Neutrino Flavor Conversions in Dense Medium: Matter Stimulation, Dispersion Relation, and Neutrino Halo

PhD Defense

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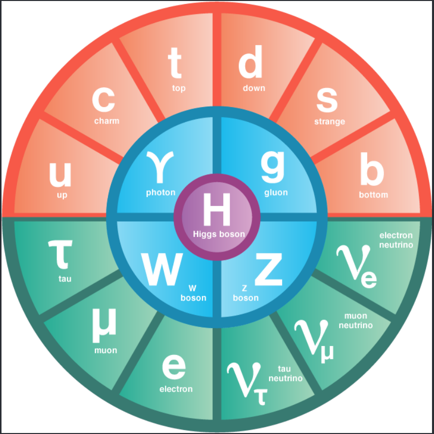
Outline for Section 1

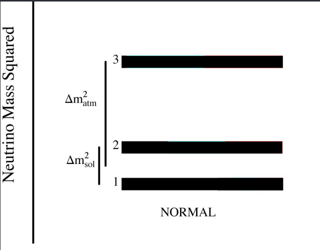
What are Neutrinos?



Elementary particles.
Source: symmetrymagazine.org

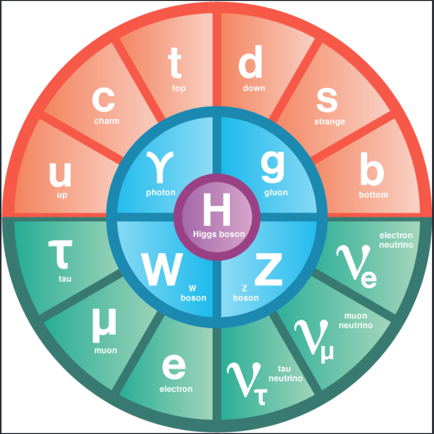
Neutrinos are

- fermions,
- electrically neutral,
- three flavors,
- none vanishing mass.



Adapted from Olga Mena & Stephen Parke (2004)

What are Neutrinos?



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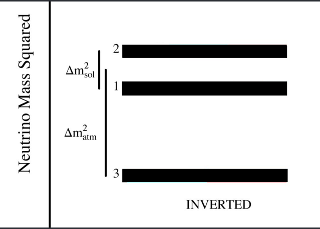

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Why Do Neutrinos Oscillate?

Flavor states are different from mass states.

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_\mathrm{v} & \sin\theta_\mathrm{v} \\ -\sin\theta_\mathrm{v} & \cos\theta_\mathrm{v} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Why Do Neutrinos Oscillate?

Equation of Motion

$$i\partial_x \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \mathbf{H} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

Why Do Neutrinos Oscillate?

Equation of Motion

$$i\partial_x \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \mathbf{H} \begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix}$$

$$\mathbf{H} = \frac{\omega_{\mathrm{v}}}{2} \left(-\cos 2\theta_{\mathrm{v}} \sigma_3 + \sin 2\theta_{\mathrm{v}} \sigma_1 \right)$$

- Oscillation frequency:

$$\omega_{\mathrm{v}} = \frac{\delta m^2}{2E} = \frac{m_2^2 - m_1^2}{2E}$$

- Mixing angle θ_{v}

Flavor Isospin

Hamiltonian: $\mathbf{H} = -\frac{\vec{\sigma}}{2} \cdot \vec{H}$

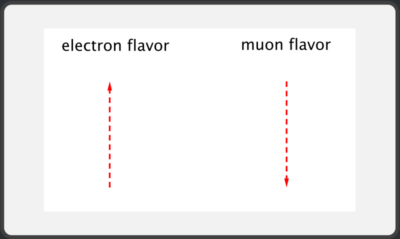
Flavor isospin: $\vec{s} = \Psi^\dagger \frac{\vec{\sigma}}{2} \Psi$

Electron flavor survival probability

$$P = \frac{1}{2} + s_3$$

Equation of motion

$$\dot{\vec{s}} = \vec{s} \times \vec{H}$$



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Equation of motion

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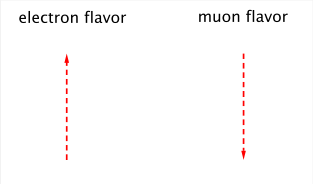
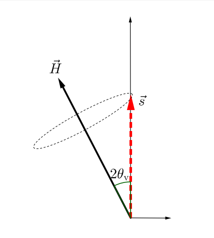
Vacuum oscillation Hamiltonian

$$\frac{\omega_v}{2} \left(-\cos 2\theta_v \sigma_3 + \sin 2\theta_v \sigma_1 \right)$$

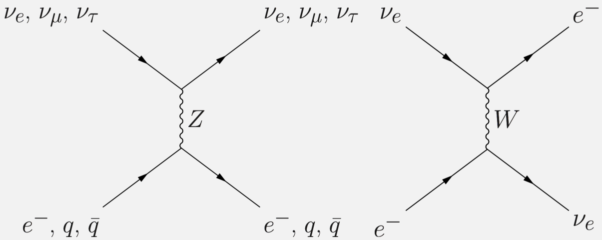
$$\rightarrow \cos 2\theta_v \begin{pmatrix} 0 \\ 0 \\ \omega_v \end{pmatrix} - \sin 2\theta_v \begin{pmatrix} \omega_v \\ 0 \\ 0 \end{pmatrix}$$





Outline for Section 2

Matter Interaction



Neutral current interaction between ν_e, ν_μ, ν_τ, and e^-, etc.

