Modified Gravity as Dark Energy

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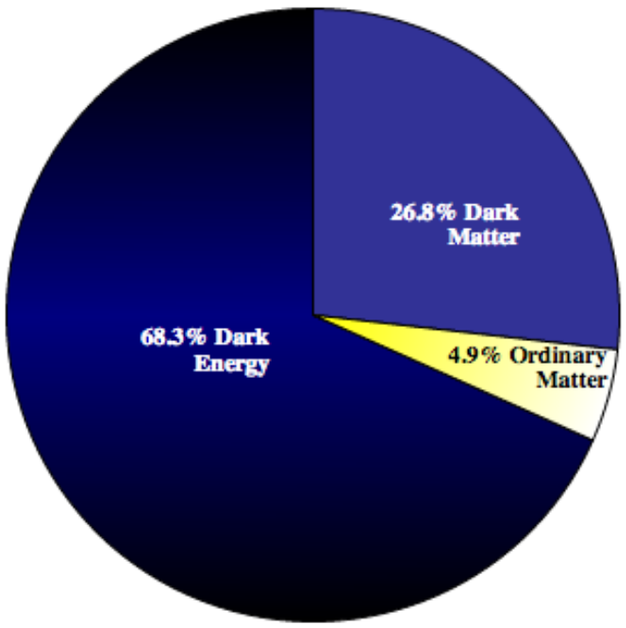
Outline

- Why?
- How to Modify Gravity?
- f(R)
- Interacting DE

Why Bother

The Pie



~~DM and~~ DE models are good

~~Dark Matter~~

Dark Energy

Why?
Detected?
Is DE exotic matter?

We know so little about DE

Equation of state?

Interactions?

Why Bother

Adding more terms
Tweaking simple terms

More Complicated

No strange matter
Expansion arise naturally

More Natural

How to Modify?

Starting from action

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\ R\, d^4x + \frac{1}{8\pi G}\int \sqrt{-g}\ (-\Lambda)\, d^4x + \int L_m(g_{\mu\nu}, \phi)\, d^4x$$

- Newton's constant
- Metric
- Ricci scalar R
- Dimensions
- Extra fields
- Transformations

f(R)

R ⟹ f(R)

$$S_{fR} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\ f(R) + S_m$$

FRW universe

$$H^2 + \dot{H} - H\frac{\dot{F}}{F} - \frac{1}{6}\frac{f}{F} = \frac{1}{-F}\frac{\kappa^2}{3}\rho_m \qquad H^2 = \frac{8\pi G}{3}\rho_m$$

$$\dot{H} - H\frac{\dot{F}}{2F} + \frac{\ddot{F}}{2F} = \frac{1}{-F}\frac{\kappa^2}{2}(1+w_m)\rho_m \qquad \dot{H} = -4\pi G(1+w_m)\rho_m$$

f(R)

Field equation

$$G_{\mu\nu} = \frac{\kappa^2}{F}\left(T^{(e)}_{\mu\nu} + T^{(m)}_{\mu\nu}\right) \qquad T^{(e)}_{\mu\nu} = \frac{1}{2\,\kappa^2}\left(f - F\,R\right) g_{\mu\nu} + \frac{1}{\kappa^2}\left(\nabla_\mu \nabla_\nu - g_{\mu\nu}\,\Box\right) F$$

Coordinate transformation

$$\tilde{G}_{\mu\nu} = \kappa^2\left(\tilde{T}^{(e)}_{\mu\nu} + \tilde{T}^{(m)}_{\mu\nu}\right)$$

$$\tilde{T}^{(e)}_{\mu\nu} = \tilde{\nabla}_\mu \phi\, \tilde{\nabla}_\nu \phi - \frac{1}{2}\,\tilde{g}_{\mu\nu}\,\tilde{g}^{\tau\sigma}\,\tilde{\nabla}_\tau \phi\, \tilde{\nabla}_\sigma \phi - \tilde{g}_{\mu\nu}\,V,$$

$$V = \frac{F\,R - f}{2\,\kappa^2\,F^2},$$

$$T^{(m)}_{\mu\nu} = \Omega^2\,\tilde{T}^{(m)}_{\mu\nu}\ .$$

f(R)

