# A Theoretical View of Saturn’s Ring

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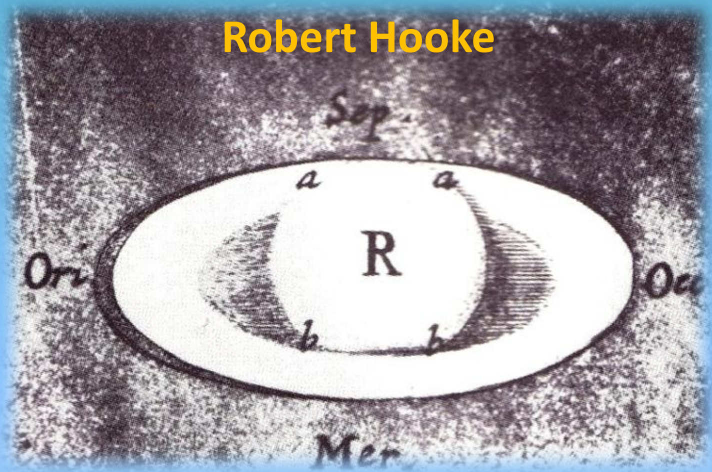
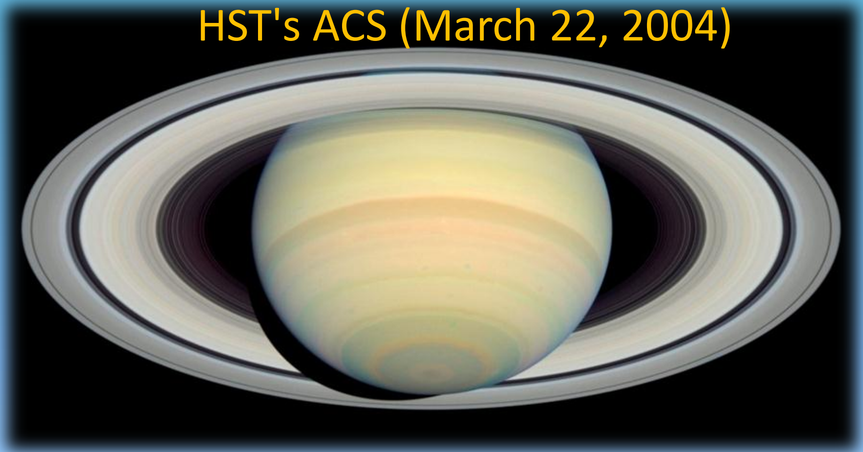
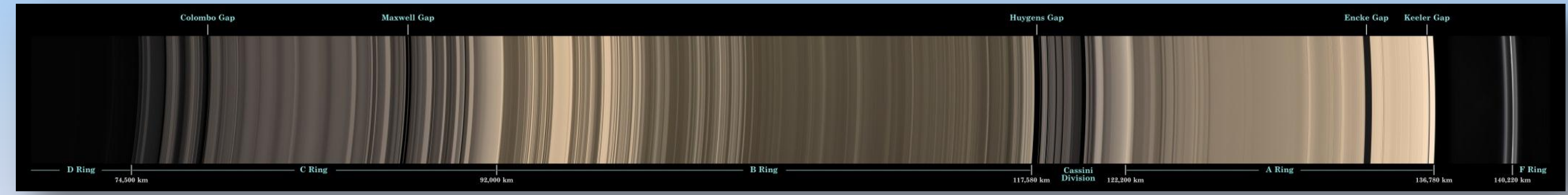
# Characteristics



**Robert Hooke**


HST's ACS (March 22, 2004)

