BILKENT UNIVERSITY

EEE 391 - BASIC SIGNALS AND SYSTEMS

MATLAB Assignment 01

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Contents

1 Fourier Series Representation of Periodic Signals

1.1 Part (a)

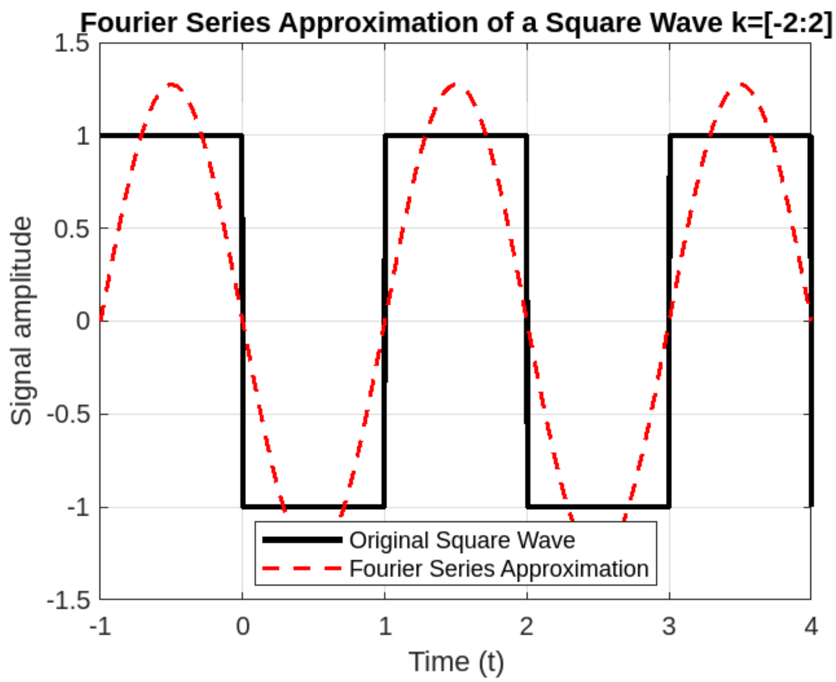


Figure 1: Fourier series approximation of the square wave with $k = \pm 2$

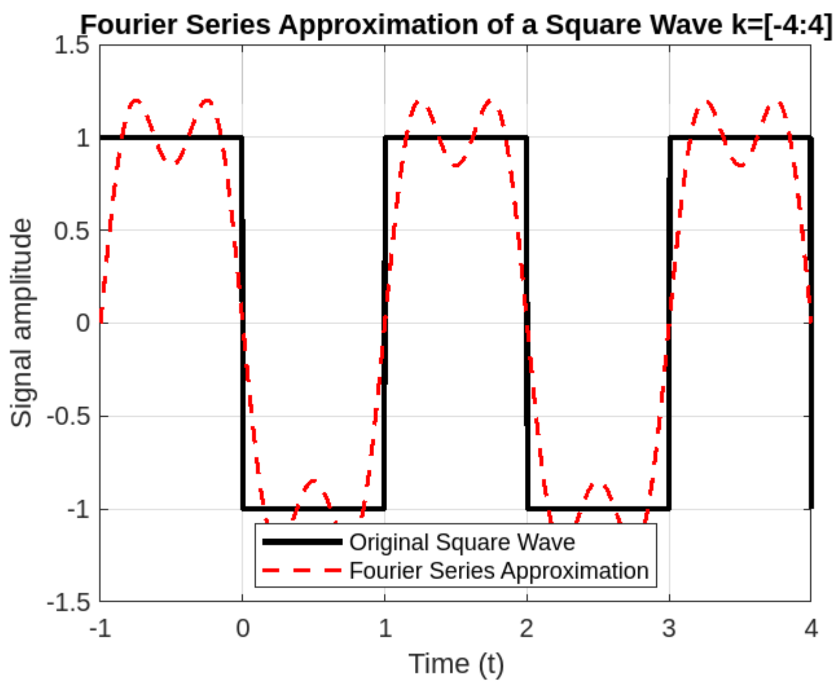


Figure 2: Fourier series approximation of the square wave with $k = \pm 4$