Conformal transformation

$$\ln(F) = \kappa \sqrt{\frac{2}{3}}\ \phi,\ \Omega^2 = F$$

$$\tilde{ds}^2 = \Omega^2\, ds^2,\ \tilde{g}_{\mu\nu} = \Omega^2\, g_{\mu\nu},\ \tilde{g}^{\mu\nu} = g^{\mu\nu} \Big/ \Omega^2$$

Conservation

$$\tilde{\nabla}^{\mu}\, \tilde{T}^{(m)}_{\mu\nu} = \frac{-\kappa}{\sqrt{6}}\, \tilde{T}^{(m)}\, \tilde{\nabla}_{\nu}\phi = \frac{-\tilde{T}^{(m)}}{2}\, \tilde{\nabla}_{\nu} \ln(F).$$

Energy-momentum tensor is not conserved.

Energy flow between gravitation effective EM tensor and matter EM tensor.

Interacting Dark Energy

Friedmann equation

$$\dot{\rho}_c + 3\,H\,\rho_c = Q_c\,,$$

$$\dot{\rho}_d + 3\,H(1+w)\,\rho_d = -Q_c\,.$$

Simple models

- $Q_c = \xi\,H\,\rho_c$
- $Q_c = \xi\,H\,\rho_d$

Qc=0: LCDM universe

Qc<0: energy flow to DE

Qc>0: energy flow to DM

Interacting Dark Energy

$$Q_c = \xi H \rho_c$$

$$\Omega_m = \Omega_{\rm m0}(1+z)^{3-\xi}$$

$$\Omega_d = \left(\Omega_{\rm d0} + \frac{\xi}{3w+\xi}\Omega_{\rm m0}\right)(1+z)^{3(1+w)} + \frac{-\xi}{\xi+3w}\Omega_m \equiv \tilde{\Omega}_{\rm d0}(1+z)^3 + \frac{-\xi}{\xi+3w}\Omega_m$$

$$Q_c = \xi H \rho_d$$

$$\Omega_m = \left(\Omega_{\rm m0} + \frac{\xi}{\xi+3w}\Omega_{\rm d0}\right)(1+z)^3 + \frac{-\xi}{\xi+3w}\Omega_d \equiv \tilde{\Omega}_{\rm m0}(1+z)^3 + \frac{-\xi}{\xi+3w}\Omega_d$$

$$\Omega_d = \Omega_{\rm d0}(1+z)^{3(1+w)+\xi}$$

1. Change the amplitude of the evolution of matter or dark energy energy density;
2. Transfer energy between DE and DM.

Interacting Dark Energy

- Models
  - **a.** ICC: $Q_c = \xi H \rho_c$ with a constant EoS
  - **b.** ICCPL: $Q_c = \xi H \rho_c$ with a CPL parametrized EoS
  - **c.** I2CC: $Q_c = \xi H \rho_d$ with a constant EoS
  - **d.** I2CCPL: $Q_c = \xi H \rho_d$ with a CPL parametrized EoS

