二维蜂窝状格子 Frustrated Heisenberg模型的 Monte Carlo方法研究

05组

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物理系10级

Outline

Frustrated Heisenberg Model

Kosterlitz–Thouless Phase

What to Investigate

MC Results

Frustrated Heisenberg Model

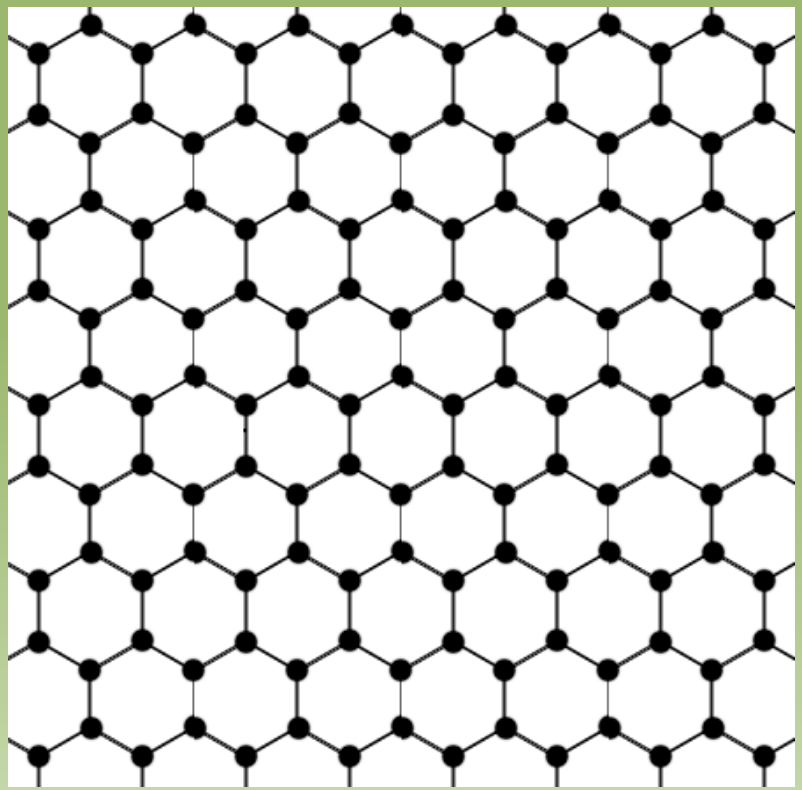
$$H = J_1 \sum_{<ij>} \vec{s}_i \cdot \vec{s}_j + J_2 \sum_{<<ij>>} \vec{s}_i \cdot \vec{s}_j$$

$$H = \frac{1}{2} J_1 \sum_{i}^{N} \vec{s}_i \cdot (\sum_{j} \vec{s}_j + \frac{J_2}{J_1} \sum_{j'} \vec{s}_{j'})$$

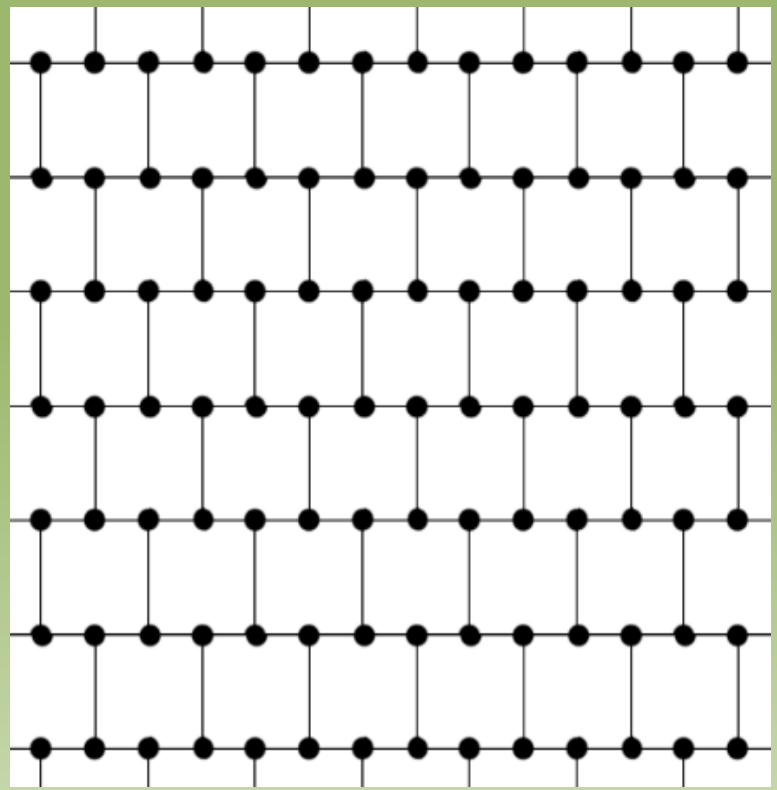
$$R = J_2 / J_1$$

<*ij*>表示最近邻格子对; <<*ij*>>表示次近邻格子对

Frustrated Heisenberg Model



蜂窝状格点分布



做同胚映射后

Frustrated Heisenberg Model

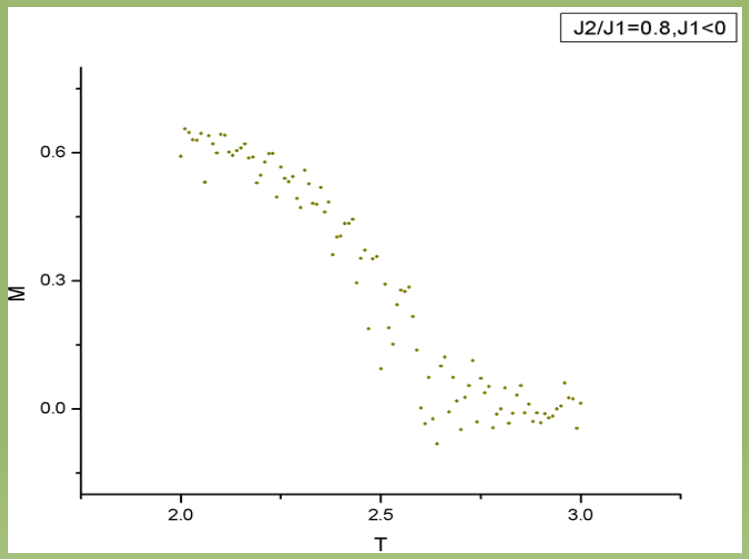
$$H = \frac{1}{2} J_1 \sum_i^N \vec{s}_i \cdot \left(\sum_j \vec{s}_j + \frac{J_2}{J_1} \sum_{j'} \vec{s}_{j'} \right)$$

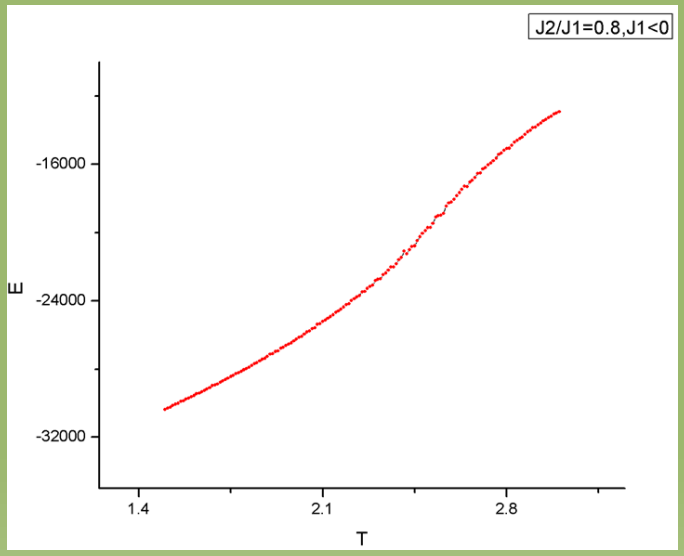
- J1<0时，存在铁磁结构；
- J1>0时，存在反铁磁结构；
- 蜂窝状晶格+连续矢量模型=>Frustrated

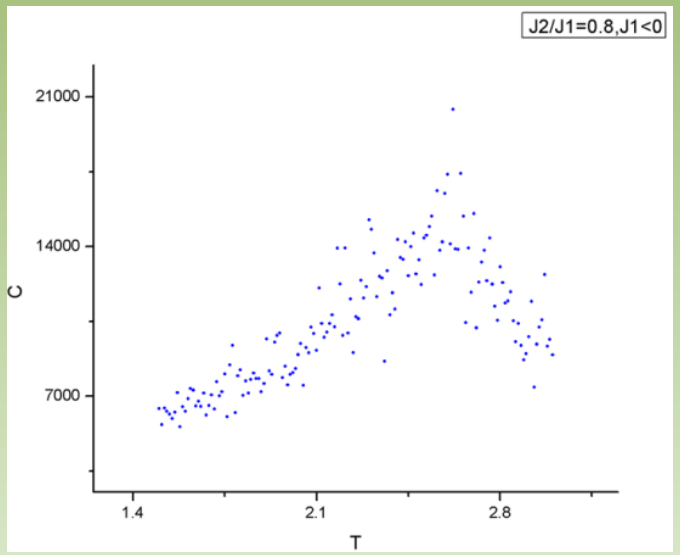
What to investigate?

