Neutrino Flavor Conversions in Dense Medium: Matter Stimulation, Dispersion Relation, and Neutrino Halo

PhD Defense

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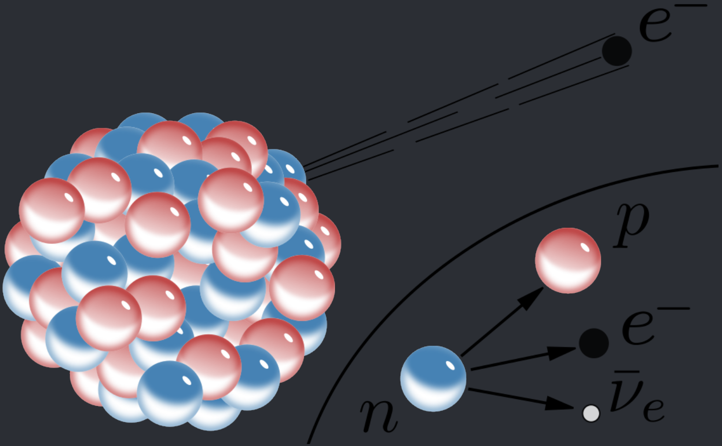
Outline for Section 1

What are Neutrinos?



Beta decay and antineutrino production. Source: Beta_Decay@Wikipedia

What are Neutrinos?

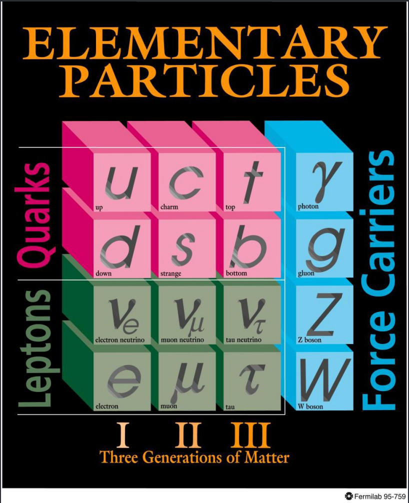
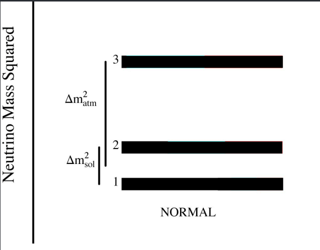


Table of elementary particles. Source: Fermilab

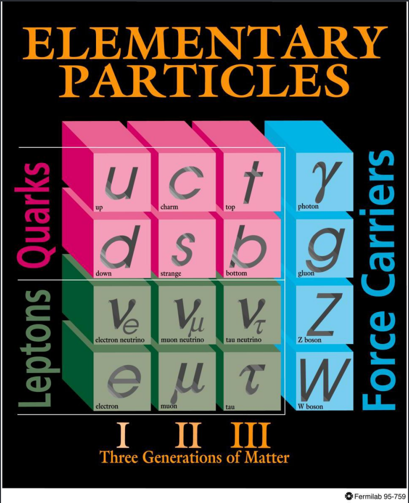
Neutrinos are

- fermions,
- electrically neutral,
- light.



Adapted from Olga Mena & Stephen Parke (2004)

What are Neutrinos?



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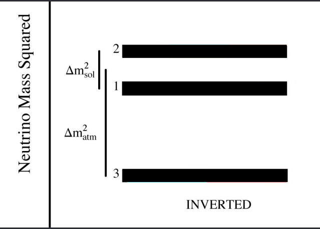

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What is Neutrino Oscillation?

Neutrino Oscillation

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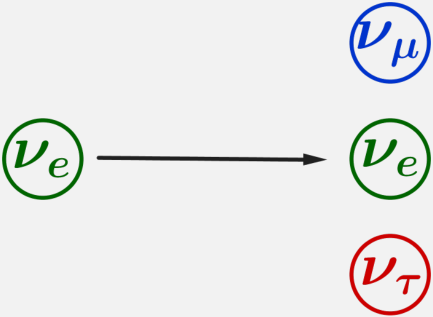
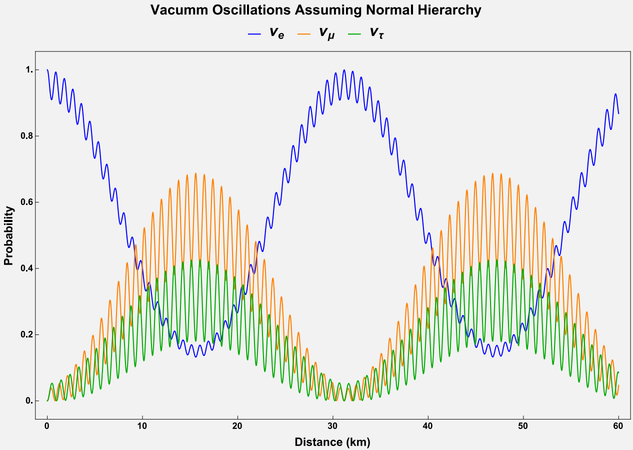
Neutrino Flavor Conversion



Neutrino Oscillations

What is Neutrino Oscillation?



