Some Notes[a)]

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Just some note

a)

I. SOME NOTES

A. Mathematical Proof that Jacobi-Anger Expansion Brings Us the Expansion onto Many Constant Matter Perturbations

We can prove it by using a inverse transformation. However, we can make this clear by the special example that $\delta(x) = A$ which is a constant matter potential on top of the background potential.

The Equation of motion becomes

$$\mathrm{i}\frac{d}{dx}\begin{pmatrix}\psi_{r1}\\ \psi_{r2}\end{pmatrix} = \left[-\frac{\omega_\mathrm{m}}{2}\sigma_3 - \frac{A}{2}\sin 2\theta_\mathrm{m}\begin{pmatrix}0 & e^{\mathrm{i}\cos 2\theta_\mathrm{m} A x}\\ e^{-\mathrm{i}\cos 2\theta_\mathrm{m} A x} & 0\end{pmatrix}\right]\begin{pmatrix}\psi_{r1}\\ \psi_{r2}\end{pmatrix}. \tag{1}$$

By comparing this equation with the Jacobi-Anger expanded equation, we find that to interpret each Jacobi-Anger expanded mode as constant matter potential perturbation requires

$$B_n = A\sin 2\theta_\mathrm{m} \tag{2}$$

$$nk_1 = A\cos 2\theta_\mathrm{m}. \tag{3}$$

To find out the relation, we need to solve A and θ_m,

$$\theta_\mathrm{m} = \arctan\frac{B_n}{nk_1} \tag{4}$$

$$A = nk_1/\cos 2\theta_\mathrm{m}. \tag{5}$$

So each mode corresponds to a constant matter potential perturbation with amplitude $nk_1/\cos 2\theta_\mathrm{m}$ and also a particular mixing angle θ_m.

However, for the purpose of the paper, I believe we could simply interpret the purpose of Jacobi-Anger expansion as expansion on to systems with infinite constant matter perturbations.