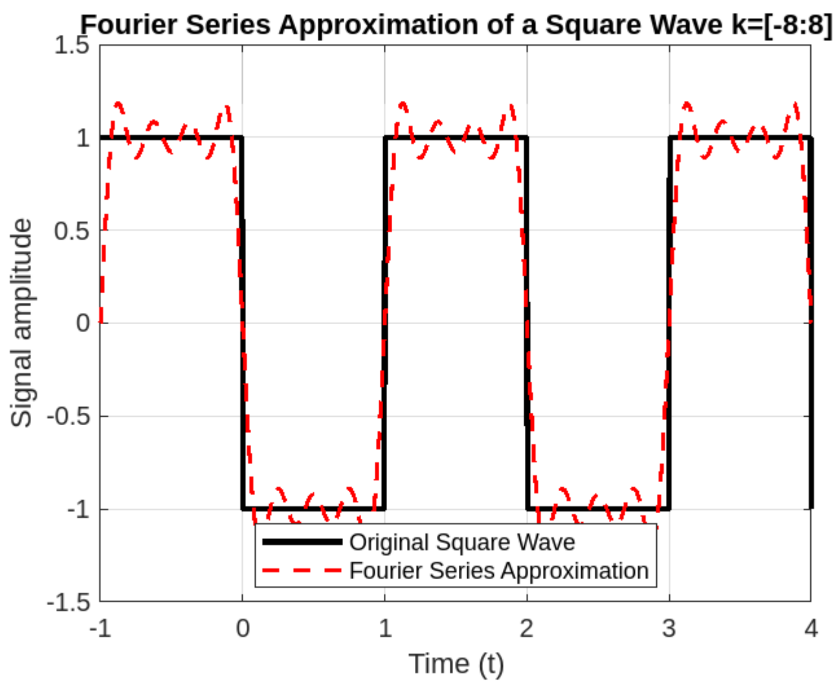


Figure 3: Fourier series approximation of the square wave with $k = \pm 8$

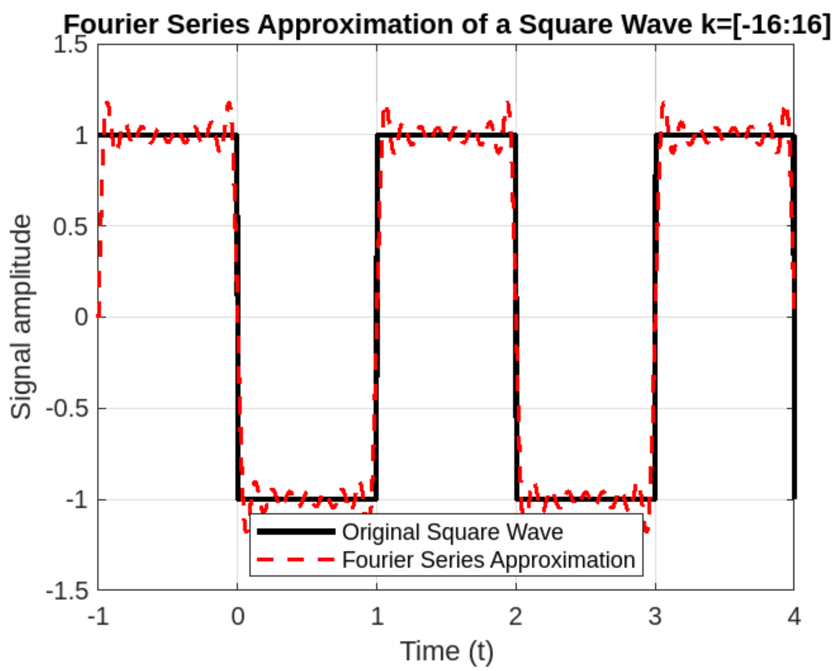


Figure 4: Fourier series approximation of the square wave with $k = \pm 16$

The convergence of the Fourier series towards the square wave signal can be analyzed by observing the approximation for different values of the harmonic index k. The square wave signal is a periodic function with discontinuities.

