

1. General

1.1 Units and Conventions

1. Energy and temperature:

$$\frac{1}{40}\text{eV} = 300\text{K}k_B. \quad (1)$$

2. Natural units: energy is related to length by

$$1\text{fm} \times 197\text{MeV} = \hbar c = 1. \quad (2)$$

3. For light, energy 1eV corresponds to wavelength $1.24\mu\text{m}$.

4. Pauli matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3)$$

5. Transformations of Pauli matrices

$$\mathbf{U}^\dagger \sigma_3 \mathbf{U} = \cos 2\theta \sigma_3 - \sin 2\theta \sigma_1 \quad (4)$$

$$\mathbf{U}^\dagger \sigma_1 \mathbf{U} = -\sin 2\theta \sigma_3 + \cos 2\theta \sigma_1 \quad (5)$$

$$\mathbf{U}^\dagger \sigma_2 \mathbf{U} = \sigma_2. \quad (6)$$

1.2 Check Results

- Are the dimensions correct?
- Does the limits of the result make sense?
- Does the result make sense when the complexity of the system is removed?
- Is it the right basis to draw a conclusion?
- Have you double checked analytical expression with other people or Mathematica?
- Have you checked numerical results with previously calculated simple results?

2. Field Theory

2.1 Equations of Motion

1. Dirac Equation

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0 \quad (7)$$

2. Klein Gordon Equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0 \quad (8)$$

3. Neutrinos

3.1 Fundamental Parameters

1. Mixing angles

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024 \quad (9)$$

$$\sin^2 2\theta_{23} > 0.95 \quad (10)$$

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010 \quad (11)$$

$$(12)$$

2. Masses (fig 1)

$$\Delta m_{12}^2 = \Delta m_{sol}^2 = 7.53^{+0.18}_{-0.18} \times 10^{-5}\text{eV}^2 \quad (13)$$

$$|\Delta m_{31}^2| = \Delta m_{atm}^2 = 2.44^{+0.06}_{-0.06} \times 10^{-3}\text{eV}^2 \quad (14)$$