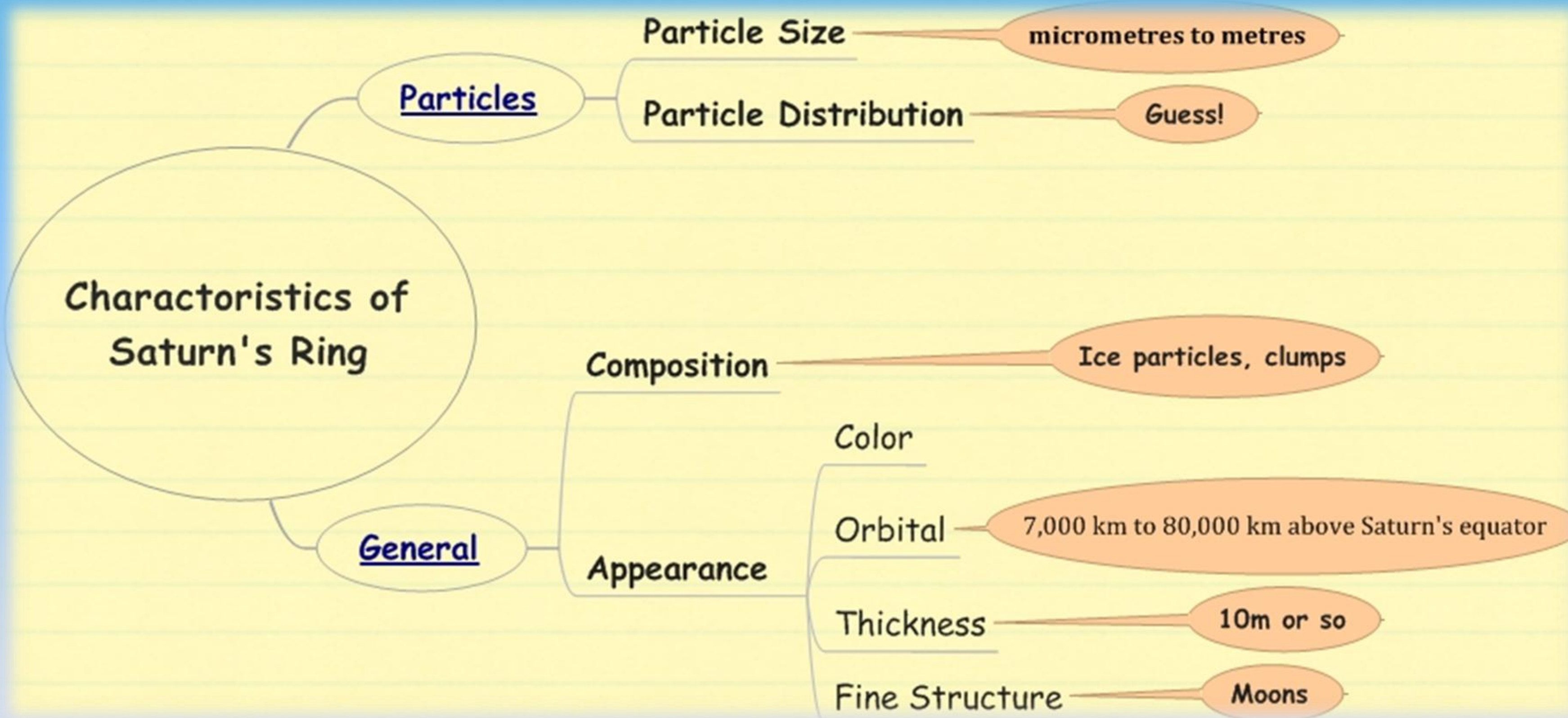
Natural-color mosaic of *Cassini* narrow-angle camera images (May 9, 2007)




Rings of Saturn. Wikipedia. 2010-11-27

# Characteristics

Knowledge we accumulated about Saturn’s Ring in the past centuries




Detailed references will be listed at the end of these slides.

