1. General

1.1 Units and Conventions

1. Energy and temperature:

$$\frac{1}{40}\mathrm{eV} = 300\mathrm{K}k_B. \tag{1}$$

2. Natural units: energy is related to length by

$$1\mathrm{fm} \times 197\mathrm{MeV} = \hbar c = 1. \tag{2}$$

3. For light, energy 1eV corresponds to wavelength $1.24\mu\mathrm{m}$.
4. Pauli matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

1.2 Check Results

1. Are the dimensions correct?
2. Does the limits of the result make sense?
3. Does the result make sense when the complexity of the system is removed?
4. Is it the right basis to draw a conclusion?

2. Field Theory

2.1 Equations of Motion

- Dirac Equation

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0 \tag{4}$$

- Klein Gordon Equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0 \tag{5}$$

3. Neutrinos

3.1 Fundamental Parameters

- Mixing angles

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024 \tag{6}$$
$$\sin^2 2\theta_{23} > 0.95 \tag{7}$$
$$\sin^2 2\theta_{13} = 0.095 \pm 0.010 \tag{8}$$
$$\tag{9}$$

- Masses (fig 1)

$$\Delta m_{12}^2 = \Delta m_{sol}^2 = 7.53_{-0.18}^{+0.18} \times 10^{-5}\mathrm{eV}^2 \tag{10}$$
$$|\Delta m_{31}^2| = \Delta m_{atm}^2 = 2.44_{-0.06}^{+0.06} \times 10^{-3}\mathrm{eV}^2 \tag{11}$$