Probabilities of finding neutrinos to be in each flavor.

Why Do Neutrinos Oscillate?

Equation of Motion

$$i\partial_x \left|\Psi\right\rangle = \hat{\mathbf{H}} \left|\Psi\right\rangle$$

- Basis: Hamiltonian diagonalized basis/mass basis/propagation basis, $\{\left|\nu_1\right\rangle, \left|\nu_2\right\rangle\}$.
- $$\mathbf{H} = -\frac{\omega_v}{2}\sigma_3, \qquad \text{where } \omega_v = \frac{\delta m^2}{2E} = \frac{m_2^2 - m_1^2}{2E}.$$
- The system can be solved given initial condition of the amplitudes of the two eigenstates $(\langle\nu_1|\Psi(0)\rangle, \langle\nu_2|\Psi(0)\rangle)^T$,

$$\begin{pmatrix} \langle\nu_1|\Psi(x)\rangle \\ \langle\nu_2|\Psi(x)\rangle \end{pmatrix} = \begin{pmatrix} \langle\nu_1|\Psi(0)\rangle \exp(i\omega_v x/2) \\ \langle\nu_2|\Psi(0)\rangle \exp(-i\omega_v x/2) \end{pmatrix}$$

Why Do Neutrinos Oscillate?

Flavor basis

Neutrino wave function in flavor basis $\{|\nu_{\mathrm{e}}\rangle, |\nu_{\mu}\rangle\}$ is related to state in energy basis $\{|\nu_1\rangle, |\nu_2\rangle\}$ through

$$\begin{pmatrix} |\nu_{\mathrm{e}}\rangle \\ |\nu_{\mu}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{v}} & \sin\theta_{\mathrm{v}} \\ -\sin\theta_{\mathrm{v}} & \cos\theta_{\mathrm{v}} \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

θ_{v}: vacuum mixing angle

Why Do Neutrinos Oscillate?

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$\theta_{\rm v}$: vacuum mixing angle

Hamiltonian **H**

Mass basis

$$\frac{\omega_{\rm v}}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -\frac{\omega_{\rm v}}{2}\boldsymbol{\sigma_3}$$

Flavor basis

$$\frac{\omega_{\rm v}}{2} \begin{pmatrix} -\cos 2\theta_{\rm v} & \sin 2\theta_{\rm v} \\ \sin 2\theta_{\rm v} & \cos 2\theta_{\rm v} \end{pmatrix} = \frac{\omega_{\rm v}}{2}\left(-\cos 2\theta_{\rm v}\boldsymbol{\sigma_3} + \sin 2\theta_{\rm v}\boldsymbol{\sigma_1}\right)$$

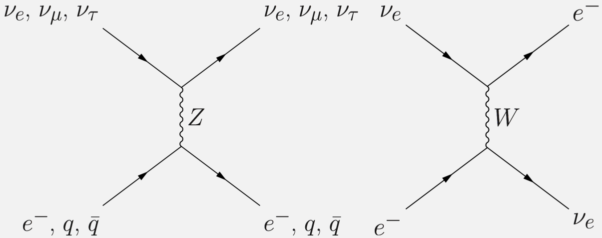
Nature of Neutrino Oscillation

Transition Probability

$$P(|\nu_{\mathrm{e}}\rangle \rightarrow \left|\nu_{\mu}\right\rangle) = \sin^2(2\theta_{\mathrm{v}})\sin^2(\omega_{\mathrm{v}} x/2)$$

- $\omega_{\mathrm{v}} = (m_2^2 - m_1^2)/2E$ determines oscillation wavelength.
- Mixing angle θ_{v} determines flavor oscillation amplitude.

Matter Interaction



Neutral current interaction between ν_e, ν_μ, ν_τ, and e^-, quarks and antiquarks.

Charged current interaction between ν_e and e^-

Matter Interaction

Hamiltonian with matter interaction in flavor basis ($\omega_{\mathrm{v}} = \delta m^2/2E$):

$$\mathbf{H} = \frac{\omega_{\mathrm{v}}}{2} \begin{pmatrix} -\cos 2\theta_{\mathrm{v}} & \sin 2\theta_{\mathrm{v}} \\ \sin 2\theta_{\mathrm{v}} & \cos 2\theta_{\mathrm{v}} \end{pmatrix} \pm \sqrt{2} G_{\mathrm{F}} n_{\mathrm{e}}(x) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

- Vacuum Hamiltonian
- Matter interaction

Matter Interaction

Hamiltonian with matter interaction in flavor basis ($\omega_{\mathrm{v}} = \delta m^2/2E$):

$$\mathbf{H} = \boxed{\frac{\omega_{\mathrm{v}}}{2}\left(-\cos 2\theta_{\mathrm{v}}\boldsymbol{\sigma_3} + \sin 2\theta_{\mathrm{v}}\boldsymbol{\sigma_1}\right)} \boxed{+\frac{\lambda(x)}{2}\boldsymbol{\sigma_3}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2}G_{\mathrm{F}}n_{\mathrm{e}}(x)$