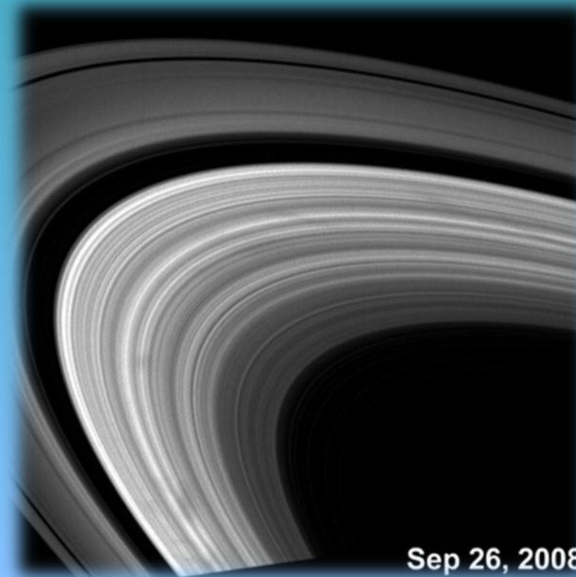
# Characteristics

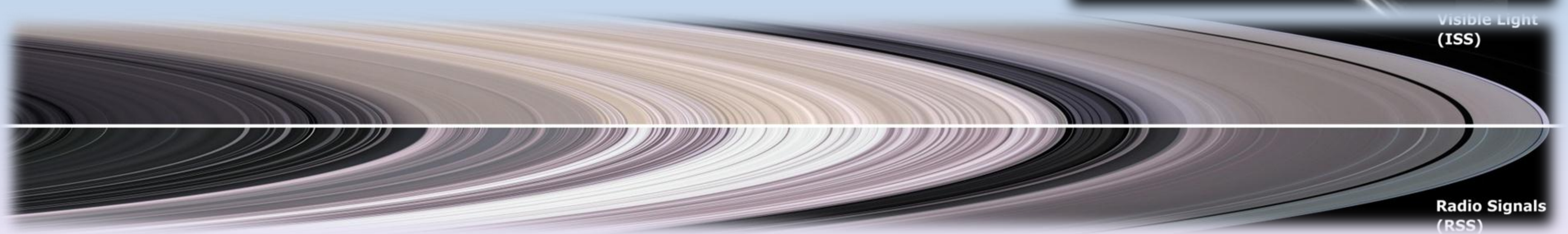
Spokes structure in B ring

Disturbance from Moons

Complete structure investigation?

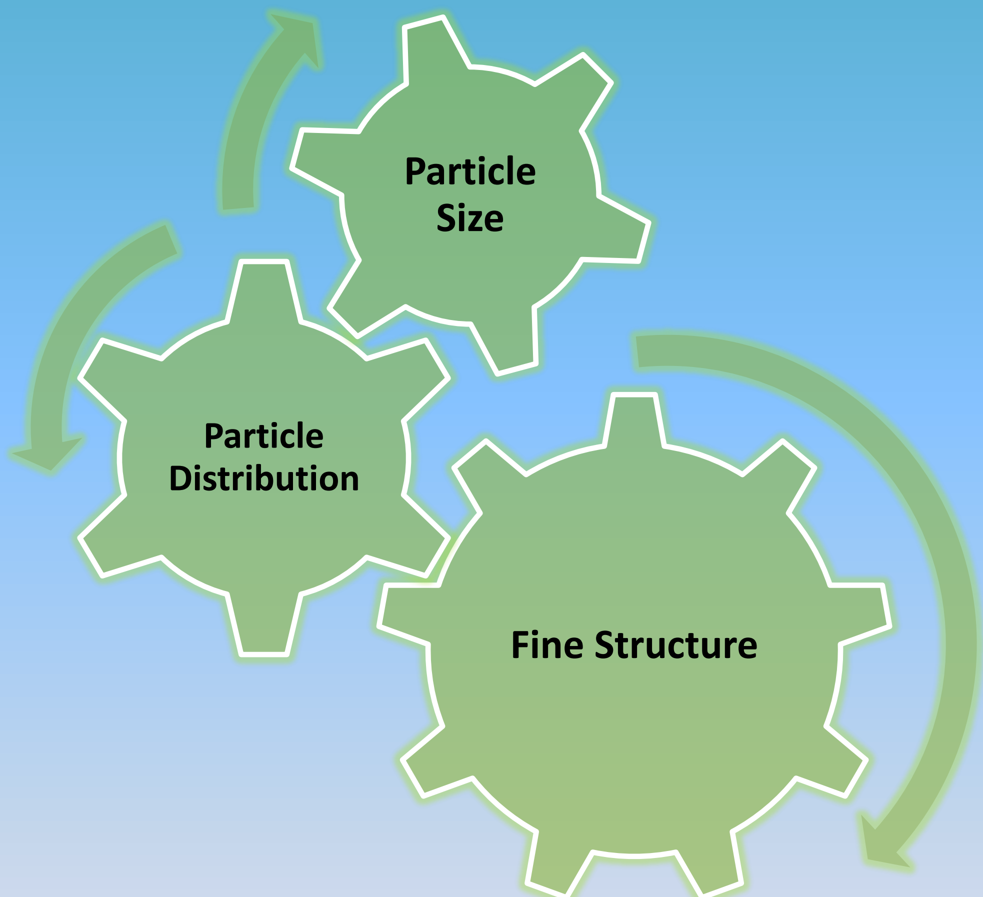
Far from complete!




