# Stimulated Neutrino Oscillations - A Rabi Oscillation View

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## Abstract

ABSTRACT PLACEHOLDER

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# I. INTRODUCTION

1. Work done before, but the physics is not clear
2. Decompose the system into Rabi oscillations

# II. NEUTRINO OSCILLATIONS AND RABI OSCILLATION

- Neutrino oscillations in matter (background matter basis)
- Rabi oscillations
- Width, detuning, and Rabi frequency, and their significance. (Relation to amplitude and oscillation wavelength)
- Interference: destruction

## A. Neutrino oscillations in matter

- The formalism
- What has been found before

Matter profile

$$\lambda(x) = \lambda_0 + \delta\lambda(x), \tag{1}$$

where

$$\lambda_0 = \sqrt{2}G_F n_{e0}. \tag{2}$$

$$\delta\lambda(x) = \sqrt{2}G_F \delta n_e(x) \tag{3}$$

We use

$$\delta\lambda = A\sin(kx). \tag{4}$$

$\boxed{\text{Question: Should we use } A\cos(kx) \text{ to make comparision with Rabi oscillation?}}$

Choose $\lambda_0 = 0.5\lambda_{\mathrm{MSW}}$. (For easier reading, specify the actual number density for some characteristic energy such as 10 MeV.)

In background basis, Hamiltonian becomes

$$H^{(m)} = -\frac{\omega_m}{2}\sigma_3 + \frac{1}{2}A\sin(kx)\cos 2\theta_m\sigma_3 - \frac{1}{2}A\sin(kx)\sin 2\theta_m\sigma_1. \tag{5}$$

At resonance, $k = \omega_m$, the $\sigma_3$ component of perturbation has no effect when the system is at resonance, as we would prove later more rigorously.

### B. Rabi oscillation

$$H_{\mathrm{R}} = -\frac{\omega_m}{2}\sigma_3 - \frac{1}{2}A_1\cos(k_1 x)\sigma_1 + \frac{1}{2}A_1\sin(k_1 x)\sigma_2, \tag{6}$$

which is equivalent to

$$H_{\mathrm{R}} = -\frac{\omega_m}{2}\sigma_3 - \frac{A_1}{2}\begin{pmatrix} 0 & e^{ik_1 x} \\ e^{-ik_1 x} & 0 \end{pmatrix} \tag{7}$$

Probability

$$P(x) = \frac{|A_1|^2}{\Omega_R^2}\sin^2\left(\Omega_R x/2\right). \tag{8}$$

Rabi frequency

$$\Omega_{\mathrm{R}} = \sqrt{|A_1|^2 + (k_1 - \omega_m)^2}, \tag{9}$$

$k_1 - \omega_m$ is the detuning.

1. Significance of $A_1$, detuning $k_1 - \omega_m$, and Rabi frequency $\Omega_{\mathrm{R}}$.

   - $|A_1|$ is the width of the resonance.
   - $Q = |k_1 - \omega_m|/|A_1|$ (relative detuning?) determines how close to exact resonance.
   - $\Omega_{\mathrm{R}}$ determines the oscillation wavelength. The order of the wavelength is determined by the width, as long as the system is not too far away from exact resonance.

2. Interference: destruction

- First mode is on exact resonance, $k_1 = \omega_m$.
- Add in a new perturbation with $k_2 \ll k_1$, so that

$$H'_{\mathrm{R}} = -\frac{\omega_m}{2}\sigma_3 - \frac{1}{2}(A_1\cos(k_1x) + A_2\cos(k_2x))\sigma_1 + \frac{1}{2}(A_1\sin(k_1x) + A_2\sin(k_2x))\sigma_2. \tag{10}$$

- The slow mode will change the energy gap, $\omega'_m = \sqrt{\omega_m^2 + A_2^2} \approx \omega_m + \frac{A_2^2}{2\omega_m}$, to significantly decrease the amplitude we need a large $A_2$ which satisfies $|\omega'_m - k_1| \gg A_1$.
- A plot showing that this approximation actually works.

