

Neutrino Oscillations in Matter

PhD Candidacy Exam

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OUTLINE

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 - What are Neutrinos
 - Neutrino Oscillations
 - Why Do Neutrinos Oscillate
2. Matter Effect
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 - Solar Neutrino Problem
 - Stimulated Neutrino Oscillations
3. Understanding Stimulated Oscillations
 - Hamiltonian, and Basis
 - Single Frequency Matter Profile
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OVERVIEW

Introduction

- What are Neutrinos

- Neutrino Oscillations

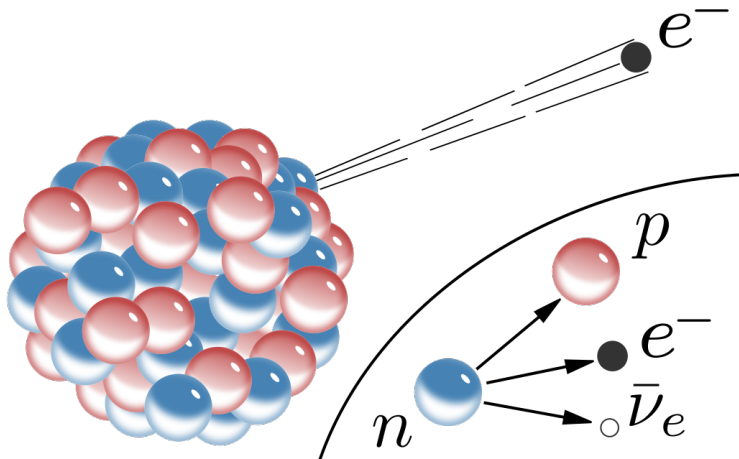
- Why Do Neutrinos Oscillate

Matter Effect

Understanding Stimulated Oscillations

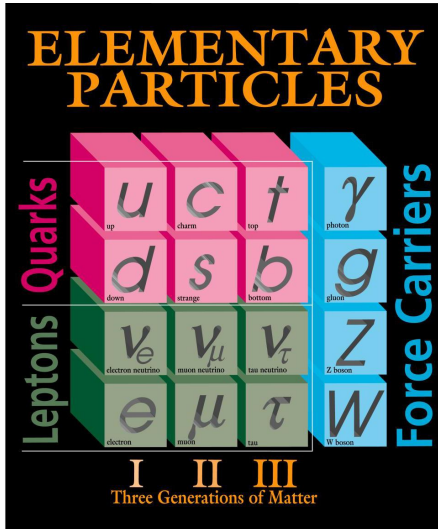
Summary & Future Work

WHAT ARE NEUTRINOS?



Beta decay and antineutrino production. Source: Beta_Decay@Wikipedia

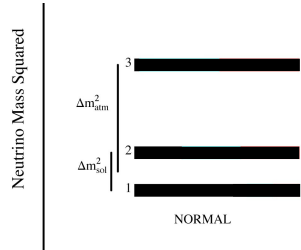
WHAT ARE NEUTRINOS?



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Neutrinos are

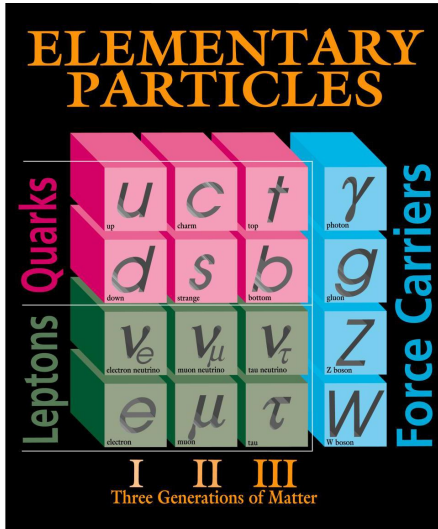
- fermions,
- electrically neutral,
- light.



Adapted from Olga Mena & Stephen Parke (2004)

Table of elementary particles. Source:
Fermilab

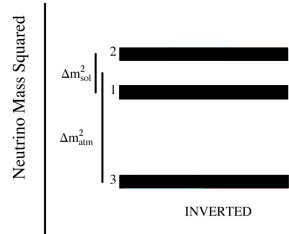
WHAT ARE NEUTRINOS?



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Neutrinos are

- fermions,
- electrically neutral,
- light.



Adapted from Olga Mena & Stephen Parke (2004)

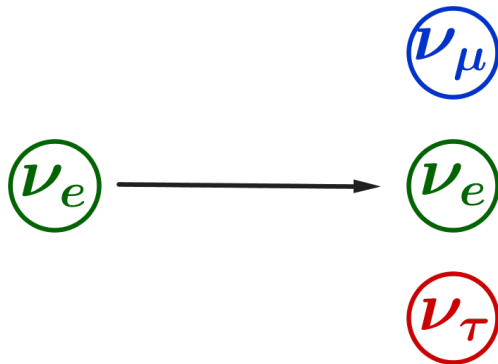
Table of elementary particles. Source:
Fermilab

WHAT IS NEUTRINO OSCILLATION?