Charged current interaction between ν_e and e^-

Matter Interaction

Hamiltonian with matter interaction in flavor basis ($\omega_{\mathrm{v}} = \delta m^2/2E$):

$$\mathbf{H} = \underbrace{\frac{\omega_{\mathrm{v}}}{2}\left(-\cos 2\theta_{\mathrm{v}}\boldsymbol{\sigma}_3 + \sin 2\theta_{\mathrm{v}}\boldsymbol{\sigma}_1\right)}_{\text{Vacuum Hamiltonian}} + \underbrace{\frac{\lambda(x)}{2}\boldsymbol{\sigma}_3}_{\text{Matter interaction}}$$

- Vacuum Hamiltonian
- Matter interaction
- $\lambda(x) = \sqrt{2}G_{\mathrm{F}}n_{\mathrm{e}}(x)$

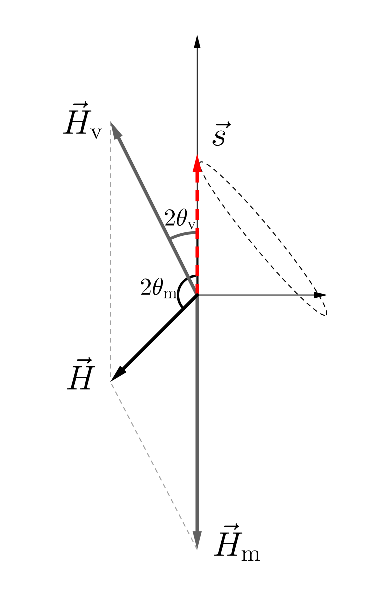
MSW Effect

$$\mathbf{H} = \boxed{\frac{\omega_{\mathrm{v}}}{2}\left(-\cos 2\theta_{\mathrm{v}}\sigma_{3} + \sin 2\theta_{\mathrm{v}}\sigma_{1}\right)}\boxed{+\frac{\lambda(x)}{2}\sigma_{3}}$$

$$\rightarrow \boxed{\omega_{\mathrm{v}}\begin{pmatrix} -\sin 2\theta_{\mathrm{v}} \\ 0 \\ \cos 2\theta_{\mathrm{v}} \end{pmatrix}}\boxed{+\begin{pmatrix} 0 \\ 0 \\ -\lambda(x) \end{pmatrix}}$$

$$= \boxed{\vec{H}_{\mathrm{v}}}\boxed{+\vec{H}_{\mathrm{m}}(x)}$$

MSW Effect



Electron flavor survival probability

$$P = \frac{1}{2} + s_3$$

Oscillation frequency in **vacuum**:

$$\omega_{\rm v} = |\vec{H}_{\rm v}|$$

Oscillation frequency in **matter**:

$$\omega_{\rm m} = |\vec{H}|$$

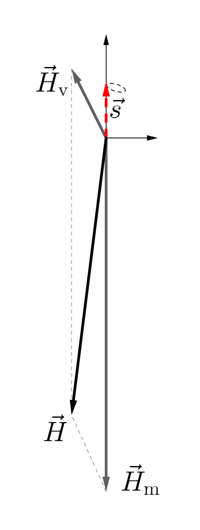
Flavor states and mass states in matter

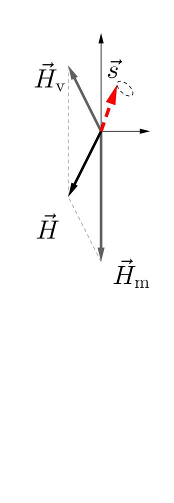
$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{\rm m} & \sin\theta_{\rm m} \\ -\sin\theta_{\rm m} & \cos\theta_{\rm m} \end{pmatrix} \begin{pmatrix} \psi_{\rm L} \\ \psi_{\rm H} \end{pmatrix}$$

