CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a) $3x + 3yj + 4 = 2j$ - $x + yj$
   If we equalize imaginary part and reel part separately:
   $3y = 2 + y$
   $2y = 2$
   y = 1
   $4x$ = -4
   x = -1
   z = -1 + j , $\mid z \mid^2 = 2$

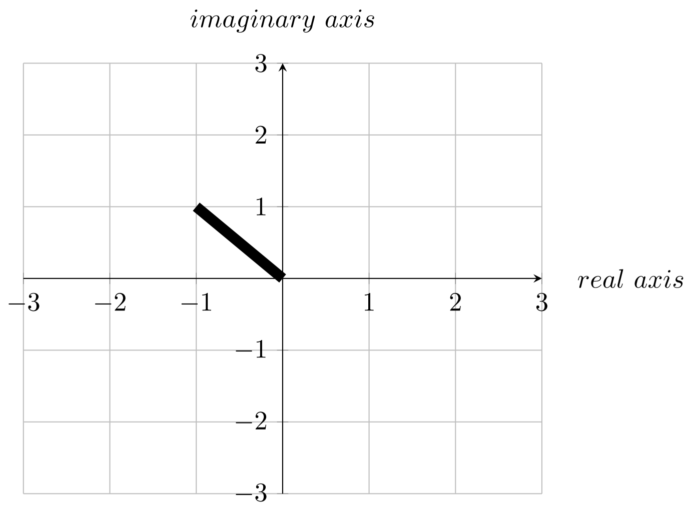

Figure 1: *complex plane.*

   (b) $r^3 e^{3j\theta} = 64j$
   $r = 4$
   $e^{j\theta} = -j$ Since $(-j)^3 = j$
   $e^{j\theta} = cos(\theta) + j.sin(\theta) = -j$
   θ = -$\frac{\pi}{2}$
   $z = 4e^{-\frac{\pi}{2}j}$

   (c) If we multiply z with the conjugate of $1 + j$, we get $\frac{-2j+2\sqrt{3}}{2} \Rightarrow$ -j + $\sqrt{3} = cos(\theta) + jsin(\theta)$
   $sin(\theta) = \frac{-1}{2}$ $cos(\theta) = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{-\pi}{6}$
   $\mid z \mid = \sqrt{1^2 + \sqrt{3}^2} = 2$

   (d) $e^{j\frac{\pi}{2}} = cos(\frac{\pi}{2}) + j.sin(\frac{\pi}{2}) = j$
   Since $sin(\frac{\pi}{2}) = 1$
   $z = -j.j = 1$
   Therefore z should be equal to 1, then in polar form $z = e^{j2\pi}$

2. This is the y(t) graph

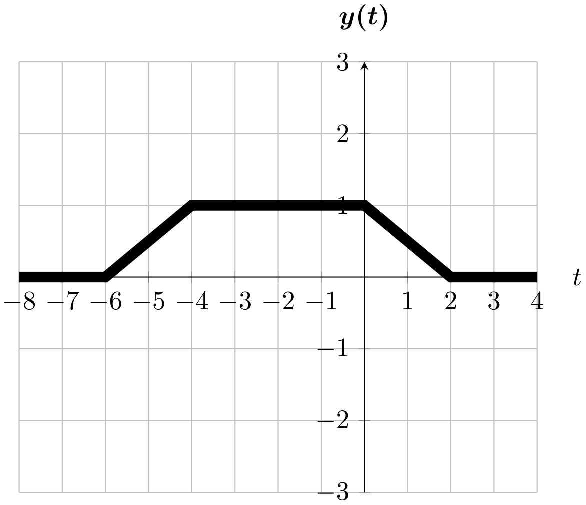

Figure 2: t vs. $y(t)$.

3. (a) $x[-n] + x[2n+1]$ graph is:

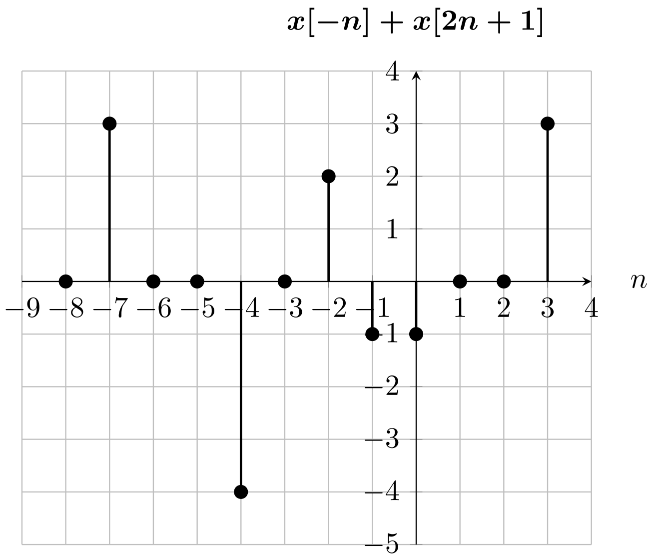

Figure 3: n vs. $x[-n] + x[2n+1]$.

