# Sets and Parameters

Let $G = (V, A)$ be a directed graph where $|V| = n + 2$, and the depot is represented by the two vertices $0$ and $n + 1$. Feasible vehicle routes then correspond to paths starting at vertex $0$ and ending at vertex $n + 1$.

Let $N$ be set of non-depot locations where $N = V\{0, n + 1\}$.

Let $A$ be the set of arcs where $A = \{(i, j) \mid i, j \in V, i \neq j\}$. Let $\delta^+(i) = \{j \mid (i, j) \in A\}$ and $\delta^-(j) = \{i \mid (i, j) \in A\}$. Note that arc $(i, j) \in A$ denotes location $j$ is visited after location $i$.

Let $K$ be the set of vehicles where $|K| = m$. Let $\bar{s}_i$ and $s_i$ be the service time at node $i \in V$ without and with helper, respectively. Note that $\bar{s}_0 = \bar{s}_{n+1} = 0$ and $s_0 = s_{n+1} = 0$.

Let $t_{ij}$ be the travel minutes from location $i \in V$ to location $j \in V \setminus \{i\}$.

Let $[a_i, b_i]$ be time window for non-depot location $i \in V \setminus \{0, n + 1\}$ where $a_i$ and $b_i$ are starting and ending times. Let $[a_0, b_0]$ and $[a_{n+1}, b_{n+1}]$ be time windows for the depot. We can define $[a_0, b_0]$ and $[a_{n+1}, b_{n+1}]$ as

$$
\begin{aligned}
a_0 &= \min_{i \in N}\{a_i - t_{0i}\}, \\
b_0 &= \max_{i \in N}\{b_i - t_{0i}\}, \\
a_{n+1} &= \min_{i \in N}\{a_i + s_i + t_{in+1}\}, \\
b_{n+1} &= \max_{i \in N}\{b_i + s_i + t_{in+1}\}.
\end{aligned}
$$

Let $q_i$ denote the demand of location $i$ and let $Q_k$ be the vehicle $k \in K$ capacity.

# Variables

Let $x_{ijk}$ be the binary variable for sequencing where

$$x_{ijk} = \begin{cases} 1, & \text{if and only if arc } (i,j) \in A \text{ is used by vehicle } k \in K, \\ 0, & \text{otherwise.} \end{cases}$$

We now provide the mathematical formulation for the RDC Network Design problem.

Let $x_{ijpbt}$ be the variable for the flow of product in pounds from node $i \in N$ to node $j \in N \setminus \{i\}$ for product $p \in P$ in BU $b \in B$ at period $t \in T$.

Let $z_i$ be the binary variable for the selection of an RDC node $i \in N_3$ where

$$z_i = \begin{cases} 1, & \text{if RDC node } i \in N_3 \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

The model **RDC-1** is formulated as,

$$\text{minimize} \sum_{i \in \{N_2 \cup N_3\}} \sum_{j \in \{N_3 \setminus \{i\} \cup N_4\}} \sum_{p \in P} \sum_{b \in B} \sum_{t \in T} \bar{c}_{ijpb} x_{ijpbt} \tag{1}$$

subject to

$$\sum_{i \in N_3} x_{ijpbt} = d_{jptbt}, \qquad \forall j \in N_4, p \in P, b \in B, t \in T, \tag{2}$$

$$\sum_{i \in \{N_1 \cup N_2\}} x_{ijpbt} = \sum_{i \in N_4} x_{jipbt}, \quad \forall j \in N_3, p \in P, b \in B, t \in T, \tag{3}$$

$$\sum_{i \in \{N_1 \cup N_2\}} \sum_{p \in P} \sum_{b \in B} \ell_p * \frac{x_{ijpbt}}{u_{ipt}} \leq v_{jt}, \qquad \forall j \in N_3, t \in T, \tag{4}$$

$$\sum_{j \in N_3} \sum_{b \in B} x_{ijpbt} \leq w_{ipt}, \qquad \forall i \in N_2, p \in P, t \in T, \tag{5}$$

$$\sum_{j \in N_4} y_{ij} = 1, \qquad \forall i \in N_3, \tag{6}$$

$$\sum_{p \in P} \sum_{t \in T} \sum_{b \in B} x_{ijpbt} \leq M * y_{ij}, \qquad \forall i \in N_3, j \in N_4, \tag{7}$$

$$\sum_{j \in N_4} y_{ij} \leq M * z_i, \qquad \forall i \in N_3, \tag{8}$$

$$x_{ijpbt} \geq 0, \qquad \begin{array}{l} \forall i \in N, j \in \{N \setminus \{i\}\}, \\ p \in P, t \in T, b \in B, \end{array} \tag{9}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in N_3, j \in N_4, \tag{10}$$

$$z_i \quad \in \quad \{0,1\} \qquad \forall i \in N_3. \tag{11}$$

The objective of the model, (1), minimizes the total transportation costs from RP to RDC to customer. In (2), total product demand for each customer should be satisfied for each period. Flow balances at the RDC are described in (3). The RDC capacity is measured in total pallets, and thus (4) describes the conversion of the flow in pounds into pallets. In (5), the flow out of the RP is constrained by its capacity for the horizon. RDC assignments to CDC are described in (6) and (7), which ensures one and only CDC is assigned to a RDC for the model horizon. The model is able to decide whether an RDC is open or closed for the model horizon which is described in (8). Lastly, nonegativity and binary constraints are described (9), (10), and (11).