As the number of terms in the series increases, the approximation resembles the original square wave more closely. However, due to the **Gibbs phenomenon**, there will be anomalies near the discontinuities, which do not completely disappear even as more terms are added. This effect can be observed in the plots generated for different values of k.

The approximation is quite poor for a few terms, especially near the discontinuities.

1.2 Part (b)

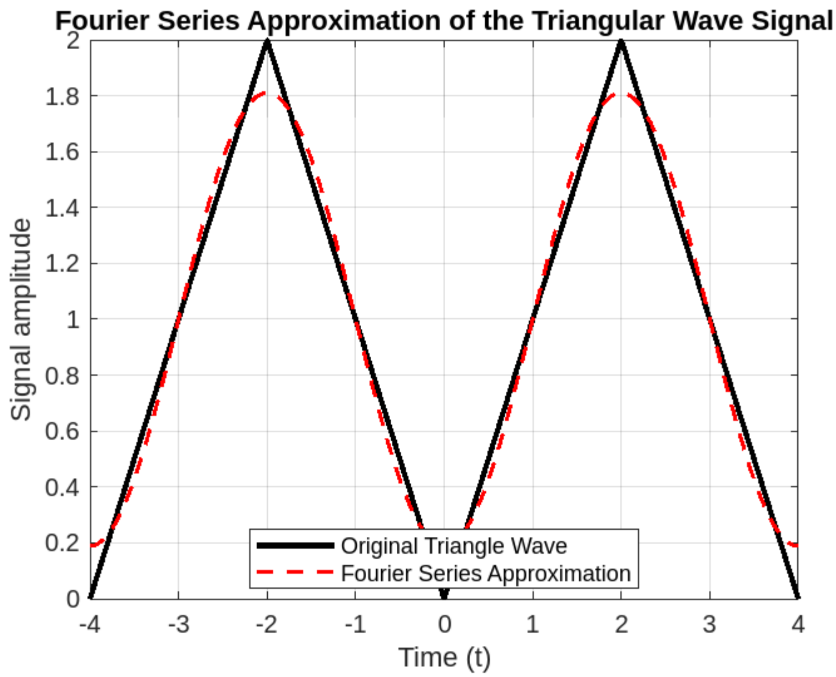