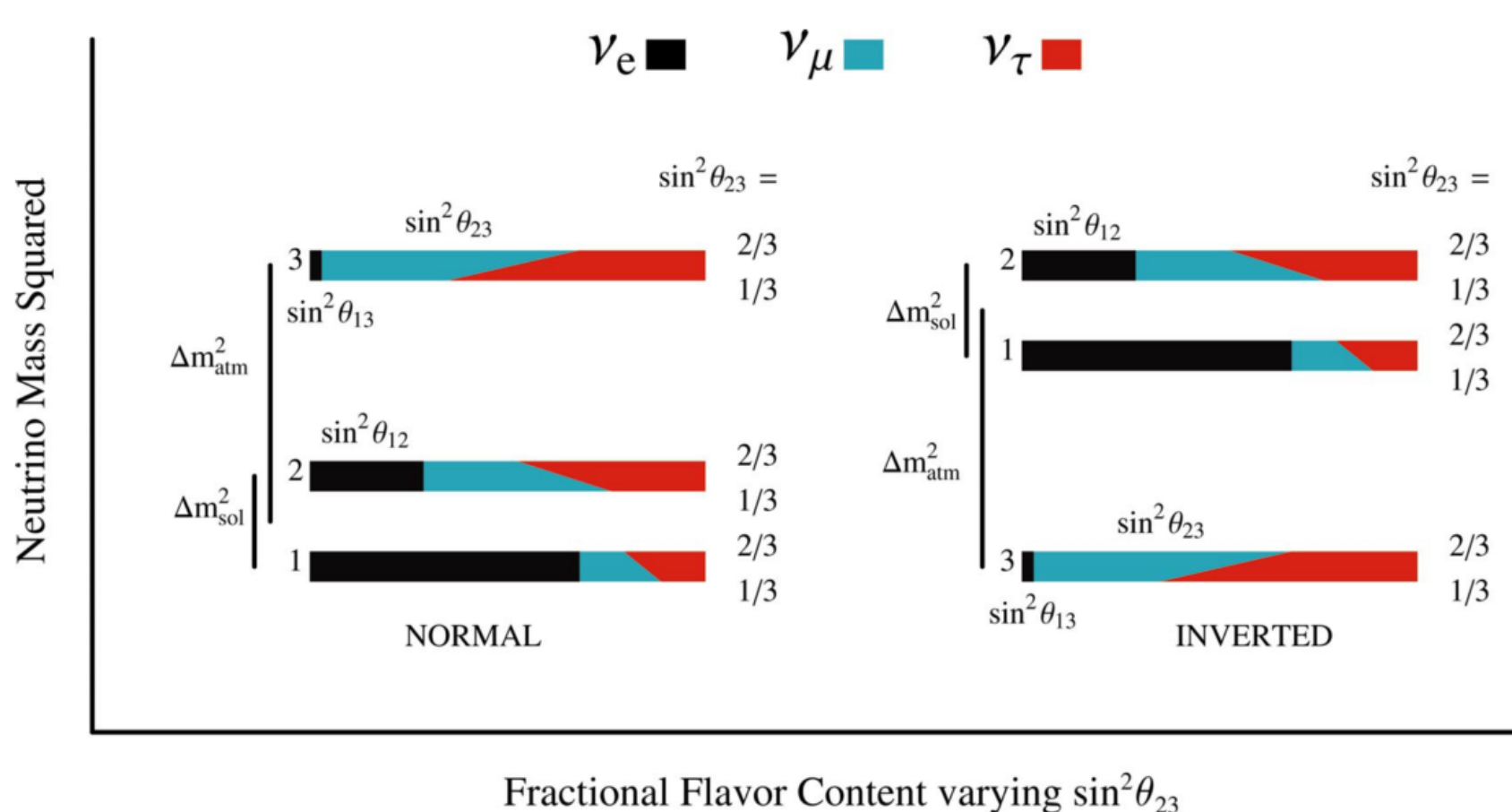


Figure 1: Neutrino mass hierarchy. Mena, O., & Parke, S. (2004). Unified graphical summary of neutrino mixing parameters. Physical Review D, 69(11), 117301.