MSW Effect

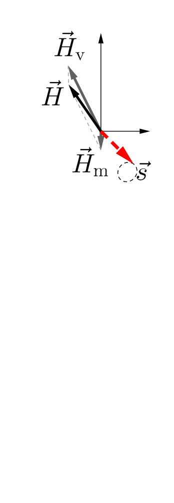
Adiabatic matter density change

Large density

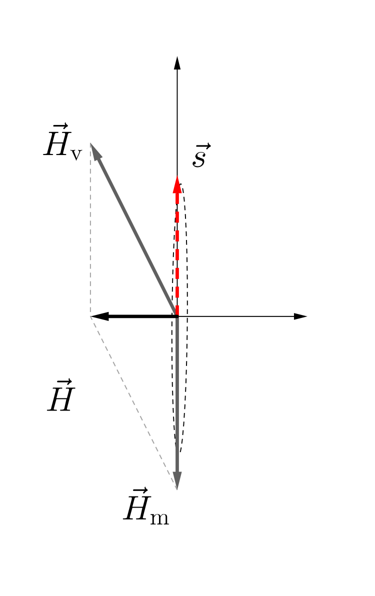




Low density



MSW Effect

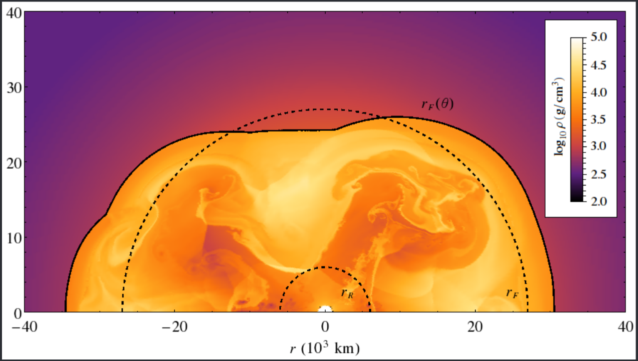


- Maximum possible flavor transition probability amplitude
- MSW Resonance
- A specific matter density

$$\sqrt{2}G_{\rm F}n_{\rm e} \equiv \omega_{\rm v}\cos 2\theta_{\rm v}$$

Supernova Matter Density Profile

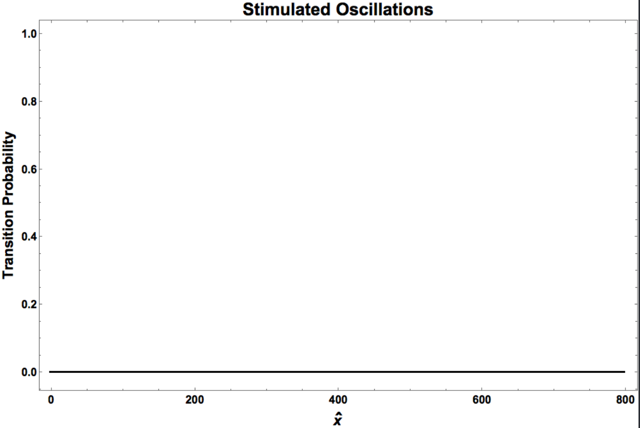
Astrophysical environments: supernovae, accretion disks etc



Supernova shock and turbulence. E. Borriello, et al (2014)

Stimulated Neutrino Flavor Conversions

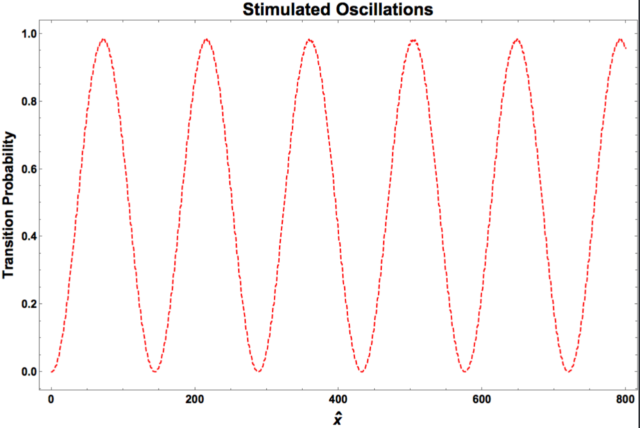
$$\lambda(x) = \lambda_0$$



Transition probabilities between mass states in matter.

Stimulated Neutrino Flavor Conversions

$$\lambda(x) = \lambda_0 + A\cos(kx)$$



P. Krastev and A. Smirnov (1989); A. Friedland et al (2006); J. Kneller et al (2013); K. Patton et al (2014);

Rabi Oscillations

Rabi Oscillation

Hamiltonian

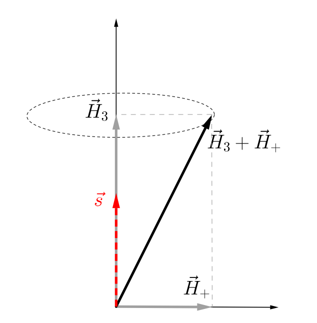
$$-\frac{\omega_{\mathrm{m}}}{2}\sigma_3 - \frac{\alpha}{2}\begin{pmatrix} 0 & e^{ikt} \\ e^{-ikt} & 0 \end{pmatrix}$$

Scheme

Rabi Oscillations

Static Frame

$$\vec{H}_3 = \omega_{\mathrm{m}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \vec{H}_+ = \alpha \begin{pmatrix} \cos(kt) \\ -\sin(kt) \\ 0 \end{pmatrix}$$