Interacting Dark Energy

Two important parameters

$$q \stackrel{\text{def}}{=} -\frac{\ddot{a}a}{\dot{a}^2}$$

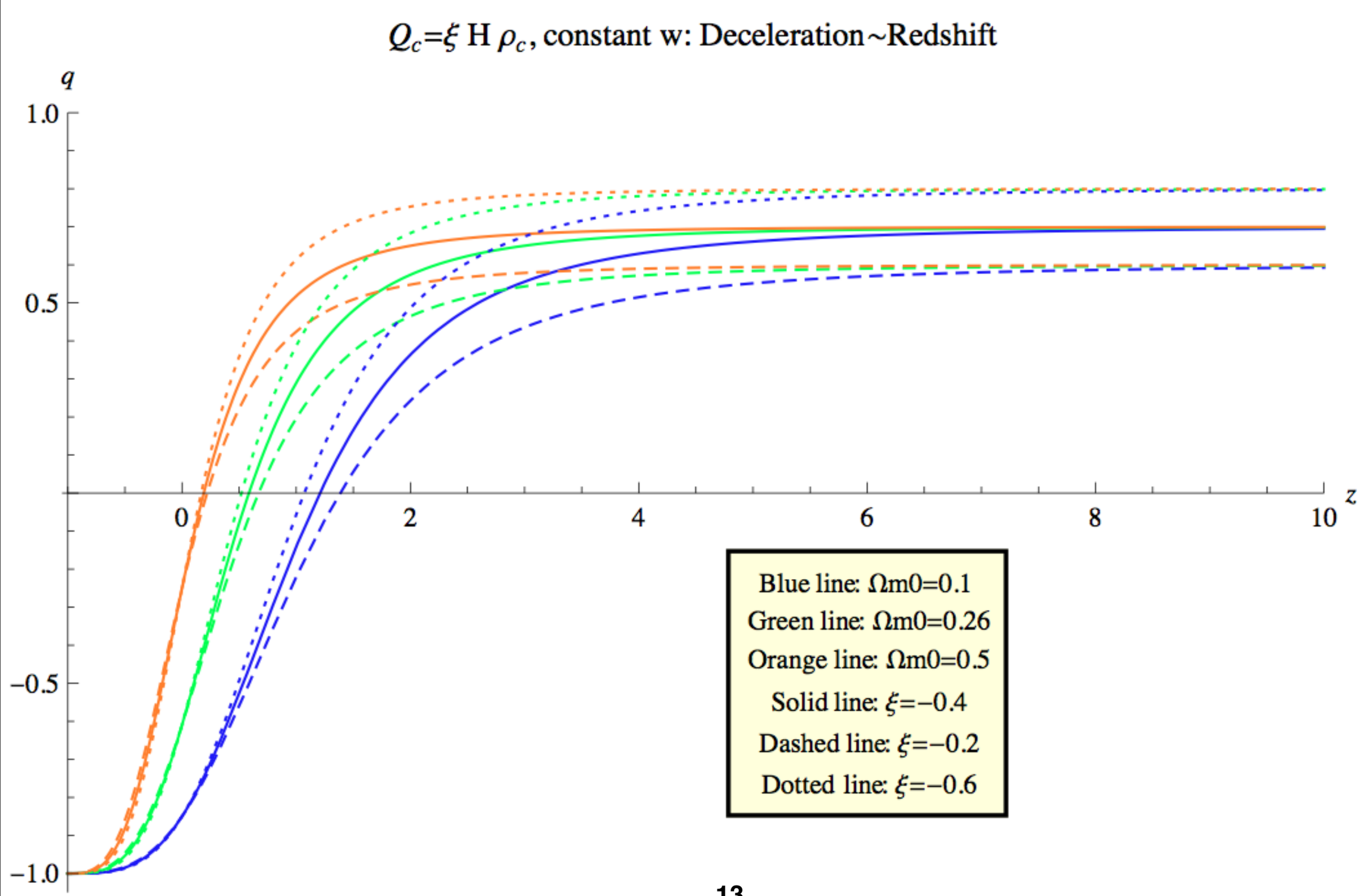
$$H(z) = \frac{\dot{a}(z)}{a(z)}, \; q(z) = -1 + \frac{1+z}{H(z)} \frac{\mathrm{d}\, H}{\mathrm{dz}}$$

