Dispersion Relation Gaps and Neutrino Flavor Instabilities in Fast Modes*

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ABSTRACT PLACEHOLDER

I. INTRODUCTION

Neutrino flavor conversions in vacuum are linear effects in Schrodinger equation, in other words, the vacuum Hamiltonian doesn't depend on the state of neutrino flavors. However, neutrinos propagate in dense neutrino media demonstrate highly nonlinear flavor transformations due to forward scattering interactions of neutrinos. Such interactions lead to flavor conversions instabilities. The technique used to investigate the nonlinear neutrino flavor conversion is linear stability analysis [1, 2]. Recent studies by I. Izaguirre, G. Raffelt, and I. Tamborra show that linear stability analysis indicates the dispersion relation [3]. They also showed that dispersion relation can be defined and calculated in linear regime of neutrino flavor conversions. Moreover, they concluded that neutrino flavor conversion instabilities occur within gaps in dispersion relation. We argue that neutrino flavor conversion instabilities is not exactly mapped to gaps in dispersion relation for discrete emission with more than two angles or continuous angular distributions of neutrino emissions. To begin with, we review reference 3 to explain the dispersion relation in Sec. II.

II. DISPERSION RELATION OF NEUTRINO FLAVOR CONVERSION

We consider two-flavor scenario of neutrino oscillations. As an initial condition, all neutrinos and antineutrinos are emitted approximately as electron flavors. For the purpose of linear stability analysis, the single particle density matrix for neutrinos is explicitly defined as

$$\rho = \frac{1}{2}\begin{pmatrix} 1 & \epsilon \\ \epsilon^* & -1 \end{pmatrix}. \qquad (1)$$

To determine the flavor evolution, Liouville equation of neutrinos is used,

$$\mathrm{i}(\partial_t + \mathbf{v}\cdot\boldsymbol{\nabla})\rho = [H, \rho]\,, \qquad (2)$$

where H is the Hamiltonian for neutrino oscillations. Density matrix and equation of motion for antineutrinos are defined in the same maner with the corresponding Hamiltonian for antineutrinos.

In principle, neutrino oscillations Hamiltonian depend on three different contributions, vacuum oscillations H_v, interactions with matter H_m, and interactions with neutrinos themselves $H_{\nu\nu}$. The concentration of this work is on fast neutrino oscillations, which would occur even without neutrino mass differences. However, the vacuum term can always be combined with matter term by redefining new matter potential. Thus neglecting vacuum term doesn't change the formalism of linear stability analysis. Vacuum oscillations term is defined

$$H_\mathrm{v} = -\frac{\omega_\mathrm{v}}{2}\sigma_3, \qquad (3)$$

where $\omega = \frac{\delta m^2}{2}$ with δm^2 being the mass squared difference in this two flavor scenario. Interactions with matter is described by matter potential

$$H_\mathrm{m} = \frac{1}{2}\lambda\sigma_3. \qquad (4)$$

where $\lambda = \sqrt{2}G_\mathrm{F}n_\mathrm{e}$. G_F is the Fermi constant and n_e is the number density of electrons in the background. In order to calculate the neutrino forward scattering, the spectrum of neutrino (antineutrinos) distributions $f_{\nu_\mathrm{e}(\bar{\nu}_\mathrm{e})}(p)$, where p is the four momentum of neutrinos (antineutrinos), is required. Following the definition of electron lepton number $G(\hat{\mathbf{v}})$ in 3, neutrino forward scattering potential is

$$H_{\nu\nu} = \sqrt{2}G_\mathrm{F}\iint \frac{\mathrm{d}\cos\theta'\mathrm{d}\phi'}{4\pi}G(\cos\theta', \phi')v^\mu v'_\mu\rho, \qquad (5)$$

where $v^\mu = (1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)^\mathrm{T}$ in spherical coordinate system.