Rabi Oscillations

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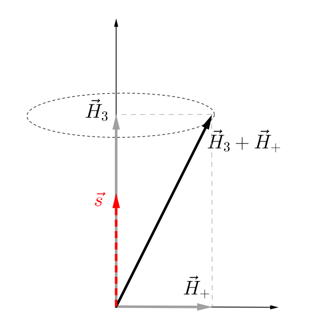
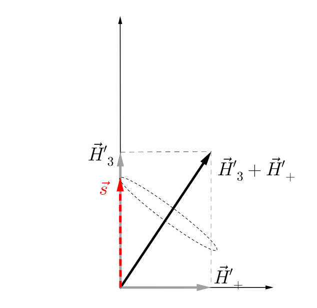


Corotating Frame

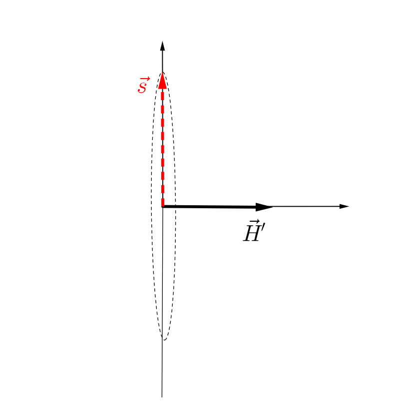
$$\vec{H}'_3 = (\omega_{\mathrm{m}} - k) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \vec{H}'_+ = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



Rabi Oscillations

Corotating Frame

$$\vec{H}'_3 = (\omega_\mathrm{m} - k)\begin{pmatrix}0\\0\\1\end{pmatrix} = 0 \Rightarrow k = \omega_\mathrm{m}$$



Rabi Oscillations

Rabi Oscillation

Hamiltonian

$$-\frac{\omega_{\mathrm{m}}}{2}\sigma_3 - \frac{\alpha}{2}\begin{pmatrix} 0 & e^{ikt} \\ e^{-ikt} & 0 \end{pmatrix}$$

Scheme

Rabi formula

$$P_{1\to 2} = \frac{1}{1 + D^2}\sin^2\left(\frac{\Omega_{\mathrm{R}}}{2}t\right).$$

Relative detuning

$$D = \left|\frac{\omega_{\mathrm{m}} - k}{\alpha}\right|.$$

Rabi frequency

$$\Omega_{\mathrm{R}} = |\alpha|\sqrt{1 + D^2}$$

Hamiltonian in Matter Basis

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_H \end{pmatrix}$$

Matter Potential

$$\lambda(x) = \lambda_0$$

Basis

matter basis:

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}}\right)\boldsymbol{\sigma_3}$$

Hamiltonian in Matter Basis

$$\begin{pmatrix} \psi_e \\ \psi_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_H \end{pmatrix}$$

Matter Potential

$$\lambda(x) = \lambda_0 + A\cos(kx)$$

Basis

Background matter basis:

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + A\cos(kx)\cos 2\theta_{\mathrm{m}}\right)\boldsymbol{\sigma_3} - \frac{A\cos(kx)}{2}\sin 2\theta_{\mathrm{m}}\boldsymbol{\sigma_1}$$

Hamiltonian in Matter Basis

Matter potential frequency

$$k \sim \omega_{\mathrm{m}}$$

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \cancel{\cos 2\theta_{\mathrm{m}} A\cos(kx)}\right)\sigma_3 - \frac{\sin 2\theta_{\mathrm{m}}}{2} A\cos(kx)\sigma_1$$

$$\rightarrow \omega_{\mathrm{m}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \alpha \begin{pmatrix} \cos(kx) \\ -\sin(kx) \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} \cos(-kx) \\ -\sin(-kx) \\ 0 \end{pmatrix}$$

$$\alpha = \frac{\sin 2\theta_{\mathrm{m}}}{2} A$$

Hamiltonian in Matter Basis

Matter potential frequency

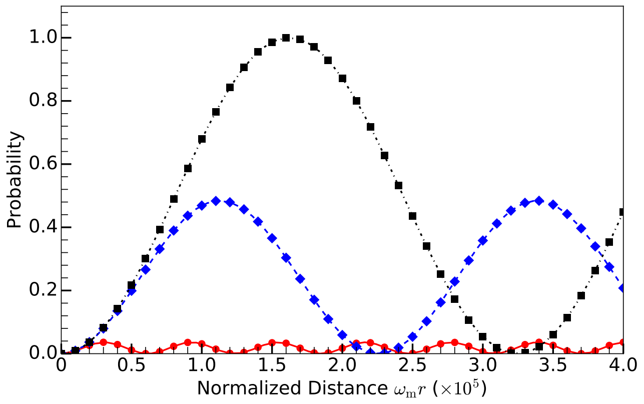
$$k \sim \omega_{\mathrm{m}}$$

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \cancel{\cos 2\theta_{\mathrm{m}} A\cos(kx)}\right)\sigma_3 - \frac{\sin 2\theta_{\mathrm{m}}}{2} A\cos(kx)\sigma_1$$

$$\to \omega_{\mathrm{m}}\begin{pmatrix}0\\0\\1\end{pmatrix} + \alpha\begin{pmatrix}\cos(kx)\\-\sin(kx)\\0\end{pmatrix} + \cancel{\alpha\begin{pmatrix}\cos(-kx)\\-\sin(-kx)\\0\end{pmatrix}}$$

$$\alpha = \frac{\sin 2\theta_{\mathrm{m}}}{2} A$$

Rabi Formula Works



Transition between two mass states in background matter potential λ_0
Lines: Rabi formula
Dots, diamonds, triangles, and squares are **full solutions without approximations** for $k = \omega_{\mathrm{m}}$, $k = (1 - 2 \times 10^{-5})\omega_{\mathrm{m}}$, and $k = (1 - 10^{-4})\omega_{\mathrm{m}}$ respectively.

