

Finite Automata

Motivation

An Example

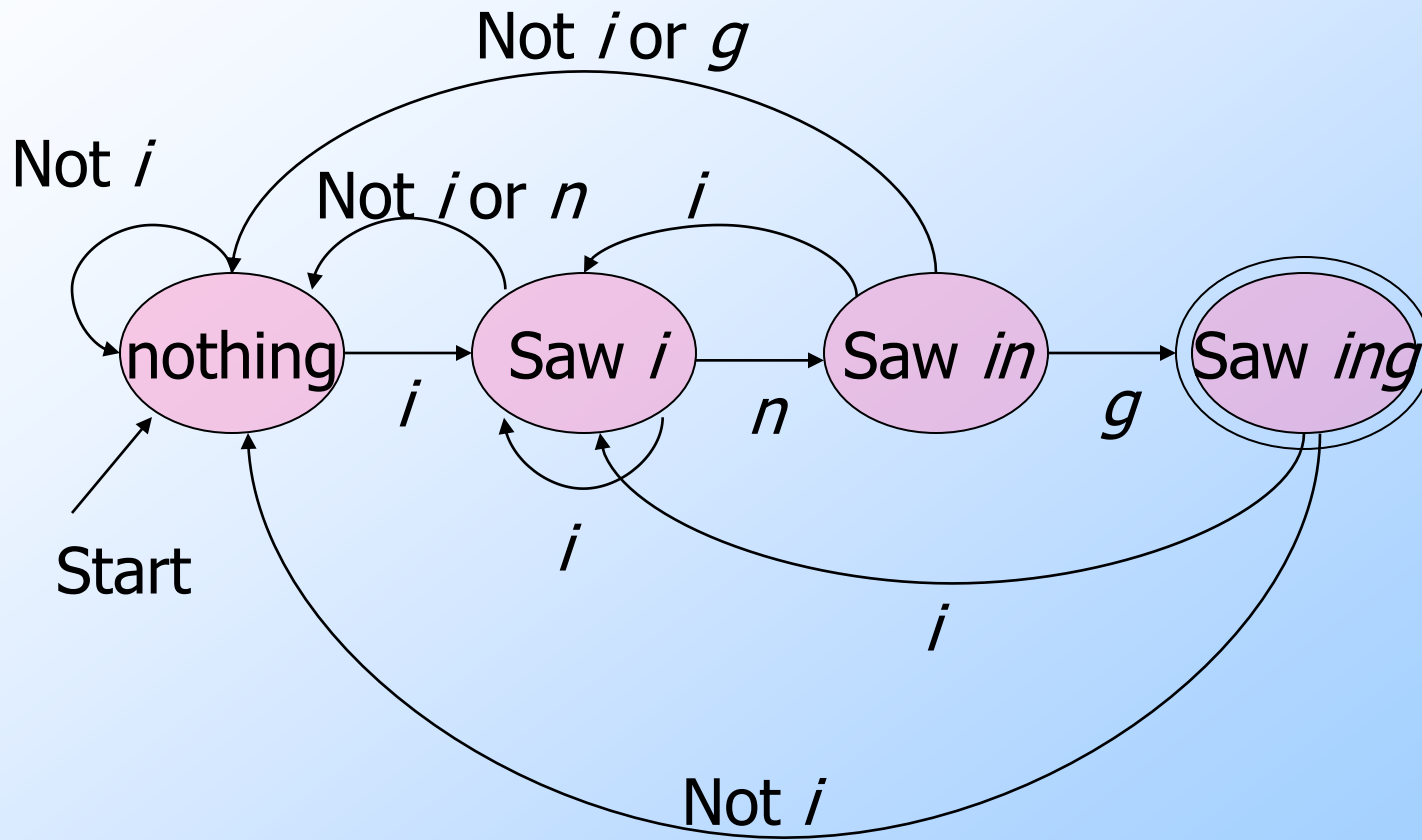
Informal Explanation

- Finite automata are finite collections of states with transition rules that take you from one state to another.
- Original application was sequential switching circuits, where the “state” was the settings of internal bits.
- Today, several kinds of software can be modeled by FA.

