CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 2

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1. (a) $y(t) = \int[x(t) - 4y(t)] \longrightarrow y'(t) = x(t) - 4y(t)$

   (b) For the equation we found on part (a) we first derive the homogeneous solution;

   $y'(t) + 4y(t) = x(t) \longrightarrow y'(t) + 4y(t) = 0$

   The solution will be of form $y_h(t) = Ae^{st}$

   Then find the characteristic polynomial $y'(t) + 4y(t) = 0 \longrightarrow s + 4 = 0$

   We have one root which is $s = -4$ so our homogeneous solution is $y_h(t) = Ae^{-4t}$

   Now, we need to solve particular solution. Let $y_p(t) = (Be^{-t} + Ce^{-2t})u(t)$

   Then we substitute $y_p(t)$ with $y(t)$ on the equation $y'(t) + 4y(t) = x(t)$

   New equation $\Rightarrow -Be^{-t} - 2Ce^{-2t} + 4Be^{-t} + 4ce^{-2t} = e^{-t} + e^{-2t}$
   $B = \frac{1}{3}$ and $C = \frac{1}{2}$

   $y_p(t) = \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t}$

   Since general solution is $y(t) = y_p(t) + y_h(t) \Rightarrow y(t) = \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t} + Ae^{-4t}$

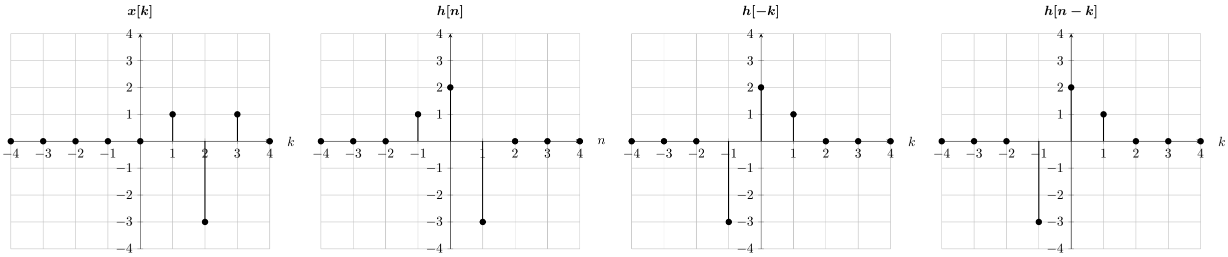
   System is initially at rest, therefore $y(t) = 0$ when $t = 0$.

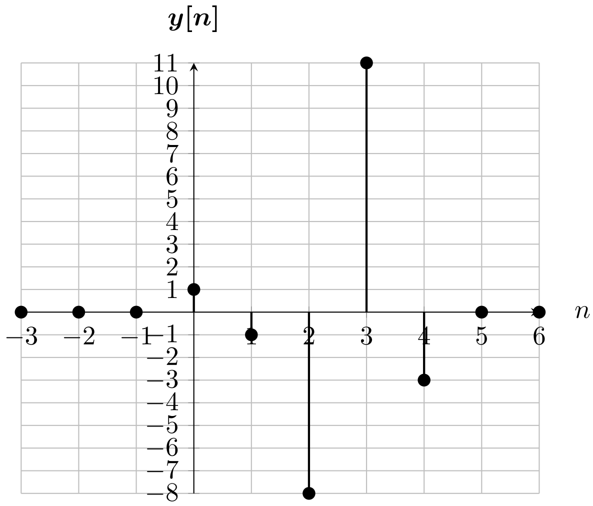
   $y(t) = \frac{1}{3}e^{0} + \frac{1}{2}e^{0} + Ae^{0} = 0$

   $\Rightarrow A = -\frac{5}{6}$

   $y(t) = \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t} - \frac{5}{6}e^{-4t}$

2. (a) By using the definition of convolution, we transform both functions and then compute for each value.





(b) $\frac{dx(t)}{dt} = \delta(t) + \delta(t-1)$

$y[n] = \frac{dx(t)}{dt} * h[t] = (\delta(t) + \delta(t-1)) * h[t]$

From the distributive property of convolution $(\delta(t) + \delta(t-1)) * h[t] = \delta(t) * h[t] + \delta(t-1) * h[t]$

Since the convolution of a function with the impulse function is itself and the convolution of a function with the shifted impulse function is the same function with the same shift, we can derive this equation.

$y[n] = h[t] + h[t-1]$

And then we substitute $h[t] = e^{-2t}\cos(t)u(t) \longrightarrow y[n] = e^{-2t}\cos(t)u(t) + e^{-2t+2}\cos(t-1)u(t-1)$

3. (a) We need to compute $y(t) = x(t) * h(t) = h(t) * x(t) = \int h(\tau)x(t-\tau)d\tau$

Since both $h(t)$ and $x(t)$ has the $u(t)$ component, we can derive the fact that their value will be 0 for $t < 0$.

$y(t) = \int_0^t e^{-3\tau}e^{\tau - t}d\tau$

$\int_0^t e^{-3\tau}e^{\tau-t}d\tau = \int_0^t e^{-2\tau}e^{-t}d\tau = e^{-t}\int_0^t e^{-2\tau}d\tau = e^{-t}(\frac{-e^{-2\tau}}{2}\ |_0^t)$

$y(t) = e^{-t}(\frac{-e^{-2t}-1}{2}) = \frac{-e^{-3t}-e^{-t}}{2}$

Since $y(t)$ have to be 0 for $t < 0$, we will multiply the derived equation with the unit step function $u(t)$.