Design a Rabi oscillation system with one mode on exact resonance and a slow modes,

$$H'_{\mathrm{R}} = -\frac{\omega_m}{2}\sigma_3 - \frac{1}{2}(A_1\cos(k_1x) + A_2\cos(k_2x))\sigma_1 + \frac{1}{2}(A_1\sin(k_1x) + A_2\sin(k_2x))\sigma_2. \tag{11}$$

where $k_1 = \omega_m$, and $k_1 \gg k_2$.
The new energy gap can be predicted

$$\omega'_m = \sqrt{\omega_m^2 + A_2^2} \tag{12}$$

Predict the critical $A_2$ that significantly reduces the transition amplitude by setting the detuning value larger than the width,

$$|k_1 - \omega'_m| \gtrsim \text{width of resonance}. \tag{13}$$

Figure **??** shows that this hypothesis works. **Maybe a plot that shows the comparison of the predicted amplitudes and numerical amplitudes for different $A_2$'s.**

## III. STIMULATED NEUTRINO OSCILLATIONS

### A. Neutrino oscillations in matter

- The formalism

FIG. 1. Reduction of transition amplitudes. Black dashed line: the system has only one perturbation which is at exact resonance; Green dash-dotted line: $A_2 = A_{2,\text{Critical}} = 0.0083666$; Blue dotted line: $A_2 = 0.01$; Red line: $A_2 = 0.02$. The markers are the probabilities predicted using Rabi formula correspondingly. Black cross is the transition probability between two background mass eigenstates for the neutrinos with matter perturbation $A\sin(kx)$.

- What has been found before

Matter profile

$$\lambda(x) = \lambda_0 + \sum_{n=1}^{N} \delta\lambda_n(x), \tag{14}$$

where

$$\lambda_0 = \sqrt{2} G_F n_{e0} \tag{15}$$

$$\delta\lambda_n(x) = \sqrt{2} G_F \delta n_{e,n}(x) \tag{16}$$

Hamiltonian becomes

$$H^{(m)} = -\frac{\omega_m}{2}\sigma_3 + \frac{\delta\lambda}{2}\cos 2\theta_m \sigma_3 - \frac{\delta\lambda}{2}\sin 2\theta_m \sigma_1. \tag{17}$$

### B. To Make connections to Rabi oscillation

- Transofrmation
- Jacobi-Anger expansion
- Interpretation of each mode

A New Basis: Hamiltonian looks like a Rabi oscillation but with some complicated perturbations. Apply rotation (name this new basis? Rabi basis?)

$$\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{i\eta(x)} \end{pmatrix} \begin{pmatrix} |\nu_{b1}\rangle \\ |\nu_{b2}\rangle \end{pmatrix}. \tag{18}$$

Hamiltonian

$$H = -\frac{\sigma_3}{2} - \frac{\delta\lambda}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix} \begin{pmatrix} \psi_{b1} \\ \psi_{b2} \end{pmatrix}, \tag{19}$$

with

$$\eta(x) = \frac{\cos 2\theta_m}{2}\int_0^x \delta\lambda(\tau)d\tau. \tag{20}$$

*1. Jacobi-Anger Expansion: Write the system into a superposition of Rabi oscillations.*

$$\begin{aligned} H =& -\frac{\omega_m}{2}\sigma_3 \\ &+ \frac{1}{2}\sum_{n_1}\cdots\sum_{n_N} \begin{pmatrix} 0 & B_{n_1,\cdots,n_N}\Phi_{n_1,\cdots,n_N} e^{i\left(\sum_a n_a k_a\right)x} \\ B^*_{n_1,\cdots,n_N}\Phi^*_{n_1,\cdots,n_N} e^{-i\left(\sum_a n_a k_a\right)x} & 0 \end{pmatrix}, \end{aligned} \tag{21}$$

where

$$B_{n_1,\cdots,n_N} = -(-i)^{\sum_a n_a}\tan 2\theta_m \left(\sum_a n_a k_a\right)\left(\prod_a J_{n_a}\left(\frac{A_a}{k_a}\cos 2\theta_m\right)\right), \tag{22}$$

$$\Phi_{n_1,\cdots,n_N} = e^{i\left(\sum_a n_a \phi_a\right)}. \tag{23}$$

For each mode the solution is

$$P = |\psi_2|^2 = |\psi_{b2}|^2 = \frac{|B_{n_1,\cdots,n_N}|}{|B_{n_1,\cdots,n_N}| + \left(\sum_i n_i k_i - \omega_m\right)^2}\sin^2\left(\frac{\Omega_{\mathrm{R}}}{2}x\right). \tag{24}$$

