NEUTRINO OSCILLATIONS IN MATTER

L. Wolfenstein (1978). Phys. Rev. D 17(9)

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OUTLINE

- Neutrino oscillation
- Nature of neutrino oscillation
- Matter Interaction
- Conclusion

NEUTRINO OSCILLATION

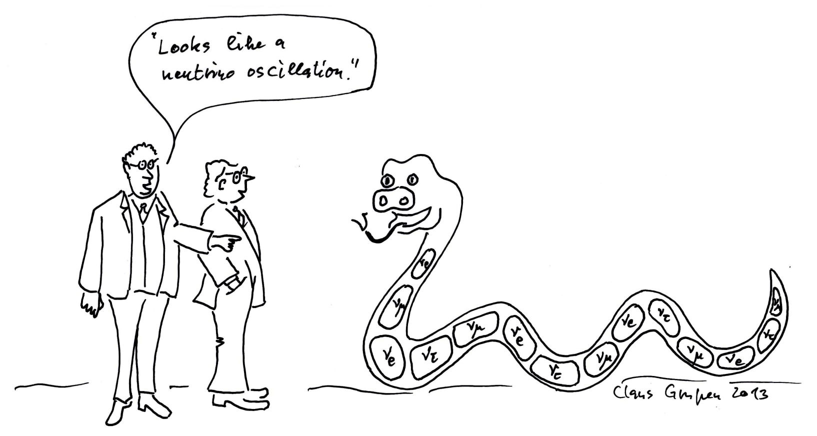

Figure: Source: http://www.hep.physik.uni-siegen.de/~grupen/

- Equation of motion

$$i\partial_t \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \Leftrightarrow i\partial_t \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\Delta E & 0 \\ 0 & \Delta E \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

  in which $\Delta E = E_2 - E_1$.
- Initial state is electron flavor $|\nu_e\rangle$.
- Neutrino flavor states $|\nu_e\rangle$ and $|\nu_x\rangle$ are different from energy eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$.

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$|\psi(t=0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

NEUTRINO OSCILLATION

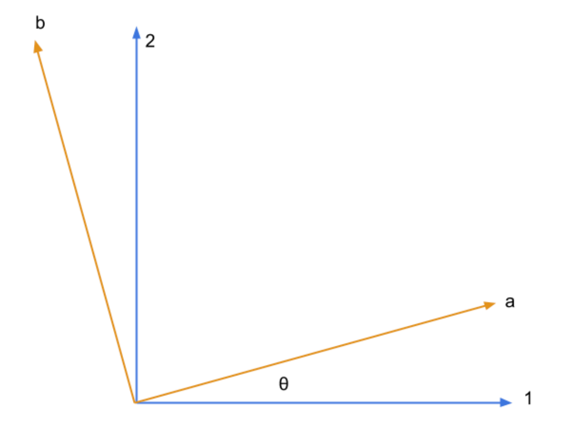

Figure: Neutrino mixing. Blue states are the VACUUM energy eigenstates while the orange states are the flavor eigenstates. Blue: electron flavor; Red: the other flavor. Source: `http://docs.neutrino.xyz/oscillations.html`

- We could write down the solution to the problem without any thinking.

$$|\psi(t)\rangle = \cos\theta\,|\nu_1\rangle\, e^{-iE_1t} + \sin\theta\,|\nu_2\rangle\, e^{-iE_2t}.$$

- The survival probability of $|\nu_e\rangle$ is

$$\begin{aligned} & P(|\nu_e\rangle \to |\nu_e\rangle) \\ &= |\langle \nu_e \mid \psi(t)\rangle|^2 \\ &= \cdots \\ &= 1 - \frac{1}{2}\sin^2(2\theta)(1 - \cos(\Delta Et)) \end{aligned}$$

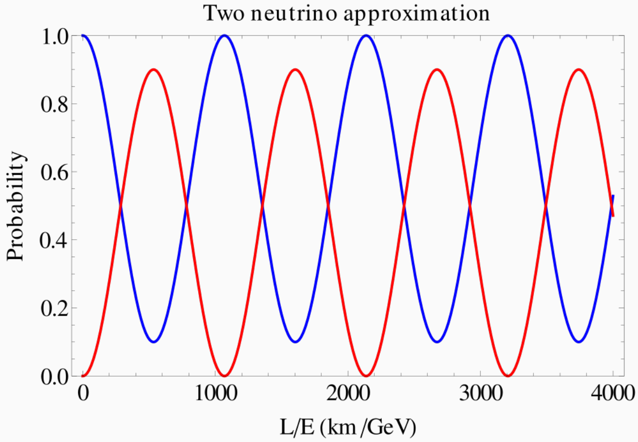

Figure: Neutrino flavor state probabilities. Source: https://en.wikipedia.org/wiki/File:Oscillations_two_neutrino.svg

NATURE OF NEUTRINO OSCILLATION

What if flavor eigenstates are the energy eigenstates?