3.2 Nuclear Reactions

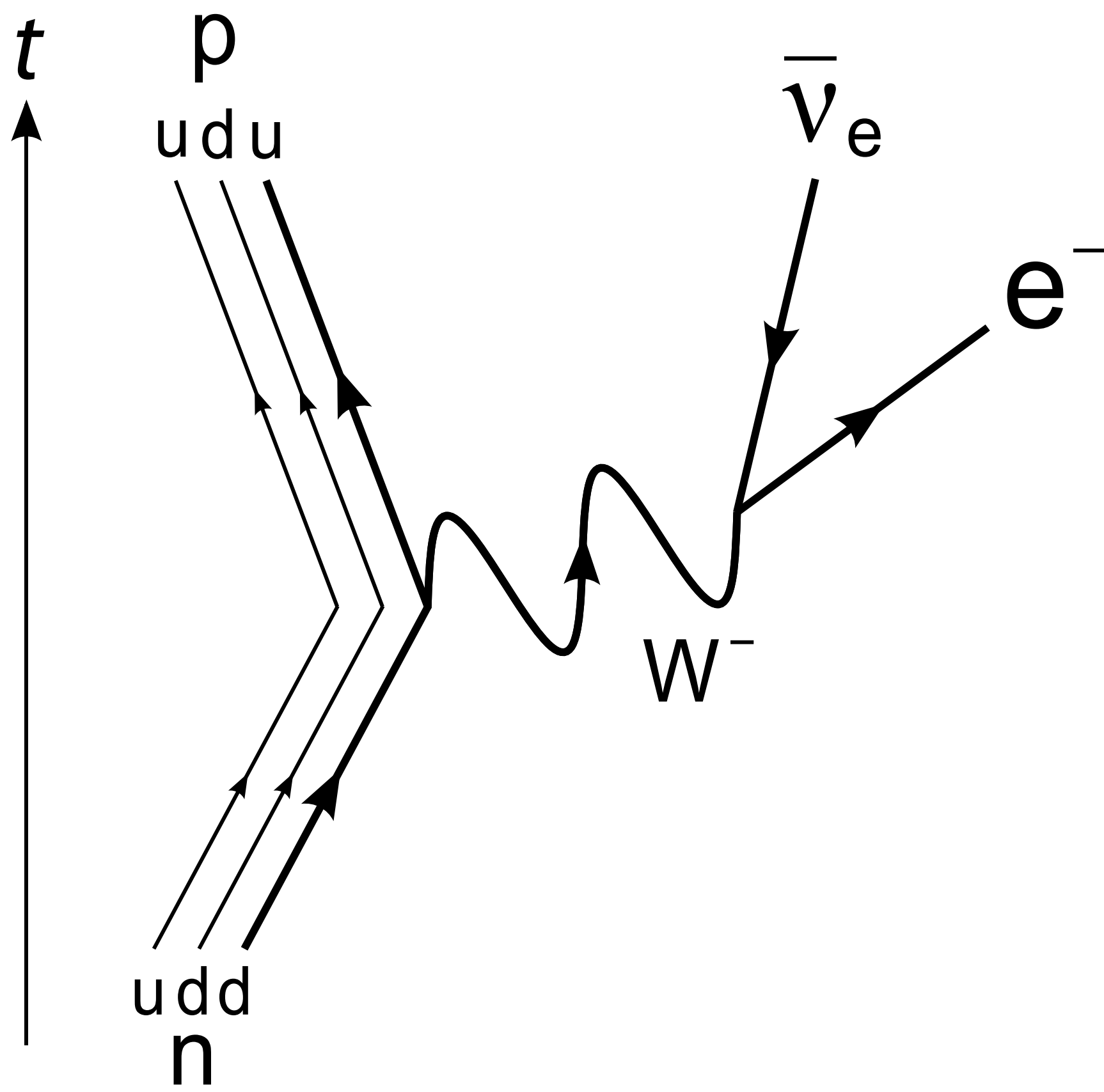


Figure 2: Feynman diagram of beta decay. The charged current weak interaction boson in this case is a W^- . Credit: Joel Holdsworth, within public domain.

Reaction	Equation	Boson
Electron emission	$\overset{A}{Z}X \rightarrow \overset{A}{Z+1}X + e^- + \bar{\nu}_e$	W^-
Positron emission	$\overset{A}{Z}X \rightarrow \overset{A}{Z-1}X + e^+ + \nu_e$	W^+
Electron capture	$\overset{A}{Z}X + e^- \rightarrow \overset{A}{Z-1}X + \nu_e$	W^+
Positron capture	$\overset{A}{Z}X + e^+ \rightarrow \overset{A}{Z+1}X + \bar{\nu}_e$	W^-
Electron annihilation	$e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e$	W^-
Electron annihilation	$e^- + e^+ \rightarrow \nu + \bar{\nu}$	Z
Neutrino capture	$\overset{A}{Z}X + \overset{(-)}{\nu}_e \rightarrow \overset{A}{Z\mp 1}X + e^\pm$	W
$e^- \nu$ scattering	$e^- + \overset{(-)}{\nu}_e \rightarrow e^- + \overset{(-)}{\nu}_e$	W^-
$e^- \nu$ scattering	$e^\pm + \overset{(-)}{\nu}_e \rightarrow e^\pm + \overset{(-)}{\nu}_e$	Z
Neutrino scattering	$\overset{A}{Z}X + \overset{(-)}{\nu} \rightarrow \overset{A}{Z}X + \overset{(-)}{\nu}$	Z
Bremsstrahlung	$N + N \rightleftharpoons N + N + \nu + \bar{\nu}$	
Annihilation	$e^+ e^- \rightleftharpoons \nu + \bar{\nu}$	
Neutrino annihilation	$\nu + \bar{\nu} \rightleftharpoons \nu + \bar{\nu}$	