Transition redshift

$$q(z)=0$$

From deceleration to acceleration

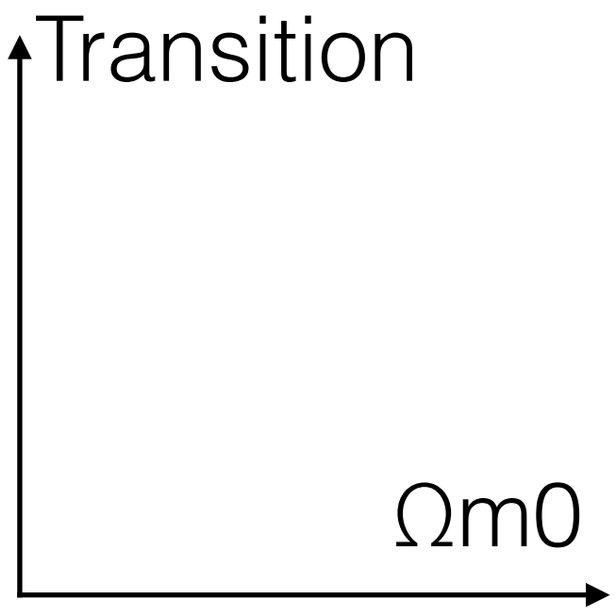
Interacting Dark Energy



Interacting Dark Energy

Flat universe, expansion history