Neutrino Oscillation

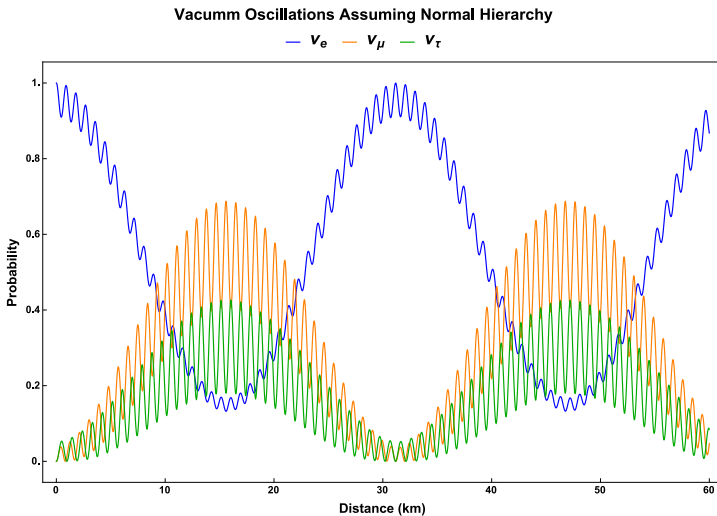
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Neutrino Flavor Conversion



Neutrino Oscillations

WHAT IS NEUTRINO OSCILLATION?



Probabilities of finding neutrinos to be in each flavor.

WHY DO NEUTRINOS OSCILLATE?

Equation of Motion

$$i\partial_x |\Psi\rangle = \hat{\mathbf{H}} |\Psi\rangle$$

- Basis: Hamiltonian diagonalized basis/mass basis/propagation basis, $\{|\nu_1\rangle, |\nu_2\rangle\}$.

►

$$\mathbf{H} = -\frac{\omega_v}{2}\sigma_3, \quad \text{where } \omega_v = \frac{\delta m^2}{2E} = \frac{m_2^2 - m_1^2}{2E}.$$

- The system can be solved given initial condition of the amplitudes of the two eigenstates $(\langle\nu_1|\Psi(0)\rangle, \langle\nu_2|\Psi(0)\rangle)^T$,

$$\begin{pmatrix} \langle\nu_1|\Psi(x)\rangle \\ \langle\nu_2|\Psi(x)\rangle \end{pmatrix} = \begin{pmatrix} \langle\nu_1|\Psi(0)\rangle \exp(i\omega_v x/2) \\ \langle\nu_2|\Psi(0)\rangle \exp(-i\omega_v x/2) \end{pmatrix}$$

WHY DO NEUTRINOS OSCILLATE?

Flavor basis

Neutrino wave function in flavor basis $\{|\nu_e\rangle, |\nu_\mu\rangle\}$ is related to state in energy basis $\{|\nu_1\rangle, |\nu_2\rangle\}$ through

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_v & \sin \theta_v \\ -\sin \theta_v & \cos \theta_v \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

θ_v : vacuum mixing angle

WHY DO NEUTRINOS OSCILLATE?

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θ_v : vacuum mixing angle

Hamiltonian H

Mass basis

$$\begin{aligned} & \frac{\omega_v}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= -\frac{\omega_v}{2} \sigma_3 \end{aligned}$$

Flavor basis

$$\begin{aligned} & \frac{\omega_v}{2} \begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix} \\ &= \frac{\omega_v}{2} (-\cos 2\theta_v \sigma_3 + \sin 2\theta_v \sigma_1) \end{aligned}$$

NATURE OF NEUTRINO OSCILLATION

Transition Probability

$$P(|\nu_e\rangle \rightarrow |\nu_\mu\rangle) = \sin^2(2\theta_v) \sin^2(\omega_v x/2)$$

- ▶ $\omega_v = (m_2^2 - m_1^2)/2E$ determines oscillation wavelength.
- ▶ Mixing angle θ_v determines flavor oscillation amplitude.

OVERVIEW

Introduction

Matter Effect

- Matter Interaction

- MSW Effect

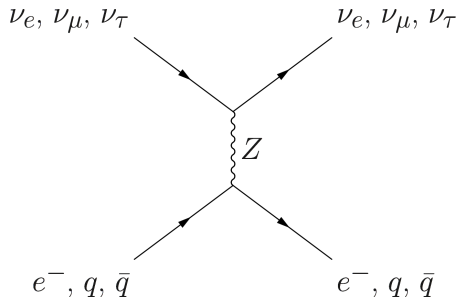
- Solar Neutrino Problem

- Stimulated Neutrino Oscillations

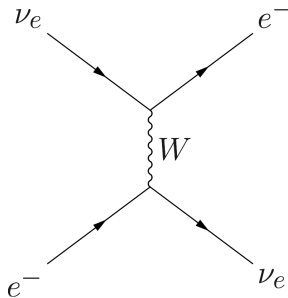
Understanding Stimulated Oscillations

Summary & Future Work

MATTER INTERACTION



Neutral current interaction between
 ν_e, ν_μ, ν_τ ,
and e^- , quarks and antiquarks.



Charged current interaction between
 ν_e and e^-

MATTER INTERACTION

Hamiltonian with matter interaction in flavor basis ($\omega_V = \delta m^2/2E$):

$$\mathbf{H} = \frac{\omega_V}{2} \begin{pmatrix} -\cos 2\theta_V & \sin 2\theta_V \\ \sin 2\theta_V & \cos 2\theta_V \end{pmatrix} \pm \sqrt{2} G_F n_e(x) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

► Vacuum Hamiltonian

► Matter interaction

MATTER INTERACTION

Hamiltonian with matter interaction in flavor basis ($\omega_v = \delta m^2/2E$):

$$\mathbf{H} = \frac{\omega_v}{2} (-\cos 2\theta_v \sigma_3 + \sin 2\theta_v \sigma_1) + \frac{\lambda(x)}{2} \sigma_3$$