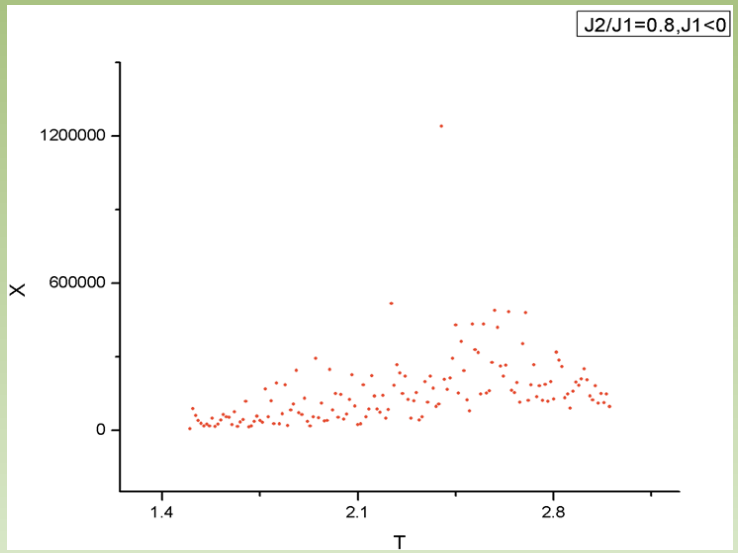
$$H = \frac{1}{2} J_1 \sum_i^N \vec{s}_i \cdot (\sum_j \vec{s}_j + \frac{J_2}{J_1} \sum_{j'} \vec{s}_{j'})$$