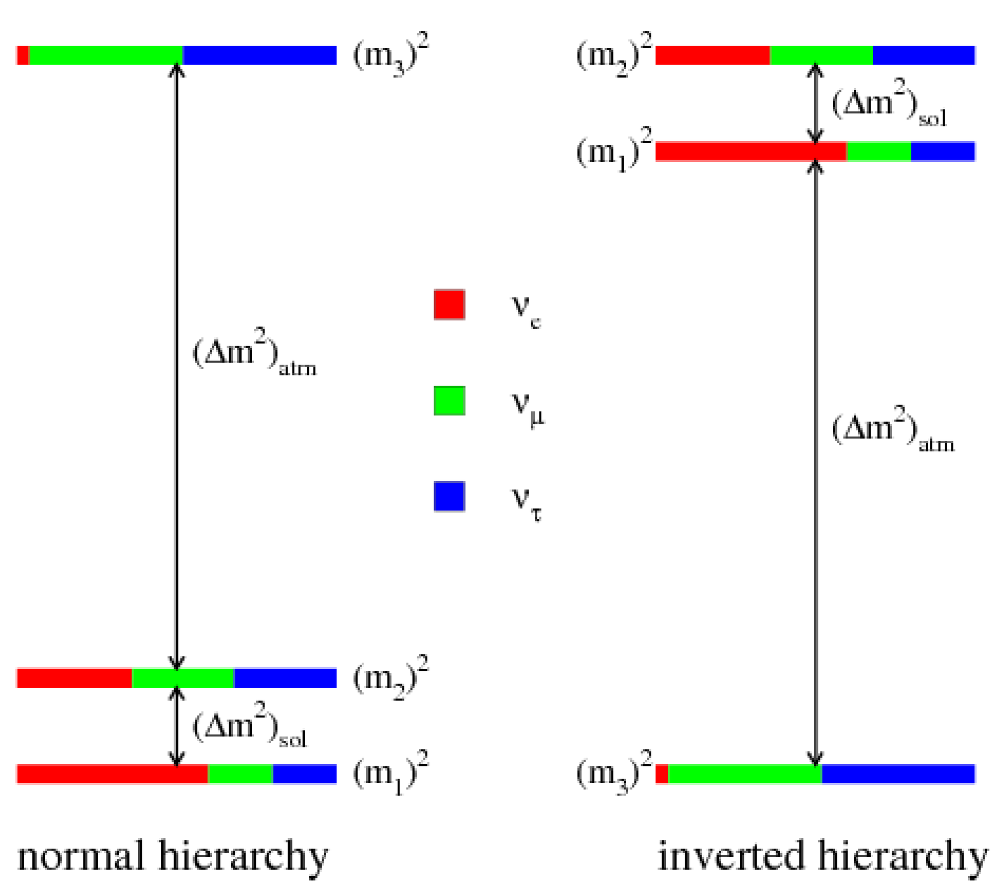

Figure 1: *Neutrino mass hierarchy. J. L. Hewett et al. Fundamental Physics at the Intensity Frontier, arXiv:1205.2671.*

3.2 Nuclear Reactions

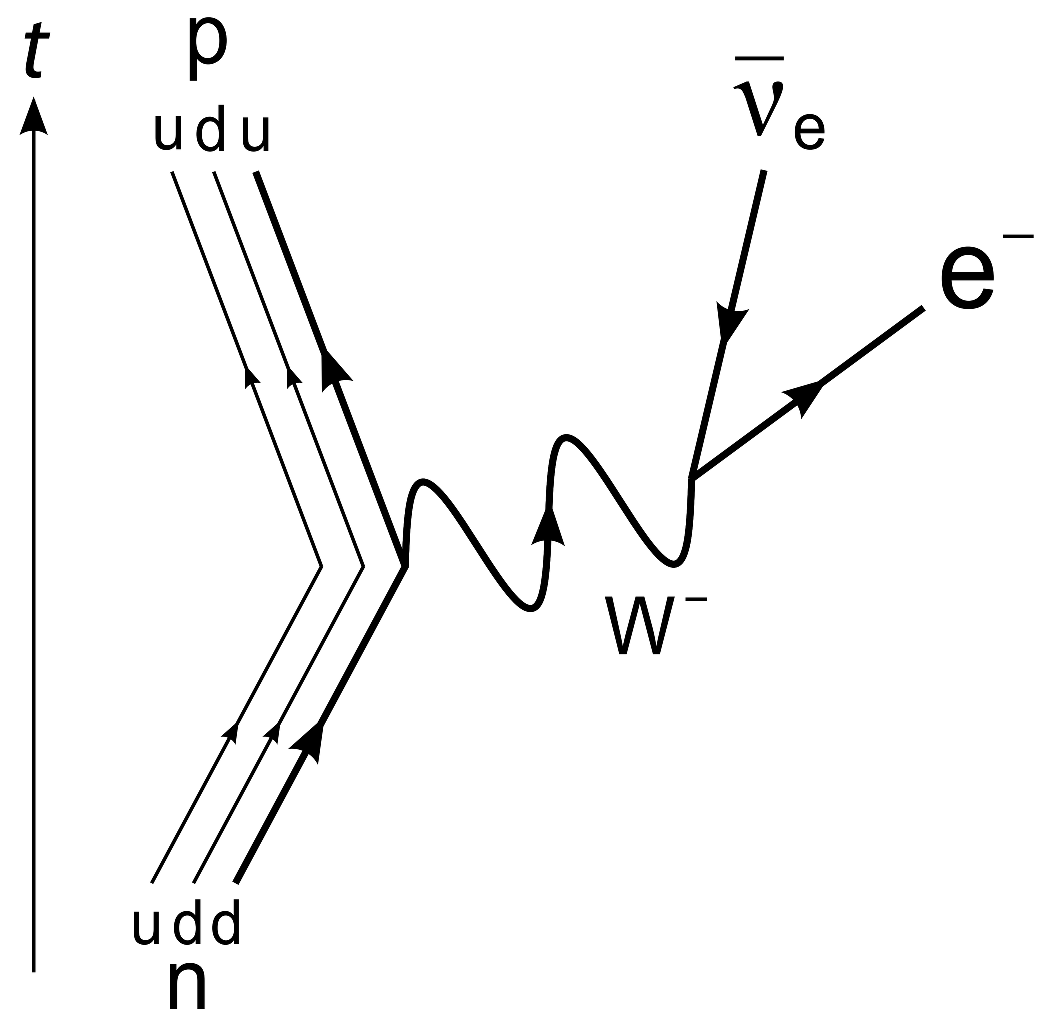

Figure 2: *Feynman diagram of beta decay. The charged current weak interaction boson in this case is a W^-. Credit: Joel Holdsworth, within public domain.*