MSW Effect

Hamiltonian in Vacuum

$$\mathbf{H}_{\mathrm{vacuum}} = \frac{\omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}}}{2} \boldsymbol{\sigma}_3 + \frac{\omega_{\mathrm{v}} \sin 2\theta_{\mathrm{v}}}{2} \boldsymbol{\sigma}_1$$

$$\begin{aligned} \mathbf{H} &= \frac{\lambda(x) - \omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}}}{2} \boldsymbol{\sigma}_3 + \frac{\omega_{\mathrm{v}} \sin 2\theta_{\mathrm{v}}}{2} \boldsymbol{\sigma}_1 \\ &= \frac{\omega_{\mathrm{m}}(x) \cos 2\theta_{\mathrm{m}}(x)}{2} \boldsymbol{\sigma}_3 + \frac{\omega_{\mathrm{m}}(x) \sin 2\theta_{\mathrm{m}}(x)}{2} \boldsymbol{\sigma}_1, \end{aligned}$$

where

$$\omega_{\mathrm{m}}(x) = \sqrt{(\lambda(x) - \omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}})^2 + \omega_{\mathrm{v}}^2 \sin^2 2\theta_{\mathrm{v}}},$$

$$\tan 2\theta_{\mathrm{m}}(x) = \frac{\omega_{\mathrm{v}} \sin 2\theta_{\mathrm{v}}}{\omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}} - \lambda(x)}.$$

MSW Effect

Constant matter profile λ_0 as an example,

Significance of θ_{m}

Define matter basis $\{|\nu_{\mathrm{L}}\rangle, |\nu_{\mathrm{H}}\rangle\}$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{m}} & \sin\theta_{\mathrm{m}} \\ -\sin\theta_{\mathrm{m}} & \cos\theta_{\mathrm{m}} \end{pmatrix} \begin{pmatrix} |\nu_{\mathrm{L}}\rangle \\ |\nu_{\mathrm{H}}\rangle \end{pmatrix}$$

In matter basis

$$\mathbf{H}_{\text{matter-basis}} = -\frac{\omega_{\mathrm{m}}}{2}\boldsymbol{\sigma_3}$$

MSW Resonance

$$\mathbf{H} = \frac{\lambda(x) - \omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}}}{2} \sigma_3 + \frac{\omega_{\mathrm{v}} \sin 2\theta_{\mathrm{v}}}{2} \sigma_1$$
$$= \frac{\omega_{\mathrm{m}}(x) \cos 2\theta_{\mathrm{m}}(x)}{2} \sigma_3 + \frac{\omega_{\mathrm{m}}(x) \sin 2\theta_{\mathrm{m}}(x)}{2} \sigma_1$$

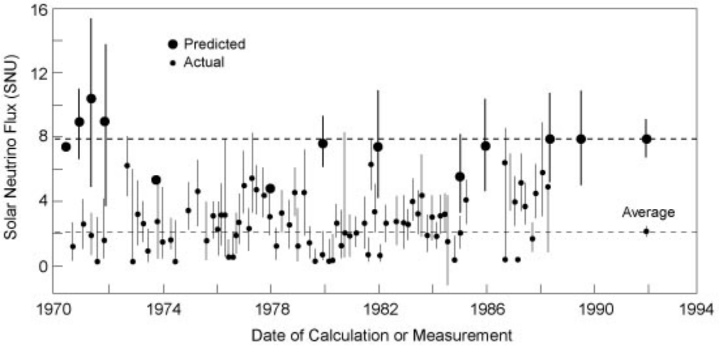
$$\tan 2\theta_{\mathrm{m}}(x) = \frac{\omega_{\mathrm{v}} \sin 2\theta_{\mathrm{v}}}{\omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}} - \lambda(x)}.$$

$$\begin{pmatrix} |\nu_{\mathrm{e}}\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{m}} & \sin\theta_{\mathrm{m}} \\ -\sin\theta_{\mathrm{m}} & \cos\theta_{\mathrm{m}} \end{pmatrix} \begin{pmatrix} |\nu_{\mathrm{L}}\rangle \\ |\nu_{\mathrm{H}}\rangle \end{pmatrix}$$

Transition Probability

$$P(|\nu_{\mathrm{e}}\rangle \to |\nu_\mu\rangle) = \sin^2(2\theta_{\mathrm{m}}) \sin^2(\omega_{\mathrm{m}} x)$$

Solar Neutrino Problem



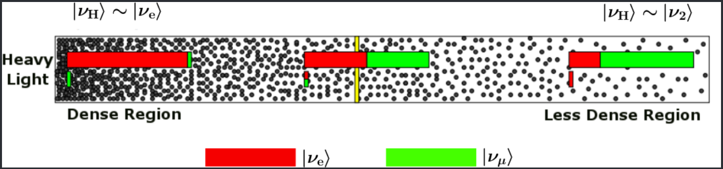
Chlorine detector (Homestake experiment) results and theory predictions. SNU: 1 event for 10^{36} target atoms per second. Kenneth R. Lang (2010)