The initial condition $|\Psi(t=0)\rangle = |\nu_e\rangle$ with eigen energy E_1. Then the state at any time is

$$|\Psi(t)\rangle = |\nu_e\rangle\, e^{-iE_1 t}.$$

Survival probability for electron flavor neutrino is

$$\begin{aligned} & P(|\nu_e\rangle \to |\nu_e\rangle) \\ &= |\langle \nu_e \mid \psi(t)\rangle|^2 \\ &= 1 \end{aligned}$$

What is the cause of oscillation?

As long as flavor eigenstates are NOT energy eigenstates, neutrino oscillations can happen.

Basis

Energy Eigenstates Diagonalized Hamiltonian

Flavor Eigenstates Eigenstates of weak interaction

Mixing Angle and Eigenenergies

Change mixing angles → change mixing amplitude
Change energy eigenstates → mixing frequency

$$P(|\nu_e\rangle \rightarrow |\nu_x\rangle) = \frac{1}{2}\sin^2(2\theta)(1 - \cos(\Delta Et))$$

MATTER INTERACTION

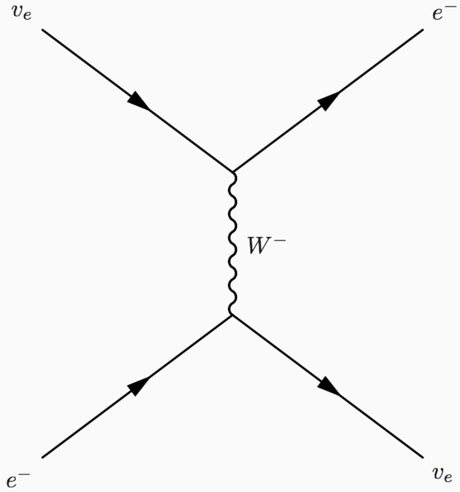

Figure: Charged current has only effect on electron flavor neutrinos.

- Interaction in flavor basis

$$V_f = \sqrt{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \to \frac{\sqrt{2}}{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Transform Vacuum Hamiltonian to flavor basis

$$H = U^{-1} H_0 U = \frac{\Delta E}{2} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- Total Hamiltonian of the problem

$$H = \frac{\Delta E}{2} \begin{pmatrix} -\cos 2\theta + \frac{\sqrt{2} G_F n_e}{\Delta E} & \sin 2\theta_V \\ \sin 2\theta_V & \sin 2\theta - \frac{\sqrt{2} G_F n_e}{\Delta E} \end{pmatrix}$$

What Changes?

1. Eigenenergies are different from Vacuum oscillation.
2. Effective mixing angle is different from Vacuum mixing angle.

New energy eigenstates

Transition probability from $|\nu_e\rangle$ to $|\nu_x\rangle$ is

$$P(|\nu_e\rangle \to |\nu_x\rangle) = \frac{1}{2}\sin^2(2\theta)(\frac{l_m}{l_v})^2(1 - \cos(\frac{2\pi}{l_m}t)),$$

where

$$l_v = \frac{2\pi}{\Delta E}$$
$$l_i = \frac{2\pi}{\sqrt{2}G_F n_e}$$
$$l_m = \frac{l_v}{\sqrt{1 + \left(\frac{l_v}{l_i}\right)^2 - 2\cos 2\theta\left(\frac{l_v}{l_i}\right)}}.$$

CONCLUSION

CONCLUSION

1. Flavor eigenstates are NOT energy eigenstates.$\Rightarrow$ Oscillation
2. Mass interaction changes the oscillation amplitude and frequency due to the change of effective mixing angle.

REFERENCES

1. Giunti, C., & Kim, C. W. (2007). Fundamentals of Neutrino Physics and Astrophysics. Oxford University Press. doi:10.1093/acprof:oso/9780198508717.001.0001
2. Wolfenstein, L. (1978). Neutrino oscillations in matter. Physical Review D, 17(9), 2369–2374. doi:10.1103/PhysRevD.17.2369
3. `http://docs.neutrino.xyz/`

BACKUPS

Backups

HIGH ENERGY APPROXIMATION

$$E = p\sqrt{1 + \frac{m^2}{p^2}} = \cdots$$

FLAVOR BASIS HAMILTONIAN

$$\mathsf{H}_{\mathsf{mf}} = \left(\frac{\Delta}{2} - \frac{\omega}{2} \cos 2\theta_{\mathsf{v}} \right) \boldsymbol{\sigma}_3 + \frac{\omega}{2} \sin 2\theta_{\mathsf{v}} \boldsymbol{\sigma}_1.$$

Vacuum Eigenstates Basis

$$H_{\text{vacuum}} = \begin{pmatrix} -\frac{\Delta E}{2} & 0 \\ 0 & \frac{\Delta E}{2} \end{pmatrix}$$

$$H_{\text{matter}} = \begin{pmatrix} -\frac{\Delta E}{2} + \frac{\sqrt{2}G_F n_e}{2}\cos 2\theta_v & \frac{\sqrt{2}G_F n_e}{2}\sin 2\theta_v \\ \frac{\sqrt{2}G_F n_e}{2}\sin 2\theta_v & \frac{\Delta E}{2} - \frac{\sqrt{2}G_F n_e}{2}\cos 2\theta_v \end{pmatrix}$$