► Vacuum Hamiltonian

► Matter interaction

► $\lambda(x) = \sqrt{2}G_F n_e(x)$

MSW EFFECT

Hamiltonian in Vacuum

$$\mathbf{H}_{\text{vacuum}} = \frac{\omega_v \cos 2\theta_v}{2} \sigma_3 + \frac{\omega_v \sin 2\theta_v}{2} \sigma_1$$

$$\begin{aligned}\mathbf{H} &= \frac{\lambda(x) - \omega_v \cos 2\theta_v}{2} \sigma_3 + \frac{\omega_v \sin 2\theta_v}{2} \sigma_1 \\ &= \frac{\omega_m(x) \cos 2\theta_m(x)}{2} \sigma_3 + \frac{\omega_m(x) \sin 2\theta_m(x)}{2} \sigma_1,\end{aligned}$$

where

$$\begin{aligned}\omega_m(x) &= \sqrt{(\lambda(x) - \omega_v \cos 2\theta_v)^2 + \omega_v^2 \sin^2 2\theta_v}, \\ \tan 2\theta_m(x) &= \frac{\omega_v \sin 2\theta_v}{\omega_v \cos 2\theta_v - \lambda(x)}.\end{aligned}$$

MSW EFFECT

Constant matter profile λ_0 as an example,

Significance of θ_m

Define matter basis $\{|\nu_L\rangle, |\nu_H\rangle\}$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} |\nu_L\rangle \\ |\nu_H\rangle \end{pmatrix}$$

In matter basis

$$\mathbf{H}_{\text{matter-basis}} = -\frac{\omega_m}{2} \sigma_3$$

MSW RESONANCE

Hamiltonian with Matter Potential

$$\begin{aligned}\mathbf{H} &= \frac{\lambda(x) - \omega_v \cos 2\theta_v}{2} \boldsymbol{\sigma}_3 + \frac{\omega_v \sin 2\theta_v}{2} \boldsymbol{\sigma}_1 \\ &= \frac{\omega_m(x) \cos 2\theta_m(x)}{2} \boldsymbol{\sigma}_3 + \frac{\omega_m(x) \sin 2\theta_m(x)}{2} \boldsymbol{\sigma}_1\end{aligned}$$

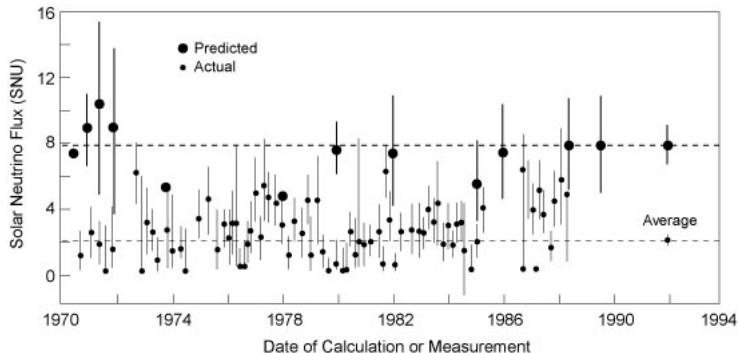
$$\tan 2\theta_m(x) = \frac{\omega_v \sin 2\theta_v}{\omega_v \cos 2\theta_v - \lambda(x)}.$$

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} |\nu_L\rangle \\ |\nu_H\rangle \end{pmatrix}$$

Transition Probability

$$P(|\nu_e\rangle \rightarrow |\nu_\mu\rangle) = \sin^2(2\theta_m) \sin^2(\omega_m x)$$

SOLAR NEUTRINO PROBLEM



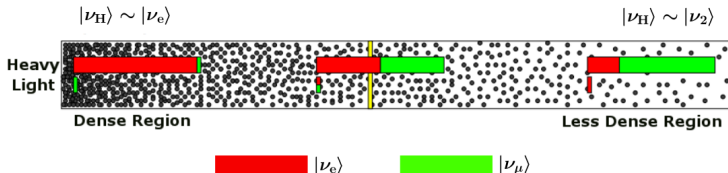
Chlorine detector (Homestake experiment) results and theory predictions.
SNU: 1 event for 10^{36} target atoms per second. Kenneth R. Lang (2010)

MSW EFFECT AND SOLAR NEUTRINOS

$$\mathbf{H} = \frac{\lambda(x) - \omega_v \cos 2\theta_v}{2} \sigma_3 + \frac{\omega_v \sin 2\theta_v}{2} \sigma_1$$

$$\begin{pmatrix} |\nu_L\rangle \\ |\nu_H\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_m & -\sin \theta_m \\ \sin \theta_m & \cos \theta_m \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

$$\mathbf{H}_{\text{matter-basis}} = -\frac{\omega_m}{2} \sigma_3$$



Yellow bar is the resonance point. Red: $|\nu_e\rangle$. Green: $|\nu_\mu\rangle$. Adapted from Smirnov, 2003.

MSW EFFECT

Suppose $\omega_v = (m_2^2 - m_1^2)/2E < 0$,

$$\mathbf{H} = -\frac{\omega_v}{2} \begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix} + \sqrt{2}G_F n_e(x) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

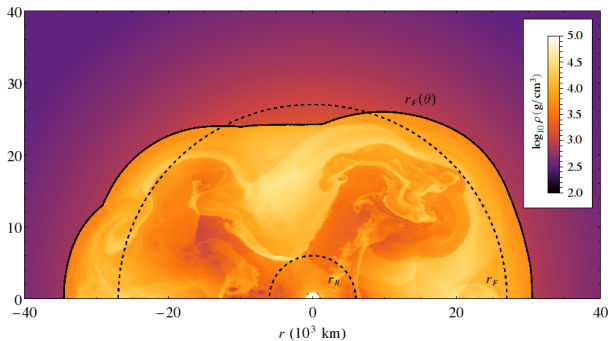


$$\mathbf{H} = \left(\frac{-\omega_v}{2} \cos 2\theta_v + \frac{\lambda(x)}{2} \right) \sigma_3 - \frac{\omega_v}{2} \sin 2\theta_v \sigma_1$$