MSW Effect and Solar Neutrinos

$$\mathbf{H} = \frac{\lambda(x) - \omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}}}{2}\sigma_3 + \frac{\omega_{\mathrm{v}} \sin 2\theta_{\mathrm{v}}}{2}\sigma_1$$

$$\begin{pmatrix} |\nu_{\mathrm{L}}\rangle \\ |\nu_{\mathrm{H}}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{\mathrm{m}} & -\sin\theta_{\mathrm{m}} \\ \sin\theta_{\mathrm{m}} & \cos\theta_{\mathrm{m}} \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

$$\mathbf{H}_{\text{matter-basis}} = -\frac{\omega_{\mathrm{m}}}{2}\sigma_3$$



Yellow bar is the resonance point. Red: $|\nu_e\rangle$. Green: $|\nu_\mu\rangle$. Adapted from Smirnov, 2003.

MSW Effect

Suppose $\omega_{\mathrm{v}} = (m_2^2 - m_1^2)/2E < 0$,

$$\mathbf{H} = -\frac{\omega_{\mathrm{v}}}{2}\begin{pmatrix} -\cos 2\theta_{\mathrm{v}} & \sin 2\theta_{\mathrm{v}} \\ \sin 2\theta_{\mathrm{v}} & \cos 2\theta_{\mathrm{v}} \end{pmatrix} + \sqrt{2} G_{\mathrm{F}} n_{\mathrm{e}}(x) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

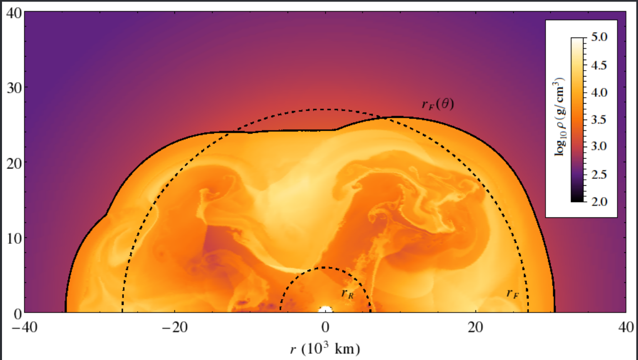
$$\downarrow$$

$$\mathbf{H} = \left(\frac{-\omega_{\mathrm{v}}}{2} \cos 2\theta_{\mathrm{v}} + \frac{\lambda(x)}{2} \right) \sigma_3 - \frac{\omega_{\mathrm{v}}}{2} \sin 2\theta_{\mathrm{v}} \sigma_1$$

Supernova Matter Density Profile

Why Do We Care

Astrophysical environments: supernovae, accretion disks etc



Supernova shock and turbulence. E. Borriello, et al (2014)

Stimulated Neutrino Oscillations

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

Basis

Background matter basis: Hamiltonian is diagonalized with only background matter profile λ_0,

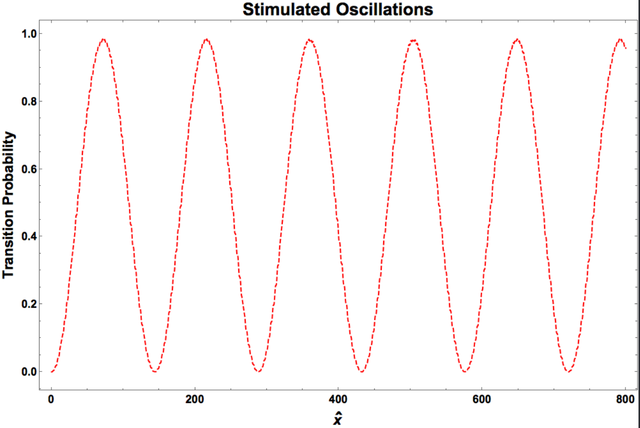
$$\mathrm{H}_{\mathrm{background}} = -\frac{\omega_{\mathrm{m}}}{2}\boldsymbol{\sigma_3}.$$

Hamiltonian

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \delta\lambda(x)\cos 2\theta_{\mathrm{m}}\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_{\mathrm{m}}\boldsymbol{\sigma_1}.$$

Stimulated Neutrino Oscillations

P. Krastev and A. Smirnov (1989); J. Kneller et al (2013);
K. Patton et al (2014);



Stimulated oscillations. $\lambda(x) = \lambda_0 + A\sin(kx)$ with $\hat{x} = \omega_{\mathrm{m}} x$, $A = 0.1\omega_{\mathrm{m}}$,

Hamiltonian, and Basis, and Rabi Oscillations

Hamiltonian in Background Matter Basis

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \delta\lambda(x)\cos 2\theta_{\mathrm{m}}\right)\sigma_3 - \frac{\delta\lambda(x)}{2}\sin 2\theta_{\mathrm{m}}\sigma_1.$$

Matter profile

$$\lambda(x) = \lambda_0 + A\cos(kx),$$

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \cos 2\theta_{\mathrm{m}} A\cos(kx)\right)\sigma_3 - \frac{\sin 2\theta_{\mathrm{m}}}{2} A\cos(kx)\sigma_1.$$