Ωm0, zt, w, xi

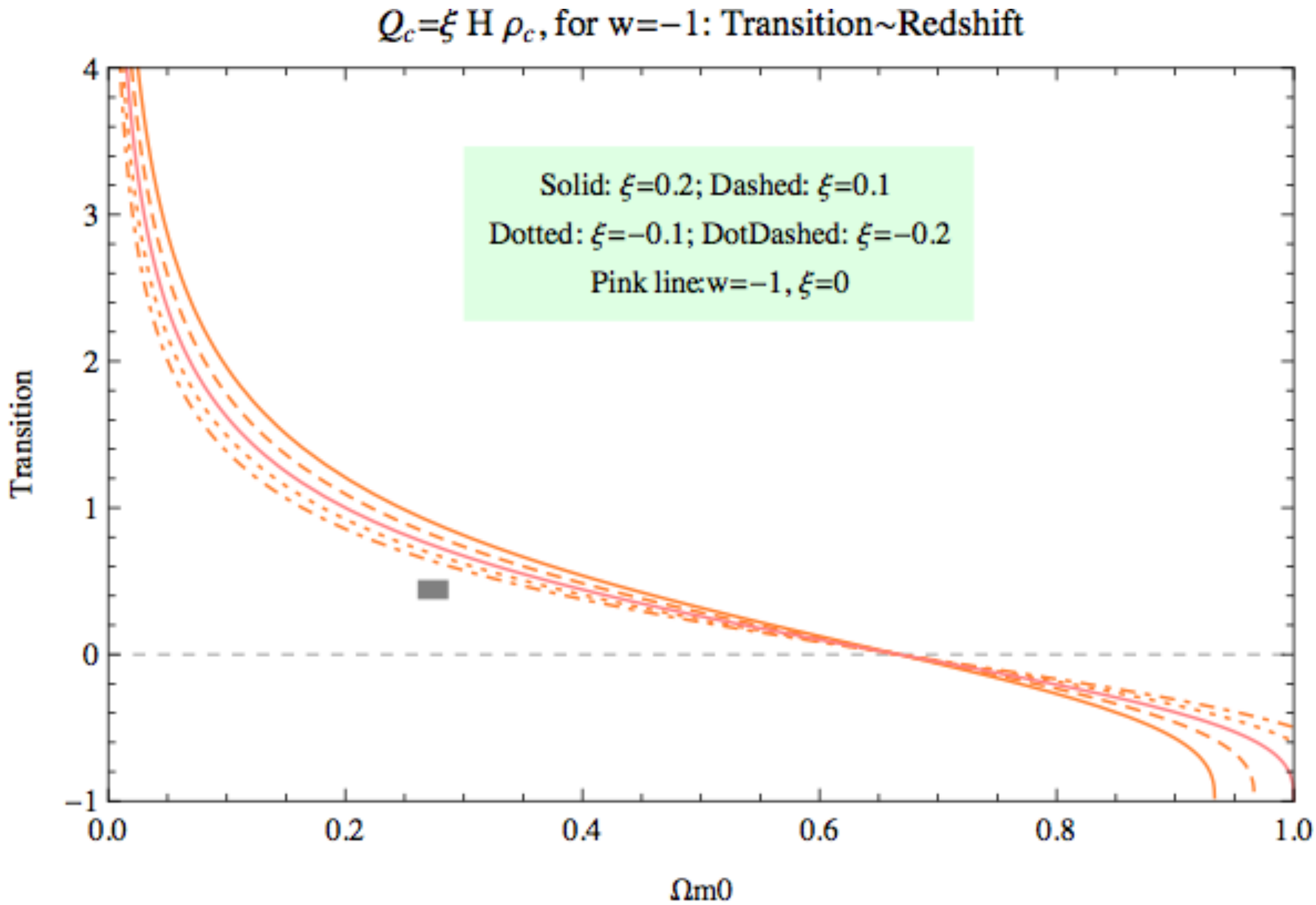


Data from observation arXiv:1205.4688

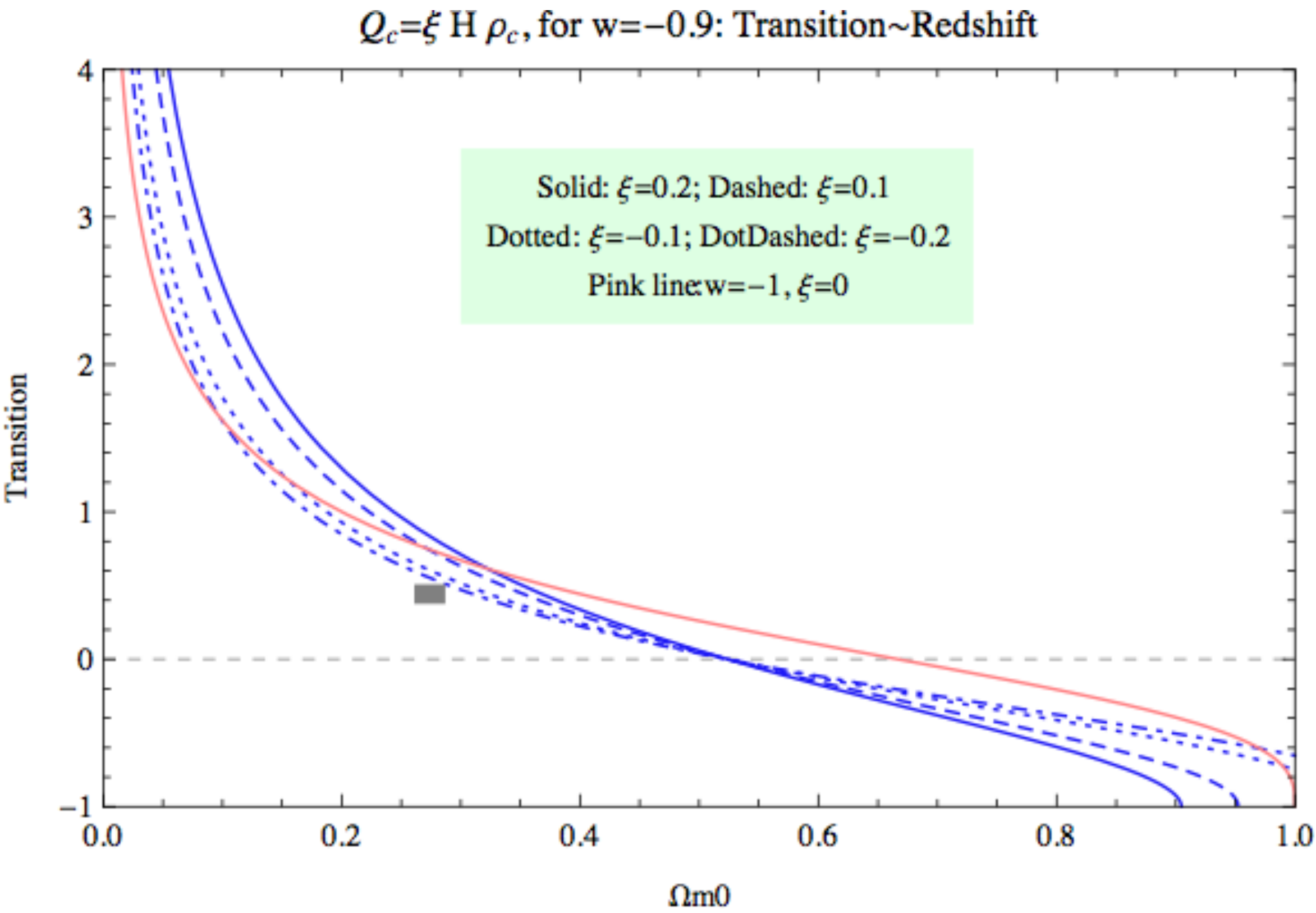
Ωm0=0.247(+0.013,-0.013)

zt=0.426(+0.082,-0.050)