Figure 5: Fourier series approximation of the triangular wave with $k = \pm 2$

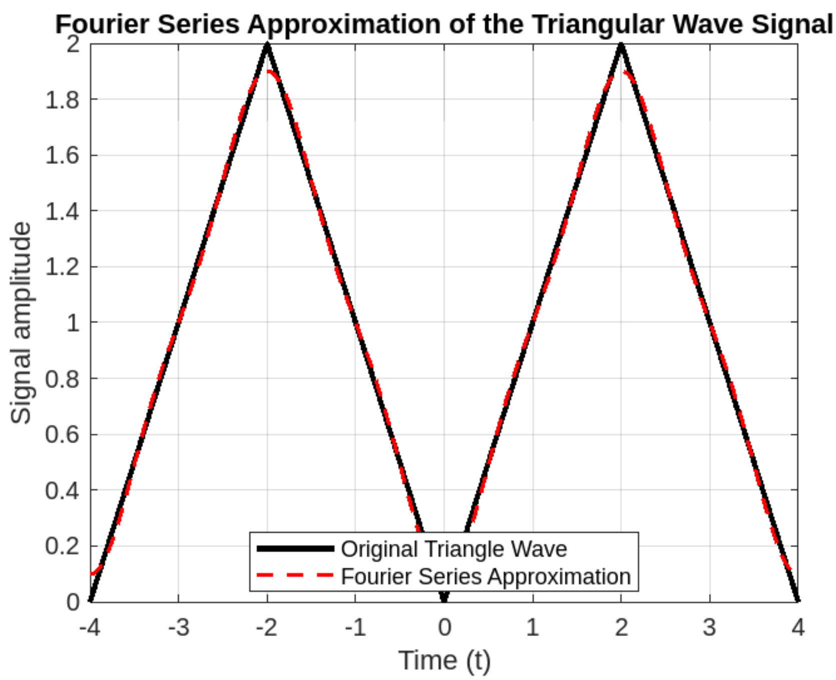


Figure 6: Fourier series approximation of the triangular wave with $k = \pm 4$

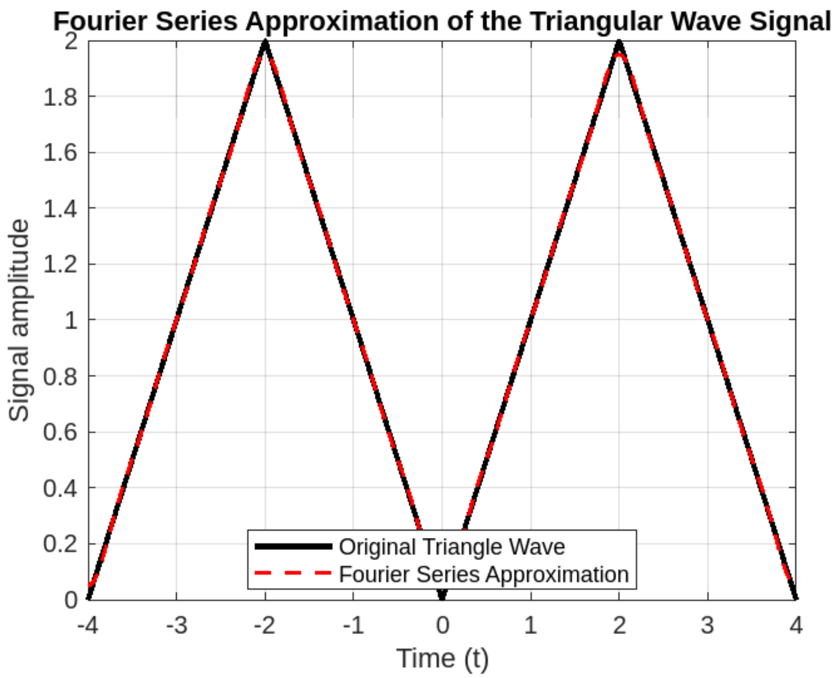


Figure 7: Fourier series approximation of the triangular wave with $k = \pm 8$

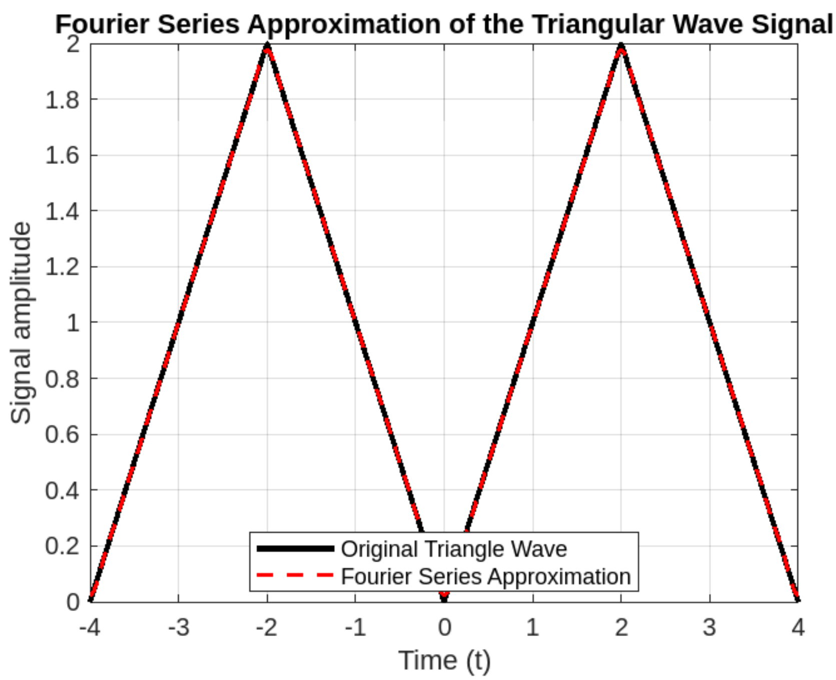


Figure 8: Fourier series approximation of the triangular wave with $k = \pm 16$

The plots show that the triangular wave signal is a piecewise linear periodic function smoother than the square wave.

