Chapter 1

Sets

1.1 Definitions

If x is an element of a set S, we write $x \in S$. This is read "x in S".

- The set of *natural numbers*, denoted $\mathbb{N}$, is the set {1, 2, 3, ...} [1] For e.g.,
  - $\{n^2 : n \in \mathbb{N}\} = \{1, 4, 9, 16, 25, \dots\}$
  - $\{n \in \mathbb{N} : 6|n\} = \{6, 12, 18, 24, 30, \dots\}$
- The set of *integers*, denoted $\mathbb{Z}$ is the set {..., -3, -2, -1, 0, 1, 2, 3, ...}. For e.g.,
  - $\{|n| : n \in \mathbb{Z}\} = \{0, 1, 2, 3, \dots\}$
  - $\{n \in \mathbb{Z} : niseven\} = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$
- The set of *rational numbers*, denoted $\mathbb{Q}$ is the set $\{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$
  - This can be read as the following:

$\mathbb{Q}$	$=$	$\{$	$\frac{a}{b}$	$:$	$a, b \in \mathbb{Z}$	$,$	$b \neq 0\}$
The rational numbers	are defined to be	the set of all	fractions of the form $\frac{a}{b}$	such that	a and b are integers	and	b is nonzero

So the definition for *rational numbers* includes $\frac{2}{3}$ and $\frac{4}{6}$ and $\frac{6}{9}$ and infinitely more representation of this same number. However, the set itself only keeps one of each element, so the duplicates of each rational number would not be included in the set.

[1] Note that it does not include 0.

The set of *real numbers*, denoted $\mathbb{R}$, is difficult to define (it would take dozens of pages to rigorously define it) but it is effectively all the numbers you can write with a decimal point. However, we can use $\mathbb{R}$ and set notation to generate and define other familiar sets:

- The set of 2 x 2 real matrices can be written:
$$\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R}\}.$$
- The xy-plane represents the set of *ordered pairs* of real numbers. This set can be written:
$$\mathbb{R}^2 = \{(x, y) : x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}.$$
- The unit circle, which is a circle of radius 1 centered at the origin, is contained inside of $\mathbb{R}^2$ and can be defined as:
$$\mathbb{S}^1 = \{(x, y) \in \mathbb{R} : x^2 + y^2 = 1\}.$$
- The closed interval [a,b] can be defined as follows:
$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}.$$
- The open interval (x,y) can be defined as:
$$(a, b) = \{x \in \mathbb{R} : a < x < b\}.$$
This applies even if $a = -\infty$ and/or $b = \infty$. The definitions for the half open intervals, (a,b] and [a, b), are similar. Also note that the open interval notation (a,b) is the same as an ordered pair, so it will be determined which is which from context.

1.2 Proving A $\subseteq$ B