Interacting Dark Energy

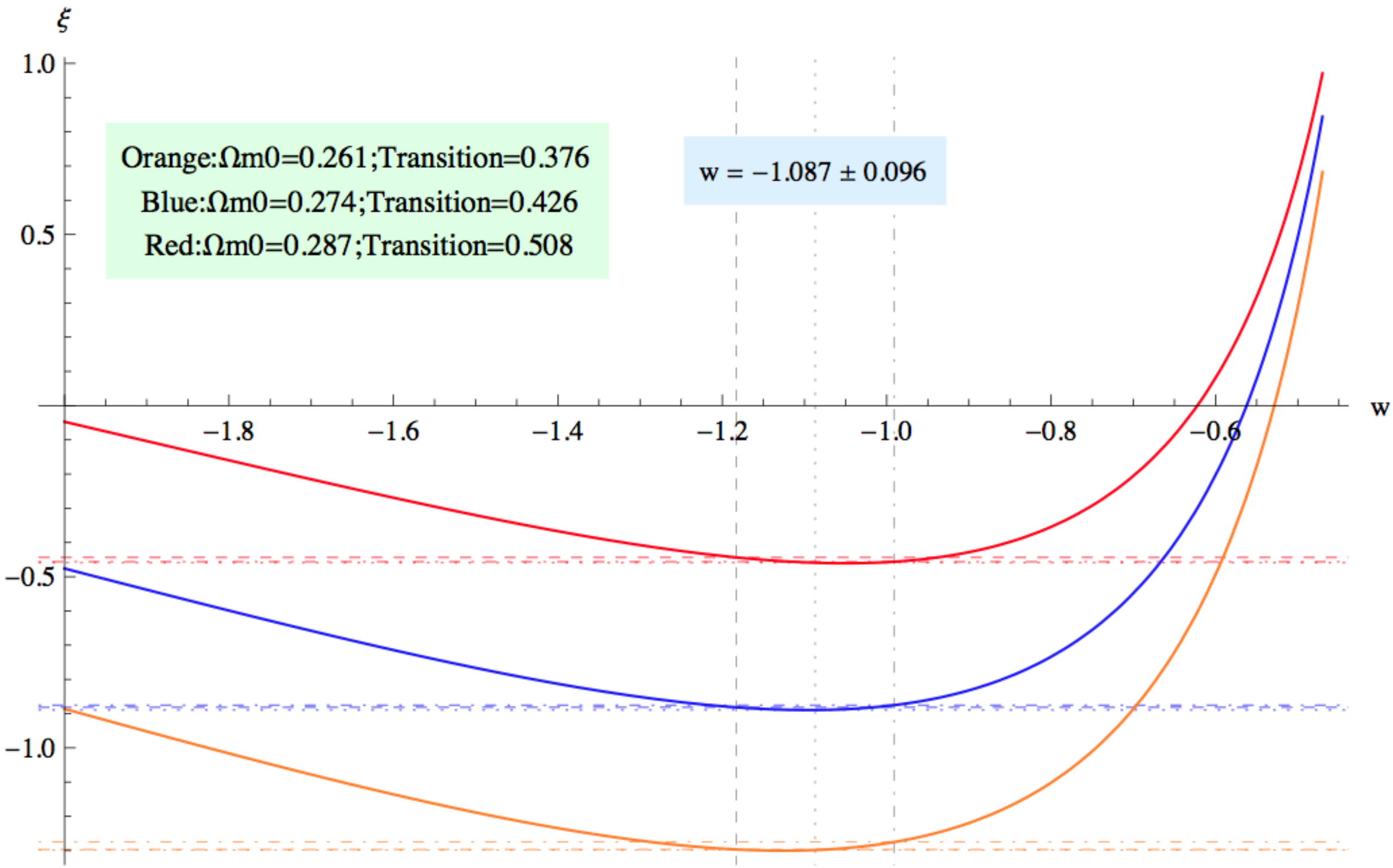


Interacting Dark Energy



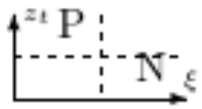
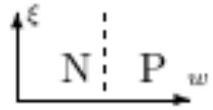
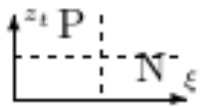
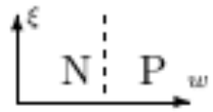
Interacting Dark Energy

