

BİL 331/531 Design and Analysis of Algorithms

HOMEWORK 2 (60 Points)

Due Date: November 11, 2022

1 [15 POINTS] WHO IS TO BE MATCHED WITH WHOM?

Recall that at some point Utku had invented a time machine and went to Roman times. You were at the beginning of your college life and most probably focused on decrypting texts encrypted with Ceaser cipher. Almost nobody shown interest in the details of the construction of the time machine. Utku has also terminated his studies on the time machine since he was quite scared of being trapped in a different time.

Mr. Bidik, however, was quite interested in the specifics of the time machine and since then have been heavily working on constructing a more stable version. He has finally managed to make the machine work properly. Recently, he traveled to year 4100AC and came back. He has given a lot of information to us about the future.

He had the chance to go to some of the lectures at a university. He said that Artificial Intelligence Engineering Departments are quite popular in the future. In the finance classes he attended, people were discussing the bankruptcy of Meta due to mis-speculation on the future of metaverse. He has also attended the algorithms classes while he was there. To his surprise, people were still interested in algorithms that match men and women. This was partly because a deadly world war, which put the future of humanity at risk.

The most fundamental issue -in the future- was to increase the size of the population and thus, when matching men and women, they did not care about the individual preferences. Rather, the problem they studied was as follows:

As input, we are given an integer array M of size n representing the ages of men in the society, and an integer array W of size n representing the ages of women in the society, and an integer k . The goal is to obtain a maximum sized matching between men and women such that no person is matched with someone whose age differs by more than k . For instance, if $k = 36$,

somebody at the age of 1286 can be matched with someone whose age is at least 1250 and at most 1322.

The output of your program should be a single integer: the size of the maximum matching.

PS: Due to advancement in the life sciences, people can live up to $2^{31} - 1$ years.

2 [15 POINTS] HOW MANY SORTIES ARE NEEDED?

Air defense systems are usually good at hunting planes that fly at a high attitude but not as good in hunting planes that fly at a low attitude. See Rust's story for some historical background on the subject matter.

United States wants to test the abilities of Russian air defense systems. They already know due to prior intelligence reports that Russian air defense systems are quite effective for altitudes higher than n . However, Pentagon wants to determine the exact precision of the Russian air defense systems.

There are several costs associated with testing the performance of Russian air defense systems. First of all, if a plane that flies at an altitude that the air defense systems can effectively handle, the plane will be shot down and the pilot gets killed. As far as the propaganda office is concerned the American society will not tolerate to lose more than 2 pilots. At a lesser cost, every such experiment will create a tension with the Russian authorities.

The goal is to determine the smallest integer k (which is clearly less than or equal to n) such that the Russian air defense systems is effective for all altitudes greater than or equal to k . For this task, you are permitted at most two planes to get hunted. You need to complete the task with fewest number of experiments. Notice that an experiment is sending a plane to the Russian air space at a fixed altitude.

The input to your program will be a single integer n . Your program should output two integers: the first one is the integer k , and the second one is the minimum number of experiments needed to find k .

Your program is going to be interactive. That is, for each experiment it will call a method called `isSurvived(int altitude)`, which returns true if altitude is less than or equal to k .

3 [15 POINTS] SORTING NORMALLY DISTRIBUTED NUMBERS

You will be given a double array consisting of n numbers that are independently drawn from a distribution defined over $(0, 1)$ whose probability density function is given as $pdf(x) = 4x^3$. Your job is to come up with a sorting algorithm that works quite fast on average time.

4 [15 POINTS] FINDING NUMBERS CLOSE TO A REFERENCE POINT

In this question, as input, you will be given an integer array A and two integers k and l (Both k and l are less than or equal to the size of A). Your program should output an array of size l consisting of the l closest integers in A to the k^{th} order statistic of A . For instance, for

$A = \{3, 2, 5, 1, 7, 10, 8, 9, 4, 6\}$, $k = 5$, and $l = 4$, your program should output the array $\{4, 5, 6, 7\}$ or $\{3, 4, 5, 6\}$ in any order.
Your program will not pass the test cases if its running time is not $O(n)$.