J2/J1=0.8（J1<0）

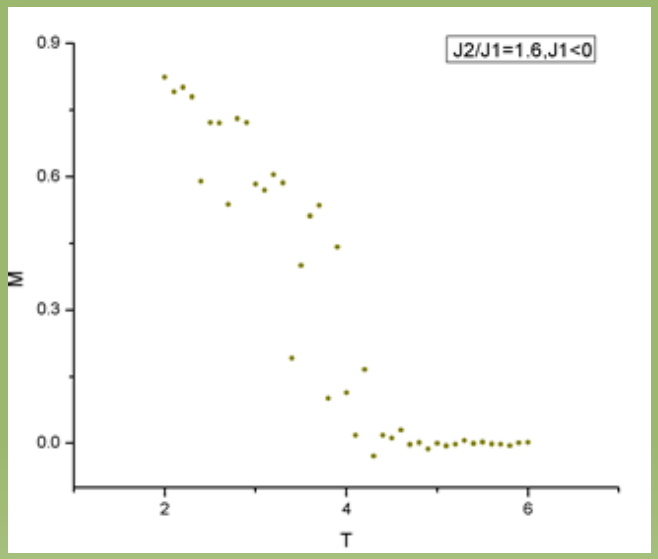

J2/J1=0.8,J1<0
M
0.6
0.3
0.0
2.0
2.5
3.0
T

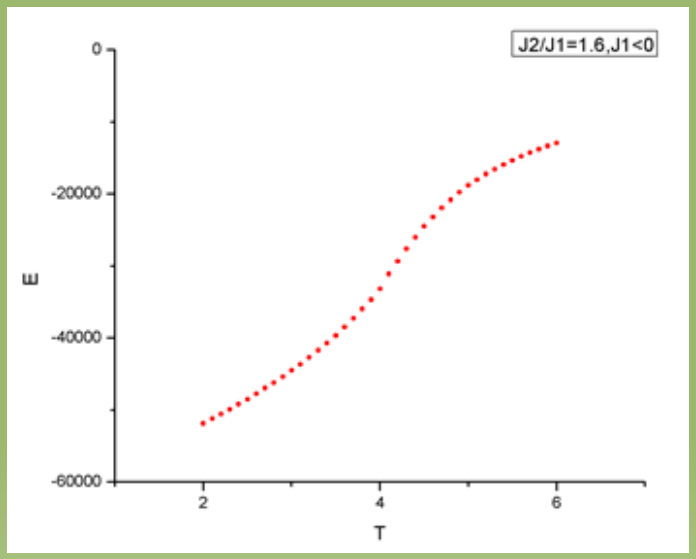

J2/J1=0.8,J1<0
E
-16000
-24000
-32000
1.4
2.1
2.8
T

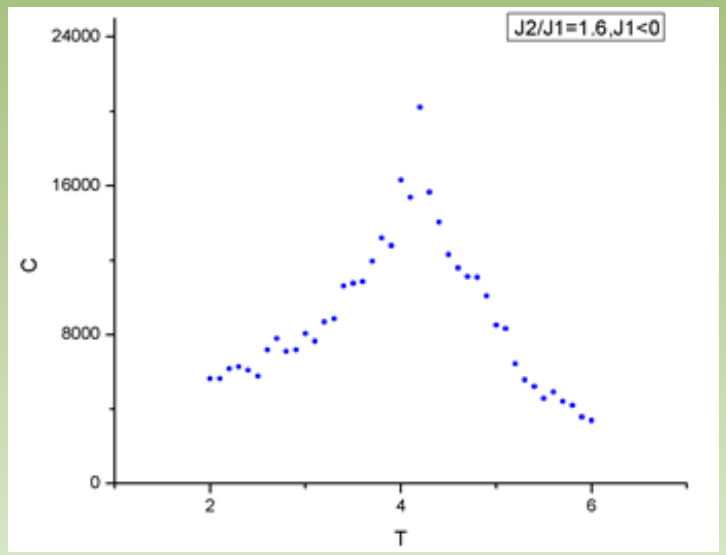

J2/J1=0.8,J1<0
C
21000
14000
7000
1.4
2.1
2.8
T

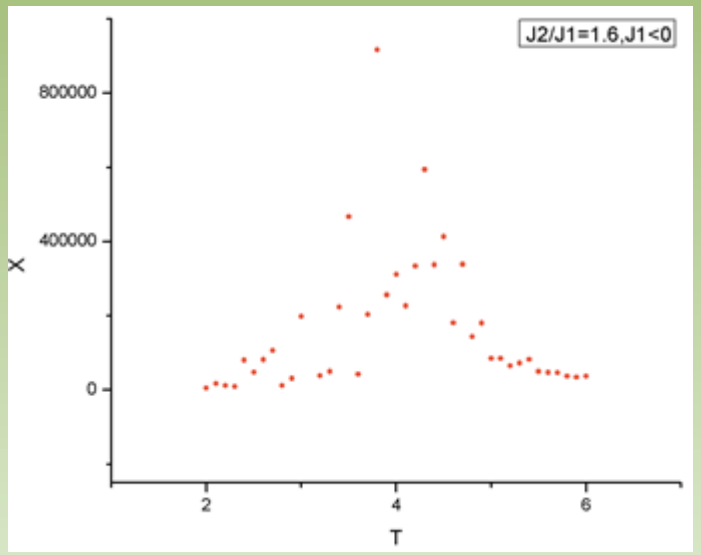

J2/J1=0.8,J1<0
X
1200000
600000
0
1.4
2.1
2.8
T

J2/J1=1.6（J1<0）

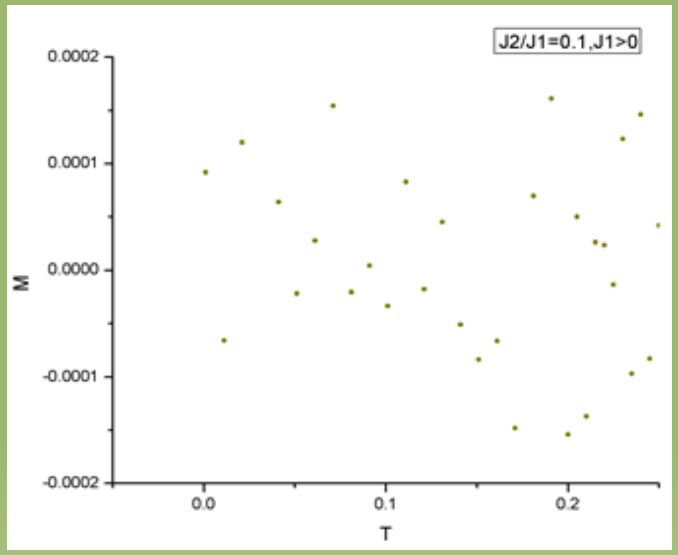

J2/J1=1.6,J1<0
M
T
0.9
0.6
0.3
0.0
2
4
6

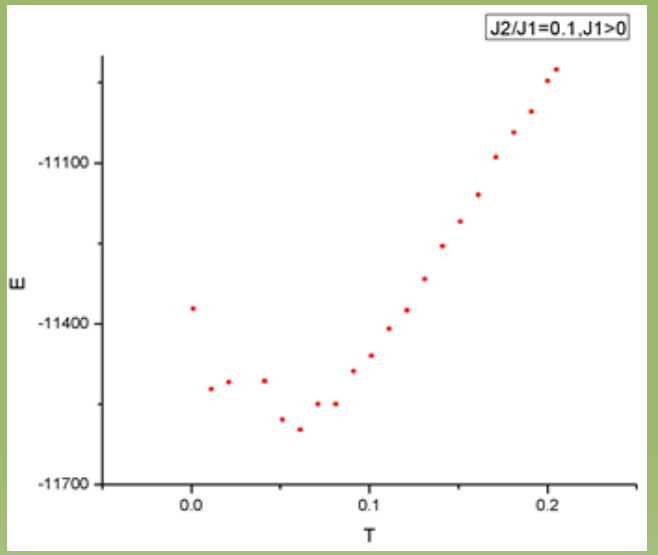

J2/J1=1.6,J1<0
E
T
0
-20000
-40000
-60000
2
4
6

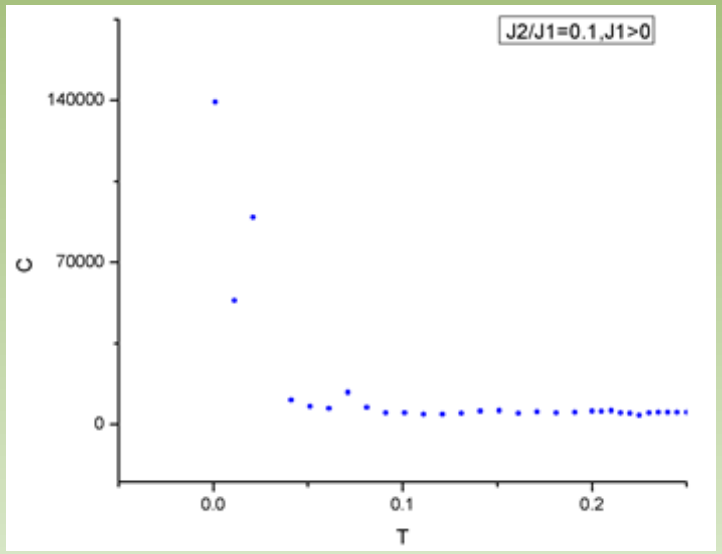

J2/J1=1.6,J1<0
C
T
24000
16000
8000
0
2
4
6

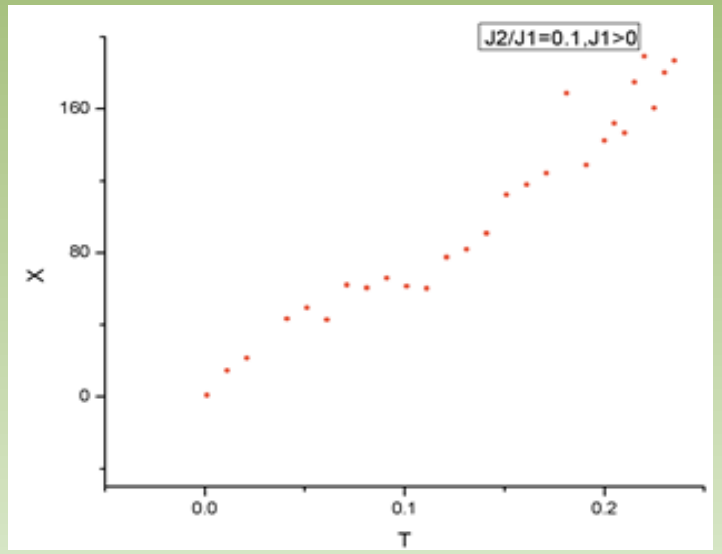

J2/J1=1.6,J1<0
X
T
800000
400000
0
2
4
6

J2/J1=0.1 (J1>0)