$Q_c = \xi \, H \, \rho_c$, constant w: Coupling Constant ~ EoS



Interacting Da

Table 1: Parameters in Different Models

Model	Parameter	Transition Behavior	Deceleration Behavior	ξ (fixed transition) Behavior
ICC	ξ		N	-
	w	CC1		
	$\Omega m0$	N	P	P
ICCPL	ξ	CC2	N	-
	$w0$	CC3		P
	$w1$	CC		P
	$\Omega m0$	N	P	
I2CC	ξ		N	-
	w	CC4		
	$\Omega m0$	N	P	P
I2CCPL	ξ	CC	N	-
	$w0$	CC		P
	$w1$	CC		P
	$\Omega m0$	N	P	

Outline

- Why Bother?
- How to Modify Gravity?
- Test of MG
- Examples of MG
- Interacting DE
- IDE and Background Evolution

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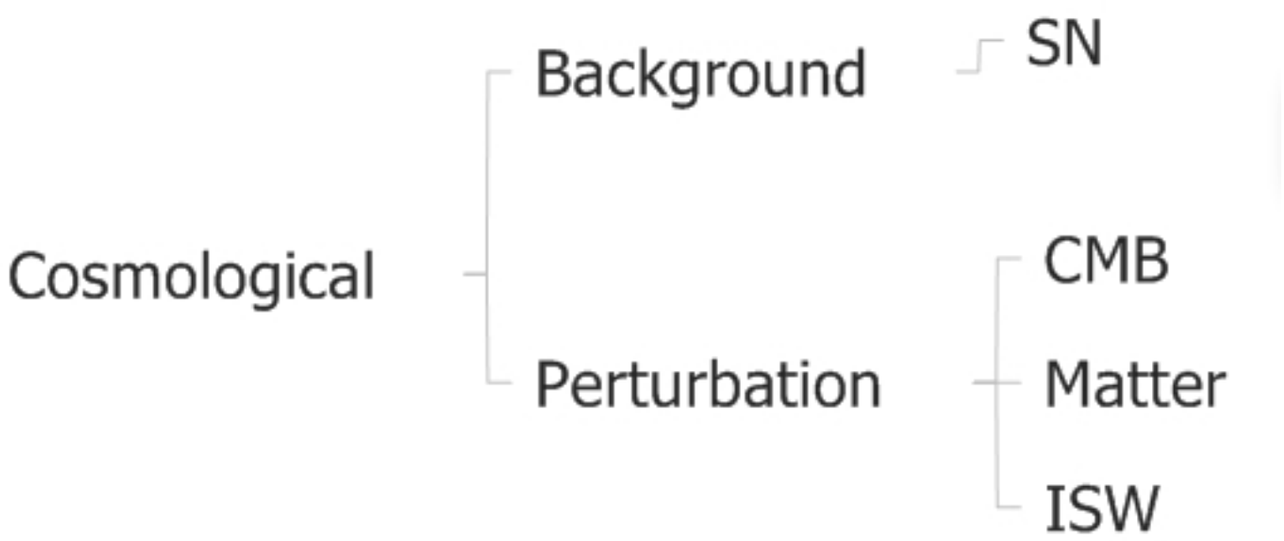
Test of MG