Rings of Saturn. Wikipedia. 2010-11-27

# Theoretical View




**Most concerned here!**

# Theoretical View

Maximum particle size

Size distribution

Thin

Ringlets

Spokes etc.

Tidal force

Gravitation

CAM

Friction

Collision

Power law

# Theoretical View

**Maximum size**

(Hint: Similar to Roche Limit)

Tensile Force

Tidal Force

$$\frac{32}{19}\frac{GM}{r^3}\rho R < T$$

$$\Rightarrow R < r^3 \times 10^{-19}\,\mathrm{m}^{-2}$$

=> r~10^7m to 10^8m => R<100km

M is the mass of Saturn; r is the orbital radius of the particles; ρ is the density of the particles (ice); R is the maximum size

Peter Goldreich, Scott Tremaine, (1982) "The dynamics of planetary rings". Ann. Rev. Astron. Astrophys. 20: 249-83.

# Theoretical View

## Size distribution

Assumption: Power law form

$$\frac{dN(R)}{d(\ln R)} = \frac{1}{R_0^2}\left(\frac{R}{R_0^2}\right)^{-p}, R_{\min} < R < R_{\max}$$

*N*(*R*) is the number of particles per unit area with radii less than R; $R_0$ is a scaled radius.

Considering the normalization condition, optical depth observation, surface density observation, occultation exp. et al., Goldreich reached a result of $R_{min}$=13cm, $R_{max}$=200m.

Richard G. French & Philip D. Nicholson (2000) did a great work on the particle size problem.

$$\frac{n}{n_0} = \left(\frac{a}{a_0}\right)^{-q}, a_{\min} < a < a_{\max}$$

Peter Goldreich, Scott Tremaine, (1982) "The dynamics of planetary rings". Ann. Rev. Astron. Astrophys. 20: 249-83.

# Theoretical View

Thickness

| | |
|---|---|
| Thin ring | momentum conservation & gravitational force |
| Thickness | Viral theorem $\left(\left\langle z^2\right\rangle = \left\langle v_z^2\right\rangle / \Omega^2\right)$: $\left\langle z^2\right\rangle^{1/2} \sim 100R$, in which R is the particle size. |

Peter Goldreich, Scott Tremaine, (1982) "The dynamics of planetary rings". Ann. Rev. Astron. Astrophys. 20: 249-83.

# Theoretical View

## Ringlets (instability)

Many ways to investigate the dynamics of rings. A simple model is to treat the rings as fluid. Fluid mechanics would be useful!

Particle drift flux at radius R: $\dot{n} = -2/(\Omega R)\,\partial g/\partial R$

In which $g \equiv 3\pi\Omega R^2 \Sigma \nu$

\Omega is the orbital angular frequency; \niu is the kinematic viscosity; \Sigma is the surface number density.

Adopt power law hypothesis $\sigma \propto \Sigma^{\delta}$

\sigma is the velocity dispersion.

