NUMERICAL METHODS, MAT202E

Homework II (Due April 8)

1- A linear system of equations arising from the finite element discretization of a beam is presented in matrix form:

$$\begin{bmatrix} 87.82557 & 0 & -43.91278 & 3659.399 & 0 & 0 \\ 0 & 813199.7 & -3659.399 & 203299.9 & 0 & 0 \\ -43.91278 & -3659.399 & 87.82557 & 0 & -43.91278 & 3659.399 \\ 3659.399 & 203299.9 & 0 & 813199.7 & -3659.399 & 203299.9 \\ 0 & 0 & -43.91278 & -3659.399 & 44.41278 & -3659.399 \\ 0 & 0 & 3659.399 & 203299.9 & -3659.399 & 406599.8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix}$$

a) Decompose the coefficient matrix into LU form using Gauss elimination.

b) Decompose the coefficient matrix into LU form using Cholesky decomposition.

c) Calculate the inverse of the coefficient matrix (use L and U matrices obtained in either part a or part b)

d) Calculate the matrix condition number of the coefficient matrix using row-sum norm.

2- Use least squares regression to fit polynomials of order 1, 2 and 3 to the data given in table (Use Gauss elimination with partial pivoting in the solution of linear equation systems). Compute the correlation coefficient for each fit. Plot 1st, 2nd, and 3rd order polynomial fits and the given discrete data.

Comment on which of these curves explain better the variability of given data.

x_i	y_i
-0.08782	0.078597
0.084523	0.308436
0.263619	0.62902
0.293331	0.90405
0.472033	0.880547
0.529981	1.072706
0.603711	1.081378
0.783421	0.849039
0.885681	0.669272
0.914265	0.449664
1.006573	0.191648
1.070965	-0.20313
1.232576	-0.42675
1.359112	-0.77451
1.327033	-0.85954
1.477735	-0.99787
1.647759	-1.06124
1.75348	-0.88792
1.882376	-0.73878
1.813837	-0.57943
1.92543	-0.2393

3- The variation of specific heats of air is presented in the table. By using this data, rebuild the given table for c_p, c_v and k between T=250K and T=500K with ΔT=10K steps by using Newton's divided difference interpolating polynomials or the Lagrange interpolating polynomials. Present your results in a new table.

Temperature, K	c_p kJ/kg·K	c_v kJ/kg·K	k
		Air	
250	1.003	0.716	1.401
300	1.005	0.718	1.400
350	1.008	0.721	1.398
400	1.013	0.726	1.395
450	1.020	0.733	1.391
500	1.029	0.742	1.387

Note:

- Always use radians for trigonometric functions.
- Writing program codes for the solution of problems is highly recommended (You can use any programming language of your choice).
- You can't use built in functions for the solution of HW questions. You need to either write program codes (advised) or do calculations by hand (not advised, will take a lot of time and effort). There is no third option.
- Present your results in a HW report. If you wrote program codes you need to add them to your submission.
- If you calculated by hand (not advised) use at least 5 significant figures in calculations and present your calculations in your HW report.