Everything We have done to GR

Cosmological Observations



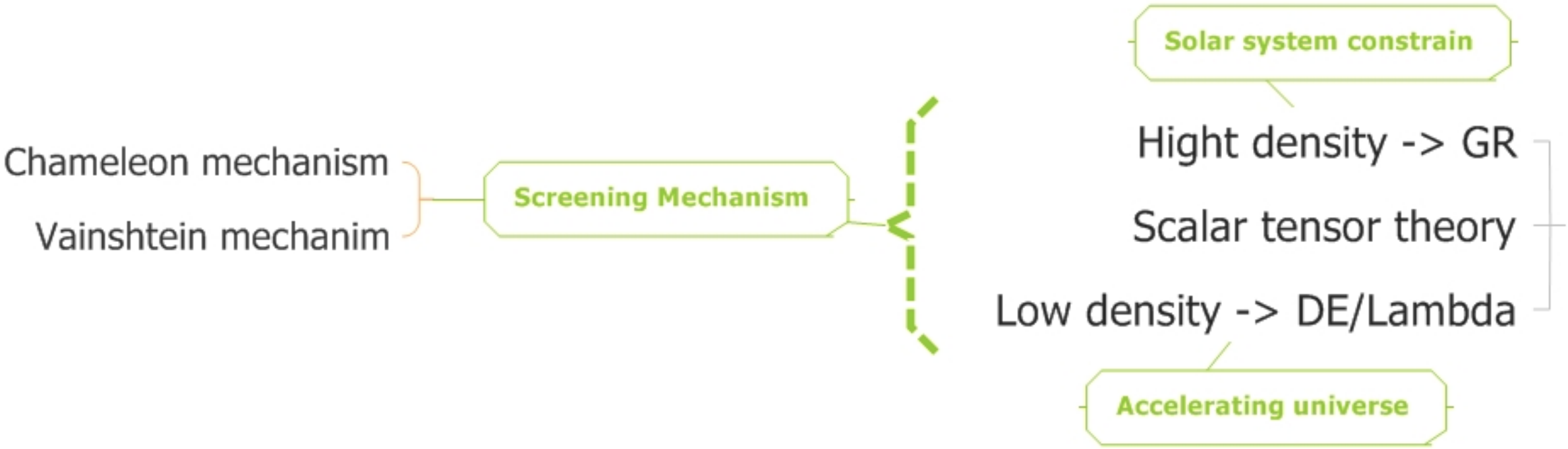
Why Bother

Hyperthesis					
Theory	**Mach**	**WEP**	**EEP**	**SEP**	**GC**
GR	Partially	Y	Y	Y	Y

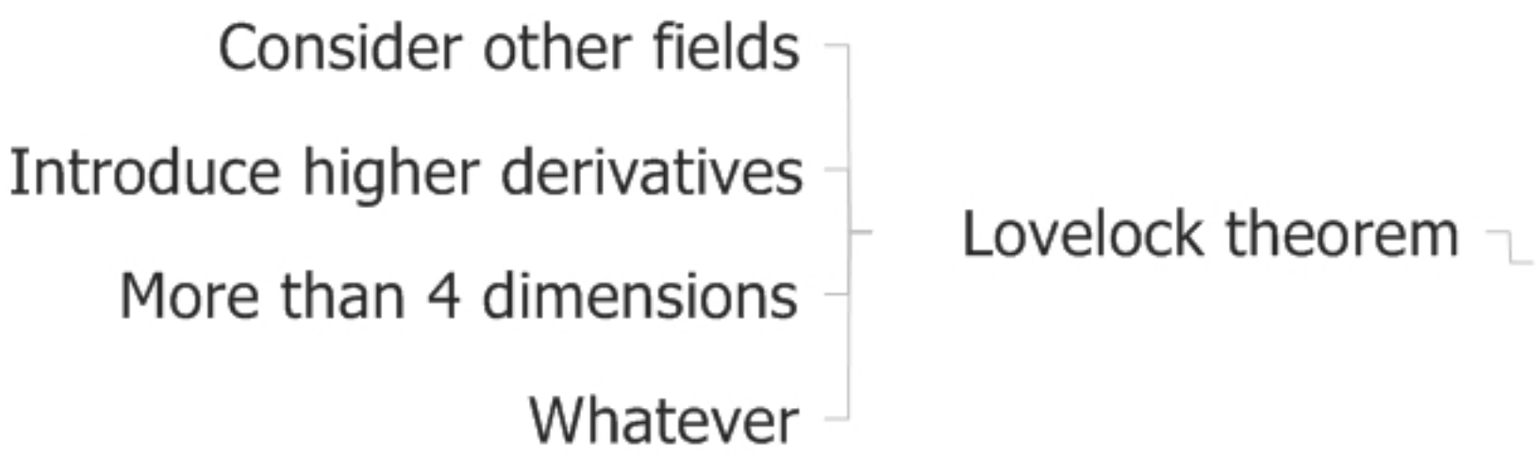
Arena		
Theory	**Connection**	**Metric**
GR	Non-torsion	Y

How to Modify?

What Should MG Look Like?



How to start a MG theory?



Why Bother

Dark Energy

Modified Gravity