

1. General

1.1 Units and Conventions

1. Energy and temperature:

$$\frac{1}{40}\text{eV} = 300\text{K}k_B. \quad (1)$$

2. Natural units: energy is related to length by

$$1\text{fm} \times 197\text{MeV} = \hbar c = 1. \quad (2)$$

3. For light, energy 1eV corresponds to wavelength $1.24\mu\text{m}$.

4. Pauli matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3)$$

1.2 Check Results

- Are the dimensions correct?
- Does the limits of the result make sense?
- Does the result make sense when the complexity of the system is removed?
- Is it the right basis to draw a conclusion?
- Have you double checked analytical expression with other people or Mathematica?
- Have you checked numerical results with previously calculated simple results?

2. Field Theory

2.1 Equations of Motion

1. Dirac Equation

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0 \quad (4)$$

2. Klein Gordon Equation

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0 \quad (5)$$

3. Neutrinos

3.1 Fundamental Parameters

1. Mixing angles

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024 \quad (6)$$

$$\sin^2 2\theta_{23} > 0.95 \quad (7)$$

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010 \quad (8)$$

$$(9)$$

2. Masses (fig 1)

$$\Delta m_{12}^2 = \Delta m_{sol}^2 = 7.53^{+0.18}_{-0.18} \times 10^{-5}\text{eV}^2 \quad (10)$$

$$|\Delta m_{31}^2| = \Delta m_{atm}^2 = 2.44^{+0.06}_{-0.06} \times 10^{-3}\text{eV}^2 \quad (11)$$