D. N.C. Lin & P. Bodenheimer, (1981) “On The Stability of Saturn’s Rings”. The Astro. J. **248**:L83-L86

# Theoretical View

$$\dot{n} = -2/(\Omega R)\,\partial g/\partial R$$

## Ringlets (instability)

Rewrite the drift fulx $$\dot{n} = B\,\partial\Sigma/\partial R$$

where $$B \equiv -2/(\Omega R)\,\partial g/\partial\Sigma$$

$$\partial g/\partial\Sigma > 0 \Rightarrow B < 0 \Rightarrow \text{stable}$$
$$\partial g/\partial\Sigma < 0 \Rightarrow B > 0 \Rightarrow \text{instable}$$

**Instable states exists!**

D. N.C. Lin & P. Bodenheimer, (1981) “On The Stability of Saturn’s Rings”. The Astro. J. **248**:L83-L86

# Theoretical View

$$\dot{n} = B\,\partial\Sigma/\partial R$$

$$B \equiv -2/(\Omega R)\,\partial g/\partial\Sigma$$

Ringlets (instability)

That formula actually tells us nothing! We have to do some approximation.

$$\partial g/\partial\Sigma \propto \partial\left(\Sigma\nu\right)/\partial\Sigma = \sigma_0^{2\delta-1}\,\partial\left(\Sigma^{2\delta}\left(1+\tau^{-2}\right)^{-1}\right)\Big/\partial\Sigma$$

\niu is substituted by $\nu = \sigma^2\Big/\left(\Omega\left(\tau+\tau^{-1}\right)\right)$ $\quad \tau = \Sigma A$

A is the cross section of a typical particle in EM scattering.

$$\delta < 0, \tau > 1 \Rightarrow \text{unstable} \qquad \Sigma \to 0 \Rightarrow stable$$

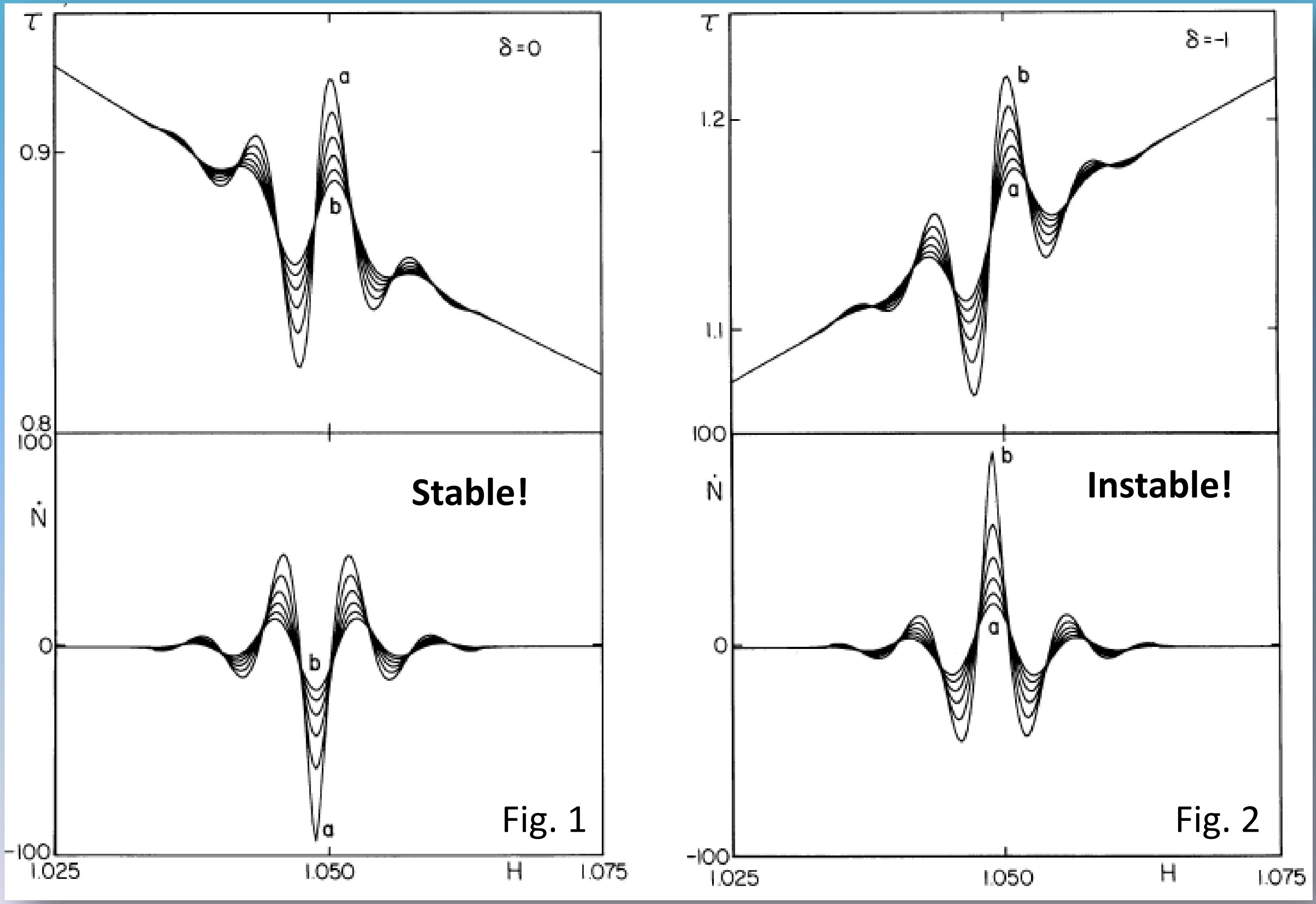
$$\delta < -1, \tau << 1 \Rightarrow \text{unstable} \qquad \delta \to \infty \Rightarrow stable$$