Single perturbation modes (width at large n limit)

Show that the width drops for higher orders probably using

$$J_n(n\operatorname{sech}\alpha) \sim \frac{e^{n(\tanh\alpha - \alpha)}}{\sqrt{2\pi n\tanh\alpha}}. \tag{25}$$

### C. The Important Factors

- Width of resonance $B$

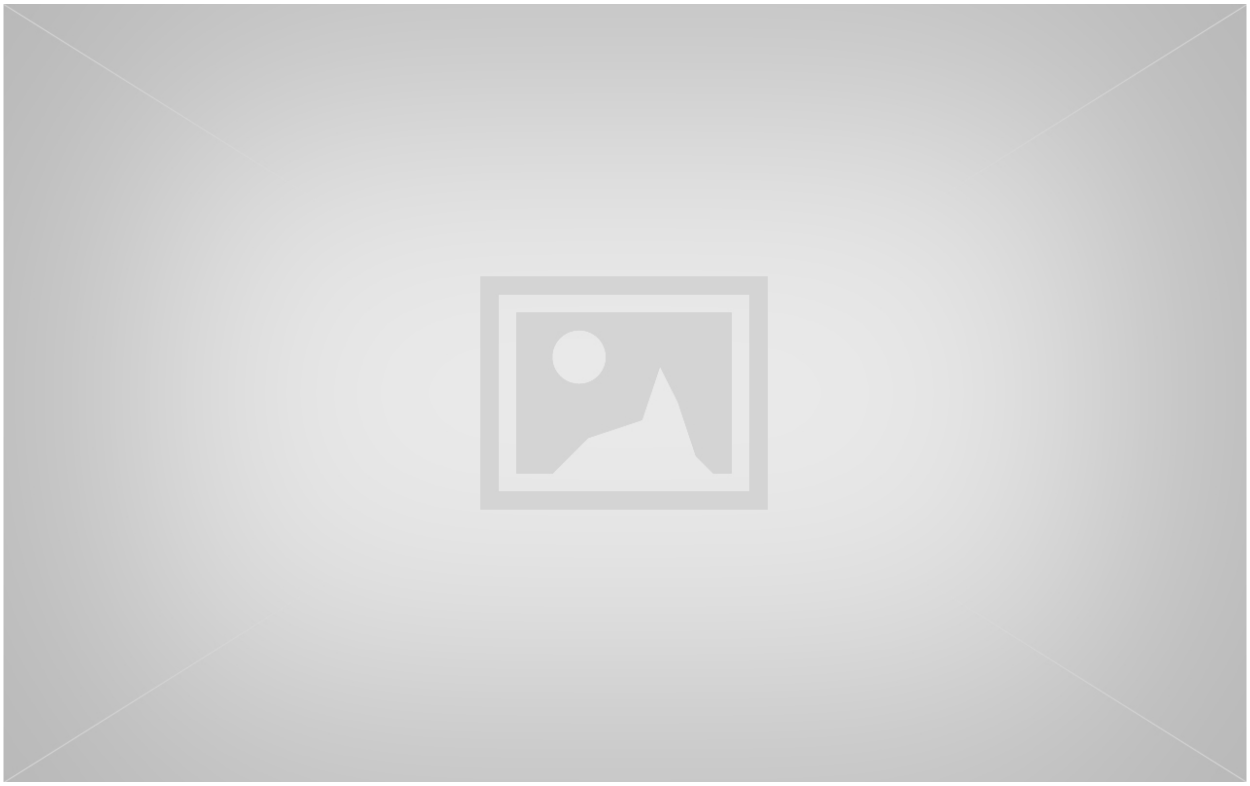

FIG. 2. Single perturbation: Resonance, modes, and width of each mode

- Deviation from exact resonance $g$, called **detuning** (value).
- Oscillation wavelength of mode (determined by Rabi frequency, which is in turn related to $B$ and $g$ ) compared to size of physical system

## IV. CONCLUSIONS

- A simple interpretation with some caveats.
- Phase of the matter profile doesn't play any role in the resonance argument.
- Realistic matter profile probably destroys the resonance due to this shift in the energy gap.