$\Rightarrow y(t) = \frac{-e^{-3t}-e^{-t}}{2}u(t)$

(b) Function $x(\tau)$ creates a $1x1$ window for the convolution, so we will analyze this convolution in three parts; $t < 1$, $1 \leq t \leq 2$ and $t > 2$.

For $t < 1$ since $x(\tau)$ is always 0 and since $h(t-\tau)$ is 0 for $t < \tau$, we can conclude that the convolution is also 0.

For $1 \leq t \leq 2$ we will compute the convolution that is seen through the window that $x(\tau)$ creates.

$u(t)$ is always 1 for $1 \leq t \leq 2$ so we can write the convolution as follows:

$y(t) = \int_1^t e^t e^{-\tau} d\tau = e^t \int_1^t e^{-\tau} d\tau = e^t(-e^{-\tau} \mid_1^t) = e^t(-e^{-t} + e^{-1}) = -1 + e^{t-1}$

For $t > 2$, $x(\tau)$ and $h(t-\tau)$ have some non-zero intersections only between the $1 \leq t \leq 2$ interval and since $u(t)$ is always 1 for $1 \leq t \leq 2$ we can write the convolution as follows:

$y(t) = \int_1^2 e^t e^{-\tau} d\tau = e^t \int_1^2 e^{-\tau} d\tau = e^t(-e^{-\tau} \mid_1^2) = e^t(-e^{-2} + e^{-1}) = -e^{t-2} + e^{t-1}$

$$y(t) = \begin{cases} 0 & \text{for } t \text{ less than } 1 \\ -1 + e^{t-1} & 1 \leq t \leq 2 \\ -e^{t-2} + e^{t-1} & \text{for } t \text{ greater than } 2 \end{cases}$$

4. (a) To find the characteristic polynomials, we can write this quadratic equation:

$r^2 - 15r + 26 = 0$

$r_1 = 2$ and $r_2 = 13$

So our general solution is $y[n] = k_1 2^n + k_2 13^n$

Because of the initial condition y[0] = 10, if we substitute n with 0 in our equation, we obtain $k_1 + k_2 = 10$.

When $n = 1$, our general solution will be $2k_1 + 13k_2 = 42$. So, if we solve these two equations together, we get $11k_2 = 22 \Rightarrow k_2 = 2$ and $k_1 = 8$.

Solution is $y[n] = 8.2^n + 2.13^n$

(b) As we did like part a), we find the characteristic polynomials.

$r^2 - 3r + 1 = 0$

Roots of this quadratic equation is $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$.

$r_1 = \frac{3+\sqrt{5}}{2}$ and $r_2 = \frac{3-\sqrt{5}}{2}$

Our solution is :

$y(n) = k_1(\frac{3+\sqrt{5}}{2})^n + k_2(\frac{3-\sqrt{5}}{2})^n$

When n = 0, $k_1 + k_2 = 1$

When n = 1, $k_1(\frac{3+\sqrt{5}}{2}) + k_2(\frac{3-\sqrt{5}}{2}) = 2 \Rightarrow k_1(3+\sqrt{5}) + k_2(3-\sqrt{5}) = 4$

$\Rightarrow 3(k_1 + k_2) + \sqrt{5}(k_1 - k_2) = 4$. Since we know that $k_1 + k_2 = 1 \Rightarrow \sqrt{5}(k_1 - k_2) = 1$.

$\Rightarrow k_1 - k_2 = \frac{\sqrt{5}}{5}$
$k_1 = \frac{\sqrt{5}+5}{10}$ and $k_2 = \frac{-\sqrt{5}+5}{10}$

So our solution is $y[n] = (\frac{\sqrt{5}+5}{10})(\frac{3+\sqrt{5}}{2})^n + (\frac{-\sqrt{5}+5}{10})(\frac{3-\sqrt{5}}{2})^n$

5. (a) For homogeneous solution we need to solve characteristic equation.
Characteristic equation of this equation is:

$r^2 + 6r + 8 = 0$

$r_1 = -2$, $r_2 = -4$

Our homogeneous solution is :

$y_h(t) = Ae^{-2t} + Be^{-4t}$

We set the equation to calculate impulse response

$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2\delta(t)$

We integrate both sides of the equation from 0 to t;

$y'(t) + 6y(t) + 8\int_0^t y(\tau)d\tau = 2u(t)$

$y'(t) = 2u(t) - 8\int_0^t y(\tau)d\tau - 6y(t)$

We know that $y(0^+) = 0$, so we can deduce that $y'(0^+) = 2u(0^+) - 8\int_0^{0^+} y(\tau)d\tau - 6y(0^+) = 2$, since the integral part is also going to be 0.

Since we have initial resting conditions, we set

$y(0^+) = A + B = 0$
$y'(0^+) = -2A - 4B = 2$

When we solve these two equations, we get $A = 1$ and $B = -1$

Hence, we get impulse response h(t) $= (e^{-2t} - e^{-4t})u(t)$

(b) i) It is causal because the $u(t)$ function ensures that any future information will be omitted, since it has zero value for $t < 0$ and we flip it in order to get $h(t-\tau)$ the convolution will always exclude any τ which is greater than t ensuring causality.

ii) It is not memoryless because the $u(t)$ component will require future information, since it has non-zero value for $t > 0$ which contradicts with being memoryless (only depending on the current value).

iii) It is stable because the $e^{-2t} - e^{-4t}$ expression is a convergent function as t goes to infinity. Thus, we can consider its' integral being a constant. Since $u(t)$ is also a constant, we can derive that for any bounded input $x(t)$, our $y(t)$ will also be bounded.

iv) It is invertible, and every LCCDE is invertible.