**Stability is connected with the power-law exponent!**

**A more precise work is done in Lin's paper!**

D. N.C. Lin & P. Bodenheimer, (1981) "On The Stability of Saturn's Rings". The Astro. J. **248**:L83-L86

# Theoretical View

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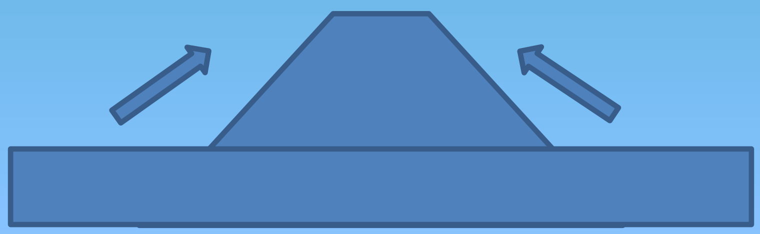
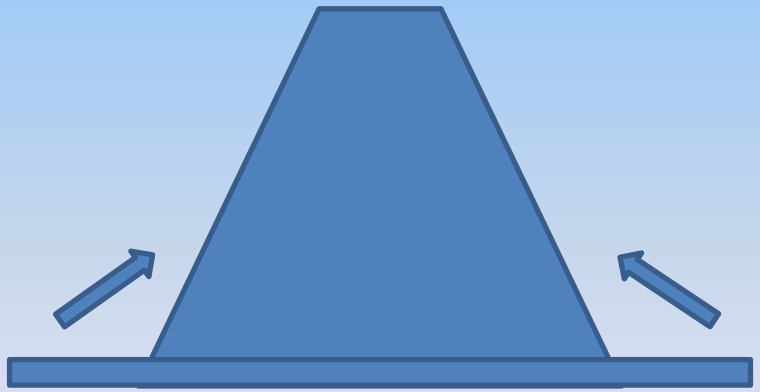
# Theoretical View



Less dissipation!
Or less production of entropy!