J2/J1=0.1,J1>0
0.0002
0.0001
0.0000
-0.0001
-0.0002
M
0.0
0.1
0.2
T

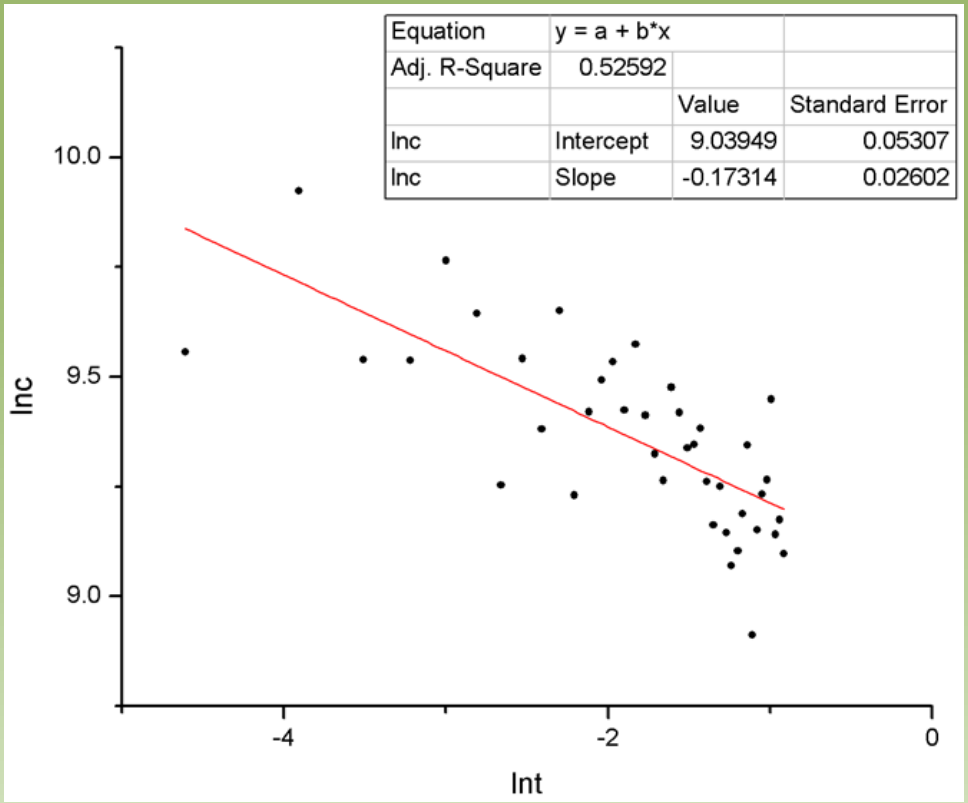

J2/J1=0.1,J1>0
-11100
-11400
-11700
E
0.0
0.1
0.2
T


J2/J1=0.1,J1>0
140000
70000
0
C
0.0
0.1
0.2
T

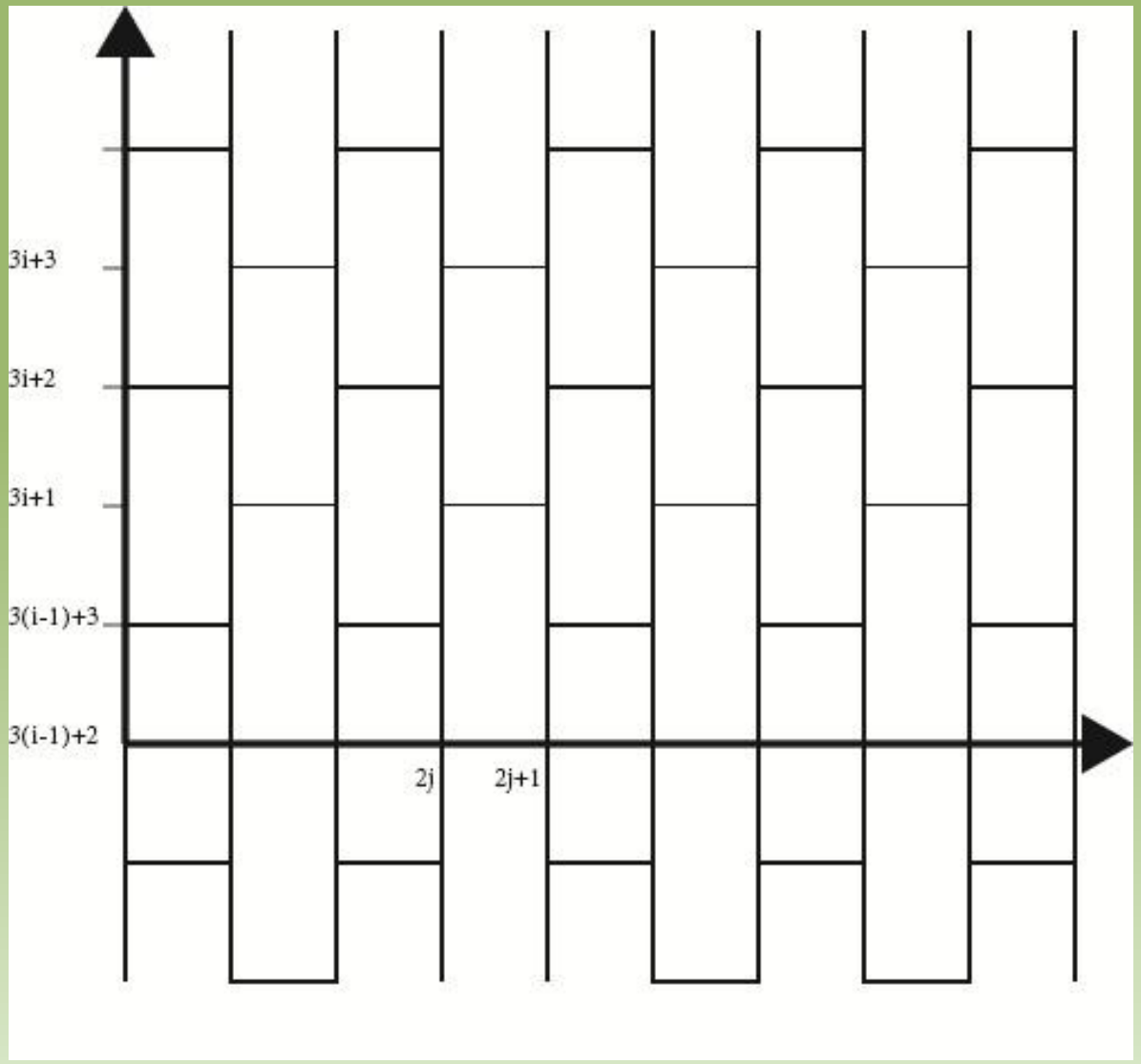

J2/J1=0.1,J1>0
160
80
0
X
0.0
0.1
0.2
T

临界指数

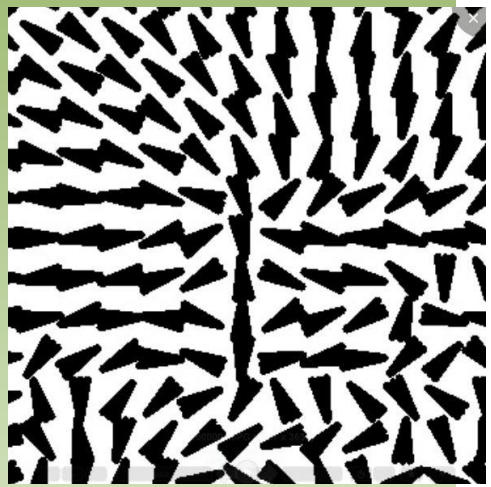
- 对J2/J1=0.8的情况进行分析，结果如图：

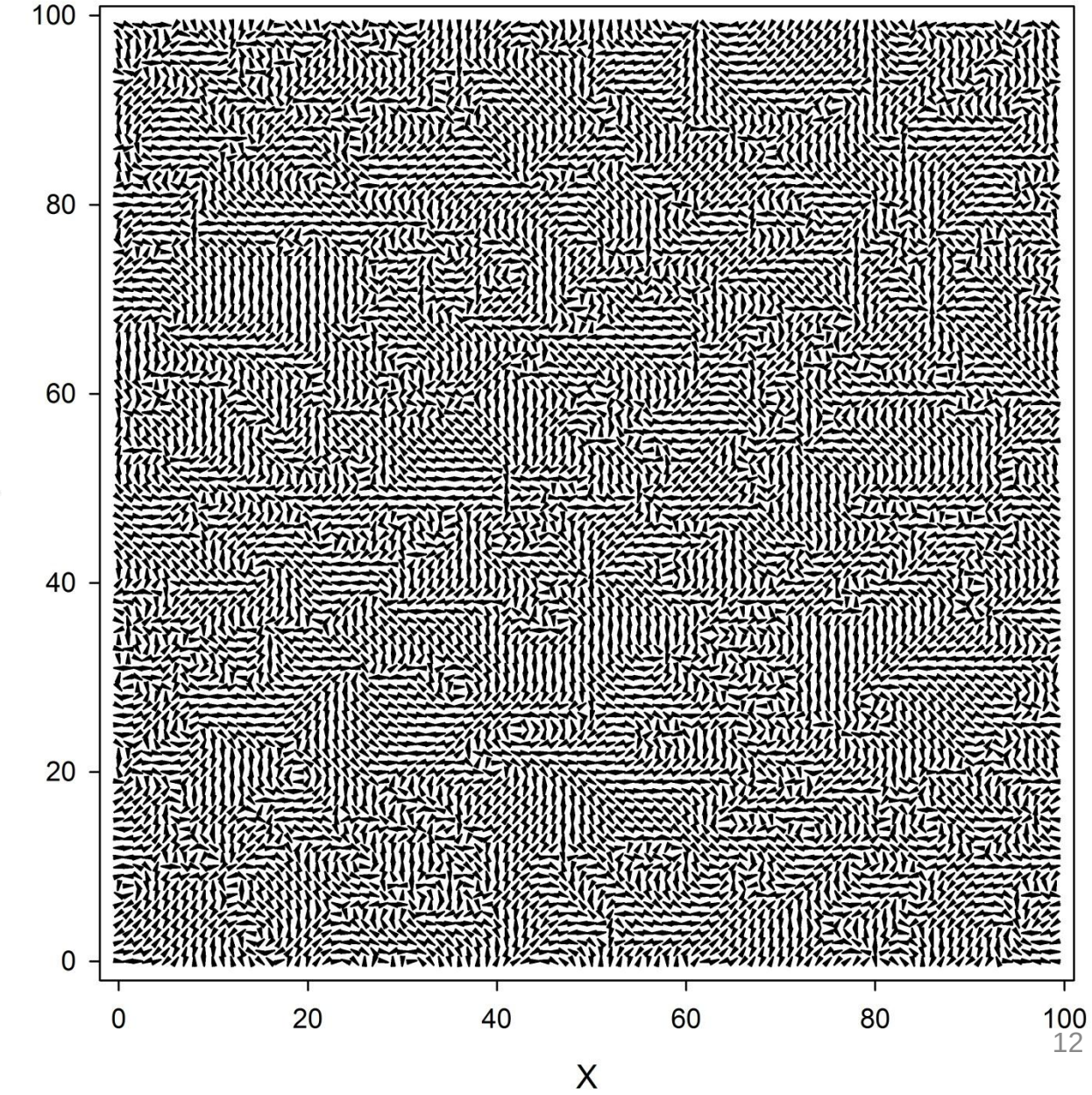


Configuration

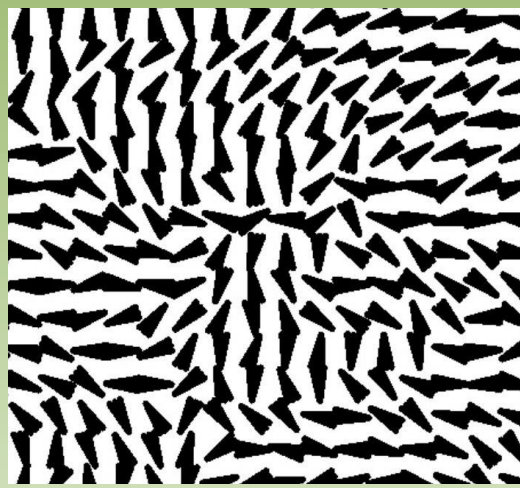


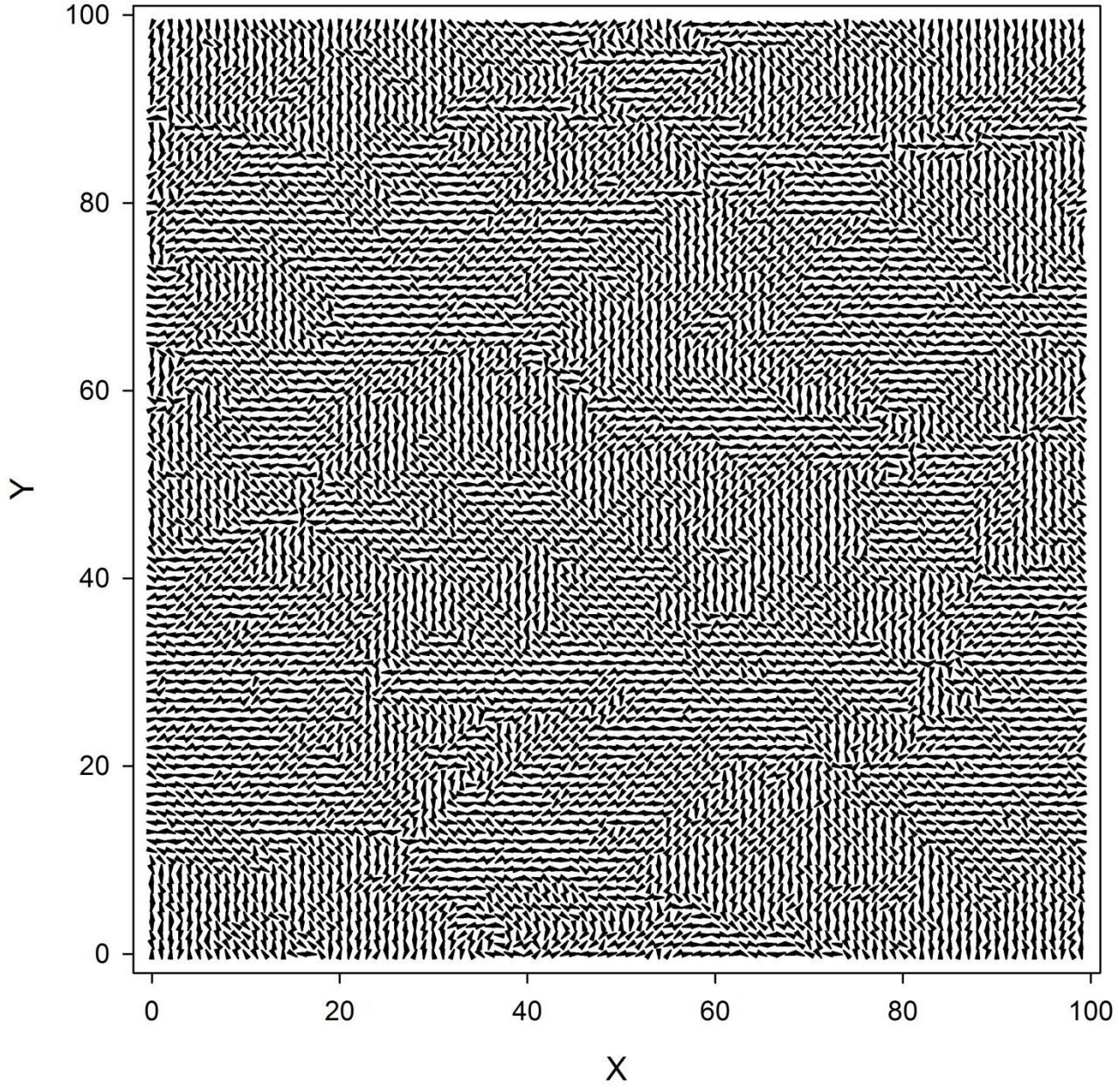
T=0.000001
R=0.1
LXL=100X100



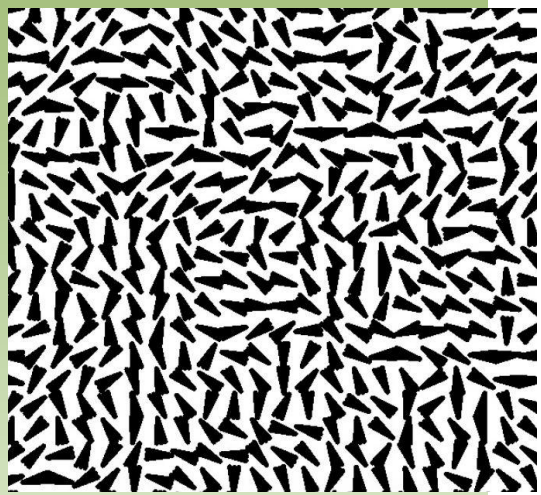


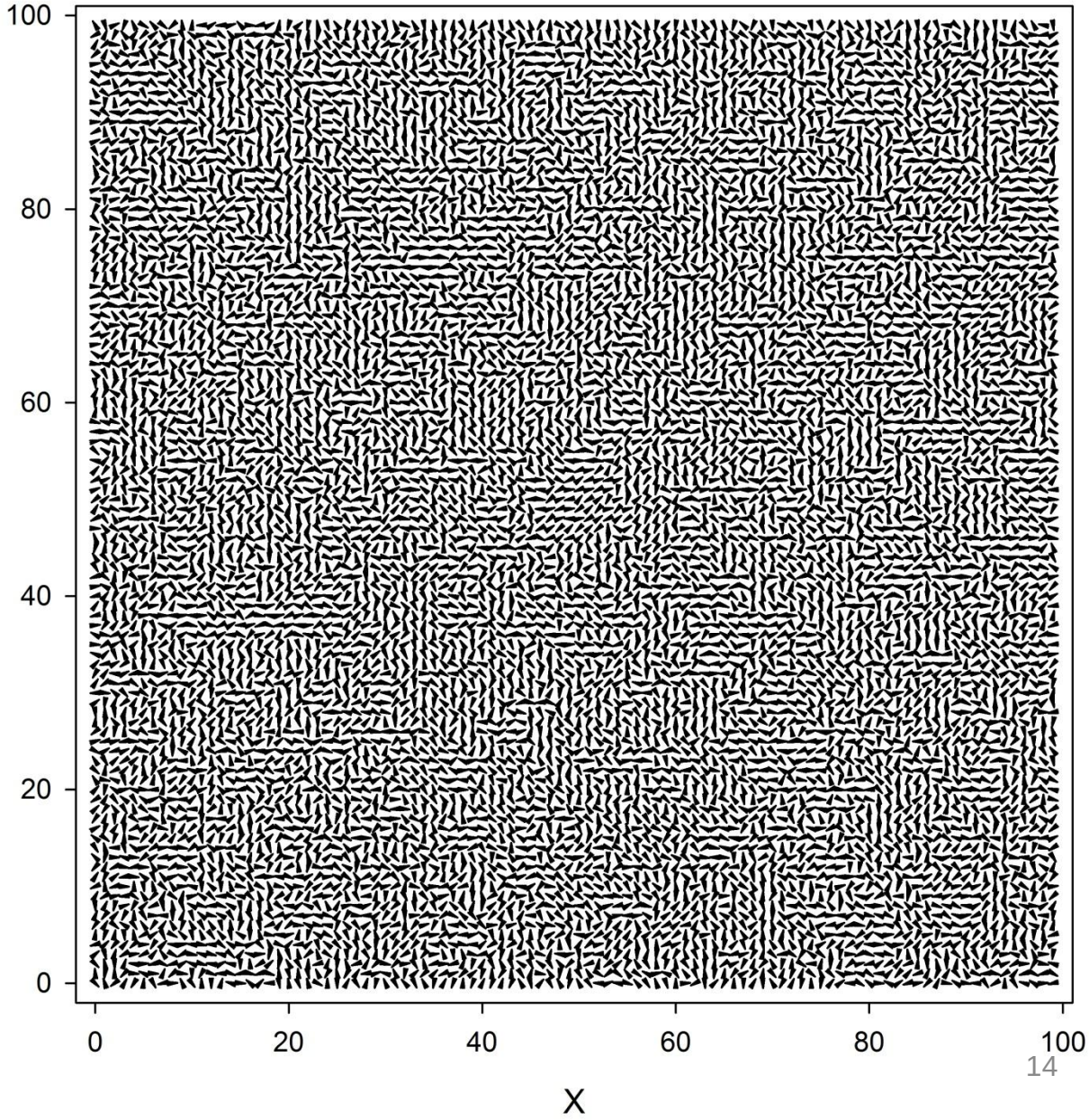
T=0.1
R=0.1
LXL=100X100



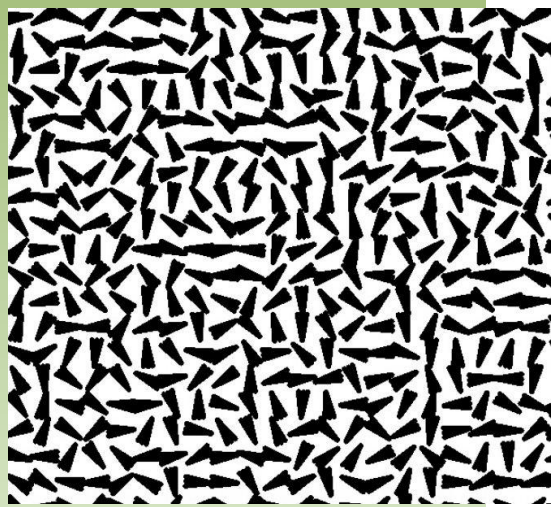


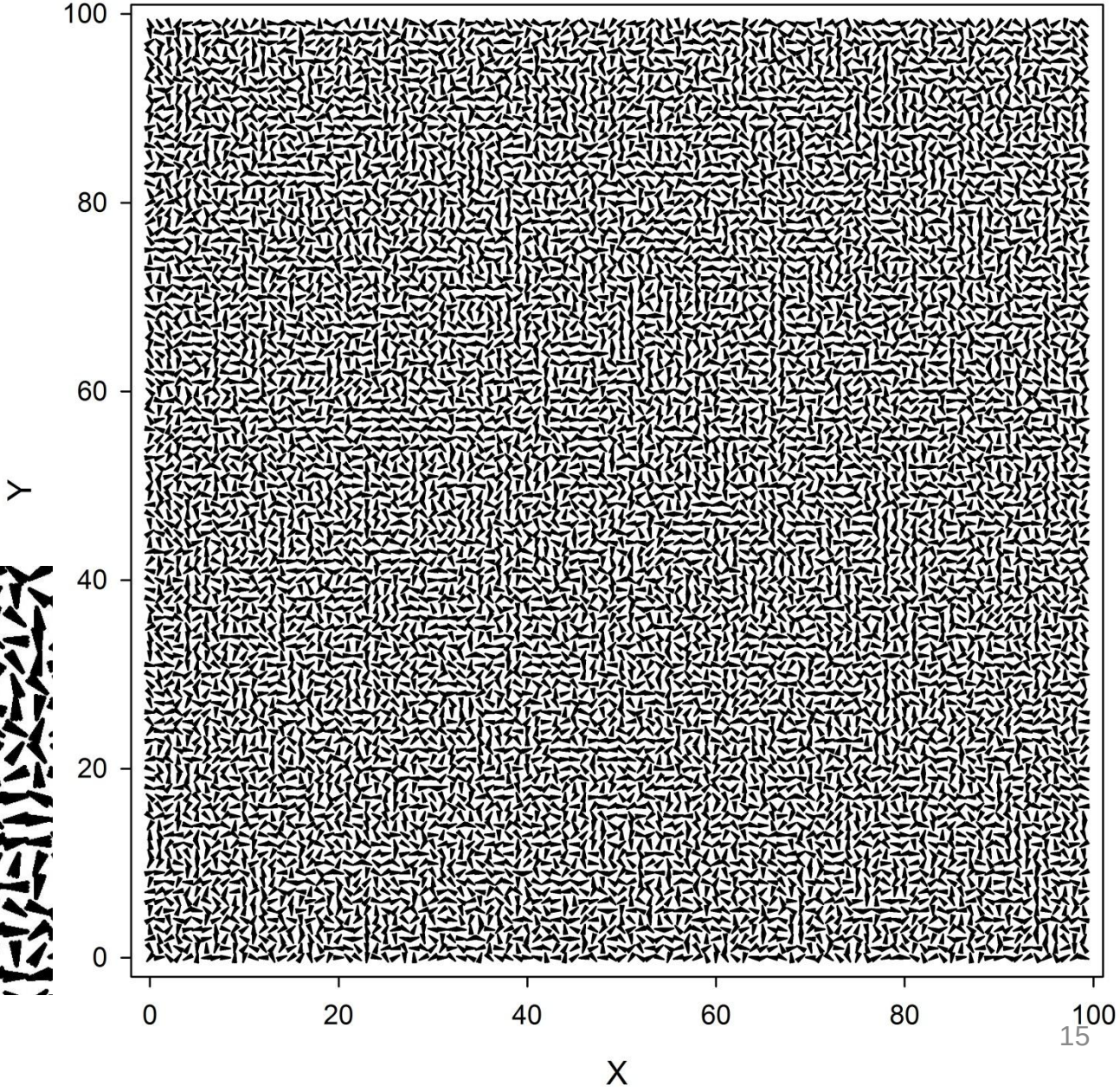
T=0.4
R=0.1
LXL=100X100





T=1.0
R=0.1
LXL=100X100





Phase Transition

朗道相变:

连续相变~对称性自发破缺。

Kosterlitz 和 DJ Thouless:

平面格点上SO(2)自旋的海森堡模型有连续相变

Coleman的定理=>一个仅有连续对称性的两维体系不能发生连续相变。

1. N.D. Mermin, H. Wagner: “Absence of Ferromagnetism or Antiferromagnetism in One- or Two-Dimensional Isotropic Heisenberg Models”, Phys. Rev. Lett. 17, 1133–1136 (1966)
2. P.C. Hohenberg: “Existence of Long-Range Order in One and Two Dimensions”,Phys. Rev. 158, 383 (1967)
3. Sidney Coleman: “There are no Goldstone bosons in two dimensions”, Commun. Math. Phys. 31, 259 (1973)
4. J. M. Kosterlitz & D. J. Thouless, “Ordering, metastability and phase transitions in two-dimensional systems“, Journal of Physics C: Solid State Physics, Vol. 6 pages 1181-1203 (1973)