Linear stability analysis of Eq. (2) for axial symmetric neutrino emission shows that

$$\det\left(\omega\mathrm{I} + \frac{1}{2}\begin{pmatrix} I_0 & 0 & 0 & -I_1 \\ 0 & -\frac{1}{2}(I_0 - I_2) & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(I_0 - I_2) & 0 \\ I_1 & 0 & 0 & -I_2 \end{pmatrix}\right) = 0, \qquad (6)$$

where

$$I_m(\theta) = \int_{\cos\theta_H}^{\cos\theta_L} d\cos\theta G(\theta)\frac{\cos^m\theta}{1 - (|k|/\omega)\cos\theta}. \qquad (7)$$

For convinience, we denote $u = \cos\theta$ later on. Two categories of solutions was found, namely the multi-azimuthal

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angle (MAA) solution and multi-zenith angle (MZA) solution. The MAA solution is only related to instabilities due to azimuthal angle. Its formal solution is

$$\omega = \frac{1}{4}(I_0 - I_2). \tag{8}$$

The MZA solution is related to both azimuthal angle and zenith angle, which has formal solutions

$$\omega = -\frac{1}{4}\left(I_0 - I_2 \pm \sqrt{(I_0 + I_2 - 2I_1)(I_0 + I_2 + 2I_1)}\right). \tag{9}$$

Eq. (6) is equivalent to dispersion relation defined in 3.

HAVE TO EXPLAIN THE IDEA OF GAP AND INSTABILITY HERE.

III. DISCRETE ZENITH ANGLE MODEL

For the purpose of illustration, we consider axial symmetric neutrino emissions with discretized zenith angles θ. In this case, the ELN is independent of azimuth angle ϕ. For neutrino emission with N zenith angles, the ELN spectrum can be written as

$$G(u) = \sum_{i=1}^{N} g_i \delta(u - u_i). \tag{10}$$

For two zenith angles $N = 2$, the MAA solution becomes an equation of hypobola for ω and k, which has asymptotic lines $\omega = ku_i$ for $i = 1, 2$. Mathematically speaking, the hyperbola equation has two solutions of ω (k) for any given real k (ω). The solutions are either real which indicates stable solutions or complex which indicates exponential growth in linear regime. On the other hand, non-existance of real solution of ω (k) for given real k (ω) is equivalent to gap in dispersion relation. Thus the equivalence of gap and instabilities is guaranteed in neutrino emission with two-zenith-angle emission.

However, this conclusion can not be generalized to arbitrary number of emission angles. As an example, we calculate the three-zenith-angle emission configuration.

Place the dispersion relation as well as LSA for three zenith angles here

Emphasize here that the number of solutions is the key.

The forbidden region is determined by the emission angles.

IV. CONTINUOUS SPECTRUM

More realistic models of supernova explosions involves continuous zenith angle ELN spectra. In this section we analyze dispersion relation and instabilities for box-shaped spectra and spectra from Garching group.

A. Instabilities at $\omega \to 0$

$$4 = \bar{I}_0 - \bar{I}_2 \tag{11}$$

$$4 = \frac{1}{k} \int G(u) \frac{1 - u^2}{\omega/k - u} \tag{12}$$

$$k_R = \frac{1}{4}\left(P \int G(u) \frac{1 - u^2}{-u}\right) \tag{13}$$

$$k_I = \frac{\pi}{4} G(0)\, \mathrm{Sign}\,(\omega)\, \mathrm{Sign}\,(\mathrm{Im}(k)) \tag{14}$$

$$|k_I| = \frac{\pi}{4} G(0)\, \mathrm{Sign}\,(\omega)\,. \tag{15}$$

V. CONCLUSION

Appendix A: Plan of the paper

- ~~Review fast mode oscillations~~
- State what has been done in Raffelt's paper.
- The conclusion is not true.
- Two beams examples to prove that the number of solutions is the key.
- Show that the continuous case is not related to gap at all. Box spectrum?
- Continuous spectrum or Garching group, data to show that we can prove the location of the instability.

But I have a question. Is it really reliable? Should I use principal value integral for box spectrum DR?

We are still not crystal clear about the relation between gap and lsa.

[1] A. Banerjee, A. Dighe, and G. Raffelt, Physical Review D **84**, 053013 (2011), arXiv:1107.2308 [hep-ph].

[2] G. Raffelt, S. Sarikas, and D. D. S. Seixas, Physical Review Letters **111**, 091101 (2013).

[3] I. Izaguirre, G. Raffelt, and I. Tamborra, Physical Review Letters **118**, 021101 (2017), arXiv:1610.01612.