Single Frequency Matter Potential Revisited

We have been making approximations.

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \cancel{\cos 2\theta_{\mathrm{m}} A\cos(kx)}\right)\sigma_3 - \frac{\sin 2\theta_{\mathrm{m}}}{2} A\cos(kx)\sigma_1$$

$$\rightarrow \omega_{\mathrm{m}}\begin{pmatrix}0\\0\\1\end{pmatrix} + \alpha\begin{pmatrix}\cos(kx)\\-\sin(kx)\\0\end{pmatrix} + \cancel{\alpha\begin{pmatrix}\cos(-kx)\\-\sin(-kx)\\0\end{pmatrix}}$$

Rabi Basis

Hamiltonian in Background Matter Basis

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + A\cos(kx)\cos 2\theta_{\mathrm{m}}\right)\boldsymbol{\sigma}_3 - \frac{A\cos(kx)}{2}\sin\theta_{\mathrm{m}}\boldsymbol{\sigma}_1.$$

A Better Basis

Define Rabi basis in which the wave function is related to wave function in background matter basis through

$$\begin{pmatrix} \psi_{\mathrm{L}} \\ \psi_{\mathrm{H}} \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{i\eta(x)} \end{pmatrix} \begin{pmatrix} \tilde{\psi}_{\mathrm{L}} \\ \tilde{\psi}_{\mathrm{H}} \end{pmatrix},$$

where

$$\eta(x) - \eta(0) = \frac{\cos 2\theta_{\mathrm{m}}}{2}\int_0^x A\cos(k\tau)d\tau.$$

Single Frequency Matter Potential

$$\lambda(x) = \lambda_0 + A\cos(kx)$$

Hamiltonian in Rabi Basis

The Hamiltonian

$$\widetilde{\mathbf{H}} = -\frac{\omega_{\mathrm{m}}}{2}\sigma_3 + \sum_{n=-\infty}^{\infty} \begin{pmatrix} 0 & \frac{1}{2}\alpha_n e^{i(nk)x} \\ \frac{1}{2}\alpha_n^* e^{-i(nk)x} & 0 \end{pmatrix}$$

where $\alpha_n = -(-i)^n nk \tan 2\theta_{\mathrm{m}} J_n(A\cos 2\theta_{\mathrm{m}}/k)$.

Single Frequency Matter Potential

$$\lambda(x) = \lambda_0 + A\cos(kx)$$

Hamiltonian in Rabi Basis

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where $\alpha_n = -(-i)^n nk \tan 2\theta_{\mathrm{m}} J_n(A\cos 2\theta_{\mathrm{m}}/k)$.

Map neutrino oscillations in single frequency matter potential to Rabi oscillations with many driving potentials.

Resonance condition for each mode: $nk = \omega_{\mathrm{m}}$

Rabi Oscillations With Multiple Driving Frequencies

Consider Rabi oscillation with two driving frequencies $k_1 = n_1 k$, $k_2 = n_2 k$

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ \omega_m \end{pmatrix} + \alpha_1 \begin{pmatrix} \cos(k_1 x) \\ -\sin(k_1 x) \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} \cos(k_2 x) \\ -\sin(k_2 x) \\ 0 \end{pmatrix}$$

Corotating frame of the second potential,

$$\vec{H} = \begin{pmatrix} 0 \\ 0 \\ \omega_m - k_2 \end{pmatrix} + \alpha_1 \begin{pmatrix} \cos(k_1 - k_2 x) \\ -\sin(k_1 - k_2 x) \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Energy gap in this frame becomes the length of the vector

$$\begin{pmatrix} 0 \\ 0 \\ \omega_m - k_2 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

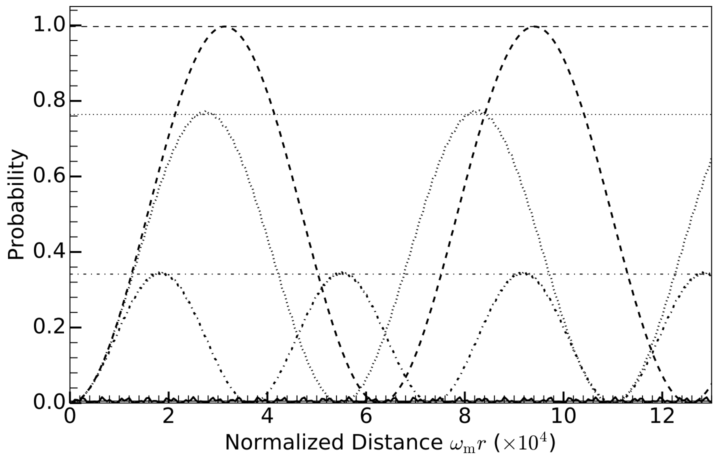
Rabi Oscillations With Multiple Driving Frequencies