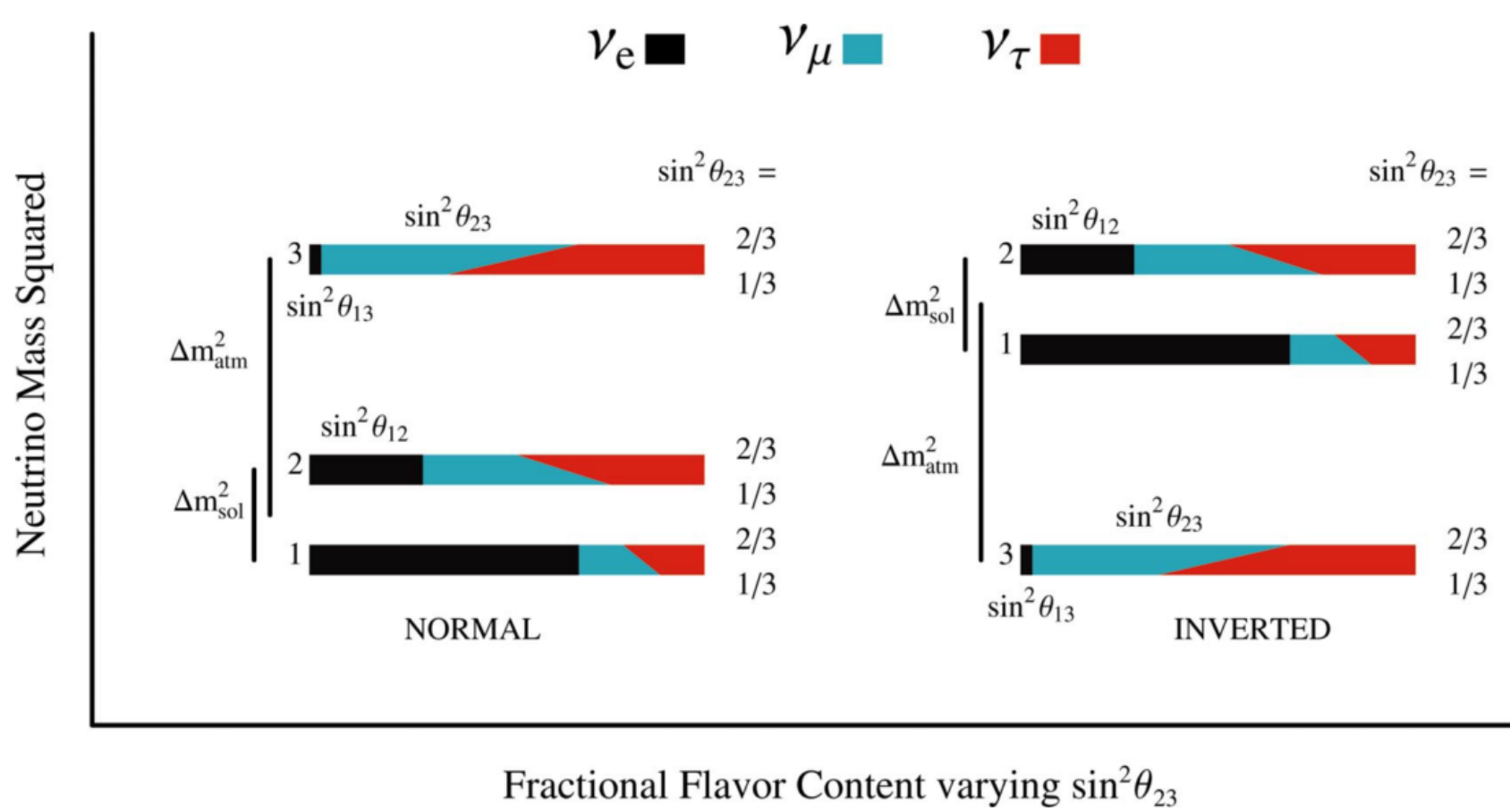


Figure 1: Neutrino mass hierarchy. Mena, O., & Parke, S. (2004). Unified graphical summary of neutrino mixing parameters. Physical Review D, 69(11), 117301.

3.2 Nuclear Reactions

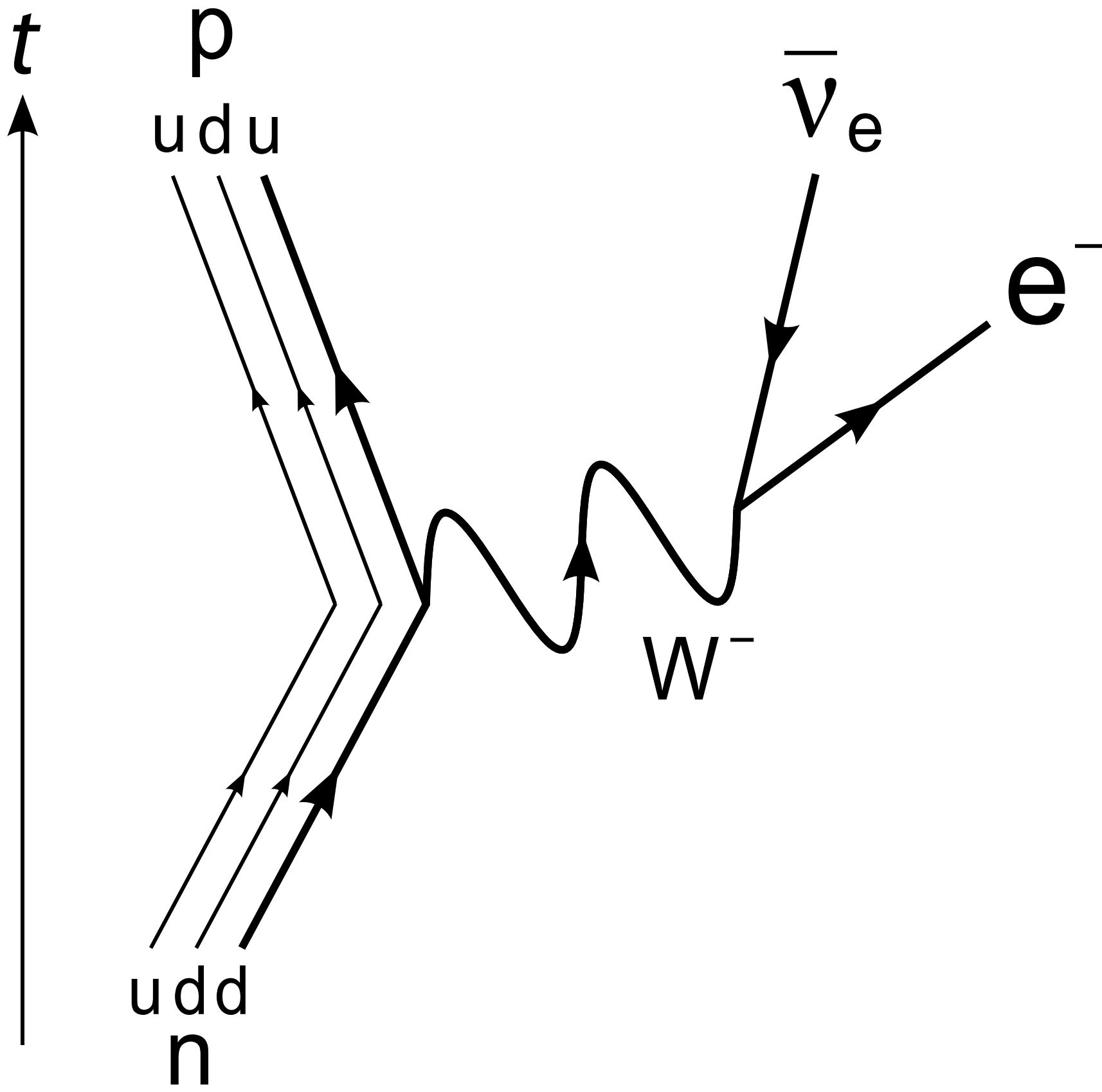


Figure 2: Feynman diagram of beta decay. The charged current weak interaction boson in this case is a W^- . Credit: Joel Holdsworth, within public domain.

