Homework 9

Problem 1: FitzHugh-Nagumo Model

The FitzHugh-Nagumo model is a system of ordinary differential equations used to describe the excitation of a neuron membrane:

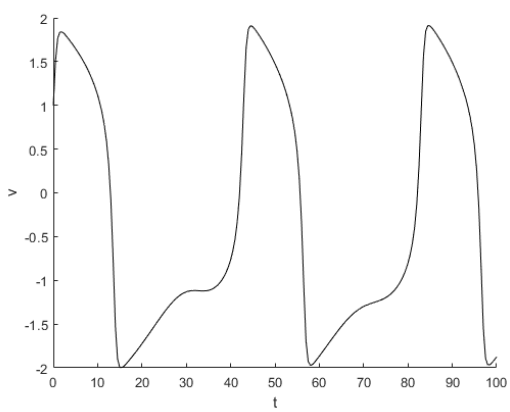
$$\dot{v} = v - \frac{1}{3}v^3 - w + I(t),$$
$$\dot{w} = \frac{a + v - bw}{\tau}$$

In this model, v is the membrane voltage and w is a variable representing the activity of several types of membrane channel proteins. The function $I(t)$ represents an external electrical current, and the parameters a, b and τ are constants controlling the channel protein activity.

In this problem, we will assume that $a = 0.7$, $b = 0.8$ and $\tau = 12.5$ and that

$$I(t) = \frac{1}{10}\left(5 + \sin\left(\frac{\pi t}{10}\right)\right).$$

Below is a plot of the solution $v(t)$ to this equation:



Notice that the solution appears roughly periodic. We will try to calculate the amplitude and period of this solution.

(a) First, solve the equation from time $t = 0$ to time $t = 100$ using a second order Runge-Kutta method and $\Delta t = 0.5$. Use the initial conditions $v(0) = 1$ and $w(0) = 0$. Save your approximation of the voltage at time 100 in `A1.dat`.

(b) Your approximation for v should have a local maximum at some time t_1 between $t = 0$ and $t = 10$, a local minimum at some time t_2 between $t = 10$ and $t = 20$ and another local maximum at some time t_3 $t = 40$ and $t = 50$. Find these times, then calculate the amplitude of v, given by $v(t_1) - v(t_2)$, and save it in `A2.dat`. Finally, calculate the period of v, given by $t_3 - t_1$, and save it in `A3.dat`.

(c) Repeat part (a) using a fourth order Runge-Kutta method. Save your approximation of the voltage at time 100 in `A4.dat`.

(d) Repeat part (b) using your fourth order approximation for v. Save the amplitude in `A5.dat` and the period in `A6.dat`.

Problem 2: Boundary Value Problem

Now consider the boundary value problem

$$\ddot{x} + x = 4\cos(5t),$$

with $x(0) = 1$ and $x(6) = 2$.

Solve this problem using a second order central difference scheme for $\ddot{x}$. That is, use the scheme

$$f''(t) \approx \frac{f(t - \Delta t) - 2f(t) + f(t + \Delta t)}{(\Delta t)^2}.$$

Use a step size of $\Delta t = 0.01$.

(a) Save the number of interior points (i.e., the number of t values not including $t = 0$ and $t = 6$) in `A7.dat`.

(b) Save your approximation of x at time $t = 3$ in `A8.dat`.

(c) Find the time at which x reaches its maximum value. Save this time in `A9.dat`.

(d) Find the time at which x reaches its minimum value. Save this time in `A10.dat`.