Relative detuning

$$D' = \left| \frac{\omega_{\mathrm{m}} - k_1}{\alpha_1} + \frac{\alpha_2^2}{2\alpha_1(\omega_{\mathrm{m}} - k_2)} \right|$$

Rabi Oscillations With Multiple Driving Frequencies

$$D' = \left| \frac{\omega_{\mathrm{m}} - k_1}{\alpha_1} + \frac{\alpha_2^2}{2\alpha_1(\omega_{\mathrm{m}} - k_2)} \right|$$



Grid lines: amplitude predicted using $1/(1 + D'^2)$

α_2, k_1 values			
Dashed	dotted	dash-dotted	solid
$10^{-2}\omega_{\mathrm{m}}, 10\omega_{\mathrm{m}}$	$10^{-2}\omega_{\mathrm{m}}, 10^{-1}\omega_{\mathrm{m}}$	$5.0 \times 10^{-2}\omega_{\mathrm{m}}, 10\omega_{\mathrm{m}}$	$5 \times 10^{-2}\omega_{\mathrm{m}}, 10^{-1}\omega_{\mathrm{m}}$

Rabi Oscillations With Multiple Driving Frequencies

Consider $k_1 = \omega_{\mathrm{m}}$

$$D' = \left| \frac{\alpha_2^2}{2\alpha_1(\omega_{\mathrm{m}} - k_2)} \right|$$

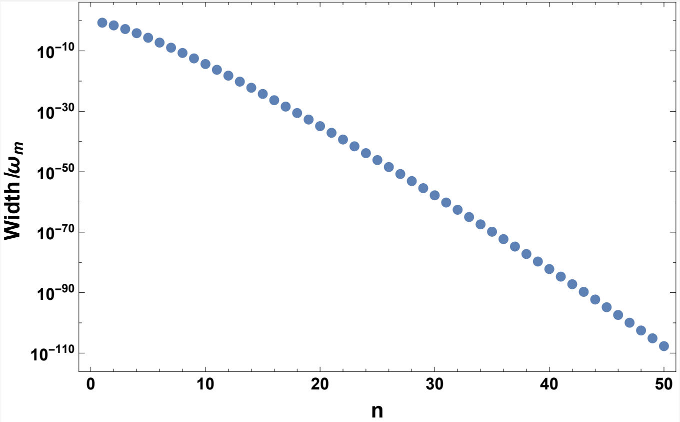
Amplitude reduces from 1 to 1/2 if

$$D' = 1 \Rightarrow \alpha_{2,\mathrm{C}} \equiv \sqrt{2|\alpha_1(k_2 - \omega_{\mathrm{m}})|}.$$

Two driving frequencies k_1, and k_2, with amplitude α_1, and α_2
For $k_1 = \omega_{\mathrm{m}}$, survival of resonance requires

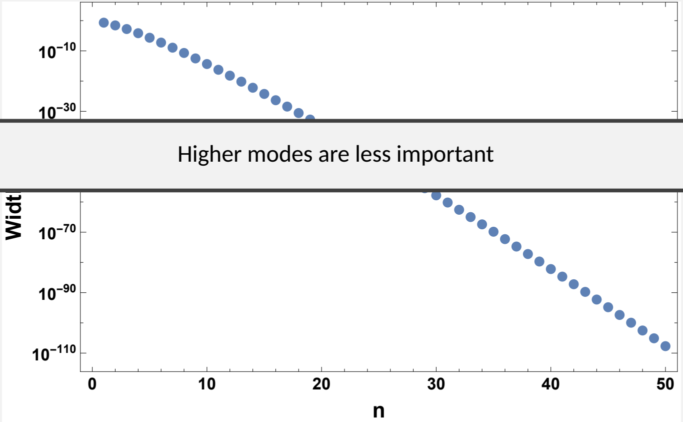
$$|\alpha_2| \ll \alpha_{2,\mathrm{C}} \equiv \sqrt{2|\alpha_1(k_2 - \omega_{\mathrm{m}})|}$$

