A SYMMETRIC THEORY OF ELECTRONS AND POSITRONS

Majorana, E. (1937). A symmetric theory of electrons and positrons.(2)
Elliott, S. R., & Franz, M. (2015). Colloquium: Majorana fermions in nuclear, particle, and solid-state physics

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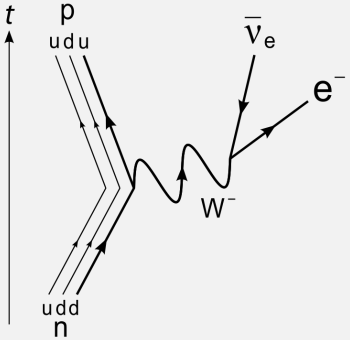
PandA @ UNM

OUTLINE

- Story of the paper
- What is Mass in Dirac Equation
- Majorana Fermion
- Consequences
- Conclusion

STORY OF THE PAPER

Beta Decay and Inverse Beta Decay



$$n \to p + e^- + \bar{\nu}_e \qquad e^- + p \to \nu_e + n$$

Do we really need to invent neutrino and antineutrino at the same time? Maybe we need only one?

Majorana's idea

$$\psi^{(c)} = \psi$$

MASS

Dirac equation

$$(i\not{\partial} - m)\Psi = 0$$

Equations for left and right components in Weyl basis

In Weyl basis, $\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$$(i\partial_t - \vec{p} \cdot \vec{\sigma})\psi_R - m_D \psi_L = 0,$$
$$(i\partial_t + \vec{p} \cdot \vec{\sigma})\psi_L - m_D \psi_R = 0.$$

(Dirac) Mass couples left and right chirality.

Lagrangian

The Lagrangian for Dirac mass is

$$
\begin{aligned}
\mathscr{L}_D &= m_D \bar{\Psi}\Psi \\
&= m_D(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) \\
&= \begin{pmatrix} \psi_L^\dagger & \psi_R^\dagger \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}
\end{aligned}
$$

(Dirac) Mass couples left and right chirality.

MAJORANA FERMION

Majorana Fermion

$$\Psi^{(c)} = \Psi.$$

What is charge conjugation?

Consider electron state $\Psi(p)$ with momentum p and position state $\Psi'(-p)$. They should obey equations

$$\gamma^\mu(i\partial_\mu + eA_\mu - m)\Psi = 0,$$
$$\gamma^\mu(i\partial_\mu - eA_\mu - m)\Psi' = 0.$$

Because they are the same in Feynman diagram.

We define a operator that relates the two states

$$\Psi' = C\Psi.$$

Weyl Representation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ -\bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \sigma^\mu = (\mathrm{I}, \sigma^i), \qquad \bar{\sigma}^\mu = (\mathrm{I}, -\sigma^i).$$

Charge conjugation operator is

$$C = i\gamma^2$$

Apply the Majorana condition $\Psi^{(c)} = \Psi$,

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \Rightarrow \Psi = \begin{pmatrix} \psi_1 \\ -i\sigma^2\psi_1^* \end{pmatrix}$$

In fact here this state is left handed. We have another result which is right handed.

Comparing Majorana EoM and Dirac EoM

Majorana fermion:

$$(i\partial_t - \vec{p}\cdot\vec{\sigma})\psi_R - im_R\sigma^2\psi_R^* = 0,$$
$$(i\partial_t + \vec{p}\cdot\vec{\sigma})\psi_L - im_L\sigma^2\psi_L^* = 0.$$

Dirac fermion:

$$(i\partial_t - \vec{p}\cdot\vec{\sigma})\psi_R - m_D\psi_L = 0,$$
$$(i\partial_t + \vec{p}\cdot\vec{\sigma})\psi_L - m_D\psi_R = 0.$$

Majorana fermion = a special constrained case of Dirac fermion.

CONSEQUENCES

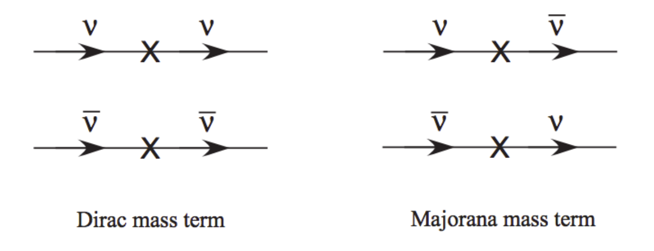


Figure: Kayser B. arXiv:hep-ph/0211134

$$\mathscr{L}_m = \frac{1}{2}\left((\bar{\nu}_L)^c \bar{\nu}_R\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} + h.c..$$

See-saw Mechanism

$$\mathscr{L}_{\mathrm{m}} = \frac{1}{2}\left((\bar{\nu}_{\mathrm{L}})^{\mathrm{c}}\bar{\nu}_{\mathrm{R}}\right)\begin{pmatrix} \mathrm{m_L} & \mathrm{m_D} \\ \mathrm{m_D} & \mathrm{m_R} \end{pmatrix}\begin{pmatrix} \nu_{\mathrm{L}} \\ (\nu_{\mathrm{R}})^{\mathrm{c}} \end{pmatrix} + \mathrm{h.c.}.$$

