MECHANICAL ENGINEERING FACULTY

MATHEMATICAL SIMULATION MODELS

SEMESTRAL ASSIGMENT

Ball&Beam

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1.ASSIGMENT

Mathematical model of the ball&beam

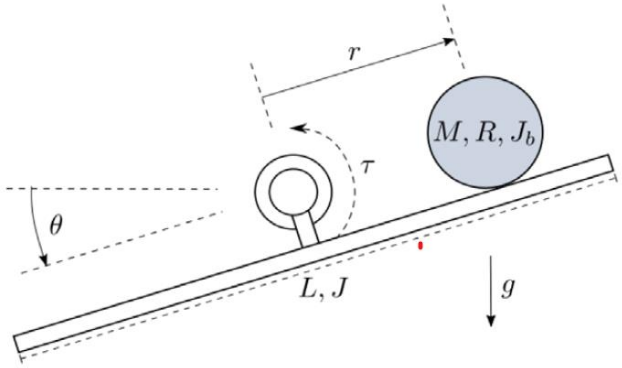

Figure1.Ball and Beam system,variables and parameters

$$0 = \left(\frac{J_b}{R^2} + M\right)\ddot{r} + Mgsin\theta - Mr\dot{\theta}^2 \qquad (1)$$

$$\tau = (Mr^2 + J + J_b)\ddot{\theta} + 2Mr\dot{r}\dot{\theta} + Mgrcos\theta \qquad (2)$$

r = Sphere Position
θ: Rail Angle
τ: Applied Torque
g: Gravity acceleration
J: Moment of inertia of the rail
Jb: Moment of inertia of the ball
m: Mass of the Sphere
R: Sphere radius

2.State Space Representation

$$X_1 = r \ , X_2 = \ \dot{r} \ , X_3 = \ \theta \ , X_4 = \ \dot{\theta} \qquad (3)$$

$$u = \tau \ , \quad y = r \qquad (4)$$

$$\dot{X}_1 = X_2 \qquad (5)$$

$$\dot{X}_2 = b\left(X_1 X_4^2 - gsin(X_3)\right) \qquad (6)$$

$$X_3 = X_1 \qquad (7)$$

$$\dot{X}_4 = \frac{(-2mX_1X_2X_4 - mgX_1\cos(X_3) + u)}{mX_1^2 + J + J_b} \qquad (8)$$

$$\mathrm{b} = \frac{\mathrm{m}}{\frac{\mathrm{J_b}}{\mathrm{r}^2} + \mathrm{m}} \qquad (9)$$

3.Linearization and Equilibrium Points

$$\mathrm{f(x,u)} = \begin{bmatrix} \mathrm{f}_1 \\ \mathrm{f}_2 \\ \mathrm{f}_3 \\ \mathrm{f}_4 \end{bmatrix} = \begin{bmatrix} X_2 \\ \mathrm{b}(X_x X_4^4 - gsin(X_3)) \\ \mathrm{X}_3 \\ \frac{(-2mX_1X_2X_4 - mgX_1\cos(X_3) + u)}{mX_1^2 + J + J_b} \end{bmatrix} \qquad (10)$$

Once the state equations f_i were obtained, the equilibrium point X_q was determined for taking each state derivative as zero x=0, and an input u_q= 0. Replacing all the values the state space matrix results in four equilibrium equations:

$$X_{2q} = 0 \qquad (11)$$

$$b\left(X_{1q} X_{4q}^2 - gsin\left(X_{3q}\right)\right) = 0 \qquad (12)$$

$$X_4 = 0 \qquad (13)$$

$$\frac{\left(-2mX_{1q}X_{2q}X_{4q} - mgX_{1q}\cos\left(X_{3q}\right) + u_q\right)}{mX_1^2 + J + J_b} = 0 \qquad (14)$$

Replacing equations (11) and (13) ,in equations (12) and (14) result in :

$$\dot{X} = \begin{bmatrix} X_{1q} \\ X_{2q} \\ X_{3q} \\ X_{4q} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

4.Linear State-Space Model

Then ,the state space matrices of the bar -sphere are obtained.For the state space of (16) ,A matrix is defined as follows:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du \qquad (16)$$

$$A = \left(\frac{\partial f_i}{\partial x_j}\right) |_{x=x_q, u=u_q} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ X_{4q}^2 & 0 & gcos(X_{3)} & 2X_{1q}X_{4q} \\ 0 & 0 & 0 & 1 \\ \frac{\left(-2mX_{1q}X_{2q}X_{4q} - mgX_{1q}\cos(X_{3q}) + u_q\right)}{mX_1^2 + J + J_b} & \frac{(2mX_{1q}X_{2q}X_{4q})}{J+J_b} & \frac{(mgX_{1q}\sin{(X_{3q})})}{J+J_b} & \frac{-(2mX_{1q}X_{2q}X_{4q})}{J+J_b} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -bg & 0 \\ 0 & 0 & 0 & 1 \\ \frac{mg}{J+J_b} & 0 & 0 & 0 \end{bmatrix}$$

Matrices B and C are as follow:

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J+J_b} \end{bmatrix} \qquad C = [1\ 0\ 0\ 0]$$

The values for the system constants are:
• Mass of the Sphere: 0,1 [kg]
• Radius of the Sphere: 0,1 [m]
• Moment of Inertia of the Sphere: 0,004 [kg m^2]
• Moment of Inertia of the Bar: 0,02083 [kg m^2]
• Acceleration of Gravity: 9,8 [m/s^2]