Different from square wave signal, the triangular wave is continuous but has points of non-differentiability at the maximum-minimum points. As the number of terms in the Fourier series increases, the approximation more closely resembles the original triangular wave. **The Gibbs phenomenon**, characterized by anomalies near discontinuities in the square wave, is less obvious in the triangular wave due to its continuous nature.

However, a minor form of **Gibbs phenomenon** can be observed at non-differentiability points, though it is significantly less severe than the square wave.

2 A Real Life Example

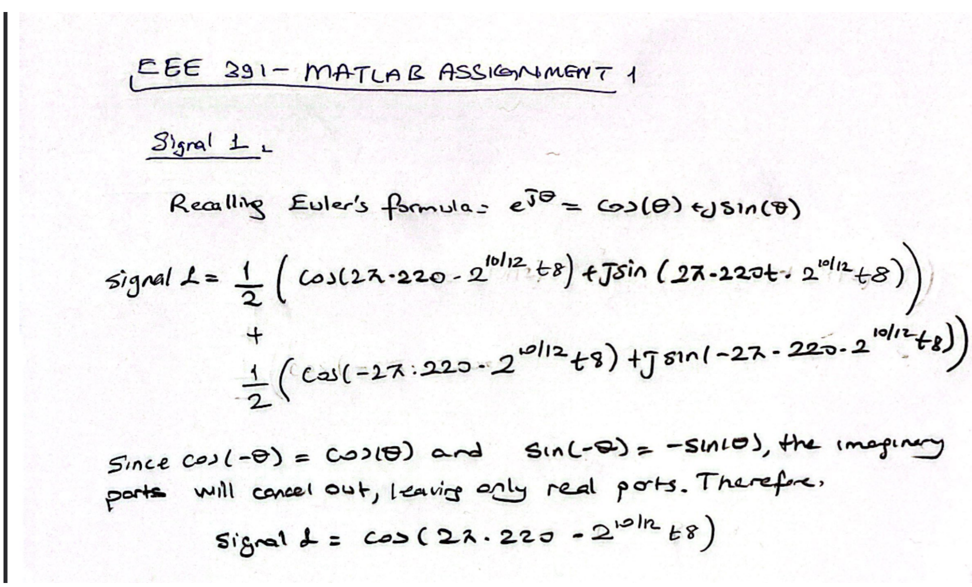
2.1 Part (a)

$$\text{signal}_1 = \frac{1}{2}e^{j(2\pi\cdot 220\cdot 2^{10/12})t8} + \frac{1}{2}e^{-j(2\pi\cdot 220\cdot 2^{10/12})t8} \tag{1}$$

$$\text{signal}_2 = \frac{1}{2}e^{j(2\pi\cdot 220\cdot 2^{6/12})t2} + \frac{1}{2}e^{-j(2\pi\cdot 220\cdot 2^{6/12})t2} \tag{2}$$

$$\text{signal}_3 = \frac{1}{2}e^{j(2\pi\cdot 220\cdot 2^{8/12})t8} + \frac{1}{2}e^{-j(2\pi\cdot 220\cdot 2^{8/12})t8} \tag{3}$$

$$\text{signal}_4 = \frac{1}{2}e^{j(2\pi\cdot 220\cdot 2^{5/12})t2} + \frac{1}{2}e^{-j(2\pi\cdot 220\cdot 2^{5/12})t2} \tag{4}$$



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Signal 1:

Recalling Euler's formula: $e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$$\text{signal 1} = \frac{1}{2}\left(\cos(2\pi\cdot 220\cdot 2^{10/12}\,t8) + j\sin(2\pi\cdot 220\cdot 2^{10/12}\,t8)\right) + \frac{1}{2}\left(\cos(-2\pi\cdot 220\cdot 2^{10/12}\,t8) + j\sin(-2\pi\cdot 220\cdot 2^{10/12}\,t8)\right)$$

Since $\cos(-\theta) = \cos(\theta)$ and $\sin(-\theta) = -\sin(\theta)$, the imaginary parts will cancel out, leaving only real parts. Therefore,

$$\text{signal 1} = \cos(2\pi\cdot 220\cdot 2^{10/12}\,t8)$$

Figure 9: Solution for Signal 1.

