Data Analysis using R

R Basics

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R is a free, not-commercial, open-source software language for statistical computing and graphics. It comes with a lot of advantages, especially for data analysis projects.



Origin

- R is based on the programming language S ('dialect of S'), which in turn is based on the core programming language C
- First version published by Ross Ihaka and Robert Gentlemen in 1993 (Name is based on their first initials)
- intention of R: creating an environment where one doesn't consciously think of programming

Ihaka and Gentleman (1996)



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General Information

- R is case sensitive
- R and his basic statistical functions can easily be extended by packages
- Comprehensive R Archive Network (CRAN) is the main-platform for packages (extensions)
- over 20,000 packages in 2021 (roughly 4,000 packages in 2013)

Open-Source and R-Core-Team (2023)

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Advantages/Strengths

- free software
- open-source: contributions from a large active and vibrant community (huge functionality)
- great data visualization options
- interfaces with many other languages (e.g. LaTeX, C, Markdown, Python, Java, HTML, CSS)
- runs on almost every platform (through C++ even on the PlayStation)

Wickham and Grolemund (2016)



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Need additional help?

- use the built-in help system in R (help("...") and ?)
- use online-forums like Stack Overflow or GitHub (developers are active here as well)
- use the documentation files provided on the official R-Website (manuals, FAQs, books)

To understand computations in R, two slogans are helpful: Everything that exists is an object. Everything that happens is a function call.

Chambers (2014), Co-Founder of S



Good coding style is like using correct punctuation. You can manage without it, but it sure makes things easier to read. [...] you really should use a consistent style. Good style is important because while your code only has one author, it'll usually have multiple readers. This is especially true when you're writing code with others.

Wickham (2019)



Prerequisites

Download the appropiate versions of R and RStudio for your operating system

- R can be downloaded via the Comprehensive R Archive Network (CRAN)
- Best integrated development environment (IDE), i. e. where you write and compile your code, is RStudio and can be downloaded here



Basic Calculations



Math operators

- +: addition
- -: subtraction
- *: multiplication (elementwise!)
- /: division
- ^: power operator (e.g. 2³)



Commands

- abs(): absolute value
- sqrt(): square root
- exp(): exponential function
- log(): natural logarithm
- factorial(): factorial (e.g. 8!)
- round(, digits = x): round to x digits

Note: These operations work on different types (e.g. scalars, vectors, matrices, ...)



Basic Calculations

```
3 + 5 * 2
## [1] 13
 (3 + 5) * 2
## [1] 16
sqrt(8.33)
## [1] 2.886174
round(sqrt(8.33), 2)
## [1] 2.89
```

Caution: Decimal numbers use a dot instead of a comma (convenient in the US and in the UK)

R-Scripts

- Within R there are several file types to create
- R-Scripts are simple text files (<file_name>.R) with a collection of commands (and comments)
- Useful to store and document your statistical analyses
- A new R-Script can be generated by clicking on File > New File > R Script
- Run code from R scripts by highlighting the code snippet and typing Ctrl + Enter



Comments

- For better documentation, you can place comments in your R-Script using the symbol #
- Everything placed behind the # and belonging to the same line is treated as a comment

```
# Multiply two by five
                                                        10 / 4 # Divide ten by four
2 * 5
                                                       ## [1] 2.5
## [1] 10
```

R-Packages

- R has a lot of built-in commands delivered by the installation of R
- Often, these built-in commands are not sufficient (or convenient) to conduct more complex statistical analyses
- Further commands can be used by installing specialized packages



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Installing

- Run install.packages() while your PC is connected to the internet
- Each package has to be installed only once

```
# Installing the package rio
install.packages("rio")
```

Loading

- If you want to use a command from a specific package you need to activate the package by running the command library()
- Packages need only to be activated once per session

```
library(rio)
```



Variables

- Variables can take all kinds of different content (vectors, matrices, data sets, regression, ...)
- Variables are created by the assignment operator
- When variables are assigned, they are shown in RStudio's environment pane
- Using = to assign variables is considered bad practice

```
# Assign value 4 to variable x
x <- 4
# Print x
x</pre>
## [1] 4
```

```
# Do some calculations with x
x * 2
```

```
## [1] 8

# Overwrite x
x <- 1</pre>
```

```
## [1] 1
```

Χ

Vectors

- Vectors are the most important data type in R
- What we normally refer to as vector is an "atomic vector" in R: A vector of elements with the same type
 - Scalars in R are atomic vectors as well (length 1 vectors)
- R allows vectors to have elements of different types, a so-called "list"
 - More complex than atomic vectors: Each element can be any type
 - Elements can differ in length
 - May contain several atomic vectors of different lengths with different types



Atomic Vectors

- Four primary types:
 - 1. Logical: TRUE or FALSE
 - 2. **Integer**: Whole numbers followed by L, e. g. 1234L
 - 3. **Double**: Decimal number defined by the floating point standard, e. g. 0.1234
 - 4. Character: Strings surrounded by " or ', e.g. "this is a string" or 'this as well'
- Integers and doubles are numeric with unique values Inf, -Inf and NaN (not a number)
- Generate vectors with c(1, 2, ...)