Table 1: Neutrino related nuclear or leptonic reactions

3.3 Neutrino Mixing

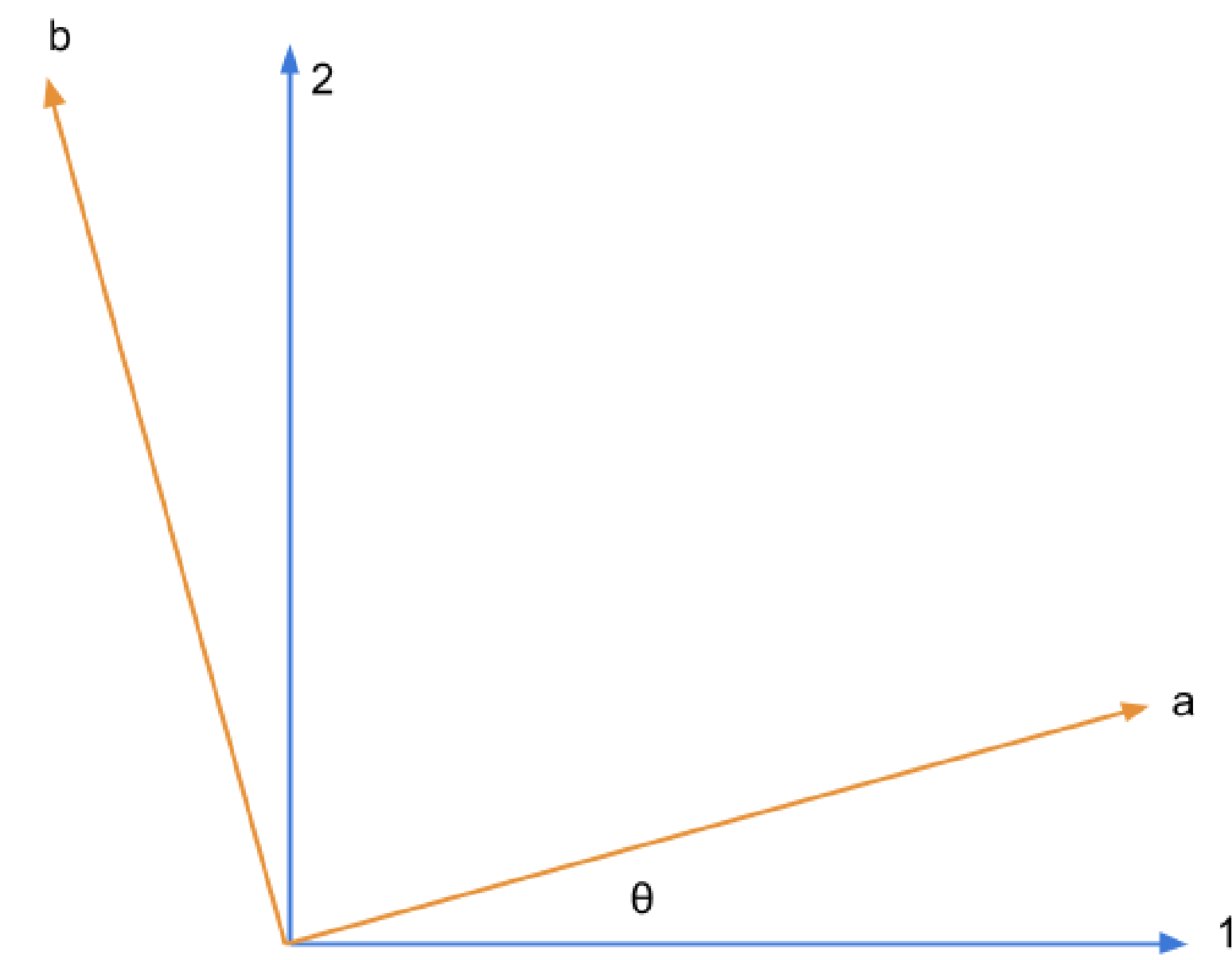


Figure 3: Neutrino mixing. Blue states $\{|1, 2\rangle\}$ are the VACUUM energy basis while the orange states $\{|a, b\rangle\}$ are the flavor basis. Blue: flavor basis; Red: propagation basis.