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**Appendix A: Interesting Results**

- Width drops for higher orders (for the systems we have)
- Width and detuning $\rightarrow$ Q Value $\rightarrow$ Amplitude of oscillation
- Width and detuning $\rightarrow$ Oscillation wavelength $\rightarrow$ Can be used to compare with the physical system
- Interference between different modes/perturbations

### Appendix B: Keypoints

- Hamiltonian in matter basis:

  Matter profile

$$\lambda(x) = \lambda_0 + \sum_{n=1}^{N} \delta\lambda_n(x), \tag{B1}$$

  where

$$\lambda_0 = \sqrt{2} G_F n_{e0} \tag{B2}$$

$$\delta\lambda_n(x) = \sqrt{2} G_F \delta n_{e,n}(x) \tag{B3}$$

  Hamiltonian becomes

$$H^{(m)} = -\frac{\omega_m}{2}\sigma_3 + \frac{\delta\lambda}{2}\cos 2\theta_m \sigma_3 - \frac{\delta\lambda}{2}\sin 2\theta_m \sigma_1. \tag{B4}$$

- A New Basis: Hamiltonian looks like a Rabi oscillation but with some complicated perturbations. Apply rotation

$$\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \begin{pmatrix} e^{-i\eta(x)} & 0 \\ 0 & e^{i\eta(x)} \end{pmatrix} \begin{pmatrix} |\nu_{b1}\rangle \\ |\nu_{b2}\rangle \end{pmatrix}. \tag{B5}$$

  Hamiltonian

$$H = -\frac{\sigma_3}{2} - \frac{\delta\lambda}{2}\sin 2\theta_m \begin{pmatrix} 0 & e^{2i\eta(x)} \\ e^{-2i\eta(x)} & 0 \end{pmatrix} \begin{pmatrix} \psi_{b1} \\ \psi_{b2} \end{pmatrix}, \tag{B6}$$

  with

$$\eta(x) = \frac{\cos 2\theta_m}{2} \int_0^x \delta\lambda(\tau) d\tau. \tag{B7}$$

- Jacobi-Anger Expansion: Write the system into a superposition of Rabi oscillations.

$$
\begin{aligned}
H = &- \frac{\omega_m}{2}\sigma_3 \\
&+ \frac{1}{2}\sum_{n_1}\cdots\sum_{n_N}\begin{pmatrix} 0 & B_{n_1,\cdots,n_N}\Phi_{n_1,\cdots,n_N}e^{i\left(\sum_a n_a k_a\right)x} \\ B^*_{n_1,\cdots,n_N}\Phi^*_{n_1,\cdots,n_N}e^{-i\left(\sum_a n_a k_a\right)x} & 0 \end{pmatrix},
\end{aligned}
\tag{B8}
$$

where

$$
B_{n_1,\cdots,n_N} = -(-i)^{\sum_a n_a}\tan 2\theta_m\left(\sum_a n_a k_a\right)\left(\prod_a J_{n_a}\left(\frac{A_a}{k_a}\cos 2\theta_m\right)\right), \tag{B9}
$$

$$
\Phi_{n_1,\cdots,n_N} = e^{i\left(\sum_a n_a\phi_a\right)}. \tag{B10}
$$

For each mode the solution is

$$
P = |\psi_2|^2 = |\psi_{b2}|^2 = \frac{|B_{n_1,\cdots,n_N}|}{|B_{n_1,\cdots,n_N}| + \left(\sum_i n_i k_i - \omega_m\right)^2}. \tag{B11}
$$

Single perturbation modes (width at large n limit)

- Each mode can be solved and explained using Rabi oscillation. Slightly different from Rabi oscillation but approximately true.

$$
H_{\mathrm{R}} = -\frac{\omega_m}{2}\sigma_3 - A\cos(kt)\sigma_1 \tag{B12}
$$

Important mode

$$
H'_{\mathrm{R}} = -\frac{\omega_m}{2}\sigma_3 - \frac{A}{2}\begin{pmatrix} 0 & e^{ikt} \\ e^{-ikt} & 0 \end{pmatrix} \tag{B13}
$$

$$
= -\frac{\omega_m}{2}\sigma_3 - \frac{A}{2}\cos(kt)\sigma_1 + \frac{A}{2}\sin(kt)\sigma_2. \tag{B14}
$$