Reaction	Equation	Boson
Electron emission	$\bar{A}_Z^A X \rightarrow \bar{A}_{Z+1}^A X + e^- + \bar{\nu}_e$	W^-
Positron emission	$\bar{A}_Z^A X \rightarrow \bar{A}_{Z-1}^A X + e^+ + \nu_e$	W^+
Electron capture	$\bar{A}_Z^A X + e^- \rightarrow \bar{A}_{Z-1}^A X + \nu_e$	W^+
Positron capture	$\bar{A}_Z^A X + e^+ \rightarrow \bar{A}_{Z+1}^A X + \bar{\nu}_e$	W^-
Electron annihilation	$e^- + e^+ \rightarrow \nu_e + \bar{\nu}_e$	W^-
Electron annihilation	$e^- + e^+ \rightarrow \nu + \bar{\nu}$	Z
Neutrino capture	$\bar{A}_Z^A X + \nu_e^{(-)} \rightarrow \bar{A}_{Z+1}^A X + e^\pm$	W
$e^- \nu$ scattering	$e^- + \nu_e^{(-)} \rightarrow e^- + \nu_e^{(-)}$	W
$e^- \nu$ scattering	$e^\pm + \nu_e^{(-)} \rightarrow e^\pm + \nu_e^{(-)}$	Z
Neutrino scattering	$\bar{A}_Z^A X + \nu_e^{(-)} \rightarrow \bar{A}_Z^A X + \nu^{(-)}$	Z
Bremsstrahlung	$N + \bar{N} \rightleftharpoons N + \bar{N} + \nu + \bar{\nu}$	
Annihilation	$e^+ e^- \rightleftharpoons \nu + \bar{\nu}$	
Neutrino annihilation	$\nu + \bar{\nu} \rightleftharpoons \nu + \bar{\nu}$	