The two basis are related to each other through a unitary matrix \mathbf{U} ,

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_x\rangle \end{pmatrix} = \mathbf{U} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}, \quad (15)$$

where

$$\mathbf{U} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (16)$$

One thing to notice is that the relation of the amplitude has the same transformation

$$\begin{pmatrix} \psi_e \\ \psi_x \end{pmatrix} = \mathbf{U} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (17)$$

where the amplitude is define as the components of a state in a certain basis.

$$|\Psi\rangle = (\psi_e \ \psi_x) \begin{pmatrix} |\nu_e\rangle \\ |\nu_x\rangle \end{pmatrix} = (\psi_1 \ \psi_2) \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}. \quad (18)$$

3.4 Hamiltonian and Basis

3.4.1 Basis

Rotation from a Hamiltonian diagonalized basis wave function Ψ_v to flavor basis wave function Ψ_f is

$$\Psi_f = R_{d2f}(\theta_x)\Psi_d, \quad (19)$$

where d can be v for vacuum eigenbasis, m for matter eigenbasis and

$$R_{d2f}(\theta_x) = \begin{pmatrix} \cos \theta_x & \sin \theta_x \\ -\sin \theta_x & \cos \theta_x \end{pmatrix}. \quad (20)$$

Matter mixing angle θ_m is determined through

$$\sin 2\theta_m = \frac{\sin 2\theta_v}{\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v} \cos 2\theta_v}}, \quad (21)$$

$$\cos 2\theta_m = \frac{\cos 2\theta_v - \lambda/\omega_v}{\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v} \cos 2\theta_v}}, \quad (22)$$

$$\tan 2\theta_m = \frac{\sin 2\theta_m}{\cos 2\theta_m - \lambda/\omega_v}, \quad (23)$$

where $\lambda = \sqrt{2}G_F n_e$.

3.4.2 Hamiltonian

With the appearance of matter perturbation $\lambda(x) = \lambda_0 + \delta\lambda(x)$ to the system, we have the Hamiltonian in vacuum basis

$$\mathbf{H}_v = -\frac{\omega_v}{2}\sigma_3 + \frac{\lambda(x)}{2}\cos 2\theta_v\sigma_3 + \frac{\lambda(x)}{2}\sin 2\theta_v\sigma_1. \quad (24)$$

In flavor basis,

$$\mathbf{H}_f = \frac{\omega}{2}(-\cos 2\theta\sigma_3 + \sin 2\theta\sigma_1) + \frac{\lambda(x)}{2}\sigma_3. \quad (25)$$

In background matter potential λ_0 basis

$$\mathbf{H}_{\bar{m}} = -\frac{\omega_m}{2}\sigma_3 + \frac{\delta\lambda(x)}{2}\cos 2\theta_m\sigma_3 - \frac{\delta\lambda}{2}\sin 2\theta_m\sigma_1, \quad (26)$$

where $\omega_m = \omega_v\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v} \cos 2\theta_v}$.

The caveat is the position/time dependence of the matter potential leads to position/time dependent transformation of the wave function which will cause an term in the Hamiltonian of the form

$$-i\mathbf{U}^\dagger(x)\frac{\partial}{\partial x}\mathbf{U}(x). \quad (27)$$

3.4.3 Physics of Neutrino Oscillation

- Nature of neutrino oscillation
- MSW effect
- Collective oscillation
- Gravitation effect on neutrino oscillations
- Kinetic equation ($\nu\bar{\nu}$ pairing correlations?)

3.4.4 Applications of Neutrino Oscillation

- Supernova shock revive
- Accretion disc
- Cosmology
- Tomography
- Detection of nuclear activities

3.4.5 Big Questions

- Are neutrinos Majorana or Dirac?
- What is the mass hierarchy?

4. Numerical

4.1 Writing Code

- Reduce quantities to dimensionless;
- Always calculate the corresponding value of reduced quantities in suitable units.

4.2 Limits

- Simple cases where analytical is possible;
- Verified numerical calculations had previously.

4.3 Zen of Python

import this

4.4 Check You Code (Neutrino)

Check the code step by step:

- Vacuum oscillation amplitude and frequency
- Constant matter potential oscillation amplitude and frequency
- MSW resonance