Sets and Parameters

Let $G = (V, A)$ be a directed graph where $|V| = n + 2$, and the depot is represented by the two vertices 0 and $n + 1$. Feasible vehicle routes then correspond to paths starting at vertex 0 and ending at vertex $n + 1$.

Let N be set of non-depot locations where $N = V\{0, n + 1\}$.

Let A be the set of arcs where $A = \{(i, j) \mid i, j \in V, i \neq j\}$. Let $\delta^+(i) = \{j \mid (i, j) \in A\}$ and $\delta^-(j) = \{i \mid (i, j) \in A\}$. Note that arc $(i, j) \in A$ denotes location j is visited after location i.

Let K be the set of vehicles where $|K| = m$. Let $\bar{s}_i$ and s_i be the service time at node $i \in V$ without and with helper, respectively. Note that $\bar{s}_0 = \bar{s}_{n+1} = 0$ and $s_0 = s_{n+1} = 0$.

Let t_{ij} be the travel minutes from location $i \in V$ to location $j \in V \setminus \{i\}$.

Let $[a_i, b_i]$ be time window for non-depot location $i \in V \setminus \{0, n+1\}$ where a_i and b_i are starting and ending times. Let $[a_0, b_0]$ and $[a_{n+1}, b_{n+1}]$ be time windows for the depot. We can define $[a_0, b_0]$ and $[a_{n+1}, b_{n+1}]$ as

$$\begin{aligned}
a_0 &= \min_{i \in N}\{a_i - t_{0i}\}, \\
b_0 &= \max_{i \in N}\{b_i - t_{0i}\}, \\
a_{n+1} &= \min_{i \in N}\{a_i + s_i + t_{in+1}\}, \\
b_{n+1} &= \max_{i \in N}\{b_i + s_i + t_{in+1}\}.
\end{aligned}$$

Let q_i denote the demand of location i and let Q_k be the vehicle $k \in K$ capacity.

Let $M_{ij} = \max\{0, b_i + s_i + t_{ij} - a_j\}$.

Let c_k, h_k, and p_k be unit transportation, helper, and team truck costs for vehicle $k \in K$.

Variables

Let x_{ijk} be the binary variable for sequencing where

$$x_{ijk} = \begin{cases} 1, & \text{if and only if arc } (i,j) \in A \text{ is used by vehicle } k \in K, \\ 0, & \text{otherwise.} \end{cases}$$

Let w_{ik} be continuous variable indicating the time at which vehicle $k \in K$ starts servicing location $i \in V$.

Let y_k be the binary variable for helper assignment

$$y_k = \begin{cases} 1, & \text{if vehicle } k \in K \text{ has a helper,} \\ 0, & \text{otherwise.} \end{cases}$$

Let z_k be the binary variable for team driver assignment where

$$z_k = \begin{cases} 1, & \text{if vehicle } k \in K \text{ has a team driver,} \\ 0, & \text{otherwise.} \end{cases}$$

Mathematical Formulation

The model is formulated as,

$$\text{minimize} \sum_{k \in K} \Big[\sum_{(i,j) \in A} c_k t_{ij} x_{ijk} + h_k y_k + q_k z_k \Big] \tag{1}$$

s.t.

$$\sum_{k \in K} \sum_{j \in \delta^+(i)} x_{ijk} = 1, \qquad \forall i \in N, \tag{2}$$

$$\sum_{j \in \delta^+(0)} x_{0jk} = 1, \qquad \forall k \in K, \tag{3}$$

$$\sum_{j \in \delta^-(i)} x_{jik} - \sum_{j \in \delta^+(i)} x_{ijk} = 0, \qquad \forall k \in K, \forall i \in N, \tag{4}$$

$$\sum_{i \in \delta^-(n+1)} x_{in+1k} = 1, \qquad \forall k \in K, \tag{5}$$

$$w_{ik} - w_{jk} - \bar{s}_i(1 - y_k) - s_i y_k - t_{ij} + M_{ij}(1 - x_{ijk}) \geq 0, \qquad \forall k \in K, \forall (i,j) \in A, \tag{6}$$