SUPERNOVA MATTER DENSITY PROFILE

Why Do We Care

Astrophysical environments: supernovae, accretion disks etc



Supernova shock and turbulence. E. Borriello, et al (2014)

$$\Delta n_e(r) = \sum_n c_n \sin(k_n r + \phi_n)$$

STIMULATED NEUTRINO OSCILLATIONS

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x)$$

Basis

Background matter basis: Hamiltonian is diagonalized with only background matter profile λ_0 ,

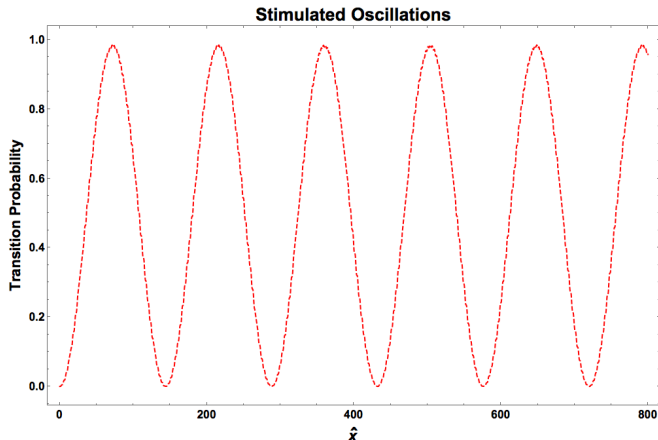
$$H_{\text{background}} = -\frac{\omega_m}{2} \sigma_3.$$

Hamiltonian

$$\mathbf{H} = \frac{1}{2} (-\omega_m + \delta\lambda(x) \cos 2\theta_m) \sigma_3 - \frac{\delta\lambda(x)}{2} \sin \theta_m \sigma_1.$$

STIMULATED NEUTRINO OSCILLATIONS

P. Krastev and A. Smirnov (1989); J. Kneller et al (2013);
K. Patton et al (2014);



Stimulated oscillations. $\lambda(x) = \lambda_0 + A \sin(kx)$ with $\hat{x} = \omega_m x$, $A = 0.1\omega_m$,
 $k = 0.995\omega_m$, $\theta_m = \pi/6$

OVERVIEW

Introduction

Matter Effect

Understanding Stimulated Oscillations

- Hamiltonian, and Basis

- Single Frequency Matter Profile

- Two-frequency Matter Profile

Summary & Future Work

UNDERSTANDING STIMULATED OSCILLATIONS

Matter profile

$$\lambda(x) = \lambda_0 + A \sin(kx),$$

Hamiltonian in Background Matter Basis

$$\mathbf{H} = \frac{1}{2} (-\omega_m + \delta\lambda(x) \cos 2\theta_m) \boldsymbol{\sigma}_3 - \frac{\delta\lambda(x)}{2} \sin \theta_m \boldsymbol{\sigma}_1.$$

A Better Basis

Define new basis $\{|\tilde{\nu}_L\rangle, |\tilde{\nu}_H\rangle\}$ is related to background matter basis $\{|\nu_L\rangle, |\nu_H\rangle\}$ through

$$\begin{pmatrix} |\nu_L\rangle \\ |\nu_H\rangle \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{i\eta(x)} \end{pmatrix} \begin{pmatrix} |\tilde{\nu}_L\rangle \\ |\tilde{\nu}_H\rangle \end{pmatrix},$$

where

$$\eta(x) - \eta(0) = -\frac{\omega_m}{2}x + \frac{\cos 2\theta_m}{2} \int_0^x \delta\lambda(\tau) d\tau.$$

SINGLE FREQUENCY MATTER PROFILE

Hamiltonian in new basis

$$\tilde{\mathbf{H}} = -\frac{\delta\lambda(x)}{2} \sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix} = \begin{pmatrix} 0 & h \\ h^* & 0 \end{pmatrix}$$

Hamiltonian in New Basis

$$\begin{aligned} h &\equiv -\frac{\delta\lambda(x)}{2} e^{2i\eta(x)} \\ &= \frac{i}{4} \left[\exp \left(i(k + \omega_m)x + i \cos 2\theta_m \frac{A}{k} \cos(kx) \right) \right. \\ &\quad \left. - \exp \left(i(-k + \omega_m)x + i \cos 2\theta_m \frac{A}{k} \cos(kx) \right) \right] \end{aligned}$$

RABI OSCILLATION

Rabi Oscillation

Hamiltonian

$$\begin{pmatrix} -\omega_0/2 & \alpha\omega_0 e^{i\omega x} \\ \alpha\omega_0 e^{-ikx} & \omega_0/2 \end{pmatrix},$$