(b) $3\delta[n-3] + (-1)\delta[n] + (-1)\delta[n+1] + 2\delta[n+2] - 4\delta[n+4] + 3\delta[n+7]$

4. (a) Let x[n] = x[N+n] and N is the smallest integer that satisfies the equation.

$cos[\frac{13\pi}{10}n] = cos[\frac{13\pi}{10}(n+N_1)] \quad \rightarrow \quad \frac{13\pi}{10}n + 2\pi k = \frac{13\pi}{10}n + \frac{13\pi}{10}N_1 \rightarrow \quad \frac{20k}{13} = N_1 \qquad N_1 = 20$

$sin[\frac{7\pi}{3}n - \frac{2\pi}{3}] = sin[\frac{7\pi}{3}(n+N_2) - \frac{2\pi}{3}] \quad \rightarrow \frac{7\pi}{3}n - \frac{2\pi}{3} + 2\pi k = \frac{7\pi}{3}n + \frac{7\pi}{3}N_2 - \frac{2\pi}{3}$

$2\pi k = \frac{7\pi}{3}N_2 \rightarrow \quad N_2 = 6$
Since the least common multiplier is 60, period N is also 60.

(b) Let x[n] = x[N+n] and N is the smallest integer that satisfies the equation.

$sin[3n - \frac{\pi}{4}] = sin[3(n+N) - \frac{\pi}{4}]$

$3N = 2\pi k \quad \rightarrow \quad$ N = k$\frac{\pi}{3}$
Since π is not a rational number, N can not be an integer. It is not periodic.

(c) It is a continuous time and sinusoidal signal. It is periodic. $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi} = \frac{2}{3}$

(d) $e^{j\omega_0 t}$ is periodic for any value of ω_0 if $T_0 = \frac{2\pi}{\omega_0}$. $T_0 = \frac{2\pi}{5}$. So it is periodic.

5. $x(t) = x_e(t) + x_o(t) \rightarrow a$
$x(-t) = x_e(-t) + x_o(-t) \quad \Rightarrow \quad x(-t) = x_e(t) - x_o(t) \rightarrow b$

If we sum up two equations a, b, we get even decomposition of $x(t)$ and if we subtract equation b from equation a, we get odd decomposition of $x(t)$.
Therefore, even decomposition $x_e(t) = \frac{1}{2}(x(t) + x(-t))$.
Odd decomposition $x_o(t) = \frac{1}{2}(x(t) - x(-t))$

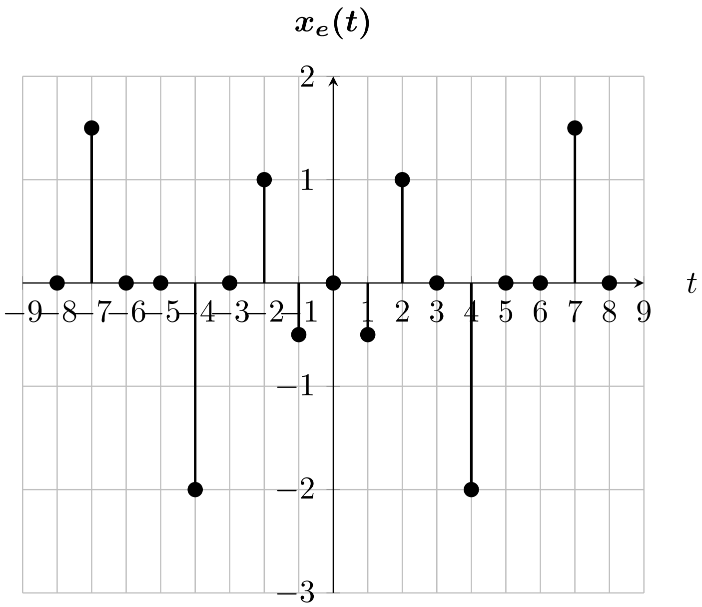

Figure 4: t vs. $x_e(t)$.

