TOPIC TWO: FUNCTIONS

In Topic One we studied the static properties of sets. Things get more dynamic when we start applying functions to those sets.

1. Definitions

Definition. $\boxed{f : A \to B}$ Given a pair of sets A and B, suppose that each element $x \in A$ is associated, in some way, to a unique element of B, which we denote $f(x)$. Then f is said to be a *function* from A to B. This is often denoted

$$f : A \to B$$

which is typically read "f from A to B."

Furthermore, A is called the *domain* of f, (can be though of as inputs of f)

B is called the *codomain* of f, and (... as a set to which all outputs belong)

The set $\{f(x) : x \in A\}$ is called the *range* of f. (... as outputs of f)

All three correspond to each other via f but they are just sets.
Range consists only of elements in the codomain that gets mapped. That is, y is in the range if there is an x in the domain that maps to it: $f(x) = y$. For e.g.,

- if $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = 2x$ then the range is $\mathbb{R}$.
- if $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = x^2$ then the range is the set of nonnegative real numbers: ie, the interval $[0, \infty)$.

A function's domain and codomain can each be any set. Here is a graphical way to write som function f:

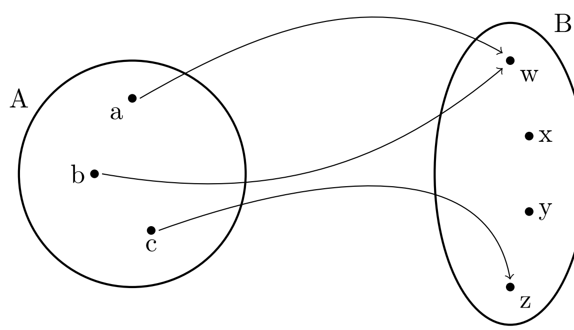

Figure 1. A function $f : A \to B$

Figure 1 above illustrates a function with *domain* {a, b, c}, *codomain* {w, x, y, z}, and *range* {w, z}.

Note. There does not exist any $k \in \{a, b, c\}$ such that $f(k) = y$, which is why y is not in the range.

For diagrams such as Figure 1 to represent a function, it would have to satisfy both existence and uniqueness aspects of a function. If it fails to do so, then it would not represent a function. For e.g., if b did not map onto any of the elements in B in Figure 1 then the diagram would not satisfy the existence condition - since b is being sent to nowhere - and thus not represent a function. Similarly, if b not only mapped onto w but also onto y simultaneously then it would not satisfy the uniqueness condition and the diagram would again not represent a function.