Hamiltonian, and Basis, and Rabi Oscillations

Rabi Oscillation

Hamiltonian

$$-\frac{\omega_0}{2}\sigma_3 - \frac{\alpha}{2}\begin{pmatrix} 0 & e^{ikt} \\ e^{-ikt} & 0 \end{pmatrix}$$

$$\frac{1}{2}\left(-\omega_{\mathrm{m}}+\cos 2\theta_{\mathrm{m}} A\cos(kx)\right)\sigma_3-\frac{\sin 2\theta_{\mathrm{m}}}{2} A\cos(kx)\sigma_1$$

Hamiltonian, and Basis, and Rabi Oscillations

Rabi Oscillation

Hamiltonian

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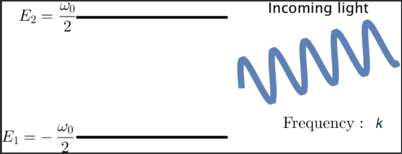


The transition probability from low energy to high energy is

$$P_{1\to 2}=\frac{\alpha^2}{\alpha^2+(\omega_0-k)^2}\sin^2\left(\frac{\Omega_{\mathrm{R}}}{2}t\right),$$

where

$$\Omega_{\mathrm{R}}=\sqrt{\alpha^2+(\omega_0-k)^2}$$

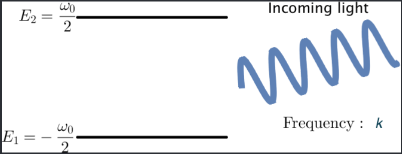
is Rabi frequency.

Hamiltonian, and Basis, and Rabi Oscillations

Rabi Oscillation

Hamiltonian

$$-\frac{\omega_0}{2}\sigma_3 - \frac{\alpha}{2}\begin{pmatrix} 0 & e^{ikt} \\ e^{-ikt} & 0 \end{pmatrix}$$



The transition probability from low energy to high energy is

$$P_{1\to 2} = \frac{1}{1+D^2}\sin^2\left(\frac{\Omega_{\mathrm{R}}}{2}t\right),$$

where

$$D = \left|\frac{\omega_0 - k}{\alpha}\right|.$$

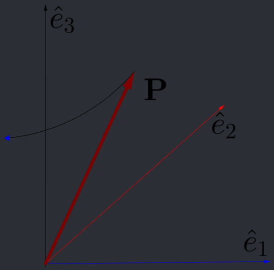
$$-\frac{\omega_0}{2}\sigma_3 - \frac{\alpha}{2}\begin{pmatrix} 0 & e^{ikt} \\ e^{-ikt} & 0 \end{pmatrix}$$

Visualizing Rabi Oscillations

$$-\frac{\omega_0}{2}\sigma_3 - \frac{\alpha}{2}\cos(kt)\sigma_1 + \frac{\alpha}{2}\sin(kt)\sigma_2$$

$$= \begin{pmatrix} \alpha\cos(kt) & -\alpha\sin(kt) & \omega_0 \end{pmatrix} \begin{pmatrix} -\sigma_1/2 \\ -\sigma_2/2 \\ -\sigma_3/2 \end{pmatrix}$$

$$= \vec{H} \cdot (-\vec{\sigma}/2)$$



$$D = \left| \frac{\omega_0 - k}{\alpha} \right|$$

is

Interferences of Rabi Oscillations

$$\begin{pmatrix} 0 \\ 0 \\ \omega_0 \end{pmatrix} + \alpha \begin{pmatrix} \cos(kt) \\ -\sin(kt) \\ 0 \end{pmatrix}$$

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \cos 2\theta_{\mathrm{m}} A \cos(kx)\right)\sigma_3 - \frac{\sin 2\theta_{\mathrm{m}}}{2} A \cos(kx)\sigma_1$$

$$\rightarrow -\frac{\omega_m}{2}\sigma_3 - \frac{A \sin 2\theta_{\mathrm{m}}}{2}\cos(kx)\sigma_1$$

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ \omega_m \end{pmatrix} + \frac{A \sin 2\theta_{\mathrm{m}}}{2}\begin{pmatrix} \cos(kx) \\ -\sin(kx) \\ 0 \end{pmatrix} + \frac{A \sin 2\theta_{\mathrm{m}}}{2}\begin{pmatrix} \cos(-kx) \\ -\sin(-kx) \\ 0 \end{pmatrix}$$

Two frequencies!