Phase Transition

长程上~自由场论
两点相之间有长程关联

低温时Vertex与Anti-vertex 结成对

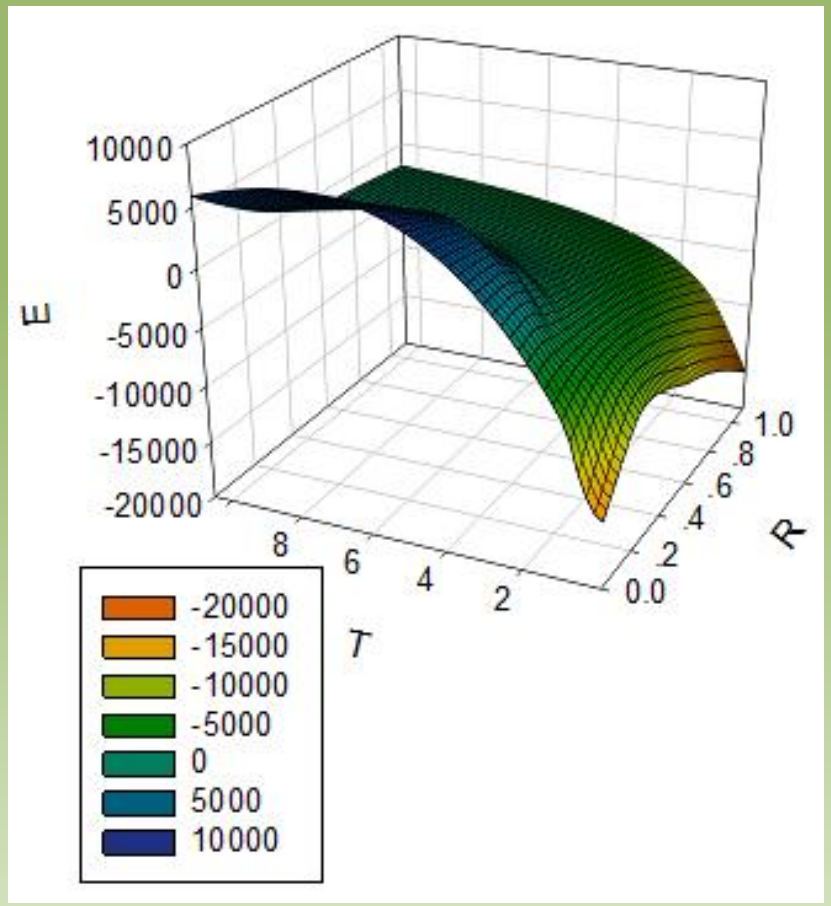
长程序

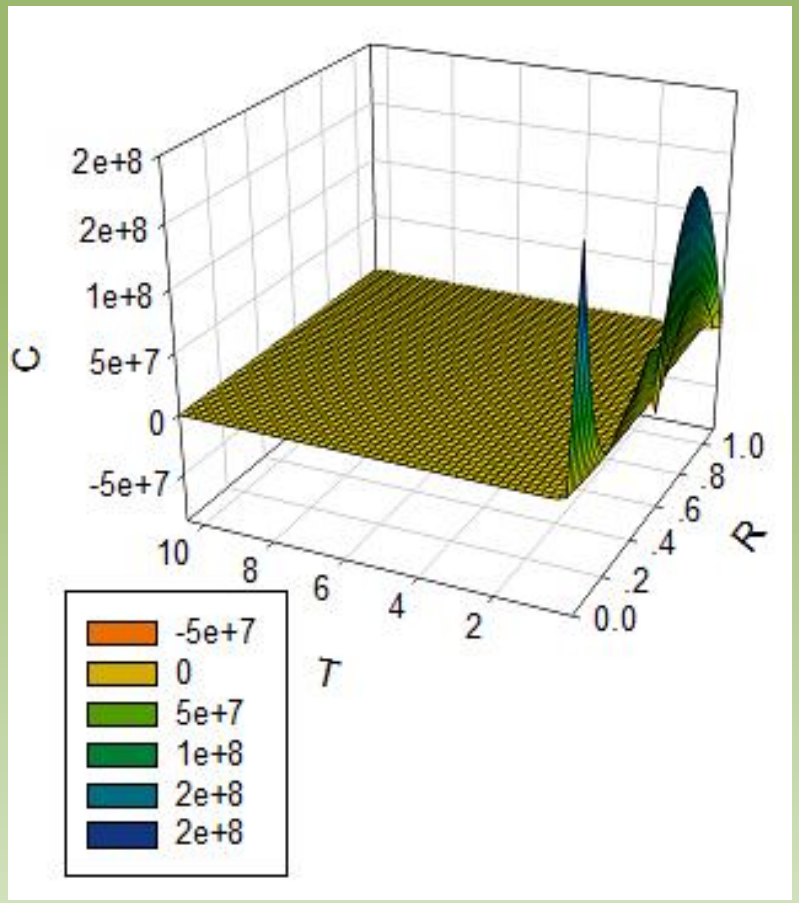
E<TS
“德拜”屏蔽

高温时vertex与anti-vertex 趋于解开

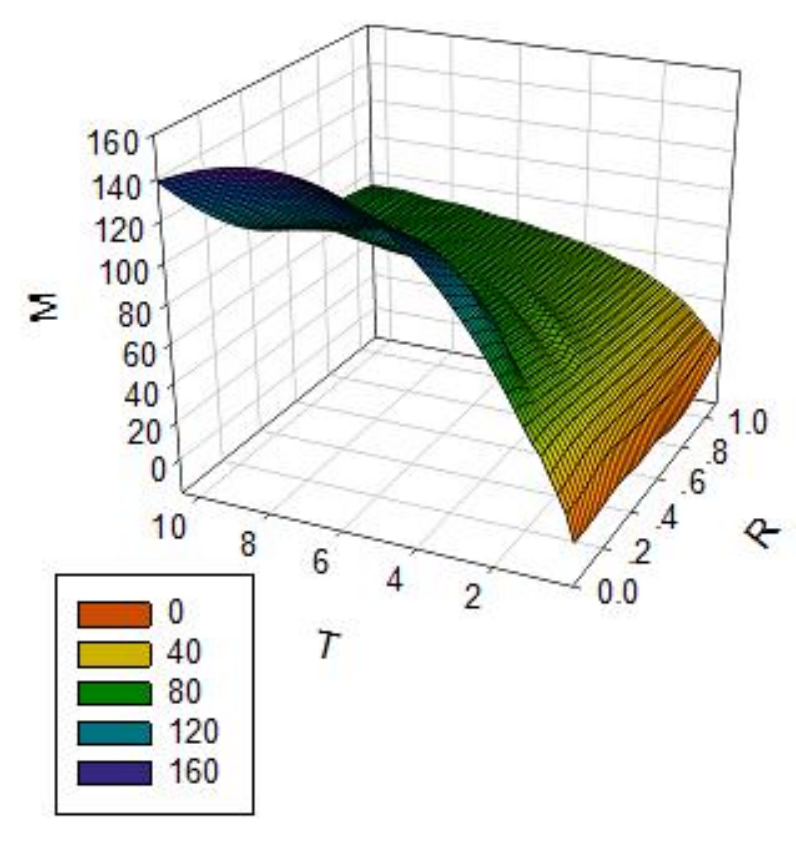
长程序破坏

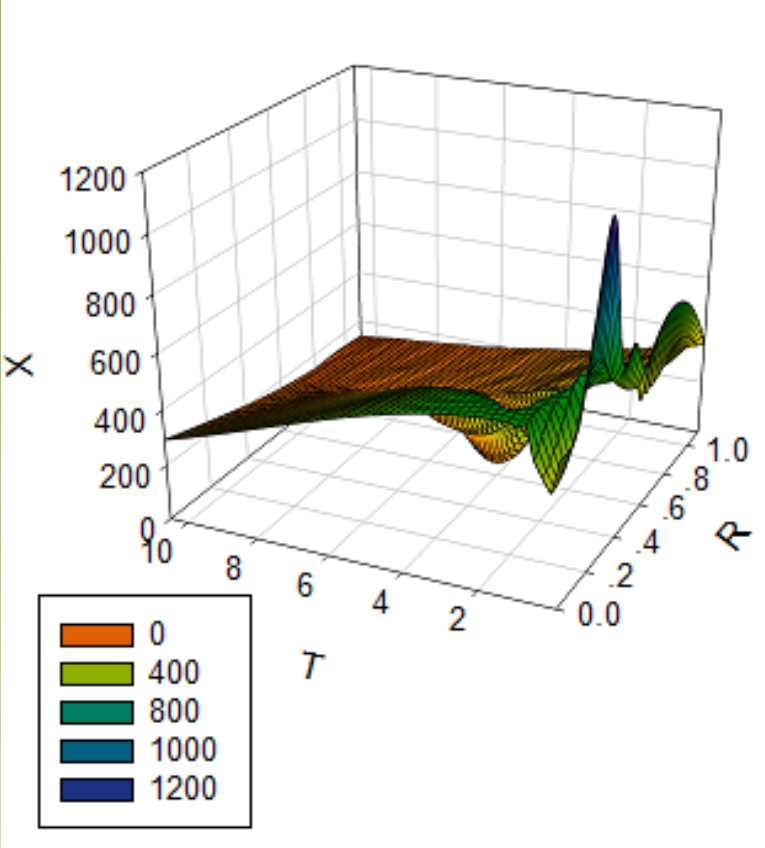