Representing FA

- Simplest representation is often a graph.
 - Nodes = states.
 - Arcs indicate state transitions.
 - Labels on arcs tell what causes the transition.

Example 1: Recognizing Strings Ending in “ing”



Automata to Code (by hand)

- In C/C++/Java:
 1. Initialize state q to start state.
 2. Loop through the string one character at a time.
 3. Make a switch statement with a case for each state for q , where each case sets q according to the transitions for that state.
 4. Accept if you end in a final state.

Example in Java

```
Scanner scan = new Scanner(System.in);
```

```
String s = scan.next();
```

```
int q = 0; ← Start state
```

```
for (char c : s.toCharArray()) { ← Loop through string s
```

```
    switch (q) {
```

```
        case 0: q = (c=='i')? 1 : 0; break;
```

```
        case 1: q = (c=='n')? 2 : ((c=='i')? 1 : 0); break;
```

```
        case 2: q = (c=='g')? 3 : ((c=='i')? 1 : 0); break;
```

```
        case 3: q = (c=='i')? 1 : 0;
```

```
    }
```

```
}
```

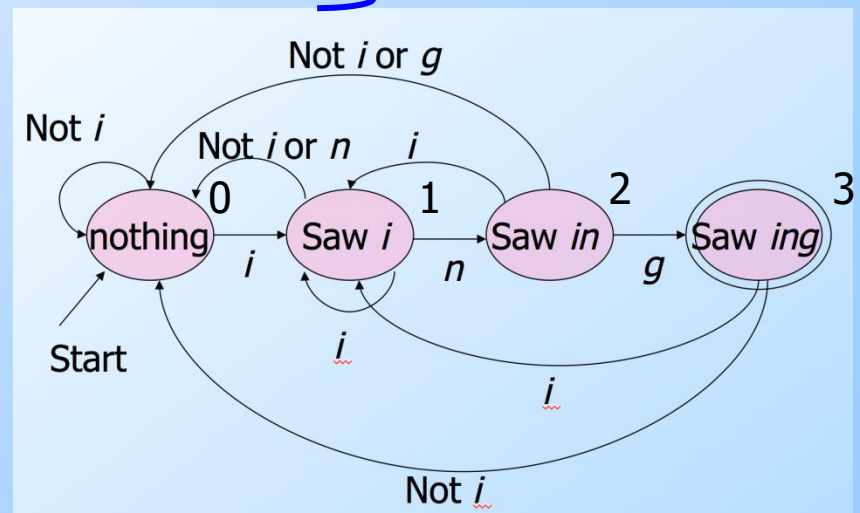
```
if (q==3) ← Final state
```

```
    System.out.println("accept.");
```

```
else
```

```
    System.out.println("reject.");
```

Transitions



Example 1: Automata to Code

```
2: /* i seen */  
   c = getNextInput();  
   if (c == 'n') goto 3;  
   else if (c == 'i') goto 2;  
   else goto 1;  
3: /* "in" seen */  
   . . .
```

Automata to Code – General

- It would be nice to have an automatic way to generate such code...
- Rather than do it by hand, a code generator takes a “regular expression” describing the pattern(s) you are looking for and produces the code for it.
 - **Example:** `. *ing` works in grep.

Example 2: Set of all strings that start with 0 over $\{0,1\}$

Example 3: Set of all strings of length 2 over $\{0,1\}$

General Comments

- Some things are easy with finite automata:
 - Substrings (...abcabc...)
 - Subsequences (...a...b...c...b...a...)
 - Modular counting (odd number of 1's)
- Some things are impossible with finite automata (we will prove this later):
 - An equal number of a's and b's
 - More 0's than 1's