D. N.C. Lin & P. Bodenheimer, (1981) “On The Stability of Saturn’s Rings”. The Astro. J. **248**:L83-L86

# Theoretical View

Ringlets (instability)

Key points here:

1. Fluid mechanic method.
2. Viscous couple.
3. Power law assumption.

Things need to be investigated:

1. Time dependent stability
2. Non-axisymmetric perturbations
3. Origin of power law
4. Lifetime of the ring (another kind of stability)

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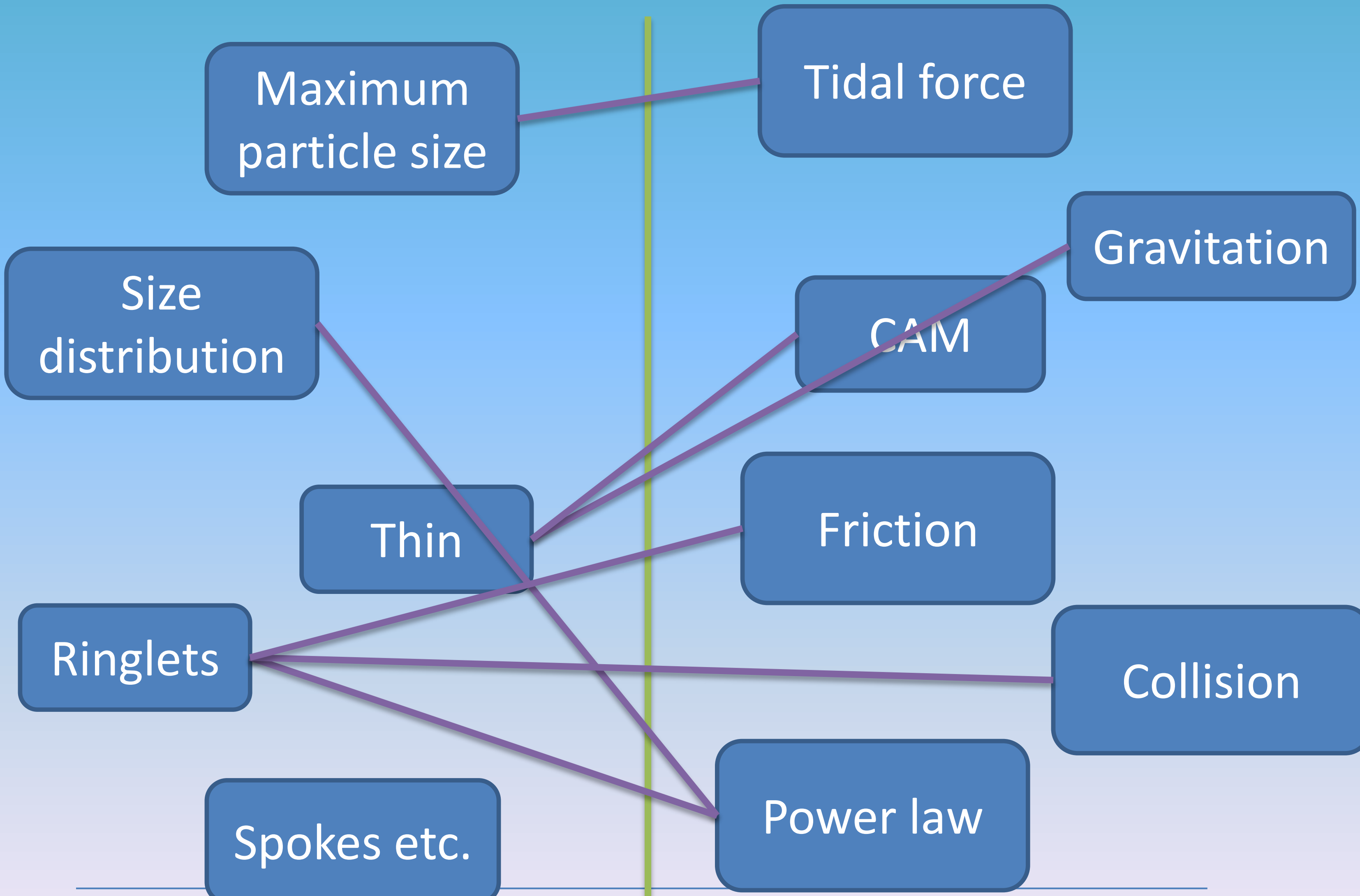
# Conclusion?




D. N.C. Lin & P. Bodenheimer, (1981) “On The Stability of Saturn’s Rings”. The Astro. J. **248**:L83-L86

# Acknowledgement

Thanks to

Luo Yin and Fu Zhaoming for their helpful discussions on computer simulation,

Professor Zhou for his inspiring lectures.