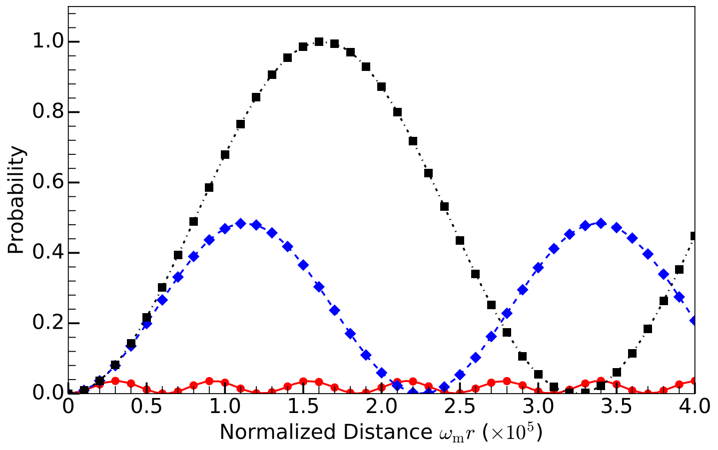
Interferences of Rabi Oscillations

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ \omega_m \end{pmatrix} + \alpha_1 \begin{pmatrix} \cos(k_1 x) \\ -\sin(k_1 x) \\ 0 \end{pmatrix} + \boxed{\alpha_2 \begin{pmatrix} \cos(k_2 x) \\ -\sin(k_2 x) \\ 0 \end{pmatrix}}$$

Corotating frame of the second frequency,

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ \omega_m - k_2 \end{pmatrix} + \alpha_1 \begin{pmatrix} \cos(k_1 - k_2 x) \\ -\sin(k_1 - K_2 x) \\ 0 \end{pmatrix} + \boxed{\alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

Rabi Formula Works



Dots, diamonds, triangles, and squares are for $k = \omega_{\mathrm{m}}$, $k = (1 - 2 \times 10^{-5})\omega_{\mathrm{m}}$, and $k = (1 - 10^{-4})\omega_{\mathrm{m}}$ respectively.

Lines: Rabi formula

Interferences of Rabi Oscillations

Matter profile with two frequencies,

$$\lambda(x) = \lambda_0 + \frac{\alpha_1}{2} \sin(k_1 x) \sigma_1$$

Understanding Stimulated Oscillations

Matter profile

$$\lambda(x) = \lambda_0 + A\sin(kx),$$

Hamiltonian in Background Matter Basis

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \delta\lambda(x)\cos 2\theta_{\mathrm{m}}\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_{\mathrm{m}}\boldsymbol{\sigma_1}.$$

A Better Basis

Define new basis $\{|\tilde{\nu}_{\mathrm{L}}\rangle, |\tilde{\nu}_{\mathrm{H}}\rangle\}$ is related to background matter basis $\{|\nu_{\mathrm{L}}\rangle, |\nu_{\mathrm{H}}\rangle\}$ through

$$\begin{pmatrix} |\nu_{\mathrm{L}}\rangle \\ |\nu_{\mathrm{H}}\rangle \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{i\eta(x)} \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_{\mathrm{L}}\rangle \\ |\tilde{\nu}_{\mathrm{H}}\rangle \end{pmatrix},$$

where

Single Frequency Matter Profile

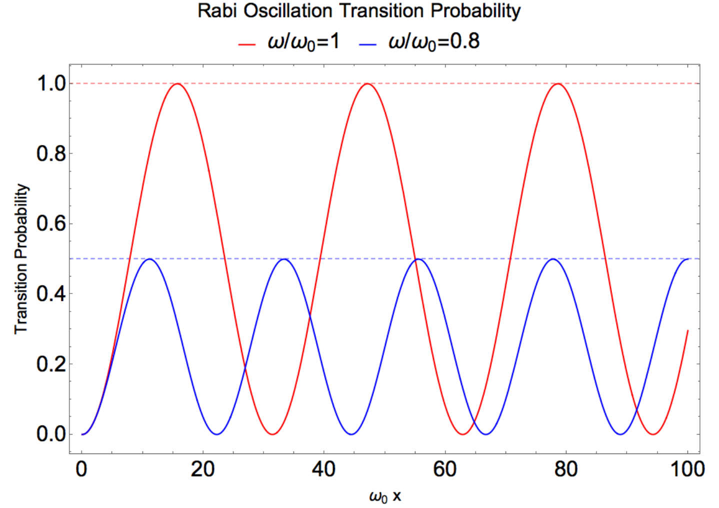
Hamiltonian in new basis

$$\widetilde{\mathbf{H}} = -\frac{\delta\lambda(x)}{2}\sin 2\theta_{\mathrm{m}} \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix} = \begin{pmatrix} 0 & h \\ h^* & 0 \end{pmatrix}$$