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Operations

```
# Generate x and z
x <- c(5, 1, 9, 3)
z <- c(4, 9, 0, 4)
```

```
# Multiplication with scalar
2 * x
```

```
## [1] 10 2 18 6
```

```
# Elementwise multiplication
x * z
```

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- Generate vectors with c(1, 2, ...)

Special Commands

```
# vector with 5 ones
rep(1, 5)

## [1] 1 1 1 1 1

# sequence from 1 to 5
seq(1, 5)

## [1] 1 2 3 4 5
```

```
1:5
```

Subsetting Vectors

- Sometimes we want to work with specific elements from a vector
 - 1. Subset via element indices
 - 2. Subset via logicals
- Specific elements can be extracted by enclosing the index with [and]



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```
# Elements 2
x[2]
## [1] 1
# Elements 1 to 3
x[1:3]
## [1] 5 1 9
# All elements except the last one
x[-4]
## [1] 5 1 9
```

Subsetting Vectors

- Sometimes we want to work with specific elements from a vector
 - 1. Subset via element indices
 - 2. Subset via logicals
- Specific elements can be extracted by enclosing the index with [and]

```
# Subset with TRUE/FALSE
x[c(TRUE, FALSE, FALSE, TRUE)]
```

```
## [1] 5 3
```

Factors

- Vector that contains predefined values
- Used to represent categorical data
- Integer vectors with the following **attributes** (metadata of the vector):
 - 1. class: "factor"
 - 2. levels: Set of allowed values, sorted into increasing order
- Created by the factor() function
- Can be ordered when setting the ordered argument of factor() to TRUE

Missing Values

- R reserves NA for missing values ("not applicable")
- Most computations with missing values return missing values

• Exception: When identity holds for all inputs, for example when computing x^0

• Use is.na() to test for missing values

```
NA * 10
```

[1] NA

```
NA^0
```

[1] 1

```
x <- c(1, NA, 2, 3)
is.na(x)
```

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[1] FALSE TRUE FALSE FALSE

- Vectors can have attributes (some metadata of the vector)
- Adding the dim attribute to an atomic vector makes it a 2-dimensional matrix
- Can be created by assigning dim() to an atomic vector or simply calling matrix()
- Can also be created by binding atomic vectors with the same data type and length column-wise using cbind() or row-wise using rbind()



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```
# Add dim attribute to atomic vector
x <- 1:6
dim(x) <- c(3, 2) # 3 rows, 2 columns
x</pre>
```

```
## [,1] [,2]
## [1,] 1 4
## [2,] 2 5
## [3,] 3 6
```

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```
# Use the matrix command to create a
# matrix from a vector
matrix(x, nrow = 3)
```

```
## [,1] [,2]
## [1,] 1 4
## [2,] 2 5
## [3,] 3 6
```

```
# Specify ncol instead of nrow
matrix(x, ncol = 2)
```

```
## [,1] [,2]
## [1,] 1 4
## [2,] 2 5
## [3,] 3 6
```

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```
# Fill matrix column-wise instead of
# row-wise
matrix(x, ncol = 2, byrow = T)
```

```
## [,1] [,2]
## [1,] 1 2
## [2,] 3 4
## [3,] 5 6
```

- Vectors can have attributes (some metadata of the vector)
- Adding the dim attribute to an atomic vector makes it a 2-dimensional matrix
- Can be created by assigning dim() to an atomic vector or simply calling matrix()
- Can also be created by binding atomic vectors with the same data type and length column-wise using cbind() or row-wise using rbind()

```
# Bind atomic vectors column-wise
cbind(1:3, 4:6)
```

```
## [,1] [,2]
## [1,] 1 4
## [2,] 2 5
## [3,] 3 6
```

