

Dispersion Relation and Neutrino Flavor Instabilities in Fast Modes*

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ABSTRACT PLACEHOLDER

I. INTRODUCTION

Neutrino flavor conversions in vacuum are linear effects in Schrodinger equation. In dense neutrino media, neutrinos demonstrate highly nonlinear flavor transformations due to forward scattering interactions of neutrinos. Such interactions lead to flavor conversions of different category from vacuum oscillations. The technique used to investigate the nonlinear effect is linear stability analysis.[1, 2] Recent studies by I. Izaguirre, G. Raffelt, and I. Tamborra show that neutrino flavor conversion instabilities is related to gaps in dispersion relation.[3] They showed that dispersion relation can be defined and calculated in linear regime of neutrino flavor conversions. We argue that neutrino flavor conversions instabilities are not exactly mapped to gaps in dispersion relation for continuous angular distributions of neutrino emissions. In Sec. II we review the formalism of linear stability analysis and dispersion relation.

II. DISPERSION RELATION OF NEUTRINO FLAVOR CONVERSION

We consider two-flavor scenario of neutrino oscillations. As an initial condition, all neutrinos and antineutrinos are emitted approximately as electron flavors. For the purpose of linear stability analysis, the single particle density matrix for neutrinos is explicitly defined as

$$\rho = \frac{1}{2} \begin{pmatrix} 1 & \epsilon \\ \epsilon^* & -1 \end{pmatrix}. \quad (1)$$

To determine the flavor evolution, Liouville equation of neutrinos is used,

$$i(\partial_t + \mathbf{v} \cdot \nabla)\rho = [H, \rho], \quad (2)$$

where H is the Hamiltonian for neutrino oscillations. Density matrix and equation of motion for antineutrinos are defined in the same maner with the corresponding Hamiltonian for antineutrinos.

In principle, neutrino oscillations Hamiltonian depend on three different contributions, vacuum oscillations H_v ,

interactions with matter H_m , and interactions with neutrinos themselves $H_{\nu\nu}$. The concentration of this work is on fast neutrino oscillations, which would occur even without neutrino mass differences. However, the vacuum term can always be combined with matter term by redefining new matter potential. Thus neglecting vacuum term doesn't change the formalism of linear stability analysis. Vacuum oscillations term is defined

$$H_v = -\frac{\omega_v}{2}\sigma_3, \quad (3)$$

where $\omega = \frac{\delta m^2}{2}$ with δm^2 being the mass squared difference in this two flavor scenario. Interactions with matter is described by matter potential

$$H_m = \frac{1}{2}\lambda\sigma_3. \quad (4)$$

where $\lambda = \sqrt{2}G_F n_e$. G_F is the Fermi constant and n_e is the number density of electrons in the background. In order to calculate the neutrino forward scattering, the spectrum of neutrino (antineutrinos) distributions $f_{\nu_e(\bar{\nu}_e)}(p)$, where p is the four momentum of neutrinos (antineutrinos), is required. Following the definition of electron lepton number $G(\hat{\mathbf{v}})$ in 3, neutrino forward scattering potential is

$$H_{\nu\nu} = \sqrt{2}G_F \iint \frac{d\cos\theta' d\phi'}{4\pi} G(\cos\theta', \phi') v^\mu v'_\mu \rho, \quad (5)$$

where $v^\mu = (1, \sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)^T$ in spherical coordinate system.

Linear stability analysis of Eq. (2) for axial symmetric neutrino emission shows that

$$\det \left(\omega I + \frac{1}{2} \begin{pmatrix} I_0 & 0 & 0 & -I_1 \\ 0 & -\frac{1}{2}(I_0 - I_2) & 0 & 0 \\ 0 & 0 & -\frac{1}{2}(I_0 - I_2) & 0 \\ I_1 & 0 & 0 & -I_2 \end{pmatrix} \right) = 0, \quad (6)$$

where

$$I_m(\theta) = \int_{\cos\theta_H}^{\cos\theta_L} d\cos\theta G(\theta) \frac{\cos^m\theta}{1 - (|k|/\omega)\cos\theta}. \quad (7)$$

For convinience, we denote $u = \cos\theta$ later on. Two categories of solutions was found, namely the multi-azimuthal angle (MAA) solution and multi-zenith angle (MZA) solution. The MAA solution is only related to instabilities due to azimuthal angle. Its formal solution is

$$\omega = \frac{1}{4}(I_0 - I_2). \quad (8)$$

* A footnote to the article title

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The MZA solution is related to both azimuthal angle and zenith angle, which has formal solutions

$$\omega = -\frac{1}{4} \left(I_0 - I_2 \pm \sqrt{(I_0 + I_2 - 2I_1)(I_0 + I_2 + 2I_1)} \right). \quad (9)$$

Eq. (6) is equivalent to dispersion relation defined in 3.

HAVE TO EXPLAIN THE IDEA OF GAP AND INSTABILITY HERE.

III. DISCRETE ZENITH ANGLE MODEL

Axially symmetric two emission angles model is determined by the electron lepton number

$$G(u) = \sum_{i=1}^2 g_i \delta(u - u_i). \quad (10)$$

The MAA solution is an equation of hypobola for ω and k , which as asymptotic lines $\omega = ku_i$ for $i = 1, 2$. The hyperbola equation has two solutions for ω given real k or two solutions for k given real ω . The solutions are either real which indicates stable solutions or complex which indicates exponential growth in linear regime. Thus the equivalence of gap and instabilities is guaranteed.

However, this conclusion can not be generalized to arbitrary number of emission angles. As an example, we calculate the three emission angles configuration.

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IV. CONTINUOUS SPECTRUM

$$4 = \bar{I}_0 - \bar{I}_2 \quad (11)$$

$$4 = \frac{1}{k} \int G(u) \frac{1 - u^2}{\omega/k - u} \quad (12)$$

$$k_R = \frac{1}{4} \left(P \int G(u) \frac{1 - u^2}{-u} \right) \quad (13)$$

$$k_I = \frac{\pi}{4} G(0) \text{Sign}(\omega) \text{Sign}(\text{Im}(k)) \quad (14)$$

$$|k_I| = \frac{\pi}{4} G(0) \text{Sign}(\omega). \quad (15)$$

V. CONCLUSION

Appendix A: Plan of the paper

- ~~Review fast mode oscillations~~
- State what has been done in Raffelt's paper.
- The conclusion is not true.
- Two beams examples to prove that the number of solutions is the key.
- Show that the continuous case is not related to gap at all. Box spectrum?
- Continuous spectrum or Garching group, data to show that we can prove the location of the instability.

But I have a question. Is it really reliable? Should I use principal value integral for box spectrum DR?

We are still not crystal clear about the relation between gap and lsa.

[1] A. Banerjee, A. Dighe, and G. Raffelt, Physical Review D **84**, 053013 (2011), arXiv:1107.2308 [hep-ph].
 [2] G. Raffelt, S. Sarikas, and D. D. S. Seixas, Physical Review Letters **111**, 091101 (2013).

[3] I. Izaguirre, G. Raffelt, and I. Tamborra, Physical Review Letters **118**, 021101 (2017), arXiv:1610.01612.