# More Ref. of The Characteristcs

A. Micro properties, i.e., details about those particles.
1. Particle size.
Particles vary from micrometres to metres.[http://www.nasa.gov/worldbook/saturn_worldbook.html].
2. Particle distribution.
The rings are roughly less denser and possess smaller particles when their orbital radii become smaller. But it is really a complex distribution if we look into its fine structure.
3. Particle ingredients.
Those particles are mainly composed of ice with little tholins or silicates. [Nicholson, P.D. and 16 co-authors (2008). "A close look at Saturn's rings with Cassini VIMS" (http:/ / adsabs. harvard. edu/ abs/ 2008Icar.
193. . 182N). Icarus 193: 182\[Dash]212. doi:10.1016/j.icarus.2007.08.036. .]
B. Marcro properties.
1. Color.
It is generally ashen seen from  Cassini spacecraft's natural color photo obtained on May 9, 2007. [http://photojournal.jpl.nasa.gov/catalog/PIA08389]
2. Optical depth.
This is important because we can get infomation about the general distribution of matter by testing it. The outcome is that the ring is subdivided into ringlets which are of high optical depth and gaps which are of low optical depth.
3. Thickness.
It is 10m or so which means the ring is one particle thick disk. [Lane, A.L.. et al. Science 215, 537-543 (1982)][Zebker, H. A. & Tyler, G. L. Science (1984)][Cornell University News Service (2005-11-10). "Researchers Find Gravitational Wakes In Saturn's Rings" (http:/ / www. sciencedaily. com/
releases/ 2005/ 11/ 051110220809. htm). ScienceDaily. . Retrieved 2008-12-24.
]
4. Orbital.
Typical radius of rings are about 1.2*10^8 meters and typical angular speed in rings is 3*10^-4 s^-1.[Null (1976)] Most of the rings are almost perfect circles except a few rings. [Porco, C.; Nicholson, P. D.; Borderies, N.; Danielson, G. E.; Goldreich, P.; Holberg, J. B.; Lane, A. L. (October 1984). "The Eccentric
Saturnian Ringlets at 1.29RS and 1.45RS". Icarus (Elsevier Science) 60 (1): 1\[Dash]16. doi:10.1016/0019-1035(84)90134-9.][Porco, C. C.; Nicholson, P. D. (November 1987). "Eccentric features in Saturn's outer C ring". Icarus (Elsevier Science) 72 (2): 437\[Dash]467.
doi:10.1016/0019-1035(87)90185-0.]
5. disturbance.
This characteristic is important for disturbance indicates additional condensation objects otherwise we have to find out a intrinsic mechanism for this phenomenon such as a theory the same as the one for spiral arm of galaxies. [Hedman, Matthew M.; Burns, Joseph A,; Showalter, Mark R. at al. (2007). "Saturn's dynamic D ring" (http:/ / ciclops. org/ media/ sp/ 2007/
2678_7440_0. pdf) (pdf). Icarus 188: 89\[Dash]107. doi:10.1016/j.icarus.2006.11.017. .][Weiss, J. W.; Porco, C. C.; Tiscareno, M. S. (11 June 2009). "Ring Edge Waves and the Masses of Nearby Satellites" (http:/ / www. iop.
org/ EJ/ abstract/ 1538-3881/ 138/ 1/ 272/ ). The Astronomical Journal (American Astronomical Society) 138 (1): 272\[Dash]286.
doi:10.1088/0004-6256/138/1/272. . Retrieved 2009-06-15.][Porco, C.C.; Baker, E.; Barbara, J., et al. (2005). "Cassini Imaging Science: Initial Results on Saturn\[CloseCurlyQuote]sRings and Small Satellites" (http:/ /
ciclops. org/ sci/ docs/ RingsSatsPaper. pdf) (pdf). Science 307 (5713): 1226\[Dash]1236. doi:10.1126/science.1108056. PMID 15731439. .](This paper is very important.) It is unexplained that the B ring has a spokes structure.