Probability

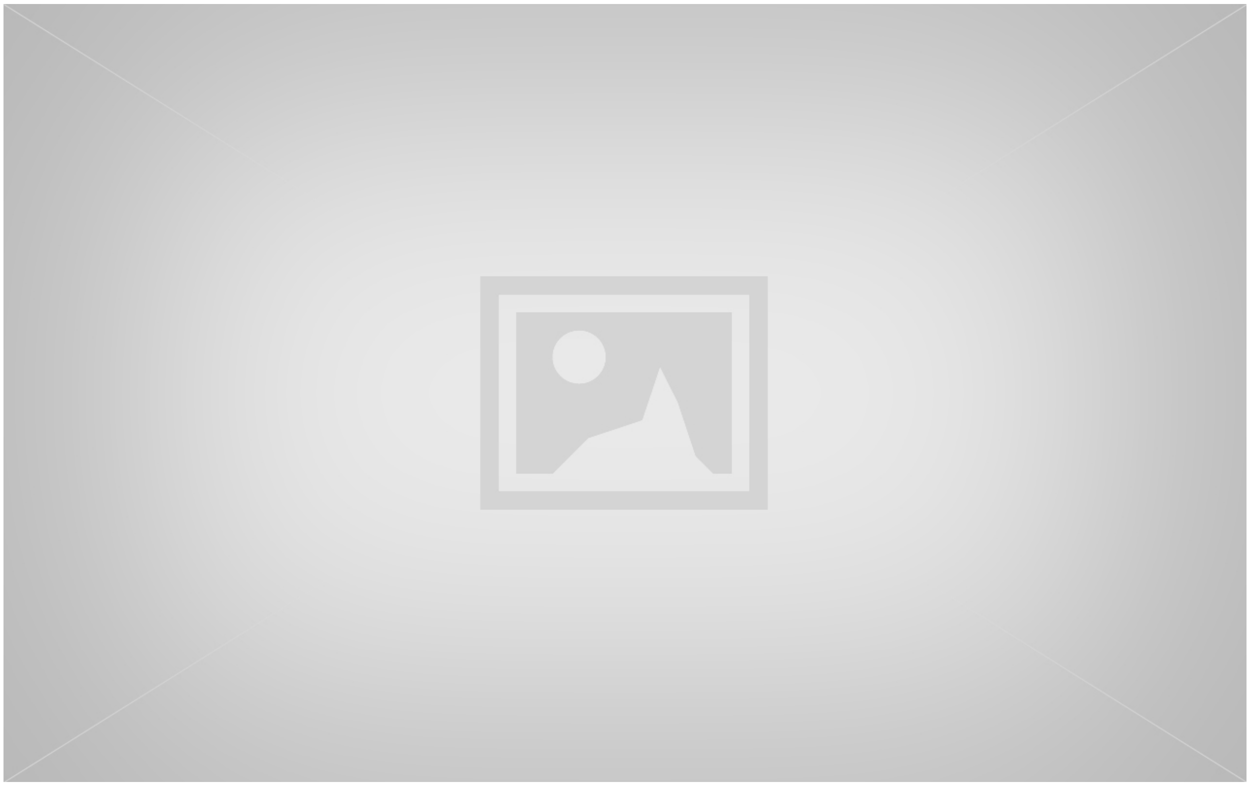

FIG. 3. Single perturbation: Resonance, modes, and width of each mode

$$P(x) = \frac{|A|^2}{|A|^2 + (k - \omega_m)^2} \sin^2\left(\sqrt{|A|^2 + (k - \omega_m)^2}\,x/2\right). \tag{B15}$$

Rabi frequency

$$\Omega_{\mathrm{R}} = \sqrt{|A|^2 + (k - \omega_m)^2}. \tag{B16}$$

1. Significance of $A$.
2. Why each mode is a Rabi oscillation? **Requires some really convincing evidence that each mode is really close to a Rabi oscillation.**

- Whether a mode is important depends on several factors.
  - Width of resonance $B$
  - Deviation from exact resonance $g$, called **detuning** (value).
  - Oscillation wavelength of mode (determined by Rabi frequency, which is in turn related to $B$ and $g$ ) compared to size of physical system