5. Non-Linear and Linear Dynamics of Ball&Beam and applying State-Feedback Controller in Matlab/Simulink

The state feedback can place the poles by altering the eigenvalues of the closed loops system matrix;

$$u = -Kx$$

```
m = 0.1;
R = 0.1;
g = 9.8;
J = 0.02083;
Jb = 0.004;

b = m * R * R / (Jb + (m *R * R));


A = [0 1 0 0
     0 0 -b*g 0
     0 0 0 1
     m*g/(J + Jb) 0 0 0]
B = [0;0;0;1/(J+Jb)]
C = [1 0 0 0];
D = [0];



p1 = -2+2i;
p2 = -2-2i;
p3 = -20;
p4 = -80;

K = place(A,B,[p1,p2,p3,p4])
```

Figure2. The script for the definition of the parameters in MATLAB

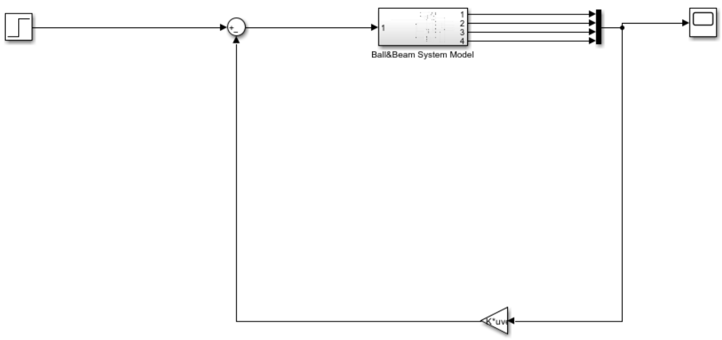

Figure3. The Simulink representation of Non-Linear System of Ball-Beam

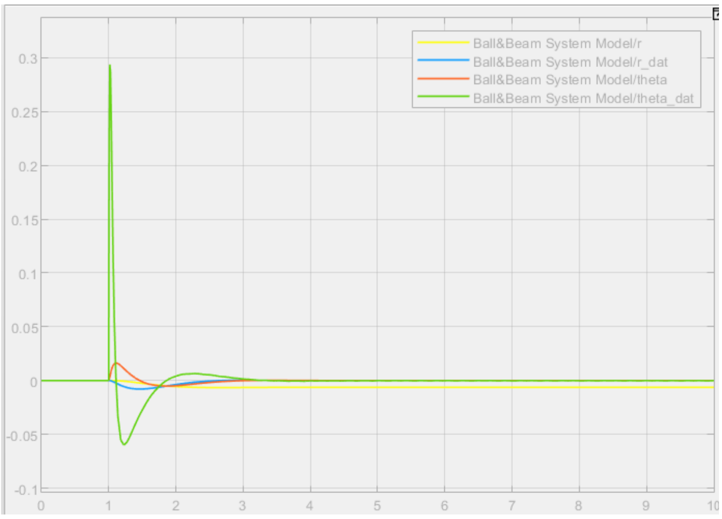

Figure 4:The graph from scope shows that non-linear equation’s states with contoller in Simulink

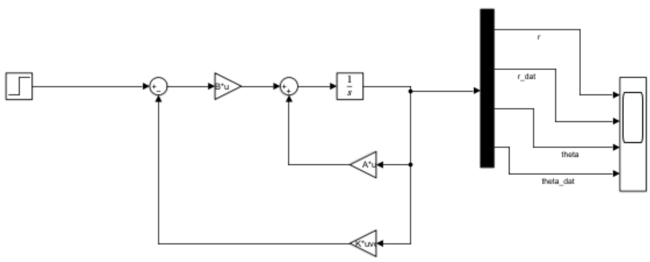

Figure5. The Simulink representation of Linear System of Ball-Beam

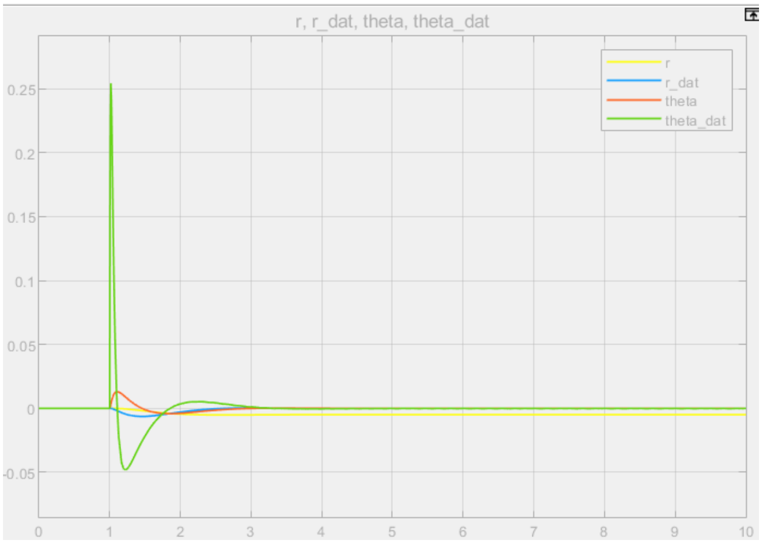

Figure 6:The graph from scope shows that linear equation's states with contoller in Simulink

6.Conclusion

Firstly,the nonlinear model for the ball-beam dynamic system, it is required to represent the nonlinear equations into the standard state space form.After that,since the goal of this particular system is to keep the ball-beam, the linearization might be considered about this equilibrium point.Lastly,state-feedback control is an optimal control technique to make the optimal control decisions have been implemented to control the nonlinear and linear ball-beam systems with continuous disturbance input. The state-feedback controller was one of the fastest because it was able to stabilize the system in around three seconds.