$$E_2 = \frac{\omega_0}{2}$$

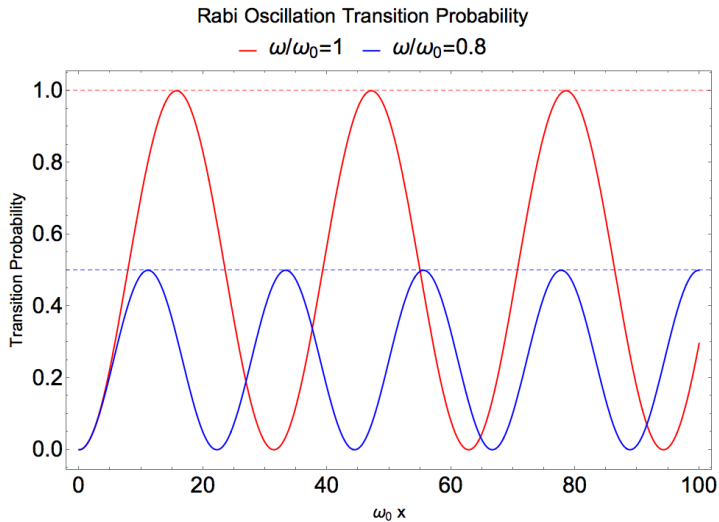
$$E_1 = -\frac{\omega_0}{2}$$

Incoming light

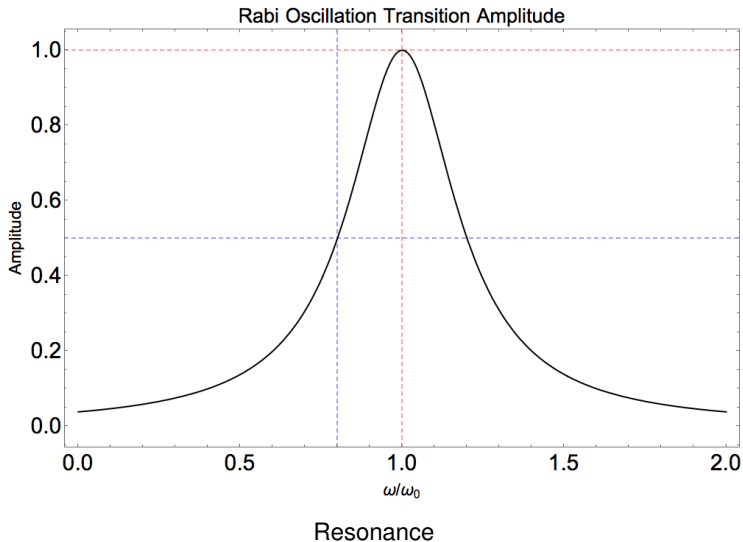


Frequency : ω

RABI OSCILLATIONS



RABI OSCILLATIONS



SINGLE FREQUENCY MATTER PROFILE

Off-diagonal Term in Our System

$$\tilde{\mathbf{H}} = \begin{pmatrix} 0 & h \\ h^* & 0 \end{pmatrix}$$

$$h \propto \left[\exp \left(i(k + \omega_m)x + i \cos 2\theta_m \frac{A}{k} \cos(kx) \right) - \exp \left(i(-k + \omega_m)x + i \cos 2\theta_m \frac{A}{k} \cos(kx) \right) \right]$$

Jacobi-Anger expansion

$$e^{i\beta \cos(kx)} = \sum_{n=-\infty}^{\infty} i^n J_n(\beta) e^{inkx},$$

where $J_n(\beta)$ are Bessel's functions of the first kind.

SINGLE FREQUENCY MATTER PROFILE

Scaled Quantities

Characteristic scale: ω_m

- ▶ $\hat{A} = A/\omega_m$
- ▶ $\hat{k} = k/\omega_m$
- ▶ $\hat{x} = \omega_m x$
- ▶ $\hat{h} = h/\omega_m$

SINGLE FREQUENCY MATTER PROFILE

Rotation Wave Approximation

The off-diagonal element of Hamiltonian

$$\tilde{\mathbf{H}} = \sum_{n=-\infty}^{\infty} \begin{pmatrix} 0 & \frac{1}{2}\hat{B}_n e^{i(n\hat{k}-1)\hat{x}} \\ \frac{1}{2}\hat{B}_n^* e^{-i(n\hat{k}-1)\hat{x}} & 0 \end{pmatrix}$$

where $\hat{B}_n = -(-i)^n n \hat{k} \tan 2\theta_m J_n(\hat{A} \cos 2\theta_m / \hat{k})$.

SINGLE FREQUENCY MATTER PROFILE

Transition Probability

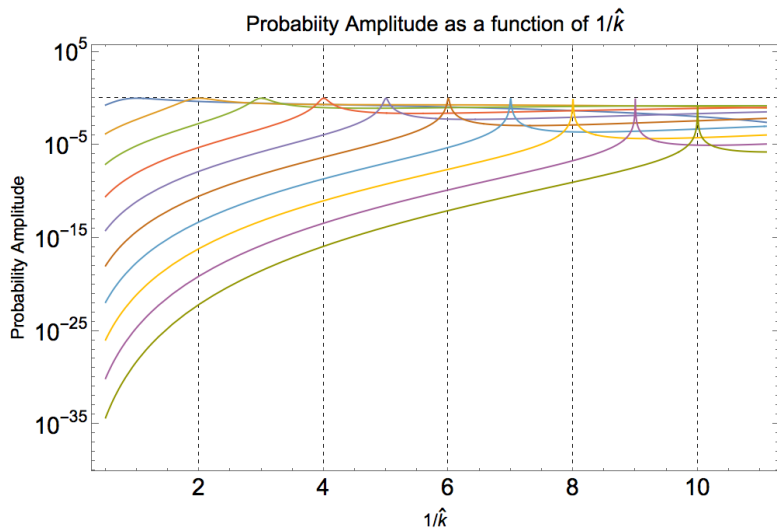
$$P_{L \rightarrow H}^{(n)} = \frac{|\hat{B}_n|^2}{|\hat{B}_n|^2 + (n\hat{k} - 1)^2} \sin^2 \left(\frac{q^{(n)}}{2} x \right),$$