Reaction	Equation	Boson
Electron emission	${}^A_ZX \to {}^A_{Z+1}X + e^- + \bar{\nu}_e$	W
Positron emission	${}^A_ZX \to {}^A_{Z-1}X + e^+ + \nu_e$	W
Electron capture	${}^A_ZX + e^- \to {}^A_{Z-1}X + \nu_e$	W
Positron capture	${}^A_ZX + e^+ \to {}^A_{Z+1}X + \bar{\nu}_e$	W
Electron annihilation	$e^- + e^+ \to \nu_e + \bar{\nu}_e$	W
Electron annihilation	$e^- + e^+ \to \nu + \bar{\nu}$	Z
Neutrino capture	${}^A_ZX + \overset{(-)}{\nu_e} \to {}^A_{Z\mp1}X + e^\pm$	W
$e^-\nu$ scattering	$e^- + \overset{(-)}{\nu_e} \to e^- + \overset{(-)}{\nu_e}$	W
$e^-\nu$ scattering	$e^\pm + \overset{(-)}{\nu_e} \to e^\pm + \overset{(-)}{\nu_e}$	Z
Neutrino scattering	${}^A_ZX + \overset{(-)}{\nu} \to {}^A_ZX + \overset{(-)}{\nu}$	Z
Bremsstrahlung	$N + N \rightleftharpoons N + N + \nu + \bar{\nu}$	
Annihilation	$e^+e^- \rightleftharpoons \nu + \bar{\nu}$	
Neutrino annihilation	$\nu + \bar{\nu} \rightleftharpoons \nu + \bar{\nu}$	