Matrix Operations

- t(): transpose
- solve():inverse
- %*%: matrix multiplication



Matrix Operations

- t(): transpose
- solve(): inverse
- %*%: matrix multiplication

```
A \leftarrow matrix(c(1, 2, 0, 3), ncol = 2)
Α
## [,1] [,2]
## [1,] 1 0
## [2,] 2 3
t(A)
## [,1] [,2]
## [1,] 1 2
## [2,] 0 3
solve(A)
##
            [,1] \qquad [,2]
## [1,] 1.0000000 0.0000000
## [2,] -0.6666667 0.3333333
```

Matrix Operations

- t(): transpose
- solve(): inverse
- %*%: matrix multiplication

Subsetting Matrices

- Specific elements can be extracted using [and]
- To distinguish the row from the column positions, a comma is used within the brackets [row, column]

```
C <- matrix(-5:6, ncol = 4)</pre>
```

```
## [,1] [,2] [,3] [,4]
## [1,] -5 -2 1 4
## [2,] -4 -1 2 5
## [3,] -3 0 3 6
```

Subsetting Matrices

- Specific elements can be extracted using [and]
- To distinguish the row from the column positions, a comma is used within the brackets [row, column]

```
C[1, ]
## [1] -5 -2 1 4
C[, 1:3]
##
       [,1] [,2] [,3]
## [1,] -5 -2
## [2,] -4 -1
## [3,] -3 0
C[1:2, 2]
## [1] -2 -1
```

Note: Depending on the matrix and choice of subset, the subset may no longer be a matrix!

Useful Statistical/Mathematical Commands

Suppose x and y are numeric vectors with same length:

Command	Description
sum(x)	Sum of x
table(x)	Frequeny table for x
mean(x)	Average of x
median(x)	Median of x
var(x)	Variance of x
sd(x)	Standard deviation of x
cov(x, y)	Covariance matrix of x and y
quantile(x)	Quantiles of x
min(x)	Minimum value of x
max(x)	Maximum value of x

Note: Some of these commands can also be applied to matrices



Lists

- Each element can be any type
- Can be constructed using list()
- Just like with atomic vectors, elements in a list can be named
- Subsetting lists is done via double brackets [[and]] or \$

```
x <- list(1:5, "a", c(TRUE, FALSE),
           seg(0.2, 0.6, by = 0.2))
typeof(x)
## [1] "list"
Χ
## [[1]]
## [1] 1 2 3 4 5
##
## [[2]]
## [1] "a"
##
## [[3]]
## [1] TRUE FALSE
##
## [[4]]
## [1] 0.2 0.4 0.6
```

Lists

- Each element can be any type
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```
x <- list(
  integer_vector = 1:5,
  character_vector = "a"
)</pre>
```

```
## $integer_vector
## [1] 1 2 3 4 5
##
## $character_vector
## [1] "a"
```

Lists

- Each element can be any type
- Can be constructed using list()
- Just like with atomic vectors, elements in a list can be named
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```
x$integer_vector

## [1] 1 2 3 4 5

x[[1]]
```

[1] 1 2 3 4 5

Data Frames

- Used when dealing with tabular data
- Essentially a named list of vectors with
 - same length of all elements
 - attributes for column names and row.names
 - and "data.frame" as the class
- Created using the data.frame() function

```
df <- data.frame(x = 1:10, y = letters[1:10])
df</pre>
```

```
##
       х у
## 1
       2 b
## 3
       3 c
       4 d
## 4
## 5
## 6
## 7
       7 g
       8 h
## 8
## 9
       9 i
## 10 10 j
```

- View(df): Opens df in a spreadsheet-style data viewer in RStudio
- summary (df): Produces summary statistics for each variable
- df\$x: Access variable with name x
- head(df): Shows first six rows of df
- tail(df): Shows last six rows of df
- nrow(df): Number of rows (observations)
- ncol(df): Number of columns (variables)
- names(df): Column names



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```
summary(df)
```

```
## x y
## Min. : 1.00 Length:10
## 1st Qu.: 3.25 Class :character
## Median : 5.50 Mode :character
## Mean : 5.50
## 3rd Qu.: 7.75
## Max. :10.00
```

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```
df$x
```

[1] 1 2 3 4 5 6 7 8 9 10

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```
nrow(df)
```

```
## [1] 10
```



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- names(df): Column names

```
names(df)
```

```
## [1] "x" "y"
```



Working with Data Frames

Create a new column:

```
df$z <- seq(0.1, 1, by = 0.1)
df</pre>
```

Working with Data Frames

Select multiple columns by name:

```
df[, c("x", "z")]
```

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Working with Data Frames

Select a subset of observations based on a condition:

- df\$x <= 5 evaluates to a vector of logicals with same length as the atomic vector df\$x
- Logical vector selects rows to keep when subsetting
- Can of course also subset using a vector of indices

```
df[df$x <= 5,]
```

Importing Data Sets

- Data sets are usually stored in specific file formats such as .csv, .xlsx or Stata's .dta
- Base R offers functions for the most common data storage formats
- For some formats we have to use packages such as haven for .dta files (or write functions ourselves)
- Import the .csv file stored under data/processed/happiness.csv, data from Wooldridge (2013)

df <- read.csv("data/processed/happiness.csv")</pre>

Note that...

- ... you can change the working directory by using the command setwd("YOUR_PATH") (recommended!)
- ... you can also change the working directory by clicking on Session > Set Working Directory > Choose Directory
- ... as a rule, R accepts "/" but does not accept "\" (which is often used on german PCs)
- ... if your working directory is specified you do not need to set the whole path for future operations (only the relative path)

Linear Regression Models in R

Let's build a linear probability model using the happiness data!

We are interested in analyzing how owning a gun has an effect on whether individuals see themselves as "very happy":

$$vhappy = \alpha + eta_1 imes educ + eta_2 imes income + eta_3 imes owngun + arepsilon,$$

where

- vhappy = 1 if the individual is very happy
- *educ*: Years of schooling completed
- *income*: Income brackets
- owngun = 1 if the individual owns a gun



Inspect the Data

```
summary(df[,c("vhappy", "educ", "income", "owngun")])
                      educ
##
      vhappy
                                   income
                                                    owngun
         :0.0000 Min. : 0.00
                                Length:17137 Length:17137
   Min.
                                Class :character Class :character
   1st Qu.:0.0000
                  1st Qu.:12.00
   Median :0.0000 Median :13.00
                                Mode :character Mode :character
   Mean
        :0.3069 Mean :13.32
   3rd Qu.:1.0000 3rd Qu.:16.00
   Max. :1.0000
                  Max. :20.00
##
                  NA's :44
```

- Contains NA need to be removed for the estimation
- income and owngun are of type character transform columns to correct format

Remove missing values using na.omit():

```
# Create a new data frame in the environment with a subset of columns used for the analysis
df_clean <- df[,c("vhappy", "educ", "income", "owngun")]

# Remove all rows containing NAs
df_clean <- na.omit(df_clean)

summary(df_clean)</pre>
```

```
educ
                                      income
##
       vhappy
                                                        owngun
   Min.
          :0.0000
                   Min. : 0.00
                                   Length:9969
                                                     Length:9969
                                   Class :character
                                                     Class :character
   1st Qu.:0.0000
                   1st Qu.:12.00
   Median :0.0000
                   Median :13.00
                                   Mode :character
                                                     Mode :character
##
   Mean :0.3074
                   Mean
                          :13.39
   3rd Qu.:1.0000
                   3rd Qu.:16.00
   Max. :1.0000
                    Max. :20.00
##
```



Recode owngun to a dummy variable using ifelse():

```
# Recode owngun to a dummy variable
df_clean$owngun <- ifelse(df_clean$owngun == "yes", 1, 0)
summary(df_clean)</pre>
```

```
vhappy
                        educ
                                     income
##
                                                        owngun
                                   Length:9969
          :0.0000
                   Min. : 0.00
                                                    Min.
                                                           :0.0000
   Min.
                                   Class :character
   1st Qu.:0.0000
                   1st Qu.:12.00
                                                    1st Qu.:0.0000
   Median :0.0000
                   Median :13.00
                                   Mode :character
                                                    Median :0.0000
   Mean :0.3074
                   Mean :13.39
                                                    Mean :0.3692
##
   3rd Qu.:1.0000
                   3rd Qu.:16.00
                                                    3rd Qu.:1.0000
##
   Max. :1.0000
                          :20.00
                                                    Max. :1.0000
                   Max.
```



The income is given in the form of income brackets and stored as a character vector:

```
# Take a look at unique values of the income
unique(df_clean$income)

## [1] "$15000 - 19999" "$10000 - 14999" "$25000 or more" "$20000 - 24999"

## [5] "$6000 to 6999" "$5000 to 5999" "$3000 to 3999" "$1000 to 2999"

## [9] "$8000 to 9999" "$7000 to 7999" "$4000 to 4999" "lt $1000"
```

Convert to factor



Convert income to an ordered factor:

```
##
       vhappy
                        educ
                                             income
                                                           owngun
   Min. :0.0000
                   Min. : 0.00
                                   $25000 or more:6422
                                                              :0.0000
                                                       Min.
   1st Qu.:0.0000
                   1st Qu.:12.00
                                   $20000 - 24999: 873
                                                      1st Qu.:0.0000
   Median :0.0000
                   Median :13.00
                                   $10000 - 14999: 813
                                                      Median :0.0000
                   Mean :13.39
   Mean :0.3074
                                   $15000 - 19999: 751
                                                      Mean :0.3692
   3rd Qu.:1.0000
                   3rd Qu.:16.00
                                   $8000 to 9999 : 266
                                                       3rd Qu.:1.0000
                                   $7000 to 7999 : 153
##
   Max. :1.0000
                   Max. :20.00
                                                       Max. :1.0000
##
                                   (Other)
                                                : 691
```



Linear Model: lm()

- The function lm() is used to fit (multivariate) linear models
- For the model defined before, we simply need:
 - 1. formula: vhappy ~ educ + income +
 owngun
 - 2. data: The df_clean data frame we prepared

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Linear Model: lm()

```
model
```

```
##
## Call:
## lm(formula = vhappy ~ educ + income + owngun, data = df_clean)
##
  Coefficients:
  (Intercept)
                       educ
                                 income.L
                                              income.Q
                                                            income.C
                                                                         income^4
##
      0.103169
                   0.009402
                                 0.046546
                                              0.091012
                                                            0.054562
                                                                         0.040568
     income^5
                   income^6
                                 income^7
                                              income^8
                                                            income^9
                                                                        income^10
                                -0.013639
##
     -0.037144
                   0.068211
                                             -0.020895
                                                           -0.007898
                                                                         0.017806
##
     income^11
                     owngun
##
     -0.034506
                   0.031326
```



Model Summary: summary()

```
summary(model)
##
## Call:
## lm(formula = vhappy ~ educ + income + owngun, data = df_clean)
##
## Residuals:
      Min
           10 Median 30
##
                                    Max
## -0.4289 -0.3506 -0.2375 0.6212 0.9223
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.103169 0.022115 4.665 3.12e-06 ***
## educ
              0.009402 0.001646 5.713 1.14e-08 ***
```



Extract Coefficients: coef()

```
coef(model)
    (Intercept)
                                  income.L
                                               income.Q
                                                             income.C
                                                                          income^4
##
                        educ
    0.103168955
                 0.009401949
                              0.046545580
                                            0.091012452
                                                         0.054561656
                                                                       0.040568424
       income^5
                                               income^8
                                                                         income^10
##
                    income^6
                                  income^7
                                                             income^9
  -0.037143658
                 0.068210608 - 0.013638591 - 0.020894643 - 0.007898189
                                                                       0.017805968
      income^11
##
                      owngun
## -0.034506034
                 0.031326131
```



Formulas

- Formulas have a special syntax in R and reference the column names given in the data
- LHS and RHS are separated by ~
- Variables to be included in the model are separated by +
- R automatically assumes an intercept: To remove the intercept, add -1 at the start or +0 at the end of the RHS
- May include transformations of the data, e. g. log()
- : indicates interactions of variables
- * denotes factor crossings, i. e. a*b is the same as a + b + a:b
- ^ is used for polynomials, i. e. a^2 translates to a*a



Formulas: Examples

Formula	Regression Model
$y \sim x1 + x2 + x3$	$y = \alpha + \beta_1 \times x_1 + \beta_2 \times x_2 + \beta_3 \times x_3 + \varepsilon$
$y \sim -1 + x1 + x2 + x3$	$y = \beta_1 \times x_1 + \beta_2 \times x_2 + \beta_3 \times x_3 + \varepsilon$
$y \sim log(x1) + x2 + x3$	$y = \alpha + \beta_1 \times \ln x_1 + \beta_2 \times x_2 + \beta_3 \times x_3 + \varepsilon$
y ~ x1*x1 + x2 + x3	$y = lpha + eta_1 imes x_1 + eta_2 imes x_1^2 + eta_3 imes x_2 + eta_4 imes x_3 + arepsilon$
y ~ x1:x1 + x2 + x3	$y = \alpha + \beta_1 \times x_1^2 + \beta_2 \times x_2 + \beta_3 \times x_3 + \varepsilon$
y ~ x1 + x2*x3	$y = lpha + eta_1 imes x_1 + eta_2 imes x_2 + eta_3 imes x_3 + eta_4 imes x_2 imes x_3 + arepsilon$

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