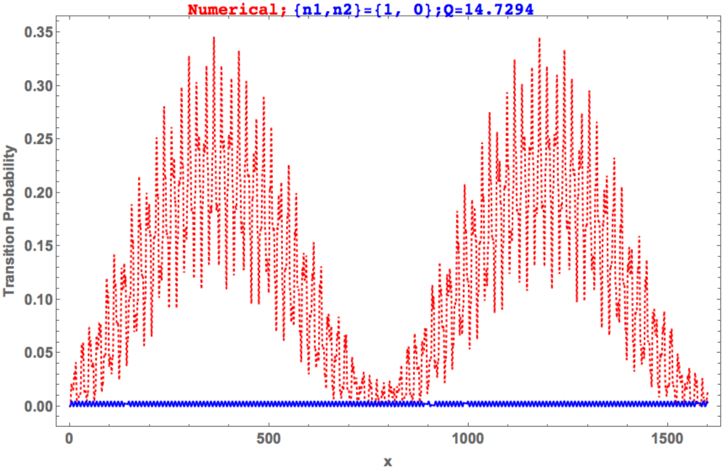
Two-frequency Matter Profile



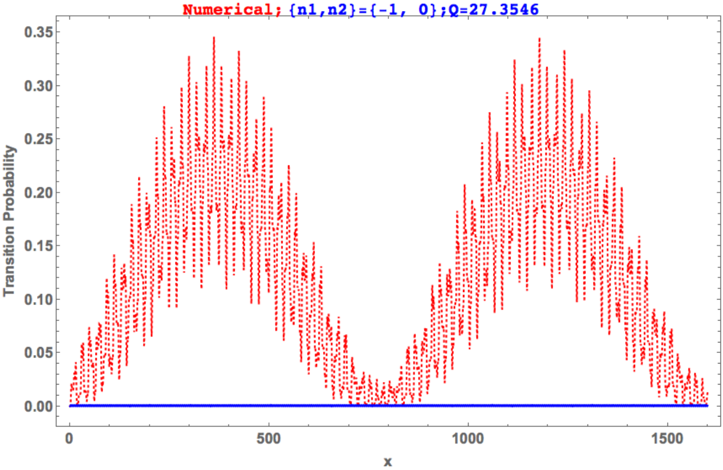
Two-frequency Matter Profile



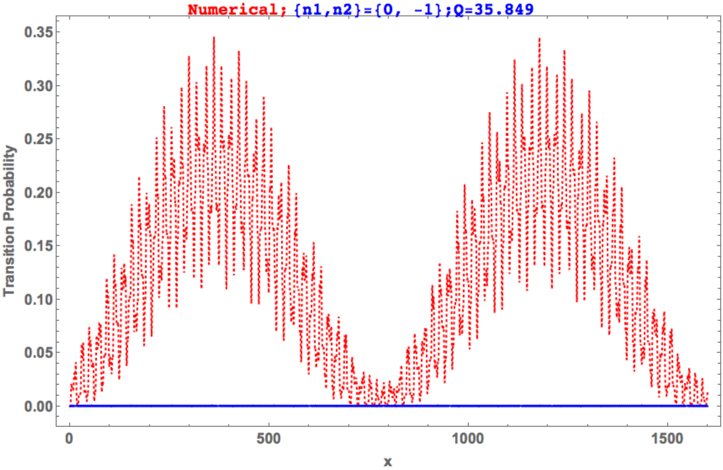
Two-frequency Matter Profile



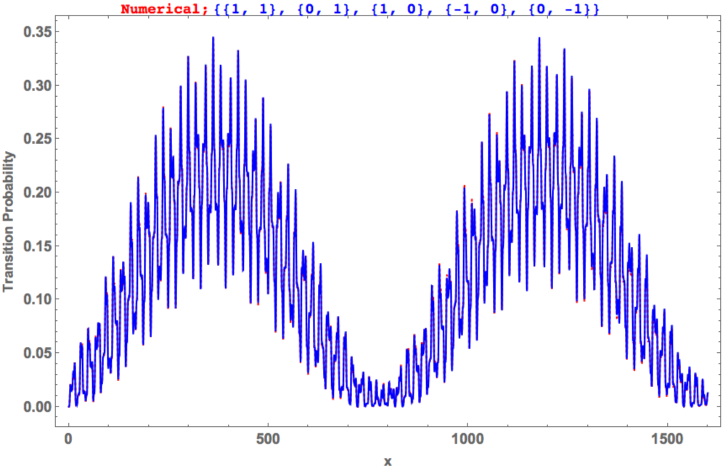
Two-frequency Matter Profile



Two-frequency Matter Profile



Two-frequency Matter Profile



Overview

Summary & Future Work

- Matter interaction depends on matter profile
- Single frequency periodic matter profile can be approximated using lowest order of RWA.
- Two frequency periodic matter profile can be calculated using some lowest orders of RWA.

- Multi-frequency matter profile

ACKNOWLEDGEMENT

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.
The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.
metropolis will automatically turn off slide numbering and progress bars for slides in the appendix.

References I