General Checking



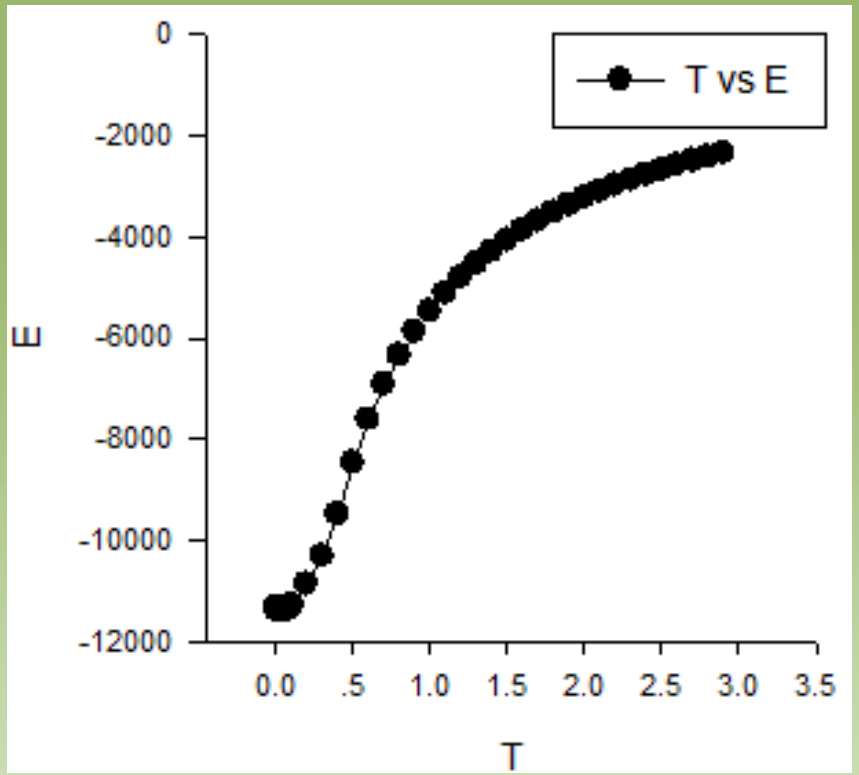


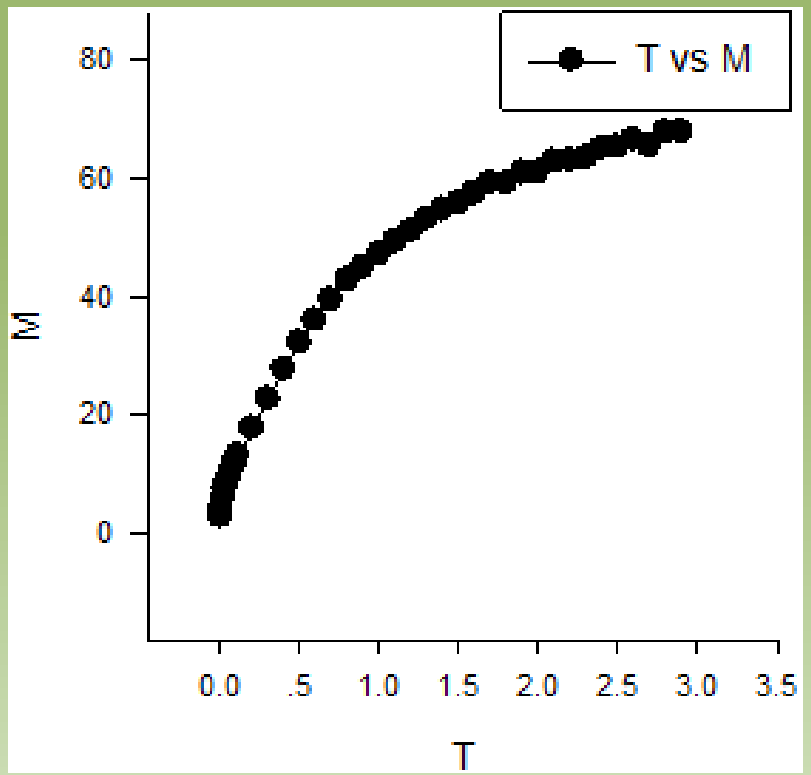
General Checking



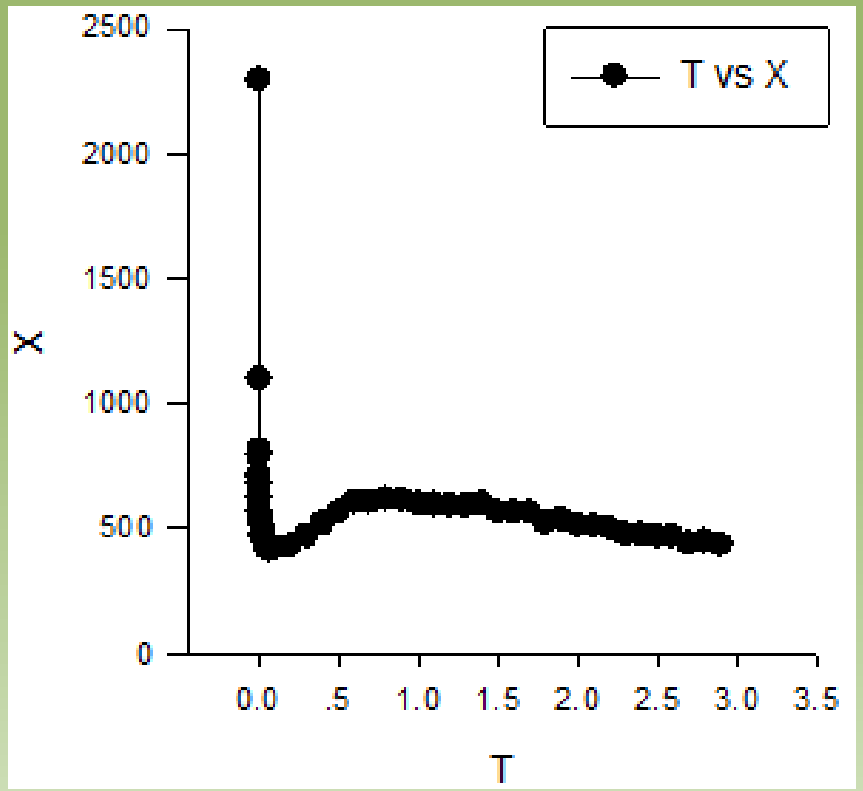


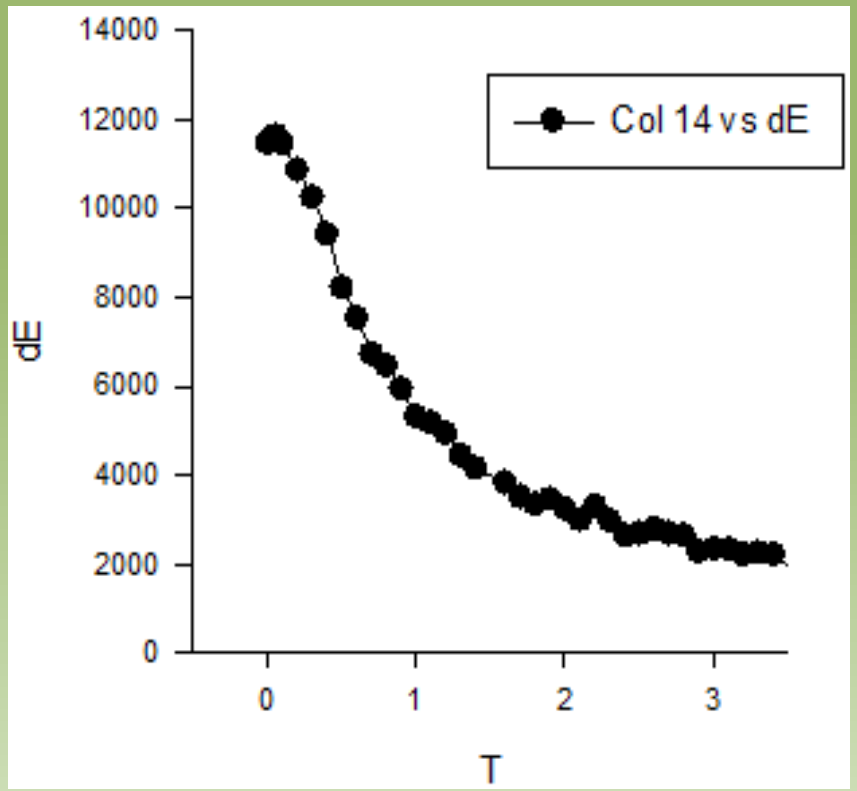
J2/J1=0.1


T vs E
E
0
-2000
-4000
-6000
-8000
-10000
-12000
0.0
.5
1.0
1.5
2.0
2.5
3.0
3.5
T