Single Frequency Matter Potential



Width of different modes given value of matter potential frequency k

Single Frequency Matter Potential



Higher modes are less important

Width of different modes given value of matter potential frequency *k*

Multiple Frequencies in Matter Potential

$$\lambda(x) = \lambda_0 + \sum_{a=1}^{N} A_a \sin(k_a x)$$

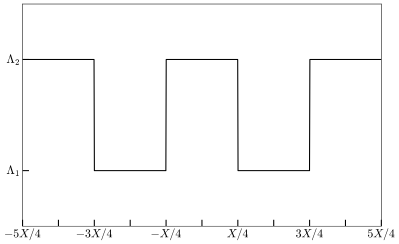
Hamiltonian in Rabi Basis

$$\widetilde{\mathbf{H}} = -\frac{\omega_{\mathrm{m}}}{2}\sigma_3 + \frac{1}{2}\sum_{n_1=-\infty}^{\infty}\cdots\sum_{n_N=-\infty}^{\infty}\begin{pmatrix} 0 & B_{\{n_a\}}\, e^{i\sum_a n_a k_a x} \\ B^*_{\{n_a\}}\, e^{-i\sum_a n_a k_a x} & 0 \end{pmatrix}$$

where

$$B_{\{n_a\}} = -(-i)^{\sum_a n_a} \tan 2\theta_m \left(\sum_a n_a k_a\right)\left(\prod_a J_{n_a}\left(\frac{A_a}{k_a}\cos 2\theta_m\right)\right)$$

Castle Wall Matter Potential



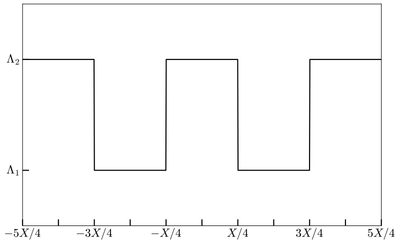
Castle wall matter profile:
$\Lambda_2 = 0.35\omega_\mathrm{v}\cos 2\theta_\mathrm{v}$, $\Lambda_1 = 0.15\omega_\mathrm{v}\cos 2\theta_\mathrm{v}$
and period $X = 2\pi/\omega_\mathrm{m}$

$$\lambda(x) = \lambda_0 + \sum_1^\infty \lambda_n \cos(k_n x)$$

where

$$\begin{aligned}\lambda_0 &= (\Lambda_1 + \Lambda_2)/2 \\ \lambda_n &= 2(-1)^n(\Lambda_1 - \Lambda_2)/(2n\pi - \pi) \\ k_n &= 2\pi(2n-1)/X\end{aligned}$$

Castle Wall Matter Potential



Castle wall matter profile:
$\Lambda_2 = 0.35\omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}}$, $\Lambda_1 = 0.15\omega_{\mathrm{v}} \cos 2\theta_{\mathrm{v}}$
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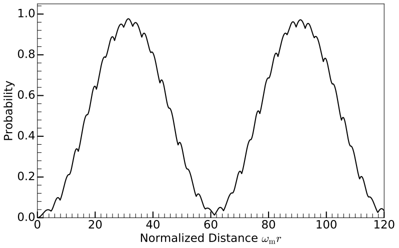
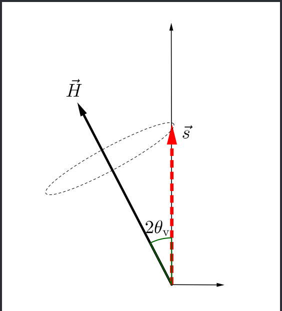


Transition probability is a Rabi resonance with small variations due to higher orders.

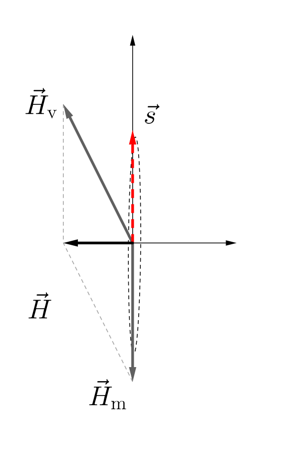
Summary of Stimulated Oscillations

1. Vacuum oscillations: flavor sates are not mass states.



Summary of Stimulated Oscillations

1. Vacuum oscillations: flavor sates are not mass states.
2. MSW resonance: matter potential cancels out the vacuum diagonal elements of the Hamiltonian.



Summary of Stimulated Oscillations

1. Vacuum oscillations: flavor sates are not mass states.
2. MSW resonance: matter potential cancels out the vacuum diagonal elements of the Hamiltonian.
3. Stimulated oscillations: variation in matter potential can cause resonances.

For matter potential

$$\lambda(x) = \lambda_0 + A\cos(kx),$$

Resonance condition

$$nk = \omega_{\mathrm{m}}$$

Summary of Stimulated Oscillations

1. Vacuum oscillations: flavor sates are not mass states.
2. MSW resonance: matter potential cancels out the vacuum diagonal elements of the Hamiltonian.
3. Stimulated oscillations: variation in matter potential can cause resonances.
4. In many cases neutrino oscillations in multi-frequency matter potential can be viewed as Rabi oscillations with few driving frequencies.