Table 1: Neutrino related nuclear or leptonic reactions

3.3 Neutrino Mixing

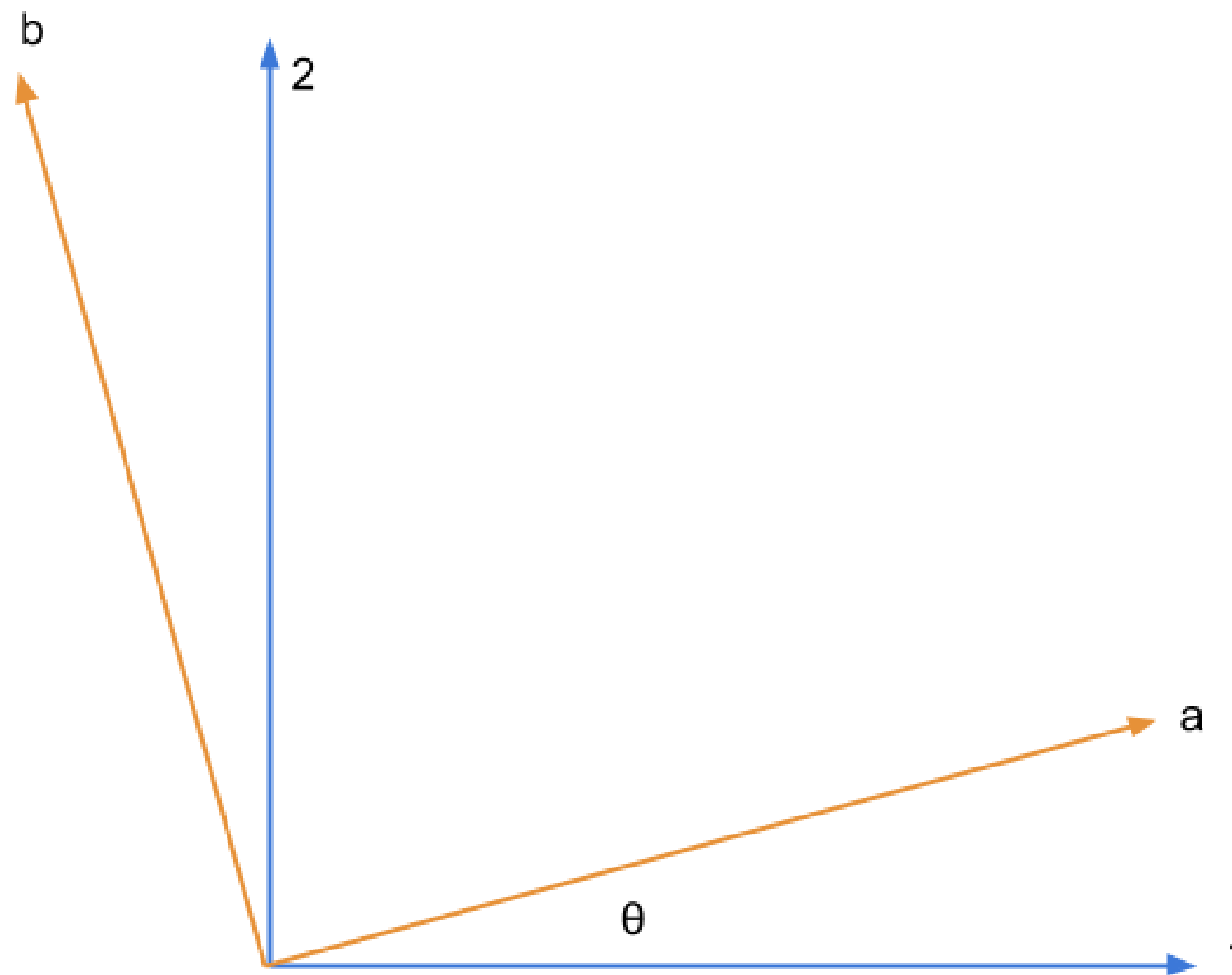


Figure 3: Neutrino mixing. Blue states ($\{1, 2\}$) are the VACUUM energy eigenstates while the orange states ($\{a, b\}$) are the flavor eigenstates. Blue: flavor states; Red: propagation states.

3.4 Hamiltonian and Basis

3.4.1 Basis

Rotation from a Hamiltonian diagonalized basis wave function Ψ_v to flavor basis wave function Ψ_f is

$$\Psi_f = R_{d2f}(\theta_x)\Psi_d, \quad (12)$$

where d can be v for vacuum eigenbasis, m for matter eigenbasis and

$$R_{d2f}(\theta_x) = \begin{pmatrix} \cos \theta_d & \sin \theta_d \\ -\sin \theta_d & \cos \theta_d \end{pmatrix}. \quad (13)$$

Matter mixing angle θ_m is determined through

$$\sin 2\theta_m = \frac{\sin 2\theta_v}{\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v} \cos 2\theta_v}}, \quad (14)$$

$$\cos 2\theta_m = \frac{\cos 2\theta_v - \lambda/\omega_v}{\sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v} \cos 2\theta_v}}, \quad (15)$$

$$\tan 2\theta_m = \frac{\sin 2\theta_m}{\cos 2\theta_m - \lambda/\omega_v}, \quad (16)$$

where $\lambda = \sqrt{2}G_F n_e$.

3.4.2 Hamiltonian

With the appearance of matter perturbation $\lambda(x) = \lambda_0 + \delta\lambda(x)$ to the system, we have the Hamiltonian in vacuum basis

$$\mathbf{H}_v = -\frac{\omega_v}{2}\sigma_3 + \frac{\lambda(x)}{2}\cos 2\theta_v\sigma_3 + \frac{\lambda(x)}{2}\sin 2\theta_v\sigma_1. \quad (17)$$

In flavor basis,

$$\mathbf{H}_f = \frac{\omega}{2}(-\cos 2\theta\sigma_3 + \sin 2\theta\sigma_1) + \frac{\lambda(x)}{2}\sigma_3. \quad (18)$$

In background matter potential λ_0 basis

$$\mathbf{H}_{\bar{m}} = -\frac{\omega_m}{2}\sigma_3 + \frac{\delta\lambda(x)}{2}\cos 2\theta_m\sigma_3 - \frac{\delta\lambda}{2}\sin 2\theta_m\sigma_1, \quad (19)$$

where $\omega_m = \omega_v \sqrt{\left(\frac{\lambda}{\omega_v}\right)^2 + 1 - 2\frac{\lambda}{\omega_v} \cos 2\theta_v}$.

The caveat is the position/time dependence of the matter potential leads to position/time dependent transformation of the wave function which will cause an term in the Hamiltonian of the form

$$-i\mathbf{U}^\dagger(x)\frac{\partial}{\partial x}\mathbf{U}(x). \quad (20)$$

3.4.3 Physics of Neutrino Oscillation

- Nature of neutrino oscillation
- MSW effect
- Collective oscillation
- Gravitation effect on neutrino oscillations
- Kinetic equation ($\nu\bar{\nu}$ pairing correlations?)

3.4.4 Applications of Neutrino Oscillation

- Supernova shock revive
- Accretion disc
- Cosmology
- Tomography
- Detection of nuclear activities

3.4.5 Big Questions

- Are neutrinos Majorana or Dirac?
- What is the mass hierarchy?

4. Numerical

4.1 Limits

- Simple cases where analytical is possible;
- Verified numerical calculations had previously.

4.2 Zen of Python

```
import this
```

4.3 Check You Code (Neutrino)

Check the code step by step:

- Vacuum oscillation amplitude and frequency
- Constant matter potential oscillation amplitude and frequency
- MSW resonance