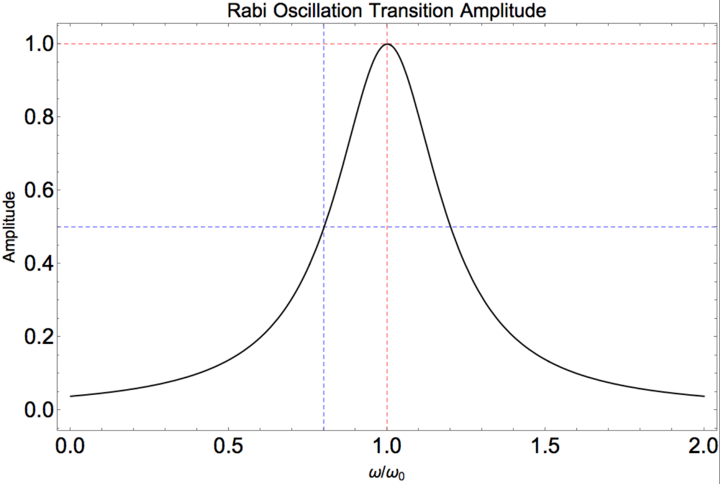
Hamiltonian in New Basis

$$\begin{aligned} h &\equiv -\frac{\delta\lambda(x)}{2} e^{2i\eta(x)} \\ &= \frac{i}{4}\Big[\exp\left(i(k+\omega_{\mathrm{m}})x + i\cos 2\theta_{\mathrm{m}}\frac{A}{k}\cos(kx)\right) \\ &\quad - \exp\left(i(-k+\omega_{\mathrm{m}})x + i\cos 2\theta_{\mathrm{m}}\frac{A}{k}\cos(kx)\right)\Big] \end{aligned}$$

Rabi Oscillations



Rabi Oscillations



Resonance

Single Frequency Matter Profile

Off-diagonal Term in Our System

$$\widetilde{\mathbf{H}} = \begin{pmatrix} 0 & h \\ h^* & 0 \end{pmatrix}$$

$$h \propto \left[\exp\left(i(k+\omega_{\mathrm{m}})x + i\cos 2\theta_{\mathrm{m}} \frac{A}{k}\cos(kx) \right) \right. \\ \left. - \exp\left(i(-k+\omega_{\mathrm{m}})x + i\cos 2\theta_{\mathrm{m}} \frac{A}{k}\cos(kx) \right) \right]$$

Jacobi-Anger expansion

$$e^{i\beta\cos(kx)} = \sum_{n=-\infty}^{\infty} i^n J_n(\beta) e^{inkx},$$

where $J_n(\beta)$ are Bessel's functions of the first kind.

Single Frequency Matter Profile

Scaled Quantities

Characteristic scale: ω_{m}

- $\hat{A} = A/\omega_{\mathrm{m}}$
- $\hat{k} = k/\omega_{\mathrm{m}}$
- $\hat{x} = \omega_{\mathrm{m}} x$
- $\hat{h} = h/\omega_{\mathrm{m}}$

Single Frequency Matter Profile

Rotation Wave Approximation

The off-diagonal element of Hamiltonian

$$\widetilde{\mathbf{H}} = \sum_{n=-\infty}^{\infty} \begin{pmatrix} 0 & \frac{1}{2}\hat{B}_n e^{i(n\hat{k}-1)\hat{x}} \\ \frac{1}{2}\hat{B}_n^* e^{-i(n\hat{k}-1)\hat{x}} & 0 \end{pmatrix}$$

where $\hat{B}_n = -(-i)^n n\hat{k} \tan 2\theta_{\mathrm{m}} J_n(\hat{A}\cos 2\theta_{\mathrm{m}}/\hat{k})$.

Single Frequency Matter Profile

Transition Probability

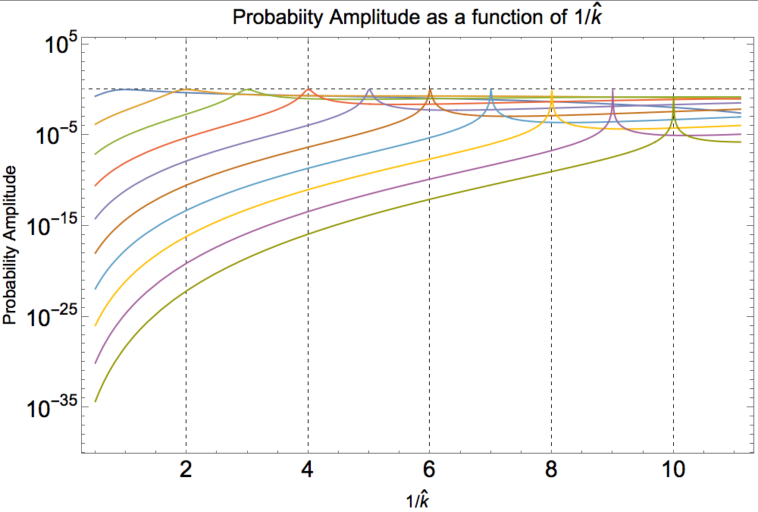
$$P^{(n)}_{\mathrm{L}\to\mathrm{H}} = \frac{\left|\hat{B}_n\right|^2}{\left|\hat{B}_n\right|^2 + (n\hat{k} - 1)^2} \sin^2\left(\frac{q^{(n)}}{2}x\right),$$

where

$$q^{(n)} = \sqrt{\left|\Gamma^{(n)}\right|^2 + (n\hat{k} - 1)^2}, \quad \text{frequency of oscillations}$$