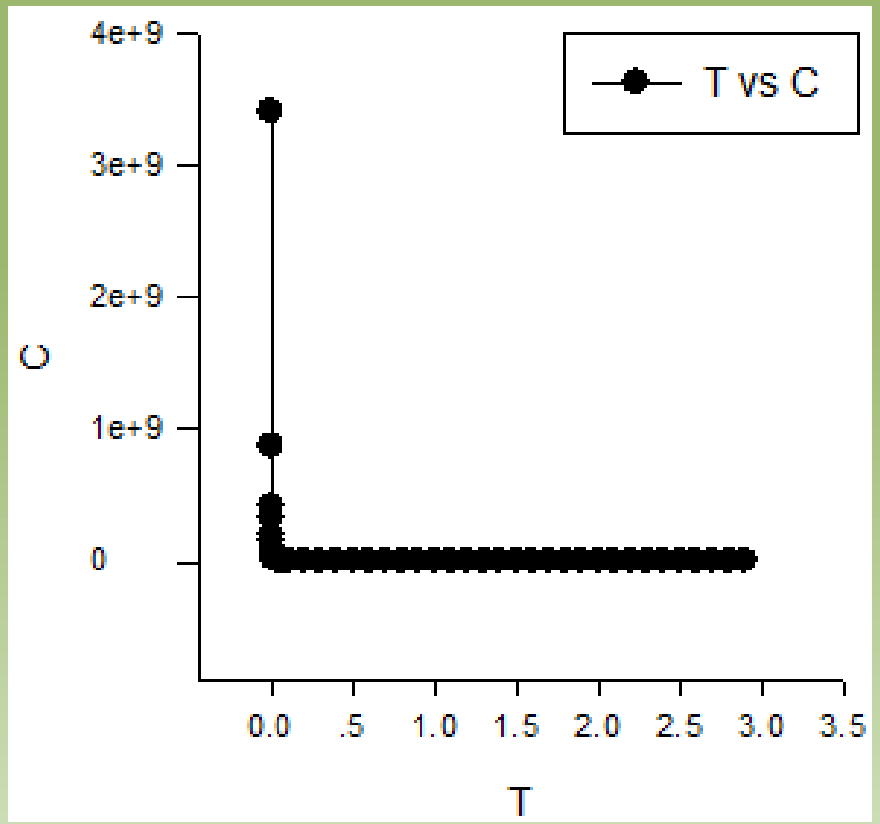

T vs M
M
80
60
40
20
0
0.0
.5
1.0
1.5
2.0
2.5
3.0
3.5
T

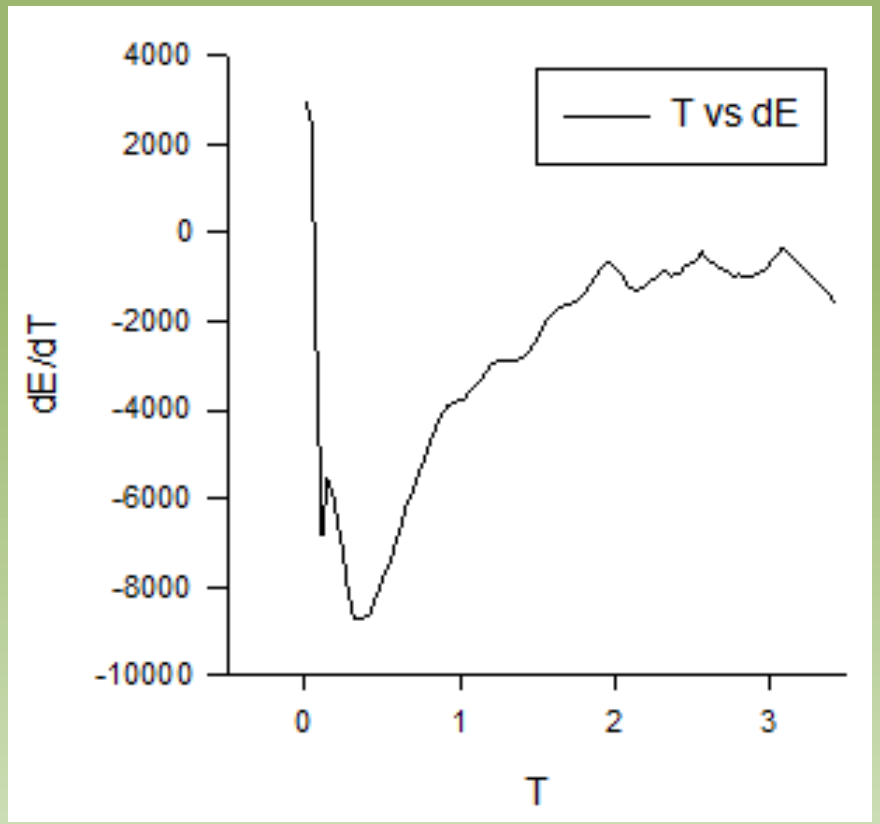
J2/J1=0.1


T vs X
X
T
2500
2000
1500
1000
500
0
0.0
.5
1.0
1.5
2.0
2.5
3.0
3.5

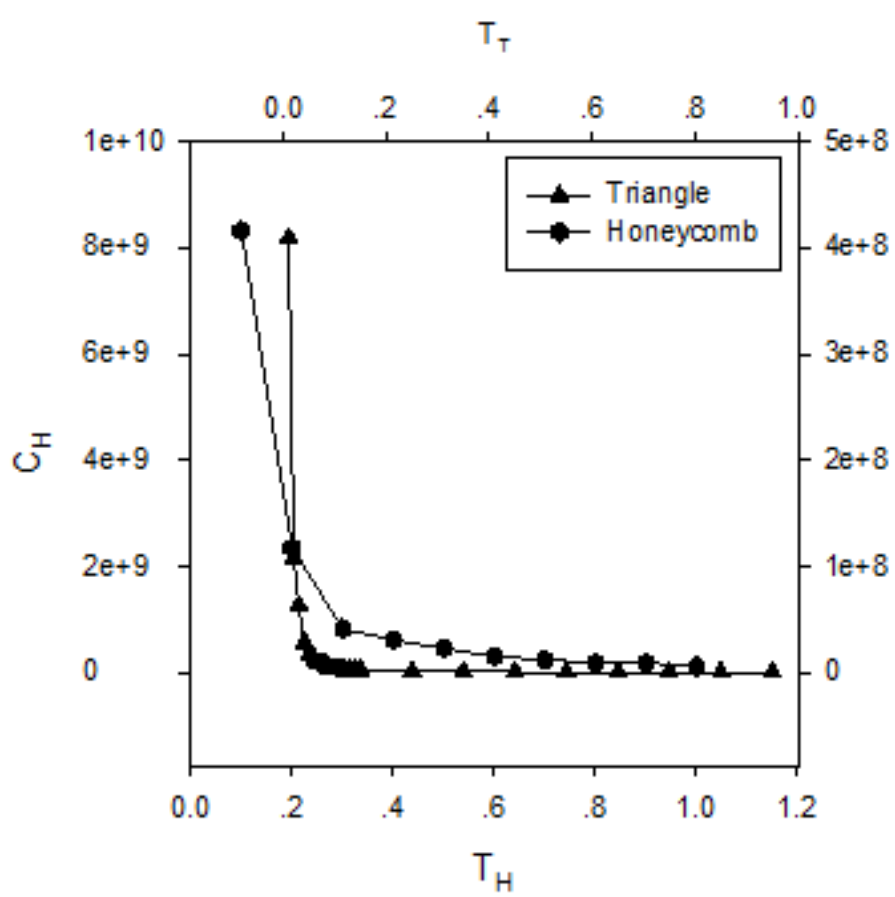

Col 14 vs dE
dE
T
14000
12000
10000
8000
6000
4000
2000
0
0
1
2
3

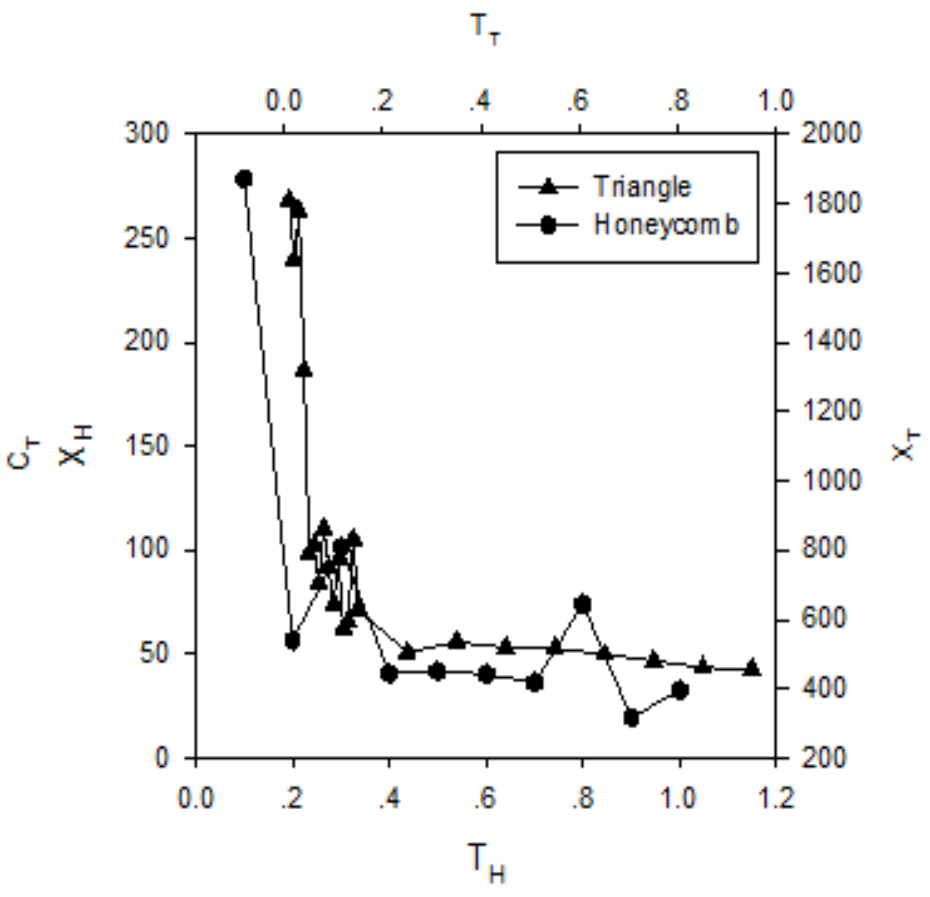
J2/J1=0.1


T vs C
4e+9
3e+9
2e+9
1e+9
0
C
0.0
.5
1.0
1.5
2.0
2.5
3.0
3.5
T


T vs dE
4000
2000
0
-2000
-4000
-6000
-8000
-10000
dE/dT
0
1
2
3
T

J2/J1=50 ⇔ Triangle Lattice (XY)





Conclusion

J2/J1>0

J2/J1对体系影响低温时小

反铁磁状态

无二级相变。其他相变？

Vertex？

J2/J1<0

二级相变?

Vertex？

J2/J1=50

Triangle Lattice (XY)

Acknowledgements

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在可视化方面给予的帮助
（OpenGL，仅在论文中提及）