where

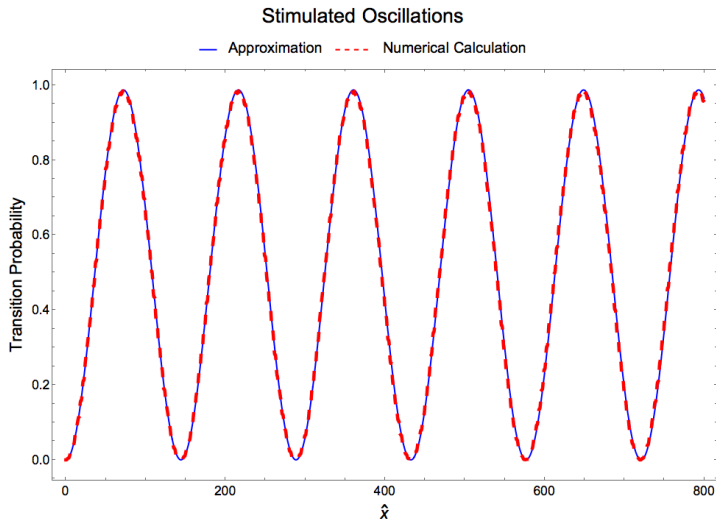
$$q^{(n)} = \sqrt{|\Gamma^{(n)}|^2 + (n\hat{k} - 1)^2}, \quad \text{frequency of oscillations}$$

$$\Gamma^{(n)} = |\hat{B}_n|, \quad \text{width of resonance } (n\hat{k} \text{ as parameter})$$

SINGLE FREQUENCY MATTER PROFILE

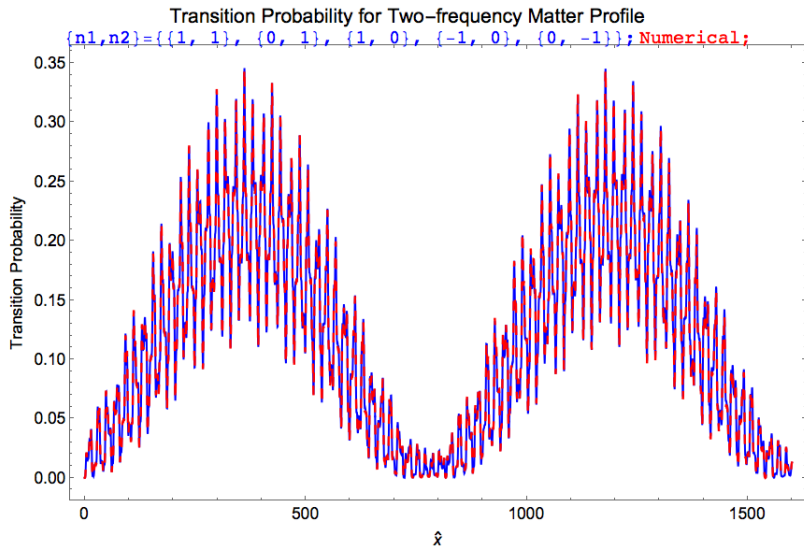


SINGLE FREQUENCY MATTER PROFILE



$$\hat{A} = 0.1, \hat{k} = 0.995, \theta_m = \pi/6$$

TWO-FREQUENCY MATTER PROFILE



$$\lambda(x) = \lambda_0 + A_1 \sin(k_1 x) + A_2 \sin(k_2 x). \quad \hat{k}_1 = 0.3, \hat{k}_2 = 0.7, A_1 = A_2 = 0.1, \\ \theta_m = \pi/5.$$

OVERVIEW

Introduction

Matter Effect

Understanding Stimulated Oscillations

Summary & Future Work

SUMMARY & FUTURE WORK

- ▶ The fact that neutrino flavor states are not mass states causes vacuum oscillations.
- ▶ MSW resonance happens when matter potential cancels out the vacuum diagonal elements of the Hamiltonian.
- ▶ Even matter profile doesn't match MSW requirement, variation in matter profile can cause resonances.
- ▶ Single frequency perturbations in matter profile is a combination of many Rabi oscillations.
- ▶ How to understand and calculate systems with multi-frequency matter profile (turbulence).
- ▶ Combine periodic or even turbulent matter profile with neutrino self-interaction.

ACKNOWLEDGEMENT

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BACKUP SLIDES

BACKUP SLIDES

PARAMETERS USED FOR VACUUM OSCILLATIONS

$$\theta_{12} = 33.36/180\pi; \theta_{13} = 8.66/180\pi; \theta_{23} = 40/180 * \pi; \delta_{cp} = 0;$$
$$m_1^2 = 0.01; m_2^2 = m_1^2 + 0.000079; E = 1\text{MeV}$$

SINGLE FREQUENCY MATTER PROFILE

Why Does It Work?

$$J_n(n \operatorname{sech} \alpha) \sim \frac{e^{-n(\alpha - \tanh \alpha)}}{\sqrt{2\pi n \tanh \alpha}}, \quad \text{for large } n$$

\Rightarrow

$$\Gamma \propto \hat{B}_n \propto \frac{e^{-n(\alpha - \tanh \alpha)}}{\sqrt{2\pi n \tanh \alpha}}$$

Small perturbation \Rightarrow Small $\hat{A} \Rightarrow$ Large $\alpha \Rightarrow$ Drops fast at large n .

TWO-FREQUENCY MATTER PROFILE

Matter Profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x), \quad \delta\lambda(x) = A_1 \sin(k_1 x) + A_2 \sin(k_2 x).$$

TWO-FREQUENCY MATTER PROFILE $\hat{h} = \sum_{n=-\infty}^{\infty} \frac{1}{2} \hat{B}_n e^{i(n\hat{k}-1)\hat{x}}$,

Hamiltonian Off-diagonal Element

Apply Jacobi-Anger expansion,

$$\hat{h} = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \frac{1}{2} \hat{B}_{n_1, n_2}(\hat{k}_1, \hat{k}_2) e^{i(n_1 \hat{k}_1 + n_2 \hat{k}_2 - 1)\hat{x}},$$

where

$$\begin{aligned} & \hat{B}_{n_1, n_2}(\hat{k}_1, \hat{k}_2) \\ &= -(-i)^{n_1+n_2} (n_1 \hat{k}_1 + n_2 \hat{k}_2) J_{n_1} \left(\frac{\hat{A}_1 \cos 2\theta_m}{\hat{k}_1} \right) J_{n_2} \left(\frac{\hat{A}_2 \cos 2\theta_m}{\hat{k}_2} \right) \end{aligned}$$

Which terms are important?