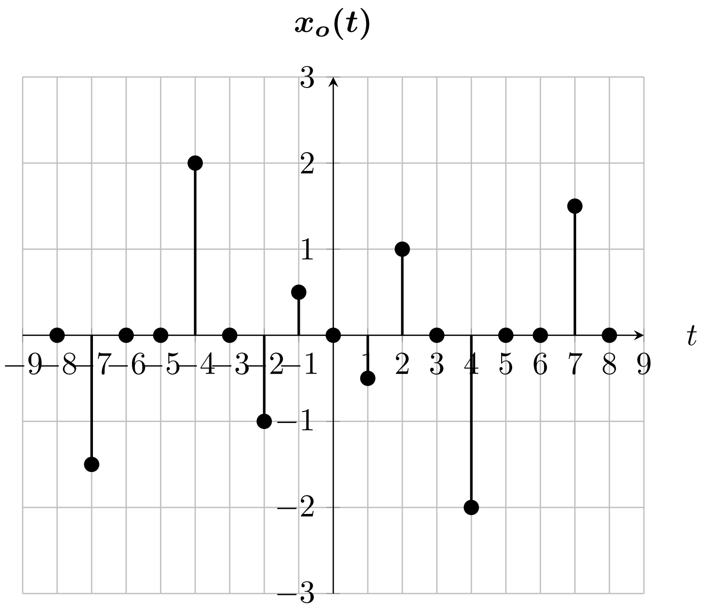

Figure 5: t vs. $x_o(t)$.

6. (a) Not memoryless because for some t the value of $2t-3$ will be different than t, hence we deduct that this system requires memory.

Stable because when $x(2t-3)$ is bounded then $y(t)$ will also be bounded.

Not causal because for some t the value of $2t-3$ will be greater than t, which means it will require future information that contradicts with causality.

Linear because if we substitute $\alpha x(2t-3)$ for $x(2t-3)$ in $y(t) = x(2t-3)$ we get $\alpha x(2t-3)$ which is also equal to the expression that results from multiplying $y(t)$ with α which is $\alpha y(t) = \alpha x(2t-3)$.

Invertible because for every $x(2t-3)$ value we have a different $y(t)$ value.

Not time invariant because substituting t with $t-t_0$ in $y(t)$ will result in $y(t-t_0) = x(2t-2t_0-3)$ which will not result in the same time shift as the input.

(b) Memoryless because the output depends only on the current value of the input t.

Unstable because although $x(t)$ is bounded, due to the multiplication by t, $y(t)$ is unbounded.

Causal because the output depends only on current value of the input t.

Linear because if we substitute $\alpha x(t)$ for $x(t)$ in $y(t) = tx(t)$ we get $t\alpha x(t)$ which is also equal to the expression that results from multiplying $y(t)$ with α which is $\alpha y(t) = \alpha t x(t)$.

Invertible because for every $x(t)$ value we have a different $y(t)$ value.

Not time invariant because substituting t with $t-t_0$ in $y(t)$ will result in $y(t-t_0) = (t-t_0)x(t-t_0)$ which will not result in the same time shift as the input.

(c) Not memoryless because for some n the value of $2n-3$ will be different than n, hence we deduct that this system requires memory.

Stable because when $x[2n-3]$ is bounded then $y[n]$ will also be bounded.

Not causal because for some n the value of $2n-3$ will be greater than n, which means it will require future information that contradicts with causality.

Linear because if we substitute $\alpha x[2n-3]$ for $x[2n-3]$ in $y[n] = x[2n-3]$ we get $\alpha x[2n-3]$ which is also equal to the expression that results from multiplying $y[n]$ with α which is $\alpha y[n] = \alpha x[2n-3]$.

Invertible because for every $x[2n-3]$ value we have a different $y[n]$ value.

Not time invariant because substituting n with $n-n_0$ in $y[n]$ will result in $y[n-n_0] = x[2n-2n_0-3]$ which will not result in the same time shift as the input.

(d) Not memoryless because the expression $x[n-k]$ requires additional information other than current input, hence requires memory.

Unstable because even if we assume that the expression $x[n-k]$ is bounded, $y(t)$ which is a summation will diverge and be unbounded.

Causal because the output depends only on the current and past input values.

Linear because if we substitute $\alpha x[n-k]$ for $x[n-k]$ in $y[n] = \Sigma_{k=1}^{\infty} x[n-k]$ we get $\Sigma_{k=1}^{\infty} \alpha x[n-k]$ which is also equal to the expression that results from multiplying $y[n]$ with α which is $\alpha y[n] = \alpha \Sigma_{k=1}^{\infty} x[n-k]$.

Not invertible because, since $y[t]$ is a summation, for some values of n we might have the same value.
For example, if we assume that $x[n]$ is the unit impulse function then we will have lots of n values with $y[n] = 0$ or $y[n] = 1$.

Time invariant because substituting n with $n-n_0$ in $y[n]$ will result in $y[n-n_0] = \Sigma_{k=1}^{\infty} x[n-n_0-k]$ which is the same time shift as the input.