We consider the case background density close to MSW resonance density. The width of the nth mode is

$$B_n = \tan 2\theta_m nk J_n(A\cos 2\theta_m/k).$$

As the background density approaches MSW resonance density, we have

$$\cos 2\theta_m \to 0.$$

We know that the Bessel function for small argument becomes

$$J_n(z) \to \frac{1}{\Gamma(n+1)}(z/2)^n, \quad \text{for small } z.$$

Then we can calculate the width for such background densities,

$$B_n = \tan 2\theta_m nk \frac{(A\cos 2\theta_m/(2k))^n}{\Gamma(n+1)} = \tan 2\theta_m k \frac{(A\cos 2\theta_m/(2k))^n}{\Gamma(n)}.$$

We simplify it to

$$B_n = \sin 2\theta_m \cos 2\theta_m \frac{k(A/2k)^n}{(n-1)!}$$

Since

$$A/2k < 1$$

we always have

$$B_1 > B_2.$$

It seems that the approximation to use only first order is quite robust. For small matter perturbation A, we always have the first mode being the most important mode.

Then the only example that I can think of to break the approximations in the second section is to choose

$$k \sim 0.5\omega_m.$$