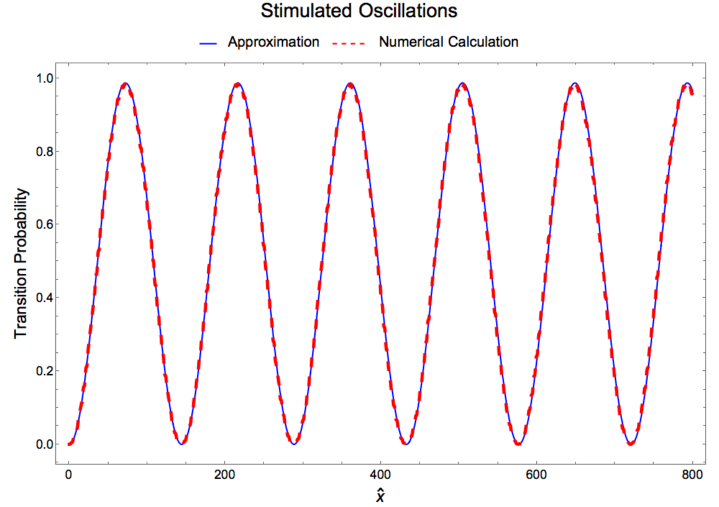
$$\Gamma^{(n)} = \left|\hat{B}_n\right|, \quad \text{width of resonance } (n\hat{k} \text{ as parameter})$$

Single Frequency Matter Profile



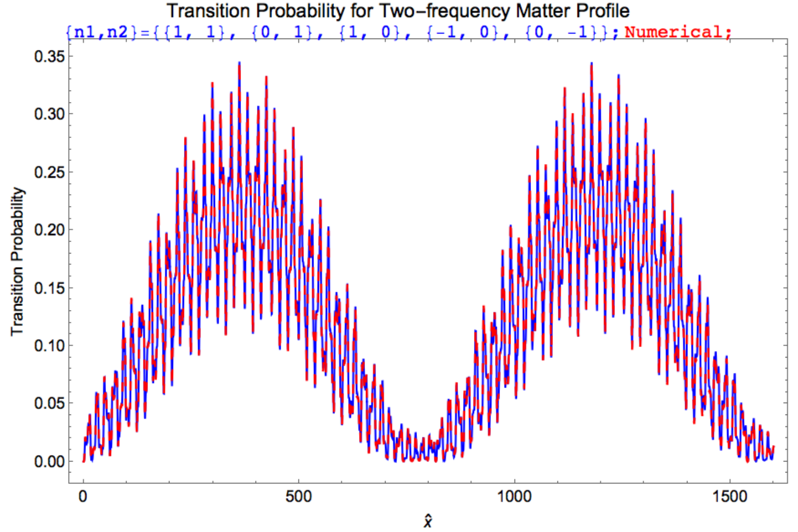
Resonances of different $n = 1/\hat{k}$.

Single Frequency Matter Profile



$$\hat{A} = 0.1,\ \hat{k} = 0.995,\ \theta_{\mathrm{m}} = \pi/6$$

Two-frequency Matter Profile



$\lambda(x) = \lambda_0 + A_1 \sin(k_1 x) + A_2 \sin(k_2 x)$. $\hat{k}_1 = 0.3$, $\hat{k}_2 = 0.7$, $A_1 = A_2 = 0.1$, $\theta_v = \pi/5$

Outline for Section 2

Summary

- The fact that neutrino flavor sates are not mass states causes vacuum oscillations.
- MSW resonance happens when matter potential cancels out the vacuum diagonal elements of the Hamiltonian.
- Even matter profile doesn’t match MSW requirement, variation in matter profile can cause resonances.
- Single frequency perturbations in matter profile is a combination of many Rabi oscillations.

- How to understand and calculate systems with multi-frequency matter profile (turbulence).
- Combine periodic or even turbulent matter profile with neutrino self-interaction.

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Citations

TeX, LaTeX, and Beamer

TeX is a programming language for the typesetting of documents. It was created by Donald Erwin Knuth in the late 1970s and it is documented in *The TeXbook* [1].

In the early 1980s, Leslie Lamport created the initial version of LaTeX, a high-level language on top of TeX, which is documented in *LaTeX: A Document Preparation System* [2]. There exists a healthy ecosystem of packages that extend the base functionality of LaTeX; *The LaTeX Companion* [3] acts as a guide through the ecosystem.

In 2003, Till Tantau created the initial version of Beamer, a LaTeX package for the creation of presentations. Beamer is documented in the *User's Guide to the Beamer Class* [4].

Bibliography

TeX, LaTeX, and Beamer

[1] Donald E. Knuth. *The TeXbook*. Addison-Wesley, 1984.

[2] Leslie Lamport. *LaTeX: A Document Preparation System*. Addison-Wesley, 1986.

[3] M. Goossens, F. Mittelbach, and A. Samarin. *The LaTeX Companion*. Addison-Wesley, 1994.

[4] Till Tantau. *User's Guide to the Beamer Class Version 3.01*. Available at `http://latex-beamer.sourceforge.net`.

[5] A. Mertz and W. Slough. Edited by B. Beeton and K. Berry. *Beamer by example* In TUGboat, Vol. 26, No. 1., pp. 68-73.