$$w_{ik} - a_i \geq 0, \qquad \forall k \in K, \forall i \in V, \tag{7}$$

$$w_{ik} - b_i \leq 0, \qquad \forall k \in K, \forall i \in V, \tag{8}$$

$$\sum_{i \in N} q_i \sum_{j \in \delta^+(i)} x_{ijk} - Q_k \leq 0, \qquad \forall k \in K, \tag{9}$$

$$y_{ij} \in \{0,1\} \quad \forall i \in N_3, j \in N_4, \tag{10}$$

$$z_i \in \{0,1\} \quad \forall i \in N_3. \tag{11}$$

We now provide the mathematical formulation for the RDC Network Design problem.

Let x_{ijpbt} be the variable for the flow of product in pounds from node $i \in N$ to node $j \in N \setminus \{i\}$ for product $p \in P$ in BU $b \in B$ at period $t \in T$.

Let z_i be the binary variable for the selection of an RDC node $i \in N_3$ where

$$z_i = \begin{cases} 1, & \text{if RDC node } i \in N_3 \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

The model **RDC-1** is formulated as,

$$\text{minimize} \sum_{i \in \{N_2 \cup N_3\}} \sum_{j \in \{N_3 \setminus \{i\} \cup N_4\}} \sum_{p \in P} \sum_{b \in B} \sum_{t \in T} \bar{c}_{ijpb} x_{ijpbt} \tag{12}$$

subject to

$$\sum_{i \in N_3} x_{ijpbt} = d_{jptbt}, \quad \forall j \in N_4, p \in P, b \in B, t \in T, \tag{13}$$

$$\sum_{i \in \{N_1 \cup N_2\}} x_{ijpbt} = \sum_{i \in N_4} x_{jipbt}, \quad \forall j \in N_3, p \in P, b \in B, t \in T, \tag{14}$$

$$\sum_{i \in \{N_1 \cup N_2\}} \sum_{p \in P} \sum_{b \in B} \ell_p * \frac{x_{ijpbt}}{u_{ipt}} \leq v_{jt}, \quad \forall j \in N_3, t \in T, \tag{15}$$

$$\sum_{j \in N_3} \sum_{b \in B} x_{ijpbt} \leq w_{ipt}, \quad \forall i \in N_2, p \in P, t \in T, \tag{16}$$

$$\sum_{j \in N_4} y_{ij} = 1, \quad \forall i \in N_3, \tag{17}$$

$$\sum_{p \in P} \sum_{t \in T} \sum_{b \in B} x_{ijpbt} \leq M * y_{ij}, \quad \forall i \in N_3, j \in N_4, \tag{18}$$

$$\sum_{j \in N_4} y_{ij} \leq M * z_i, \quad \forall i \in N_3, \tag{19}$$

$$x_{ijpbt} \geq 0, \quad \forall i \in N, j \in \{N \setminus \{i\}\}, p \in P, t \in T, b \in B, \tag{20}$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in N_3, j \in N_4, \tag{21}$$

$$z_i \in \{0, 1\} \quad \forall i \in N_3. \tag{22}$$

The objective of the model, (12), minimizes the total transportation costs from RP to RDC to customer. In (13), total product demand for each customer should be satisfied

for each period. Flow balances at the RDC are described in (14). The RDC capacity is measured in total pallets, and thus (15) describes the conversion of the flow in pounds into pallets. In (16), the flow out of the RP is constrained by its capacity for the horizon. RDC assignments to CDC are described in (17) and (18), which ensures one and only CDC is assigned to a RDC for the model horizon. The model is able to decide whether an RDC is open or closed for the model horizon which is described in (19). Lastly, nonegativity and binary constraints are described (20), (21), and (22).