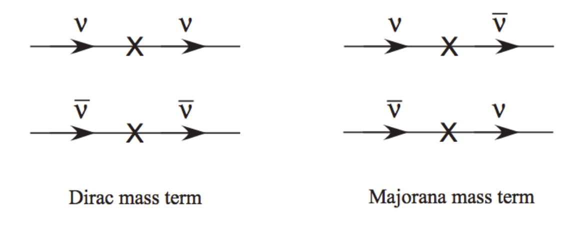
Requre

$$\mathrm{m_L} = 0$$
$$\mathrm{m_R} \gg \mathrm{m_D}.$$

Diagonalize the mass matrix, we have the measurable masses, which are

$$\mathrm{m_1} = \mathrm{m_R}$$
$$\mathrm{m_2} = \mathrm{m_D^2}/\mathrm{m_R}.$$

$$\psi^{(c)} = \psi.$$



1. Ettore Majorana Scientific Papers. pp. 201–33. doi:10.1007/978-3-540-48095-2_10.
2. Wilczek, F. (2009). Majorana returns. Nature Physics, 5(9), 614–618. doi:10.1038/nphys1380
3. Kayser, B. (2002). Neutrino Mass, Mixing, and Flavor Change. http://arxiv.org/abs/hep-ph/0211134.
4. Elliott, S. R., & Franz, M. (2015). Colloquium: Majorana fermions in nuclear, particle, and solid-state physics. Reviews of Modern Physics, 87(March), 137–163. doi:10.1103/RevModPhys.87.137
5. `http://docs.neutrino.xyz/mass.html#majorana-particle`

Backups

In Dirac equation, $\mathrm{m} \to 0$ leads to decoupling of left and right.

Lagrangian with Dirac mass term is

$$\mathscr{L} = \frac{\mathrm{i}}{2}\bar{\Psi}\overleftrightarrow{\partial}\Psi - \mathrm{m}\bar{\Psi}\Psi.$$

Using action principle,

$$\frac{\partial \mathscr{L}}{\partial \bar{\Psi}} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \bar{\Psi})} = 0$$

and the fact that

$$\frac{\partial \mathscr{L}}{\partial \bar{\Psi}} = \frac{\mathrm{i}}{2}\not{\partial}\Psi - \mathrm{m}\Psi$$
$$\frac{\partial \mathscr{L}}{\partial(\partial_\mu \bar{\Psi})} = -\frac{\mathrm{i}}{2}\gamma^\mu \Psi$$

I have the equation of motion,

$$\frac{i}{2}\not{\partial}\Psi - m\Psi + \frac{i}{2}\partial_\mu\gamma^\mu\Psi = 0,$$

which simplifies to

$$(i\not{\partial} - m)\Psi = 0.$$

Its conjugate is

$$\bar{\Psi}(\overleftarrow{\not{\partial}} + m) = 0.$$

A state is composed of two spinors

$$\Psi \rightarrow \begin{pmatrix} \xi \\ \eta \end{pmatrix}$$

Recall we have a chirality operator,

$$\mathrm{P}_{\pm} = \frac{1}{2}(1 \pm \gamma^5)$$

The Lagrangian using the chirality states are

$$\mathscr{L} = \frac{\mathrm{i}}{2}\bar{\Psi}_{\mathrm{R}}\mathrm{i}\overleftrightarrow{\not{\partial}}\Psi_{\mathrm{R}} + \frac{\mathrm{i}}{2}\bar{\Psi}_{\mathrm{L}}\mathrm{i}\overleftrightarrow{\not{\partial}}\Psi_{\mathrm{L}} - \mathrm{m}\left(\bar{\Psi}_{\mathrm{R}}\Psi_{\mathrm{L}} + \bar{\Psi}_{\mathrm{L}}\Psi_{\mathrm{R}}\right).$$

It is obvious that mass couples left and right in Dirac theory.

Using

$$\bar{\Psi} = \Psi^\dagger \gamma^0,$$

the Dirac mass term is

$$\mathscr{L}_{\mathrm{Dirac}} = -m(\xi^\dagger \eta + \eta^\dagger \xi).$$

Majorana mass term is

$$\mathscr{L}_{\mathrm{Majorana}} = \frac{1}{2}\left(\mathrm{m}\eta^{\dagger}\epsilon\eta - \mathrm{m}^{*}\mathrm{eta}^{\mathsf{T}}\epsilon\eta\right).$$

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^1 = \begin{pmatrix} \mathrm{i}\sigma^3 & 0 \\ 0 & \mathrm{i}\sigma^3 \end{pmatrix},$$

$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} -\mathrm{i}\sigma^1 & 0 \\ 0 & -\mathrm{i}\sigma^1 \end{pmatrix},$$

$$\gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}.$$

$$\gamma^0 = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & -\mathrm{I} \end{pmatrix},$$
$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},$$
$$\gamma^5 = \begin{pmatrix} 0 & \mathrm{I} \\ \mathrm{I} & 0 \end{pmatrix}.$$

CREATION AND ANNIHILATION

Field	Effect on ν	Effect on $\bar{\nu}$
$\nu_{\mathrm{L,R}}$	Annihilation	Creation
$\bar{\nu}_{\mathrm{L,R}}$	Creation	Annihilation
$\nu_{\mathrm{L,R}}^{(\mathrm{c})}$	Creation	Annihilation
$\overline{\nu_{\mathrm{L,R}}}^{(\mathrm{c})}$	Annihilation	Creation