- Interference between each modes can cause destruction.
  - Slow rotating perturbation

    Construct system of two perturbations

$$-\frac{\omega_m}{2}\sigma_3 - \frac{1}{2}\left(A_1\cos(k_1 t) + A_2\cos(k_2 t)\right)\sigma_1 + \frac{1}{2}\left(A_1\sin(k_1 t) + A_2\cos(k_2 t)\right)\sigma_2. \tag{B17}$$

    with $k_1 \gg k_2$

$$\omega'_m = \sqrt{\omega_m^2 + A_2^2} \tag{B18}$$

    Predict the critical $A_2$ that significantly reduces the transition amplitude,

$$|k_1 - \omega'_m| \gtrsim \text{width of resonance}. \tag{B19}$$

    Figures showing that this hypothesis is true.

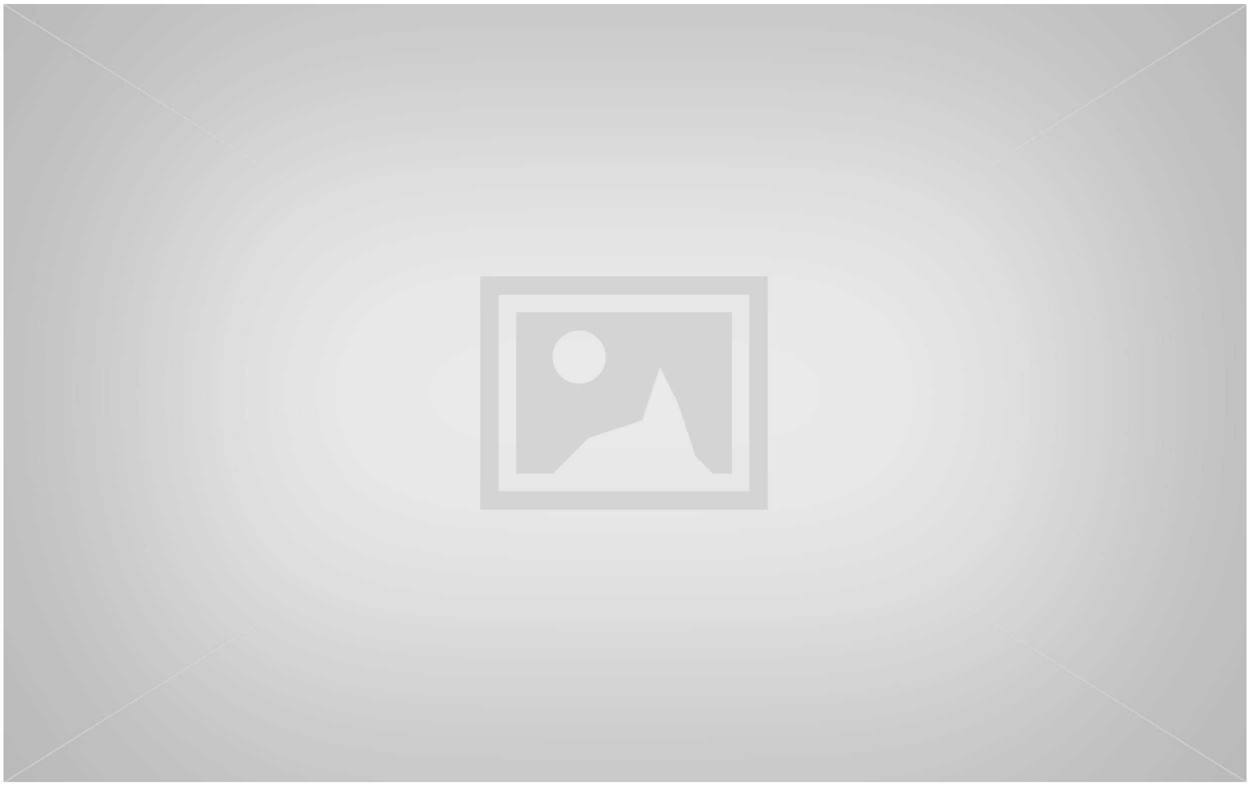

FIG. 4. Reduction of transition amplitude. The mode at resonance; Adding a second mode could destroy the resonance if the condition is satisfied.