Outline for Section 3

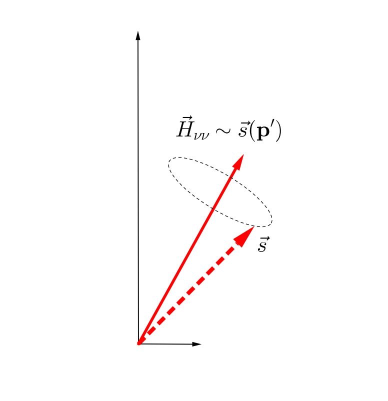
Neutrino Self-interactions

Interaction Hamiltonian $\mathbf{H}_{\nu\nu}$

$$\sqrt{2}G_{\mathrm{F}}(1-\hat{p}\cdot\hat{p}')\rho(\mathbf{p}')$$

In Flavor Isospin space

$$2\sqrt{2}G_{\mathrm{F}}(1-\hat{p}\cdot\hat{p}')\vec{s}(\mathbf{p}')$$



Linear Stability Analysis

Dispersion Relation

Outline for Section 4

Neutrino Halo

Flavor Isospin

Relaxation Scheme

Numerical Method

Outline for Section 5

Summary

- The fact that neutrino flavor sates are not mass states causes vacuum oscillations.
- MSW resonance happens when matter potential cancels out the vacuum diagonal elements of the Hamiltonian.
- Even matter profile doesn't match MSW requirement, variation in matter profile can cause resonances.
- Single frequency perturbations in matter profile is a combination of many Rabi oscillations.

- How to understand and calculate systems with multi-frequency matter profile (turbulence).
- Combine periodic or even turbulent matter profile with neutrino self-interaction.

Acknowledgement

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Citations

TeX, LaTeX, and Beamer

TeX is a programming language for the typesetting of documents. It was created by Donald Erwin Knuth in the late 1970s and it is documented in *The TeXbook* [1].
In the early 1980s, Leslie Lamport created the initial version of LaTeX, a high-level language on top of TeX, which is documented in *LaTeX: A Document Preparation System* [2]. There exists a healthy ecosystem of packages that extend the base functionality of LaTeX; *The LaTeX Companion* [3] acts as a guide through the ecosystem.
In 2003, Till Tantau created the initial version of Beamer, a LaTeX package for the creation of presentations. Beamer is documented in the *User's Guide to the Beamer Class* [4].

Bibliography

TEX, LATEX, and Beamer

[1] Donald E. Knuth. *The TEXbook*. Addison-Wesley, 1984.

[2] Leslie Lamport. *LATEX: A Document Preparation System*. Addison-Wesley, 1986.

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[4] Till Tantau. *User's Guide to the Beamer Class Version 3.01*. Available at `http://latex-beamer.sourceforge.net`.

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Hamiltonian, and Basis, and Rabi Oscillations

Hamiltonian in Background Matter Basis

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \delta\lambda(x)\cos 2\theta_{\mathrm{m}}\right)\sigma_3 - \frac{\delta\lambda(x)}{2}\sin 2\theta_{\mathrm{m}}\sigma_1 .$$

Matter profile

$$\lambda(x) = \lambda_0 + A\cos(kx),$$

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \cos 2\theta_{\mathrm{m}} A\cos(kx)\right)\sigma_3 - \frac{\sin 2\theta_{\mathrm{m}}}{2} A\cos(kx)\sigma_1 .$$

Stimulated Neutrino Oscillations

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

Basis

Background matter basis: Hamiltonian is diagonalized with only background matter profile λ_0,

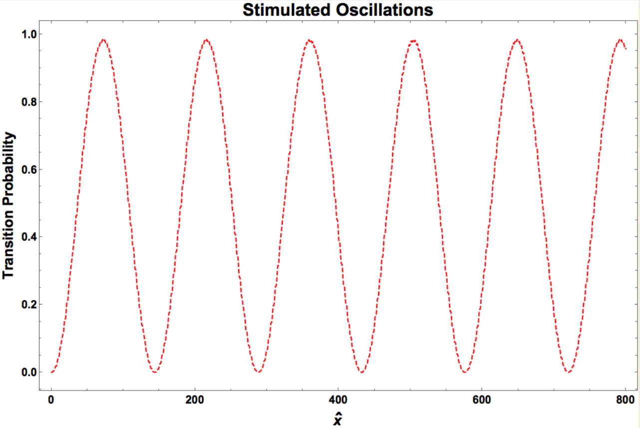
$$\mathrm{H}_{\mathrm{background}} = -\frac{\omega_{\mathrm{m}}}{2}\boldsymbol{\sigma_3}.$$

Hamiltonian

$$\mathbf{H} = \frac{1}{2}\left(-\omega_{\mathrm{m}} + \delta\lambda(x)\cos 2\theta_{\mathrm{m}}\right)\boldsymbol{\sigma_3} - \frac{\delta\lambda(x)}{2}\sin\theta_{\mathrm{m}}\boldsymbol{\sigma_1}.$$

Stimulated Neutrino Oscillations

P. Krastev and A. Smirnov (1989); J. Kneller et al (2013);
K. Patton et al (2014);



Stimulated oscillations. $\lambda(x) = \lambda_0 + A\sin(kx)$ with $\hat{x} = \omega_{\rm m} x$, $A = 0.1\omega_{\rm m}$, $k = 0.995\omega_{\rm m}$, $\theta_{\rm m} = \pi/6$