Table 1: *Neutrino related nuclear or leptonic reactions*

3.3 Neutrino Mixing

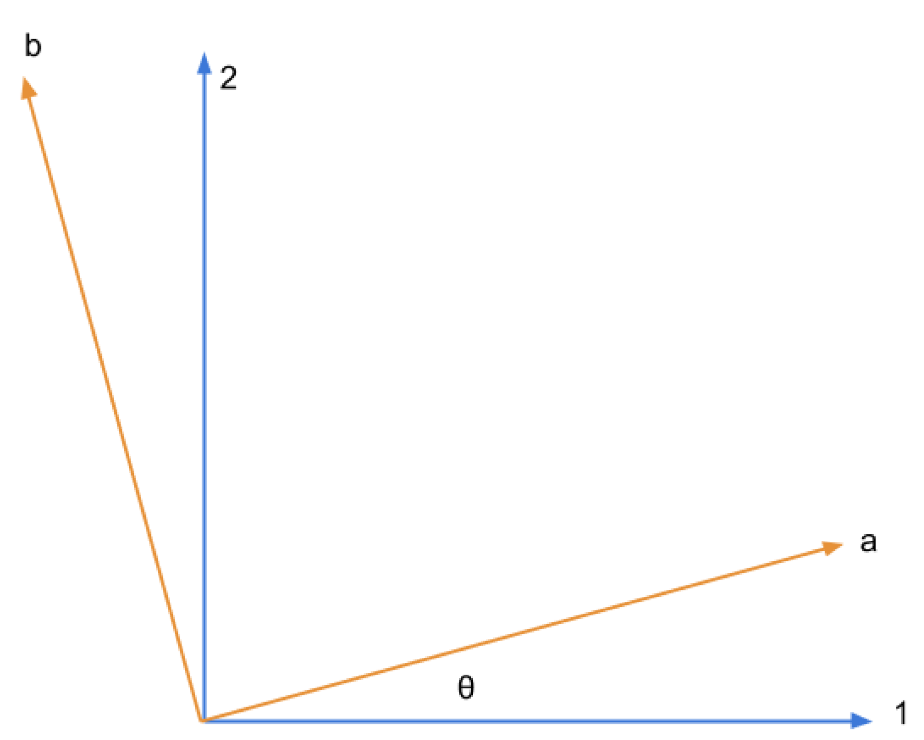

Figure 3: *Neutrino mixing. Blue states ($\{1, 2\}$) are the VACUUM energy eigenstates while the orange states ($\{a, b\}$) are the flavor eigenstates. Blue: flavor states; Red: propagation states.*

3.4 Hamiltonian and Basis

3.4.1 Basis

Rotation from a Hamiltonian diagonalized basis wave function Ψ_v to flavor basis wave function Ψ_f is

$$\Psi_f = R_{d2f}(\theta_x)\Psi_d, \tag{12}$$

where d can be v for vacuum eigenbasis, m for matter eigenbasis and

$$R_{d2f}(\theta_x) = \begin{pmatrix} \cos\theta_d & \sin\theta_d \\ -\sin\theta_d & \cos\theta_d \end{pmatrix}. \tag{13}$$

Matter mixing angle θ_m is determined through

$$\sin 2\theta_m = \frac{\sin 2\theta_v}{\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v}\cos 2\theta_v}}, \tag{14}$$

$$\cos 2\theta_m = \frac{\cos 2\theta_v - \lambda/\omega_v}{\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v}\cos 2\theta_v}}, \tag{15}$$

$$\tan 2\theta_m = \frac{\sin 2\theta_m}{\cos 2\theta_m - \lambda/\omega_v}, \tag{16}$$

where $\lambda = \sqrt{2}G_F n_e$.

3.4.2 Hamiltonian

With the appearance of matter perturbation $\lambda(x) = \lambda_0 + \delta\lambda(x)$ to the system, we have the Hamiltonian in vacuum basis

$$\mathbf{H_v} = -\frac{\omega_v}{2}\sigma_3 + \frac{\lambda(x)}{2}\cos 2\theta_v\sigma_3 + \frac{\lambda(x)}{2}\sin 2\theta_v\sigma_1. \tag{17}$$

In flavor basis,

$$\mathbf{H_f} = \frac{\omega}{2}(-\cos 2\theta\sigma_3 + \sin 2\theta\sigma_1) + \frac{\lambda(x)}{2}\boldsymbol{\sigma_3}. \tag{18}$$

In background matter potential λ_0 basis

$$\mathbf{H_{\tilde{m}}} = -\frac{\omega_m}{2}\sigma_3 + \frac{\delta\lambda(x)}{2}\cos 2\theta_m\sigma_3 - \frac{\delta\lambda}{2}\sin 2\theta_m\sigma_1, \tag{19}$$

where $\omega_m = \omega_v\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v}\cos 2\theta_v}$.

The caveat is the position/time dependence of the matter potential leads to position/time dependent transformation of the wave function which will cause an term in the Hamiltonian of the form

$$-i\mathbf{U}^\dagger(x)\frac{\partial}{\partial x}\mathbf{U}(x). \tag{20}$$

3.4.3 Physics of Neutrino Oscillation

1. Nature of neutrino oscillation
2. MSW effect
3. Collective oscillation

3.4.4 Big Questions

1. Are neutrinos Majorana or Dirac?
2. What is the mass hierarchy?

4. Numerical

4.1 Check You Code

Check the code step by step:

- Vacuum oscillation amplitude and frequency
- Constant matter potential oscillation amplitude and frequency
- MSW resonance

4.2 Zen of Python

```
import this
```

2015-11-10