TWO-FREQUENCY MATTER PROFILE

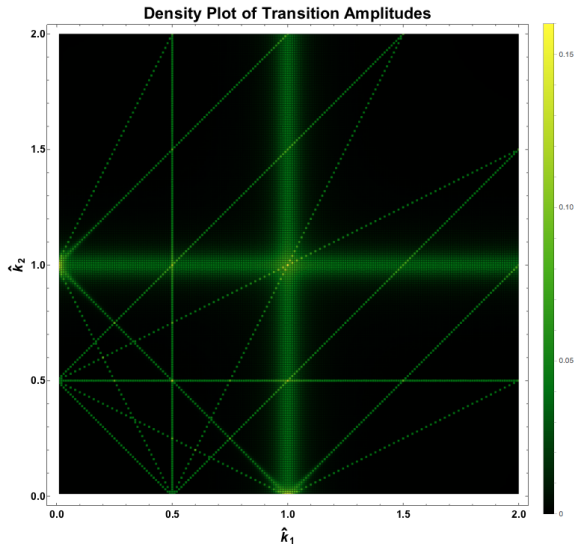
Resonance Lines

There are still resonances, i.e., (almost) zero phases, on lines

$$n_{1,0}\hat{k}_1 + n_{2,0}\hat{k}_2 - 1 = 0$$

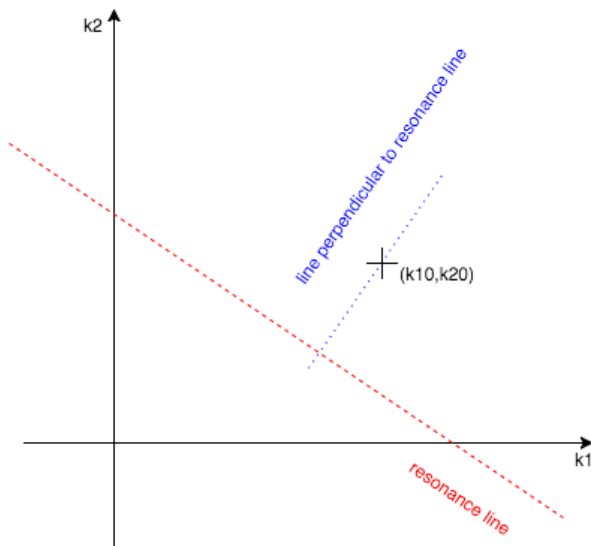
in $\{\hat{k}_1, \hat{k}_2\}$ plane. \Rightarrow Resonance width for each point on resonance lines.

TWO-FREQUENCY MATTER PROFILE $\hat{h} \equiv \sum_{n_1} \sum_{n_2} \frac{1}{2} \hat{B}_{n_1, n_2}(\hat{k}_1, \hat{k}_2) e^{i(n_1 \hat{k}_1 + n_2 \hat{k}_2 - 1)\hat{x}}$



Density plot of transition amplitudes calculated using only one term out of the whole summation in Hamiltonian. $n_1, n_2 \in [-2, 2]$

TWO-FREQUENCY MATTER PROFILE



Resonance line, distance to resonance, and width

TWO-FREQUENCY MATTER PROFILE

Width

$$\Gamma_2 = \frac{\hat{B}_{n_1, n_2}(\hat{k}_{1, \text{intercept}}, \hat{k}_{2, \text{intercept}})}{\sqrt{n_1^2 + n_2^2}}.$$