Signal 2:

$$\text{Signal } 2 = \frac{1}{2}\left(\cos(2\pi \cdot 220 \cdot 2^{6/12} t2) + j\sin(2\pi \cdot 220 \cdot 2^{6/12} \cdot t2)\right)$$
$$+$$
$$\frac{1}{2}\left(\cos(-2\pi \cdot 220 \cdot 2^{6/12} t2) + j\sin(-2\pi \cdot 220 \cdot 2^{6/12} t2)\right)$$

$$\text{Signal } 2 = \cos(2\pi \cdot 220 \cdot 2^{6/12} t2)$$

Figure 10: Solution for Signal 2.

Signal 3:

$$\text{Signal } 3 = \frac{1}{2}\left(\cos(2\pi \cdot 220 \cdot 2^{8/12} t8) + j\sin(2\pi \cdot 220 \cdot 2^{8/12} t8)\right)$$
$$+$$
$$\frac{1}{2}\left(\cos(-2\pi \cdot 220 \cdot 2^{8/12} t8) + j\sin(-2\pi \cdot 220 \cdot 2^{8/12} t8)\right)$$

$$\text{Signal } 3 = \cos(2\pi \cdot 220 \; 2^{8/12} t8)$$

Figure 11: Solution for Signal 3.

Signal 4:

$$\text{Signal } 4 = \frac{1}{2}\left(\cos(2\pi \cdot 220 \cdot 2^{5/12} t2) + j\sin(2\pi \cdot 220 \cdot 2^{5/12} t2)\right)$$
$$+$$
$$\frac{1}{2}\left(\cos(-2\pi \cdot 220 \cdot 2^{5/12} t2) + j\sin(2\pi \cdot 220 \cdot 2^{5/12} t2)\right)$$

$$\text{Signal } 4 = \cos(2\pi \cdot 220 \cdot 2^{5/12} t2)$$

Figure 12: Solution for Signal 4.

2.2 Part (b)

The Corresponding MATLAB expressions for the given signals are as follows:

```
% Signal 1
signal1 = cos(2 * pi * 220 * 2^(10/12) * t8);

% Signal 2
signal2 = cos(2 * pi * 220 * 2^(6/12) * t2);

% Signal 3
signal3 = cos(2 * pi * 220 * 2^(8/12) * t8);

% Signal 4
signal4 = cos(2 * pi * 220 * 2^(5/12) * t2);
```

2.3 Part (c)

The corresponding MATLAB code for the given variables in (c) is as follows:

```
% Defining the sampling frequency
fs = 8000;

% Defining note length
n1 = 2;

% Defining time vectors for eight-note and half-note
t8 = (0:1/fs:n1/8-1/fs);
t2 = (0:1/fs:n1/2-1/fs);

% Defining silence
sd = zeros(1,round(length(t8)/10));

% Defining rest
rest = zeros(1,length(t8));
```

2.4 Part (d), Part (e)

By using these notes, which are expressed in part (b), and the parameters, which are explained in part (c), we get this MATLAB code which creates sound for the song array.

```
% Define the sampling frequency
fs = 8000;

% Define note lengths for an eight note and a half note
n1 = 2;
t8 = (0:1/fs:n1/8-1/fs);
t2 = (0:1/fs:n1/2-1/fs);

% Defining the frequences
f1 = 220 * 2^(10/12);
f2 = 220 * 2^(6/12);
f3 = 220 * 2^(8/12);
f4 = 220 * 2^(5/12);

% Creating the notes
signal1 = cos(2 * pi * f1 * t8);
signal2 = cos(2 * pi * f2 * t2);
signal3 = cos(2 * pi * f3 * t8);
signal4 = cos(2 * pi * f4 * t2);

% Defining silence
sd = zeros(1,round(length(t8)/10));

% Defining rest
rest = zeros(1,length(t8));

% Combining the notes
song = [signal1, sd, signal1, sd, signal2, sd,
rest, sd, signal3, sd, signal3, sd, signal4];

% Play the song
sound(song, fs);
```