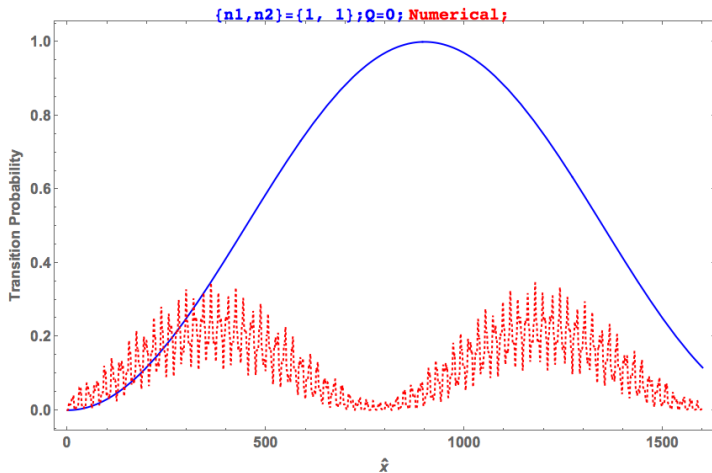
Distance to Resonance Line

$$d = \frac{|n_1 \hat{k}_{10} + n_2 \hat{k}_{20} - 1|}{\sqrt{n_1^2 + n_2^2}}.$$

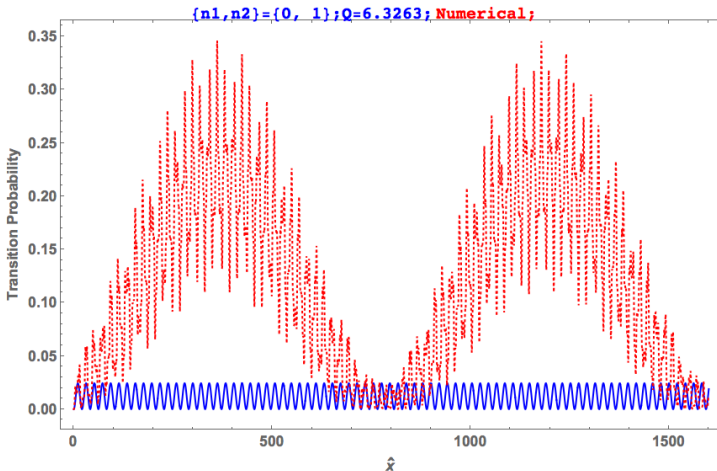
Distance to Resonance Width Ratio

$$Q_2 = \frac{d}{\Gamma_2}.$$

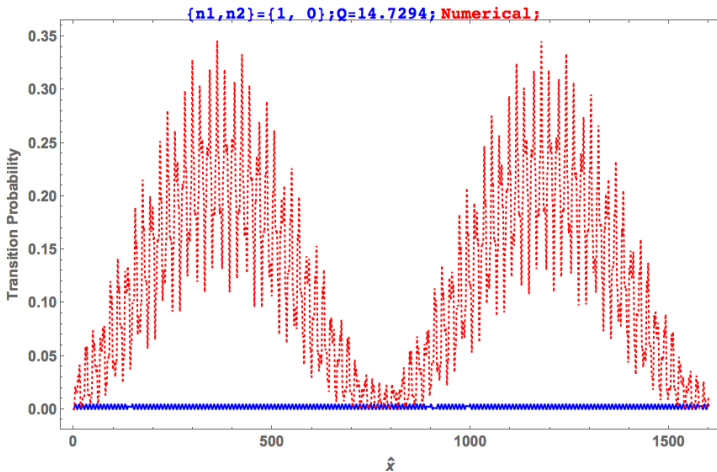
TWO-FREQUENCY MATTER PROFILE



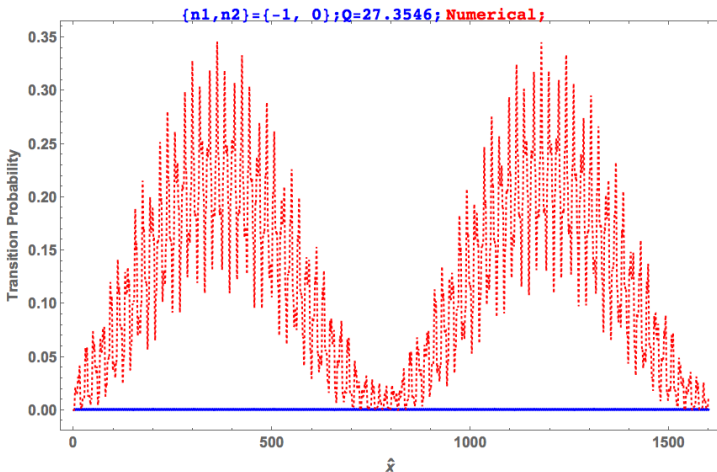
TWO-FREQUENCY MATTER PROFILE



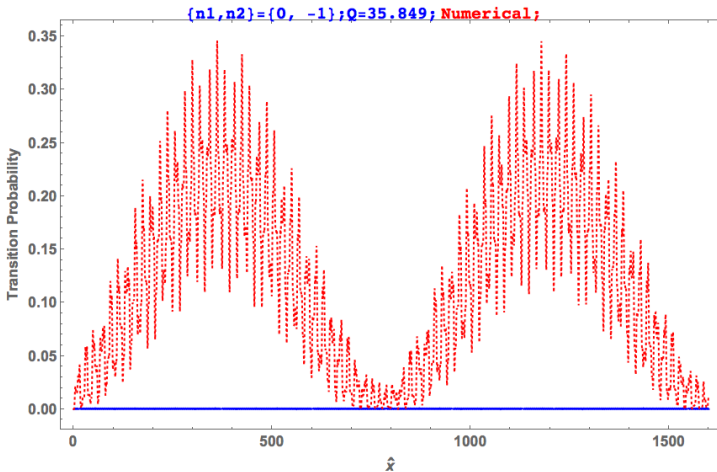
TWO-FREQUENCY MATTER PROFILE



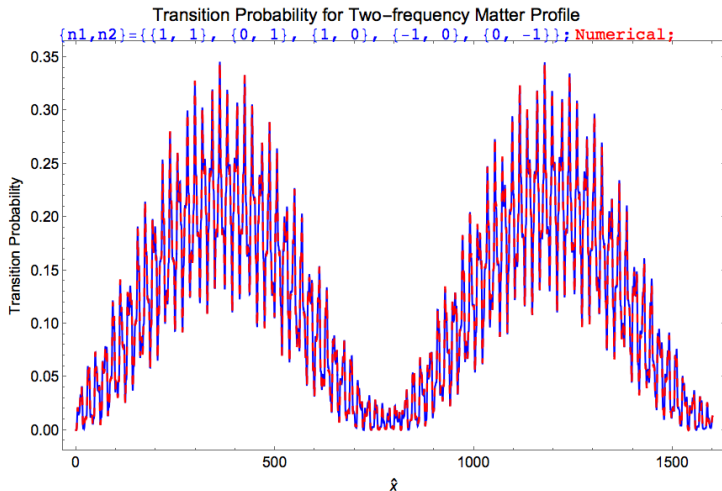
TWO-FREQUENCY MATTER PROFILE



TWO-FREQUENCY MATTER PROFILE



TWO-FREQUENCY MATTER PROFILE



BESSEL'S FUNCTION

$$J_n(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(m+n+1)} \left(\frac{\beta}{2}\right)^{2m+n}$$

REFERENCES I